

USE OF SPREADSHEETS FOR RAINFALL-RUNOFF CALCULATIONS

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ABSTRACT

The nature of rainfall-runoff calculations necessitates the use of different equations for calculating losses and several response functions to route the effective rainfall hyetograph through the catchment. The efficient calculation facilities of spreadsheets are used to develop several models for calculating effective rainfall and outflow hydrograph. The models use recently developed theories in rainfall runoff modelling. Several options are made available for the user to choose from, e.g. several infiltration equations and several response functions. The spreadsheet models are used to plot the rainfall hyetograph and the outflow hydrograph, and perform the iterative calculations encountered in some equations. The models are also suitable for conducting sensitivity analysis, calibration and verification. The spreadsheet model is used also to derive the unit hydrograph from measured data using different methods. Simple examples are used to demonstrate the application of the developed models.

NOMENCLATURE

β	:	negative deviation
η	:	porosity
θ	:	positive deviation
ψ	:	wetting front soil suction head
$\Delta\theta$:	change in moisture content
θ_e	:	effective porosity
θ_r	:	residual moisture content
A	:	watershed area
B	:	catchment breadth
c_1, c_2, k, M', k', K''	:	constants
CN	:	dimensionless curve number
f_0	:	initial infiltration rate

$F(t)$:	cumulative infiltration
f_c	:	final infiltration rate
$I(\tau)$:	input time function
I_a	:	initial abstraction
i_{eff}	:	effective rainfall intensity
K	:	decay time constant
K'	:	hydraulic conductivity
L	:	flow length
M	:	number of pulses
n'	:	Manning's roughness coefficient
P	:	precipitation
Q	:	discharge
r_d	:	direct runoff depth
R_m	:	rainfall depth in time interval m
S	:	potential storage
s	:	slope of overland flow
S'	:	sorptivity
s_e	:	effective saturation
t_b	:	time base
t_c	:	time of concentration
t_p	:	time to peak
U	:	unit hydrograph
$u(t)$:	unit response function
y_2	:	flow depth
y_d	:	surface depression storage

INTRODUCTION

Hydrologic phenomena are extremely complex, and may be represented in a simplified way by means of hydrological models. These models may be divided into two categories: physical models which represent the system on a reduced scale, and abstract models which represent the system in a mathematical form. In the latter category, the system operation is described by a set of equations linking the input and the output variables. These variables may be functions of space and time, and they may also be probabilistic or deterministic. Deterministic models can be lumped where the system is spatially averaged, or distributed where model variables are defined as functions of space dimensions.

Since the early 1960s, a host of deterministic hydrologic simulation models have been developed (1). These models include event simulation models for modeling a single rainfall-runoff event and continuous simulation models, which have soil moisture accounting procedures to simulate runoff from rainfall in hourly or daily intervals over long time periods. Examples of event simulation models include: the U.S. Army Corps of Engineers (1981) HEC-1 flood hydrograph model, The Soil Conservation Service (1965) TR-20 computer program for project hydrology; The U.S. environmental Protection Agency (1977) SWMM storm water management model; and Illinois State Water Survey ILLUDAS model. Examples of continuous simulation models include: the U.S. National Weather Service runoff forecast System (1985), the U.S. Army Corps of engineers (1976) STORM model and the U.S. Army Corps of Engineers (1972) SSARR streamflow synthesis and reservoir regulation model.

Spreadsheets have proven to be efficient engineering calculation tools. Several papers have been published to illustrate the use of spreadsheets in water resources. Some of these papers are reported in (2). Rainfall-runoff modelling can be performed efficiently using the different features of spreadsheets. The typical calculations are made in table form where the rainfall hyetograph and outflow hydrograph are functions of time. Some model parameters have to be calibrated. Trial and error or optimization techniques can be used to choose the most suitable model parameters to represent the actual situation (3,4). In case of scarce data, several models may be compared to choose a model capable of representing the particular rainfall-runoff process (5). The latter computational tasks can be carried out efficiently by spreadsheets.

The paper describes the different types of deterministic lumped rainfall-runoff models, and the different mathematical expressions for calculating the losses due to infiltration, and the methods for routing the rainfall through the catchment to obtain the outflow hydrograph. The principle of the unit hydrograph is also reviewed. The spreadsheet models are then demonstrated to show why they are considered attractive computational environment for simulating rainfall-runoff processes, and to illustrate their importance as a learning tool for engineering students and practicing engineers.

RAINFALL RUNOFF MODELS

Event simulation models may be grouped in two general classifications (6). One approach uses the concept of effective rainfall in which a loss model is assumed. The loss model calculates the infiltration losses. Losses other than infiltration like interception and depression storage can be estimated and added to the infiltration losses. The effective rainfall is then used as input to a routing model to produce the runoff hydrograph using a convolution algorithm which uses response functions of different shapes. An alternative approach which might be termed a surface water budget model, incorporates the loss mechanism into the catchment model. In this way, the incident rainfall hyetograph is used as input and the estimation of infiltration and other losses is made an integral part of the calculation of runoff.

The Losses Sub-model

The effective rainfall hyetograph may be determined from the rainfall hyetograph in one of two ways, depending on the availability of streamflow data.

The ϕ -index is used when such data is available, and the infiltration equations or the SCS method are used when the data are not available. The ϕ -index is a simple alternative for calculating effective rainfall, which is a constant rate of abstractions that will yield an excess rainfall hyetograph with a total depth equal the direct runoff over the watershed. The value of ϕ is determined from the equation:

$$r_d = \sum_{m=1}^M (R_m - \phi \Delta t)$$

where R_m is the rainfall depth in time interval m , r_d is the direct runoff depth. The latter equation is solved by trial and error (1).

There are different infiltration equations. Horton's equation assumes that infiltration begins at some rate f_o and exponentially decreases until it reaches a constant rate f_c :

$$f(t) = f_c + (f_o - f_c)e^{-Kt}$$

where K is the decay time constant of unit T^{-1} .

Philip's equation uses a parameter called sorptivity S' , which is a function of the soil suction potential, and K' the hydraulic conductivity. The equation for f reads:

$$f(t) = \frac{1}{2} S' t^{-1/2} + K'$$

Green-Ampt's equation has the form:

$$f(t) = K' \left(\frac{\psi \Delta \theta}{F(t)} + 1 \right)$$

where ψ is the wetting front soil suction head, $\Delta \theta$ is the change in moisture content.

The SCS method uses the following equation to calculate the cumulative effective rainfall:

$$P_e = \frac{(P - I_a)^2}{P - I_a + S}$$

where I_a is the initial abstraction and S is the potential storage calculated from the equation:

$$S = \frac{C_1}{CN} - C_2$$

where CN is a dimensionless curve number defined such that $0 < CN < 100$, c_1 and c_2 are constants depending on the system of units. More details about the infiltration equations can be found in (1).

The Routing Sub-model

The interaction between rainfall and runoff on a watershed is analyzed by viewing the watershed as a lumped linear system. The basic principles for linear system operations are the principle of proportionality and principle of superposition.

If a system receives an input of unit amount applied instantaneously at time τ , the response of the system at a later time t is described by the unit instantaneous response function $u(t-\tau)$, where $(t-\tau)$ is the time lag since the input is applied. The response to the complete input time function $I(\tau)$ can be found by integrating the instantaneous response function to its constituent input:

$$Q(t) = \int_0^t I(\tau)u(t-\tau)d\tau$$

On a discrete time domain, if the input function is a series of M pulses of constant rate, the discrete-time version of the convolution integral for a linear system is:

$$Q_n = \sum_{m=1}^{n \leq M} P_m U_{n-m+1}$$

where :

$$P_m = \int_{(m-1)\Delta t}^{m\Delta t} I(\tau)dt$$

$$Q_n = Q(n\Delta t)$$

$$U_{n-m+1} = \frac{1}{\Delta t} \int_{(n-m)\Delta t}^{(n-m+1)\Delta t} u(l)dl$$

It can be seen that the rainfall is represented by the depth of precipitation falling during the time interval, while the outflow is represented as the instantaneous value of flow rate at the end of the time interval.

The following are some unit response functions that are used in practice and reported in (6). The results of the spreadsheet model were compared with those of the MIDUSS model (6) to check the correctness of calculations.

Rectangular Response Function

The response function is assumed to be rectangular with a dynamically varying time base equal to the time of concentration t_c defined as the time of flow from the farthest point on the watershed to the outlet. It is computed using the kinematic wave equation as:

$$t_c = k' \left(\frac{Ln'}{\sqrt{s}} \right)^{0.6} i_{\text{eff}}^{-0.4}$$

where L is flow length, n' is the Manning's roughness coefficient, s is the slope of overland flow, i_{eff} is the effective rainfall intensity, and k' is a constant which depends on the system of units. The ordinate of the response function is given by $u_{\text{max}} = A/t_c$ so that the evaluation of a discretized form of the convolution integral is relatively straightforward.

Triangular Response Function

A very common and popular technique proposed by the Soil Conservation Service uses a triangular response function in which the time to peak t_p and the time base t_b are given by :

$$t_p = 0.6t_c + \frac{\Delta t}{2}$$

$$t_b = \left(\frac{8}{3} \right) t_p$$

The triangular option used here has a dynamic value of t_c which varies nonlinearly in much the same way as for the rectangular response function. For each time step the effective rainfall intensity is known and the triangular response function for the corresponding time of concentration t_c is discretized and multiplied by the effective rainfall.

Linear Reservoir Response Function

A more complex response function was suggested by Pederson (7) and is used frequently for urban catchments. The shape of the unit hydrograph is obtained as the response of a single linear reservoir to a rectangular pulse of rainfall of unit volume and duration Δt . The storage coefficient of the linear reservoir is taken to be 0.5 t_c , in which the maximum rainfall

intensity is used. The resulting unit hydrograph comprises a steeply rising limb over the time step Δt followed by an exponential decay.

The SWMM Model

This method employs the surface budget approach. The incident rainfall intensity is the input to the control volume on the surface of the plane; the output is a combination of the runoff Q and the infiltration f . Considering a unit breadth of the catchment, the continuity and dynamic equations which have to be solved are as follows:

$$iL = \left(fL + \frac{Q}{B} \right) + L \frac{\Delta y}{\Delta t}$$

$$Q = B \frac{M'}{n'} s^{0.5} (y_2 - y_d)^{5/3}$$

where B is the catchment breadth, M' is a constant depending on the system of units, and y_d is the surface depression storage. The two equations are implicit functions of y_2 and may be solved by the Newton-Raphson method.

THE UNIT HYDROGRAPH

The unit hydrograph is defined as a direct runoff hydrograph for a specific location resulting from a unit depth effective rainfall generated uniformly over the drainage area at a constant rate for an effective duration.

The discrete convolution equation allows the computation of direct runoff Q as a function of P and U . The reverse process, called deconvolution is needed to derive a unit hydrograph U given P and Q . Deconvolution may be used to derive the unit hydrograph from a complex multi-peaked hydrograph, but the possibility of errors or nonlinearity in the data is greater than for a single-peaked hydrograph. The deconvolution equations may be solved by different methods. The solution by linear regression produces the least squares error between the observed and estimated outflow.

Linear programming (LP) is also an alternative method of solving for the unit hydrograph, that minimizes the absolute value of error between observed and estimated flows and ensures that all entries of the unit

hydrograph are nonnegative. The linear program model to solve for the unit hydrograph can be stated as follows:

Minimize:

$$Z = \sum_{n=1}^N (\theta_n + \beta_n)$$

subject to the constraints:

$$\sum_{m=1}^n P_m U_{n-m+1} + [\theta_n] - [\beta_n] = [Q_n]$$

and

$$\sum_{m=1}^M U_m = K''$$

where θ_n and β_n are the positive and negative deviations respectively between the actual and simulated direct runoff, and K'' is a constant which converts the units of excess rainfall hyetograph into units of direct runoff hydrograph.

THE SPREADSHEET MODEL

Electronic spreadsheets consist of rows and columns, where the calculations are performed using cell references instead of variable names as in high-level computer languages like FORTRAN, PASCAL, etc. Another characteristic of the spreadsheet is that the mathematical expression (formula) and the numerical results (data) are located in the same cell. The advantages of using spreadsheets, which are easier to learn, include the visibility and efficiency of calculations, the ease of 'what-if' analysis, and the availability of instant graphics. Detailed explanation of spreadsheet applications in science and engineering can be found in (8).

The spreadsheet, or notebook designed to carry out the rainfall runoff calculations mentioned above is arranged in three pages, page A for losses sub-model where the effective rainfall is calculated, page B for routing sub-model. The SWMM model is included also in page B. Page C comprises

unit hydrograph calculations. The pages can be connected together to automatically incorporate the results of one page as input to another page. Graphs generated within the spreadsheet can be used to show the result of choosing a specific model and compare its output with other models and with measured data for the purpose of calibration or verification. The following tables give a general description of each page with sample calculations, where the contents of some important cells are shown to explain how the calculations are done.

In page A, the losses sub-model includes the ϕ -index method, the SCS method, the Horton's, Philip's and Green-Ampt's equations. The input data is the rainfall hyetograph, the infiltration equation parameters and the output is the effective rainfall hyetograph. A hypothetical storm is used to illustrate the application of the Horton's equation. The calculation takes care also of the depression storage as a first demand on the effective rainfall where the depression must be filled before the runoff can start. Tables (A-1) through (A-3) show the different losses sub-models. The cell contents show the mathematical formulas using cell references. The \$ sign is used for mixed referencing of a cell to fix its row, column or both when copying or repeating the formula. Table (A-1) shows the calculations to determine the effective rainfall given the streamflow data, and demonstrates the use of circular calculation, which is an important feature of spreadsheets that allows an automatic calculation of ϕ and alleviate the engineer from the cumbersome operation of trial and error reported in (1). Circular calculation is utilized also in Table (A-3) to find the infiltration losses using the Green-Ampt's equation, where the cumulative infiltration $F(t)$ is calculated from the implicit equations,

$$F(t) = K't + \psi\Delta\theta \ln\left(1 + \frac{F(t)}{\psi\Delta\theta}\right)$$

$$\Delta\theta = (1 - s_e)\theta_e$$

$$\theta_e = \eta - \theta_r$$

where s_e is the effective saturation, θ_e is the effective porosity, η is porosity, and θ_r is the residual moisture content. The input parameters will be K', t, ψ and $\Delta\theta$, and the output is $F(t)$.

Table (A-1). The Losses Sub-model

A	T	U	V	W	X	Y	Z
2	Phi-index Method						
3							
4	time	i	outflow	ieff			
5	hr	in	cfs	in			
6	0.00						
7	0.50	0.15		0.000			
8	1.00	0.26		0.000			
9	1.50	1.33	428	1.060			
10	2.00	2.20	1923	1.930	area	7.03	mi2
11	2.50	2.08	5297	1.810	Vol	7.84E+07	ft3
12	3.00	0.20	9131	0.000	rd	4.800	in
13	3.50	0.09	10625	0.000	φ	0.53994	in/hr
14	4.00		7834				
15	4.50		3921				
16	5.00		1846				
17	5.50		1402				
18	6.00		830				
19	6.50		313				
20							
21	Cell Y12	+Y11/(Y10*5280^2)*12					
22	Cell W10	@IF(U10<=\$Y\$13,0,U10-\$Y\$13*0.5)					
23							
24							
25							

Table (A-2). The Losses Sub-model (continued)

A	A	B	C	D	E	F
1	Horton's Equation					
2						
3	fo	6 mm/hr				
4	fc	2 mm/hr				
5	K	0.25 hr-1				
6	ds	0 mm				
7	delt	5 min				
8						
9	Time	i	f	ieff		
10	min	mm/hr	mm/hr	mm/hr		
11	5	4	4.87	0.00		
12	10	8	4.05	3.54		
13	15	6	3.47	2.24		
14	20	2	3.05	0.00		
15						
16	Cell C12	+ $B\$4 + (B\$3 - B\$4) * @EXP(-A12 / B\$5 / 60)$				
17						
18						
19						
20	SCS Method					
21						
22	CN	99.220				
23	S	1.997 mm				
24	la	0.200 mm				
25						
26	Time	i	icum	ieffcum		ieff
27	min	mm/hr	mm/hr	mm/hr		mm/hr
28	5	4	4	2.49		2.49
29	10	8	12	10.09		7.60
30	15	6	18	16.00		5.91
31	20	2	20	17.99		1.98
32						
33	Cell C30	$(C30 - B\$23)^2 / (C30 + B\$22 - B\$23)$				
34						
35						
36						
37						

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Table (A-3). The Losses Sub-model (continued)

A	J	K	L	M	N	O	P	
1	Philip's Equation							
2								
3	S'	0.05 cm/hr(-.5)						
4	K'	0.5 cm/hr						
5	delt	5 min						
6								
7	Time	i	F	f	ieff			
8	min	mm/hr	mm	mm/hr	mm/hr			
9	5	4	0.56	5.87	0.00			
10	10	8	1.04	5.61	2.26			
11	15	6	1.50	5.50	0.44			
12	20	2	1.96	5.43	0.00			
13								
14	Cell L12	(\$K\$3*(J12/60)^(0.5)+\$K\$4*J12/60)*10						
15								
16								
17								
18	Green-Ampt's Equation							
19								
20	θ	0.434		se	0.3			
21	ψ	8.89 cm		Δθ	0.3038			
22	K'	0.34 cm/s		delt	5 min			
23								
24	Time	i	F	f	ieff	F	Function	
25	min	mm/hr	mm	mm/hr	mm/hr	cm		
26	5	4	4.10	6.77	0.00	0.41	0.41	
27	10	8	5.92	5.94	2.06	0.59	0.59	
28	15	6	7.35	5.58	0.42	0.74	0.74	
29	20	2	8.60	5.36	0.00	0.86	0.86	
30								
31	Cell P31	+\$K\$25*J30/60+\$K\$24*\$N\$24*@LN(1+O30/(\$K\$24*\$N\$24))						
32	Cell M31	+\$K\$25*(\$K\$24*\$N\$24/L30+1)*60/\$N\$25						
33								
34								
35								
36								

In page B, the routing sub-model includes the rectangular, triangular, and single linear reservoir response functions in addition to the SWMM model. The same hyetograph used in the page A will be used here also for illustration. Table (B-1) shows the rectangular and triangular response functions, where the response from each rainfall intensity is calculated in a separate column and the outflow is calculated by summing the columns row by row. These columns are shown for the rectangular response only and hidden for the rest of the responses. The cell formulas for the individual response and the total outflow are shown at the bottom of each section. Table (B-2) shows the single linear reservoir response and the SWMM model. Circular calculation is used here also to solve for y_2 instead of using the Newton Raphson method as reported in (6). Figure (1) shows the rainfall hyetograph and the different outflow hydrographs calculated using the different response functions. The considerable differences between the various responses show the importance of choosing a suitable model and the need for model calibration and verification (3,4,5).

Page C is devoted to unit hydrograph calculations using the methods mentioned above to derive the unit hydrograph, i.e. matrix solution, linear regression, and linear programming. Table (C-1) shows an example of using linear regression to derive the 1/2-hr unit hydrograph for the given rainfall hyetograph (inches) and outflow hydrograph (cfs). The table shows the result obtained using the linear regression facility in the spreadsheet. The constraint matrix for the LP model, shown in Table (C-2), comprises the rainfall matrix (11x9) plus a positive and a negative unit matrices which are the coefficients of the θ and β vectors representing the positive and negative deviations respectively, in addition to the constraint ensuring that the unit hydrograph represents one unit of direct runoff. The size of the LP constraint matrix is therefore $(11+1=12) \times (9+2*11=31)$. The constant K'' is 9073.4. Both methods produce the same results.

CONCLUSIONS

The paper reviews a wide range of rainfall-runoff models and uses spreadsheets as the computational tool to carry out the calculations. The efficiency and visibility of spreadsheet calculations make it attractive to both engineering students and practicing engineers. The instant visual graphical presentation of results allows the comparison of several combinations of losses and routing sub-models to choose the most suitable model for a specific location.

Use of Spreadsheets For Rainfall-Runoff Calculations

Table (B-1). The Routing Sub-model

B	A	B	C	D	E	F	G	H
3	Rectangular Response							
4								
5	L	2000 m		slope	0.014			
6	n	0.013		constant	6.989			
7	area	600 hec		delt	5 min			
8								
9	Time	ieff	tc	outflow	out1	out2	out3	out4
10	min	mm/hr	min	m3/s				
11	5	4	102.03	0.327	0.327			
12	10	8	77.33	1.189	0.327	0.862		
13	15	6	86.76	1.765	0.327	0.862	0.576	
14	20	2	134.63	1.889	0.327	0.862	0.576	0.124
15	25			1.889	0.327	0.862	0.576	0.124
16	30			1.889	0.327	0.862	0.576	0.124
17	35			1.889	0.327	0.862	0.576	0.124
18								
19	Cell E12 @IF(A12<\$C\$11,\$B\$7/\$B:\$C\$11*\$B\$11*\$E\$7/360,0)							
20	Cell D12 @SUM(E12..H12)							
21								
22	Triangular Response							
23								
24	L	2000 m		slope	0.014			
25	n	0.013		constant	6.989			
26	area	600 hec		delt	5 min			
27								
28	Time	ieff	tc	tp	tb	umax	const	outflow
29	min	mm/hr	min	min	min	m2/min		m3/s
30	5	4	102.03	63.72	169.92	70622	5.8E-05	0.031
31	10	8	77.33	48.90	130.39	92033	2.6E-04	0.166
32	15	6	86.76	54.55	145.48	82488	1.4E-04	0.365
33	20	2	134.63	83.28	222.08	54035	1.3E-05	0.572
34	25							0.779
35	30							0.987
36								
37	Cell I12 @IF(A30<E\$30,@IF(A30<D\$30,G\$30*(E\$30-D\$30)*A30,							
38	G\$30*D\$30*(E\$30-A30)),0)							
39	Cell H12 @SUM(I30..L30)							
40								
41								
42								

Table (B-2). The Routing Sub-model (continued)

B	J	K	L	M	N	O	P	
3	Single Linear Reservoir							
4								
5	L	2000 m		slope	0.014			
6	n	0.013		constant	6.989			
7	area	600 hec		delt	5 min			
8								
9	Time	ieff	tc	outflow				
10	min	mm/hr	min	m3/s				
11	5	4	102.03	0.000				
12	10	8	77.33	1.617				
13	15	6	86.76	2.634				
14	20	2	134.63	2.719				
15	25			2.389				
16	30			2.099				
17								
18	Cell N11	1-@EXP(-\$N\$7/(0.5*@MIN(\$B:\$L\$11..\$L\$14))))*\$K\$7*\$K11/36						
19	Cell M11	@SUM(N11..Q11)						
20								
21								
22								
23	The SWMM Model							
24								
25	L	2000 m		slope	0.014			
26	n	0.013		constant	6.989			
27	area	600 hec		delt	5 min			
28	fc	0 mm/hr		K	0.25 hr-1			
29	fo	0 mm/hr						
30								
31	Time	ieff	outflow	y2	Function			
32	min	mm/hr	m3/s	mm				
33	5	4	0.056	0.387	0.387			
34	10	8	0.291	1.039	1.039			
35	15	6	0.544	1.511	1.511			
36	25	2	0.627	1.647	1.647			
37	30		0.608	1.616	1.616			
38	35		0.590	1.587	1.587			
39								
40	Cell N33	+ \$N\$27/60*(+K33+@IF(K33<\$K\$28,0,-\$K\$29-\$N\$28/\$N\$27*60						
41		*(1-@EXP(-\$N\$27/\$N\$28/60)))-1/\$K\$26*\$N\$25^(0.5)						
42		*(M33/1000)^(5/3)/\$K\$25*1000*3600)+L33						
43								
44								
45								

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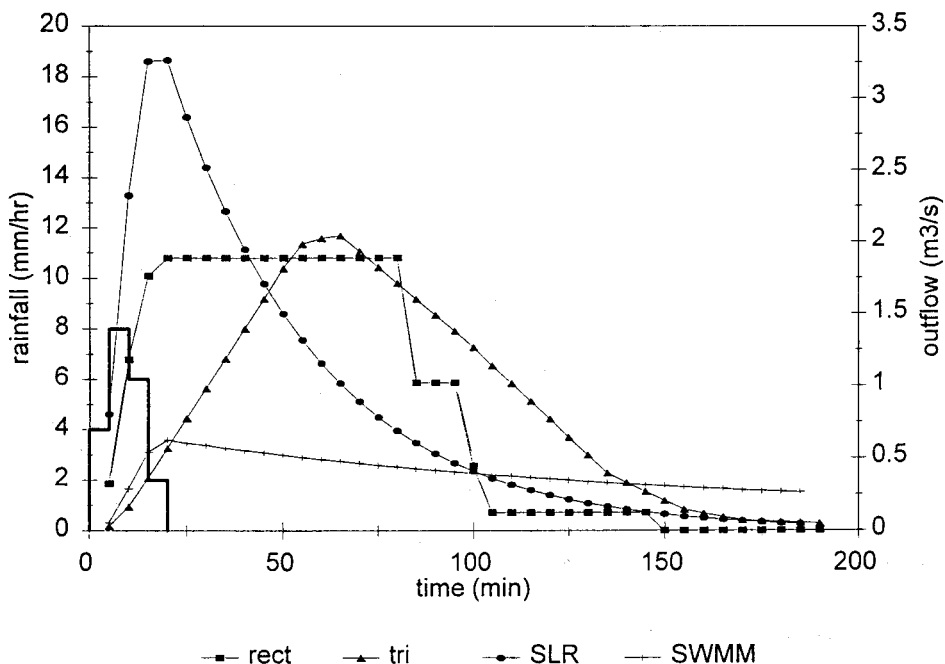


Fig. (1). Comparison among outflow hydrographs

Table (C-1). Derivation of Unit Hydrograph Using Linear Regression

rainfall matrix										RHS	Xcoeff
2	0	0	0	0	0	0	0	0	0	808	404
3	2	0	0	0	0	0	0	0	0	3370	1079
1	3	2	0	0	0	0	0	0	0	8327	2343
0	1	3	2	0	0	0	0	0	0	13120	2506
0	0	1	3	2	0	0	0	0	0	12781	1460
0	0	0	1	3	2	0	0	0	0	7792	453
0	0	0	0	1	3	2	0	0	0	3581	381
0	0	0	0	0	1	3	2	0	0	2144	274
0	0	0	0	0	0	1	3	2	0	1549	173
0	0	0	0	0	0	0	1	3	0	793	
0	0	0	0	0	0	0	0	1	1	173	

Regression Output:

Constant	0
Std Err of Y Est	0
R Squared	1
No. of Observations	11
Degrees of Freedom	1

Table (C-2). Constraint Matrix for the LP Model

constraint matrix coefficients										RHS						
unit hydrograph					positive deviation					negative deviation						
2					1											808
3	2					1										3370
1	3	2					1									8327
	1	3	2					1								13120
		1	3	2					1							12781
			1	3	2					1						7792
				1	3	2					1					3581
					1	3	2					1				2144
						1	3	2					1			1549
							1	3						1		793
								1							1	173
																9073.4

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