Vienna University of Economics & B.A.
Department of Economics Working Paper Series

Output specific efficiencies: The case of UK private secondary schools

Dieter Gstach * Andrew Somers ** Susanne Warning ¶

Working Paper No. 84
February 2003

Abstract — *Based on regularly published data we quantitatively assess the efficiency of UK secondary, private schools in providing quantity vs. quality of graduates on a per output basis. In economic terms the primary question is whether indeed an increase in the quantity of graduates with the observed inputs would be associated with a deterioration of average quality of graduates. The estimation framework is a new, statistically enriched type of Data Envelopment Analysis as detailed in Gstach (2002) to account for output-specific efficiencies. The results indicate that quantity clearly dominates quality as performance distinguishing criteria amongst sample schools, i.e. on average quantity efficiency is low while quality efficiency is high. The results also provide evidence that the abilities of schools to provide quantity resp. quality are positively correlated. These findings indicate considerable scope for increasing the number of graduates without sacrificing average graduation quality through improved school management.*

Keywords: Efficiency, Economics of Schooling, Human Capital

JEL-Classification: I29, C11, C15

*Corresponding author: Vienna University of Economics, Dept. VW6, Augasse 2-6, 1090 Vienna, Austria, e-mail: dieter.gstach@wu-wien.ac.at, fax: +43 1 31336-755.
**Haverstock School, Crogsland Rd, Camden, London, NW1 8AS.
¶University of Konstanz, Fach D 144, 78457 Konstanz, Germany.

1 Introduction

The issue of quality vs. quantity of education is a focal point in the persisting debate about secondary schooling in the UK just as in many other countries.[1] A common stance taken in this debate is, that attempts to improve quality of graduates with given means would necessarily require a decrease in their quantity. This simplifying view about the tradeoff between quantity and quality takes for granted, that, in economic terms, schools already transform their inputs efficiently into outputs, the basic forms of which are quality and quantity of graduates. The present paper aims to quantify this type of efficiency and thus to assess the relevance of this quality vs. quantity issue.

Rankings amongst schools are important reference points in this debate about education policy. One such ranking, the so called the FT-1000 league table, is yearly published by the Financial Times (FT) for UK secondary schools. Rankings purely based on quality criteria, like the FT-1000 league tables, by their very construction generally support the simplifying tradeoff view expressed above. But they also motivate attempts of schools to increase quality with given effort levels to improve their ranking. Such attempts then are likely to come indeed at the cost of quantity, thereby lending further support to this simplifying view. So rankings ignoring the dual nature of schooling output are of little help in assessing the relevance of the quality vs. quantity issue and may be more concealing than revealing. Here instead efficiency scores are estimated, which take this principal duality into account. The corresponding ranking is then contrasted with the FT-1000 ranking.

Efficiency will be evaluated relative to observed best practice amongst schools. To determine the latter *Data Envelopment Analysis* (DEA) will be used as in many related studies. But best practice, for given inputs, may be represented by very different schools: Some producing high quality graduates but only a small quantity of them and some producing high quantities of graduates but with low quality. A particular aim of this paper is to provide a statistical rational for choosing particular reference schools from those representing best practice. This choice involves consideration of output-specific efficiencies, which were rarely ever calculated in applied efficiency analysis and if so, were devoid of statistical plausibility. So the novel methodological contribution of this paper is a statistical foundation of output specific efficiencies.

Parents prepared to pay considerable amounts of tuition and looking for best value for money are likely to study school rankings more closely than other parents. Furthermore the amount of tuition payed reflects to some extent the socio-economic background of a schools typical pupil, an important contributing factor to pupils performance. No variable like 'tuition' is available from our dataset to capture the corresponding characteristics of pupils in state schools. So we focus on analyzing private schools in a parental perspective, where tuition expenditures are compared with outcome in the basic forms of graduation probability and graduation quality.

For estimation purposes we will rely solely on the very dataset underlying the FT-1000 league tables to show the potential for extracting much more information from this dataset

[1] A good and regularly updated account of the UK debate is to be found on the webpage of the Department for Education `www.dfee.gov.uk` and the webpage of The Times Educational Supplement `www.tes.co.uk`.

than do the FT-1000 league tables. In particular the relative per-output-efficiency for each of 237 private, secondary UK schools contained within this dataset will be estimated. It should be emphasized, that we try to *quantify* efficiency rather than to *explain* performance differentials. Studies of explanatory require richer data sets (to control for the operating environment of schools and for pupils characteristics at entry) than the one provided along with the FT-1000 league tables.

The paper is organized as follows: In Section 2 related literature on schooling efficiency will be reviewed briefly. The model for the data generating process is outlined in section 3 along with some comments on the involved concept of target output-ratios. Section 4 describes the data and the variable specification used in the analysis. Section 5 presents the results and Section 6 summarizes.

2 A brief review of related literature

Hanushek (1986) surveys 147 studies of educational production functions, all aiming at comparative measurement of school performance. He characterizes the "achievements of individual students" as the output of the educational process resulting from consuming inputs related to school characteristics, teachers qualifications and curricula. For state schools he proposes to use expenditures to proxy these inputs. Analogously we proxy them by the tuition parents have to pay in our case of private schools. With regard to proper output measures Hanushek distinguishes between two general groups of educational output: qualitative output (presented mostly by test scores) and quantitative output (like school continuation rates or attendance rates).[2] Wenger (2000), making the same point about the two basic education outputs, adds school completion to the list of potential quantity output measures. He shows that test scores and school completion are indeed substitutes in a production theoretic perspective.

The only comparative study of school performance explicitly using quantitative and qualitative output and using an analytical framework similar to ours is Ray and Mukherjee (1998). They use test scores to proxy output quality and percentages of graduating students continuing further education to represent output quantity. The latter measure, like earnings proposed early on in Mincer (1974) and used in applied analysis for example in Behrman and Birdsall (1983), refers to post schooling experiences of former pupils. The FT-dataset does not contain such information and therefore our quantity output measure refers to experiences during schooling time.

According to the quantity/quality criteria mentioned above the following survey of other relevant studies is divided into three groups: The first one only considering quality outputs, the second one focusing on quantity aspects and the third one including both types of characteristics, although often not under the labels of quantity and quality outputs.

The first group only considers qualitative output measures. [3] Amongst these studies are

[2] The notion of a qualitative output here means that the variable reflects qualitative aspects of education. It does not mean that the variable is restricted in the statistical sense to a nominal or ordinal scale.

[3] For internal evaluations of schools performed by the UK Department of Education so called *valued-added* models are being used, where quality aspects of pupils on entry are compared to quality aspects of the same cohort at later stages of schooling. In such models quality variables thus enter as inputs as

Bessent and Bessent (1980), Authella M. Bessent and Reagan (1982), Authella M. Bessent and Long (1984), Chalos (1997) and Ruggiero (1999), which all treat school performance in a production theoretic fashion. While they all use different output measures, their common input variable is expenditures per pupil, in line with our approach. Some authors additionally use the ratio of administrative to instructional expenditures as input variable like Bessent and Bessent (1980), Mayston and Jesson (1988) or Chalos (1997). We do not have such data and therefore could not follow this approach.

The studies within the second group refer explicitly to quantity output measures and use DEA for efficiency evaluation, so they are closely related in a basic methodological aspect to the present study. These studies include Mancebon and Moliero (2000) (for primary schools in Hampshire), who found neither significant influence of the proportion of girls in the class nor the size of school, while religious orientation (Church of England) is found to have significant positive influence on efficiency. Our findings by and large stand opposite to these findings as will be shown. Like Mancebon and Moliero (2000) also Noulas and Ketkar (1998), McCarthy and Yaisawarng (1993), Engert (1996) and (Färe, Grosskopf and Weber (1989)) interpret pass-levels at different exams as quantity outputs.

The third group of studies includes qualitative as well as quantitative outputs and therefore is closest in spirit to ours. Thanassoulis (1996b), Thanassoulis (1996a) and Thanassoulis and Dunstan (1994) are particularly relevant here, as they also deal with secondary schools in the UK (although looking at state schools rather than private schools). They consider placement ratios to jobs after graduation as quantity output and test scores as quality output. Behrman and Birdsall (1983) also incorporate quality and quantity aspects of schooling into their analysis (regression of earning functions), but their target is not efficiency evaluation but rather to compare social rates of return for these two basic types of outputs.

In terms of variable specification the study most closely related to ours, along with Ray and Mukherjee (1998) as mentioned above, is Ruggiero and Vitalino (1999). They analyze cost efficiency of New York State schools with a variable returns-to-scale DEA model and use expenditures per pupil as single input, quantity output variables based on dropout and graduation rates and qualitative output variables based on test scores. One result from their study and contrasting ours, is that school size (measured as total pupils) does not significantly influence efficiency. Lovell, Walters and Wood (1994) examine secondary schools with regression analysis and find a positive influence of school size on efficiency as we do. They also find that the religious orientation (catholic vs. non-catholic in their paper) has a negative impact on efficiency in the long term, in basic accordance with our findings.

Regarding the statistical concepts employed here the interested reader is referred to Gstach (2002) for more details. A related approach, as it also employs Markov Chain Monte Carlo techniques within a Bayesian framework for efficiency estimation, is given in van den Broeck, Koop, Osiewalski and Steel (1994), but it does not cover the multiple-output case investigated here.

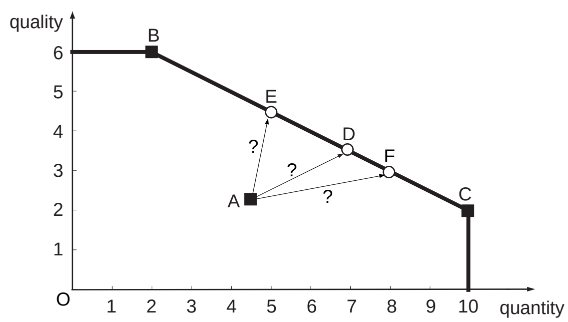
well as outputs.

3 Modelling issues

Output-specific efficiency estimation as adopted here is a two-step procedure: First the *best practice* to transform inputs into outputs is determined via traditional *Data Envelopment Analysis* (DEA), more exactly via an output oriented BCC model as developed in Banker, Charnes and Cooper (1984). So the assumption maintained throughout this paper as in all DEA studies is that we deal with non-noisy data. Under this assumption the DEA estimator for *best practice* is a consistent estimator of the production technology as proven in Banker (1993) and Korostelev, Simar and Tsybakov (1995) under very general conditions.

The second step is to estimate the parameters of the output-specific efficiency distribution (along with the parameters of the distribution of a latent variable as explained below) which describes the population of schools. Given these population parameter estimates we then calculate the conditional expectation of school-specific and output-specific efficiencies to come up with two efficiency rankings of schools based on the performance with regard to quality resp. quantity. Within this second estimation step target output-ratios are used as auxiliary statistical devices (latent variables) to describe the data generating process. Whether or not schools indeed formulate such target output-ratios is irrelevant, because they are logical complements to output-specific efficiencies in the data generating process.

Figure 1: Geometrical representation of the basic issues



To fix ideas take a look at Figure 1. It depicts the combinations of quantity and quality produced by three schools, labelled A,B resp. C (the solid squares) with some common amount of tuition. The DEA best practice estimate corresponding to this data set is the fat lined convex envelopment around the observed data points. So in terms of efficiency we can not distinguish between schools B and C (both represent best practice) but we find A to produce clearly insufficient amounts of both outputs. Three (of infinitely many) options to measure the efficiency of school A are depicted graphically relative to best practice at frontier points D,E resp. F (the white circles). Measured relative to point D (as in standard DEA applications) we find the maximum *proportional* expansion of both outputs of school A as ratio between the lengths $\overline{OA}/\overline{OD}$. So relative to point D quantity efficiency and quality efficiency of school A would be equal and around 64%.

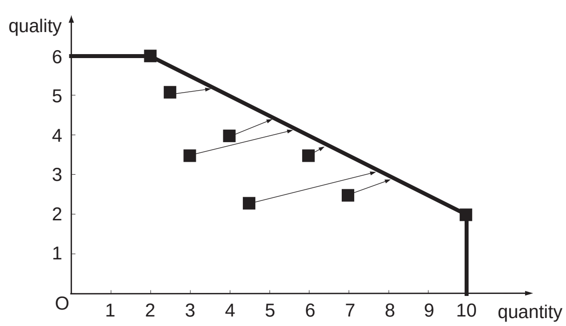
Evaluated relative to point E matters are different: Now quantity efficiency of school A (measured as ratio of quantity of A vs. quantity of E) appears rather high at around 90%

whereas quality efficiency (measured as ratio of quality of A vs. quality of E) is fairly low at around 50%. So if it could be argued, that point E is the most plausible reference point, we should conclude that the school primarily fails in providing quality for given tuition, while the deficiency in the quantity aspect is comparatively negligible. The picture would be the opposite using F as reference point rather than E.

Note that each of the three potential reference points is associated with a unique output-ratio (quantity over quality): For E this is 5/4.5, for D it is 7/3.5 and for F it is 8/3. In other words: Each combination of output-specific efficiencies one could possibly measure relative to the best practice is uniquely associated with a certain output-ratio, which identifies the particular reference point. The genesis of observed outputs may therefore statistically be described in terms of a best practice frontier and two random quantities: a pair of output-specific efficiencies and an output-ratio. The latter is labelled *target output-ratio* owing to its role in this description of the data generating process.

For given parameters describing the involved distributions of output-specific efficiencies and target output-ratios, one can calculate the *expected* reference point conditional on observed outputs and inputs. Such *expected* reference points will be estimated here and used for output-specific efficiency evaluation of individual schools. So our reference points have a clear statistical meaning unlike those calculated in traditional DEA variants involving output-specific efficiencies. The required estimation of the distributional parameters boils down to a likelihood comparison between sets of projections of observed outputs upon the best practice frontier. Two such sets of projections are given as arrows in the left resp. right hand graph in Figure 2 for a fictitious sample of observed output combinations (the solid squares). The question is, under which of these two projections the sample of observations appears more likely. The answer to this question has immediate consequences for output-specific efficiencies: Average quantity-efficiency in the left hand graph is rather low and average quality-efficiency rather high and vice versa for the right hand graph. In the actual estimation process the likelihood of thousands of such sets of projections are compared as described in detail in Gstach (2002).

Figure 2: Two sets of projections for a given sample of observations



Without going into estimation details here we just give a brief formal statement of the data generating process. Denote the maximum amount of output s which can be produced from input vector $x_i \in \mathbb{R}^R_+$ when the target output-ratio is m_i as $\mathrm{y}^F_s(x_i, m_i) \in \mathbb{R}_{++}$ for $s = 1, 2$ (we deal with only two outputs here). These $\mathrm{y}^F_s(x_i, m_i)$ functions describe the schooling production technology and in the present application result from a first step

DEA to determine best practice. Denoting output-specific efficiency terms as $\varepsilon_{is} \in \mathbb{R}$ for $s = 1, 2$ the statistical generation of output observations from inputs, target output-ratios and efficiency terms as envisioned here is

$$y_{is} = \mathrm{y}_s^F(x_i, m_i) \Big/ \left(1 + e^{-\varepsilon_{is}}\right) \quad \text{for} \quad s = 1, 2 \tag{1}$$

Using definition $\tilde{\varepsilon}_{is} = 1/(1-e^{-\varepsilon_{is}})$ one could write relationship (1) in more conventional form as $y_{is} = \mathrm{y}_s^F(x_i, m_i)\,\tilde{\varepsilon}_{is}$ for $s = 1, 2$, with $\tilde{\varepsilon}_{is}$ now being a traditional efficiency score from the [0,1]-domain. The reason for using logit transformed efficiency scores and formulation (1) instead is that this enables to model the efficiency terms $\varepsilon_i = \{\varepsilon_{i1}, \varepsilon_{i2}\}$ as bivariate Normal distributed random variables, which are easier to handle than bivariate distributions on the [0,1]×[0,1]-domain. Likewise also the target output-ratios will be represented in logit-form, so that their statistical properties can be described via an univariate Normal distribution. The distributional assumptions imposed are

$$m_i \overset{iid}{\sim} \mathcal{N}(m_i; \mu_M, \sigma_M) \tag{2}$$

$$\varepsilon_i \overset{iid}{\sim} \mathcal{N}_2(\varepsilon_i; \mu_E, \Sigma_E), \tag{3}$$

where $\mathcal{N}$ resp. $\mathcal{N}_2$ denote the univariate resp. bivariate Normal distributions and *iid* means identically and independently (across schools) distributed. This parametric structure of efficiency and target output-ratio distributions is the driving force for identifying eventual efficiency differences in producing the two outputs considered here. Because of this superimposed parametric structure (unnecessary for standard DEA), the present approach may be reminiscent of stochastic frontier analysis (SFA), but there the parametrization serves a completely different purpose, which is to distinguish between efficiency and noise.

4 Data

The data for our analysis of 237 private, secondary schools in the UK were published in the Financial Times (FT) on October 10th 2000 (along with data for state schools). Summary statistics for these data are given in Table 1. Based on this data set the FT generates the FT-1000 league table, ranking 1000 schools solely according to average grades of graduating pupils. We want to improve upon these tables by better exploiting the information contained in the FT data set. This is achieved by simultaneously considering quality and quantity aspects of school performance along with tuition. This will give a richer performance picture based on two complementary rankings, one based on the efficiency of providing quantity and one based on the efficiency in providing quality.

Following the advice of Hanushek (1986) we model schooling technology using *tuition* as single input. Tuition is what all parents have to pay for the education of their children in private secondary schools. Some parents consider the option of boarding their kids at the school location in which case they will pay additional attention to boarding fees. The primary reason for not including the latter into the input cost variable is that some parents necessarily have to incur such costs while others don't and consequently it would be unfair to assess school performance from a parental perspective based on such unequal ground.

A further reason is that only 40% of kids are boarding in school offered facilities. And finally tuition and boarding fees are highly correlated (0.86), so that we might consider the tuition costs as roughly proportional to the sum of tuition plus boarding, the alternative input variable.

The funds available to any school are used to cover various types of expenditures. So any given amount of tuition may be associated with rather different spending habits of schools. But however the funds are employed, the ultimate goal of schools in an economic perspective is to provide graduation quality and quantity, as stated for example Wenger (2000). So for our purpose of establishing a ranking in terms of quality and quantity the distinctive ways of school to spend tuition is immaterial.

Our first output variable is the ratio *candidates/pupils* which we consider as indicative of the schools ability to transform pupils into graduation candidates, i.e. as proxy for graduation quantity. For given tuition it is clearly desirable to achieve a given average grading quality with the highest possible candidates/pupils-ratio, as this implies less waste of human capital during the curriculum. With fixed cohort size across all seven forms (the sixth form actually consists of two years of schooling) this ratio would be roughly 14.3%, while the actual average across all sample schools is 13%. So, unsurprisingly, the typical school in our sample has net-dropout. But our sample also contains 27 schools with ratios higher than 20% and 19 schools with ratios less than 5%. This indicates considerable shifts of pupils amongst schools and should lead to the observation of marked tradeoffs between quantity and quality in the form of some high quantity /low quality schools being efficient just like some low quantity /high quality schools.

The second output used is supposed to proxy quality: *points per candidate* (average A-level grade). Correlation of this variable with an alternative quality variable *points per entry* is 0.94 and using it instead of points/candidate does not change the results qualitatively.[4] So we will only report results for the points/candidate figures as quality indices.

Table 1: Data description

	Candidates	Total Pupils	%Male	%Boarding
MEAN	62	468	47	40
STD	40	222	34	28
MIN	10	101	0	0
MAX	208	1421	100	100

	Tuition = Input	Quantity = Output 1	Quality = Output 2
MEAN	8513	0.130	20.55
STD	2095	0.053	4.29
MIN	3471	0.027	10.38
MAX	16455	0.283	32.76

Quantity=Candidates/Pupils, Quality=Points/Candidate

In modelling schooling technology one would like to control for different socioeconomic backgrounds or for entry performance of pupils to filter out the actual contribution of a

[4] In generating the FT-1000 league tables both of these alternative quality variables are used to construct a weighted performance index to rank the schools.

school to pupils graduation performance. The body of data available with the FT-1000 league tables prevents this because it lacks corresponding variables with the exception of *tuition*. The latter variable to some extent reflects parents wealth and so covers at least one socioeconomic aspect. But the available data allow to distinguish the sample schools by size (measured as total number of pupils), by gender proportions, by percentage of boarding students and by faith. For some corresponding subgroups, which are also regularly referenced in the public debate, we calculate the respective mean efficiencies.

5 Results

We will only present estimation results here as far as they seem relevant for practical purposes. Regarding the parameter estimates themselves we only report the estimated correlation coefficient associated with covariance matrix Σ_E from distribution (3). This correlation between the two efficiency terms was estimated to be around 0.64 and found to be significantly positive. So the provision of good quality grades comes along with provision of relatively high quantity of grades. This result definitely does not support the often expressed view, that a schools ability to provide good grading quality would necessarily be associated with the inability to provide high quantities of graduates. Instead the correlation estimate of 0.64 provides some evidence, that a typical school tends to perform relatively well (compared to other schools) in both respects or in neither of them. While this finding does not question the basic tradeoff between quantity and quality in a technical sense (substitution possibilities along the frontier) it clearly indicates, that management efforts to provide efficient grading quality resp. quantity of UK private secondary schools should not be seen as substitutes.

Table 2: Correlation Estimator

	Mean	Median	STD	Min	Max
ρ	0.64	0.65	0.11	0.21	0.88

ρ: correlation between ε_{i1} and ε_{i2},
STD: standard deviation of sampled ρ values.

The other main findings for practical purposes are displayed in Table 3. As can be seen the average efficiency across schools of providing grading quality is far higher (0.89) than the average efficiency of providing grading quantity (0.59). In terms of targets this means that the typical UK private secondary school comes much closer to meet its quality target than its quantity target. The debate over secondary schooling issues in the UK lead to the critique that secondary schools would too willingly sacrifice quantity for quality in order to achieve higher ranks in the FT-1000 league tables which are solely based on quality variables. Our finding gives clear support to this critique although the latter is usually targeting non-private schools, while we solely investigate private schools. On the other hand our findings quantitatively substantiate the typical reputation of private schools of providing high grading quality rather than quantity.

Table 4 displays the rank correlations between the various types of performance measures: The FT-1000 league scores (FTS), radial DEA efficiency scores (RE), quantity-specific

Table 3: Sample Mean Efficiencies

	Mean	Median	STD	Min	Max
Radial-Efficiency	0.77	0.78	0.14	0.39	1.00
Quantity-Efficiency	0.59	0.59	0.18	0.16	0.98
Quality-Efficiency	0.89	0.90	0.08	0.54	1.00

(NE) and quality-specific efficiency scores (LE). The correlation between RE and FTS is 0.81. This is quite high given the fact that the DEA estimated technology relates the achievements of schools to inputs (tuition) in contrast to the FT-scores, which ignore inputs completely. The correlation being as high the standard radial efficiencies from DEA seem not to provide much additional insight into systematic performance problems beyond the FT-scores. The story with output-specific scores is different. In particular the correlation between ranks according to NE resp. FTS is as low as 0.53, clearly reflecting the lack of quantity considerations in the FT-scores. The correlation between the ranks according to FTS resp. LE is unsurprisingly higher at 0.72, not too far from the corresponding figure 0.81 for radial efficiencies.

Table 4: Rank correlations

	RE	NE	LE
FTS	0.81	0.53	0.72
RE		0.73	0.91
NE			0.93

FTS: scores from FT-1000 league tables. RE: radial efficiencies from output-oriented DEA with variable returns to scale. NE resp. LE: quantity-specific resp. quality-specific efficiencies under the same technique.

Detailed figures for data, estimated performance measures and associated ranks for six schools are given in Table 5 to highlight some marked differences in ranks depending on the type of performance measure used. For example the school ranked 8 according to FTS (in the subsample of private schools only) falls back to rank 54 in a quantity efficiency perspective, mainly because it only provides about average grading quantity at average costs and despite providing very good grading quality.[5] An opposite argument applies to the school ranked 134 according to FTS in our subsample. It could dramatically improve its relative position to rank 9 according to the various efficiency measures, because it provides superior grading quantity along with average grading quality at average costs.

Regarding the relationship between performance scores and gender of pupils we find significantly higher average efficiency of pure boy schools vs. pure girl schools in achieving

[5] Note that this deterioration also applies to the quality-efficiency rank of this school which is only 43. The reason for this result is that the statistically plausible target output-ratios of ALL schools are shifted towards more quantity relative to the actually observed output-ratios. This shift necessarily brings along improved quality efficiency for ALL schools (although to different degrees) leaving little room for the school, originally ranked 8, to improve its relative position in this regard.

Table 5: Results for selected schools

data			efficiencies			ranks according to			
TUIT	QN	QL	RE	NE	LE	FTS	RE	NE	LE
12150	0.15	29.76	0.92	0.65	0.94	4	37	101	87
10050	0.17	27.25	0.88	0.77	0.96	8	54	54	43
7170	0.13	26.18	0.97	0.84	0.98	22	15	32	20
13590	0.24	25.27	0.95	0.93	0.99	33	21	15	16
10221	0.28	19.99	1.00	1.00	1.00	134	9	9	9
8319	0.21	18.91	0.91	0.90	0.97	165	44	18	27

TUIT: tuition; QN: quantity; QL: quality; FTS: FT-1000 scores, RE: radial efficiencies; NE resp. LE: quantity-specific resp. quality-specific efficiencies.

quantity and quality goals alike (see Table 6). Furthermore mixed schools perform worse than gender separated schools. Note that the most marked difference between pure boy and pure girl schools occurs in terms of quantity efficiency: 0.79 vs. 0.60, while difference in quality efficiency is rather small: 0.96 vs. 0.91. So primarily the quantity-efficiency accounts for the observed differences amongst these two groups of schools. Interpreting the quantity efficiency as reciprocal of selectivity we may thus state that it is much greater selectivity of pure girl schools which makes them appear inefficient from a quantity perspective. This story comes out even clearer if looking at the weighted average efficiencies, when the weights are the percentage of boys resp. girls of total pupils. Then the difference between a typical boy school and a typical girl school is negligible in radial or quality-efficiency perspective, but considerable in magnitude (0.65 vs. 0.57) and significant from the quantity efficiency perspective. So this is another instance, where output-specific efficiencies tell more than the usual radial efficiency scores.

Table 6: Schools according to sex

	n	FTS	RE	NE	LE
boy schools	19	1.21	.89	.79	.96
mixed schools	168	0.99	.74	.59	.89
girl schools	50	1.13	.81	.60	.91
boy% weighted schools	-	1.02	.77	.65	.90
girl% weighted schools	-	1.05	.77	.57	.89

n=number of observations in each category.

A common distinguishing criteria of observations in efficiency analysis is the size of the analyzed units. Here we consider the total number of pupils per school as size indicator and look separately at the various performance scores of the group of schools with less than 400 pupils and the group with at least 400 pupils. As Table 7 shows, all measures indicate a performance advantage of large schools versus small schools. But while the FT-scores, purely quality based performance measures, suggest that bigger schools perform 13% better on average in quality terms, a more detailed analysis accounting also for inputs and output quantity reveals a meager 4% advantage of bigger schools relative to smaller ones in terms of quality efficiency. On the other hand the output-specific efficiency analysis shows a 13% advantage of the bigger schools in terms of quantity efficiency. So once again we find: What primarily distinguishes schools is their ability to cope with the quantity issue rather than their ability of achieving high average grades for the A-levels. Obviously the different skills necessary to accomplish the second task varies much less across schools than the skills necessary to come up with efficient quantity figures.

Table 7: Schools according to size

	n	FTS	RE	NE	LE
total pupils < 400	106	0.97	.72	.56	.88
total pupils ≥ 400	131	1.10	.81	.64	.92
lead of schools ≥ 400	-	13.2%	13.0%	13.0%	4.0%

n=number of observations in each category.

A hot topic within the current debate in the UK is whether church schools or Non Denominational schools perform better, an issue discussed recently for example in Dean (2001). The Prime Minister and the Secretary of Education have proposed massive investments in church schools based on the argument that church schools would typically perform better than other types of schools. The evidence from our analysis, presented in table 8, does not support this argument. While Church of England schools have significantly higher average efficiencies than interdenominational and non-conformist schools,[6] they are in fact outperformed on average by non-denominational schools for all types of efficiency measures considered here. But also with respect to faith the performance differences between the various types of school is most pronounced for quantity-efficiency with averages varying between 52% and 66%, while in quality terms the differences appear much smaller in magnitude with per-type-averages lying in the narrow range between 87% and 92%.

Table 8: Schools according to faith

	n	FTS	RE	NE	LE
Church of England	131	1.07	.78	.64	.91
Interdenominational	40	0.96	.72	.54	.87
Roman Catholic	20	1.02	.78	.60	.90
Non-Denominational	17	1.07	.81	.66	.92
Non-Conformist	22	1.00	.73	.52	.87

n=number of observations in each category.

The relationship between the percentage of boarding pupils and school efficiency looks positive from all perspectives: Schools with higher percentages of boarding pupils tend to perform better than schools with lower percentages of boarding schools. But in magnitude only the quantity-efficiency differences seem noteworthy (0.66 for a typical boarding school vs. 0.57 for a typical non-boarding school), while neither FT-1000 performance scores nor radial efficiencies nor quality efficiencies are much affected by differences in the proportion of boarding pupils. This result is rather plausible when, as may be argued, private schools feel more obliged to boarding pupils rather than non-boarding pupils because the former

[6] Significance levels are those achieved in non-parametric Wilcoxon-Mann-Whitney tests for equal means of distributions, which lie for this comparison between 90% and 99%.

contribute much more than the latter to the economic well being of the school via boarding fees.

Table 9: Schools according to percentage of boarding pupils

	FTS	RE	NE	LE
boarding% weighted schools	1.06	.77	.66	.91
non-boarding% weighted schools	1.03	.77	.57	.89

6 Summary

In this paper a new type of efficiency estimation methodology was applied to investigate quantity and quality efficiency of UK private secondary schools. Quantity and quality improvement potentials are estimated simultaneously based on an economic model of the schooling process which accounts for the substitution possibilities between quantity and quality. As quality indicator average scores per pupil at the final A-level exams were used and as quantity indicator the ratio of candidates for these exams to total pupils over all grades. Lacking other type of data tuition was considered as sole input. This set of variables captures the essential characteristics of schooling decisions in a parental perspective, when value for money is at stake. The data set used is the one published regularly by the Financial Times along with the corresponding ranking amongst UK secondary schools (the so called FT-1000 league tables).

The outstanding result is that the 237 private schools analyzed have an average efficiency in producing quantities of graduates of only 59% vs. the best schools within this sample, while at the same time achieving 89% average efficiency in quality terms. From an aggregate economic point of view this finding shows large quantity improvement potentials without any need to sacrifice the quality of graduates and thus contrasts with repeated opposite claims in the UK schooling debate.

We find furthermore that individual schools abilities to cope with quantity and quality issues are strongly positively correlated. So a typical school either provides good quality graduates and also a high quantity of them (relative to the pool of pupils a school can draw from) or it performs badly in both of these respects. This casts further doubts about the claim schools could only improve the quantity of graduates at the cost of lower quality of A-level graduates.

We also calculated average efficiencies for certain subgroups of schools. With regard to religious orientation we find that non-denominational schools perform better than any other subgroup (including Church of England schools and Catholic Schools) with respect to both output categories. This result contrasts with another wisdom often expressed in the UK secondary schooling debate which has led to primarily subsidizing church schools on grounds of their postulated better performance. With regard to gender we find that boy schools are on average more efficiently managed when it comes to quantity than girl schools, while the difference in quality respect is negligible. The former result indicates significantly higher selectivity in girl schools than in boy schools, because our quantity variable may be interpreted as the reciprocal of a schools selectivity (driving pupils out of the curriculum before they become candidates to the final exams). While factors not included in our analysis may contribute to this finding, it nevertheless points towards the problem of a waste of human capital particularly in the form of girls intellectual capabilities through inefficient school managements. Finally we also find better performance in quality and quantity respects of large schools (with more than 400 pupils) than of small schools (less than 400), but again the big difference occurs in the quantity dimension, whereas in terms of graduates quality it is negligible.

The school ranking published in the Financial Times is only based on a quality variable and ignores tuition and the quantity dimension. Unsurprisingly the FT-1000 league table gives a rather different picture of schools relative performance than our findings, although we use the very body of data underlying the calculation of the FT-1000 league tables. Thus in the light of the presented analysis the information about school performance that is available to parents and pupils from the FT-1000 league tables seems fairly limited. Looking additionally at different dimensions of school performance and tuition as shown here, parents and pupils can get a much more accurate picture of how any individual school may be giving value for money.

Acknowledgements

We gratefully acknowledge many helpful and stimulating comments on earlier drafts of this paper from the discussants at EWEPA7, Oviedo, Sept. 2001 and at the conference of the Verein für Socialpolitik, Magdeburg, Sept. 2001.

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