Engineering Journal of Qatar University, Vol. 2, 1989.

"A MODEL REFERENCE ADAPTIVE CONTROL SCHEME WITH EXTERNAL CONTROLLERS"

By

Riyadh Al-Salman, R. Nagarajan and **Dakhil H. Jerwe**

Electrical Engineering Department,
University of Basrah, Iraq.

ABSTRACT

A new model reference adaptive control scheme, suitable for linear multivariable discrete time systems, is proposed. A set of parameter adaptation algorithms are developed based on the validity of perfect matching between the model and the plant, and on the availability of complete state vector. Arrangements for linear state error feedback, and time varying adaptive loop gains are made in the proposed scheme. It is shown that the scheme with such arrangements offers increased rate of decay of adaptive system errors towards the origin of the error space. A problem of on-line matrix inversion, encountered in implementing the proposed adaptation algorithms, is solved by modifying the structure of one of the adaptive controllers.

1. INTRODUCTION

The plant of a Model Reference Adaptive Control (MRAC) system can be controlled in two ways: (i) by a set of controllers linked to each of the plant parameters which are assumed to be accessible for direct adjustment, (ii) by a set of controllers connected externally to the plant. The second approach has more applications since it does not require the accessibility of plant parameters.

The MRAC system is, further, classified mostly into two configurations: series-parallel and parallel. In the first configuration the model is necessarily to be a hardware, circuits formed by discrete components.

This configuration is employed in the design of external controllers for single-input single output plants [Udink Ten Kate, 1978]. This design approach is restricted to plants having no numerator dynamics and requires the estimated maximum value of one of the plant parameters. In the parallel configuration of the MRAC system, the model can be another physical system, perhaps a prototype of the plant. This configuration has also been considered in the formulation of parameter adaptation laws for direct controllers in multi-input system [Sebakhy 1976]. Adaptation by direct controllers is not always possible since the state variable representation of the plant may result in matrices having elements as functions of more than one parameter of the plant. Sebakhy's method, if extended for the design of external controllers leads to a problem of on-line matrix inversion, and requires the maximum values of the elements of input distribution matrix of the plant.

In the above referred works, and in many others, the parameter adaptation algorithms employ certain time invariant gain matrices. Though the convergence of the algorithms has been ensured, high overshoots and large settling times have been observed in parameter adaptive responses. It has been shown that these undesirable effects can be minimized by employing time varying gain matrices (matrix gain sequence) [Subbayyan and Nagarajan 1978]. Such a scheme results in a more flexible adaptive control system than the one employing time invariant gain matrices in the adaptive loops.

This paper considers the above mentioned desirable features of gain sequences in the adaptive loops, and proposes a new, asymptotically stable MRAC scheme for multivariable, discrete time systems. The proposed scheme is suitable for parallel configuration of MRAC system with external controllers, and it does not assume any apriori knowledge of the bounds on plant parameters. A set of parameter adaptation algorithms are developed, based on the validity of perfect matching between the model and the plant, and on the availability of complete state vector. The application of perfect matching conditions requires on-line inversion of a matrix equals to the number of inputs of the plant. This undesirable requirement is circumvented by modifying the structure of one of the adaptive controllers. A set of loop gain sequences is employed in the scheme, and a method for choosing an optimum set of such gain sequences is presented.

The scheme also utilises a linear state error feedback. It is shown that additional improvement in the rate of decay of adaptive system errors is achieved by proper selection of the state error feedback matrix. The effectiveness of the proposed scheme is illustrated by an example.

2. PROBLEM STATEMENT

A linear m input, nth order plant, for which a set of adaptive controllers are to be defined, is described by the dynamical equation where A_p and B_p are, respectively $(n\times n)$ and $(n\times m)$ unknown plant matrices. $x_p(k)$ is assumed to be accessible for precise measurement.

$$x_p(k+1) = A_p x_p(k) + B_p u_p(k) \tag{1}$$

The linear m input, nth order completely controllable model, whose state variable responses due to a reference input $u_m(k)$ are the desired state variable responses of the plant, is described by.

$$x_m(k+1) = A_m x_m(k) + B_m u_m(k) \tag{2}$$

where A_m and B_m are appropriately dimensioned known model matrices.

The plant input $u_p(k)$ is controlled by a set of controllers, and is defined as

$$u_p(k) = Q(k+1) \left[u_m(k) + F^T(k+1)\, x_p(k) + P^T e(k) \right] \tag{3}$$

where

$Q(k+1)$ and $F(k+1)$ are, respectively, $(m\times m)$ and $(n\times m)$ dimensioned adaptive controllers.
$e(k)$ is the $(n\times 1)$ state error vector such that

$$e(k) = x_m(k) - x_p(k) \tag{4}$$

and P is a known (nxm) constant matrix.

The problem of MRAC system is to modify on-line the elements of Q(k+1) and F(k+1) based on a set of parameter adaptation algorithms, to achieve

$$\underset{k \longrightarrow \infty}{Lt} \; e(k) = 0 \tag{5}$$

$$\underset{k \longrightarrow \infty}{Lt} \; [\, F^* - F(k) \;\vdots\; Q^* - Q(k) \,] = 0 \tag{6}$$

F* and Q* are the correct controller matrices, (unknown) which when incorporated in the adaptive system, perfectly match x_p (k) with x_m (m)

THE ERROR MODEL

The dynamics of state error is obtained from equation (1) to (4) as

$$e(k+1) = [\, A_m - B_m P^T \,]\, e(k) + [\, A_m - A_p - B_m F^T(k+1) \,]$$

$$x_p(k) + [\, B_m Q^{-1}(k+1) - B_p \,]\, u_p(k) \tag{7}$$

In order to achieve the requirements given in equations (5) and (6), the plant and the model are asumed to satisfy the conditions of perfect matching [Chan 1973, Narendra and Kudva 1974].

$$A_p = A_m - B_m (F^*)^T \tag{8}$$

$$B_p = B_m (Q^*)^{-1} \tag{9}$$

The conditions of perfect matching require the existances of $(Q^*)^{-1}$. Q(k) has to be non singular for all k. Substitution of equations (8) and (9) in equation (7) leads to.

$$e(k+1) = [A_m - B_m P^T]\, e(k) + B_m\, \theta^T(k+1)\, z(k) \tag{10}$$

where the $(m\times(n+m))$ parameter alignment error matrix

$$\theta^T(k+1) = [\,(F^*)^T - F^T(k+1) \;\vdots\; Q^{-1}(k+1) - (Q^*)^{-1}\,] \tag{11}$$

$$z(k) = \begin{bmatrix} z_1(k) \\ z_2(k) \\ \vdots \\ z_f(k) \end{bmatrix} = \begin{bmatrix} x_p(k) \\ ---- \\ u_p(k) \end{bmatrix} \quad ; \quad f = n+m \tag{12}$$

It is noted that the error model represented by equation (10) is different from that of model reference system reported earlier [Sebakhy 1976] which does not use state error feedback. The matrix P in equation (10) can be freely chosen so that the eigenvalues of $[A_m - B_m P^T]$ are placed at any desired locations in the z−plane. The dynamics of the state error e(k) are thus controlled by this eigenvalue assignment.

3. THE ADAPTATION ALGORITHMS

The proposed adaptation algorithms are derived using the gradient estimation procedure. Let $J(\theta)$ be a quadratic index defined as $\theta(k)$ as

$$J(\theta) = Tr\,[\,\theta^T(k)\,\theta(k)\,] \tag{13}$$

The' gradient matrix' of the index $J(\theta)$ is obtained by [Graupe, 1976].

$$\nabla J(\theta) = \frac{\partial J(\theta)}{\partial \theta(k)} = 2\,\theta(k) \tag{14}$$

Since $\theta(k)$ is unknown, $\nabla J(\theta)$ cannot be computed. However, an estimate of gradient matrix is determined by computing an estimate on $\theta(k)$.

Defining

$$s(k) = e(k) - [A_m - B_m P^T]\, e(k-1) \tag{15}$$

equation (10) is written as

$$s(k) = B_m\, \theta^T(k)\, z(k-1) \tag{16}$$

An estimate on $\theta(k)$, denoted by $\theta(k)$, is then derived from equation (16) as

$$\tilde{\theta}^T(k) = \alpha(k)\, (B_m^{+})\, s(k)\, z^T(k-1) \tag{17}$$

where $\alpha(k) > 0$ is a scalar function of $z(k-1)$, and

$$(B_m^{+}) = [B_m^T B_m]^{-1} B_m^T \tag{18}$$

is the $(m \times n)$ left pseudo inverse of B_m.

Equation (17), substituted in (14), yields an estimate on $\nabla J(\theta)$ as

$$\nabla J(\tilde{\theta}) = 2\alpha(k)\, z(k)\, s^T(k)\, (B_m^{+})^T \tag{19}$$

The estimated gradient matrix $\nabla J(\theta)$ is now employed in the gradient estimation approach [Mendel 1974] to form an algorithm for modifying the elements of $\theta(k)$.

The algorithm is of the form

$$\theta(k+1) = \theta(k) - 2\alpha(k)\ R(k) \circledast [z(k-1)\ s^T(k)\ (B_m^{+})^T] \tag{20}$$

where R(k) is a $(f \times m)$ matrix gain sequence with itse elements $r_{ij}(k) > 0$, i = 1, 2,... f, j = 1,2...m, and the symbol – stands for the "element by element matrix product" operator [Carroll,, 1974].

The required parameter adaptation algorithms are, then, derived from equations (11) and (20) as

$$F(k+1) = F(k) + 2\alpha(k)\ R_1(k) \circledast [x_p(k-1)\ s^T(k)\ (B_m^{+})^T] \tag{21}$$

$$(Q^{-1}(k+1))^T = (Q^{-1}(k))^T - 2\alpha(k)\ R_2(k)$$

$$\circledast [u_p(k-1)\ s^T(k)\ (B_m^{+})^T \tag{22}$$

where $R_i(k)$ and R_2 (k) are, respectively, $(n \times m)$ and $(m \times m)$ dimensioned matrix gain sequences such that

$$R(k) = \begin{bmatrix} R_1(k) \\ --- \\ R_2(k) \end{bmatrix} \tag{23}$$

A correct choice of the scalar $\alpha(k)$ and the gain sequences, $R_1(k)$ and $R_2(k)$, of equations (21) and (22) can yield an asymptotically stable MRAC scheme with increased rate of decay of adaptive system errors towards the origin of the error space.

4. ASYMPTOTIC STABILITY OF THE MRAC SCHEME

Lyapunov's direct method is employed to ensure the asymptotic stability of the developed MRAC scheme. A candidate for the Lyapunov function, V(k), is considered as

$$V(k) = Tr\ [\ \hat{R}^T(k)\ [\ \theta(k) \circledast \theta(k)\ ]\ ] \tag{24}$$

where $R^T(k)$ is the "element by element inverse matrix" such that

$$R(k) \circledast \hat{R}(k) = E \tag{25}$$

and all elements of E are unity

The first forward difference Δ V(k) is obtained as

$$\begin{aligned} \Delta V(k) &= Tr\ [\ \hat{R}^T(k+1)\ [\ \theta(k+1) \circledast \theta(k+1)] \\ &\qquad - \hat{R}^T(k)\ [\theta(k) \circledast \theta(k)]\ ] \\ &= g(k) + h(k) \end{aligned} \tag{26}$$

where

$$g(k) = Tr\ [\ [\ \hat{R}^T(k+1) - \hat{R}^T(k)]\ [\ \theta(k+1) \ast \theta(k+1)]\ ] \tag{27}$$

$$\begin{aligned} h(k) = Tr\ [\ [\ \hat{R}^R(k)\ [\ \theta(k+1) \circledast \theta(k+1) \\ - \theta(k) \circledast \theta(k)]]\ ] \end{aligned} \tag{28}$$

The rate of convergence of $\theta(k)$ towards zero is related to the rate with which V(k) goes to zero. Hence it is advantageous if h(k) and g(k) are made individually negative definite for all k.

g(k) is made negative definite for all $\theta(k+1) \neq 0$ by constraining the elements of R(k) as $r_{ij}(k) < r_{ij}(k+1)$.

Defining

$$p(k) = (B_m^{+})\ s(k) \tag{29}$$
$$= \theta(k)\ z(k-1)$$

and using equation (20) in (28)

$$h(k) = \mathrm{Tr}\ [\ 4\ \alpha\ (k)\ \hat{R}^T(k)\ [\ \alpha\ (k)\ R(k) \circledast R(k) \circledast [z(k-1)\ p^T(k)] \circledast [z\ (k-1)\ p^T(k)] - R(k) \circledast [z(k-1)\ p^T(k)] \circledast \theta(k)]\ ] \tag{30}$$

Employing the properties of element by element matrix product in sequence as listed in Appendix A, equation (30) is simplified to [Riyadh Al-Salman, 1981].

$$h(k) = \mathrm{Tr}\ [\ 4\alpha\ (k)\ [\ \alpha\ (k)\ R^T(k)\ [\ z(k-1) \circledast z(k-1)] - v\ ]\ [p^T(k) \circledast p^T(k)\ ]\ ] \tag{31}$$

when the scalar weightage α (k) is selected as

$$\alpha\ (k) \lesssim \frac{1}{z^T(k-1)\ z(k-1)} \tag{32}$$

and, when all elements of R (k) are negative definite as long as
$p(k) = \theta^T(k)\, z(k-1) \neq 0$.
Thus h(k) and g(k) are individually negative definite, and V(k) becomes a Lyapunov function.

The situation when $p(k) = 0$ is considered in the Appendix B, and the proposed MRAC scheme is proved to be asymptotically stable in (θ , e) space when

(i) $(A_m - B_m P^T)$ has its eigenvalues within unit circle.
(ii) (A_m , B_m) is a completely controllable pair, and
(iii) u_m (k) is sufficiently general.

5. ALGORITHMS WITH OPTIMUM WEIGHTAGES

Without loss of information regarding asymptotic stability of the proposed MRAC scheme, h(k) may be considered of having m different weightages α_i (k), i =1,2,... m. Equation (31) can be rewritten with these weightages as

$$h(k) = 4 \sum_{j=1}^{m} [\alpha_j^2(k)\; r_j^T(k)\; [z(k-1) \circledast z(k-1)]\; p_j^2(k) - \alpha_j(k)\; p_j^2(k)]$$

$$= \sum_{j=1}^{m} h_j(k) \tag{33}$$

where

$R_j(k)$ is the jth column of R(k), and

P_j is the jth element of p(k)

Minimization of $h_j(k)$ with respect to α_j (k) offers a set of optimum weightages

$$\alpha_j^*(k) = \frac{1}{2r_j^T(k)\ [\ z(k-1) \circledast z(k-1)\ ]}, \quad J=1,2 \ldots m \tag{34}$$

The parameter adaptation algorithms, equations (21) and (22) are, then, expressed as

$$F(k+1) = F(k) + [R_1(k)\ D(k)\ ] \circledast [x_p(k-1)\ s^T(k)\ (B_m^+)^T] \tag{35}$$

$$Q^{-1}(k+1) = Q^{-1}(k) - [\ R_2(k)\ D(k)\ ]^T \circledast [(B_m^+)\ s(k)\ u_p^T(k-1)] \tag{36}$$

where R_1 (k) and R_2(k) are as in equation (23), and

$$D(k) = 2\ \mathrm{diag}\ (\ \alpha_1^*(k),\ \alpha_2^*(k),\ \ldots\ \alpha_m^*(k) \tag{37}$$

The parameter adaptation algorithm with optimum scalar weightage can be readily derived in the same lines as those indicated in [Riyadh 1985].

These algorithms are obtained as

$$F(k+1) = F(k) + \alpha^*(k)\ [\ R_1(k)] \circledast [x_p(k-1)\ s^T(k)(B_m^+)^T\ ] \tag{38}$$

$$Q^{-1}(k+1) = Q^{-1}(k) - \alpha^*(k)\ [R_2(k)]^T \circledast [(B_m^+)\ s(k)\ u_p^T(k-1)] \tag{39}$$

where

$$\alpha^{*}(k) = \frac{s^{T}(k)\ s(k)}{2\ s^{T}(k)\ T(k)\ s(k)} \tag{40}$$

$$T(k) = \text{diag}\ [\ t_{1}(k)\ t_{2}(k)\ \ldots\ t_{n}(k)]\ \ldots$$

and

$$t_{i}(k) = \sum_{j=1}^{f} z_{j}^{2}(k-1)\ r_{ij}(k) \tag{41}$$

6. IMPLEMENTATION OF THE ADAPTATION ALGORITHMS

Equations 21 and 22 with proper weightages suggested in Section 5 offer an asymptotically stable MRAC scheme. Implementation of equation 21 for modifying the adaptive controller F(k) is easy. But modifying the elements of Q(k) requires Δ Q(k) which is to be computed from equation Defining the first forward difference of Q^{-1} (k) as

$$\Delta Q^{-1}(k) = 2\ \alpha\ (k)\ R_{2}^{T}(k) \circledast [\ u_{p}(k-1)\ s^{T}(k)\ (B_{m}^{+})^{T}]^{T} \tag{42}$$

then

$$Q(k+1) = [\ Q^{-1}(k) + \Delta Q^{-1}(k)]^{-1} \tag{43}$$

and

$$\Delta Q(k) = [\ Q^{-1}(k) + \Delta Q^{-1}(k)\ ]^{-1} - Q(k) \tag{44}$$

$$= Q(k) \left[(\ I + Q(k)\ \Delta Q^{-1}(k)\ )^{-1} - I \right] \tag{45}$$

Equation 43 or 45, along with 42 can be implemented for modifying Q(k). However, such an implementation requires additional on-line computation due to inversion of a matrix of (mxm) dimension. This problem can be circumuented by considering Fig. 1(a), $u_p(k)$, of Fig. 1 (a) is evaluated as

$$\hat{u}_p(k) = u_p(k) + W(k)\,\hat{u}_p(k) \tag{46}$$

$$= [\,I - W(k)\,]^{-1}\, u_p(k) \tag{47}$$

If W (k) is selected as

$$W(k) = [\,I - Q^{-1}(k)\,] \tag{48}$$

then

$$\hat{u}_p(k) = Q(k)\, u_p(k) \tag{49}$$

which is the required control signal to be injected into the plant.

Hence, a modification of the structure and location of Q(k) in the adaptive loop solves the problem of on - line matrix inversion. Fig. 1(b), show the MRAC scheme with the proposed modification of the adaptive controller Q(k).

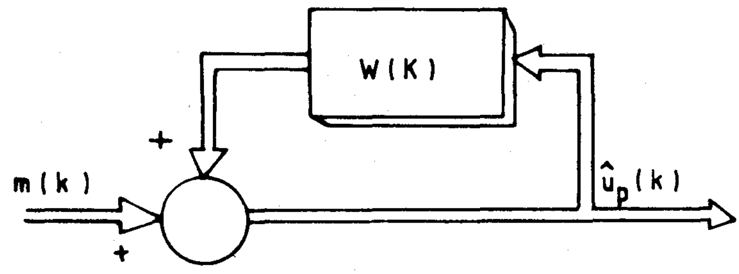

Fig. (1.a) : A Suggested Modification to Solve the Inverse Problem.

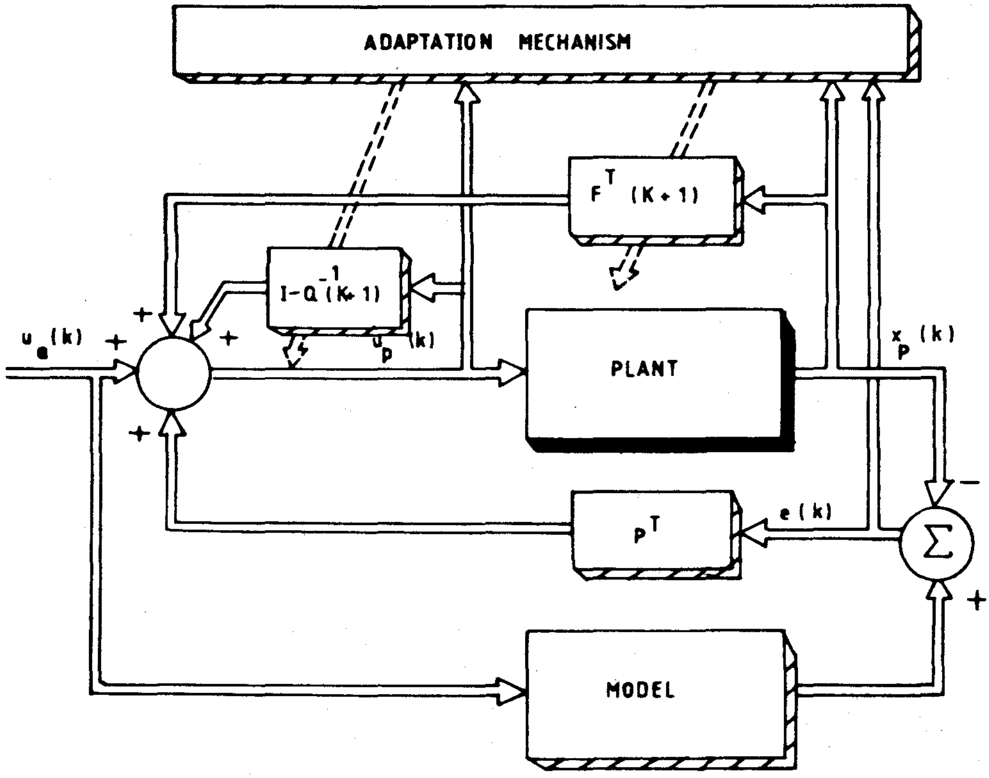

Fig. (1.b) : The Proposed MRAC Scheme With External Controllers.

ILLUSTRATIVE EXAMPLE

The following example illustrates the application of the algorithms developed in this paper.

Let the model matrices be

$$A_m = \begin{bmatrix} 0.6 & 0.16 \\ 0.5 & 0.62 \end{bmatrix} \quad ; \quad B_m = \begin{bmatrix} 0.8 \\ 1.6 \end{bmatrix}$$

The model has the poles at 0.893 and 0.327.

The input $u_m(k)$ is a sequence of rectangular pulses switching between -1 and $+1$ with a frequency corresponding to 2 sample periods.

For the purpose of simulation, the plant matrices are considered as

$$A_p = \begin{bmatrix} 0.36 & 0.08 \\ 0.5 & 0.02 \end{bmatrix} \quad ; \quad B_p = \begin{bmatrix} 0.4 \\ 0.8 \end{bmatrix}$$

The controller matrices at the start of adaptation are assumed to be

$$T(0) = [0 \quad 0] \quad ; \quad Q(0) = 1$$

(a) Equations 21 and 22 for improved convergence, 38 and 39 for optimum convergence with scalar weightages and optimum convergence with matrix weightages (equations 35 and 36) are employed for parameter adaptation.

The following fixed gain matrix and matrix gain sequences are used in the algorithms

(1) $R^T(k) = R_L^T = [0.002 \quad 0.00027 \quad 0.0044]$

(2) $R^T(k) = R_L^H = [0.11 \quad 0.032 \quad 0.4]$

(3) $R^T(k) = R^T(k) + kH^T \quad k = 0, 1, 2, \ldots$

Where

H^T = [0.003252   0.000605   0.006127]

(4) P^T = [0.0   0.0] , so that the eigenvalues of

$[A_m - b_m p^T]$ are placed at (0.893,   0.327)

(b) The same example is again considered but with the following state error feedback matrix

P^T = [0.54   0.29], so that the eigenvalues of

$[A_m - b_m p^T]$ are placed at (0.4 , 0.0) far from the respective poles of model.

A computer program obtainable from authors (on request) is utilised for simulation and its flow chart is as given in Fig.2.

Tabes 1 and 2 compare the responses of controller parameters and the responses of adaptive system errors at k = 1000. It can be verified from both tables that the algorithm for optimum convergence with matrix gain sequences is offering fastest response.

The adaptive responses of the controller parameters, and the responses of adaptive system errors with matrix weightage are presented in Fig. 3 through Fig. 6. The responses utilizing the gains as given above are indicated, respectively, as 1, 2 and 3 in the figures. The figures indicate that the degree of stability of the adaptive scheme is appreciable when the scheme employs matrix gain sequences and linear state error feedback.

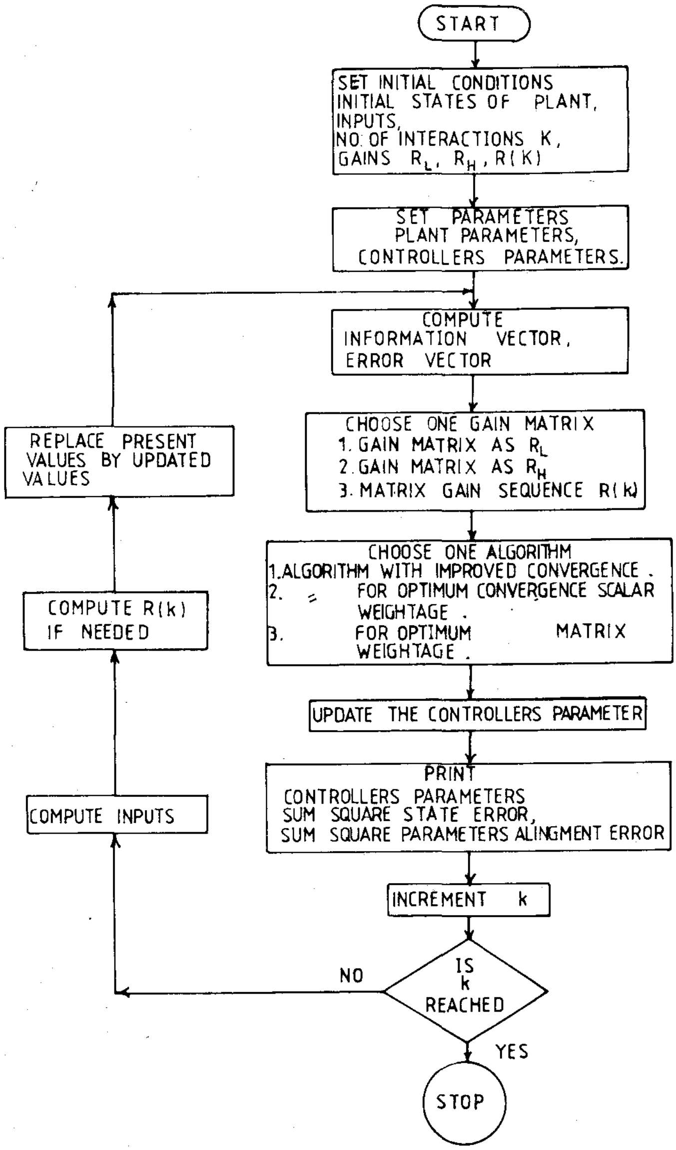

Fig. (2) : Flowchart of simulation program.

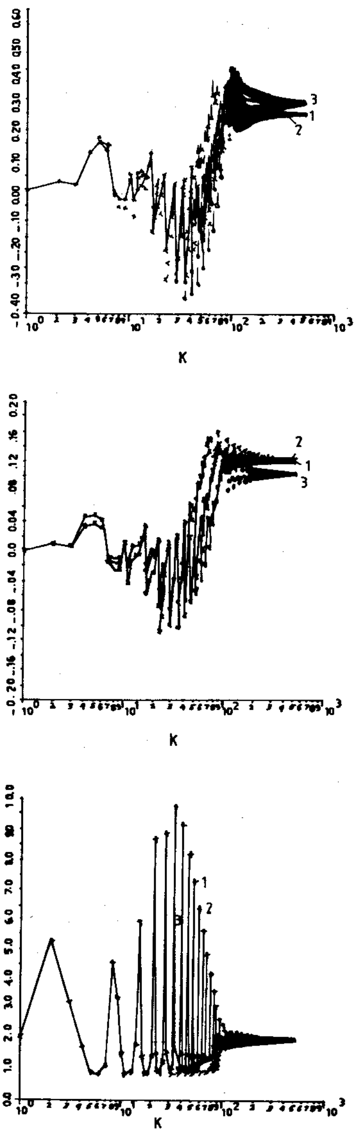

Fig. (3) : Adaptive responses of controller parameters (Optimum convergence with matrix weightage. P = 0).

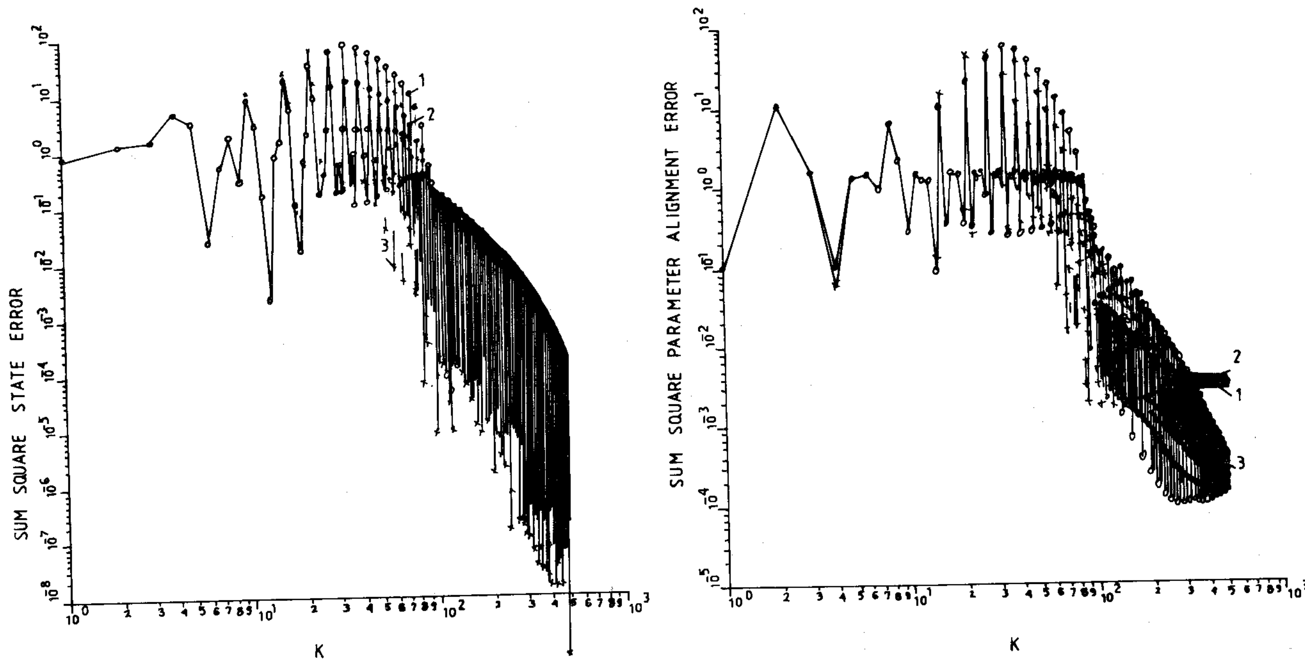

Fig. (4) : Responses of sum-square state error.
(Optimum convergence with matrix weightage. P = 0).

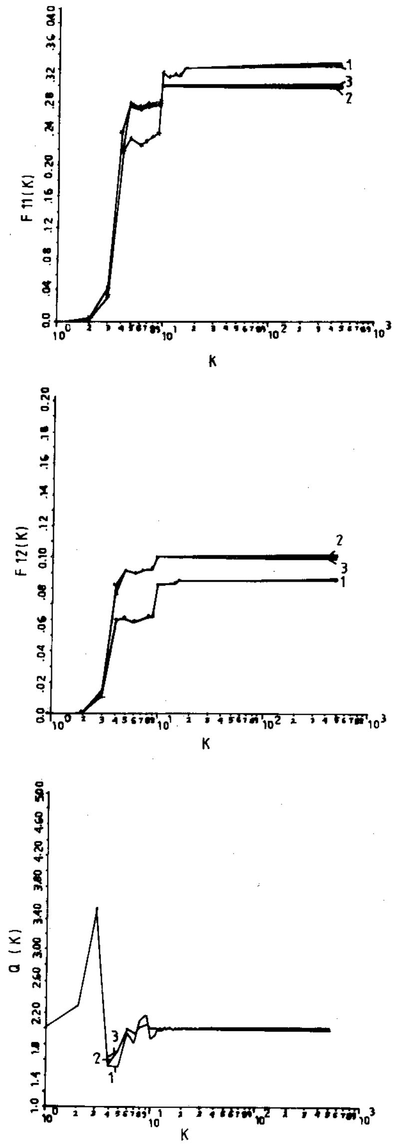

Fig. (5) – Adaptive responses of controller parameters (Optimum convergence with matrix weightage. P ≠ 0).

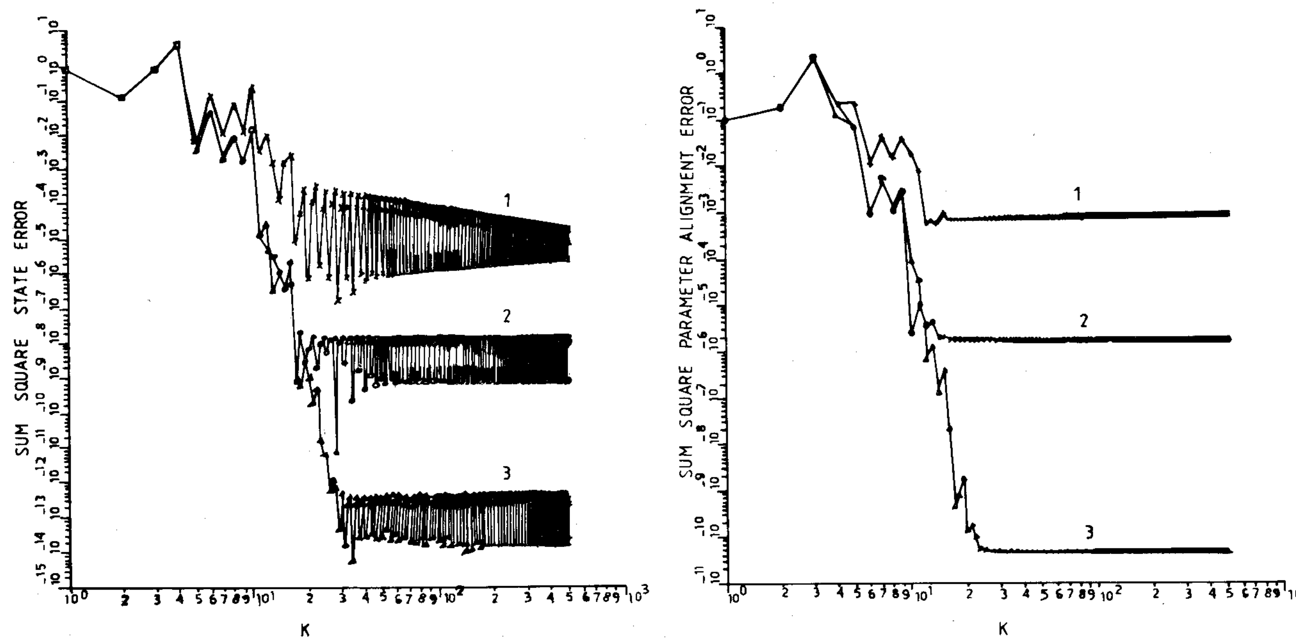

Fig. (6) – **Responses of sum-square parameter alignment error.**
(Optimum convergence with matrix weightage. $P \neq 0$).

Table (1)

Responses of the controllor parameters and responses of adaptive system errors without error feodback matrix ($P^T = [0 \quad 0]$) at itoration k = 1000

PARAMETERS & SQUARE ERRORS	IMPROVED CONVERGENCE			OPTIMUM CONVERGENCE WITH SCALAR WEIGHTAGE			OPTIMUM CONVERGENCE WITH MATIX WEIGHTAGE		
	R_L	R_H	R(k)	R_L	R_H	R(k)	R_L	R_H	R(k)
F_{11}(k)	0.2164	0.0960	0.1155	0.2886	0.2493	0.2523	0.2529	0.2490	0.2881
F_{21}(k)	0.0584	0.2061	0.1960	0.1058	0.1260	0.1241	0.1239	0.1258	0.1060
Q(k)	1.5121	2.0817	2.0792	2.0004	1.9992	1.9951	1.9992	1.9953	1.9956
$\sum e^2(k)$	2.88×10^{-1}	4.47×10^{-2}	4.72×10^{-3}	2.19×10^{-3}	1.73×10^{-5}	7.14×10^{-7}	1.16×10^{-7}	2.63×10^{-7}	2.26×10^{-8}
$\sum O^2(k)$	2.47×10^{-1}	5.96×10^{-2}	4.95×10^{-2}	1.65×10^{-4}	3.25×10^{-3}	2.88×10^{-3}	2.81×10^{-3}	3.29×10^{-3}	1.79×10^{-4}

$\sum e^2(k)$ = Sum – square state error

$\sum O^2(k)$ = Sum – square parameter alignment error

Table (2)

Responses of the controllor parameters and responses of adaptive system errors with error feodback matrix (P^T = [0.54 0.29]) at itoration k = 1000

PARAMETERS & SQUARE ERRORS	IMPROVED CONVERGENCE			OPTIMUM CONVERGENCE WITH SCALAR WEIGHTAGE			OPTIMUM CONVERGENCE WITH MATIX WEIGHTAGE		
	R_L	R_H	R(k)	R_L	R_H	R(k)	R_L	R_H	R(k)
F_{11} (k)	0.1051	0.2861	0.2913	0.3265	0.2988	2.9480	0.3350	0.2989	0.3000
F_{21} (k)	0.0298	0.1071	0.1045	0.0860	0.1006	0.1025	0.0820	0.1006	0.1000
Q(k)	1.9909	1.9988	1.9992	2.0011	1.9999	1.9995	2.0016	1.9999	2.0000
$\sum e^2(k)$	4.39x10	6.28x10	2.51x10	9.52x10	7.23x10	6.63x10	7.25x10	7.51	1.65x10
$\sum O^2(k)$	4.30×10^{-2}	9.43×10^{-4}	9.70×10^{-5}	8.97×10^{-5}	3.34×10^{-5}	3.34×10^{-5}	1.61×10^{-6}	1.59×10^{-6}	$4,56.10^{-11}$

$\sum e^2(k)$ = Sum − square state error

$\sum O^2(k)$ = Sum − square parameter alignment error

7. CONCLUSION

A new model reference adaptive control (MRAC) scheme is proposed in this work and a set of parameter adaptation algorithms are developed. The algorithms introduce certain matrix gain sequences (R(k) D(k)) in the adaptive loops resulting in an increased rate of decay of parameter alignment error, O(k). The elements of R(k) are to be monotonically increasing functions of k with their maximum values less than unity, and can be selected independent of each other. The elements of D(k) are nonlinear functions of signals measured from the adaptive system. Another feature of the developed MRAC scheme is the possibility of controlling the dynamics of state error by the matrix P. Proper selection of the elements of P leads to additional improvement in the structure of one of the controllers is suggested to solve the problem of on-line matrix inversion encountered in the implementation of the scheme. Though the proposed scheme demands more on-line computations due to the introduction of P. D(k) and R(k), it promises an improved degree of stability in (θ, e) space.

REFERENCES

1. **Carroll, R.l., 1974.** : IEEE Trans. Autom. Control, 19, 246.

2. **Chan, Y.T., 1973.** : Proc. Joint Automatic Control Conference, Paper 10.5, 287.

3. **Graupe, D., 1976.** : Identification of Systems (New York; Rober E Krieger).

4. **Kudva, P., Narendra, K.S., 1974.** : IEEE Trans. Autom. Control, 19, 549.

5. **Mendel, J.M., 1974.** : IEEE Trans. autom. control, 19, 820.

6. **Narendra, K.S.,** and **Kudva, P., 1974.** : IEEE Trans. Syst, Man and Cyber, 4, 542 and 552.

7. **Riyadh Al-Salman, 1981.** : M.Sc. Thesis, College of Engg., Univ. Basrah.

8. **Sebakhy, O.A., 1976.** : Int. J. Control, 23, 799.

9. **Subbayyan, R., Nagarajan, R., 1978.** : IEEE Trans. autom. control, 23, 726.

10. **Udink Ten Kate, 1978.** : Int. J. Control, 28, 241.

11. **Riyadh Al-Salman, R. Nagarajan** and **Dakhil H. Jerew.** : Arabian Journal for Science and Engineering, April, 1985.

APPENDIX A

The following properties of element by element matrix product are use in simplifying equation (30).

(i) $\mathrm{Tr}\,[\,A\,(B \circledast C)\,] = \mathrm{Tr}\,[\,(A \circledast B^T)\,C\,]$ (A.1)

(ii) $A \circledast \hat{A} = E$ (A.2)

(iii) $(xy^T) \circledast (xy^T) = (x \circledast x)\ \ (y^T \circledast y^T)$ (A.3)

(iv) $\mathrm{Tr}\,[\,E\,(xx^TA) \circledast A)\,] = \mathrm{Tr}\,[\,(A^Tx)\ \ (x^TA)\,]$ (A.4)

(v) $\mathrm{Tr}\,[\,xx^T\,] = \mathrm{Tr}\,[\,(x^T \circledast x^T)\,]$ (A.5)

where the matrices A, B,C and E, and vectors x, y and v are appropriately dimensioned; all the elements of E and v are unity; $\hat{A}$, the element by element inverse of A, exists when A has no zero element.

APPENDIX B

The state error feedback matrix P is chosen such that $(A_m - B_mP^T)$ has its eigenvalues within the unit circle in the complex plane.

When

$$p(k) = \theta^T(k)\ z(k-1) = 0 \quad \text{(B.1)}$$

it can be shown, from equation (10), that [Sebakhy, 1976].

$$\underset{k \to \infty}{\mathrm{Lt}}\ e(k) = \underset{k \to \infty}{\mathrm{Lt}}\ [x_p(k) - x_m(k)\ ] = 0 \quad \text{(B.2)}$$

Let δ (k) and β (k) be the submatrices of θ be the submatrices of θ (k) such that

$$\theta^T(k) = [\ \delta^T(k)\ \vdots\ \beta^T(k)\ ] \quad \text{(B.3)}$$

Employing equations (3), (12) and (B.3) in equation (B.1)

$$\delta^T(k)\ x_p(k-1) + \beta^T(k)\ [\ Q(k)\ [\ u_m(k-1) + F^T(k)\ x_p(k-1) + P^T e(k-1)\ ]\ ] = 0 \tag{B.4}$$

From equation (B.2), equation (B.4) is written as

$$[\ \delta^T(k) + \beta^T(k)\ Q(k)\ F^T(k)]\ x_m(k-1) + [\ \beta^T(k)\ Q(k)]\ u_m(k-1) = 0 \tag{B.5}$$

When the pair (A_m, B_m) is completely controllable, and the input $u_m(k)$ is sufficiently general, equation (B.5) results in [Kudva and Narendra 1974]

$$\beta^T(k)\ Q(k) = 0 \tag{B.6}$$

and

$$\delta^T(k) + \beta^T(k)\ Q(k)\ F^T(k) = 0 \tag{B.7}$$

Since Q(k) is non-singular for all k, equations (B.6) and (B.7) reduce to

$$\theta(k) = 0 \tag{B.8}$$