CO-CONSTRUCTING DECIMAL NUMBER KNOWLEDGE

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Abstract

The purpose of this research was to help a class of Year 6 students to build rich connected knowledge of decimal numbers. Most students in this class displayed considerably weak understanding of decimal concepts. Therefore, the students were unable to adaptively use or generate accurate strategies to meet task demands involving decimal numbers. This difficulty is well documented to be a worldwide problem that continues to persist despite decades of extensive research and reform recommendations.

Research literature shows that the decimal domain is one of the most difficult in mathematics to learn and teach. This research opens new ways of thinking for educators about why students resist attempts to promote understanding. It does this by drawing upon two main fields of research. The first is through Students Participating As Research Co-constructors (SPARC) to unveil the reality of the students for whom the learning experience is intended. SPARC was developed to create a community of learners to support each other to successfully break the failure cycle. The second is through taking a multidimensional approach to reflect on the complexities of knowledge acquisition. This was achieved by focusing not only on the cognitive aspects of processes that underlie decimal numbers, but also on the interplay between cognition, affect and conation.

In brief, as literature shows, the inherent difficulty of decimals, together with factors such as dissonance between prior and new knowledge, influences the ease with which students cognitively process decimal numbers. The affective responses that emerge from the experience of difficulty causes a cascade of shifting emotions from avoidance and/or embarrassment to genuine interest. Hence, conation was found to be important for achieving desired learning outcomes.

The research design combines action research, case study, and aspects of teaching experiment to address the complex and multifaceted nature of knowledge acquisition. A series of metacognitive teaching strategies were used to co-construct/ deconstruct mathematical knowledge and pedagogical practices. This included whole-class, peer and independent knowledge building. Error listings were used to make students aware of their own errors in their declarative knowledge. Co-analysis

of errors with peers prompted students to seek metacognitive knowledge about how cognition works from different perspectives to improve understanding and practice. Peer teaching and problem posing were used to co-explore procedural knowledge of successful methods (heuristics and strategies) for achieving specific learning goals. Community of inquiry was used to question perceptions about conditional knowledge in relation to the external factors that influenced the learning and teaching strategies.

The research clearly showed that students had improved cognitively, affectively and conatively by participating as co-constructors (as evident in pre/post-tests and observations). This research identified that in general, students fear failure. Students' emotional responses often dominate over cognition in predicting conative attitude and resistive action when experiencing mathematical difficulty. To deflate fears, an effective approach used by students during peer teaching was to use success before understanding. Students claimed that by structuring their task approach to first build confidence and a sense of efficacy through the experience of success, they were able to motivate their peers to develop an interest in wanting to understand.

This research offers significant insights for educational practice and advances the field in research on decimal knowledge through its focus on students as co-researchers and co-constructors of knowledge and the interaction between cognition, affect and conation. It contributes to an understanding of how students process decimal knowledge beyond cognition and develop a sense of self as teachers, learners and researchers of mathematics.

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List of Abbreviations

ACARA	Australian Curriculum, Assessment and Reporting Authority
LAB	Linear Arithmetic Blocks
MAB	Multi-base Arithmetic Blocks
NAPLAN	National Assessment Program Literacy and Numeracy
NCTM	National Council of Teachers of Mathematics
PISA	Programme for International Student Assessment
SPARC	Students participating as research co-constructors
TIMSS	Third International Mathematics and Science Study
ZPD	Zone of proximal development

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: ____________________

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Chapter 1: Introduction

The main purpose of this thesis is to improve knowledge and pedagogical practices to address students' difficulties with acquisition of decimal number knowledge. Concerns about students' decimal number knowledge have been well documented in worldwide research literature that dates back several decades (Hiebert & Wearne, 1985; Reichert, 1940).

Recent literature shows that many of the mathematical difficulties have persisted despite extensive research and reform recommendations (Roche, 2010; Shaughnessy, 2011; Steinle & Stacey, 2004; Ubuz & Yayan, 2010). Much of this research knowledge has been inferred from cognitive analysis of students' responses by researchers. To develop further insights into students' difficulties with decimals, this research follows two main lines of thinking.

The first is that this research is not only based on the cognitive element (knowing/processing) of the decimal knowledge acquisition process. Rather, it also takes into consideration the influence of affect (feelings/emotions) and conation (coordinating/translating cognition and affect into action). Cognition, affect and conation are viewed as being inextricably intertwined with acquisition of knowledge (Demetriou & Wilson, 2009). Cognition refers to the mental processes involving thinking, learning, and memory in the process of gaining knowledge. Affect refers to emotional responses such as feelings and motivation. Conation is concerned with taking action by focusing and maintaining persistent effort in order to achieve maximal production in performance of a task (Forgas, 2008).

The second is that theory can be developed with students as co-researchers. Therefore students are not simply subjects of research, but can be active participants in co-exploring two main things simultaneously: (a) students' knowledge and application of decimal concepts; and (b) effective teaching and learning practices to facilitate decimal knowledge acquisition.

In doing so, this research provides theories for learning and teaching of decimal knowledge and for developing mathematics pedagogy in general. It also expounds understanding from the perspective of the missing voice of students (Cook-Sather, 2006).

This chapter outlines the background (section 1.1) and definitions of common terms (section 1.2). The chapter continues with an outline of the overarching and process objectives of the research (section 1.3), followed by the context (section 1.4) and significance of this research (section 1.5). Finally, it includes an outline of the remaining chapters of the thesis (section 1.6).

1.1 BACKGROUND

In this section, I outline my reasons for undertaking this thesis and my beliefs about learning, teaching and researching that explain my approach to this research. The reason for selecting decimals as the focus of the research is also discussed.

1.1.1 Personal interest in research

My desire to conduct this research stemmed from a long-standing interest in mathematics and alternative approaches to pedagogy. The need for alternative pedagogical processes is implicit in my own cultural subjectivities as a learner from a non-mainstream islander cultural upbringing.

I started this research soon after completing my education course. However, after my appointment as a teacher, students changed my approach, my objectives and my feelings about what is important in research. This change was driven by a deep empathetic concern for the way in which mathematics education makes many of my students feel about themselves as people.

With experience in working with students, it became evident that learning is a personal and internal process. Feelings of achievement and failure that are felt by students during the learning process were highlighted. My classroom observations provided insight into how students deal with these feelings of failure and success.

I found many students who are comparatively slower in processing mathematical concepts than others, or experiencing confusion, are often found to adopt countervailing defence mechanisms. Given that most students attributed smartness to those who are good at maths, defence mechanisms served to maintain stability in themselves and relationships with peers.

Indeed, covering up is a well-practised skill in classroom environments, designed to prevent embarrassment. Hence, the prospect of sharing this inner world or taking risks in learning mathematics publicly is understandably unattractive or even a terrifying proposition to some.

These observations of students' reality in a classroom raise important questions related to what it means to research about someone. It offers a challenge to ethically elicit students to disclose their private world without having the need to mislead and to maintain social grace, thus collecting research data that is a true representation of what the subject thinks and knows and not simply what is perceived by the subject to be the right thing to say to avoid humiliation.

1.1.2 Why decimals?

Research posits the decimal domain as one of the most complex and problematic areas of mathematics for students (Irwin, 2001; Moss & Case, 1999; Okazaki & Koyama, 2005; Peled & Shahbari, 2009).

There is consensus amongst researchers that cognitive conflict resulting from prior knowledge of rote rules and computational procedures for whole numbers has led to a number of misconceptions about decimal numbers (Steinle & Stacey, 2004). Since decimal number sense is more complex than whole number sense, researchers (Goldstone, Landy, & Son, 2010; Hiebert & Wearne, 1986) argue that students need to restructure their schema to acquire decimal number knowledge of the core rules and principles as well as make interrelations in the decimal domain. This conundrum created by the complexity inherent in decimal number conceptions opened a rich arena for me to research, test and develop new ways of acquiring decimal number knowledge with my students.

In my teaching experience, I found that when given the confidence and opportunity, students with misconceptions or non-normative thinking are able to articulate an argument from a point of view which is well thought out and supported by their cultural and/or prior knowledge. Given that difficulties in the decimal domain have been linked directly to students' prior knowledge (Steinle & Stacey, 2004), without understanding a student's mathematical standpoint, it is difficult to ascertain why a student is resisting cognitive change even after substantial amounts of training or instruction.

To this end, this research argues that pedagogical research must explain not only how students' thinking changes with learning, but also the factors that cause students to resist change. As such, data produced to inform research need to wrestle with one another to reveal how tensions implicate changes for mathematical learning to occur.

1.2 DEFINITIONS

1.2.1 Defining mathematical terms

Decimal number

The term *decimal number* in this research uses the definition proposed by Steinle and Stacey (2004) to refer to base-10 numbers that are written with a decimal point, whether or not the number is greater or less than one. The decimal number describes the notation in which it is written rather than the abstract number itself. For example, expressions such as 23.0, 0.5, 0.23, and 1.23 are referred to as decimal numbers as opposed to 23, 277, and 0, which are referred to as whole numbers.

The decimal number is written as a row of digits, with each position in the row corresponding to a certain power of 10. In other words, the position of each digit conveys the multiplier (a power of ten) to be used with that digit—each position has a value ten times that of the position to its right. More detailed description of decimals is provided in section 2.2.

Decimal conception

Conceptual knowledge (Rittle-Johnson & Koedinger, 2005; Schneider & Stern, 2010) refers to students' knowledge of core mathematical rules and principles that underlie the structure of decimals (described in section 2.2.2). It involves understanding of the interrelations among various forms of linguistic, symbolic, concrete, pictorial, and real-world representations of part/part and part/whole quantities.

Conceptual knowledge about decimals includes an understanding of the role of the decimal point symbol, the place value of individual digits involved, and the arrangement of digits in determining the value of the number (Resnick et al., 1989).

1.2.2 Defining SPARC

SPARC refers to Students Participating As Research Co-constructors. SPARC is a conception that emerged during the implementation of the research. It took shape from a recognised need and the potential of drawing together different student and teacher expertise and perspectives (Willis, 2011) in addressing the objectives of this research.

Given that the overarching purpose for doing this research was to both gain new knowledge and improve practice, this research took the view that the best way to do both these things was for the students to be complicit in research and teaching as well as learning.

Within this framework, SPARC acknowledges that, where people are concerned, unveiling the cognitive processing that underlies a person's mathematical thinking is riddled with several social details which can be easily skewed to project the perception that is desired at the time by the researched unless the researched is complicit with the objectives of research.

Therefore, the effectiveness of SPARC relies on its ability to improvise and adapt responsively to alter the mindset of students from institutionalised subjects of education to interested co-partners in learning to learn (Claxton, 2006). To do this, I found that SPARC had to undo what the past five years of school had done in constructing what it means to be a good student.

The term *teacher* in relation to SPARC refers to the role of teacher-researcher as being what McWilliam (2009) describes as the "meddler in the middle instead of a guide on the side" (p. 264).

Description of the terms *teacher as researcher* and *student co-researchers* vary in literature depending on the nature and purpose of the research. Teacher as researcher in some action research literature has a central practical purpose of improving own practice (Trent, 2003). Others advocate collaborative approaches in which teachers research with other colleagues or students as co-researchers (Bland & Atweh, 2007; Fielding & McGregor, 2005; Kemmis & McTaggart, 2000) to bring about social and educational change. These descriptions can be equally applied to teachers who are improving their teaching and to students who are improving their learning.

The term *student as co-researcher* in relation to SPARC refers to students actively participating in congenial learning cultures to develop autonomy in assessment of their own learning (Willis, 2011), and to co-construct and co-produce mathematical knowledge as co-researchers (Bland & Atweh, 2007; Brousseau, 2008; Claxton, 2006; Fielding & McGregor, 2005; Vygotsky, 1978).

As a process in which students gather data on themselves, SPARC extends students' roles within the research from subject/learner to peer teacher and data gatherer. SPARC is characterised by the interchangeability and plurality of roles the students and teacher assume to give richness to participation and engagement in the learning, teaching and research process.

Bland and Atweh (2007) explain that in students as co-researchers approaches, student participation can "vary from mere data to full participants in the planning, conducting and writing about the research activity" (p. 6). The level of student participation is generally based on the research objective/s and the researcher's beliefs about who should be central to the research (Fielding & McGregor, 2005).

This move towards reflexive critique offers a robust alternative to traditional views of research (Trent, 2003), particularly to those who wish to see theory play a more inclusive role for the researched. However, as cautioned by Gergen (2006), such critique also runs the risk of reducing "complex problems to single dimensions, such that the voices of those not represented by the binary in question are silenced" (p. 303).

Instead of placing emphasis on whose voice is more important (students' or teacher's), this research views importance as a dynamic process where importance constantly shifts from teacher to students, to both, according to the situation/s. SPARC centralises the position of both the teacher-researcher and student co-researchers in joint pursuit of gaining new knowledge and improving their own learning and teaching practice to facilitate change. That is, research that is purposeful, ongoing, reflective, and seeks a deeper level of understanding. Within this framework, the student and teacher are seen as cognitive participants rather than subjects of/to research. In the SPARC process, both teacher and students are considered as learners, teachers, researchers and agents of change.

1.2.3 Defining metacognition and proficiency

The term *metacognition* in this research refers to two main focus components: metacognitive knowledge and regulation (Brown, 1987; Flavell, 1976).

The term *metacognition* in this research refers to two main focus components: metacognitive knowledge and regulation (Brown, 1987; Flavell, 1976). Within this construct, of metacognition knowledge refers to students' awareness of what they know about themselves and others as cognitive processors. Regulation refers to the activities used to regulate and oversee learning. Processes such as planning, monitoring of the progress of processing, effort allocation and strategy use are linked to metacognition (Brown, 1987; Schreiber, 2005). An important difference to the use of term metacognition in this research is that it emphasises the affect and emotional response in its description (section 2.3.6; section 7.2.3).

Proficiency is defined initially according to students' results in the diagnostic pre-test performance (entry knowledge). However, the term proficiency during the teaching experiment defines itself on the basis of students' demonstration of stronger evidence related to two capabilities: (a) decimal-knowledge-related proficiency; and (b) practice-related proficiency.

Decimal-knowledge proficiency is understood in this research as possessing a body of mathematical decimal knowledge within the adaptive framework of creative and analytical ability to extract, analyse and apply that knowledge (Robbins, 2009). This includes students' demonstration of declarative, procedural, metacognitive, structural, and strategic knowledge with regard to decimal numbers (Baturo, 1998).

- *Declarative* refers to factual decimal domain knowledge.
- *Procedural* is the knowledge required for the execution of mathematical steps.
- *Metacognitive* is self-regulatory knowledge and metacognitive skills when applying mathematical knowledge (Flavell, 1976).
- *Structural knowledge* refers to the organisation and interconnections among declarative facts within the learner's knowledge network (Baturo, 1998).

- *Strategic knowledge* refers to adaptive selection of strategies which supports shifting strategies as a consequence of changes in task demands of the situational parameters involving decimal numbers.

It should be noted that the same terms (declarative and procedural knowledge) are used by Schreiber (2005) for metacognitive sub-processes (section 2.3.6), however the differences in meanings can be identified by their contextualised use. *Practice proficiency* is understood in terms of the knowledge students have gained of themselves as co-learners and teachers, of task requirements, and of specific strategy use to deliberately select, control and monitor strategies needed to achieve desired learning goals.

As such, proficiency levels are determined by students' capacity to notice when they are not learning and their ability to seek strategic solutions when faced with difficulties. That is, their awareness of the knowledge and skills they do or do not possess and use of appropriate strategies to actively implement or acquire them. They are thus self-directed and goal-oriented (Bruner, 1990; Flavell, 1976; Robbins, 2009).

1.3 PURPOSES

This section outlines the purpose of this research in terms of: (a) the overarching objectives of the research; and (b) the process objectives to achieve the overarching objectives. An explanation of how these objectives are drawn from the literature review is further discussed in Chapter 2.

1.3.1 Overarching objectives

The overarching objectives of this research are to develop theories that:

1. Explain Middle Years students' cognitive processes underlying conceptions of decimal numbers.
2. Explain the effect and the effectiveness of metacognitive-based teaching and learning approaches in which the teacher as researcher and students as co-researchers work collaboratively to co-produce knowledge (i.e., SPARC).

1.3.2 Process objectives

To achieve these objectives, the process objectives are to:

3. Determine students' entry knowledge of decimal concepts using a diagnostic pre-test and interviews.
4. Compare levels of proficiency among students (see section 1.2.3 for definition of proficiency) in decimal-related knowledge and understanding.
5. Co-explore the effectiveness of a teaching experiment to improve students' decimal knowledge using metacognitive teaching strategies.
6. Co-construct theories for the overarching objectives that explain behaviours and practices observed during the teaching experiment to obtain desired outcomes.
7. Draw implications for learning, teaching and researching students' cognitive abilities.

1.4 CONTEXT

This research is situated in the context of a pedagogically difficult mathematical domain of decimal numbers (Desmet, Mussolinb, & Gregoireb, 2010; Stacey et al., 2001). It supports the current curriculum objectives of teaching students decimal knowledge for deep mathematical understanding (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2012; National Council of Teachers of Mathematics [NCTM], 2000; Queensland Studies Authority, 2012).

1.4.1 Context of curriculum paradox

Learning and teaching of decimal concepts and skills places this research within the context of a curriculum paradox where there is an apparent contradiction in perception between educators and students about what is student centred, interesting and engaging. This is exemplified in recent media articles that make claims such as, "one in two school children find science and maths too difficult or too boring to study despite seven in 10 wanting science-based careers" (Bowater, 2012, para.1)

Informed by research, the Queensland mathematics curriculum (ACARA, 2012; Queensland Studies Authority, 2012) provides a very detailed scope and sequence of number concepts leading to acquisition of decimal knowledge. What should be taught, when and what is important for students to have opportunities to learn are clearly defined. It describes the knowledge that students need in terms of concepts and facts and procedures. Holistic approaches that include the use of concrete materials, verbal, written, visual and real-world connections to decimal knowledge are suggested.

In line with the curriculum, teachers in Queensland classrooms use activities with manipulatives, games and a myriad of approaches to make decimals meaningful to students (ACARA, 2012; NCTM, 2000). Such reform initiatives suggest that the wealth of rich mathematical activities availed to students and professional development given to teachers to support creativity in mathematical pedagogy will actively engage students. In contrast, students maintain the view that not only decimals, but mathematics in general is difficult and boring. Claims are made that maths education is in crisis and that the number of students interested in studying maths is dropping (Simos, 2010; Williams, 2010).

Schoonmaker (2007) explains that, partly, such problems in relation to the implementation of the mathematics curriculum have resulted due to the highly political agenda in the current high-stakes, testing climate. This suggests that to a large extent it is the implementation rather than the content that is of concern.

Schools (such as mine) have employed curriculum advisors to oversee the adherence to the curriculum. Detailed weekly scripted lesson plans are prepared prior to the start of the term to ensure consistency across the school for the purpose of accountability. In this research, assumptions made in teacher plans are explored and adapted to the needs of the students by consulting students to obtain their opinion of the learning experiences.

1.4.2 Context of impoverished teacher knowledge

This research is situated in a context where difficulties in knowledge and understanding of decimal numbers are not just limited to students but, also, many teachers (Stacey et al., 2001).

Research literature (e.g., Ubuz & Yayan, 2010) reveals concerning weaknesses in some teachers' mathematical construct of decimal numbers, as well as difficulty in explaining decimal numbers to students and why algorithms work. Becker and Lin's

(2005) research shows that some teachers cannot carry out fractional computation procedures correctly, even when they have correct answers. The fact that some mathematics teachers themselves have limited understanding of decimals might explain the practice of relying on the memorised procedures.

Researchers (e.g., Moody 2011; Widjaja, Stacey, & Steinle, 2011) also point out that there is a paucity of literature in relation to overcoming teachers' difficulties with decimals. The fact that teachers would most likely share their misconceptions with their students underscores the need to improve teachers' understanding of decimals. This provides an impetus for research into designing and exploring teaching strategies to improve teachers' pedagogical content knowledge of decimals and practices in teaching.

Informal discussions with my teacher colleagues suggest that, despite an increase in professional development opportunities offered to teachers, they are not always useful because either they are not always timely or they do not address the localised individual needs of the teachers or their students.

1.4.3 Context of students in the shadows

This research contextualises research in collaborative practices (Ritchie & Rigano, 2007) that brings students out of the shadows of the teacher as the only expert, to include the differences in students' expertise and perspectives as co-researchers.

Limited empirical work can be found where students were involved in the true sense of participatory action research in relation to decimal knowledge acquisition. Research studies that engaged students in collaborative or self-analysis of causes for misconceptions in relation to the mathematics of decimals were difficult to find. This research draws on the assumption that, without this voice of the students, any decimal pedagogical knowledge remains only in the shadows of mathematics learning.

While other learning areas such as literacy education offer innovative learning and teaching strategies such as Philosophical Community of Inquiry (Knight & Collins, 2010) to elicit critical metacognition through philosophy-based forums, classroom mathematics often stops at strategies and skills to find the correct answer. My classroom observation indicates that this ends the inquiry for some students with different perspectives. They opt to safe places of following procedures and preferring to work from the pages of textbooks rather than deeply engaging in critical thinking and reasoning (Engelbrecht, Bergsten, & Kagesten, 2009).

This research takes the view that mathematics is a dynamic multi-layered domain where many aspects of mathematical understanding require an interactive platform that changes the classroom process from individual into a collective search for analysis and/or solutions to specific mathematical problems or concepts.

As Vygotsky (1978) reminds us, mathematics reasoning calls for negotiations, construction, deconstruction and synthesis where the role of the teacher and students is to fashion solutions to mathematical problems through a process of facilitated dialogue and interrogation of knowledge. Drawing on the underpinning of this proposition by Vygotsky (1978), this research recognises that the learning of mathematics enacted in pedagogy has three central aims.

The first is to convey a body of knowledge that may be new or contradictory/complementary to an individual student's prior understanding. Second is to foster active participation by students in their own learning. Third is to encourage students to metacognitively reflect on and use critically the knowledge they encounter. As such, student learning is perceived to be actively interactive at two levels: (a) students act as a part of the classroom community to co-produce knowledge about practices and difficulties in conceptualising mathematics; and (b) at other times, each student is a single central protagonist facing seemingly intractable mathematical predicament.

The assumption is made that problems can be discerned and solved collectively when students are able to articulate their position. At both levels the centrality of student voice is emphasised in instigating change in pedagogical practices and mathematical knowledge.

1.5 SIGNIFICANCE OF THIS RESEARCH

The significance of this research is threefold: (a) the importance of decimal knowledge for real-world interactions; (b) to address a pedagogical need; and (c) to fill gaps in research literature about decimal knowledge acquisition as well as the practices used to acquire the knowledge.

This research fills gaps in literature by including students as co-researchers to provide insight into their perspectives about teaching and learning experiences. This understanding is further deepened by consideration of influences of cognition, affect and conation on decimal knowledge acquisition processes.

1.5.1 Use in the real world

This research is significant because it contributes to knowledge that is needed to empower students' ability to communicate, process and interpret information in the real world. Since Australia is a metricated society, decimal numbers pervade students' everyday interactions—providing comparative quantitative information. For example, when comparing items of interest, in making critical judgements about products through quantitative analysis of ingredients in labels of products, and when analysing information used for persuasive purposes in advertisements.

Kordi, Mansournia, Rostami, and Maffulli (2011) found in their review of sports medicine literature that there are several inconsistencies in reporting of number of decimal places. They point out that the precision of data plays an important role in determining the precision of summary statistics such as mean, median and standard deviation. This need for understanding and ability to use numbers and quantitative methods is strongly supported by the educational curriculum authorities (ACARA, 2012; NCTM, 2000; Queensland Studies Authority, 2012).

Steinle and Pierce (2006) point out the importance and urgency in having deep understanding of decimal numbers by drawing our attention to media headlines such as "Baby died after 'decimal' error" (p.161). Their research show that 40% of the 355 student nurses made fundamental errors when comparing decimal numbers in their pre-test, indicating an alarming improvised understanding of decimal concepts. Similarly, to pursue other careers such as engineering, science, business and computer technologies, many measurements and calculations require the use of decimals, which are of higher precision than whole numbers (Liu, 2005).

1.5.2 Addressing a pedagogical need

This research is significant because it addresses a pedagogical need made evident by the extent and number of students' difficulties, as well as teachers' difficulties with teaching decimal concepts (Irwin, 2001; Moss & Case, 2002;

Okazaki & Koyama, 2005). The urgency for research is indicated by the persistence of misconceptions that have resulted from students' difficulties with understanding decimal numbers (Widjaja et al., 2011).

Research points out that if these mathematical difficulties are not addressed, they often continue to adulthood (Ubuz & Yayan, 2010). As discussed in section 1.4.2, the urgent need for pedagogical guidance is even further heightened by research findings that have revealed just how weak many teachers' knowledge is on mathematics and, in particular, decimals.

1.5.3 Filling gaps in knowledge

In view of the pedagogical difficulties with decimal knowledge acquisition (Steinle & Stacey, 2004; section 2.2.3), there is a clear indication of gaps in knowledge.

Student voice as co-researchers

This research contributes to fill gaps in pedagogical knowledge because it reflects a pragmatic student and teacher collaborative process (SPARC) in exploring learning, teaching and researching strategies. Drawing on the student and teacher participation as co-researchers, this research views informers of research as co-creators of reality and emphasises experiential knowing, dialogue, and reflective action, with knowledge arising from this action.

Dialogic and transformative peer learning and teaching methods are used to foster authentic dialogue, a sense of trust and a critical appreciation of the research process. As co-researchers, the participants are actively engaged in and will benefit from the educative nature of the research process. This nature of engagement supports a commitment towards the authenticity of research.

A tripartite view of experience (cognitive, affective, conative)

Findings in this research revealed the multidimensional nature of individual differences in learning as not being strictly cognitive; rather, as combining cognitive, conative, and affective functions (Reitan & Wolfson, 2000). However, it is difficult to find research literature in relation to decimals which addresses these functions in unison.

Most of the research literature found, focuses on identifying decimal difficulties and teaching strategies to eliminate conceptual difficulties experienced by students. Hence, this research offers fresh ways of viewing the pedagogical difficulties pertaining to decimals. It provokes thought on the influence of affect on cognition in predicting conative attitude and action.

1.6 OVERVIEW OF CHAPTERS

Chapter 1: Introduction provides an overview of the focus and approach of this research. It provides background, definition of terms, purposes, context, significance, and outlines the contents of each chapter.

Chapter 2: Literature Review examines pertinent literature relating to the decimal domain, learning, teaching and researching. Gaps in literature are also identified. This chapter concludes with implications for the theoretical framework of a teaching experiment for this research.

Chapter 3: Research Design describes methodology, participants, instruments, procedure, analysis and limitations.

Chapter 4: Results Stage 1 describes findings about students' entry knowledge at the start of the research from a diagnostic decimal number concepts pre-test and interviews.

Chapter 5: Results Stage 2 describes findings about changes in students' decimal knowledge during the teaching experiment and students' retention of knowledge in a post-test.

Chapter 6: Analysis is drawn from the results in the light of literature to develop theory in relation to teaching, learning and researching about decimal number knowledge acquisition.

Chapter 7: Conclusion describes overall findings and draws implications for practice and further research.

Chapter 2: Literature Review

2.1 OVERVIEW

The purpose of this thesis is to improve students' decimal number knowledge and practices for decimal number knowledge acquisition. Thus, this chapter reviews literature to delineate various theoretical positions and inform the construction of a theoretical framework that: (a) explains how Middle Years students construct decimal number knowledge; and (b) explains how teaching and learning practices could be improved to acquire decimal number knowledge.

Literature reviewed is organised in this chapter to address five main topics:

1. *The mathematics of decimals* discusses the history, structure, conceptions and common misconceptions about decimal numbers (section 2.2).
2. *Learning theories* examines and considers expansive and integrative ways of thinking about learning (section 2.3).
3. *Teaching recommendations for decimals* makes connections between the *Australian Curriculum: Mathematics* (ACARA, 2012) and the interpretation of the curriculum in the classroom (section 2.4).
4. *Metacognitive teaching strategies* explores various strategies to inform a conceptual framework for improving students' decimal knowledge in this research (section 2.5).
5. *Affective influences on teaching and learning* discusses how affects influence decimal knowledge acquisition (section 2.6).

This is followed by a summary of the literature review (section 2.7).

2.2 THE MATHEMATICS OF DECIMALS

2.2.1 The history of decimal

Drawing on Smith and Karpinski's (2004) account, the history of the Hindu-Arabic decimal system can be traced back to having originated in India with the Hindus (around 300–200 B.C.). The Hindu numeral system is speculated to have been adopted by Persian and Arab scientists (in around A.D. 800), brought to Spain (about A.D. 800–900) and the rest of Europe (in about A.D. 1100). The important innovation in the Arabic system was the use of positional notation to indicate place values. The creation of a symbol for zero made it possible to differentiate numbers such as 11, 101, and 1001, without the use of additional symbols. Therefore, all numbers could be expressed in terms of only ten symbols, the numerals from 1 to 9 plus 0. Positional notation is said to have greatly simplified all forms of written numerical calculation. The first recorded use of the system in Europe was in A.D. 976. The notation of the modern-day decimal system is credited to the work of Fibonacci in A.D. 1202.

Decimal is derived from the Latin *decem* (10) and Greek *deka* (10). It is considered likely that the choice of base 10 is connected to the instinctual use of the number of human fingers. In fact, the word digit is from the Latin *digitus* and means finger. The base 10 system is still evident in modern usage not only in the logical structure of the decimal system, but also in the English names for numbers.

In our current society, the decimal system is widely used in various systems employing numbers. The metric system of weights and measures, used in most of the world, is based on the decimal system, as are most systems of national currency. In the course of history, the decimal system finally overshadowed all other positional number systems, and it is now found in all technologically advanced nations.

2.2.2 Structure of the decimal system

The decimal number system groups numbers based on 10 different symbols called digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) that can be used to form a variety of numbers of different values. The digits are written as a row of digits, with each position in the row corresponding to a certain power of 10 specifying its place value. For example, in a number such as 777, each digit has a different place value (in this case 700, 70, and 7). Hence a multiplicative (e.g., 7×100, 7×10 and 7×1) and additive (700+70+7) relationship exists between the digits to form the number. The additive aspect of decimal number establishes the fact that numbers are composed of other numbers of different sizes (Krebs, Squire, & Bryant, 2003) and can be decomposed to add up exactly to ‘n’ (in this case 777).

Figure 2.1 shows the pattern of 10 times smaller or larger relationship between place values in a decimal number system depending on the position of a number as being on the left or right of another number. Numbers greater and less than one consist of sums of groups of tens where the groups are arranged according to how many groups of tens are present (10^0 as ones, 10^1 as tens, 10^2 as hundreds and so on). For example,

$$7532 = (7 \times 10^3) + (5 \times 10^2) + (3 \times 10^1) + (2 \times 10^0) = 7000 + 500 + 30 + 2$$

In comparison, numbers less than one are represented by a sequence of numbers to the right of a decimal point and multiplied by 10 raised to a negative power, starting from negative 1. In moving to the right, the place value decreases in value, each being one tenth as small as the previous place value. Thus, each successive digit to the right of the decimal point denotes the number of tenths, hundredths, thousandths, and so forth.

For example,

$$0.7532 = (7 \times 10^{-1}) + (5 \times 10^{-2}) + (3 \times 10^{-3}) + (2 \times 10^{-4}) = 0.7 + 0.05 + 0.003 + 0.0002$$

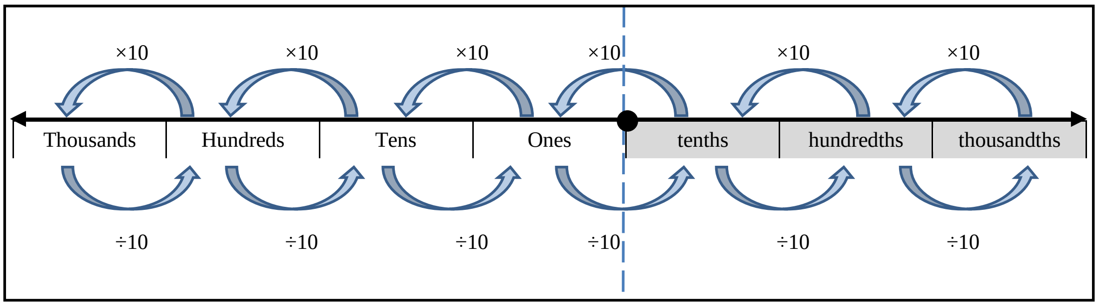


Figure 2.1 Place names and multiplicative relationship between adjacent places.

A semantic mathematical pattern can be seen to exist between ones, tens and hundreds as the place value increases based on the number of digits starting from the ones place value. However, this pattern is not as clear with decimals as students tend to use the decimal point as the central point of the pattern rather than the ones place value.

Baturo (1998) explains that, in comparison to whole numbers, decimal numbers add an extra dimension of measurement which requires considerable flexibility and complexity in thinking. This places cognitive demands which make decimal number acquisition a challenge for most students. When a whole is partitioned into hundredths or thousandths they can be recorded or renamed in other equivalence forms. This requires students to have the ability to transform and keep track of transformations.

The structural focus in understanding decimal concepts used in this research takes the view of Mulligan, English, and Robertson (2010) that structural thinking is more than simply recognising elements or properties of a relationship. Rather, it is having a deeper awareness of how those properties are used, explicated or connected.

2.2.3 Mathematical difficulties with decimal numbers

Research in mathematics education documented for over twenty years (Okazaki & Koyama, 2005) has shown that the decimal domain is not straightforward. Rather, it is notoriously difficult for students to learn, and for educators to teach (Cramer, Post, & delMas 2002; Glasgow, Ragan, Fields, Reys, & Wasman, 2000; Irwin, 2001; Moss & Case, 1999; Okazaki & Koyama, 2005; Smith, Solomon, & Carey, 2005). Although most studies were concerned with the deep processes related to the mathematics in the decimal domain, it is clear that students' difficulties have not been resolved.

A worldwide problem

Thomson, De Bortoli, Nicholas, Hillman, and Buckley (2010) claim that students' lack of understanding of what a decimal number means and the relationship between decimals and fractions is clearly articulated in the Third International Mathematics and Science Study (TIMSS). This was conducted in the years 1999, 2003 and 2007, highlighting a concerning worldwide pedagogical problem. TIMSS involves more than 60 countries from around the world and uses a curriculum model that is characterised by three main considerations: (a) the decimal knowledge that society intends for students to learn and how the education system should be organised to facilitate this learning; (b) what is actually taught in classrooms, the characteristics of those teaching it, and how it is taught; and (c) what it is that students have learnt, and what they think about these subjects.

Irwin's (2003) analysis of the test results suggested that students' approaches to operating with decimals were without understanding and mechanistically procedural. Analysis of Year 8 students' responses to the following TIMSS test item exemplifies students' lack of conceptual understanding of the value of decimal numbers and how this value is represented symbolically. The approach taken to solve the problem illustrates students' attempts to make sense of decimal numbers by relating to existing incompatible knowledge structures.

48 × 5 can be 24 × 10. What is 48 × 0.5?

A number of students changed the problem to 24 × 0.10 or 24 × 10. The first error treats the decimal point as a dot that symbolises two sets of unrelated numbers on either side of the dot. The other error disregards the decimal point. Both are common errors that show that students do not understand decimals. Furthermore, as Irwin (2003) points out, the seriousness of this problem is that since the students were in Year 8, they were not likely to receive any further instruction in decimals to rectify their understanding.

An extensive array of difficulties and mathematical misconceptions have been identified and documented in research literature pertaining to both conceptual and computational aspects of decimal numbers (Vamvakoussi & Vosniadou, 2010) that explain findings in international and local assessments of students' decimal knowledge.

2.2.4 Students' misconceptions about decimals

Early studies of sources of decimal misconceptions (Nesher & Peled, 1986; Resnick et al., 1989) identified three main paths of thinking: (a) whole-number conception such as longer decimal digits were perceived as larger numbers; (b) common fractions and place value conceptions incorrectly extended to interpret decimal digits where shorter decimal digits were perceived as the larger number; and (c) zero conceptions where place value connections were not understood in terms of the role of zero. More recent comprehensive longitudinal studies (e.g., Lee, Brown, & Orrill, 2011; Stacey, 2005) indicate that many misconceptions persist despite students' progression to higher grade levels.

Other research findings have also confirmed and further defined the variations in misconceptions. Some of these include:

- Inadequate understanding of the meaning of symbols embedded in the decimal notation (Baturo, 1998; MacDonald, 2008).

- Conceptualising the dense nature of decimals—that infinite numbers exist between any two successive integers, including between 0 and 1 (Shaughnessy, 2011; Smith, Solomon, & Carey, 2005).
- Difficulty in placing a decimal number such as 0.685 on a number line that goes from 0 to 1 (Rittle-Johnson, Siegler, & Alibali, 2001).
- Not knowing the meaning or purpose of the decimal point, therefore, treating digits on either side of the decimal point as two sets of whole numbers (Baturo, 1998; MacDonald, 2008).
- Difficulty with ordering two decimals such as 0.42 and 0.402 (Widjaja et al., 2011).
- Uncertainty about place value of digits in a decimal number, hence ignoring the decimal point to suggest that 0.56 is greater than 0.7 because 56 is greater than 7 (Stacey et al., 2001).
- Confusing decimal numbers with fractions and negative numbers, such as seeing 1/5 as 1.5 even though they know that 1/5 is less than one and 1.5 is more than one (Stacey et al., 2001).
- Thinking that fractions and decimals cannot occur together in a single expression, like 0.5 + 1/2, or that one representation cannot be changed to the other (from 1/2 to 0.5) within a given problem (Moss & Case, 2002; Okazaki & Koyama, 2005; Shaughnessy, 2011).
- Comparing the size of decimal numbers (Steinle & Stacey, 2004) and applying the inappropriate "more digits make bigger" rule (e.g., 0.1814 > 0.385).

Steinle and Stacey (2004) proposed a classification system for categorising many of these misconceptions as patterns of incorrect responses in a Decimal Comparison Test (DCT). Four groups termed as behaviours were identified as:

- Longer-is-Larger (L behaviour), choosing the decimal with the most digits after the decimal point as the largest.
- Shorter-is-Larger (S behaviour), choosing the decimal with the fewest digits after the decimal point as the largest.
- Apparent Expert (A behaviour), comparing "straightforward" pairs of decimals correctly with or without full understanding.

- Unclassified (U behaviour), indicating behaviour that does not fit in L, S, or A behaviour.

These four behaviours were further categorised as twelve ways of thinking which have been used by researchers (Widjaja et al., 2011) and teachers to reveal students' misconceptions.

Graeber and Johnson (1991) noted that characteristics of misconceptions about decimals were self-evident (one does not feel the need to prove them), coercive (one is compelled to use them in an initial response) and widespread among both naïve learners and more academically able students.

2.2.5 Decimal misconceptions extended to teachers

The content of the mathematics curriculum for developing students' decimal number knowledge and understanding is outlined by the Queensland Studies Authority (QSA, 2012) and the Australian Curriculum, Assessment and Reporting Authority (ACARA, 2012) documents. Expected learning requirements are arranged by broad organisers (strands) and presented as content descriptions, which describe what teachers are expected to teach at each year level. These descriptions form a scope and sequence of building students' knowledge to facilitate decimal understanding in students with increasing sophistication. Content elaborations are also provided as support for teachers.

Stigler and Hiebert (1999) argue that evidence from comparative analysis of international studies of teaching (TIMSS) indicates that teaching is one of the major factors related to students' low mathematics achievement. According to the National Council of Teachers of Mathematics (NCTM, 2000), "Effective teaching requires knowing and understanding mathematics, students as learners, and pedagogical strategies" (p. 17). In other words, to make decimal ideas accessible to all students, teachers would need to be able to understand the pedagogical problems and the mathematics of the decimal domain deeply and flexibly so they can help students create useful cognitive maps, relate one idea to another, and address misconceptions.

However, a significant concern highlighted by literature is that many of the misconceptions discussed in section 2.2.4 have also been found in many teachers (e.g., Holmes, 2012; Ubuz & Yayan, 2010). These findings indicate that many teachers have inadequate knowledge for teaching primary school mathematics, particularly the kind of teaching demanded by recent reforms. Such concern for the mathematics curriculum in general was also articulated in Queensland's response to the draft K(P)-10 Australian Curriculum (QSA, 2010), stating "for inexperienced teachers, in the face of increased technical language and content, the elaborations may become a default curriculum" (p. 17).

In summary of these findings in the literature, the problem of decimal numeration education is twofold. The first is in the conceptualisation of the mathematics in decimal numeration and the second is in providing teachers with the right kind of knowledge and skills to address classroom needs, so that they can provide the depth of knowledge or sophistication of skills that teaching or learning about decimals necessitates.

2.3 LEARNING THEORIES

This research takes an expansive and integrative approach of using elements from learning models to situate theory with emerging needs observed in practice. It draws on a blend of aspects of theories to include phenomenographic perspectives, theories of cognitive constructivism, information processing and social constructivism to inform the complex nature of learning. In doing this, it also reflects on the ways in which theories overlap, have their proponents and critics, and have their advantages and disadvantages.

2.3.1 Integrative approach

A matrix of perspectives on learning theories underlies advances in research and teaching experiments that define models of how decimal knowledge is acquired, and account for teaching strategies proposed by researchers. Most recent publications in mathematics education adopt constructivism and situated learning as two main lines of learning models informing practice (e.g., Isotani et al., 2011; Rittle-Johnson & Koedinger, 2005; Schneider & Stern, 2010).

The definition of learning has become considerably more complex and diverse in research literature since the 1960s and 1970s, when it was essentially based on behaviourism and cognitivism (Säljö, 2010). More recently, the focus of conceptions of learning has been from a phenomenographic perspective (Reimer, 2006).

This research draws on aspects of phenomenographic perspectives (Reimer, 2006) by seeking to understand the qualitatively different ways that students experience, understand and make sense of learning. This includes issues that concern how students perceive and reciprocally respond to its affordances mentally, physically, emotionally, psychologically, and socially (Alexander, 2009).

Review of the vastness of conceptions in literature about learning highlights the interactional complexity that constitutes learning, including learner characteristics, what is to be learnt, and the context and situations in which learning occurs at any one point of time and over time. The literature review suggests that there are some disagreements and debate among the separate theories and models of learning. However, this conflict is seen by this research to serve as an advantage for questioning and reflecting on action that is argued by one theory and opposed by another. Therefore, differences among theories provide avenues for opening minds to different ways of thinking rather than being restricted within the boundaries of one pre-packaged way of looking at learning.

Useful to this research is the collection of principles as givens, or as common sense or well-established axioms of learning by Alexander (2009). These principles perceive learning as: (a) change; (b) inevitable, essential, and ubiquitous; (c) able to be resisted; (d) possibly being disadvantageous; (e) tacit and incidental as well as conscious and intentional; (f) framed by our humanness; (g) referring to both a process and a product; (h) different at different points in time; and (i) interactional.

As pointed out by Alexander (2009), embedded in these principles of learning are also characteristics that stand as salient attributes of this complex but elusive construct. Importantly, the principles highlight the nature of learning as being dynamic and in continual flux. They reframe the experience, identification and critical reflection of difficulties with learning decimals discussed in sections 2.2.3–2.2.5 as an essential and inseparable part of learning.

2.3.2 Constructivism

Most decimal education related research has adopted a constructivist framework in the analysis of decimal education and in forming theories about learning. This research draws on constructivist views of students as constructors of their own cognitive tools, as well as of their external realities. It adds to the understanding that learning is a process of making sense of an experience by drawing on prior knowledge and experiences, reflection on personal epistemologies and beliefs. As such, decimal knowledge construction can be related to the four tenets for constructivism (Tobin & Tippins, 1993) as:

1. Acquisition of decimal knowledge depends on prior knowledge—past constructions that exist as a mental framework that can be deconstructed and reconstructed through experiences and interactions with the environment.
2. Constructions of concepts are a result of the processes of assimilation of familiar information into students' mental framework and accommodation when the information is not familiar to develop a higher level theory or logic.
3. Learning has to be continuously constructed and enriched by investigation, predicting, imagining and introspective manipulation of information and invention.
4. Meaningful learning involves reflective learning that seeks to resolve cognitive conflicts (such as being able to see the differences and commonalities between whole and decimal numbers) by reorganising prior framework or understanding to make sense and connections between prior and new decimal knowledge.

Constructivism is a theory about learning based on epistemological questions such as: What is knowledge? How is knowledge acquired? How do we know what we know? Constructivism theories from the psychological or sociological underpinning do not stand in unison in all their assumptions about knowledge as an individual or a collective process.

Constructivist theory falls within two main lines of thought, social constructivism and cognitive constructivism. Cognitive constructivism explains how the individual learner understands things, in terms of developmental stages and learning styles, whereas, social constructivism emphasises how meanings and understandings grow out of social encounters (Vygotsky, 1978).

2.3.3 Cognitive view to learning

Schema perspective

Using the concept of schema, some researchers interested in understanding students' difficulties in processing decimal knowledge (e.g., Baturo, 1998; Moody, 2008) have based their analysis on mental models of students' knowledge structures (schema) and processes for using this knowledge (mental operations).

In order to understand the various mental models of cognition proposed by researchers, it is useful to understand their intellectual predecessor. The concept of schema draws on Piaget's (1985) idea that students must construct their own knowledge through experience to enable them to create schemas—mental models of the world. Schemas are where mental actions are organised to assimilate information by interpreting situations using prior knowledge to engage in the activity and accomplish a learning goal. This view of conceptual schemas suggests that learning a new conception of numbers does not only involve addition of new decimal-related information into existing whole-number knowledge structure; it also involves a major restructuring in the existing conceptual system.

This restructuring of schemas is said to occur through complementary processes: assimilation (bringing in new knowledge to their own schemas) and accommodation (changing schemas to accommodate the new information) as a search for equilibration (Wadsworth, 2004). Equilibration is suggested to occur when students shift from one stage of concept thought and development to another and is manifested with a cognitive conflict, a state of mental unbalance or disequilibrium in trying to make sense of information students are receiving.

Disequilibrium is explained as a state of being uncomfortable when adjustment to thinking (schema) is made to resolve conflict and become more comfortable (Powell, 2009). This adjustment process occurs when new information fits into existing memory (Crossland, 2010).

Researchers (Desmet et al., 2010; Vamvakoussi & Vosniadou, 2010) found that a common difficulty with decimal knowledge was that students assimilated decimal knowledge into existing whole-number schemas which interfered with their construction of a correct and adequate schema for decimals.

The term conceptual change was used to understand a variety of types of changes that needed to occur in the content and organisation of decimal concepts. From this conceptual change framework, difficulty in learning a new decimal concept (such as differences in symbolisation, ordering, the nature of the unit and the procedures for operating with decimals) is suggested to arise. This is argued to result from the fact that decimal knowledge is in conflict with a robust conceptual structure of whole numbers that was already in place (Vamvakoussi & Vosniadou, 2010). Hence, the conceptual change approach to students' understanding of decimals is useful in focusing attention on the precise differences between the whole-number and decimal conceptual frameworks.

Prior knowledge interference

Constructivist theories, conceptual change theories and the nature of mathematical difficulties (see section 2.2) experienced by students highlight the importance of prior knowledge in constructing decimal knowledge.

Research findings show that students' prior knowledge of whole numbers is foundational to decimal number understanding (Irwin, 2001; Moss & Case, 2002). On the other hand, there is also considerable evidence (e.g., Baturo, 1998; MacDonald, 2008; Resnick et al., 1989) that a substantial amount of students' prior knowledge of whole numbers is incompatible with decimal number concepts. Hence, prior knowledge has been found to interfere with the conceptualisation of decimal.

Peled and Shahbari (2009) explain that one of the main problems is that decimals are deceptive on the surface. This leads to errors due to students thinking that "decimals are similar to whole numbers and have similar (extended) algorithms for the basic operations" (p. 74). Research literature suggests that this confusion caused by the incongruences in decimal and whole numbers is evident in primary and secondary school students as well as in students at tertiary education level (Vamvakoussi & Vosniadou, 2010).

Merenluoto's (2003) research shows the progression in students' knowledge and understanding of numbers. Classroom experiences in the early years are focused on developing rich conceptual understanding of whole numbers and operations of addition, subtraction, multiplication and division. This knowledge is strengthened by linguistic operations of counting, knowing that the next number is one more and also that it is the name of the next number. In fact it is understandable how early years whole-number thinking may lead to conceptualisation of parts as a smaller whole rather than part/s of a whole relationship without explicit decimal fraction language in classroom mathematics.

Multiplication operation is understood as repeated addition (Iuculano & Butterworth, 2011). A common belief is that multiplication should always yield a number that is necessarily higher than those with which we started and a division should result in a smaller number. Merenluoto (2003) points out that "representations, language, rules of operations and of order for decimals are essentially different compared to respective rules of whole numbers" (p. 297). Thus, students experience cognitive conflict in accommodating new rules and logic that are different from the already firmly established knowledge about numbers and operations, leading to systematic problems and misconceptions in learning of decimal number concepts and skills.

Other researchers in decimal number education have also emphasised the role of cognitive conflict as a central catalyst for initiating the restructuring or reorganisation of existing ideas to accommodate new information (Huang, 2008; Liu, 2005; Vamvakoussi & Vosniadou, 2010).

2.3.4 Information processing view to learning

Elements of information processing theory (Reiser & Dempsey, 2007) add to the understanding of cognition in decimals in that it focuses on mental processes. These include: perception, recognition, imagining, remembering, thinking, judging, reasoning, problem solving, conceptualising, planning and application. As such, cognitive information processing theory provokes thinking about strategies that facilitate or hinder learning and memory.

Useful to this research in planning and teaching strategies are suggestions such as the use of graphic organisers, definition of words and feedback to encourage meaningful practice. The informational and educational value of feedback is argued by Reiser and Dempsey (2007) to have two important functions: (a) feedback provides the learner with some type of response so that they know if their answer is correct or incorrect; and (b) feedback can be used to provide corrective answers/responses to incorrect answers/responses.

Hetherington and Parke (1999) provide further insight into the information processing conceptions in relation to change, thinking, self-modification and task constraints.

Change refers to four critical mechanisms that work together to bring about change in students' cognitive skills: encoding, strategy construction, automatisation, and generalisation.

Thinking refers to when the student perceives, encodes, represents, and stores information from the environment in the mind or retrieves that information. This also includes responding to any constraints or limitations on memory processes.

Self-modification is seen as a necessity for development. As discussed with Piaget's (1985) theory of cognitive development, the information-processing approach holds that students play an active role in their own development. Through self-modification, students uses knowledge and strategies that have been acquired from earlier problem solution to modify their responses to a new situation or problem. In this way, newer and more sophisticated responses are built from prior knowledge.

Task constraints refer to the nature of the task in affecting a student's performance. Thus a student may possess the basic ability necessary to perform a particular task when it is presented in a simple form, without unnecessary complexities. However, if extra or misleading information is added to the same task, the student may become confused and be unable to perform it.

2.3.5 Social constructivist view to learning

Social interdependence

Cognitive constructivism focuses on individual cognition and differentiates from social constructivist perspectives by shifting its focus to the interdependence of social and cultural processes, where learning is co-constructed and negotiated actively through social experiences (Lave & Wenger, 1991; Vygotsky, 1978). Social constructivism places emphasis on the role of language (Vygotsky, 1978), the impact of collaboration, negotiation of meaning (Glasersfeld, 1989), apprenticeship (Brown, Collins, & Duguid, 1989), interactions with the social and physical contexts and the influence of communities of practice (Lave & Wenger, 1991) on cognitive development of individuals.

Those espousing social constructivist pedagogy (Lave & Wenger, 1991; Perera, 2011; Vygotsky, 1978; Wenger, 1998) highlight the importance of social interactions in any learning process. Social discourse is argued to be the mechanism through which transformation of ideas takes place. Perera (2011) explains that social constructivism does not separate individual cognition from social activity and mainly accentuates the group as a creator of knowledge. This however does not neglect the individual.

Perera (2011) discusses the twin processes of knowledge building as externalisation through collective reflection and internalisation of knowledge through a conscientious practice. He argues that although the group co-creates knowledge, the learner constructs own meaning when knowledge is internalised on an individual capacity. Hence, collaborative knowledge construction starts with personal understanding and then highlights social knowledge building finishing off with the individual. From this view, the individual and the group cognition cannot be separated in social constructivist thinking.

Construction with others

In explaining the construction that occurs with others, sociocultural literature related to pedagogical strategies commonly makes reference to, and expands on, Vygotsky's (1978) notion of *Zone of proximal development* (ZPD). Two interpretations of his ZPD were identified in conventional classroom views of learning by Pane (2010).

The first interpretation of ZPD used Vygotsky's (1978) explanation—as the distance between the problem-solving abilities of a learner working alone and that learner's problem-solving abilities when assisted by or collaborating with more experienced people. Pane (2010) explains that using this Vygotsky's (1978) ZPD interpretation, teaching is thought of as scaffolding, where explicit support is slowly relinquished until the learner can perform the task independently.

An example of scaffolding use in teaching decimal computation would be to solve a decimal computation problem together, demonstrating the procedure and explaining the reasoning behind the steps used. This would be followed with simple problems for practice and slowly increasing difficulty of subsequent problems.

In the second interpretation, the ZPD is the distance between cultural (understood) knowledge provided by the socio-historical context (instruction) and active knowledge owned by the student. This interpretation is based on Vygotsky's (1978) "distinction between scientific and everyday concepts, and that a mature concept is achieved when the scientific and everyday versions have merged" (p. 48).

Conceptual change from sociocultural perspective

In extending the conceptual change approach (discussed in section 2.3.3) to sociocultural perspectives, Huang (2008) suggests that it is possible to stimulate cognitive conflict by involving students in decimal-related tasks and providing them with feedback on the contradictions that arise between new evidence and prior thinking. It is thought that student production of evidence is an important factor in initiating steps towards conflict resolution.

Researchers (e.g., Desmet et al., 2010; Yıldız, Taskin, Avdin, & Koqce, 2011) argue that for conceptual change to occur, cognitive processes of adapting and restructuring are needed. This is because students are only able to interact with new information to the degree that the information is comprehensible, coherent, plausible, and rhetorically compelling according to their existing conceptual models.

Sociocultural perspective of conceptual change literature related to decimal knowledge (Desmet et al., 2010; Huang, 2008; Liu, 2005; Yıldız et al., 2011) offers a logical approach to developing students' understanding of decimal numbers. It adds to the understanding of the social nature of learning; importantly, that conceptual change is not simply cognitive. Rather, conceptual change is interwoven with the social, contextual characteristics of the student.

Furthermore, for changes to occur, the conceptual change approach assumes that students are metacognitively aware (Flavell, 1976) of their own conceptual systems and able to recognise, monitor, reorganise and evaluate inconsistencies in knowledge to make conceptual changes. However, as Limon (2001) points out, metacognitive awareness of conflicts is not enough for conceptual change. Rather, consideration of motivational factors, epistemological beliefs, prior knowledge, values and attitudes, learning strategies and cognitive engagement, and reasoning strategies, as well as sociological factors, is needed for conceptual change.

Communities of practice perspective

Supporters of a sociocultural perspective of mathematics education argue that a close relationship exists between cognitive skills, cultural technology and societal institutions and communities of practice through which understandings and practices are developed (Lave & Wenger, 1991; Pane 2010). Learning, from this perspective, is a collective activity where the knowledge that is acquired is knowledge produced within the group. And the process of acquiring knowledge is social—it is learnt through participation within the group and through the adoption of shared practices. The theory on community of practice deals with concepts such as participation, relationships, activity, practice, and context (Lave & Wenger, 1991) to explain the social and situated character of learning.

According to Lave and Wenger (1991), learning is the increasing participation in a community of practice. Participation in communities of practice is argued to encourage community membership and social identity transformation. The process of learning is said to begin with legitimate peripheral participation, where a newcomer is allowed access to a practice but spends some time at the periphery of the practice and then gradually moves to the centre and becomes a full participant.

Learning, social and academic identity transformation, agency, and new cultural forms are said to develop from resistance to constraints in classrooms or schooling structures (Varenne, 2008). As such, in communities of practice such as classrooms, learning is argued to occur through the transformative potential of negotiated, but not necessarily comfortable, interactions among the members of communities of practice (Lave & Wenger, 1991).

Wenger (1998) describes three dimensions of the relation by which practice is the source of coherence of a community as: (a) mutual engagement (characterised by engaged diversity, doing things together, relationships, social complexity, community maintenance); (b) joint enterprise (characterised by negotiated enterprise, mutual accountability, interpretations, rhythms, local response); and (c) a shared repertoire (characterised by stories, styles, artefacts, tools, discourses, concepts, historical events).

In summation, the community focuses on the social production of meaning (Wenger, 1998). Learning is argued to be achieved by members of a community of practice who are actively and mutually engaged in a shared interest of knowledge construction (Dumitru, 2012). This view raises questions for practice such as: Does mathematics learning in a working classroom truly have such sustained joint engagement? How does it involve or invite mutual accountability beyond the use of assessment tasks? Is there evidence of a shared and negotiated repertoire of artefacts, tools, and discourses?

2.3.6 Metacognitive view to learning

In view of the nature of students' mathematical difficulties (section 2.2), metacognition (section 1.2.3) is given prominence in this research. Metacognition is seen by this research as a necessity to develop students' awareness of their own mental processes about decimals and the practices used to acquire this knowledge. This includes not only an awareness of what one is thinking (Flavell, 1979) and how one is emotionally reacting to the difficulty experienced, but also evaluating the thought process and possibly choosing to change it. Flavell (1979) states that "metacognitive knowledge consists primarily of knowledge or beliefs about what factors (person, task and strategy) or variables act and interact in what ways to affect the course and outcome of cognitive enterprises" (p. 907).

In terms of the person factor, Flavell (1979) is referring to everything that we can come to believe about the nature of ourselves and others as a cognitive processor (e.g., *I am bad at maths*). The task factor refers to information available and demands of the cognitive mathematical task (e.g., *the test question was ill-structured*). The strategy factor refers to strategies that are likely to be effective.

Schreiber (2005) adds to this understanding by proposing three main sub-processes of metacognitive knowledge that need to be considered in teaching strategies:

- *Declarative knowledge* (Flavell, 1979) that refers to the conscious use of strategies that accompany planning, monitoring, and controlling processes.
- *Procedural knowledge* (Baker & Brown, 1980) about knowing the general flow of these processes such as planning, selecting, monitoring, evaluating, and debugging to reflectively use strategies.
- *Conditional knowledge* (Schreiber, 2005) about knowing why and when to apply the strategies available in a learning situation.

Brown's (1987) view of metacognitive knowledge brings to the fore the concept of knowledge as activities that involve conscious reflection on our cognitive abilities and activities. This is supported by Zimmerman (2000) who argues that self-reflection has a central role in achieving self-regulation in learning. In his view, it is divided into two components: self-judgement and self-reaction. Self-judgement involves evaluating performance and attributing causal significance to the results. Self-reaction includes satisfaction with our performance and adaptive-defensive inferences about how our needs alter the self-regulatory approach during subsequent efforts to learn and perform.

2.4 TEACHING RECOMMENDATIONS FOR DECIMALS

Research studies have emphasised the important role of teachers in the successful implementation of any educational program. Two important prerequisites are highlighted: (a) knowing how to teach for the 21st Century; and (b) knowing what to teach.

The detrimental effects resulting from classroom teachers' weak knowledge and understanding of decimals are well documented in research literature (e.g., Lee, Brown, & Orrill, 2011; Steinle & Pierce, 2006; Ubuz & Yayan, 2010). These researchers suggest that if students are going to obtain a satisfactory level of achievement, teachers must examine seriously how they teach.

2.4.1 Teaching for the society in the 21st Century

Proponents of knowing how to teach in the 21st Century point to a workforce reality that demands students and workers who are independent thinkers, problem solvers, and decision makers.

Silva's (2009) analysis of the debate about the 21st Century skill descriptors suggests that many of the skills are not new but newly important. Other skill descriptors call for new ways of interacting with knowledge, to take control of one's own learning beyond the constraints of the classroom and teacher expertise. This includes using technology such as synchronous and asynchronous online interactions as a modality for enriching/consolidating understanding of decimals.

Of particular interest to developing such autonomy in students is the work of Claxton (2006) who describes teaching in terms of expanding students' capacity to learn. His ideas about learning how to learn frame students' difficulty with decimals as an opportunity for learning rather than a problem. Claxton (2006) argues that knowing what to do when faced with situations for which students are not specifically prepared is an educational focus that is important.

Implicit within this conception are assumptions that students need to have knowledge and understanding of how they learn and, further, the capacity to motivate and regulate their actions and behaviours to learn. Biggs' (1985) definition of a meta-learner describes the capacities of such learners as "one who is aware of their motives, task demands and personal cognitive resources and exert control over strategies used" (p. 127).

In line with Claxton's (2006) views for preparing students for the 21st Century, McWilliam (2008) also presents a challenge to teachers to unlearn how to teach. Within this context, she describes the need for a shift in the role of teachers from their conventional/traditional role as "sage-on-the-stage and guide-on-the-side to meddler-in-the-middle" (p. 2). The term meddler is defined as having the following qualities: respect rich, structure rich, conversation rich, information rich, and challenge rich; and as having skills that include: being academic functional, aesthetic digital, and dynamic interactive. She refers to a classroom environment as one that is seriously playful, epistemologically agile and low threat high challenge.

As discursive analysts (e.g., Gee, 2005) have argued, identities of teacher and students are not defined in the abstract, but in the context of practice, and it is through exploring the dynamic relationship between practice and construction that we may appreciate the creativity in identity.

Discrepancies in students' and teachers' perceptions

Both Claxton's (2006) and McWilliam's (2008) recommendations for teaching suggest that teaching does not automatically lead to learning. Rather, students' and teachers' perceptions of what constitutes learning influence learning and learning behaviour and eventually learning outcomes. Importantly, the perceptions define the identity of students as being both teachers and learners.

There is an underlying view that through reflective or thoughtful practices, discrepancies between teachers and students would be revealed, negotiated and resolved. Nevertheless, it is common practice that teachers develop lessons for students without including them in the lesson planning process (Könings, Brand-Gruwel, & van Merriënboer, 2010).

Studies (Holt, Denny, Capps, & de Vore, 2005; Watkins 2004) show that teachers are often not sufficiently aware of the discrepancies between students' and teachers' perceptions. A lack of congruency in perception of instruction is said to cause detrimental and divergent behaviour patterns in the teaching–learning environment. Könings et al. (2010) add that differences between the intentions of teachers' teaching plans for an educational teaching experiment and the interpretation of the students can cause a mismatch and, as a result, lead to suboptimal use of the intervention.

Furthermore, research findings show that, when confronted with discrepancies between their own and students' perceptions, teacher awareness did not necessarily guarantee that those perceptions would actually be taken into account (Holt et al., 2005). Instead, teachers generally reduced these discrepancies by rejecting the student feedback as invalid or by changing their own perceptions rather than changing the lessons.

2.4.2 Information sources for teaching decimals

Research literature offers teachers a range of different ideas for teaching decimal knowledge. These suggestions are based on different beliefs, inquiry methods and philosophies on the chosen aspect of decimal domain under study by the researcher. For example: hands-on approach to teaching decimals (Schneiderman, 2008); everyday knowledge of decimals to enhance understanding (Irwin, 2001); cognitive conflict strategy (Huang, 2008); direct instruction (Small, 2011); and connecting equivalence understanding (Muzheve & Capraro, 2012).

In building conceptual understanding, studies have used various visual models and concrete manipulatives to link abstract mathematical ideas and build understanding of decimal concepts (Lee et al., 2011; Moody, 2011; Roche, 2010) such as the relationship between equivalents of decimal values (Livy, 2011).

In terms of the content of teaching decimal concepts, the *Australian Curriculum: Mathematics* (ACARA, 2012) proposes the inclusion of a comprehensive list of concepts. This includes: the meaning of decimals, position value of decimals, assimilation/dissimilation of decimals, value comparison of decimals, measurement unit conversion, density of decimals, relations of decimals with common fractions and representations of fractional value.

In planning learning experiences to develop students' knowledge of these decimal concepts, teachers rely on several sources to build their knowledge and inform their practice. Some of these include:

- Literature on the most recent recommendations for teaching mathematics advocated by advisory professional organisations such as ACARA, The National Council of Teachers of Mathematics, the Department of Education and Training and the State Authorities Board.
- Educational resources such as textbooks and online teaching programs developed to support the most recent curriculum.
- Professional development opportunities, where one or more perspectives are advocated to engage teachers in metacognition and creative thinking.
- Peer collaborative discussions.
- Local and national assessment tasks to model instructions so that their students are ensured success.
- School advisory systems.

How much information is gained and how it is used—in consultation (or not) with students' feedback and input—depends on the individual ethos of the teacher and the flexibility of school policies.

2.4.3 Australian curriculum for teaching decimals

Informed by research and the interpretations of research by curriculum developers, the *Australian Curriculum: Mathematics* (ACARA, 2012) provides guidance to teachers with the planning, teaching, and assessment of decimal understanding. The core foundational knowledge for decimal is covered under the content descriptions of the Number and Algebra strand.

The other two content strands (Measurement and Geometry, and Statistics and Probability) also require relational decimal understanding. For example, Number and Measurement are connected to decimals by their decimal relationship. Converting measurement units requires conceptual understanding of quantities or sizes in relation to place value. Fractions, percent and probability are connected as they all describe parts of a whole.

To ensure an increasing sophistication of decimal knowledge acquisition over the years of schooling, proficiency strands (Understanding, Fluency, Problem Solving, and Reasoning) have been incorporated into the content descriptions of the three content strands described above that describe how the decimal knowledge content is to be explored or developed. The ACARA (2012) handbook for the National Assessment Program Literacy and Numeracy (NAPLAN) tests provides teachers with samples of assessment tasks to assess students' expected year level decimal knowledge at a national level.

Assumptions of the curriculum

Literature on the *Australian Curriculum: Mathematics* (ACARA, 2012) sets well-researched standards and guidelines with clear statements advocating learning for understanding decimals. The comprehensive elaborations of knowledge, skills and processes have the potential of providing explicit guidance to teachers who have difficulties with mathematics (as described in section 2.2.5). The detailed scope and sequence lends itself to whole-school consistency in curriculum objectives.

However, how the curriculum is used depends on the individual interpretation of the teacher or school curriculum advisors. My discussions with colleagues indicate that what is missing is that on its own, the curriculum does not necessarily educate or build awareness in teachers about the theories behind the objectives stated in the decimal curriculum.

2.4.4 Referent resources for conceptual knowledge

In constructing students' decimal knowledge, researchers have shown that not only syntactic procedural rules of strategies but semantic conceptual knowledge of relations and meaning is necessary (e.g., Baturo, 1998; Hiebert & Wearne 1986; Iuculano & Butterworth, 2011). Based on research recommendations, school mathematics reform initiatives put particular emphasis on strategies to develop conceptual knowledge (ACARA, 2012).

Engaging students in mathematics through multiple representations—pictorial, symbolic or concrete referents such as base ten blocks—and grounding activities in real-world contexts is advocated to develop conceptual understanding. The use of specific referents is argued to develop targeted conceptual knowledge. For example Livy (2011) advocated the use of concrete models rather than simply pictorial models for developing understanding between the different representations of fractional value. Other concrete models such as Multi-base Arithmetic Blocks, Linear Arithmetic Blocks (Helme & Stacey, 2000) and newer models such as Decipipes (Moody, 2011) are recommended for place value conceptualisation.

The teachers' choice of referents to develop students' knowledge is considered as important. For example, Lachance and Confrey (2003) pointed out that most activities developed by researchers and educators used a fraction referent that did not transfer easily to decimals. Furthermore, they showed that students did better at fraction in comparison to decimal items in the TIMSS. This indicated that students did not make connections between existing prior fraction knowledge and decimals.

Lee, Brown, and Orrill's (2011) findings reveal that some teachers have difficulty with making validated choices for using referents. This indicates referents are often used by teachers as a process, rather than for opening conversations about abstraction and generalisation of concepts. The use of more than one type of referent is also contested by some teachers with the argument that multiple representatives confuse students (Lesh, Post, & Behr, 1987).

Research literature on students' views on using multiple types of referents or the type of referent most preferred by students was difficult to find.

2.4.5 Metacognition for teaching practice

Mental models of students' knowledge structures (schema) and processes for using this knowledge (mental operations) described by research literature provide very explicit information about the complex and ambivalent relationship between students' prior knowledge of whole numbers and accommodation of contradictory decimal knowledge (section 2.3).

Using these models, and related research literature, strategies for conceptual change have been suggested by curriculum authorities (ACARA, 2012; QSA, 2012). However, as demonstrated in section 1.5.3, there are clearly gaps in the literature; it does not explain why theories and even pragmatic teaching strategies for teaching mathematics do not evidence the same success found in research studies in most actual classrooms. It is therefore assumed that fundamental elements must be lost in interpretation or implementation of classroom practices in synergy with theories.

These disparities highlight the necessity for metacognitive (Flavell, 1979; Schreiber, 2005) monitoring prompts for subjective assessment of one's own practices to bring synergy between theoretical knowledge and practical teaching application.

2.5 METACOGNITIVE TEACHING STRATEGIES

This section discusses the implications of research literature for the metacognitive approach taken by this research. This is followed by discussions on teaching strategies to facilitate metacognition (error analysis, community of inquiry, peer teaching and problem posing). A list of each of the metacognitive teaching strategy and their learning experience objectives are provided in Appendix B.

2.5.1 Metacognitive approach to teaching strategies

The need for a metacognitive approach to learning, teaching and researching is well illustrated by literature reviewed from cognitive, social and critical perspectives. This includes the implications drawn from the literature for the history of mathematical difficulties experienced by students (Baturo, 1998; Hiebert & Wearne, 1985; Moss & Case, 1999; Widjaja et al., 2011); the social focus (Dumitru, 2012; Topping, 2005) of learning in terms of shared social responsibility (Atweh & Brady, 2009); social justice (Gee, 2005), cultural diversity and equity in participation (Bland & Atweh, 2007; Cook-Sather, 2006); the expectations of the curriculum in preparing students for the present and future through maths education (ACARA, 2012; Silva, 2009); and individual and personal interpretations made by students in constructing their own reality within these factors (Gee, 2005; Lave & Wenger, 1991).

While these agendas have different foci and have some variances in their conclusions, they share common foci in highlighting the usefulness of metacognition when seeking improvement in areas of interest. Therefore, metacognition provides a framework for a teaching and learning approach in this research to: (a) co-investigate students' pre-instructional conceptions of decimal numbers and how these conceptions develop through the use of metacognitive teaching strategies; and (b) co-evaluate practices to improve strategies for learning.

As a metacognitive approach, the teaching strategies focus on developing three main metacognitive factors: awareness, evaluation, and regulation.

- Awareness relates to students' consideration of where they are at the start, during, and at the end of a learning experience in terms of their mathematical knowledge, attitudes, beliefs and emotions and what has to be done in order to successfully engage in learning.
- Evaluation refers to judgements students make about their own thinking processes, capacities and limitations.
- Regulation occurs when students draw upon their awareness of their own knowledge and skills as well as hindrances (such as confidence, preferred ways of learning, values and dispositions) to plan, self-correct and set goals to facilitate metacognitive activity.

2.5.2 Error identification and analysis

As discussed in section 2.2, the extent and variations of misconceptions leading to errors in the decimal domain is a noteworthy cause of concern echoed in most research literature. This poses the questions: How are errors unlearnt? What are the cognitive mechanisms that enable students to detect and correct their own errors in practice? Cognitive theories (section 2.3.3) propose ideas about how students can use error detection, analysis, revision and correction to learn.

This research proposes that first, students must be aware of their errors to learn from them (Borasi, 1994). This awareness is particularly important in the decimal domain where prior knowledge stands in conflict with decimal knowledge (section 2.2.4). As such, metacognitive awareness and regulation is central to learning from errors.

According to Fernandez-Duque, Baird, and Posner (2000), metacognitive regulation refers to two processes that coordinate cognition: (a) cognitive monitoring (e.g., error detection, source monitoring in memory retrieval); and (b) cognitive control (e.g., conflict resolution, error correction, inhibitory control, planning, resource allocation). Both these processes fit in well with the decimal domain where error monitoring and error detection are fundamental to activate conceptual change needed for students to accommodate new decimal knowledge concepts and rules to their existing whole-number knowledge (section 2.2).

The prevalence of errors resulting from misconceptions about decimals (section 2.2.4) suggests that students are unable to internally evaluate their own performance to detect errors in the absence of external feedback. This theory also explains why students resist accommodation of new decimal knowledge (Vamvakoussi & Vosniadou, 2010). Unawareness of the errors they have made leads to a self-assessment of their understanding rendering the need for accommodation unnecessary.

Literature from behaviourist learning theory (Skinner, 1968) adds to the understanding of how errors in decimal knowledge can be approached. Skinner developed a learning theory which describes learning as the formation of associations between responses. A stimulus is that which is produced as a reaction to an individual organism. A response is the behaviour which is produced as a reaction to a stimulus. Skinner says that behaviour becomes a habit when a specific stimulus elicits an automatic response.

Two functions of learning from error analysis can be generated from behaviourist learning theories to elicit metacognition for this research: (a) contrastive error analysis, and (b) error identification. The basic concept behind contrastive error analysis is that a structural picture of concept and rules related to decimal numbers can be constructed which can then be used in direct comparison with the structural picture of whole numbers. Through a process of mapping one system onto another, similarities and differences could be identified. Identifying the differences would lead to a better understanding and prediction of the potential problems and misconceptions that a student would experience. This contrastive error analysis can also be used to improve learning practices by contrasting errors made in practice and considering the benefits of alternatives.

The importance of error identification is that errors can be used for learning. Errors provide evidence of the student's strategy and competence in the decimal domain (Durkin & Rittle-Johnson, 2012). These errors also define and point to students' concepts that inhibit understanding. Corder (1981) provided three classifications for errors:

1. *Pre-systematic*: errors that occur before the student has realised any system for classifying mathematical knowledge being learnt (the student can neither correct nor explain this type of error).

2. *Systematic*: errors that occur after the learner has noticed a system and error consistently occurs (the student can explain but not correct the error). The second classification also relies on other difficulties such as complexity of the structure of language and symbols used that encourages overgeneralisation, incomplete application of rules, and failure to learn conditions for rule application.
3. *Post-systematic*: errors that occur when a student is consistent in his or her recognition of systems (the student can explain and correct the error). Therefore this type of error is seen as a mistake in this research.

These classifications provide an insight into the different types of possible errors. However, as Luu, Collins, and Tucker (2000) point out, a metacognitive approach to errors not only triggers cognition, many different levels of emotions may also be triggered depending on students' personality traits.

2.5.3 Community of inquiry

Sociocultural theories (section 2.3.5) highlight the centrality of interactions with others in learning. The community of inquiry draws from elements of communities of practice (Lave & Wenger, 1991), social learning theories (Bandura, 1977) and social interaction (Vygotsky, 1978).

Researchers describe community of inquiry as a classroom approach to learning that focuses on fostering critical, creative, and caring thinking in ways that educate emotions and help participants in making informed judgements that account more fully for others' perspectives (Brubaker, 2012; Dumitru, 2012; Kennedy, 2004; Lipman, 1998).

Argumentation through collaborative reasoning has been recognised as a key element in developing the ability to reflect and make rational choices among competing alternatives (Mercier, 2011). As cognitive conflict theorists suggest, the very idea of confronting cognitions suggests a particular theory about what constitutes learning (Huang, 2008; Liu, 2005; Merenluoto & Lehtinen, 2004).

Reznitskaya (2007) argues that generating evidence and offering relevant reasons, counter arguments, and rebuttals are critical for learning. With the current emphasis on structural semantic understanding of mathematics (Baturo, 1998), this research takes the perspective that collaborative argumentation is imperative in mathematics, particularly in the domain of decimals where misconceptions have been seen to persist and argumentation could be used to assist students to navigate through their thinking.

This view is supported and integrates cognition, metacognition and social learning theories. It is a theory with strong connections to the neo-Piagetian notion of social conflict in catalysing learning (Alexander, 2009). The cognition theory on schema suggests that knowledge consists of mental structures or schemas and learning involves generation and modification of these schemas, and successful transfer entails accessing and applying relevant structures (as discussed in section 2.3). To explain the acquisition of decimal knowledge through argumentation, a social learning perspective (Vygotsky, 1978) is taken which suggests that through socialisation into argumentative discourse in dialogic collective settings, metacognitive processes (as discussed in section 2.5.1) are activated.

Dialogue in this research takes form as a philosophical inquiry analysis with the intent to cultivate metacognitive understanding of decimal concepts through the open sharing of perspectives about decimal knowledge as well as the learning and teaching practices used to facilitate the acquisition of decimal knowledge.

However, this research also acknowledges that confrontational dialogue is not always easy or pleasant for students experiencing difficulty. Reed (2006) warns that a culture of cynicism and complacency can ensue as students discern a disparity between their own knowledge and those of others. This suggests that management of argumentation is critical in encouraging enthusiasm and commitment to enter into dialogue with others.

2.5.4 Peer teaching and problem posing

Research literature shows that peer teaching engages students actively in several learning experiences that include analysing methods of teaching as well as the effectiveness of the methods (Topping, 2005; Wagner & Gansemer-Topf, 2005).

Vygotsky and Piaget developed two widely accepted theories of learning (Piaget, 1985; Vygotsky, 1978) that contribute to our understanding of the process for peer teaching. Vygotsky believed advanced or more knowledgeable peers, teachers, or other adults are capable of assisting and directing the learner in such a way as to promote a learning dialogue. As the learner engages with more knowledgeable individuals, the learner is able to begin the process of co-construction of knowledge. During the co-construction of knowledge, the learner is able to acquire the skills and confidence necessary to begin the process of concept proficiency (Robbins, 2009).

Piaget (1985) believed that interaction between individuals occurred on a continuum from constraint to cooperation. When learners engage in a task with a person whom they are obligated to obey, the learner feels a sense of constraint which can alter or discourage the learning process. On the other hand, when learners communicate with a peer, a feeling of cooperation emerges, forming a foundation for retained learning (Velez, 2011).

Within this context, problem posing is a natural part of teaching and learning in terms of generating a new problem, reformulating or restating the problem, or formulating a new problem by modifying the goals or conditions of an already solved problem (Silver, 1994).

The findings of Toluk-Ucar (2009) from research of pre-service teachers showed that prior to engaging in problem posing, most of the subjects felt confident in their knowledge and ability to teach fractions. However their views considerably changed after engaging in problem-posing activities. They claimed that problem posing had helped them understand fractions better and realise memorised rules, and that knowing how to carry out procedures was not a good account of knowing mathematics. This finding not only highlights the potential of problem posing but also its benefits to students as peer teacher and peer tutee.

2.6 AFFECTIVE INFLUENCES ON LEARNING AND TEACHING

Researchers (Brown, 1987; Clore & Storbeck, 2006) claim that affect filters and regulates our thinking and actions, influencing how we learn and teach. Affect in this research refers to emotions, beliefs, and attitudes including hopes, intentions, and dispositions (Alsop & Watts, 2003).

This section briefly discusses the change in direction taken by this research to consider the role of affect in decimal knowledge acquisition (section 2.6.1). The overlap between cognition and affect is explored from the perspective of neuroscience and neuropsychology (section 2.6.2). The overlap between cognition and affect in pedagogical literature is then presented (section 2.6.3). This is followed by a discussion on affective and cognitive regulation strategies (section 2.6.4).

2.6.1 A change in direction of this research to include affect

Most pedagogical-based research on decimals has primarily focused on topics such as how students reason, and their conceptions and misconceptions about decimal concepts (Rittle-Johnson, Siegler, & Alibali, 2001; Schneider, Grabner, & Paetsch, 2009). Ideas on how students could construct an understanding of decimal concepts and what types of activities (Stacey, 2005; Steinle & Stacey, 2004) could support these constructions have been linked to cognitive outcomes.

Similarly, the initial intention of this research was to focus only on students' cognitive and metacognitive processes that underlie decimal numbers. When probing more deeply into students' thought processes and actions during the implementation of this research (Chapters 4 and 5), the students changed my thinking. That is, to realise that for this research, students' cognition could not be understood in isolation of affect.

The importance of affect in learning decimal concepts was empirically justified (Chapters 4 and 5; section 6.3). Students' affect was found to drive attention and conation which in turn was noticed to drive knowledge acquisition and memory. While high levels of anxiety were observed to hinder motivation and confidence, and thus encourage a surface approach to learning, moderate levels were found to motivate learning and performance (Hancock, 2001; Putwain & Best, 2011; Weiner, 2011). This suggests that the relationship between affective and cognitive responses is not as straightforward as it might appear, and therefore merits further investigation into the role of affective constructs.

2.6.2 Overlap between cognitive and affective neuroscience

Neuropsychology and neuroscience research literature supports the change in direction of this research to a more balanced analysis of the role that affect might play in the basic mechanisms of cognition (Alsop & Watts, 2003; Davidson, 2000; Forgas, 2008; Izard, 2011; Moss & Damasio, 2001). These researchers argue not just for the importance, but actually for the necessity of including a serious account of affect in any systematic analysis of cognition. Davidson (2000) explains that emotion has evolved to facilitate the organism's adaptation to complex challenges. He states that "cognition would be rudderless without the accompaniment of emotion, just as emotion would be primitive without the participation of cognition" (p. 90).

Researchers (Davidson, 2000; Izard, 2011; Moss & Damasio, 2001) argue that one of the strongest evidence for considering both cognition and affect is that the circuitry of affect and the circuitry of cognition at least partially overlap. Anatomical facts provided by the researchers show that the brain circuitry of cognition and emotion is not segregated. Some researchers such as Davidson (2000) argue that, in fact, there are no parts of the brain dedicated exclusively to cognition and others to affects.

Moss and Damasio (2001) argue that emotions and the feelings that follow emotions during learning are an integral part of laying down long-term memory, and for reasoning and directing choices during decision making. Recollections from our past and reflections upon our future are noted to give rise to emotions such as fear or happiness in anticipation of positive or negative future outcomes.

Signs of anxiety observed during the administration of the diagnostics test (Chapter 4) have been associated in research literature (Hancock, 2001; Putwain & Best, 2011; Weiner, 2011) with narrowing of attention, heightened emphasis on negative information, interference with working memory and general impairment of cognitive processing ability. Emotions are said to impact on students' performance by changing brain dopamine levels affecting the consolidation of episodic memory—directing attentional processes, facilitating or impeding self-regulation of learning and performance (Hancock, 2001; Lyons & Beilock, 2011; Putwain & Best, 2011; Sparks, 2011; Weiner, 2011).

2.6.3 Overlap between cognition and affect in pedagogy

The role of affects in learning is well expressed by Claxton (1989) who states, that "cognition doesn't matter if you're scared, depressed or bored" (p. 155). Literature (Alsop & Watts, 2003) suggests that affects have considerable influence on what happens in a classroom and the nature of students' and teachers' practices. Thus, concerns about students' claims of boredom and growing lack of interest in mathematics (section 1.4.1), as well as teachers' lack of confidence in teaching decimal concepts (section 2.2.5), provide some indications that mathematics pedagogy is not only cognitively driven but also affectively.

Researchers (e.g., Demetriou & Wilson, 2009; Forgas, 2008) argue that students' affective reactions and mathematical reasoning are deeply intertwined. Cognitive processes determine emotional reactions and, in turn, affective states influence how students remember, perceive, and interpret situations and execute interpersonal behaviours.

Teacher and student attitudes and beliefs about their own capacities have been found to be closely associated with a wide range of learning and teaching behaviours (Bandura, 1977). Affects are closely associated with motivation and metacognition (Keith & Frese, 2005). Drawing upon theoretical accounts, attitudes towards learning mathematical ideas form a central concept in this respect. For example, beliefs about self-efficacy play a major role in whether or not people are motivated and able to adopt and follow systematic metacognitive strategies (Bandura, 1977). Students who are low in self-efficacy are easily discouraged by challenges and failures, and they tend not to apply appropriate self-regulatory goals (Sun, 2012).

Bandura (1977) noted that individuals avoid activities in which they judge themselves as having inadequate coping abilities, and engage in situations where they judge themselves capable. Furthermore, they are affected and deeply saddened by their perceived inefficacy to gain highly valued outcomes; thus, they have a memory bias for past failures and other negative events. Bandura (1977) hypothesised that evoked memories related to failure are likely to lower efficacy beliefs, whereas one's self-efficacy judgements may rise due to thoughts of success and feelings of wellbeing.

Demetriou and Wilson's (2009) list of other researchers that have attempted to synthesise affect and cognition include: Bloom (1981), who developed a taxonomy of the affective domain which attempts to describe how individuals' affective reactions are the basis for cognitive shifts; and Stepans' (1996) conceptual change model which places students in an environment that encourages them to confront their own preconceptions and those of their classmates, then work towards resolution and conceptual change. These perspectives in pedagogical contexts demonstrate affect and cognition as being mutually constitutive.

2.6.4 Affect and cognitive regulation

Learning as described by Alsop and Watts (2003) involves “moving from the familiar to the unfamiliar, traversing the emotional quagmire of success, self-doubt, challenge and classroom identity” (p. 1043). This “risky business” (Claxton, 1991, p. 99) of learning highlights the need for regulation of emotion and control of cognition. From a general standpoint, the control of cognition and regulation of emotion aim to accomplish similar tasks. This involves the recognition of internal or external events that are not consistent with an internal goal state, and the subsequent change of these events (Clore & Storbeck, 2006).

Several teaching and learning strategies for cognitive control are found in literature (section 2.5). These cognitive strategies provide understanding of the process by which relevant information is selected and information processing is organised to make sure that thought and behaviour are consistent with internal goals and plans.

Less is known about the emotion regulation strategies students choose to use when confronted with difficulties in learning in a classroom (with or without the presence of others). Yet, emotion is argued to be a more powerful determinant of our behaviour than our brain’s logical processes. Alsop and Watts (2003) point out that emotions that arise during learning have the potential to overwhelm thinking and concentration rendering intellectual efforts ineffective. At the other extreme are feelings of excitement, enthusiasm and confidence that can powerfully draw students into a flow of eagerness to learn.

Scarcity in research literature may be explained by the situated nature of emotions. Psychology literature views regulation of emotions as highly adaptive experiences based on how a student attends to a situation and evaluates it in terms of consistency with internal goals (Clore & Storbeck, 2006; Lyons & Beilock, 2011). Research literature (Moss & Damasio, 2001; Izard, 2011) suggests that differences in mechanisms of emotion regulation depend on the specific individual as opposed to the type of emotion regulation, and are thought to be predictive of certain aspects of personality.

2.7 SUMMARY

In summary, the review of research literature offers some reflections that, in a broad sense, invite critical exploration of cognitive processes as well as the challenge of understanding how affects might play a key role in the difficulties both teachers and students have with decimal concepts. In doing so it raises some general questions for this research including:

- What views do students hold about their difficulties with decimal concepts?
- How might we make students aware of the misconceptions they have in their thinking about decimals?
- How might students react (cognitively and affectively) to their awareness of difficulties?
- In what ways might we structure learning and teaching to encourage students to confront their difficulties and regulate their emotions, attitudes and beliefs to optimise their learning potential?
- What happens when students and teachers work together to plan, learn and teach using a pedagogical approach that prioritises both emotion and cognition?

The literature review offers insight into the various misconceptions students have with decimals (section 2.2.4). The analysis of misconceptions presented by researchers suggests that students have superficial structural knowledge of concepts underlying decimal numbers (Baturo, 1998; Lee, Brown, & Orrill, 2011; Nesher & Peled, 1986; Resnick et al., 1989; Stacey, 2005; Steinle & Stacey, 2004). Furthermore, these misconceptions are suggested to be resistant to change, as evidenced by documented problems that extend well beyond school students to adults including teachers, nurses and other professionals leading to critical errors in their roles (Holmes, 2012; Ubuz & Yayan, 2010).

While literature learning theories (section 2.3) show differences in pedagogical ideologies and perspectives about learning, they emphasise the social, cognitive and affective nature of learning. These theories infer learning as a complex mixture of knowledge about learning, attitudes, beliefs and capacities and skills to think and act on thinking in ways that make use of one’s knowledge about self with others.

Socio-constructivist and metacognitive variables are heavily cited in mathematics-education related literature. Variables such as self-regulation, monitoring, and collaborative interactions have been linked to cognitive learning outcomes. Differences among students in how students learn and the role of prior knowledge in knowledge construction indicate that learning entails methodical pluralism which should start from where students are at. While the influence of affect is implied by psychology-based research literature as a critical determinant of students' conative ability to persevere with difficulties with decimal concepts, it is unclear why research literature on the synthesis of cognition and affect is limited.

Literature on teaching recommendations (section 2.4) also implies a cognitive and affective focus for developing students' mathematical capacities in the perspective of 21st Century objectives (section 2.4.1). These capacities include more than numeracy skills capacities; they include capacities such as being independent thinkers, problem solvers, decision makers and emotionally literate (ACARA, 2012; section 2.4.3). This objective suggests that an important skill that students need is knowing what to do when they don't know what to do (Claxton, 2006).

From these theoretical frameworks offered in literature, this research took a co-constructive approach (SPARC) to learning, teaching and researching. This meant that teacher and student assumed roles as co-directors and co-editors of knowledge where knowledge is not just seen as a demonstration of skills, but actively co-constructed.

The teaching strategies (section 2.5) selected for this research were based on building cognitive and affective capacities such as building rapport to foster comfortable depth in engagement, agility to learn and resiliency to persevere.

The continual reflective assessment of success in decimal knowledge acquisition was established through content, procedural, cognitive, affective, and practice analysis.

- Content analysis to determine students' declarative knowledge through research instruments such as pre-tests and post-tests to determine the presence and frequency of certain terms in order to reveal what students know about decimals.

- Procedural analysis to determine students' mental models of procedural knowledge through focusing on both implicit and explicit factors as to not only how, but also why certain actions are performed.
- Cognitive analysis to describe and compare changes in knowledge structures of content as well as relationships between concepts.
- Affect analysis to determine the influence of emotions, beliefs and attitudes on cognition and map changes in beliefs, attitudes and emotions related to the learning experience.
- Practice analysis to determine the effectiveness of teaching, learning and researching practices and to make changes needed.

As a result, this research contributes to research knowledge about how students cognitively process decimal knowledge as well how students might develop a sense of self as a teacher, learner and researcher in mathematics.

Chapter 3: Research Design

This chapter describes the research design which was trialled as a means of teaching, learning and data gathering to achieve the objectives of this research stated in section 1.3 of Chapter 1.

This chapter discusses the overall theoretical framework of methodology and the multiple method approach taken by this research with references made to research literature used (section 3.1). The participants are described (section 3.2); a brief overall procedure is then provided in relation to the results discussed in Chapters 4 and 5 (section 3.3); instruments are listed and justified, and administering procedures are given (section 3.4). This is followed by explanations of the teaching strategies (section 3.5) and methods used for data analysis (section 3.6). Finally, ethical considerations are discussed (section 3.7).

3.1 THEORETICAL FRAMEWORK OF METHODOLOGY

3.1.1 Overview of methodology

This research is predominantly qualitative and interpretive with some quantitative references to pre- and post-test comparisons. Therefore, quantitative data is not presented as detailed descriptive statistics. However, references to data collected from test results draw on some elements of mixed methods (Creswell, Plano-Clark, Gutmann, & Hanson, 2003; Waszak & Sines, 2003) to make critical judgements through triangulation (Tashakkori & Creswell, 2008) of data produced from both approaches.

Qualitative (Burns, 2000) and interpretative (Erickson, 1986) methodology provided a means to gain depth of understanding of students' cognitive processes underlying decimal numbers. The design is qualitative in the socially constructed nature of reality, the intimate relationship between the researcher and what is studied, and the situational constraints that shape inquiry (Burns, 2000; Denzin, 2006; Tashakkori & Creswell, 2008).

It is interpretive and collaborative in its focus on understanding transformative learning and teaching in terms of the teacher and students as co-researchers;

furthermore, in its emphasis on research processes, conditions and resources through which reality is apprehended, understood, organised and conveyed in everyday life (Reason & Bradbury, 2008).

It is intervening in that findings from reflective dual action research cycles (Kemmis & McTaggart, 2000) are used to develop and trial metacognitive teaching and learning strategies to remediate students' difficulties with decimal numbers. It is process focused in that it seeks to trace both an individual's and a group's learning by understanding successive patterns in the reasoning and thinking displayed and the impact of varied instructional strategies on that reasoning and learning.

The data collecting approach is collegial (SPARC) and considers the student, the teacher and the context in the process of learning and finding out together. Hence, the processes are iterative (Cobb, Confrey, diSessa, Lehrer, & Schäuble, 2003; Kemmis & McTaggart, 2000) in that they involve linked design–analysis–redesign cycles that move towards learning, teaching and research process improvement.

This research includes a combination of action research, case study (one class) and teaching experiment (illustrated in Figure 3.1 and further described in section 3.1.2).

3.1.2 Multiple research method approach

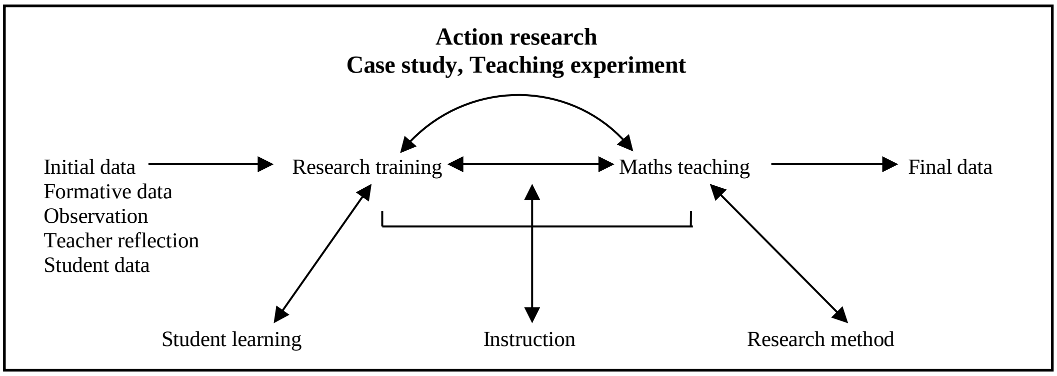


Figure 3.1 Overview of research methodology.

This research draws on multiple theories (illustrated in Figure 3.1) to adapt research methods and provide a theoretical framework for generating, selecting, and validating research processes that support the objectives of this research (diSessa & Cobb, 2004; Waszak & Sines, 2003).

A combination of methods was used, namely: (a) action research as the overarching method; (b) case study (one class); and (c) aspects of teaching experiment. This selection of research methods is based on insights from the review of literature (Chapter 2) about how students learn and how researchers can study the underlying cognitive process of development of the individual students. Learning theories draw on cognitive, affective, socio-constructivist perspectives to address social, psychological, sociological and didactical influences on learning.

Action research

Action research (Bland & Atweh, 2007; Kemmis & McTaggart, 2000; Reason & Bradbury, 2008) fits in well with the objectives of this research, because it pursues the dual outcomes of taking pragmatic actions to improve own practice by bringing about change (in teacher and students) and gaining new knowledge about teaching and learning of decimal numbers.

Understanding of action research used by this research is encapsulated by Carr and Kemmis (1986) who describe action research as a form of self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own practices, their understanding of these practices, and the situations in which the practices are carried out.

The participatory and collaborative focus of action research in this research (SPARC) extends students' responsibility to being the co-transformer rather than the reader of objectives. This approach is drawn on the assumption that without engaging students as co-researchers and co-constructors (Atweh & Brady, 2009; Cook-Sather, 2006) of decimal knowledge, there would be no basis for coming to shared understanding of the objectives of this research.

This approach fits in with Reason and Bradbury's (2000) definition of action research as a participative and democratic process that seeks to do research with, for and by students. That is, to redress the balance of power in knowledge creation and to do this in an educative manner that increases participants' capacity to engage in inquiry.

To this end, the overall purpose of action research in this research was to collaboratively improve practice through analysis of data about emergent and immanent properties of researching, teaching, and learning mathematics by co-researchers that hold different world views. It is assumed that through co-production of knowledge with students as co-researchers, emancipation might follow (Atweh & Brady, 2009; Reason & Bradbury 2008).

Case study (one class)

The purpose of using case study method (Yin, 1994) in this research was threefold. The first was that case study enabled focused in-depth analysis of data collected from students who were representatives of different levels of proficiency, hence allowing for an in-depth understanding and thick descriptions of students' cognitive processing within the context of learning and teaching decimal numbers. Second, it provided a systematic framework for looking at events, collecting data, analysing information, and reporting the results. Third, it generated and tested hypotheses that emerged through research (Bitektine, 2008; Waszak & Sines, 2003; Yin, 1994).

While action research provided the means to study changes that resulted from intervention through a number of reflective stages (Chapter 5), in contrast, case study was teacher centred, involving observation of students during the processes of action research. As such the two methods complemented each other by providing opportunities for triangulation of data and strengthening of the validity of the research.

Teaching experiment

The purpose of using teaching experiment (Brown, 1992; Cobb & Steffe, 1983; Denzin, 2006) in this research was to improve students' decimal knowledge by trialling a series of interventionist teaching strategies (Appendix B) that were both pragmatic and theory driven. These teaching strategies were used to investigate students' pre-instructional conceptions of decimal numbers and how these conceptions developed as a result of the intervention (Chapter 5). As such, the purpose of the teaching experiment was not simply to find instructional sequences that teach the mathematical construct of decimal numbers, but also to come to an analysis about how instructional sequences may or may not have cognitively broadened and stimulated the students' thinking about decimal numbers.

A common concern in literature of the large amount of data generated from the teaching experiment (Cobb & Steffe, 1983) was managed by using case study strategies (Yin, 1994) in this research. Since teaching experiment involved several in-class interventions, it was necessary to collect information about the students' mathematical activity between the sessions so that other influences could be considered when interpreting data. To do this, an action research cycle of plan, act, observe, and reflect (Kemmis & McTaggart, 2000) was used to organise and present data from the teaching experiment research.

As illustrated in the arguments above, the three overarching research methods were used to complement each other, minimise weaknesses of individual methods and strengthen the interpretive potential of individual methods.

3.2 PARTICIPANTS

As interpretative, qualitative research incorporating action research, a relatively small sample of student participants was studied in depth rather than attempting to achieve a representative sample.

The participants in this research included the teacher, teaching assistant and 23 Year 6 students aged between 10 and 12 years from a co-educational catholic school located in the city of Redcliffe. All students in one class participated in this research as part of the classroom mathematics curriculum in Year 6.

Twelve students were initially selected for in-depth analysis of cognitive growth and cognitive processing resulting from participation in the study. An extra student (U5) was added to the low-proficiency group at a later stage. Selection of students as case study focus group participants was based on students' test results and findings from interviews to ensure representation from three different levels of proficiency. That is: four proficient students (H1, H2, H3, H4); four semi-proficient students (S1, S2, S3, S4); and five low-proficiency students (U1, U2, U3, U4, U5).

Situational analysis indicated that students within the focus groups came from diverse backgrounds and differed in terms of social, emotional and behavioural needs. For example, a student (H1) from the proficient category was on individualised learning plans as part of the ascertainment process for Autism Spectrum Disorder. A student from the low-proficiency category (U5) had a history of documented difficulties that included high anxiety and learning difficulties in literacy and numeracy. Therefore the process of inclusion for participation for this student occurred differently from others. Although the focus group of students was representative of diversity in the class, this research acknowledges that it cannot be assumed to represent the wider population.

All remaining students in the class maintained active participation in the research and contributed towards data collection of class performance (Appendix E). These students were coded as C1, C2 and so on.

3.3 PROCEDURE

The procedure used by this research involved two stages that are presented in two separate chapters:

- Chapter 4 *(Stage 1)*: Students' entry knowledge.
- Chapter 5 *(Stage 2)*: Changes in students' knowledge during and after the research.

Stage 1 is presented in Chapter 4 which details and discusses results in relation to students' entry mathematical decimal knowledge and affective reactions related to this knowledge. Results are presented in chronological order to include: administration of a diagnostic pre-test related to decimal concepts; observation of students' reaction to the difficulty experienced when attempting the test items; and discussion with students about their experience and feelings about the test.

Results from interviews that were conducted to clarify interpretations of students' responses to test items are also presented in this stage. Results obtained from marking the diagnostic tests were triangulated with interview results to inform Stage 2.

Stage 2 is presented in Chapter 5 which details and discusses results from the implementation of a teaching experiment that consisted of a series of metacognitive teaching strategies. These metacognitive teaching strategies (see section 3.5) were developed in light of findings from Stage 1 and research literature. Results from a post-test are also presented in this stage to evaluate the effectiveness of the teaching experiment and students' retention of knowledge. The results from this stage were used to determine the evolutionary changes in students' knowledge during and after the teaching experiment.

Results from both stages are presented using Kemmis and McTaggart's (1988) model of action research cycles (Plan, Act, Observe and Reflect) as described in section 4.1.1.

3.4 DATA GATHERING INSTRUMENTS

This section outlines the purpose and administering procedure of instruments used to collect data for this research. Instruments used during the two stages of the research included: situational analysis (section 3.4.1); pre, immediate and post-tests (section 3.4.2); interviews (section 3.4.3); e-portfolio (section 3.4.4); and artefacts (section 3.4.5). The conceptual framework for metacognitive teaching strategies used during the teaching experiment is described in section 3.5.

3.4.1 Situational analysis

Purpose

Priori situational analysis (Skilbeck, 1984) of external and internal factors provided situated understanding of the context of the research. Information about specific needs, experience, motivation and strengths of students enabled the construction of a framework for a set of observable issues and interpretation of data in the research.

Procedure

External factors highlighted by the review of literature related to: (a) factors mediating acquisition of mathematical concepts and skills related to decimal numbers; (b) pedagogical, psychological, sociological issues influencing the discourse of teaching and learning; and (c) research paradigms, their assumptions, philosophical, theoretical, instrumental, and methodological foundations to provide a framework for a collegial research approach. Triangulation of knowledge gained from the review was used to make correlations between theoretical perspectives, mathematics curriculum and the school's policies.

Internal factors examined the needs of the student participants. This was achieved by examination of documented learning history, background information, discussion with learning support staff and past teachers and critical reflections of my experiences with students as their class teacher.

As a co-researcher, self-reflection of my teaching practices, values, subjectivities, perception of students' abilities and an overall impression of the school ethos was achieved through journalling and dialogue with the head of school.

3.4.2 Decimal diagnostic test

Purpose

An established diagnostic test (Appendix A) related to decimal concepts (Baturo, 1998) was used for the pre- and post-tests. The theoretical foundations on which Baturo (1998) has constructed and used this test in her longitudinal studies of Year 6 presents a strong rationale for the suitability of this test for the purposes of this research.

The diagnostic served three main purposes.

As a diagnostic pre-test, it provided data on students' entry knowledge on decimals and concepts and skills that underpin decimal understanding. It also pinpointed gaps in knowledge, difficulties and misconceptions. The varying nature of questions devised by Baturo (1998) in test items made it possible for responses to be crosschecked for understanding, hence improving the quality of data collected because it gave an indication whether students had made careless mistakes or used routine responses with little understanding of why the response fits the solution.

As a teaching tool, it was extensively used during the implementation of teaching experiments as a stimulus resource. Each test item in the diagnostic test was organised by Baturo (1998) to focus on a specific decimal-related concept, thus allowing it to be used as an assessment as well as a teaching instrument.

As a diagnostic post-test, it provided data on students' retention and improvements in decimal knowledge, as well as the effectiveness of teaching interventions. Areas of mathematical difficulty that persisted were also discerned.

Procedure

The pre-test was implemented at the start of the research. The test was used throughout the teaching experiment for deconstruction and construction of decimal concepts within each test item. The post-test was given seven months after the research to test students' retention of knowledge.

3.4.3 Interviews

Purpose

Semi-structured interviews were used for clarification of test responses and elaboration of students' thinking to further establish their level of proficiency.

Ad hoc teacher/student interviews were used to open communication with students to ensure I could provide support for students and challenge their practice.

Procedure

One-on-one semi-structured interviews were used after the diagnostic test was marked. When seeking clarification from students, think-aloud technique (Johnston, Bottsford-Miller, & Thompson, 2006) was used to gain insight into students' thinking during the process of solving problems. Since the cognitive load of problem solving and speaking while thinking aloud was difficult for some subjects, a two-step process was used. In this method of think aloud, first data were collected in real time by asking participants to think aloud with minimum probing to avoid distractions. When silences continued for several seconds, neutral cues (such as keep talking) were used to encourage subjects to think aloud while also ensuring not to bias the data by adding external ideas to the internal processes of subjects. The second step of this method involved asking follow-up questions to clarify any unclear data derived from think-aloud techniques.

Ad hoc teacher/student interviews were used through the research. They were unplanned, unstructured and open-ended. Specific questions emerged as the interview unfolded and the questioning depended on the particular need of the student at that point of time.

3.4.4 E-portfolio

Purpose

E-portfolio was used as an instrument for a systematic and purposeful collection of work and achievement documentation. The e-portfolio was a computer and Web-based repository management system that stored students' learning, teaching and research documents. This included lesson plans for peer teaching, their collection of resources, research, test results and reflections that were used for personal and sharing purposes. Therefore the e-portfolio was used as an evidence-based tool that documented student work and engaged students in a process of continuous research, reflection and collaborative analysis of learning. The e-portfolios were personalised, which documented learning portfolios and demonstrated individual and collaborative learning processes.

Procedure

Students were given training on computer research and computer skills from the start of the year. Students were guided step by step in creating their own e-portfolio accounts. Students were given guided practice with managing, editing and using their portfolio. A manual was placed in their school web folder to provide extra support if needed.

3.4.5 Artefacts

Purpose

Artefacts and field notes collected for this research primarily included researcher's observational field notes, students' working journals (paper and digital), pre and post-test scores and lesson plans. Artefacts and field notes provided useful data for assessing causes influencing data through triangulation and for stimulating questions or challenging data collected from other instruments. Further, they provided richness and detail to the research.

Procedure

The field notes were organised as four files: (a) transcript file that recorded collaborative conversations; (b) teacher file that contained the reflections of the teacher; (c) student file that contained the reflections of students; and (d) analytic file which identified and discussed the conceptual issues and emergent themes.

3.5 TEACHING STRATEGIES

Metacognitive teaching strategies (section 2.5; Appendix B) were used as instruments to improve students' knowledge (intervention) in light of findings about students' difficulties with decimal number concepts (Chapter 4). This section discusses how the metacognitive teaching strategies were adapted and used (procedure). The teaching strategies included:

1. Error identification and analysis
2. Knowledge building

3. Peer teaching
4. Problem posing
5. Community of inquiry.

3.5.1 Error identification and analysis

Purpose

Students' diagnostic pre-tests were used as the main source for locating and analysing their own and others' errors. Errors in diagnostic tests provided students with the necessary cognitive conflict stimulants (Limon, 2001) to make them aware of their individualised misconceptions, gaps in knowledge and flaws in reasoning. This is because error analysis (Borasi, 1994) requires analysis of both the errors and solutions, unpacking the structure and interrelatedness of decimal knowledge.

Considerations were made in acknowledgement of the assumption that the absence of error did not necessarily reflect proficiency or competency. Students could arrive to the correct responses by making a guess or use thinking that generated a correct answer without having a real understanding of how and why (e.g., when comparing values like 3.51 and 3.2 to decide which is of greater value—by ignoring the decimal point or looking at the length of the number).

Procedure

Error identification and analysis was achieved through multiple teaching strategies: (a) students individually located, listed and reviewed their errors; (b) errors were discussed as a whole class; (c) errors were further analysed through peer teaching; and (d) reflections about errors were achieved through a community of inquiry. The processes used for setting up and engaging students in learning from errors involved the following actions:

- Reviewing own test paper to make distinctions between mistakes and errors.
- Clustering errors into patterns such as errors in multiplication of decimal numbers, difficulties with tasks that involved hundredths, etc.
- Ranking errors according to knowledge, for example students may have partial understanding of some test items and in other cases may not have any understanding.

- Determining patterns in error by noting how many different kinds there were within each category (such as those described in section 2.2.4, e.g., place value, position, etc.).
- Analysing causes, consequences and context of errors by questioning, reflection, critical thinking, and reasoning strategies.

It should be noted that the term *mistake* refers to a performance error in that it is a failure to utilise a known system correctly and can be eliminated with caution. On the other hand errors are a result of misconceptions or lack of structural knowledge necessary for the task (Durkin & Rittle-Johnson, 2012).

3.5.2 Knowledge building

Purpose

Knowledge building used was anchored on an interactive whole-class learning (in relation to decimal concepts) based on Vygotsky's (1978) zone of proximal development (section 2.3) to provide scaffolds and build students' knowledge and understanding.

Procedure

Learning experiences during knowledge building were designed purposefully to focus on the specific group of decimal concepts and skills targeted by the diagnostic test items. This involved: identifying and connecting students' prior knowledge; modelling of processes, procedures and strategies with explanations of mathematical reasoning; and guided practice with scaffolds based on observed needs.

3.5.3 Peer teaching

Purpose

One-on-one peer teaching was used to assist students with analysing errors made in the pre-test and learn from the errors. Peer teaching strategies in this research unite cognitive and social learning theories (section 2.3) to facilitate and deepen students' understanding of decimal concepts. Peer teaching involves concurrent learning from and with a peer or even relearning, while teaching another (Piaget, 1985; Velez, 2011; Vygotsky, 1978).

Thus, peer teaching provides a means to learn decimal concepts as well as about learning through teaching. This includes developing co-responsibility in learning, self-efficacy, trust and active participation as co-constructors of knowledge (section 2.6.3; Bandura, 1977; Miller, James, & Miller, 2001).

Procedure

Test items from the diagnostic test were used as the stimulus for peer teaching. Errors made were analysed with a more able peer to achieve proficiency in the decimal concept targeted by the test item. When proficiency was achieved reciprocal peer teaching with an equally able peer was used to further enrich decimal knowledge and understanding. Processes used for peer teaching included:

- Selecting and matching peers in terms of needs, disposition, proficiency and goals.
- Identifying and establishing goals for peer teaching in relation to mathematical objectives for the peer teaching session.
- Analysing errors by isolating and connecting concepts to located gaps in knowledge or how knowledge could be extended.
- Measuring and evaluating progress to decide when and what type of feedback to give to peers to ensure optimal learning by avoiding dependency through scaffolding and fading strategies (Enkenberg, 1994).
- Reflecting on the success or difficulties experienced in relation to mathematics as well as with peer teaching.

3.5.4 Problem posing

Purpose

Problem posing (English, 2003) was used: (a) to metacognitively focus students' thinking about the mathematical relationship among concepts within and situations related to decimal numbers needed for formulation of questions; (b) to promote interest, confidence and excitement in learning mathematics (Silver, 1994); (c) to gain insight into students' cognitive processing (Leung, 1996) of decimal numbers through analysis of the quality of questions students produced; and (d) as a form of immediate tests for assessing knowledge and ability to apply knowledge so that immediate feedback could be given in relation to the identified situated and personalised difficulty. Hence problem posing occurred before, during, or after solving a given problem.

More specifically, problem posing was purposefully used during peer teaching and during community of inquiry.

During peer teaching students posed questions for practice of mathematical knowledge and skills gained; to assess own and peer understandings through application and formulation of questions; and to promote deconstruction of implicit mathematical concepts which is needed to formulate questions.

Problem posing in a community of inquiry was to prompt critical thinking in dialogue and reflection; and to strengthen argumentation of perspectives during the exploration of mathematical concepts and practices.

Procedure

Problem posing involved generating new questions or reformulating familiar questions in relation to the mathematical concepts and skills under discussion. Problems were posed with either some variations of problem conditions or goals of the original problem or in novel contexts.

Students were required to critically examine their mathematical knowledge in the context of a problem, and identify possible solution paths to the problem. With observed increasing proficiency, students were encouraged to become more creative in posing questions such as through games and other contexts that suited the learning style or preference of their peer.

3.5.5 Community of inquiry

Purpose

The community of inquiry was used to initiate the restructuring and establishment of a reflective and democratic teaching culture in the classroom (Cam, 2008; Lipman, 1998). Authority was negotiated within the community to construct relations of mutual interdependence (Brubaker, 2012). Specifically, it was used for developing respect and focusing on the problem rather than people so that all students could feel valued, irrespective of having the ability to generate more right or wrong answers. Thus, it provided a forum where assumptions, perspectives, and meaning structures could be comfortably challenged through critical reflection and dialogical communication (Gregory, 2007).

Such setting is assumed by this research to be particularly powerful for deconstructing and constructing decimal knowledge and in explicating students' thought process. Verbalising misconceptions about decimals, listening to alternative reasoning and accounting for ideas are assumed to lead to better understanding (Dumitru, 2012; Gregory, 2007; Mercier, 2011); moreover, creating a setting where students are prompted to think for themselves.

Procedure

The community of inquiry was introduced to students at the start of the year and used in all key learning areas so that relationships and climate of the class could be established.

The objective of the community of inquiry was explained to students as a means to: (a) listen to one another with respect; (b) build on one another's ideas; (c) challenge one another with reasons for opinions; (d) assist each other in drawing inferences from what has been said; (e) seek to identify one another's assumptions; and (f) change sides with reasons to prompt understanding of reasoning of an alternative thought.

The objective of dialogue in the community of inquiry was explained to foster sound judgement through dialectical argumentation and distributed thinking. Communication in terms of argumentation (Kennedy, 2004) was defined to include strategies for restating, clarifying, connecting, calling for elaboration, providing counter arguments to suggest new directions, and asking for examples.

Internet links to literature on strategies commonly used for active participation in community of inquiry for children were given to students to guide their own research so that they could develop a richer understanding of the purpose, practices and objectives of a community of inquiry.

Using a community of inquiry, students' findings about the community were discussed, and rules of participation were negotiated and constructed. Regular ongoing critical reflection was used to evaluate its effectiveness, value and make changes to practices used. Students were encouraged to engage in ongoing research to assist with refinement of practices used in the community of inquiry.

3.5.6 Summary

In summary, the overall framework for the metacognitive teaching strategies used for this research (Chapter 5) is illustrated in Figure 3.2. These strategies are discussed in detail in section 2.5 and elaboration of these strategies in relation to their learning experience objectives is provided in Appendix B.

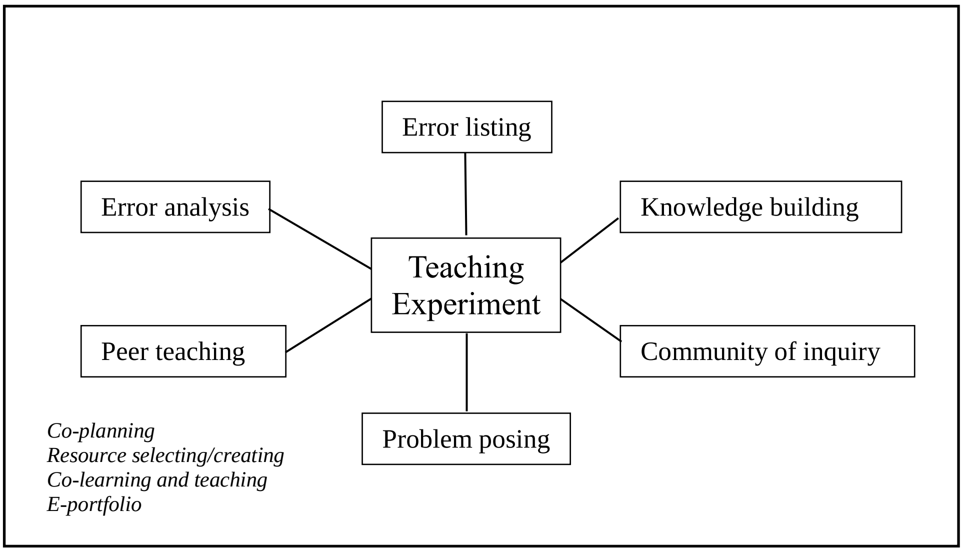


Figure 3.2 Metacognitive teaching strategies.

3.6 DATA ANALYSIS

As a mixed method research, data analysis included both quantitative and qualitative data. Quantitative methods were used to organise data from the diagnostic pre- and post-tests so that trends and patterns in students' errors could be pinpointed and growth in understanding could be easily monitored. Qualitative methods (Burns, 2000) provided avenues for exploring perspectives, experiences and feelings of students which led to deeper understanding of how these elements influence the way students think and learn.

It should be noted that the research was mainly qualitative. Therefore, although the steps involved in the analysis are shown separately below, the analysis from quantitative data was used in the thick descriptions in the analysis of qualitative data.

3.6.1 Qualitative analysis

Qualitative data consisted of detailed descriptions of situations, events, interactions, direct quotations from students about their experiences, attitudes, beliefs and thoughts in excerpts of journals, e-portfolio and records of academic histories.

Qualitative data were collected as open-ended narratives without predetermined or standardised categories. The process for systematically analysing the qualitative data collected involved the following steps.

Step one involved combining and organising data from all sources (e.g., quantitative test analysis, interviews, tests, and artefacts). The data were divided into two stages: (a) data that determined students' entry knowledge, and (b) data that determined changes in students as a result of teaching experiments.

Step two involved organising and presenting data as thick descriptions using action research cycles (plans, actions, observations and reflections).

Step three involved identifying recurring themes within data collected by reading thick descriptions and identifying connections between findings.

Step four involved theory building by analysing data with reference to the research literature in order to generate theories in relation to the objectives of the research.

3.6.2 Quantitative analysis

Analysis of quantitative data involved the following steps.

Step one involved a series of analyses to test the integrity of data obtained from the diagnostic test. Validity was checked by interviewing students to clarify interpretations of student responses. Research credibility was tested by triangulation of data from different sources to identify: (a) inconsistencies in the data within each data collection instrument, (b) unreasonable entries, and (c) impossible entries.

Step two involved conducting descriptive analyses for each test item question. Descriptive statistics were used to obtain measures of central tendency and measures of dispersion.

Step three involved making comparative analysis of data of whole-class differences to identify trends and patterns in data and organise data into categories of proficiency groups.

3.6.3 Triangulation of quantitative and qualitative method

As discussed in section 2.6, complex differences in how students learn entail methodical pluralism. No single data source can fully explain students' cognitive processing underlying decimal knowledge acquisition. Therefore an analytical approach of triangulation was used by this research to integrate quantitative and qualitative data to synthesise and interpret data.

Combining information from quantitative and qualitative data provided a means to evaluate outcomes in relation to the objectives of this research (section 1.3). By examining information collected by different methods, by different persons, findings were used to corroborate each other and reduce the effect of both systematic bias and random error present in any single situation during the research. Triangulation was also used to obtain rapid understanding of the situation and facilitate timely, appropriate decisions for actions taken in this research.

3.7 ETHICAL CONSIDERATIONS

This study involved observations of and participation of students under the age of 18 years conducted in a school setting within the course of usual teaching activities as a teaching experiment. As such, the *National Statement on Ethical Conduct in Human Research* (NHMRC, 2007) guidelines were used for this research. In line with the guidelines, this research was approved as a low risk study by the Queensland University of Technology Ethics Committee.

Consent forms to conduct the research were sent to the Archdiocese of Brisbane Catholic Education, the head of campus, principal of the school, and parents of participants. Arrangements were made to discuss the research with the head of school and progress reports including findings were communicated periodically throughout the research. A consent form was sent home with students outlining the objectives of the research, explaining student participation in the research and the purpose of instruments such as audiotapes of students' interactions, tests and other artefacts that related the students' work to the research. A parent/guardian information night was organised to clarify any questions and address any concerns.

It was also important that each of the twelve focus participants was willing to participate in providing in-depth data about their cognitive processes and development. For this reason, the research was discussed with students and students' reactivity towards research methods addressed in the initial stages of the research process.

Concern for students and, in particular, concern for the way mathematics can make students feel about their value amongst others is firmly embedded in the principles of SPARC. Therefore ongoing collaborative reflection on each research protocol was designed to ensure that respect for the dignity and wellbeing of the participants was considered to take precedence over the expected benefits to knowledge. The ethics of potentially sensitive issues such as categorising students in terms of proficiency were considered carefully. Categorisation of proficiency was used as a tool to monitor growth in knowledge and emphasise interdependence as the vehicle that facilitates proficiency instead of competition.

Chapter 4: Results Stage 1: Entry Knowledge

4.1 OVERVIEW

The results for this research are presented in two evolutionary stages:

1. Stage 1: Entry knowledge and practices.
2. Stage 2: Changes in knowledge and practices during and after the implementation of the teaching experiment.

This chapter details and discusses the results obtained from Stage 1 that determined students' entry knowledge. First, the action research cycles used to organise and present results in this chapter are explained (section 4.1). This is followed by results from the administration of the diagnostic pre-test (section 4.2) and discussion on students' thoughts about the test (section 4.3). Students were then introduced to the objectives of this research (section 4.4). Pre-test papers were marked (section 4.5), test results were checked by students (section 4.6) and discrepancies in marking were resolved through interviews (section 4.7). Finally, data collected were triangulated to draw implications about students' proficiency at entry of the research (section 4.8).

4.1.1 Organisation of results

Kemmis and McTaggart's (1988) model of action research cycles (Plan, Act, Observe and Reflect) is used to organise descriptions of the results.

The Plan cycle represents a prospective action informed by: (a) mathematical objectives; (b) students' knowledge and experiences from prior practice; (c) social and material constraints in the situation; and (d) flexibility to adapt to unforeseen circumstances.

The Action cycle recognises practice as ideas-in-action and uses action as a platform for the further development of actions. Action is guided by planning in the sense that it looks back to planning for its rationale, but is critically informed action as it is not completely controlled by plans because it takes place in real time. Action is thus fluid and dynamic, requiring instant decisions about what is to be done and the exercise of practical judgement.

The Observation cycle has the function of making inferences about the effect of the plan, action/s and unexpected events.

The Reflection cycle does not simply draw on recall of perceived effects of actions but also actively seeks to make sense of processes, problems, issues and constraints of strategic actions. It takes account of the variety of perspectives possible in the situation in relation to the circumstances in which they arise. In some situations, references to research literature are made to indicate how research literature was used to prompt deeper critical reflection.

It should be noted that the plan, act, observe and reflect cycles are not used strictly. Often there are elements of each of these cycles interwoven into each other to give meaning to the interdependent nature of this research.

4.1.2 Sequencing of results

The results are presented in a chronological order to follow the natural progression of the flow of cycles in the research. This approach is used to provide the reader with information about influences of occurrences in between events that is needed for more accurate translation and account of events. For example, test administration is presented in section 4.2 and marking and analysis of the test is not mentioned until section 4.5 where it chronologically occurred.

This flow of cycles illustrates the social reality within a pragmatic classroom context where conditions are not always linear, predictable or stable. Rather, plans and actions are continuously reshaped by messy, unpredictable and complex patterns of social, cultural and emotional experiences that are internal or external to the classroom.

Whenever possible, qualitative data collected and presented as results in this research not only takes into consideration verbatim content during dialogic interactions, but also non-verbal interaction and gestures, pauses, and inflection that influence how interpretations are made.

Direct quotes are written in non-bold *italic* fonts. To indicate who is speaking, C is used for students and T for teacher. Where there is more than one student in dialogue, numbering such as C1, C2 is used.

When more specific comparative analysis of students from focus groups is presented, each student in the focus group is referred to according to the category and level of proficiency within the same category such as H1, H2 (where H1 refers to the student more proficient than H2) and so on. Students in the proficient category are denoted as H, semi-proficient as S and low in proficiency as U.

Results from this chapter provide an important starting point for this research because they provide an insight into students' prior knowledge on entry to the research. As highlighted by research literature, students' prior knowledge is the cause of many misconceptions and difficulties with decimal knowledge acquisition (Moss & Case, 2002; Steinle & Stacey, 2004).

Therefore data collected from results in this stage serves several purposes. These include: finding out what students know and how they think; categorising and selecting students as representatives of proficiency for the focused case study; and informing the teaching experiment in Stage 2.

4.1.3 Preparation prior to starting research

In preparing a plan and setting a date to commence research, I took account of data collected from situational analysis. This included:

- Checking if students were familiar with concepts in the test. Examination of students' maths textbooks and workbook entries from the previous year level indicated that even though some of the questions were structured differently, students had been introduced to decimal concepts within the test.
- Assessing students' readiness for some of the teaching strategies used in the intervention teaching experiment. For example, training for developing skills in communicating through dialogic argumentation in a community of inquiry (Cam, 2008; Lipman, 1998) was introduced to students at the start of the year (10 weeks prior to the start of this research). Similarly, training in using computers as cognitive tools also commenced at the start of the school year so that students were able to research, learn, organise and process information using computers.

4.2 DURING PRE-TEST ADMINISTRATION

4.2.1 Plan

In view of the situational analysis, the objectives for commencement of research were to:

- Administer the diagnostic pre-test to determine students' entry levels of proficiency in relation to specific decimal concepts and skills.
- Observe students during the test to ensure active participation; identify threats that would compromise students' performance in tests, hence affecting its internal validity; and make responsive adjustments to the test and test conditions if needed.

4.2.2 Action

The diagnostic test was administered. There was no explanation given about this research and therefore, the test was perceived by students as any other class test with the same test conditions. In view of observed students' distress and extent of anxiety in relation to the perceived difficulty of the test items, the test was stopped. The test was re-administered after calming students and some adjustments to the conditions of the test were made.

Students were calmed through reassurance that results in the test would be used to improve understanding and not simply to give *a bad mark in the report card*. Instead it would be used to find out what they knew and to determine how teaching could be directed to eliminate the areas of difficulties. Humour and tone of voice was used to further reduce tension without compromising clear boundaries for conventional test rules.

Modifications to test conditions that were made included making allowances for use of calculators for students who did not know their times tables. Students were asked to place an asterisk symbol next to the test responses where they used calculators to assist with the analysis of results.

Time limits were increased allowing students an additional 30-45 minutes. This allowed all students, with fast and slow knowledge processing ability, to demonstrate their knowledge and skills to their potential.

Any assistance provided, including assistance given in reading and interpreting questions, was recorded to assist with analysis of results.

4.2.3 Observation

During the perusal of the test, students displayed high levels of distress and anxiety about the difficulty of the test and seemed overly preoccupied with the possibility of failure. This anxiety was attributed mainly to concerns that results from tests would influence their grading in report cards and worry about possible relational cost with peers, teachers and parents should failure occur.

Most students who considered themselves as high performing seemed more distressed but discreet about experiencing difficulty. Reactions from these students included being teary, shaky voice, comments such as *it is easy* (when facial expressions showed otherwise) and being fidgety. There was a clear indication that some of these students were experiencing the feeling of shame as indicated by their collapse of shoulders and movement of head and eyes in a downward motion, as though to escape the gaze of others.

Students who considered themselves as *dumb at maths* seemed more accepting of the difficulty and less stressed. However, it seemed that these students had already given up before the test had started.

Stopping the test and renegotiating some of the terms had a positive effect in refocusing students and encouraging most students to make an attempt in answering test questions. Some students were observed to use various types of strategies for management of their perceived test difficulty. For example, one student handed in the test paper 15 minutes after the test started. The student was observed to appear as if he was working hard on his test paper but was in fact waiting for a reasonable time so as not to draw attention of his peers to handing in the test. With a small amount of assistance, he was able to attempt more questions.

C: Can I hand it in now? I have finished, that’s all I can do.

T: Have you checked your answers?

C: Yes.

T: Did you attempt all the questions?

C: Yes, er… I… I…only two… last year me and Erin were in the dumb group so we didn't have to do the same tests as everyone else.

T: Ah, I can't imagine you getting out of it that easy with Mrs T, well I will have to teach you extra then hmm… (joking smile). Can you give it one more go so that I am sure you have done your best; let's go through this question together (I worked through a question without giving the answer).

Another student felt that he did not do well and was reluctant to hand over the test paper. Initially he claimed that he had already handed his test paper in. When the paper was found in his desk, he claimed that he had not finished and requested to take it home to complete. He was upset (teary, stuttering) about the possibility of failing the test and needed settling before being sent on the morning break.

4.2.4 Reflection

Research literature (Hancock, 2001; Putwain & Best, 2011; Weiner, 2011) characterises the observed test anxiety as a relatively stable personality trait. As demonstrated by students' reactions, test anxieties have the potential to generate debilitating psychological, physiological, and behavioural responses. This includes students' approach, engagement, problems in recalling material and difficulty in reading and comprehending instructions.

Students' personal standards of performance were related to active coping. A common pattern noted among students who believed that they should be high performing was, they were excessively concerned with their mistakes, and believed others would be as highly critical of them. Students who had set low personal standards had accepted disengagement as a preferable response based on beliefs of inability.

These highly aversive effects were not only observed to be integrally associated with avoidance and withdrawal but also had implications on students' wellbeing and capacity to enjoy learning. It was clear that students had learnt to define failure as an unacceptable event that carried negative implications for their self-worth and relational security.

In summary, an important implication from these findings is that students are afraid of failure and place a high priority on not failing. This leads to deterrents of participation such as anxiety, perceptions of low control, unstable self-esteem, pessimism, decreased intrinsic motivation, and decreased quality of engagement in the task.

Another adverse effect of fear of failure was students' attempts to produce false data by using cheating strategies. This finding has far-reaching implications, not just on the data from the pre-test, but on the need to have students complicit in research to obtain data that is authentic and valid.

In reflection on these findings, and findings in decimal literature which claim that cognitive conflict is important for decimal knowledge acquisition (Desmet, Mussolinb, & Gregoireb, 2010; Vamvakoussi & Vosniadou, 2010), a question that emerged was, therefore, *how do we manage the feelings of fear of the conflict?*

4.3 DISCUSSION ABOUT THE PRE-TEST

4.3.1 Plan

A follow-up class discussion in a community of inquiry was conducted immediately after the diagnostic test for the following objectives: (a) to gain insight into students' perspectives; (b) to gain further understanding for interpretation of data collected during the administration of the test; (c) to assess students' ability and willingness to reflect about their experiences; (d) to initiate the process of developing students' self-awareness of own personal views and those of others; and (e) to generate data that could be used to trace changes in thinking and affective responses.

4.3.2 Action

Three discussion topics were listed as follows:

1. Identify personal and shared experience under the subheading of thoughts and feelings about the diagnostic test.
2. Consider possible purpose/s for tests.
3. Reflect on different perspectives about the effectiveness of tests in measuring knowledge and understanding.

These topics were read and linked to the following focus questions to assist students with comprehension of the topics.

1. What did you think or feel about the test?
2. How could a test be used as a learning resource?

3. Does a maths test really measure what a student knows?

Each question was read out and students were asked to think about answers to these questions independently without speaking to anyone. Time and momentum of this activity was managed by giving students 15-30 seconds for each question (depending on my observation of students' body language to indicate active engagement in thinking and readiness for next question).

Students were told to write down their answers in their journals before coming on the floor to discuss their responses with others. Time limits were given for each question to avoid students spending too much time on one question and not being ready for the discussion. Students were reminded and encouraged to value all perspectives.

> *T: I will give you a few seconds to think about these questions, working without a sound so that you don't distract anyone, and then I will give you time to write your thoughts before we come together and share our own ideas. This way we can make sure that you all work out what you honestly think without being influenced by what someone else says.*

A concept map was drawn collaboratively listing and linking students' thoughts and ideas.

4.3.3 Observation

For the first question, "What did you think or feel about the test?", students reported feeling anxious, unsatisfied, frustrated, uncomfortable, tired, confused and upset. In agreement with my observation during the test, students' responses echoed a common theme related to being afraid. Afraid of what others will think of them and afraid of disappointing everyone who wanted them to do better. They claimed that the main motivating and reassuring comment that I had made was that others had found it difficult:

> *S: I felt so much better when you said that other people like even the year seveners found it hard. I thought if I can do some this than, yeah man…I am beating some of the year seveners.*

For the second question, "How could a test be used as a learning resource?", the most immediate response to this question was a reiteration of the comment I had made to students during the test, which was, *to help me know what to teach.* Other responses echoed perceived expectations such as *make us work harder*, *makes us revise and memorise our work*, *find out where I am going wrong*.

The body language and tone in which these responses were given suggested that students were responding as if it was a quiz question with a predetermined right or wrong answer. Further into discussion, students revealed their personal inner thoughts. They admitted that they rarely reviewed their test after it was handed back to them, and that the only object of interest to them were the marks allocated. In fact most students agreed that if possible, they *never wanted to see the test again*.

In response to the third question, "Does a maths test really measure what a student knows?", students' comments included:

> *C1: Yes, because how else would you know if you know the work or not?*
>
> *C2: I agree with C1, because in maths it's not like the other subjects where you can guess and get away with it.*
>
> *C3: No, I don't agree because what if you know and your work but just forgot for some reason, but can remember it later.*
>
> *C4: I agree with C2 and um… C1 was it? Anyway, I agree because if you know something, it means you remember, and if you forgot you actually don't know how to do the maths problem do you.*
>
> *C5: So does knowing mean that you are not allowed to forget like when you are sick or something and can't think straight?*
>
> *T: Are you saying that being able to measure students' knowledge in maths is different from other subjects?*
>
> *C1: Yeah because maths is sort of different… like rules and remembering your table and stuff.*

This discussion generated other perspectives such:

- *Yes, if the maths test covered the topic it was testing.*
- *Yes, only if the questions made sense and the teacher taught it to us well then it would be fair.*
- *No, because you say that maths is not just about procedures without thinking, but that is all it is in the test. It is a memory test about the method taught and how fast you are to finish in time.*

- *No, because it is not real except in the classroom. I think a real test would be to see if we um, kind of get put in a situation and see if we used the maths don't you think?*

Students seemed to find most comfort in talking about their thoughts and experiences that were shared by others. Initially students made responses that they perceived as safe and in agreement to cues that were given from others (who were considered more knowledgeable or socially popular).

During the discussion, most students shifted between contributing to discussion, listening, and tuning out depending on who was speaking and the personal relevance they associated with the context of conversation. A clear shift in bringing out their internal conversations with others was observed once students became absorbed in the topic.

4.3.4 Reflection

Students' reflections on their test experience were concept mapped as shown in Figure 4.1.

Figure 4.1 Students' reflections on test experience.

Students' reflections on how they could improve in the tests involved ideas about working harder in class, revising work, getting extra help from parents and the teacher (Figure 4.2). However, most students claimed that giving up was the most likely option they would take. None of the students suggested any clear ideas that involved learning from critical reflection of errors or areas of difficulties.

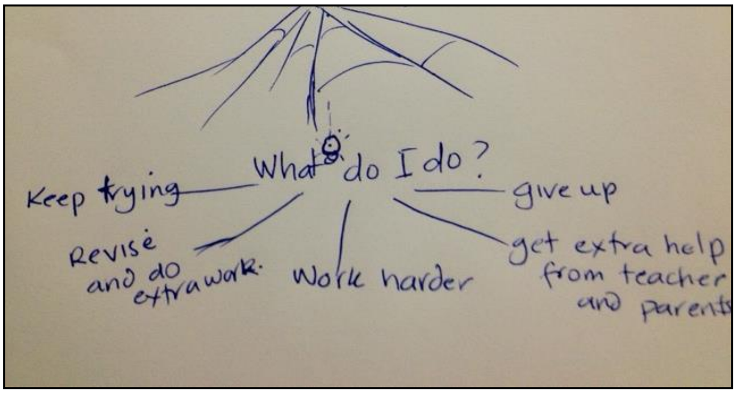


Figure 4.2 Students' reflections on improving test performance.

Students' responses during the discussion on the tests made it evident that their desire to perform well at tests was closely tied to extrinsic rather than intrinsic purposes; for example, the social implications of what others would think of them if they performed poorly. Students' reasons for doing well echoed suggestions that are often known to be given to children, such as, *If you do well, you will get a good job* or *I will give you a prize.*

Competition and a need to constantly prove worth seemed to be interwoven in their reflective thought processes. This influenced their participation in the discussion. Some students were absorbed in creating a framework for justification and found it difficult to respond in dialogue from a critical reflective position. Others were not always sincere with revealing their thoughts.

Therefore it is reasonable to conclude that, in a sense, all dialogue has a potentially artificial side that can result in subtle compliance with the perceived classroom ideology, resulting in the adoption of moulded or scripted selves.

As I stated in my personal journal*:*

> *I am not sure which students consider themselves as being the winning insiders in this discourse, those students that can maintain disengaged and have superior skills in camouflaging as to not reveal their inner thoughts or those that are undaunted by their emotions to express their ideas openly.*

4.4 INTRODUCING STUDENTS TO RESEARCH

4.4.1 Plan

This session was to introduce students to the research and their role as co-researchers and co-constructors. Therefore the plan consisted of the following objectives:

- Unpacking pertinent definitions relating to participation as co-researchers, about what participation in research entails and how it could be enacted in reality.
- Making contextualised and personal meaning of the research objectives by making connections to critical reflections generated from the previous sessions in relation to the diagnostic test experience.
- Drawing on relevant aspects of the literature review to enrich the rationale for co-research and increase the depth of students' knowledge base about research.
- Focusing students on the problem rather than the person (personal flaw/deficit in their abilities).

4.4.2 Action

Students were introduced to this research with the following focus questions and statements written on the board. *Why bother with participating in research? Aren't I already participating?*

The concept map created in the previous session that listed students' thoughts and experiences in relation to the diagnostic test was briefly revisited.

Links were made about students' perception of difficulty to those found in research literature. This was done to distance students from their focus on personal abilities to viewing difficulties in terms of global problems and focusing on the problems rather than the person.

The significance of the research was related to the use of decimals in everyday use and in situations such as in medical treatments where quantitative accuracy was essential in response to students' comments in the previous session about: *what is the point of decimals anyway!*

Co-participation in research to improve knowledge and practices was proposed. The dual objectives where clearly defined in terms of:

- Objective 1: Improve decimal knowledge based on students' identification of difficulty experienced in test.
- Objective 2: Improve learning practice based on students' identification of the need for making *boring learning* more efficient and *fun*.

The concept of participation was co-defined by students working in pairs and small groups to: list keywords related to participation; list synonyms of participation using a computer search (to expand their thoughts about participation). Collective attributes (e.g., cooperation, respect, empathy and sharing) and personal attributes (e.g., confidence, honesty) for co-participation were also listed. Students' ideas on participation were then discussed with the whole class.

The significance of co-participation was illustrated through a game. A commonly played game called Chinese whispers was used to show the effect of speaking for someone else (one person conveying the voices of others). This game was used to show how speaking for someone else can be populated with own intention and accent of the speaker.

4.4.3 Observation

Discussion with students about findings in research literature was observed to provide leverage in mediating and distancing students from focusing on their own difficulties to a shared global experience.

Is this real or are we just making it up? Students' comments and body language suggested that they already had preconceptions that research was for teachers and adults. The prospect of researching with a teacher was received with disbelief.

Students had very little difficulty in listing synonyms for participation. Interestingly, all groups stated the importance of equal participation. However, observation of students working in groups showed that, in practice, participation was not equal. Even when conscious attempts were made to include all students by assigning roles within the group, some students automatically took more dominant roles.

4.4.4 Reflection

In reflection students narrowed down four important elements of participation as: *(a) being interested; (b) sharing responsibility and wanting to share the responsibility so that others don't have to keep pushing you to do stuff; (c) giving your full attention; and (d) doing your best.*

Students were easily able to identify the attributes needed for participation. However they were also able to identify that their lists of attributes were idealistic, and that other exigent factors could influence students' participation.

In reflection, students pointed out that in reality, listening attentively to others who they didn't like was difficult. Often they were not always genuinely interested in the opinion of others unless it affirmed or gave their ideas greater significance. Without deliberate intent, they often felt inclined to encourage passive participation by cutting short others with their own anecdotes and opinions believing that other students would benefit from their expertise and experience.

A culture of dependence on experts and leaders based on perceptions of popularity within the groups was noticed as being deeply ingrained. This expert dependence was also prominent whenever I was involved. Clearly students perceived the teacher as the ultimate evaluator and with the right knowledge to pass judgement of them being a failure or success. Hence students continuously carefully observed my verbal and non-verbal cues to inform their guess of my position to ideas and would alter their thinking to match mine. Constant switching of positions and making students aware that my perspective was not fixed encouraged them to take greater responsibility in constructing and communicating their own ideas.

These reflections supported observation of patterns in student behaviours, where some students were noticed to dominate group tasks. Some students found difficulty in figuring out autonomous centred tasks while others were quicker and took advantage of the opportunity to take leading roles in activities. Similarly, speed at which different students could answer particular types of question deterred others from participating. Connections to personal experience and talents in specific areas also contributed to the unevenness in participation. Therefore democratic participation is problematised by the individuality of the participants within groups.

4.5 MARKING PRE-TEST PAPERS

4.5.1 Plan

The objective of marking the pre-test papers was to gain insight into students' performance on each of the test item as well as overall performance. Working out shown was used to makes inferences about students' understanding and knowledge about decimal numbers.

4.5.2 Action

Test papers were collected and marked. When marking the test papers, a small dot was placed next to incorrect answers. The question number was circled for ambiguous responses to indicate the need for further clarification. Marks were allocated to correct answers and the total score was written on the front page. A second independent marker who did not know the students also checked my marking. These results were recorded and placed on a spreadsheet in Microsoft Excel for analysis of data. Students were categorised into three groups (as levels of proficiency) based on the results obtained in the test.

4.5.3 Observation

The overall results in the pre-test were generally poor with very distinct differences in performance especially between students in the proficient and low in proficiency categories. Test items that required clarification seemed to be related to four main queries:

1. Students getting one question correct and getting a very similar question incorrect particularly in test item 3 (where students were required to multiply and divide with decimal numbers).

2. Almost all students did very well with test item 4 where a 'yes or no' answer was required. I was not certain if students had made a lucky guess or truly understood the concepts. Similarly in other questions a query was placed when I felt that a correct answer did not always provide clear and convincing evidence of the nature of the student's underlying reasoning process.

3. Although an answer results from a student's reasoning process, a correct answer may be the outcome of incorrect reasoning.

4. Some answers to questions seemed to suggest that the student had misinterpreted the question. For example, for the question:

   5.346 ×10

   The student wrote:

   5.346 × 10 = (5 × 10) + (0.346 × 10)

   This response suggested that the student had an understanding of the distributive law related to the question. Therefore clarification was needed to find out if the student had misinterpreted the question as asking for an expansion rather than the solution to the problem.

Other questions that required clarification were related to the legibility of texts such as two decimal points placed in a number or two answers which were not clear as working or final answer.

4.5.4 Reflection

Marking the pre-test provided considerable insight into how students understand questions and apply prior knowledge to solve decimal problems. I found that when marking the test papers, there were advantages and disadvantages in knowing the students.

For example, when interpreting a student’s incorrect responses, I was more inclined to leniency towards students that I had expected to do better, hence possibly advantaging some students over the others. On the other hand, knowing the students gave me the advantage of being able to more accurately interpret incorrect responses and determine if the cause of the error was due to lack of knowledge or actually from careless mistakes and misunderstanding the questions.

In view of this reflection, an additional action of having an independent marker to review marking was required to reduce any possibilities of bias in marking.

4.6 CHECKING FOR MISTAKES

4.6.1 Plan

Since the objective of the test was to find out students’ entry knowledge, the process of checking for mistakes was to eliminate any accidental or careless mistakes, which had the potential of giving false indication or misrepresentation of students' knowledge.

4.6.2 Action

To eliminate any errors and bias in marking, the pre-test papers were checked by an independent marker who had no knowledge of the students.

Students were also given the test papers to check accuracy of results. To do this, students were seated separately (as in test situation) and given the test paper back, and asked to check for any careless mistakes. The purpose of checking and the process was explained.

Students were guided in their review of each question because it was anticipated that not all students would show vigilance in checking their work. The questions were read out aloud and students were expected to locate mistakes and highlight the question number.

On completion of checking students were asked to go back to questions highlighted and write brief explanatory notes to assist with justifications for mistakes during the interview. When ready, students were to write their name on the board to assist with the interview process.

4.6.3 Observation

The first and most common question asked by many students as soon as they received their marked test paper was, *what did you get?* The only students who were noticed to be looking through their test items to locate where marks were lost, were the two students who had scored the highest marks in the class.

The other common comment was, *I know I am gonna do really bad in this test.* Emotional response to low test results were more evident in students that were accustomed to high achievement. A dislike for maths was voiced by most students.

Reminding students that the marks given were not their final results and that through the review process marks would be altered positively changed most students' interest in the test. Although this initial spark of interest generated compliancy in considering going through the test, most students had difficulty or became disengaged in the process when asked to write explanatory notes. Most students claimed that they did not know what or how to write notes.

4.6.4 Reflection

Clearly reviewing answers in a test was new to students. Students claimed that the activity was useful in that, *we would never have done it ourselves so we would not have known where we were going wrong.*

In reflection, students were able to identify that their dislike for maths was related to performance results. They also identified their reluctance in reviewing related to several reasons. Most of these were related to ability such as being unsure of their knowledge so seeing no point in the review. Others were related to emotions—*why would anyone want to look at something they did bad in? You already know you did bad and it just makes you feel more bad and ashamed!* There were also some mixed feelings about the interview preparation explanatory note taking activity—*I know where I made the mistake, but I don't know how to write or explain it.*

4.7 INTERVIEWS FOR CLARIFICATION

4.7.1 Plan

To conduct one-on-one interviews with students to collaboratively clarify assumptions and misjudgements made about students' knowledge; and generate data that more accurately represented students' performance results in the diagnostic pre-test.

4.7.2 Action

The interview process commenced immediately after the process used for checking for mistakes. Students' order of interview was determined using the list that was made during the marking of the test and the list made by students on the board requesting an interview. This order was determined by estimated time needed and consideration of students that I felt would be best interviewed early to maintain interest.

The interviews were conducted. Expectations of explanations given by students during this one-on-one interview included students' presentation of a logical warranted argument of mistakes made through conjecturing, supporting or jotting down procedures, representing mathematical concepts, and using language and symbols of mathematics accurately.

4.7.3 Observation

The opportunity to improve marks was a strong motivator for some students to participate in the interview:

> *I um, hid the paper in my desk as soon as I got it because I didn't want anyone to see it or think about it ever again…until you said we had a chance to change our marks. That never happens normally…usually there is nothing we can do if we fail…it is history. It's just bad luck if we had bad day like you are sick or something!*

Others, particularly those who were more accepting of their low scores, showed reluctance in participating in the interview. Similarly, students who achieved high scores were also reluctant to participate with the fear of uncertainty of knowledge and in particular being found out of not knowing.

During the interviews, the following patterns in data collected were noted from detailed transcripts of observations:

- It was not uncommon for students to stop and self-correct during an explanation and say, *No, er…that can't be right*. Therefore, some students were metacognitively exploring mathematical ideas through their explanatory talk during the interviews.
- Not all students had the language skills to articulate their thought process. The proficient students in particular felt that most of their thinking was automated through previous experience and stated that they did not know how to explain the processes they used.
- Students who were accustomed to high achievement seemed to be more distressed and nervous about making mistakes and sought creative excuses to justify and hide any weakness in knowledge.
- Some students used a range of strategies during their explanations to shield against being wrong. Words such as *I think*, *maybe*, *about*, *around*, *probably* or *what do you mean, I know this, I just can't remember*, etc. as strategies to elicit clues to get the right answer.
- Students were socially skilled and attempted to frame questions to elicit answers and relied on observations of my facial expressions and body language for social interactive cues to achieve objectives.

S: Well I thought this was the right answer because when I asked you during the test if you could show me how to do this question and if my answer was right, um… you said I can't show you how to do this question during a test, but you had that look teachers normally have and you can tell if an answer is right or wrong.

4.7.4 Reflection

Both the pencil-and-paper diagnostic test and the interviews about responses made in the test revealed that many variations of misconceptions existed in students' mental structures about decimals numbers and use of rules for computing with decimal numbers. Most of the misconceptions were consistent with those found by other researchers (section 2.2.4).

In terms of evaluating the effectiveness of the interview process, a conversational approach seemed to have provided a more natural environment for students to present their explanations. As a researcher, I found the interviews not only useful for clarification of responses but also to observe the creative ways in which students attempt to mask their lack of knowledge. A limitation of the interviews was that only certain students with the necessary disposition were willing to come forward and argue their perspectives and actively seek to have their test reviewed.

Interviews revealed that diverse factors cause large differences in student performance across modalities, of paper-based versus oral interview modes. For example, when faced with difficulties in solving maths problems, students could easily skip to the next question, whereas during interviews most students wanted to demonstrate or prove some understanding of the question. On the other hand, some students felt under greater pressure and frustration from finding difficulties in explaining their thinking. This experienced increased anxiety had a negative effect on student performance.

In review of patterns in data collected, it became evident that in the pencil-and-paper test students had relied more on recall of knowledge in questions that were procedurally familiar to them, whereas during interviews, students' responses were based more on understanding of mathematical concepts, which was needed to explain their reasoning process involved in following procedures used. Therefore interviews provided critical information in assessing students' levels of proficiency.

It was found that correct answers did not always accurately reflect students' knowledge or processes used to answer a question.

To avoid disappointment of students who lost marks due to the interview process and defuse further fixation on test scores, the reviewed test scores were not given to students. Instead students were given an explanation about the diagnostic purposes of the test.

4.8 TRIANGULATION OF DATA

4.8.1 Plan

The objectives for this activity were to:

- Analyse data through triangulation to increase the validity of data in determining the accuracy of students' test results and their levels of proficiency.
- Select a focus case study group to serve as representatives of each of the proficiency level categories (section 3.2).

4.8.2 Action

Triangulation was used to facilitate validation of data through cross verification of students' test performance results (from initial teacher marking of items) and data collected from interviews. Inconsistencies in the data within each data collection instrument and unreasonable or impossible entries were checked. Conclusions were drawn from the triangulation of data to alter and determine students' final test result.

Final results were entered into a Microsoft Excel spreadsheet. Comparative analysis of data was made in relation to individual and whole-class differences in each of the test items. Analysis of overall performance was used to obtain measures of central tendency and measures of dispersion in data.

The analysis was used to categorise students into three levels of proficiency. Triangulation of all data sources including observation and reflection cycles was used for consideration of students as representations for the case study focus group.

Students were asked to record their own test results using Microsoft Excel and generate graphs to illustrate the patterns and differences in results in relation to each test item.

4.8.3 Observation

Data triangulation

In triangulating quantitative data from test results and qualitative data from interviews, some discrepancies were found. It was clear that some students had correct answers in the test but had superficial understanding of concepts. Their accounts of justification during the interviews suggested that they relied on memory of well-practiced procedures rather than understanding.

As illustrated in the transcript below, when asked to explain reasoning used to answer the following test item, the student's response suggested that student's knowledge and understanding was weak despite being able to give the correct answer for one of the questions.

Question 4b) 0.02 × 100 4e) 0.2 × 100

When a student was asked why he had given one correct answer and got the other one requiring the same knowledge incorrect his response was:

I forgot which way the decimal point jumped. Hundred has two zeros (pointing out that both questions were multiplied by 100) so I thought if I jumped two places this way (showing movement to the right) and moved two places the other one this way (showing movement to the left) then I would get at least one of them right.

On the other hand, some students who performed poorly in the test showed evidence of using greater levels of reasoning despite arriving to incorrect answers (which was generally due to gaps in prior knowledge):

I didn't really have a clue to start with, but I looked at 0.02 and guessed it would be two 2 tenth since 2 tens looks the same with one zero next to it on the other side of 2. Then I thought 2 and tenths of something will be two time the amount then tenths probably meant I will have to divide it by 10.

Data analysis

Summary of comparative analysis (Figure 4.3 and Table 4.1) shows the spread of results between case study students in each of the three proficiency levels (described in sections 1.2.3 and 3.2) as well as how they are positioned within the whole-class mean.

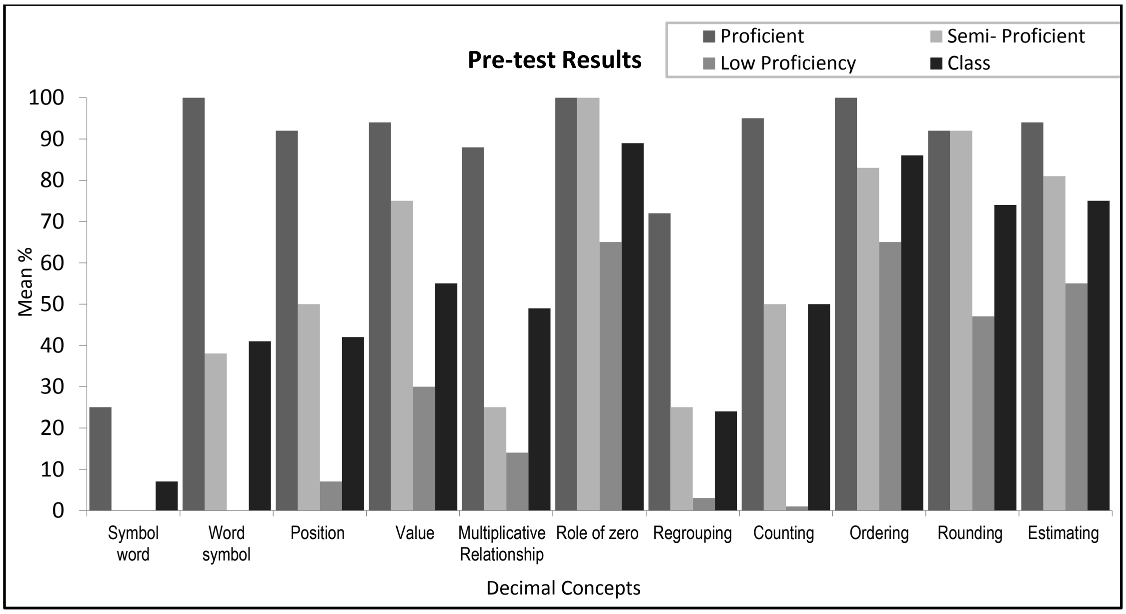


Figure 4.3 Distribution patterns of the performance categories.

Table 4.1

Class Mean in Pre-Test Items

Test item		**Concept**	**Proficient %**	**Semi-proficient %**	**Low Proficiency %**	**Class %**
Item 1	NI	Symbol to word	25	0	0	7
		Word to symbol	100	38	0	43
Item 2	PV	Place value – position	92	50	8	43
		Place value – value	94	75	31	59
Item 3		Multiplicative relationship (MR)	88	25	10	49
Item 4		Role of zero	100	100	56	89
Item 5	R	Regrouping	72	25	6	25
Item 6	C	Counting	95	50	0	62
	O	Ordering – sequence	100	83	75	86
Item 7	R	Rounding	92	92	33	74
	E	Estimating – shapes	94	81	50	75

Examination of individual areas of decimal knowledge tested, revealed that performance was generally poor in relation to:

1. Identifying decimal numbers using language to symbol relationships.
2. Regrouping—one out of 23 students was able to answer the question *write 3 tenths 6 hundredths 17 thousandths*.

More detailed results (in Appendix E) show that positional understanding of place value and multiplicative relationship were also low despite the class mean being between 43% – 49% as a result of a few students lifting the class mean obscuring the difficulties of other students. The range of results was found to be from 94% to 8%. This highlights the large variance of performance of students in the class.

4.8.4 Reflection

In categorising students into proficiency levels, the triangulation of data was found beneficial. The differences between students and the pattern in data spread within proficiency categories is illustrated in Figure 4.4.

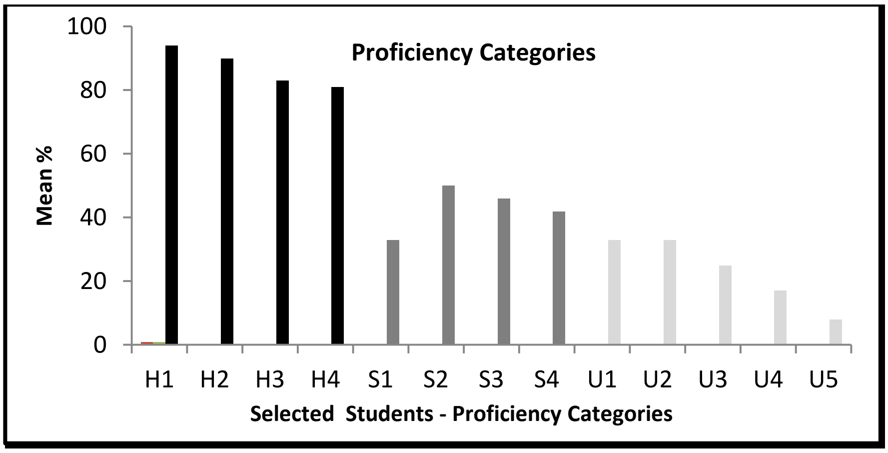


Figure 4.4 Mean score distribution between students in the proficiency categories.
Note. H = Proficient; S = Semi-proficient; U = Low proficiency

The graph shows that students in the proficient category did distinctly better than the semi-proficient and low-proficiency students. However, on the basis of triangulation of data, S1 was placed in the semi-proficient category despite having equally low test scores as two students in the low-proficiency category (U1 and U2).

The analysis of the overall spread of results illuminated the variances in students' performance. The sample consisted of a population of 23 students where the average mark scored was 25 out of 48 with a standard deviation of 11. This indicated that there was a large variation in individual marks. The implication of these differences between students for data collection in this research is that rich knowledge can be obtained about how individual students differ in their thinking. If the data spread were small, then it would have been difficult to determine individual differences.

A bell curve (Figure 4.5) illustrates this distribution of student test results in relation to the measures of central tendency.

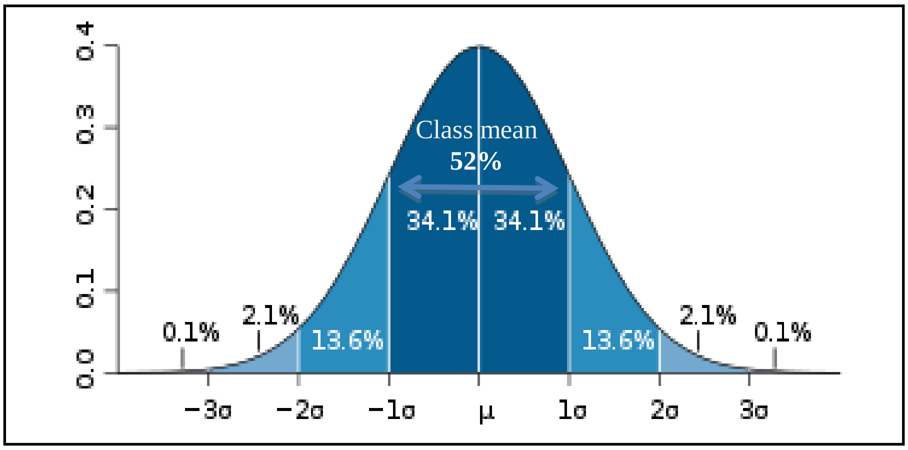


Figure 4.5 Class position on the bell curve.

As shown in the graph, the bell curve indicated that 68% of the class fell between the mean ± 1 standard deviation. In other words, 15 out of 23 students acquired marks between 14 and 36 out of 48. This measure of spread showed that the class mean (52%) was not well represented in relation to the central tendency or grouping of data. This was assumed to be due to the large variation in marks amongst students.

The graph also shows that the test was very well designed by Baturo (1998) in its thoroughness to test a wide range of concepts embedded in the decimal number domain. Therefore it is confirmed that the test will be a very valuable resource for addressing the wide range of interrelated decimal concepts needed for metacognitive reconstruction of knowledge.

4.9 SUMMARY

In summary, overall findings indicated that students did not understand thoroughly the underlying concepts of decimal numbers. Particular areas of difficulty were number identification, place value and regrouping. Interviews with students about their responses to test items confirmed that students did not have a sense of the quantitative value of decimal numbers nor any understanding of the place value of each decimal place. Thus students' approaches to test items were merely applications of fragments of remembered ideas and procedures that were used without understanding. The test results also revealed the extensive differences in levels of proficiency among students that may have implications on their confidence and ability to learn the same content or at the same rate as others

Students' reactions to the difficulty of the test elucidated the role of affects on students' cognition. Students' description of intense emotions that they experienced during the test highlighted the debilitating effects that can result from students' perceptions of difficulty. This includes psychological, physiological, and behavioural responses that affect students' engagement and ability to engage. A dislike for maths was found to be related to students' perception of their ability to do maths.

Triangulation of results indicated that instruments such as tests on their own provided limited data. However, when used together with interviews, critical information can be revealed. For example, students' application of procedures or a lucky guess without understanding can give a false reading of students' knowledge if the correct answer is given in a test. Similarly a student can be skilled to elicit the answers from the interviewer. Therefore the use of multiple instruments is useful for obtaining validity and accuracy in data.

Chapter 5: Results Stage 2: Teaching Experiment

5.1 OVERVIEW

This stage reports on the implementation of a teaching experiment that consists of a series of metacognitive teaching strategies to improve, enrich and change the way students think about decimal number concepts and skills. To this end, this chapter discusses decimal knowledge acquisition in terms of knowledge and practices.

This chapter details and discusses the results obtained from the simultaneous use of metacognitive teaching strategies that target specific decimal concepts, namely: number identification, in relation to the meaning of the decimal notation (sections 5.2 and 5.3); place value, in relation to position, value and the role of zero (section 5.4); multiplicative relationship (section 5.5); regrouping (section 5.6); and counting, approximating and estimating (section 5.7). Finally, results from the post-test are discussed (section 5.8) and analysis of results is used to determine the effectiveness of this research (section 5.9).

5.1.1 Organisation of learning sequence

Results in this chapter are organised as sessions. Each session consists of an analysis drawn from Stage 1 followed by cycles (plan, action, observation and reflection) that are targeted to address a specific decimal concept.

Session 1 introduces to students decimal number identification concepts and metacognitive teaching strategies (Appendix B). To establish both the foundations in decimals and train students in metacognitive teaching strategies, this session is lengthy and divided into two parts (session 1a and session 1b).

Session 2 consolidates place value concepts introduced in a previous session and further builds students' knowledge and understanding of position, value and the role of zero in place value concepts.

Session 3 focuses on the multiplicative relationship in decimal numbers in the base 10 number system.

Session 4 focuses on regrouping of decimal numbers which allows students flexibility in problem solving knowing that the number can be renamed without changing the value of the given number.

Session 5 focuses on counting and estimating to consolidate and build students' understanding of order within places, equivalence and base.

Since concepts in all sessions build on each other, sessions vary in length as it was found that progressively students required less time for learning once the foundational knowledge was acquired.

5.1.2 Background in relation to findings in Stage 1

In Stage 1, students' entry knowledge was determined. It was found that students had many misconceptions and gaps in their understanding of decimal concepts. Findings revealed that students did not have a sense of the quantitative value of decimal numbers nor any understanding of the place value of each decimal place. Thus students' approaches to test items were merely the application of fragments of remembered ideas and procedures that were used without understanding.

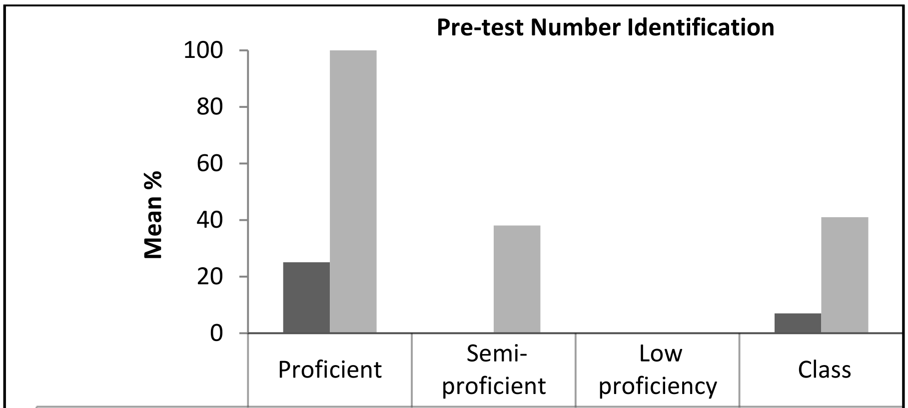
Students' reactions to the difficulty of the test showed that students get frustrated when they cannot provide answers to questions easily and quickly. They experience anxiety and get confused when confronted by contradictions, misconceptions, and salient contrasts.

There were various differences in levels of proficiency among students. These differences in proficiency had implications for students' confidence and ability to learn the same content at the same rate as others. However, this research takes the view that effortful approach to attempting difficult decimal tasks is inevitably accompanied by the natural steps of making mistakes and recovering from them. Therefore metacognitive teaching strategies used in Stage 2 aim to facilitate learning to learn by co-constructing decimal knowledge as well as practices

5.2 SESSION 1A: NUMBER IDENTIFICATION (LANGUAGE–SYMBOL RELATIONS)

5.2.1 Analysis of number identification concepts

Analysis of findings from Stage 1 suggested that students in all three categorised levels of proficiency performed particularly poorly in the symbol to language relationship test items for number identification.



	Proficient	Semi-proficient	Low proficiency	Class
Symbol to word	25	0	0	7
Word to symbol	100	38	0	41

Figure 5.1 Performance in Number Identification test items.

As highlighted in Figure 5.1, the class mean percentage was very low when converting number symbols to words using mathematical language conventions. Students were better able to covert words to symbols.

Analysis of test results suggested that poor test results may be a consequence of three main reasons. The first is inadequate structural (Baturo, 1998) understanding of the decimal number.

Second is lack of familiarity of the correct language conventions for referents due to the use of substitute terms used in spoken language, in particular the symbol used to denote a decimal point. Most students referred to the decimal point as *point* rather than *and*.

Third is uncertainty about the interpretation of the question as a result of experiences with alternate written or spoken representations. This therefore led to varied answers, such as in the test item asking *write 3.826 in words,* students wrote: *Three point eight two six; Three ones, eight tenths, two hundredths point, six thousandths; Three point eight hundred and twenty-six thousandths.*

Although these answers do not follow the expected convention, they make sense to students and can be used to translate symbols to words accurately. This partly explains why some students have become unclear about the correct mathematical conventions of converting symbols into words. However, students' difficulties were evidently a result of more than confusion with question structure and unfamiliarity with language.

Analysis of test responses revealed evidence of misconceptions and gaps in students' foundational knowledge needed for structural understanding of decimal numbers. Only two students in the class were able to provide the correct answer for the following questions:

Write these numbers in words.

a. 3.826 *b. 607.012*

Strategies used to answer the test items varied considerably between students. Further probing during one-on-one interviews indicated that the most common strategies used by students were based on the following ways of thinking:

- Viewing each digit at a time without thinking about the value of what the entire number represents. As claimed by a student: *I find it is so much easier to say each place value …so that I don't get muddled up...*
- Not knowing the purpose or function of the decimal system to be able to meaningfully describe the value of a decimal number.
- Viewing the numbers on either side of the decimal point as two groups of whole numbers.
- Ignoring the decimal point and treating the decimal point as a comma separating whole numbers; and adding the *th* sound to decimal numbers incorrectly. For example for one test item (*Write 3.826 numbers in words*) students wrote: *Three thousandths eight hundred and twenty-six tenths (I used ths for the first and last number to show that it was a decimal number).*
- Weak conceptual understanding of place value of decimal numbers leading to a reliance on memory or generalised whole-number understanding.

  Transferring whole-number thinking such as using the number of digits to determine place value name. For example, in responding to test item *1b:*

Write this number in words 607.012 students wrote*: six hundred and twelve hundredths* by assuming the place value names following the decimal point mirrored those used in whole numbers starting with a *oneth* column after the decimal point.

In reflection of these findings, it was decided that the first session would require lengthy knowledge building to consolidate fundamental mathematical knowledge. It would also require the use of metacognitive teaching strategies to make students aware of the gaps and misconception in their knowledge so that they could be addressed.

5.2.2 Plan

The objective of this session was to: (a) co-develop effective practices for learning from errors; and (b) co-develop, reinforce and enrich students' knowledge and understanding about the symbol and language conventions used for identification of decimal numbers.

The mathematical objectives were drawn from the analysis of students' responses to number identification test items. To this end, the objectives were to improve students' ability to:

1. Understand the structure of a decimal number in terms of the quantitative positional place value of each digit in a decimal number and the function of the decimal point.
2. Clarify misconceptions and conflicts with prior understandings by becoming metacognitively aware of the differences between conventional and non-conventional use of language related to decimal numbers.
3. Communicate effectively using mathematical symbols and vocabulary by writing decimal numbers in numerical form when given a decimal number (written or orally) in words; and writing or reading a number in words when given a decimal number in numerical form.
4. Identify ways in which decimal language and symbols appear in everyday life.

Materials

- Diagnostic test and class workbook (journal).
- Number line (for interactive whiteboard).
- Number expanders (electronic version and printed templates).
- Place value charts and Multi-base Arithmetic Blocks (MAB).
- Concrete and visual referent resources (objects with decimal symbols such as food labels, advertisements from newspapers such as interest rates and petrol prices, thermometer to take temperatures, scale for weights, measuring cylinder to pour out drinks, tape measure and measurements from the sports events).

5.2.3 Action

The four main actions taken in this session were:

1. Training in understanding of learning from errors.
2. Listing errors in relation to number identification.
3. Building students' foundational knowledge in relation to number identification.
4. Reflecting on the practice of learning from errors and knowledge about decimal number identification acquired in this session.

Training for learning from errors

The rationale for error listing was co-constructed with students by directing the class discussion with the following focus questions:

How is error analysis commonly used by teachers and researchers? Would it be of a greater or equal benefit for students to conduct the analysis process?

The potential benefit of errors in prompting metacognition was established on the basis that the student must analyse both the problem and the solutions as well as consider alternatives. Students were challenged with the proposal that errors were an integral and natural part of learning rather than an indicator of failure.

Listing errors

Students were asked to make a list (on the board) of errors made in relation to test items on number identification. When listing own errors on the board, students were expected to check if the error was already listed and to place a tick next to it (to record the frequency of occurrence of the same type of error).

Errors found in my review of research literature were also added to the list to broaden the scope of analysis of the types of errors that are made in relation to number identification.

Knowledge building

The errors listed were used as a stimulus for building students' foundational knowledge and understanding embedded in the number identification conception. This process involved the following activities:

- Students were given one minute to independently jot down notes explaining the process they used to answer the question.
- Reasons for variations were considered, such as:

  Here is a number 32.09, how would you say it? If you were in a quiz show and had no idea of the answer, what strategies could you draw upon to make the best guess? What if it was the other way around and you were asked to write forty-five hundred and thirty-two thousandths?
- Mind maps showing students' thinking were drawn on the board in relation to the concepts and skills that could contribute to finding a solution.

In recognition of students' uncertainty about their understanding of decimal numbers a class discussion on fundamental concepts and underlying concepts was explored. This consisted of the following concepts:

- Defining a digit and number sets.
- The base 10 decimal system.
- Differences between whole and decimal numbers in relation to the base 10 system.
- The mathematical purpose and use of decimal numbers in everyday living.
- Mathematical connections between the symbols and language used to define a decimal number in relation to place value.

To further develop meaningful understanding, the discussion was extended to include three main forms of awareness: (a) language, (b) context, and (c) referent.

To develop *language awareness*, students explored examples of how mathematical language and symbols served as a necessary form of communication in terms of informing, misinforming or the clarity of messages sent and received.

To develop *context awareness* exploration involved word problems, mathematical sentences and oral or written modes as well as contexts that made sense and didn't make sense. Activities using resources such as tape measures, weight scales, measuring cylinders, advertisements and images with decimal numbers were used to create situations that needed accurate reading and writing with decimal numbers. Discussion also included contexts where different situations may give rise to differences in clarity of understanding. *What if…? How is this different from this context? Give me a situation when you would use...*

To develop *referent awareness* examples were used to highlight the need for accurate measurements of quantities—such as monitoring the ph level of the classroom fish tank where a change in one whole number (7.5 to 6.5) would represent a change in alkalinity or acidity by 100 times.

The number identification question in the test was revisited as an opportunity for students to use and test knowledge gained in this session and to review their initial approach to questions. *Can you apply what you have learnt?* Consistent checking of understanding was used to find out which concepts required further elaboration or reteaching by questioning, asking for examples and reasoning as well as why students may agree or disagree with suggested answers.

Drama and music based class games were used to reinforce concepts. *How could you reinforce your knowledge with others?* For example, students were given place value names, another group of students were given number values and I was the decimal dot/point.

Students moved to the correct places when the music was stopped and a number was called. Digits were rearranged in different place value positions. Students were asked to volunteer for roles and other options (such as calling out numbers and writing numbers and words on the board for those who needed more time for the consolidation of concepts).

A few minutes were given for supervised independent practice of new knowledge prior to commencement of peer teaching. Students were also encouraged to make any entries of reflection in their journal.

5.2.4 Observation

Training in learning from errors

Students had no difficulties with providing strong arguments about how errors could be useful for learning. However, most students felt that only a teacher or adult would generally have the expertise to analyse the errors properly.

Listing errors

When listing errors, students showed a lack of confidence in their ability to decide if the error was already listed on the board or not. They sought my affirmation prior to listing the errors, even when they were correct in their decisions about whether they should place a tick next to an already listed error, or list the error as a new error.

Knowledge building

During knowledge building, the large differences among students' in their depth of decimal-related mathematical knowledge and understanding was evident. When asked if they knew how many numbers there are between 0 and 1, some responses included *10, 100, 1000* indicating gaps and uncertainty in knowledge.

During the discussion on identifying decimal numbers commonly used in everyday lives, some students used different representations of fractional values to describe situations. There was a clear indication that some students knew that there were different representations of fractional values but did not know how they were related to or different from each other.

Students showed fragmented understanding of fractional representations and confusion in connecting bits of information and learning strategies used in prior experiences. For example, when trying to explain a situation, a student claimed that he was better able to visualise and use language to accurately describe the size and value in terms of ½ and ¾ instead of 0.5 and 0.75.

The student explained that:

½ is easy because you can half anything. It's just cutting into two parts. You can half 10 and get 5 which is a whole number. But when you say 0.5 you have to cut up things into ten and then try and think um…how 0.5 is the same as 5/10 which is same as ½! It is really confusing.

And then you have something like 0.75. If you didn't just know it was same as ¾, you couldn't just look at 0.75 and say that looks like ¾! After that you have to try and imagine what 75 out of hundred looks like. Some teachers try and teach you with number lines which is even more confusing because if you draw a number line from 1-10, do you put half as in the middle of the number line at the half way point or do you put it like 0.5 which is between 0 and 1.

Another student attempted to use the number line to explain 1/5 and 1/6. The student converted 1/5 to 2/10 (0.2 on the number line) but became confused when he attempted the conversion of 1/6 to a decimal number:

Nothing times 6 gives 10 or 100 or 1000 so it is impossible? Wait, you can cut up the number line in 6 parts and shade one part….um, nah… that won't work.

It is easier to picture in my head if I said I have to break it up as one between five (1/5) or one between six (1/6). Anyway, who says okay, I am going to break this up in tenths or hundredths or thousandths and then share it out equally do they? Just makes your head hurt!

Students better understood hundredths when they made relational references to contextual situations such as money in terms of 100th of a dollar or commonly used percentages such as 50% or 100%. Discussion revealed that most students had limited confidence in their relational understanding between a decimal and percentage (e.g., they could not easily conceptualise 23.5% as a decimal).

During exploration of real-world uses of decimal numbers, a student offered the situation where her mark in a test was 23.5%. The problem observed was that some students were attempting to use their MAB block experiences to make sense of percentage. They interpreted the symbols used in 23.5% as 23.5 was the number, the percentage sign told them that it was out of a hundred, therefore, it would be logical to select the MAB grid with 100 squares and shade 23 squares and an additional ½ of one square. Therefore 23.5% must be 23.5 in decimal form.

When explanations were given on how to convert 23.5% into decimal using division, most students were observed to experience two main difficulties. The first was that many students were very anxious about dividing and even more worried about dividing with decimals. This affected their confidence in thinking about solutions. Second, solutions (such as convert to 235) given by proficient students did not always make sense to other students. They did not understand why 235 could be divided by 1000 in this situation to convert 23.5% to a decimal number easily. Students indicated that recalling practised procedures for solving problems took priority over understanding. *I know how to divide because I was taught how to do it, but I never really thought about why the steps I am using works.*

Students' references to decimal knowledge learning experiences suggested that various teaching strategies had been used by teachers to introduce students to decimal knowledge. This had included introductions to symbols, concrete models, real-life situations, pictures, and spoken language to develop decimal knowledge. However, only a very few students were able to make relational understanding between decimal concepts and materials used for teaching. Most saw the use of materials such as MAB activities and decimal concepts as two separate and different activities.

5.2.5 Reflection

Juxtaposing correct conventions with incorrect student interpretations not only provided students with a clearer picture of errors within their own thought processes, but also expanded understanding on thought processes behind the correct conventions.

In reflection, students stated that *we always thought that it's the teacher's job to look at errors and that kind of stuff.* Students' perception of the type of analysis teachers did was *marking the answer right or wrong and sometimes putting some comments in red pen. Some teachers write all over the paper with red pen.* Another student stated:

> *I don't know, but I think the way they analyse it is to explain how they did the marking. I am pretty sure no one reads it when they get the test paper back; we just look at the marks. You would only read it if you are going to fight with the teacher to get the marks changed.*

Students claimed that initially they felt uncomfortable about listing their errors on the board for everyone to see:

> *You don't know what everyone will think of you and it's kind of shamefully embarrassing when you are supposed to be smart. After you started listing the errors from your uni thing* [research literature], *I was okay with it.*

Students' reflections highlight how students define themselves in roles of students as learners within a social context where knowledge acquisition is not the only thing of concern. Perceptions of what others will think of them are given prominence. It also suggests that they safeguard themselves, by taking passive roles and assigning responsibility to others (*it is teacher's stuff*).

Investigating students' prior knowledge highlighted that: (a) students are not always aware of what they know—that is, they lack meta-knowledge; (b) students' confidence in their knowledge determines their readiness in revealing their prior knowledge; and (c) the emotional reaction they attach to prior knowledge influences their motivation to learn. Therefore, initiating the activation of prior knowledge was not simple. Rather it required a variety of mechanisms as proposed by the teaching strategies used in this research (section 3.5) that were focused on metacognition in terms of awareness and regulation.

In reflection, students attributed some of their difficulties with decimal acquisition to past teaching and learning practices. Their passivity in learning (referred to as *boring)* was attributed to differences between teachers' and students' ideas of what are interesting and engaging ways to learn, and students' lack of choice. For example, in the opinion of students, the teacher's idea of real-world context for learning is often wordy problem-solving situations. This generally requires comprehension of context, translations of situations to mathematical terms and applications of knowledge (which they were already having difficulty with):

> *I don't know, I read the first sentence about ten times. Teachers try and make it interesting by making the problems about kids and things we like, but I get bored the moment I see lots of writing and I can't think even if I try.*

Students' opinions about experiences in constructivist group activities highlighted the problem that emerged when the teacher and students did not have common goals in learning:

Group work is good because it is not boring if you get to work with your friends, you can get away with not being able to do the work if you play it right.

I never learnt anything in group work because the smart kids do all the work anyway and then they get angry with us for doing nothing or having to work with us because we don't get things and can't keep up with them.

Students' comments about common resources selected by teachers to develop conceptual understanding of concepts suggested that the resources were not always suited to the students' needs and added to their anxiety. Students' reflections about the use of MAB blocks with place value charts suggested that despite extensive use in previous years, some students had difficulty understanding their purpose:

I don't know why we have to colour in those squares and do MAB blocks. It's probably okay if you are like in year one or something and can't do real maths but come on, surely we don't have to keep doing that now. We did lots of MAB blocks and filled in the charts last year in year 5, it was so boring so we just made bridges and stuff when the teacher wasn't looking.

In view of these reflections, it was decided that during peer teaching, students would be given the freedom to select and use resources that best suited their needs. To ensure the quality, practicality and suitability of resources it was decided that checking by an adult teacher would be necessary. It was also collaboratively decided that it would be useful for students to co-plan follow-up lessons with their peers to explore decimal concepts.

5.3 SESSION 1B: NUMBER IDENTIFICATION

5.3.1 Plan

The objective of the second part of this session on number identification was to collaboratively: (a) co-develop effective practices for collaboratively learning with peers through peer teaching, error analysis, problem posing and making critical reflections in a community of inquiry; and (b) co-develop, reinforce, and enrich students' knowledge and understanding about the symbol and language conventions used for identification of decimal numbers.

To achieve these objectives, the plan consisted of:

1. Training and organising students for peer teaching by peer matching which involved pairing students with peers to facilitate optimal progression through the zone of proximal development (Vygotsky, 1978).
2. Training in analysing errors and using error analysis with peers to acquire decimal knowledge by: co-constructing guidelines for questioning and identifying errors with peers; and co-exploring errors made in the pre-test to identify conceptions and causes of misconceptions.
3. Training in strategies for problem posing and using problem posing to apply, test and refine decimal knowledge by: exploring different types and contexts for problems; and co-developing and applying problem posing and solving strategies to decimal concepts with peers.
4. Training in strategies for community of inquiry and using community of inquiry to engage in critical reflections about decimal knowledge and the learning and teaching practices used to acquire knowledge.

5.3.2 Action

A series of actions was taken in this session to provide students with training in using metacognitive strategies for learning and teaching to improve practices and gain decimal knowledge. This included: matching peers for peer teaching; co-analysing errors and problem posing with peers; and making critical reflections about the knowledge gained and the practices used to gain knowledge in a community of inquiry.

Peer matching

Students were consulted and their assistance was sought for matching peers. The following issues and ideas were discussed and considered to guide students with their decisions about peer matching:

- More able students were necessary for progression through the zone of proximal development.
- Pairing proficient students with less proficient students could reduce their opportunities in terms of sharing and being challenged by others that were equally proficient.

- The effect of forced pairing of students and other social factors such as conflicts in personalities and disposition that may interfere with collaborative peer teaching.
- Feeling of obligation in selecting friends out of loyalty or social enjoyment rather than for optimal learning as well as exclusion or inequitable participatory situations for some students.

In conclusion of the discussion, students paired for peer teaching using the following transcribed co-generated suggestions:

- *Students who already know the work (most proficient) should test out their knowledge and skill with another proficient student. Since they will be more likely to have finished faster than everyone else, they could help anyone needing help.*
- *Students that are finding the work really hard (low in proficiency) can work with the teacher assistant. They will be guided as a small group to start with, and we can go help teach them one-on-one once we know that they understand our work.*
- *Students that are semi-proficient can work with someone that is slightly better and you (the teacher) can help us. If it doesn't work out we can change in the next session.*
- *Students requiring a third person to assist with clarification or any mathematical questions were to write their names on the board so that other proficient students/teacher could provide the assistance needed*.

It was decided that changes in pairing of students may be negotiated if seen beneficial. My trust in students' ability to make their decisions and choices based on creating conditions for optimal learning rather than social reasons was emphasised.

Co-analysing errors and problem posing with peers

Students were given guidelines as listed in Table 5.1 for analysing errors during peer teaching. These were discussed and modelled using examples. However, it was recommended that students used the guidelines as ideas rather than as a procedure so that peer learning was personalised and focused on the needs of the peers rather than on a set of steps.

Table 5.1

Guidelines for Detection of Type of Errors

	Guiding questions	Identifying errors
Reading	Please read the question to me.	Does not recognise key words or symbols.
Comprehension	(What do you mean when you say tenths/hundredths/thousandths?) (a) Point to a word or symbol. What does this word/symbol mean? (b) Tell me what the question is asking you to do.	Can read the problems well but cannot comprehend the meaning of the words, symbols or question.
Transformation	Tell or show me how you start to find an answer to this question.	Cannot transform sentences into mathematical forms.
Processing skills	Show me how you get the answer. Tell me what you are doing as you work. (Let peer work on a piece of paper.)	Has gaps in knowledge for complete understanding or applying incorrect knowledge (such as whole-number thinking).
Encoding ability	Write down the answer to the question.	Writes the answer incorrectly.
Careless	Obtains correct answer in second attempt or able to spot own mistakes.	Answer does not match correct reasoning given for answer.

Types of errors and the use of problem posing to elicit levels of understanding and salient features of the problem situation were explored. Students were asked to create a 30-second television advertisement/comedy skit for promoting the use of conventional decimal language and symbols. This activity was integrated with other key learning areas (such as a media production including story boarding, editing, drama and music).

Community of inquiry

The community of inquiry was used to engage students in critical reflection. To prepare students for this activity, a general class discussion was used. Following the discussion students made their own personal reflections in their journal before coming together as a community of inquiry.

During the whole-class discussion, a mind map was collaboratively drawn (as a class) listing the strategies that were used for teaching and learning decimal numbers: (error listing, error analysis, knowledge building as class, problem posing, peer teaching and e-portfolio).

A second concept map was drawn listing mathematical concepts related to symbols and language for decimal number identification (place value of individual digits, place value in relation with others, position of symbols and its connection to language). Links were made to show how the students made connections between decimal concepts and strategies to learn and teach.

Students were asked to independently reflect and write their own personal opinions in their journals about: what they had learnt, what they found most beneficial (with reasons) and how their learning and teaching practices could be improved. Students were also asked to write 2–5 questions for the community of inquiry.

At the start of the community of inquiry, strategies and co-developed procedures for equitable participation (see Appendix C) in a community were briefly reviewed. The focus of the community of inquiry was decided to be on the effectiveness of teaching and learning strategies used in this session for learning about decimal number identification concepts.

5.3.3 Observation

During the process of matching students with peers, initially students wanted to work with friends. Some students displayed anxiety about being left out, or feeling worried about being allocated peers that they did not want to be with. However, after expressing to students that I trusted their judgements in selecting peers in consideration of guidelines given for matching peers (section 5.3.2), there was a clear change in attitude. Their conversations changed, from offering justification as to why they should be paired with someone they would like to work with, to the responsibilities they would take to ensure that peer matching worked:

> *I want to work with Erin, but I know we will talk too much and distract each other, so we better not.*

Such comments indicated that students were actively and genuinely seeking to contribute to an optimal learning situation. Students taking ownership of selecting and working with peers was also observed to have positive effects on how they worked with each other.

Co-analysing errors and problem posing with peers

Initially some students attempted to rigidly follow the guidelines that were given in preparing students for peer teaching (section 5.3.2). However, once students became engaged in the process, they seemed more focused on the task rather than the process.

Different approaches to peer teaching were observed:

- *We took turns to use the guidelines* [given during peer teaching preparation] *step by step and ticked the parts as we finished it and put a cross next to the ones that weren't relevant to us.*
- *I showed my peer how I did it and then we went through his work and he had to tell me where he went wrong and how to fix his.*
- *I didn't show her how to do it but I gave her the correct answer and she had to tell me how she worked it out and explain to me what she could do to get to the same answer.*

Students were observed to be naturally adjusting their focus to the localised needs of their peer. Contrary to recommendations given in literature to first build conceptual understanding, peer tutors seemed to place an emphasis on creating steps to follow and practice. For example:

Example 1: Steps with visual assistance using a place value chart (semi-proficient student)

We drew a place value chart and wrote down the following steps

1. *Say the whole-number digits, by groups, starting at the left of the number.*
2. *Put 'and' for the decimal point.*
3. *Start at the decimal point and read it like you did with the whole numbers.*
4. *Look at the last decimal place and see where it ends up in the place value chart to name the decimal number*.

Example 2: Detailed steps (given to a student low in proficiency)

H2: Can you see what is happening here, you start from the beginning of the number and you say the number as I point to it, okay, now since there is a dot you call the dot 'and' and since there is more than one number past the decimal point you read it together as eight hundred and twenty-six thousandths. Okay you do this one for me (writing down 23.03).

U4: Is it two…

H2: Um, you have to read numbers together not one at a time; here this might help you [drawing a place value chart]. *Let's put the numbers in the chart.*

Example 3: Steps with fewer scaffolds (semi-proficient student)

Say I have 3.16,

- *you say the three*
- *the point is the and*
- *take notice of the last number because it is what you name.*

Example 4: Through recall of steps (semi-proficient student)

You tell me how you would say this number 2.880 step by step.

Once peer tutors felt that their peers were able to reproduce the steps independently, they proceeded to explore the reasoning behind the steps. This seemed to be an effective strategy. The success of students in being able to get the right answers using the steps was observed to have a significant effect on their motivation and confidence. *Can you give me another one?*

Interestingly, when peers commenced exploring reasoning behind the steps, in some cases the roles reversed where the peer tutee was better able to assist with developing conceptual reasoning for procedures used.

Observation during problem posing suggested that positive peer learning and teaching relations were beginning to develop among students as a result of peer teaching. H2 stated, *he is much smarter than he thinks and he even got all the answers correct when I made a mistake and got one wrong during problem posing*. Students' attempts at problem posing in multiple contexts revealed that students' conceptual knowledge was fragmented and poorly linked to conditions of applicability.

All students showed great excitement about creating the 30-second television advertisement/comedy skit for promoting the use of conventional decimal language and symbols. Students in the low-proficiency category were particularly animated by the project and were noticed to enjoy the opportunity of offering their talents to the groups.

The reflective journal writing activity was observed to be the least desired task by most students. Students claimed that they found writing to express their thoughts difficult.

Can we just use it to record our work and write a bit on it because it is too hard to write things going into your head because it is always changes anyway? Every time I read it, I think, it is not right and that I should write it all again.

When you are writing things that are personal like feelings and stuff, you don't want to be honest because you always worry that someone will find it and read it and think dah….talking about it like now is more fun.

Most students' comments in their reflective journal were focused on their enjoyment of the peer teaching experience. Written comments about the how the experience was beneficial were vague, such as–*I learnt a lot*. However, when engaged in a dialogic discussion in the community of inquiry, students were more descriptive and better able to articulate their thoughts, feeling and experiences.

5.3.4 Reflection

Important findings in this session included: (a) the need for metacognitive awareness in revealing habitually automated learning and teaching practices; (b) the role of error identification and analysis in developing a metacognitive awareness of own prior knowledge; and (c) reaching students through peer teaching.

Review of transcripts from observations and students' reflections elucidated patterns in teaching and learning practices that had become habitual and carried out without thinking much about their intent. For example, prior knowledge inquiry is a commonly used practice at the start of lessons. Strategies such as whole-class discussions and memory-related reviewing and testing strategies are commonly used in mathematics lessons to gain access into students' prior knowledge. Individual and group entries into KWL (What I know, want to learn and learnt) charts are used to provide a reflective framework for constructing meaning from new material. Yet, as shown in the following transcript, students' awareness of their own knowledge and that of others is often limited.

We always have to fill those KWL sheets, but it doesn't really mean anything to me. I fill it in but I don't always know what to write. Anyway how can you write about what you want to know, if you don't know, what you don't know!

> *I also think that, the main problem is that it is not coming from us, it is coming from the teacher. With error listing and error analysis we got to see what we know and don't know, so it meant a lot more than just answering teachers' questions or filling in a sheet*

From this awareness of prior knowledge, students' comments suggested that they developed a deeper awareness of the value and intent of a teaching experience.

Peer teaching had very obvious advantages of engaging in more personalised, careful and deliberate processing of prior knowledge. However, monitoring students in terms of the accuracy and effectiveness of their teaching was difficult. As highlighted from observations during error analysis, peers that performed well in the test did not necessarily have the understanding required to answer questions asked by peers. Therefore there was a risk that misconceptions could be further reinforced.

There was a high demand for teacher attention because the process of error analysis required expert understanding of the correct use of strategies in cases that involved contesting strategies. It was often difficult to respond and intervene without knowing the dialogue that had led to the questions or query. Therefore it was difficult to determine what types of questioning or the right amount of hints should be given to direct students in co-constructing meaning.

Despite these potential problems, error analysis and problem posing were particularly rich contexts for meaningful learning during peer teaching. In both these approaches, students were not provided with knowledge in its final form where the responsibility of the learner was simply to internalise the ready-made. Instead, students had to pull apart concepts to understand why errors were made or discover through questioning, evaluating, testing and problem posing.

Students' reflections were mainly directed on positive aspects of peer teaching. In considering disadvantages or difficulties experienced during peer teaching, students commented that the difficulties they experienced were related to the following issues: (a) lacking in confidence, *my peer was a bit scared to do things and it took me a little while before he would do it*; (b) difficulties with being able to answer all the questions asked by peers, *I know how to do it my way but I couldn't answer her question*; (c) difficulty with explaining, *sometimes it is hard to explain things because you just do it automatically*; and (d) expert expectation, *sometimes it* *is hard to say you don't know something when everyone else expects you to know everything*.

Overall, students claimed that peer teaching had several benefits. Students agreed on three main benefits of peer teaching. The first of these was that they learnt from peer teaching:

> *I thought it will be boring and it will be a waste of time since I already know the work, but I actually learnt a lot by trying to explain. No offence, but you do learn a lot more from peers teaching. You don't even think that you are learning when you start talking, asking questions and trying to help each other. I think you learn a lot more from teaching because you have to think about the work to ask and answer questions.*

The second benefit was the feeling of doing something enjoyable. Alsop and Watts (2003) explain that such feelings are important in giving relevance to tasks and driving conative actions for learning:

> *It's funny because I was learning but it didn't feel like it because it was fun. She is my best friend so helping her made the work a lot of fun and also made me feel good.*

The third benefit identified by students was the sense that they were becoming more responsible and had some control over their learning:

> *We would never have bothered reviewing our work before, what was the point? It wasn't as if we got a chance to fix up our work and get a better mark, we just moved on to the next topic.*

Other benefits mentioned were factors such as developing friendships and respect for students who had a history of performing poorly in school-based tests.

The most common comments about the community of inquiry were:

> *I enjoyed it because you get to say things and hear everyone else's opinion. It makes you think about people's ideas differently. Some of their ideas are really clever! The bad thing was probably we didn't get enough time*.

Student proposals for improvements

In conclusion on reflection the following changes were proposed by students. To improve mathematical knowledge, students were to talk with peers using mathematical language to get familiar with concepts. To improve peer teaching practices it was suggested by students that planning follow-up lessons with their peers would help them become prepared for the lesson. To do this, a separate session would be needed where students could access the computers for ideas and resources. The plan was to be checked for suitability and further guidance.

Since it was pointed out by students that predetermined concrete materials and teacher-selected resources were not always productive for stimulating meaningful learning experiences, students would have the option of finding and selecting materials as found needed. The materials would be checked for suitability by the teacher:

> *We can put the resources that we like in a box in the middle of the classroom. Whoever we are teaching can help us decide on the resource that works for them.*

It was decided that students would first work in the classroom and would be allocated a specific time on the computer to avoid spending the entire peer session on computers. If further time was needed on the computers, this time could be negotiated.

5.4 SESSION 2: PLACE VALUE (POSITION, VALUE AND ROLE OF ZERO)

This session addresses students' place value concepts (position, value and role of zero). The presence of the symbol zero in decimal numbers was observed to cause some confusion for many students when identifying numbers in the previous session. Misunderstanding the role of zero was observed to result in errors such as writing *seven hundred and seven* as *7007*.

Essentially, students did not understand that the position of a digit determined its value (or non-value, where the digit is a zero). Due to the significant role of zero in determining place value, the test items on place value and the role of zero were explored together in this session.

5.4.1 Analysis of place value and role of zero

Analysis of data from pre-test results (Figure 5.2) suggested that semi-proficient and low-proficiency students had more difficulties with value-related than position-related place value test items. On closer analysis of test responses, a substantial gap could be seen between the semi-proficient and low-proficiency students, particularly in understanding the value of a number where more than procedural knowledge was required to actually visualise the value of a number.

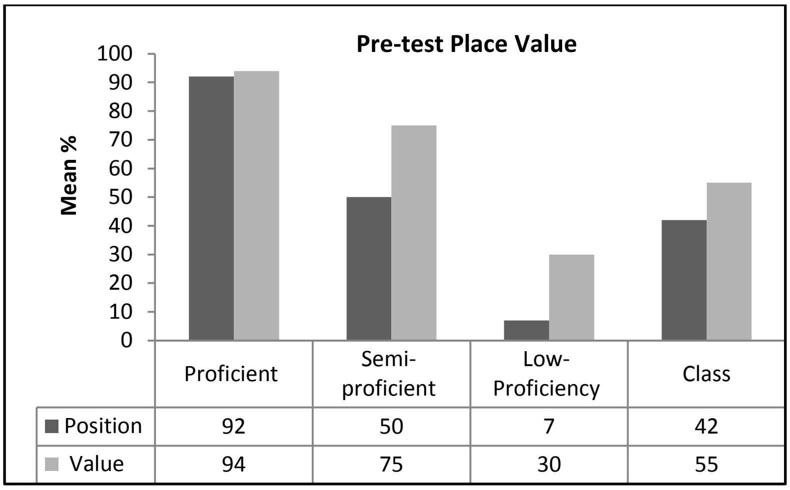


Figure 5.2 Performance in Place Value test items.

Observation of students’ reasoning in test responses and interviews revealed that proficient students had a conceptual understanding of the value of the entire number, whereas the semi-proficient and low-proficiency students thought of numbers as individual units that belonged in specific columns of the memorised place value chart. The difference between semi- and low-proficiency students was that semi-proficient students were better able to relate the question to the memorised place value chart and recall procedures that could assist in responding to the question. There were two common errors.

The first was related to the pre-test item 1b: *write the number that has 7 tens and 7 tenths.* A common response to this question was *7 tens and 7 tenths = 70.07.* Interviews with students indicated that the reason for this error was that students were relating the number of digits to symmetrical place value name. They were forgetting that there was no *oneths* column and did not understand why there was no *oneths* column.

The second most common error was in the pre-test item 3c: *In 625.078 the 50 is worth?* Common answers were: *there are no 50; 50 is worth hundreds; 50 is worth 5 ones and 0 tenths; 50 is worth hundreds 50 ones*. Baturo (1998) explained the difficulty in such questions as a result of the complexities involved in identifying singleton units (unitising) and identifying composite units and unit-of-units and being able to change one's perception of the unit (reunitising).

Analysis: Role of zero

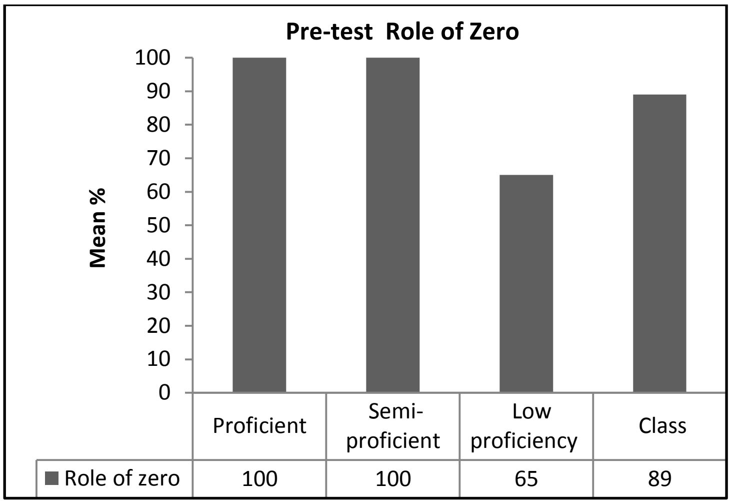


Figure 5.3 Performance in Role of Zero test items.

Class mean (89%) suggested that most students in the class performed well in the place value test items in relation to the role of zero (in Figure 5.3). However, analysis of interview transcripts revealed considerable difficulties when zero was present in numbers. A possible explanation for the discrepancies in test results compared to their actual understanding could have been the result of the way in which the questions were structured. The *yes* and *no* nature of the questions permitted students to make a guess and give correct answers.

5.4.2 Plan

Drawing on the analysis (section 5.4.1) the mathematical objectives for this session were to develop students' ability to:

1. Understand the reasoning behind the common procedural strategies observed being used (such as lining up of numbers in place value charts) for working out place values of numbers by:
   (a) Developing a rich understanding of the base-10 structure of the number system.
   (b) Exploring the significance of zero in interpreting numbers.

2. Pose problems and apply knowledge flexibly in multiple contexts by creative activities such as investigating the history of zero/base ten place value ideas. *Imagine that you are one of those mathematicians that have come up with an idea and everyone thinks that you are mad and that the society will be fine without your ideas. Present a debate for both sides.*

Note: This activity was integrated with objectives of other key learning areas and not described in this research in detail due to the length of explanations needed to provide a complete understanding of the experience.

Materials

- Diagnostic test, workbook (journal).
- Mini-white boards and pens.
- Computer lab for access.
- Place value charts, digit cards and other student-selected materials.

5.4.3 Action

Actions in this session consisted of building on awareness of existing knowledge about place value by discussing and mapping students' thinking on the board. Questions in relation to the mind map created were encouraged and listed for inquiry. The sequence of actions that were taken are listed in Table 5.2.

Table 5.2

Sequence of Actions for Place Value Knowledge

Entry knowledge	(1) Error listing and error identification.
Knowledge building	(2) Review of knowledge from previous session. (3) Building knowledge on place value.
Collaborative learning and teaching	(4) Planning peer teaching lesson by drawing on errors listed and new knowledge gained from the previous two steps. (5) Consolidation and enrichment through peer interactions (analysis of errors in pre-test items, problem posing for assessing and refining understanding).
Reflection	(6) Reflecting as community of inquiry.

From this starting point, introduction to new knowledge, connections between knowledge and refinement of knowledge about place value were made. Students listed the errors made in the pre-test using a chart that made differences between whole and decimal numbers explicit (Appendix D). It was collaboratively decided that it would be useful for students to organise their collection of data as shown in Table 5.3 when analysing errors with peers during peer teaching.

Table 5.3

Error Listing Process

Pre-test item 1a	*Write the number that has 5 tenths, 2 hundredths, 9 ones, 3 thousandths*
Error made	*5 tenths, 2 hundredths, 9 ones, 3 thousandths = 9.325*
Explanation of reasoning used	*I was thinking one number at a time starting from 9…I knew that 9 ones was on this side of the decimal point and thought the next biggest number was 3 thousandths then 2 hundredths then 5 tenths.*
Analysis	Confusing whole-number use of place value names and order of the place value names.

Number identification knowledge gained from previous session was reviewed as a warm-up quiz to assess knowledge and retention of knowledge (whole class). Drawing on findings about students difficulties (section 5.4.1), the reasons why there is no oneths was explored.

What is oneths? This question was explored by using students' prior knowledge of fractions to conceptualise why oneths would be 1/1 and that 1/1 already exists, therefore another ones place was not needed.

From this understanding of the meaning of the ones place value, the perceived symmetry of place value names was reviewed. That is, ones being the centre of symmetry rather than the decimal point—so that it made sense to have one, 10, 100 on the left and 1/10, 1/100 on the right. Furthermore, when put as decimals, why 0.001 is named as thousandths even though it has only two zeros instead of 3. A diagram was used to further illustrate how each place value corresponds to a power of ten and divides or multiplies by ten.

Students were asked to consolidate concepts explored in discussion through peer pairing to explain their understanding to each other, provide each other immediate feedback, pose questions and seek for elaboration.

Whole-class discussion and peer learning was used for: elaboration on understanding of zero in decimal numbers; and identifying numbers by writing and reading decimal numbers in expanded form. To do this, the following focus questions were used: *What does this expansion tell us about the meaning of the* *number? How could this number be used to give meaning to a context?* The question leading inquiry on the role of zero was: *Is zero nothing?*

History was used to illustrate the significance of zero and the difficulties/ ambiguities that would emerge if there was no zero (such as prior to 400 B.C., when they relied on the context of the number to tell them if 216 were intended or 2106). Students were given time to experiment with posing problems with zero in decimal numbers.

Using the errors listed, students were allocated time to use knowledge gained from the knowledge building session to plan, locate resources suited for their peers and prepare for peer teaching. In creating a plan for peer teaching students were guided to use findings from the analysis to select appropriate resources, types of problems and activities that would enrich, refine and test understanding.

A community of inquiry was used for reflection on mathematical knowledge gained and the effectiveness of strategies used for teaching and learning.

5.4.4 Observation

Prior knowledge about place value

When questions were asked at the start of this session, most students were able to answer questions related to place value of digits within numbers easily. However, the strategies they were using to respond to questions did not clearly show a rich conceptual understanding of place value. Most students still seemed to be relying on mental recall of the place value chart and focusing on the procedural strategy to work out the language and symbols used to represent decimal numbers…*I looked at the last digit and it is in 100ths place so it is 75 hundredths.*

When exploring students' practices of adding a oneths column to increase the symmetry in place value columns, students seemed to find it easier to conceptualise fractions rather than decimals. Therefore students were found to draw on their prior knowledge of common fractions to answer the question: *What is oneths?* Students suggested that oneths should be 1/10. From this understanding students could understand why oneths would be 1/1 and that 1/1 already exists, therefore another ones place was not needed.

On the other hand, some prior knowledge was found to cause confusion and misconception. For example, some students thought time was base 10 and had difficulty understanding why money was not base 100 when 100 cents makes a dollar. Similarly, when problem posing some students attempted to use base 60 time measurements from the swimming carnival in their questions with the perception time was written as decimal numbers. Most of these difficulties had emerged from making incorrect linkages between fragmented parts of knowledge.

Knowledge about the role of zero

Class discussion revealed that the impact of trailing and leading zeros in numbers was well established. Most students referred to a rule taught that zeros to the left did not hold any value for whole numbers and zeros to the right did not hold any value for decimal numbers. However questions suggested that not all students understood the reasons behind the rule.

A common response to the question *what is zero,* was *zero is nothing*. This explained why students suggested that zero was *really not importan*t and can be just left out. It was pointed out that even when talking about numbers, zero is never mentioned in a number language (e.g., 0.72, 702, and 7.200).

A student stated that when using mental computation strategies, he was taught that crossing out the zeros and getting rid of them when dividing made it easy to work out. He did not know why it worked—hence, reinforcing the concept that the purpose of zero was insignificant. The student said that he was unclear about the role of zero beyond recollection of procedures often applied to numbers with zeros.

Some semi-proficient and most low in proficiency students stated that they had not even considered if zero was important or not. None of the students in the class could explain why you cannot divide by zero. The following transcript describes students' thinking:

C1: It does not make sense, if you have 5 things and had no one to give it to (zero), then you dividing it with no one and should have the same amount.

T: What is the act of division?

C2: It is sharing something with others.

T: So what does the answer part of a division question represent?

C2: The answer is the amount given away so….the answer should be zero.

T: You are breaking the original number into zero parts and you can't break things into zero parts so you can't divide by zero.

C4: Then it would be the same for multiplying or adding or subtracting if there is nothing to multiply so it can't be done because you have done nothing.

T: Think about what multiplication means. It means how many parts of that original number you have multiplied.

C5: I kind of get it but it is making my head hurt to think about it.

Learning from planning lesson

In preparing lessons and collecting resources from online sources, students were observed to be learning from questioning and attempting to work out which concepts were related to place value.

Most students were able to select place value specific tasks without a great deal of assistance. The types and sources of resources selected varied from YouTube videos, problem solving worksheets and interactive online place value games offering challenges at different levels.

Students found the e-portfolio (section 3.3.3) useful for organising and sharing resources during their planning of lessons. Some students also printed worksheets to assist with ideas for problem posing. These were shared with students who were having difficulties with writing their own problems. Resources such as place value charts were downloaded and printed and laminated so that students having difficulty could use the charts.

Most students who tested their resources prior to teaching were better able to use them efficiently during peer teaching. Students who required enrichment during peer teaching were also found to benefit from the resources collocated by students to test their knowledge through games and varied types of problems.

Substantial improvement in students' confidence during problem posing was noticed. Access to resources during problem posing provided useful cues for constructing more varied and creative types of questions. An example of problems posed by adapting resources is illustrated in Table 5.4.

Table 5.4

Example of Place Value Activity

Decimal places change by a factor of 10. For example, let's look at the number **625.7** below ***6*** ***2*** ***5.*** ***7*** *hundreds* *tens* *ones* *tenths*	*100 (hundreds)* *10 (tens)* *1 (ones)* *1/10 (tenths)*	*6×100 hundreds* *2×10 tens* *5×1 ones* *7×0.1 tenths*	*What is the rule (or relationship) between the different place values?* *What would be the next bigger place value after tenths?* *What would be the next smaller place value after ones?*

It was interesting that a few students selected some of the resources (such as the base 10 blocks and shaded grids) that they had strongly protested against using in previous sessions:

> *I don't know, but it was boring and I hated it when we had to work with the blocks in class and fill those charts. Probably because we did not really know what we were doing and we did not care. Now I am using it probably because it was drummed into me so much. Actually, I am using it differently, I can't really explain it, but it makes sense to me now and it didn't before.*

It was observed that in some situations the difficulty of the resources encouraged both peers to work together in understanding the resources. There were some surprises where the less proficient was able to better explain the resource than the tutor.

Error analysis during peer teaching

Co-error analysis of test items related to place value revealed many of the difficulties that were found in number identification. Examples of some of the common errors are summarised in Table 5.5.

Table 5.5

Co-Error Analysis of Place Value Test Items

Pre-test item 1a	Write the number that has 5 tenths, 2 hundredths, 9 ones, 3 thousandths
Common error 1	*5 tenths, 2 hundredths, 9 ones, 3 thousandths = 9.325*
Students' reasoning	*I was thinking one number at a time starting from 9…I knew that 9 ones was on this side of the decimal point and thought the next biggest number was 3 thousandths then 2 hundredths then 5 tenths.*
Analysis	Confusing whole-number use of place value names and order of the place value names
Common error 2	*5 tenths, 2 hundredths, 9 ones, 3 thousandths = 3.259 or .3259 or 3259*
Students' reasoning	*I wrote the number (like it was whole numbers) and because it had the 'ths' then I put a decimal point in front of the entire number to show that it's a decimal number.*
Analysis	Placed the decimal point behind 3 digits since three numbers had the *th*. Using whole-number thinking, ignoring the decimal point or unsure of its position in relation to place value knowledge.

Pre-test item 1b	Write the number that has 7 tens, 7 tenths
Common error 1	*7 tens, 7 tenths = 4.9*
Students' reasoning	*7 tens = 7 × 10 = 70 … because 7 tenths = 7/10 = 0.7* *Therefore, 7.0 × 0.7 = 4.9*
Analysis	This error shows that the student has an understanding of tenths and tens but made an error in the interpretation of question.
Common error 2	*7 tens, 7 tenths = 70.07*
Students' reasoning	*7 tens is 7 × 10 = 70 and 7 tenths = 0.07 because you know that it is on the decimal side because of the 'ths' in the tenths.*
Analysis	Focusing more on *th* sound rather than the positional place value, hence seeing the decimal point as a mirror, reflecting the place value to the left of the decimal point on to the right side fractional aspect of the decimal number.
Common error 3	*7 tens, 7 tenths = 0.77*
Students' reasoning	*I just guessed that seven tens means seventy and 7 tenths means it is 7 parts of seventy. I put a decimal number in front of 77 to show this.*
Analysis	Focusing on length, familiar language and representation of two-digit number to assume that *th* in tenths indicates that it is a decimal number and the ten indicates two digits.

Pre-test item 1c	Write the number that has 2 thousandths, 8 tenths
Common error 1	*2 thousandths, 8 tenths = 0.8002*
Students' reasoning	*I keep thinking that thousand is three numbers long so thousandths will be the same.*
Analysis	Confusion with whole-number thinking.
Common error 2	*2 thousandths, 8 tenths = 2080*
Students' reasoning	*I thought it meant 2000 and 80 which is 2000 + 80 = 2080.*
Analysis	Whole-number thinking and ignoring the language used to describe the decimal fraction values of the number.
Common error 3	*2 thousandths, 8 tenths = 0.2008*
Students' reasoning	*I added 8 to 2000 = 0.2008*
Analysis	Saw the question as having two separate numbers that were required to be added.

Pre-test item 2	Ring the number in which the 7 is worth the most. 94 376 70.523 1762 1.762
Common error 1	*The 7 is worth the most in 70*.523
Students' reasoning	*When you look at the numbers, the first and second number look the largest and 7 comes first in the second number so 7 is worth most in the second number. It was the only number where 7 came first giving it the appearance that it would be of the highest value.*
Analysis	Ignoring the decimal point and treating the number as if it was a whole.
Common error 2	*The 7 is worth the most in 94 376*
Students' reasoning	*It is worth the most out of all the other numbers.*
Analysis	Focusing on the value of the entire number without considering what the question was asking.

Pre-test item 3	In 625.078: a) the 2 is worth? b) the 8 is worth? c) the 50 is worth?
Common answer	*There is no 50*
Students' reasoning	*The question was confusing. Why does it ask worth? Should it be represent? There was also debate among peers 50 could be fifty tenths or five tens and zero ones should still be marked correct.*
Analysis	Students had most difficulty in analysing test item 3c. They felt that the question was confusing and ambiguous, however the question led to substantive debate that: *There is no 50 but in 50, five is tens and 0 is ones. You can't have 50 because you carry over—so that question is not right.* Another argument was *50 could be 5 ones or 50 tenths.*

Pre-test item 5	Write yes or no to each of the following. [Role of zero] a) Does 7.2 have the same value as 7.020? b) Does 7.2 have the same value as 07.2? c) Does 7.2 have the same value as 7.200? d) Does 7.2 have the same value as 0.72?
Common answer	There were very few students who made errors in this test item.
Students' reasoning	*I could guess most of the answers. I had a chance of getting it right. I did not know if it was a trick question because 7.2 and 7.200 looked like different numbers.*
Analysis	The yes and no nature of these questions permitted student to make a guess and give correct answers (without understanding).

There was a noticeably large change in students' knowledge observed as a result of the previous session on number identification. Students' interest in analysing errors was motivated by their interest in finding resources for peer teaching. Having co-planned their lesson on place value, students were excited about peer teaching and seemed to be confident. As such, they were more creative and intuitive in creating strategies to explain concepts to peers.

For example, when explaining the place value concept, students found that underlining the ones' place value in number assisted their peer to better visualise the symmetry in place value names. To avoid dependency on charts, peer teachers were observed to be initially using place value charts as scaffolds, and then progressively encouraging peers to attempt tasks from memory of place value names. Other peer teams were using the chart to check answers. *Okay, put the numbers in the chart and tell me if you got it right.*

5.4.5 Reflection

There were three main findings of note: (a) the influence of prior knowledge; (b) the influence of sharing responsibility in learning on learning; and (c) the influence of success in motivating students in tasks.

Students' use of prior knowledge highlighted its role in developing conception or misconceptions about decimal numbers. This was illustrated by students' use of their understanding of common fraction to work out why there wasn't a need for a oneth place value column. On the other hand, confusion and misconceptions such as associating the decimal point to a base 10 system created problems when applied to time (base 60). The implication of this finding is that learning has to start from where students are at. Important to this process in a classroom situation is that all students are at different starting points.

Students taking part in the learning process not simply as a learner but also a teacher made considerable difference to students' commitment to learning. As a teacher they assumed different responsibilities of planning, finding resources and adjusting strategies for teaching. Students claimed that planning was a worthwhile metacognitive activity:

> *I don't think I have ever learnt as much as I did from trying to prepare lessons. This is because you can't just choose anything, but you have to work out why and how it will work. If it doesn't work then it makes you think even more on how to make it work. It was hard work but fun and the hard worth it when my resources worked with getting my peer understand the work. Even though I am smart at maths it had lots of stuff in it that I never knew.*

This last comment also highlighted the role of success in motivation. Peer students' use of success was also noticed in the previous session (section 5.3.3) where students were found to motivate their peers by providing them with steps. This was then followed by practice in application of steps so that the success gained from the repetition could then be used to motivate students to seek understanding.

In critical reflection, the problems with students participating in the planning process were also considered during the community of inquiry. The main problem identified was that the process of selecting and using resources relied heavily on the individual student's mathematical knowledge and skills to retrieve, interpret and adapt resources for learning and teaching purposes. For these reasons, management was difficult with the students viewing different resources that were unfamiliar to them. There were also potential risks of using unfamiliar resources without being able to provide adequate supervision.

However, in view of the gains of student interest in seeking appealing resources, assessing suitability and trying to understand the content, it was decided that the advantages outweighed the disadvantages. This approach also fitted in well with the sense-making focus of the *Australian Curriculum: Mathematics* reforms (ACARA, 2012; QSA, 2012) in the value of blending teaching approaches with students' interests to build mathematical connections.

Student proposals for improvements

The following proposals were suggested for trial in the next session to improve learning and teaching practices for decimal knowledge acquisition.

1. To address the demand on expert opinion to assess the suitability of resources, it was proposed that:

    (a) Some resources we can't decide on till we are doing the work to find out what our peer needs help with, but we can look for most of the resource before we do peer teaching.

    (b) To work out what resources we might need, we can plan the lesson before peer teaching so that it can be checked.

2. Some students felt that other students were better at finding and creating resources. Therefore it was proposed that:

    (a) E- Portfolio will be used to place resources rating and brief description of resource that could be accessed by all students.

    (b) Maybe we can put a rating each time someone uses it so we know how good it is, like the movies.

    (c) After peer teaching session we can show it to the rest of the class so that anyone who wants to use in the next session can.

    (d) We can get our peer tutee to help teach it to everyone which will give us practice in using it.

    (e) If it works out resources may be used as rotations so if needed students could access other resources.

3. Some students found the resources frustrating, attributing their difficulties to lack of ability and knowledge.

    (a) Provide additional support and pre-testing of resources prior to peer teaching session.

5.5 SESSION 3: MULTIPLICATIVE RELATIONSHIP

5.5.1 Analysis of multiplicative relationship concepts

As shown by the patterns in the spread of data in Figure 5.4, there was a noticeable difference in performance was noticed between the proficient and semi-proficient students in their understanding of multiplicative relationship in decimal numbers.

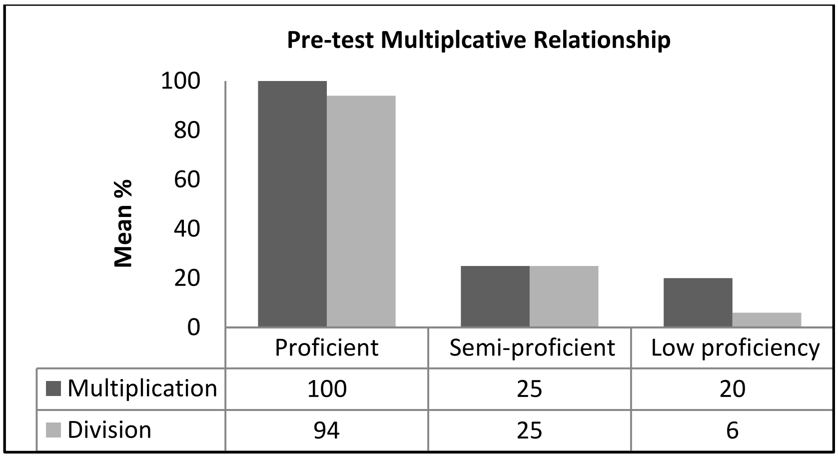


Figure 5.4 Performance in Multiplicative Relationship test items.

Case study students categorised low in proficiency and semi-proficient had similar levels of proficiency in multiplication. However, it was their performance results in division that separated the low from semi-proficiency. Proficient students had a good grasp of multiplication with some errors in division of decimal numbers. A common problem observed in division was the misunderstanding that a smaller number cannot be divided by a larger number and using commutative properties of multiplication such as 8 × 10 = 10 × 8 for division. Hence, some students switched numbers (e.g., *8 ÷ 10 to 10 ÷ 8*).

Overall, examination of strategies used by students to answer the multiplication and division test items suggested that, although some students knew computation procedures to answer questions accurately, it was unclear if they had an understanding beyond the recall of steps. Most students used the long multiplication method to answer the test items and seemed unaware of the multiplicative structure that allows for much easier ways to multiply or divide decimal numbers by 10, 100 and so on.

5.5.2 Plan

Drawing on the analysis (section 5.5.1), the mathematical objectives for this session were to develop students' ability to:

- Understand foundational multiplicative structural knowledge that underlies the concept of place value of decimal numbers.
- Make links and motivate the extension of the multiplicative structure from natural to decimal numbers.
- Understand the multiplicative structures of natural numbers and decimal numbers, by underlining the analogies and differences.
- Break routinised use of procedural knowledge for computation with decimal number to provide an understanding of the reason behind procedures.
- Estimate or calculate exact answers and make conjectures about procedures and their motivating rationales.
- Pose problems and solve problems by applying knowledge with understanding and flexibility.
- Participate in the following class task: Plan the end of decimal party. Create a list of estimated quantities of items and food ingredients needed. In small groups of 2-3 students use an online shopping site and get the best buy. Check for two-for-one prices and cost of bulk buys in comparison to smaller quantities to get the best value for the allocated party class fund.

Materials

- Diagnostic test and workbook (journal).
- Computer lab for access to the internet.
- Calculator, laminated place value charts and digit cards.
- Measuring equipment (scale, measuring tape, measuring cylinder).
- Shopping receipts.
- Student-selected materials.

5.5.3 Action

A series of actions were taken involving the use of metacognitive teaching as described in Table 5.6.

Table 5.6

Sequence of Actions

Entry knowledge	(1)	Error listing
	(2)	Review of knowledge from previous session
Knowledge building	(3)	Addressing gaps in existing knowledge
	(4)	Elaboration and refinement of decimal knowledge
Collaborative learning and teaching	(5)	Multiplicative structure of the decimal number
	(6)	Peer teaching (review and analysis of errors in pre-test items)
Reflection	(7)	Problem posing (assessing and refining understanding)
	(8)	Reflecting as community of inquiry

Error listing followed the same format as in previous sessions. Place value knowledge gained from previous sessions was reviewed as a warm up to the session.

The multiplicative structure embedded in the decimal number system was illustrated to students using diagrams and an attempt was made to show how the multiplicative structure could be used in the context of the test items.

However, in view of findings that indicated large gaps in knowledge and understanding of number operations, concepts and skills in relation to explanations on the multiplicative structure and computation were revised. To do this, actions were focused on: (a) understanding the meaning of the number operations; (b) revision of whole-number operation skills to build knowledge to reinforce students' ability to perform single and multiple-digit calculations; and (c) extending understanding to decimals numbers in relation to its multiplicative structure.

The definition of number operations (addition, subtraction, multiplication and division) and the correct operations needed in different contexts were used together to give meaning to operations.

Revising and building students' knowledge of computation skills was achieved by first drawing on students' prior knowledge to list known strategies used for multiplication and division. Starting with familiar strategies, single and multiple-digit calculations were performed.

Other strategies were introduced to develop flexibility in thinking and break students' focus on procedural thinking. To do this, three or more strategies were organised side by side for the same question so mathematical connections between them could be made. From this comparative analysis a rationale for their use was drawn. Students were given time to practise using multiple strategies and for rating their preference and showing the rationale for their preference in relation to context.

The use of estimation strategies was encouraged. An example of an activity used to encourage intuitive use of estimation skills was: *Suppose you were required to multiply two measurements with the following decimal numbers 30.25 × 3.6. What I want you to first ask your peer, is to make estimation. This way your peer will be able to reason that 30 × 3 = 90 and more. From here your peer will know where the decimal should be and more importantly know why the decimal is where it is.*

Students were asked to pose decimal-related questions using the four operations. *Use your calculator to work out the answer and write it down leaving the decimal point out. Give it to your peer and see if they can locate the correct place to position the decimal place.* For example,

$$201.34 - 78.89 = \qquad 97.56 \times 2.18 = \qquad 87/1.23 =$$

The estimation tasks were used to further lead students into thinking about decimal numbers and the quantities they represented. From this understanding, common misconceptions that can be attributed to overgeneralisation were explored: for example, that multiplication always makes bigger and division makes smaller, and that division is always of the larger number by the smaller.

The multiplicative structure was reviewed in relation to students' use of the strategy of: *I hop the decimal point this way when it is division and the other way for multiplication.* The movement of the position of numbers in their respective place value was explained to clarify students' misconception of the decimal hopping.

Students were given practice and consolidation of concepts by the following activities:

- Providing students with 10 multiplication and division items (e.g., 0.5×10, 0.5×10×10, 0.5÷10, 0.5÷10÷10) and asking students to explain how they predicted a multiplicative shift.
- Asking students to use a calculator to select the operation in changing items such as 5 tenths to 5 ones then changing 5 ones to 5 hundredths.

- Giving place value charts to students who needed additional assistance. Peer collaboration and peer teaching were also used to consolidate the multiplicative concepts.
- Working with students in small groups and using Baturo's (1998) diagram (Figure 5.5) to illustrate the multiplicative structure embedded in the decimal number system.

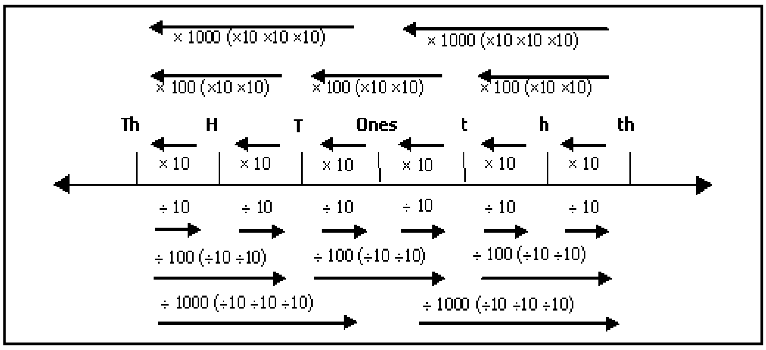


Figure 5.5 Multiplicative structure embedded in the decimal number system (Baturo, 1998).

Peer teaching was used as in previous sessions to further enrich students' understanding in the context of the errors in related test items:

- Errors in test items were analysed.
- Peers identified weakness in understanding to set objectives and plan lesson.
- Students implemented their lessons and used problem posing for refinement in understanding.

The community of inquiry was used to open a whole-class reflective forum on knowledge gained and difficulties and success experienced to make informed proposals for the next session.

5.5.4 Observation

During the explanation of the multiplicative structure embedded in the decimal number system it became evident that despite being able to give correct answers to questions such as 0.7 × 10, students did not really understand the reasoning behind the strategies they were using.

When working in small groups, it was clear that some students found the conceptualising of multiplicative structure difficult (Figure 5.6).

It kind of makes sense and kind of doesn't! It is confusing because I thought the numbers go from large to small and stops at the decimal place to go from small from large. Therefore in my head I expect two arrows facing each other direction of at the decimal point and not going all in one direction. But now that I look at it carefully it is still going from more to less after the decimal point.

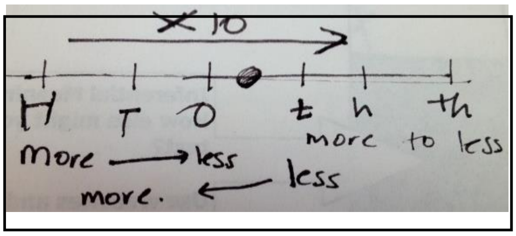


Figure 5.6 Student conception of multiplicative structure.

Use of examples and additional diagrams (as shown in Table 5.7) were found necessary to accommodate variations in students' ability to conceptualise the multiplicative structure. Supplementary knowledge of the additive structuring of decimal numbers also needed to be clarified.

Table 5.7

Multiplicative Structure Explanation

×1000	×100	×10	×1	.	/10	/100	/1000
1	8	2	3		1	2	1
Thousands	Hundreds	Tens	Ones		tenths	hundredths	thousandths

Observations indicated that beyond the procedure of computational skills, students had very little understanding of what multiplication and division really meant. Less than half the students in the class were confident about their automaticity in math facts.

Two students struggled with simple multiplication such as multiplying by two or doubling numbers. They were counting using their fingers to add, subtract and multiply and drawing lines and grouping them to divide. Without automaticity of number facts students had difficulty factoring, estimating, and multiplying single or multiple-digit computational problems. Estimation was feared as a process with the perception that it generated inaccurate answers. Previous learning experiences were associated with confusions:

I hate estimating because I always get it wrong because I don't know if they want me to round it up to ones, tens, and hundreds!

Two main computational procedures were used by students. The first was long multiplication where the individual digits in each place value were multiplied separately and the answers added together. *I pretend that there isn't a decimal place and just do the multiplication the long way till the end and just count the number of decimal places.* The second was making numbers larger or smaller by moving the decimal point. This procedure created several confusions as shown in the following examples.

Initially students were reluctant and lacked trust in new strategies introduced for multiplication. Observation showed that students who were generally high achieving were inclined to use the familiar long multiplication strategy to check if answers obtained were correct. However, with familiarity gained through practice of using multiple strategies side by side, students' preference of strategies became more flexible.

Error analysis of students' responses to the pre-test items indicated the errors were a result of various causes. As shown in the following example, confusion with the interpretation of the question was a common problem:

For the test item a) 0.7 × 10, I thought that the question was asking for a response which showed multiplicative relationship, which I thought meant that I had to show the multiplication relation to an equivalent division. So I wrote 0.7 × 10 = 70 ÷ 10, since 0.7 × 10 = 70 ÷ 10 both give the answer 7.

Lack of familiarity and gaps in computational knowledge were also attributed as causes of students' difficulty with test items:

When I did the test, I only could do one digit multiplication and division we haven't been taught how to do it with two numbers. So when I looked at the questions and saw that some of them had four digits, I was lost!

Examples of some of the common errors co-analysed in relation to multiplicative test items are summarised in Table 5.8.

Table 5.8

Co-Error Analysis of Multiplicative Relationship Test Items

Pre-test item 4a	0.7 × 10
Common error 1	*0.070*
Students' reasoning	*I couldn't really remember which side the decimal point jumps.*
Analysis	Using methods without understanding that the decimal point is fixed.
Common error 2	*0.70*
Students' reasoning	*I did the steps I was taught like for 7 × 10 = 70, so I took the zero from 10 and put next to 0.7*
Analysis	Applying procedures learnt for whole numbers without understanding the relevance of place value.

Pre-test item 4e	0.2 × 100
Common error 1	*0.200*
Students' reasoning	*I didn't know how to line up the decimal numbers, so I counted three decimal places for 0.2 × 100 because I put the 100 under 0.2.*
Analysis	Reliance on procedure and difficulty in recalling steps.

Pre-test item 4d	8 ÷ 10
Common error 1	*not possible*
Students' reasoning	*If you have eight, you don't have enough to give it to ten people so I thought this was a trick question.*
Common error 2	*8 ÷ 10 = 1 rem2*
Students' reasoning	*I divided 10 by 8.*
Analysis	Using multiplication and addition commutative properties without understanding why it cannot be used for division.

Observations during peer teaching highlighted how students individually adapted their practices to address their difficulties. Some students (semi/low in proficiency) avoided mental strategies for computation (even though they understood the processes) because they trusted strategies where they could see the process. Therefore they preferred finger counting and resisted peer-given learning strategies such as decomposition that involves reconstructing the answer based on the retrieval of a partial sum. For instance, the problem 8 + 7 might be solved by retrieving the answer to 8 +8 and then subtracting 1 mentally.

Similar behaviours were also noticed with some proficient students who preferred to use the more familiar long multiplication instead of estimation to check the reasonableness of an answer.

Students low in proficiency were observed to have four main common difficulties with computation: (a) they lacked the automaticity in recalling number facts and used tedious strategies such as finger counting; (b) they had difficulty with multistep procedures *(I can't hold numbers in my head and move it around)*; (c) they were unable to engage in word problems due to the required comprehension needed to identify and isolate relevant information as well as manipulate pertinent mathematical concepts to deduce solutions; and (d) when faced with the discomfort of learning they prefer to shut down and stop listening.

Peer tutors were found to be proactive and intuitively aware of their peers' difficulties. Efficient strategies were used by peer tutors such as: providing resources such as times table sheet; practice of steps sequenced to progressively increase in difficulty; and linking performance to effort and attributing difficulty to external issues such as—*it is a really hard problem*.

Emotional support provided by peers was evident with comments such as: *okay don't worry about that, what about we try this one which is more fun* or simply diffusing the tension by asking their peer to *just make a guess.*

5.5.5 Reflection

There were three main findings of note in this session: (a) students' perception of proficiency as being the speed and accuracy with which they could reproduce procedures; (b) balance of both rote and meaningful learning; and (c) adaptive practices dominated by resistance when faced with unfamiliarity or difficulty.

Observations of students' approaches to difficulties with foundational knowledge led me to conclude that many students saw mathematics proficiency mainly in terms of the speed and ease with which they could memorise and the accuracy of remembering rules for a process. However, this session highlighted the need for a balance of both rote and meaningful learning for retention and understanding of knowledge and skills.

Students' difficulty with instant recall of number facts affected students' confidence in their ability, speed and accuracy in computation tasks. This finding is consistent with Slava's (2011) research, which argues that without the ability to retrieve facts directly or automatically, students are likely to experience a high cognitive load as they perform a range of complex tasks. Review of students' history suggested that considerable effort was made by teachers to assist students with learning their number facts. They had used several strategies such as visual and concrete objects, regular practice and developing awareness of patterns in numbers to encourage proficiency in number fact. Therefore it was difficult to ascertain if difficulties in storing arithmetic facts in or accessing them from long-term memory was a result of lack of engagement during instruction of number facts or due to actual disability.

In reflection students identified that some of their adaptive practices when faced with difficulties were beneficial and others were detrimental to learning. For example, finger counting as a more trusted strategy than experimenting with mental strategies:

> *I can see it when I count my fingers to add or subtract, but if you just moving numbers in your head you can't be sure. In a way I know it slows me down and makes me less smart but it is hard to change.*

This reliance on trust of methods was also evidenced in high achieving students who checked their work using more familiar long multiplication instead of estimation in checking reasonableness of answers. Despite awareness of the inefficiency of strategies students claimed that an internally felt resistance dominates over taking risks when faced with unfamiliar or difficult tasks.

Students claimed that two main things were helpful in overcoming resistance: (a) practice, and (b) support of others. Practice provided familiarity that was needed to reduce students' uncertainty of success. An example (section 5.3.3) of how this strategy was used effectively was when students were given practice to use more strategies to answer the same question. With practice students developed a greater trust in their ability to obtain the correct answers, hence, were more able to take risks to use strategies flexibly. However as explained in the following comment by a student, practice requires motivation:

> *When you don't* know *something or find it hard, most of the time you end up disliking it, so the last thing you want to do is practice! If you have someone like my peer who helped me with my work, you can make it fun.*

This statement points out the complexity of learning where elements such as resistance, practice, trust, support of others and success contribute towards how knowledge is acquired.

5.6 SESSION 4: REGROUPING

5.6.1 Analysis of regrouping concepts

As shown by the patterns in data in Figure 5.7, the class mean suggested that most students' entry knowledge of regrouping was poor.

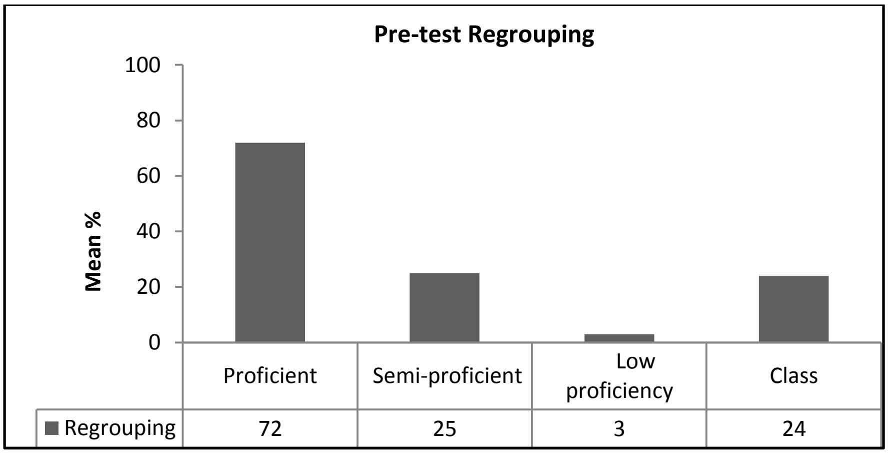


Figure 5.7 Performance in Regrouping test items.

Examination of responses to the regrouping test items revealed that only one student in the class was able to provide the correct answer to the pre-test item (2d):

3 tenths 6 hundredths 17 thousandths =………….

Another pre-test item that generated the most incorrect answers was item (1):

71 tenths 4 thousandths =……….

Most responses to these test items suggested that students were unclear of what the question was asking and were unaware of regrouping needed to answer these test items. As shown in findings from the previous session (Session 3), students' weak structural understanding of the decimal number contributed to difficulties with regrouping.

Examination of responses to regrouping pre-test items suggested that a common approach taken by students was listing the numbers stated and inserting a decimal point (such as *71 tenths 4 thousandths = 0.714, 7.14)*. Overall three out of four case study students in the proficient category performed reasonably well.

Students in the low-proficiency category were unable to answer the question with only one student guessing one out of the six pre-test items correctly.

5.6.2 Plan

Drawing on the analysis (section 5.6.1), the mathematical objectives for this session were to develop students' ability to:

- Understand the additive structure, equivalence, place value and base features of decimal numbers that are needed for regrouping of decimals (Baturo, 1978).
- Think flexibly about the notion of conservation in regrouping. Determining whether two numbers are the same or different in value, changing the name of a number without changing its value.
- Pose problems and solve problems by applying knowledge with understanding and flexibility.

Materials

Student-selected materials.

5.6.3 Action

The series of actions taken in this session consisted of:

- Brief revision of prior sessions—collaboratively mapping main ideas covered in previous sessions.
- Error listing as in previous sessions.
- Probing students' understanding during whole-class discussion on the following five semantic features for regrouping decimal numbers (Baturo,1978):

  (a) additive structure (e.g., 27t = 20t+7t);

  (b) multiplicative structure within places (e.g., 27t = 27×1t);

  (c) equivalence (10t = 1 one);

  (d) place value; and

  (e) base

- Students were given time to independently review their errors in the test.
- Co-construction of lesson plans with peers.

- Peer teaching—problem posing.
- Community of inquiry: reflections on students' perception of progress.

5.6.4 Observation

Students were able to independently review and correct errors made in the regrouping test items below.

Question 1: Write what's missing in each of the following

2.614	⇨		*ones*		*tenths*		*thousandths*
70.365	⇨	*703*		*65*			
………	⇨	*71*	*tenths*	*4*	*thousandths*		
2.007	⇨	*200*		*7*			

Only two students (in the low-proficiency category) had some uncertainty about regrouping. Most students needed only brief instructions to understand the regrouping process. Some students independently reviewed their work from the previous session and did not require any assistance. Students, particularly those at low levels of proficiency, found it difficult to regroup mentally. During peer teaching, effective peers were observed to build understanding by regrouping using simple problems with increasing complexity as their peer tutee progressed:

> *I got my peer to break up the problem and set it out like an adding problem on the grid paper so that she could see the regrouping process. After a bit of practice I think she will be able to do it in her head too.*

Some students at low levels of proficiency found problem posing in relation to regrouping difficult. They relied heavily on copying the questions in test and simply changing numbers. More proficient students were more able to experiment with problem posing to gain understanding.

5.6.5 Reflection

Important findings in this session included: (a) the success of previous sessions in building students' decimal knowledge; (b) acceptance of different ways of learning; and (c) the importance of success in motivating understanding.

The change in students' ability to review their test items on regrouping confidently and independently indicated that processes used in the previous sessions were effective in developing students' knowledge and understanding of decimal numbers. *I think I am getting it!*

Some students from the low-proficiency category continued to avoid using mental strategies and found motivation and success through a written and procedural approach to learning. However students argued *that it still a good thing because it just the way they learn. If you can accept that, then you can help them accept that it is not babyish and that it okay as long as you can do the work, that's all that matters.*

Peer tutors said that for their peers, it was more effective to first find a set of steps that their peers could follow, practice and memorise. Once they had memorised the steps and found success in applying the steps to similar problems, their peers became more confident and motivated to listen to explanations behind the procedure. They claimed that teaching the concepts first and then developing strategies did not work in most cases because their peer became bored and frustrated with the explanations.

Other students disagreed and claimed that explanation before co-constructing steps worked better with their peers.

This session highlighted the value of peer teaching in providing focused individualised learning opportunities with peers who knew the students well enough to provide the right prompts to support learning. In this session, students' references to syntactic and semantic knowledge suggested that students were starting to see mathematics as knowing the correct procedures as well as understanding the meaning of the procedures.

5.7 SESSION 5: COUNTING AND APPROXIMATING/ESTIMATING

In this session the test items explored included: (a) counting, and (b) approximating/estimating.

5.7.1 Analysis of counting concepts

Patterns of data distribution in pre-test results (as shown in Figure 5.8) of student entry knowledge suggested that students low in proficiency had considerable difficulties with counting when asked to add 1 thousandths more, but did considerably well when asked to continue a sequence.

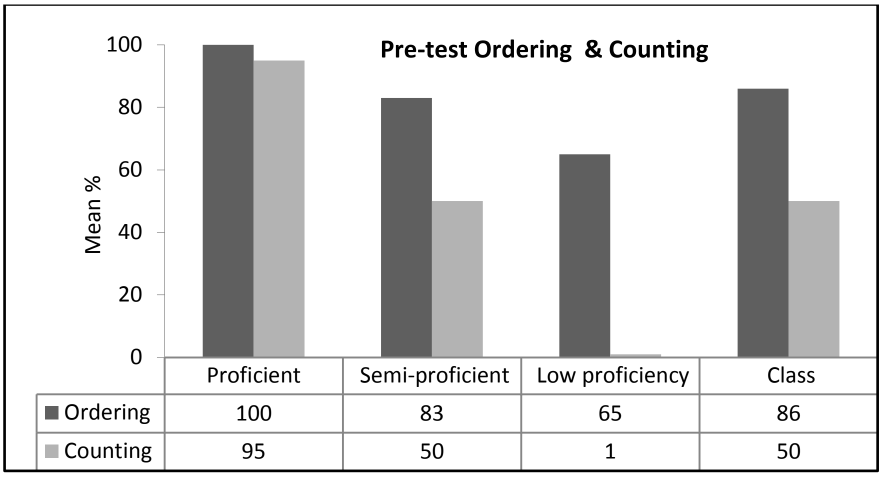


Figure 5.8 Performance in Ordering and Counting test items.

Examination of responses in relation to pre-test items for the counting suggested that this difference was because students could answer the question by counting on forwards or backwards without needing to consider the entire number. For example, when completing the following sequences, students were simply able to focus on the last one or two digits and count on.

8.527, 8.528, 8.529 ……………, ………… counting on 27,28,29,30 and so on

On the other hand, students had more difficulty when asked to: *Write the number that is 1 thousandth more* because first students needed to know the position of the thousandths place and how to regroup numbers if needed.

(a) 3.563……… (b) 5.269……… ……… (d) 4.591 ………… …………

As shown in the regrouping pre-test items in the previous session, without understanding of semantic features of place value and conservation in regrouping, students were unable to answer these questions. Hence, some students wrote *there is no thousandths* in test item: *(c) 0.09…. (e) 6….*

5.7.2 Analysis of approximating and estimating concepts

Analysis of overall test results for approximating and estimating (as shown in Figure 5.9) suggested that most students were able to approximate and estimate given decimal numbers to the nearest whole number and identify the fraction shaded in a shape (non-prototypic).

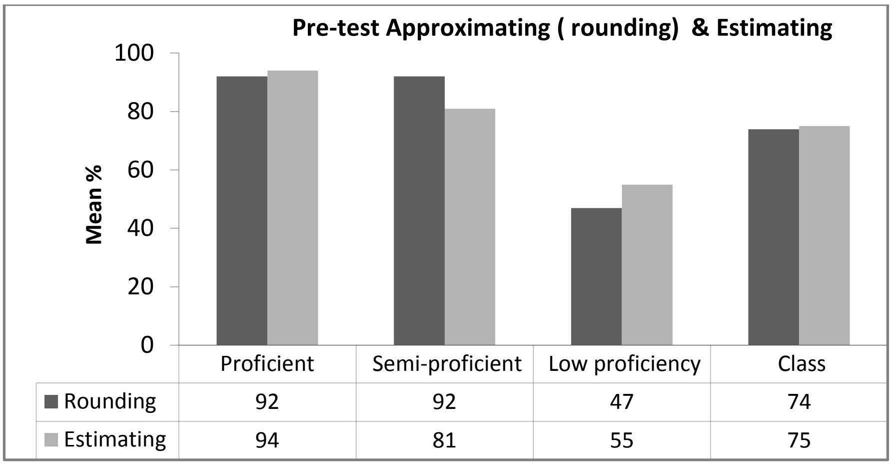
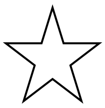
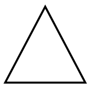


Figure 5.9 Performance in Approximating and Estimating test items.

Responses in the pre-test suggested that the pre-test item for rounding with the most incorrect answers was: *Round number 0.347 to the nearest whole number.* Most students wrote 1 assuming that 0 meant nothing, in which case 0.347 could not be rounded down to 0.

When approximating and estimating the following pre-test items, students seemed to have two different difficulties.

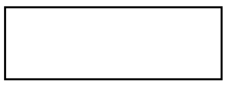
(*2a) Colour 0.935 of this shape* 

(2b) Colour 0.457 of this shape 

For test item (2a), some students had difficulty conceptualising what part of a whole was *0.935.* Yet they were more familiar with (2b) *0.457 being close to half.*

For test item (2b), some students' difficulties seemed to be related to how students divided the shape. Students who divided the triangle shape horizontally had more difficulty than those who divided the shape vertically.

Other difficulties included misunderstanding or misreading the question. For example, in the pre-test item below, most errors were a result of shading half of the rectangle.

This is 0.521 of a shape – draw the whole 

A possible cause for students' errors could be a result of students relating previous experiences with shading shapes divided into equal parts for fractional understanding to this question. Therefore this process of shading had become automated and students read only part of the question.

5.7.3 Plan

Drawing on the analysis (section 5.7.1), the mathematical objectives for this session were to develop students' ability to:

- Conceptualise the part-whole place value of decimal numbers by rounding, approximating and estimating in symbolic or pictorial form that include non-prototypic shapes.
- Pose problems and apply knowledge to count, order, compare and estimate place values of decimal number using novel contexts.
- Participate in the following class task: Role play setting up a tuckshop selling items to three decimal places (e.g., $2.199 rather than $2.20). Explain the benefits you might gain.

Materials

- Computer access (to generate shapes).
- Number line.
- Measuring instruments (tape measure, scale, marked volume cylinder).
- Cards with various numbers to allow ordering or counting on.
- Student-selected materials.

5.7.4 Action

The series of actions taken in this session consisted of:

- Brief revision of prior sessions—collaboratively mapping main ideas covered in previous sessions.
- Error listing.
- Independent error correction.
- Knowledge building: whole-class exploration with ordering numbers (before, after and between numbers).
- Activity: Fuel stations price petrol with a decimal to three places rather than as money to 2 places (e.g., $2.699). This gets consumers to read the price as $2.69 rather than the $2.70 that it really is. Will you argue to support this current practice or force the change to pricing to a full cent?

- To reduce dependency of students on more confident members of the group, the question was explained and students attempted the question independently by stating if they agree or disagree with a reason for their position prior to moving into groups.
- Celebration banquet planned in Session 3.
- Reflective discussion on how decimal knowledge could be retained and integrated in everyday classroom learning.

5.7.5 Observation

Students had no difficulty with independently self-correcting errors made initially in the approximating and estimating pre-test items. Only two students required some prompting for counting 1 thousandth more. Peer tutors were keen to help their peers and with the strategy of lining up the numbers in place value columns and adding 1 thousandth. They asked: *Where is the thousandths column? So where should we put the 1?* Once the process was made visible by seeing it written down, students were better able to comprehend the question and understand the process.

When attempting the context activity, it was observed that although most students were confident about their knowledge, when the familiar practised structures were absent, the problem was perceived as difficult. This affected the students' engagement and confidence. However, when the question was explained orally (with very little addition to what was written) students seemed to be able to comprehend the task better.

While most students agreed with the suggestion given by the activity, others question the argument given and did not agree with the statement that the price will be read as \$2.69 rather than \$2.70. They claimed that *when you remove a place value you would normally round it up. The third number would force you to read it up as \$2.70 rather than \$2.69.* Another argument disputing the suggestion was stated by a student:

> *I don't agree with it either, it doesn't make sense because by changing money to three decimals, because it is longer people will make the mistake and think it is more.*

5.7.6 Reflection

We learnt lots and what is really strange is that it didn't feel like learning because it was fun. In reflection of this statement students identified that fun included many of the elements that they would normally define as not being fun such as hard work, experiences of difficulty and persistence despite feelings of resistance:

> *Yet, this was fun because: It was working with together and learning from each other instead of being scared about not being smart like other people; we could help each other; we got to choose and help with planning and picking good activities; we get a chance to show that we can learn and do better instead of just stopping with the bad mark in the test.*

These reflections highlight the potential of students to empathise, create and learn when they feel they have some control over decisions made about how they could acquire knowledge. The efficiency and passion with which students created, implemented and experimented with knowledge showed that, whether theoretically considered sound practice or not, students were learning, making mistakes and refining ideas to make it more enjoyable and beneficial:

> *I didn't want to be part of the research at first. I was scared that I will have to work with the smart kids and they go really fast. When teachers put me with them in groups before it was really hard, I never really did anything but copy. I think what I really learnt to look busy and tried to get into groups where people will help me by giving the answers. It was different when I worked with other people in this decimal thing I actually learnt heaps.*

Such statements describe the lived experiences of students and explain that even the most well intended teaching strategies can be rendered counterproductive without consultation and complicity of the students who are the recipient of teacher actions. Students explained that the difference in the approach of students participating as research co-constructors (SPARC) was that:

> *You listened to us if we didn't want to do something. You still made us learn it, and probably even work harder, but you made it fun. Normally teachers get cranky if we disagree with anything.*

The success of the teaching experiment was further highlighted by students' interest in seeking strategies for retaining the knowledge gained. Voluntarily, students suggested that they would add an extra part to their journal entries on the cell city they were building for science to describe the mathematics they used.

5.8 DELAYED POST-TEST

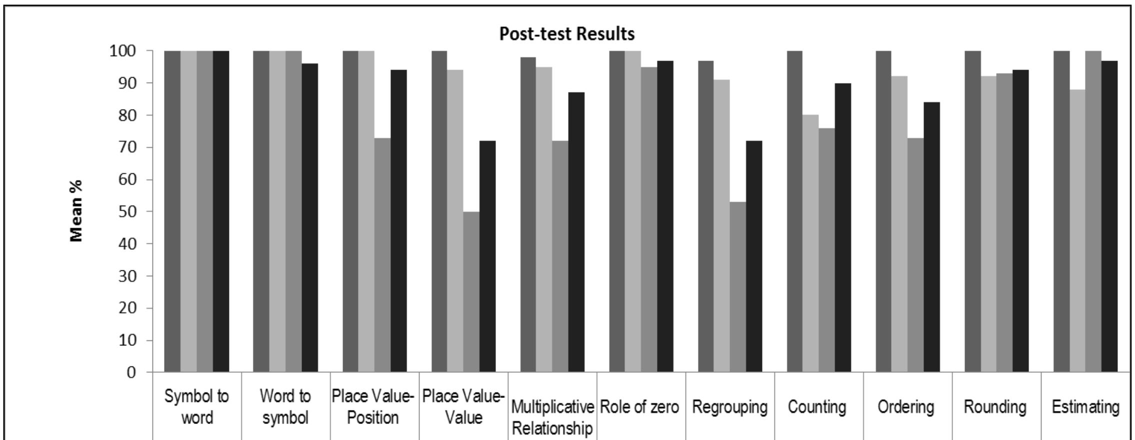
5.8.1 Plan

A delayed post-test was implemented seven months later to test students' improvement and retention of decimal knowledge.

5.8.2 Observation

In comparison to the observations made during the pre-test, students were less apprehensive and more confident during the post-test. In fact some students said that they were excited and interested in finding out how much they had improved.

Students also stated that they set new and higher expectations and still felt some distress and anger: *I felt angry that I couldn't remember things even though I knew I would do much better than I did in the pre-test. I wanted to get 100%.* However, in contrast to students' reactions and comments made during the pre-test, most students' requests for help and comments were related to decimal concepts rather than the difficulty they were experiencing.

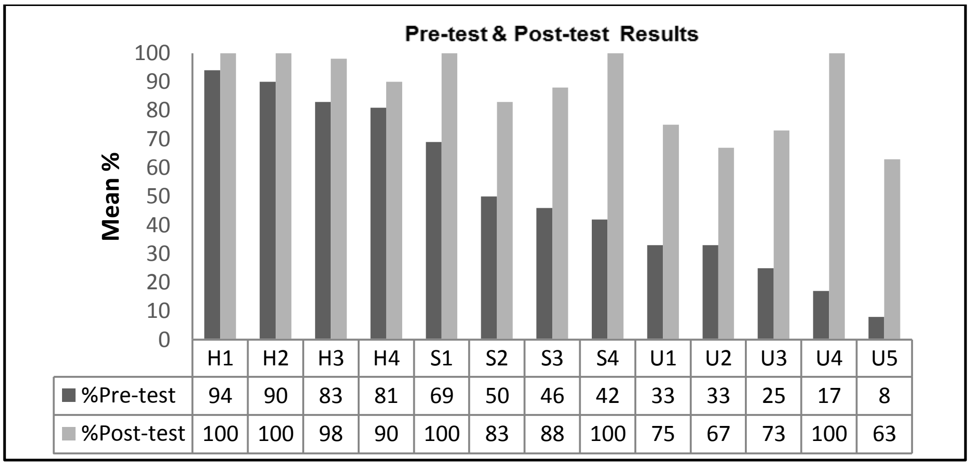


	Symbol to word	Word to symbol	Place Value-Position	Place Value-Value	Multiplicative Relationship	Role of zero	Regrouping	Counting	Ordering	Rounding	Estimating
Proficient	100	100	100	100	98	100	97	100	100	100	100
Semi- Proficient	100	100	100	94	95	100	91	80	92	92	88
Low Proficiency	100	100	73	50	72	95	53	76	73	93	100
Class	100	96	94	72	87	97	72	90	84	94	97

Figure 5.10 Post-test results.

Test results indicated substantial improvements in students' performance in the post-test (Figure 5.10) in comparison to pre-test results (Figure 4.3).

Changes in proficiency of individual students are illustrated in Figure 5.11.This micro-view of students' individual performance in the post-test shows a clearer picture of changes in performance that occurred in individual students.



	H1	H2	H3	H4	S1	S2	S3	S4	U1	U2	U3	U4	U5
%Pre-test	94	90	83	81	69	50	46	42	33	33	25	17	8
%Post-test	100	100	98	90	100	83	88	100	75	67	73	100	63

Figure 5.11 Changes in proficiency levels.

A detailed side-by-side comparison of results from the pre- and post-tests is provided in Appendix E. Overall, for number identification test items, there was an increase in accuracy of answers of all students. Some students made the same error as in the pre-test: *7 tens, 7 tenths = 7.7*. However, these students were able independently to correct their errors without assistance and were also able to provide reasons to justify their error as a careless mistake.

Examination of the post-test responses suggested that two students had some difficulties with the multiplicative and place value test items where multistep thinking was required. For multiplicative relationships, most students were able to give accurate answers but not all students were able to explain the multiplicative structure guiding the process that was used. It was unclear if automaticity of computation skills through regular practice had caused students to stop thinking and forget the reason behind the procedure explored during the teaching experiments.

For regrouping test items, post-test responses suggested that not all students were confident with grouping units by powers of 10 and partitioning units by powers of 10 and mentally keeping track of the transformations required for regrouping. This difficulty in mentally regrouping was also seen when students attempted counting test items where they were required to add one thousandth more. However, they were able to recall and use the place value chart and processes taught by peers by writing down the process as shown in Figure 5.12.

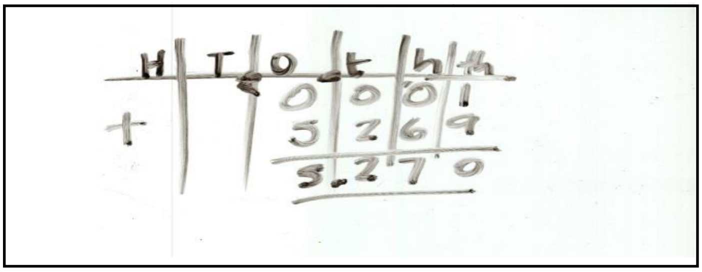


Figure 5.12 A student's use of jotting for tracking regrouping processes.

For post-test items on estimating and approximating, the incorrect responses were easily self-corrected by students suggesting that they were a result of careless mistakes rather than errors.

5.8.3 Reflection

> *Probably the biggest difference was that we wanted to know how we would go in the test instead of thinking about doing the test for the report card.*

Students stated that their positive attitude towards the test was, *because we had so much fun with learning decimal and that it was something we were involved in.*

As discussed (section 5.8.2), comparison of results from post and pre-test results suggested considerable improvement in knowledge and understanding. Looking back, it was students' change in interest and application of decimal knowledge in other contexts after the teaching experiment ended that were the most significant indicators of improvement.

The post-test results on their own had some limitations in that the familiarity of questions and ability to recall well-practised processes could have influenced performance. However students' application of decimals in problem posing, explanations during peer teaching and arguments during community of inquiry provided data that supported the results in the post-test.

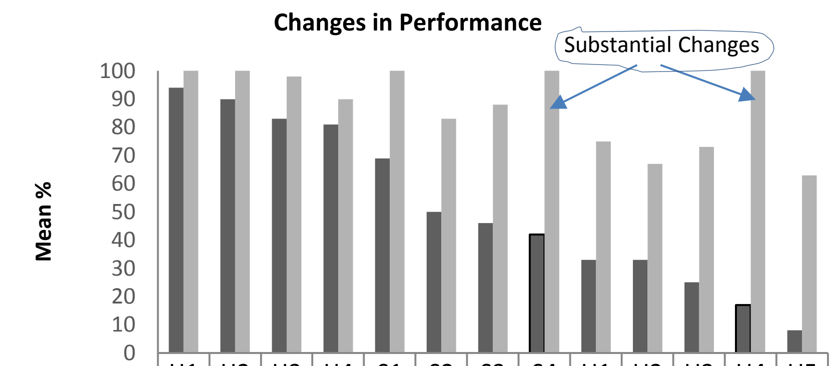
5.9 POST-TEST ANALYSIS FOR EFFECTIVENESS OF RESEARCH

This section discusses the overall results (section 5.9.1) and is followed by pre- and post-test comparisons on the four main areas of difficulty revealed in the pre-test: (a) number identification; (b) place value; (c) multiplicative structure of decimal numbers; and (d) regrouping and renaming.

5.9.1 Overall results

Side-by-side comparisons of pre- and post-test results (Appendix E) showed that all students improved considerably in all areas as a result of the teaching experiment. Figure 5.13 illustrates the differences between students' performance in the post and pre-test.

Two students in particular made substantial improvements with results that were equivalent to students that were placed in the proficient category (H) at the start of the research. One of these students was initially categorised as being low in proficiency (U4) and the other student semi-proficient (S4). Both of the students demonstrated considerably lower levels of understanding during the initial interview.



	H1	H2	H3	H4	S1	S2	S3	S4	U1	U2	U3	U4	U5
%Pre-test	94	90	83	81	69	50	46	42	33	33	25	17	8
%Post-test	100	100	98	90	100	83	88	100	75	67	73	100	63

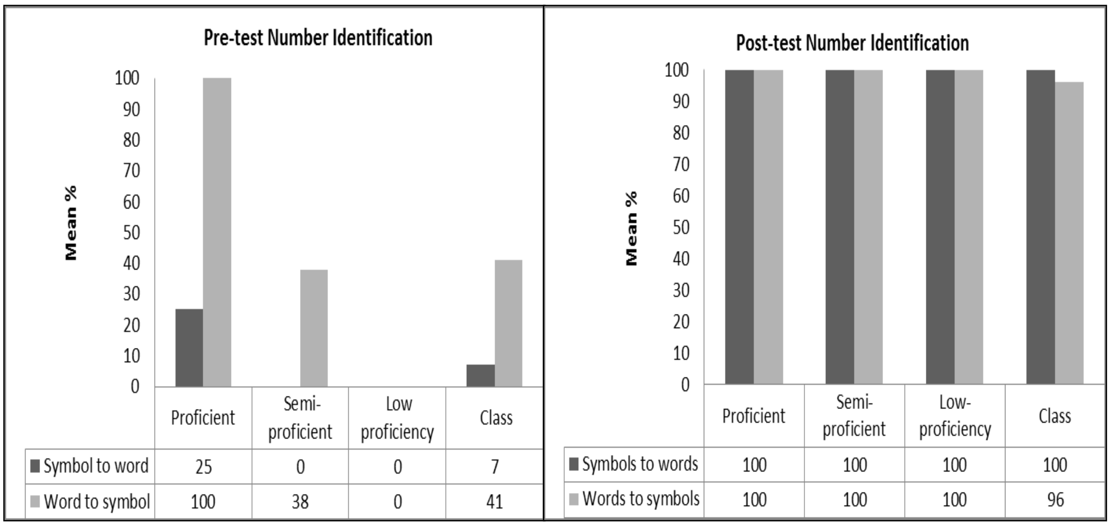
Figure 5.13 Pre-test and post-test performance comparison.

According to the students the main reasons for the change was confidence and active engagement in the learning process. U4 stated that repeated failure had caused disengagement *(shut down)* and a sense of despondency towards learning or trying. According to the students, interest was renewed because the research was engaging and they believed that learning was made within their reach.

5.9.2 Number identification

One of the greatest improvements in test results was in number identification (Figure 5.14). Students claimed that number identification became simpler due to familiarity and regular use:

> *Even though we found it painful and found saying numbers like two and three tenths instead of two point three, after a while we got used to it.*



	Proficient	Semi-proficient	Low proficiency	Class
Symbol to word	25	0	0	7
Word to symbol	100	38	0	41

	Proficient	Semi-proficient	Low-proficiency	Class
Symbols to words	100	100	100	100
Words to symbols	100	100	100	96

Figure 5.14 Pre-test and post-test Number Identification comparison.

The results show that initially students found the process of identifying symbol to word more difficult than word to symbol. Students improved across both areas with only a small drop in converting word to symbol. This improvement was also evident during the teaching experiment. For example, students questioned and identified the problem created by ambiguity of number names when calling out numbers orally, such as:

> *What is one hundred and 9 tenths times ten?*

Students pointed out that *called out orally one hundred and 9 tenths* could be interpreted as 100.9 or 10.9.

Some students were noticed to make references to the test items to support their view of ambiguity in the naming and use of *and* as the convention to denote the decimal point in decimal numbers (*Test item 3c: In 625.078 the 50 is worth?)*. These actions suggested that students were actively engaged and thinking about mathematics with an intrinsic interest in understanding. It also showed that students were developing a greater flexibility in understanding how names of decimal numbers could be changed without changing their value.

5.9.3 Place value

Post-test results suggested that there was substantial improvement in students' performance in the place value test items (Figure 5.15).

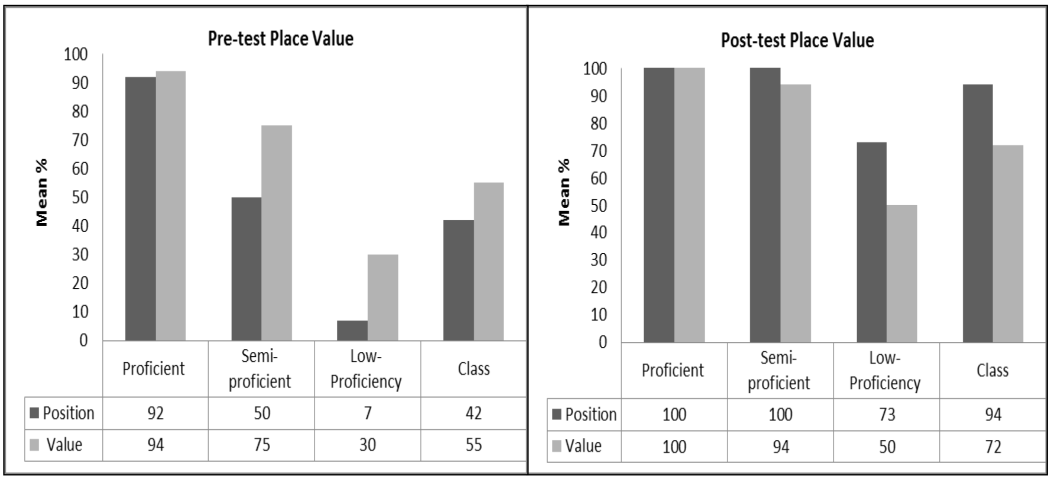


Figure 5.15 Pre-test and post-test Place Value comparison.

Examination of their working showed that a small number of students continued to rely on drawing place value charts and placing individual digits as one of the strategies taught by their peers. Most of these students had difficulty with remembering and felt that they had *forgotten everything*. However with a few hints most students regained their confidence and were able to recall and justify their answers accurately.

The area of most difficulty was renaming using place value understanding. Student said that they understood that the names of decimal numbers could be changed without changing its value, but had difficulty with remembering the process of how 50 in *625.078* could be written as 50 tenths because *it is not something you would use often.*

Despite the experience of some difficulty students were found to be interested in questioning and seeking understanding. This was different from earlier reactions where students had passively attributed difficulty to lack of ability.

5.9.4 Multiplicative understanding

Comparative analysis of multiplicative mean results in pre- and post-test results showed overall improvement (Figure 5.16). Deeper analysis of test responses suggested that not all, but a few students in the low-proficiency group still had some difficulty with remembering the multiplicative structure concepts. Students that got all answers correct lifted the mean for the category.

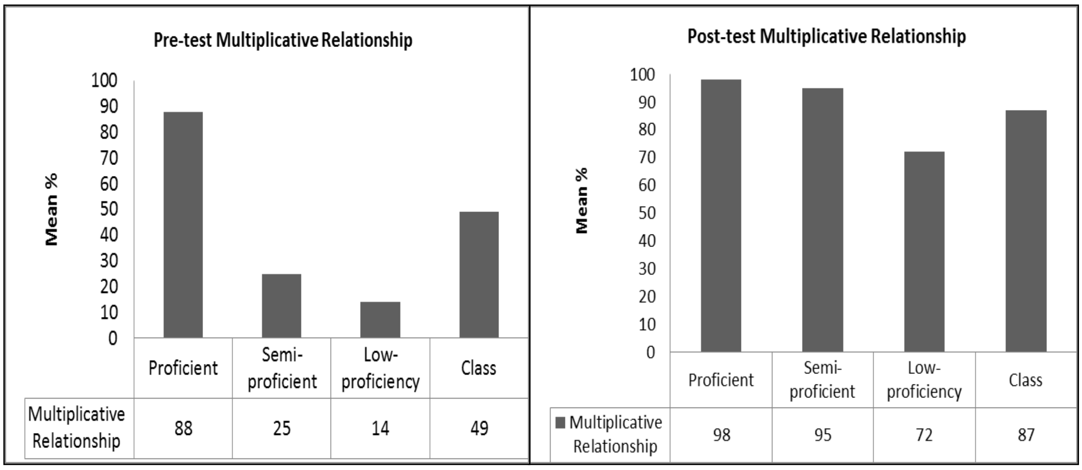


Figure 5.16 Pre-test and post-test Multiplicative Relationship comparison.

Students' jotting of working suggested that these students reverted to using the idea of jumping the decimal point to the right and left. From students' explanations, it was difficult to ascertain if students' difficulty was memory related or due to seeking the safety of ingrained strategies.

5.9.5 Regrouping

For regrouping, all proficiency groups showed improvement (Figure 5.17). The difficulty experienced by students mainly in the low-proficiency category seemed to be with conceptualising the regrouping process in their minds. However, the strategy of using algorithms and lining up numbers in a place value chart that made the process visible for students was found effective. It was interesting that students did not want the option of using ready-made laminated place value charts—*it is better when I draw it out because it helps me think when I am drawing it.*

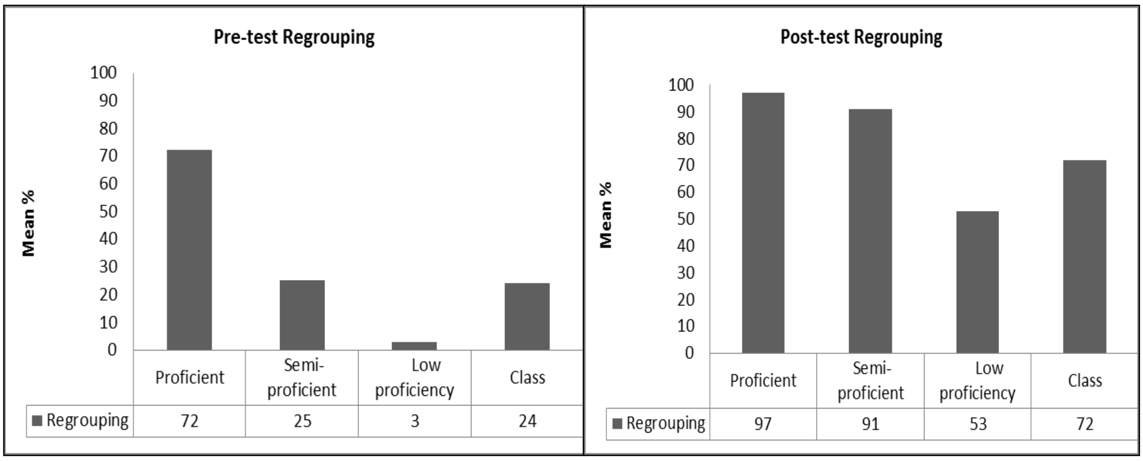


Figure 5.17 Pre-test and post-test Regrouping comparison.

5.10 SUMMARY

The improvements in students' decimal knowledge evidenced in the comparison between pre- and post-tests (Appendix E) clearly indicated that the teaching experiments were effective.

Explorations during teaching experiments showed that one of the most prominent problems with learning decimals stems from the perception that decimals are simply an extension to whole numbers and that syntactic whole-number rules can be applied to decimals. Despite teaching approaches intended to build a sound understanding of decimals based on place value notions, students were confused. Therefore the metacognitive teaching strategies used in this research were effective in making similarities and differences between whole numbers and decimals explicit, allowing students to make more accurate connections between their prior and decimal knowledge.

The teaching experiment was particularly effective in individualising learning starting from where students were at. It was found that: proficient students were more likely to use strategic knowledge to solve problems when experiencing difficulty; semi-proficient students used declarative, procedural or schematic knowledge; and low-proficiency students tended to rely on procedural knowledge. Post-test and observations during the teaching experiments indicated that these were not static preferences. Some students made extensive changes in their task approach when self-efficacy and metacognition increased and resistance decreased.

Differences in performance and the extremities of changes in proficiency levels based on test results highlight the weakness in tests for measuring immediate cognition rather than a student's capacity to learn from a given experience. As shown in Figure 5.11, some students made extensive leaps in levels of proficiency despite having initially performed poorly in the pre-test and the interview (Chapter 4) that confirmed their lack of proficiency. It is likely that these students' performance in the interview was affected by their perception and confidence of their own ability due to the value they placed on their test results. This initial perception or lack of confidence in their own ability was observed to filter through the first few sessions—where students made correct responses but were not confident in the responses. There was a discrepancy between the subjective and objective correctness of their answers. With consistent feedback that confirmed the correctness, students were able to reduce the discrepancy between their perceived and actual ability. Hence what is central to accurate assessment of students' performance is the students' ability to adjust their subjective assessments of their knowledge to reveal their capacity to acquire knowledge.

The positive changes in students' decimal knowledge evidenced in the post-test signifies the potential of using metacognitive teaching strategies in mediating individual resistance to teaching and learning experiences. It elucidates the significance of shared responsibility which is inherent in the co-constructive/analyst approach taken by this research. As students put it, *it changed me.*

Chapter 6: Analysis

6.1 CHAPTER OVERVIEW

"Education is not filling a bucket but lighting a fire."—William Butler

The two main objectives of this research were to improve students' decimal knowledge and co-improve practices used to learn and teach decimal number concepts. The overall results from the teaching experiment and pre- and post-tests showed that, at the end of this research, students had improved cognitively, affectively and conatively.

This chapter first presents four central themes drawn from results (Chapters 4 and 5) regarding the factors that influenced students' knowledge and practice:

1. Students' mathematical difficulties (section 6.2).
2. Resistance to learning (section 6.3).
3. Metacognition and learning (section 6.4).
4. Success before understanding (section 6.5).

Since the themes are interwoven, at times references are made to other themes within each section. The themes are analysed using research literature in relation to the objectives of this research (section 1.3). Conclusions are also drawn from students' input.

Central themes

Figure 6.1 illustrates the interactions of themes and provides an overview of this chapter. It shows the initial cycle of students' mathematical difficulties and resistance. Co-researching/learning/teaching (SPARC), metacognitive knowledge and experiences of success are suggested to motivate metacognitive actions. This argument is based on the assumption that metacognition develops a conscious awareness of own thinking and practices through collaborative inquiry and feedback. The experience of success was also noticed to play a critical role in encouraging self-efficacy and student agency, which then motivated students conatively to engage in reflecting and thinking about their thinking. As shown, the cycle of mathematical difficulties and resistance was replaced with a cycle of metacognition and success.

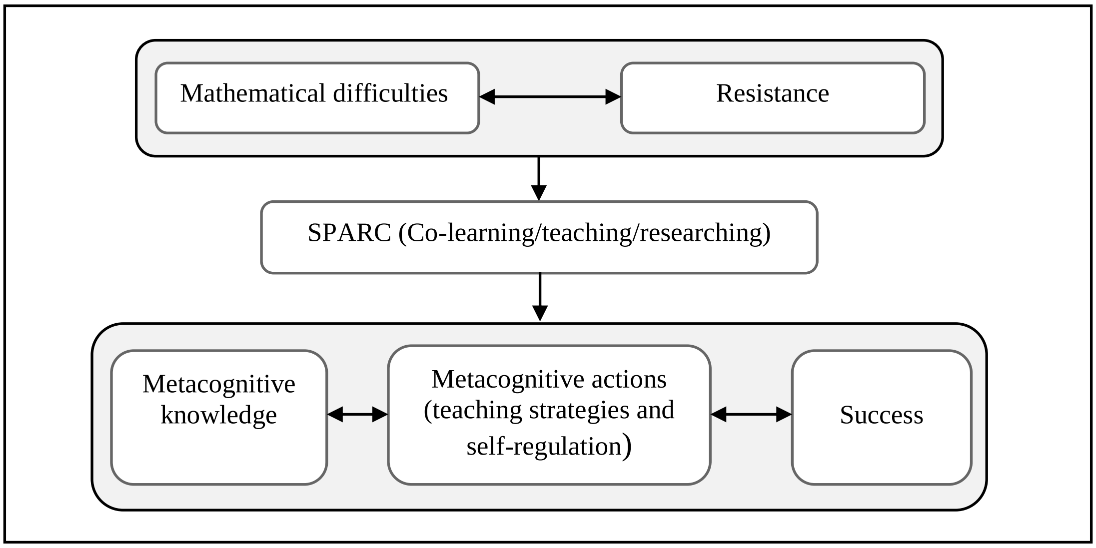


Figure 6.1 Interaction of themes in decimal knowledge acquisition.

6.2 STUDENTS' MATHEMATICAL DIFFICULTIES

This section discusses results (Chapters 4 and 5) in relation to students' difficulties with decimal knowledge acquisition and their remediation through the SPARC approach of co-constructing interventions to improve students' decimal knowledge and motivation, and to repair disengagement.

Although students' mathematical difficulties is an extensive topic, three interdependent factors selected for co-analysis of students' difficulties in this section are: (a) knowledge and understanding; (b) memory; and (c) task approach.

6.2.1 Knowledge and understanding

Analysis of test results (Chapters 4 and 5) revealed several mathematical misconceptions and gaps in prior knowledge that interfered with students' ability to meaningfully make sense of decimal knowledge. Misconceptions were found to be actively defended and resistant to change rather than being momentary conjectures that could be quickly discarded.

Many of the students' misconceptions found in this research showed similar patterns of errors in thinking to those described in other research literature (section 2.2.4; Lee et al., 2011; Nesher & Peled, 1986; Stacey, 2005). Paths of thinking errors included: zero conceptions where place value connections were not understood in terms of the role of zero; common fractions and place value conceptions incorrectly extended to interpret decimal digits; and conceptions of operations, where multiplication commutative laws were incorrectly applied to division (section 5.5).

A common and noticeable practice was applying whole-number rules with a strong reliance on syntactic rules based on whole numbers to construct decimal concepts (Hiebert & Wearne, 1985; Moody, 2011; Steinle & Stacey, 2004). In fact, many students did not simply lack structural understanding of decimal numbers (Baturo, 2002; Resnick et al., 1989), but also had only a superficial understanding of whole numbers.

Weak prior knowledge was found to be one of the most noteworthy factors contributing to students' difficulty with understanding decimal concepts (Baturo, 1998; Peled & Shahbari, 2009). Gaps in students' prior knowledge needed for understanding decimal numbers was clearly evident at the entry stage (Chapter 4) of the research. Students' difficulty with differentiating between whole-number and decimal concepts (Vamvakoussi & Vosniadou, 2010) also explained why many students did not recognise the need for reorganisation of their whole-number thinking. That is, students were operating with systems that were consistent with their internal schemata.

Research literature says that the reason for this difficulty is that disequilibrium is avoided when students fail to recognise any contradiction and simply assimilate new material into their previous way of thinking (Moody 2011; Piaget, 1985). As such, attempts to evoke cognitive conflict (Vamvakoussi & Vosniadou, 2010) are resisted and disregarded. This explains why some students who attempted to comply with a new procedure, temporarily appeared to understand but reverted to their prior thinking (such as the *decimal hopping* identified in section 5.5.3).

Students relied on prior knowledge to construct their understanding of the decimals by drawing on analogies of whole numbers which were more familiar. This reliance on prior knowledge backfired when students approached decimal concepts that could not be described sufficiently with their prior whole-number knowledge. Without sufficient and appropriate prior knowledge, students were found to be unable to inductively construct or make correct connections between knowledge.

A comparative and metacognitive approach was found useful in developing students' awareness between whole and decimal numbers. Students claimed that listing and analysing their own errors in the table adapted from Resnick et al (1989) (Appendix D) was effective in making differences between whole and decimal numbers explicit:

> *I think the table was really good because when we listed our errors we had to really examine it to see if the value increases or not. I had to take more notice of the columns, check what zero was doing to the number and the sound of starting and ending sound of the name.*

In assessing students' understanding and pinpointing students' mathematical difficulties, the diagnostic test was useful. However on its own it provided limited information. Interviews and the combined use of teaching and learning instruments served to provide deeper insights. Analysis of results during peer teaching sessions show that students' depth of understanding was often better known when students were teaching concepts to their peers rather than learning or doing a test (e.g., section 5.3.3).

Students' understanding was observed to influence the ways in which they learnt and taught. For example, during peer teaching of problems, it was observed that peer tutors varied the number of digits in the decimal component of the number when posing questions. Tutors with weaker understanding used this process to provide practice of a skill/concept with some variation to problems.

Tutors who were deeper thinkers were able to foresee how misconceptions such as "more number of digits means greater value" could be reinforced if the number was not varied. For example, comparing 3.27 and 27.47 allows students to apply this misconception without realising the difference between decimal places and whole-number places in context of the value of a number. On the other hand, using varying number of digits in relation to the position of the decimal places brings students' misconceptions to the forefront. For example, in using 3.27 and 2.747, students were able to see that the number with the most digits does not necessarily have the larger value.

6.2.2 Memory

Peer teaching strategies used by students showed that the role of memory in learning (Crossland, 2010) was identified and addressed by students with adaptive strategies such as using written processes, using visual cues and practice.

Being able to remember was perceived by students as an indicator of their ability to learn mathematics. It was noticed to have a large influence on their confidence and self-efficacy (Bandura, 1977):

> *We missed out on learning what everyone else was doing because we couldn't remember maths stuff like the times tables so we went to the dumb group to do easier work.*

Students who lacked confidence in their ability to remember were also found to resist learning new strategies. They relied on strategies taught in earlier years of learning that were ineffective and time consuming for more complex computation tasks (e.g., drawing lines and grouping them together for division). Students were observed to become tired with the lengthy process and became frustrated with not having enough time to answer questions.

Memory-related difficulties included not remembering:

- Basic maths facts and computation procedures (e.g., section 5.4.4).
- The concepts that explain the formulation or use of computation strategies: *I jumped the decimal to the right in this question and to the left in the next question so that I would get at least one answer right.*
- Previously encountered patterns and concepts such as place value patterns (Stacey & Steinle, 2006).
- What they were doing in the middle of a maths problem such as when mentally regrouping (e.g., section 5.8).
- Names of symbols such as the decimal point and names of the place value positions such tenths or hundredths (e.g., section 5.3).
- The relationship between symbol, language and concrete representations (e.g., section 5.2).

Peer tutors claimed that they found three main strategies in dealing with students' difficulty with remembering. The first was to use scaffolds as described by Vygotsky (1978) and contexts for practice of application of knowledge. Scaffolds were adapted to suit the need of the student. For example, in the case of difficulty with number facts, peer tutors encouraged peers to use times table charts or calculators:

At first my peer had to look at the times chart all the time, but after a while she actually started to try and remember it. I think that's because when your mind is focused on something you want to solve, the easy bits of the times table just comes in your head.

In this instance it was the peer tutee who chose to remove the scaffolds. In other situations such as using place value charts, it was the peer tutors who selectively removed scaffolds progressively.

The second strategy that peer tutors found useful was chunking concepts into small manageable steps (Winstead, 2004). Students claimed the chunking process was more effective when done together by the peer tutor with their peer tutee:

This is what we have to do, now you tell me where you can remember up to and then we will practice that bit and then do the next.

Peer tutors claimed that success through small steps motivated their peer tutees to then proceed to develop understanding of the process that was giving them success.

The third strategy that peer tutors found effective was based on the observation that some students with attention and memory-related difficulties were observed to find moving numbers and ideas mentally difficult. As a result they lost confidence and showed fatigue during instruction. Therefore despite research recommendations (McIntosh, 2005) of using mental computation before teaching the algorithm, peer tutors found that jotting down the algorithmic process rather than focusing on mental computation was more effective. They argued that it was not a matter of using mental computation or algorithm first, but what worked.

In consensus with the students' view, learning style theory (Dunn & Griggs, 2003) also advocates methodical pluralism and argues that students should initially be instructed according to the method best suited for their needs. This can, thereafter, be expanded to include secondary style preferences.

6.2.3 Task approach

Students varied in their cognitive styles and affective resiliency which together influenced their approach to tasks. Learning style preference (Riding & Rayner, 1998) was found to be a fairly stable individual preference for organising and representing information. However students' task approach was noticed to vary and develop over time (e.g., memory strategies and emotional and cognitive strategies).

As discussed in section 6.2.1, familiarity and schema developed from previous experiences was found to be one of the strongest predictors of students' task approach. For example,

> *... if I was asked to do 2 × 0.5, I would switch it around to 0.5 × 2. The answer one makes more sense because it is more than what we started with. But if we went 2 × 0.5 then we actually dividing and not multiplying but we have a multiplication sign which is just confusing.*

The example shows that students' approach to tasks is a conscious or unconscious choice made by students as to how to process given information and demands of a learning activity. An interesting question is *how do students make the choice in task approach*? In this respect metacognition becomes central.

A problem that was identified was that activating metacognition was often masked by automaticity. In fact automaticity of rules, accuracy and the speed at which the rules could be used to solve the problem was perceived as proficiency by most students. Therefore, students argued that trying to understand, question or experiment was inefficient or an unnecessary waste of time that could lead to possibilities of confusion and wrong answers. This was illustrated during peer teaching with some peer tutors giving instructions such as:

> *Can you stop thinking so much, just follow the steps!*

As a result of this preference for surface learning, when the questions were not replicated, and familiarity of question structure was not present, it was not uncommon for students to find difficulty in recognising the disjunctive elements of new problems.

Observations revealed that from this speed learning approach to tasks, students had developed strategies such as compartmentalising bits of information. They perceived various strategies offered to them by teachers as stand-alone procedures. Students claimed that there wasn't any time spent in finding connections between strategies or how they could be used flexibly to best match the context of the activity.

For example, Area model had been used to graphically illustrate the commutative and distributive properties of multiplication. Lattice multiplication had also been introduced. Yet students were unaware that the Lattice multiplication was algorithmically identical to the traditional long multiplication method, but broke the process into smaller steps.

By using procedures without understanding, students made errors in their thinking. For example, when given the problem *8 ÷ 10* (section 5.5.4), students argued that eight could not be divided by ten and therefore applied the commutative properties of multiplication to division (10 ÷ 8). The misconception of multiplying to create a larger number compared to division is a concept that goes against what they experienced with whole numbers (e.g., $10 \times 8 > 10 \div 8$). Therefore students believed that $10 \times 0.8 > 10 \div 0.8$ whereas in reality multiplication $10 \times 0.8 = 8$ while division $10 \div 0.8 = 12.5$, thus the opposite is true.

In the opinion of students, a strategy they found effective in provoking metacognition during the teaching experiment was practising and working with various strategies side by side (section 5.5). This included the activity of drawing relational connections between concepts within strategies and giving a rating for preferences in the context of the problem. Students said that concepts embedded within the strategies became better understood, for example, when solving the problem 5×48 using three strategies:

- The place value partitioning concepts could be seen when using long multiplication.

  $5 \times 48 = (5 \times 40) + (5 \times 8)$

- Compensation method showed students how they could think about numbers flexibly to make computation less tedious.

  $5 \times 48 = (5 \times 50) - (5 \times 2)$

- Doubling and halving strategies showed students the balance and equivalence principles embedded in mathematical concepts.

Observation showed that students initially approached a task by first using the most familiar strategy. However, with practice and confidence changes in students' preferences for strategies were noticed.

6.2.4 Summary

In summary, Figure 6.2 illustrates the interdependence of memory, knowledge and understanding, and task approach.

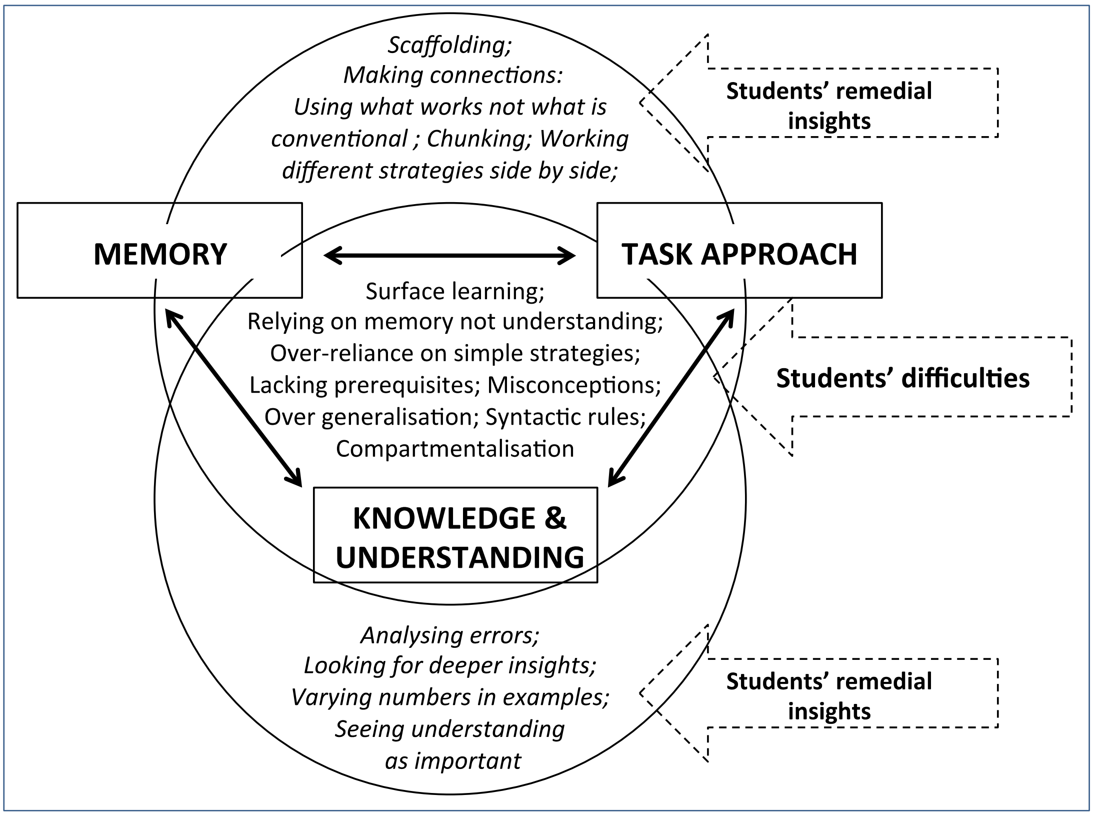


Figure 6.2 Interdependent factors in students' mathematical difficulties and remediation.

The ability to remember and what is remembered determines how students adapt their approach to tasks. Understanding and knowledge can occur only when a student's memory contains a relevant knowledge base—information to which a new idea can be connected (Crossland, 2010). With a better understanding and knowledge of the conditions of learning, and more precise knowledge of how task approach can affect learning in a positive or negative way, teachers' and students' consciousness of learning may be expanded.

Figure 6.2 lists difficulties that impede students' ability to acquire deep understanding of decimal concepts as well as the insights of students for remediation of these difficulties.

Interestingly, the SPARC approach used by this research, which invites students to be involved in the co-authoring process of remediation of students' difficulties, reveals differences in the route that was taken by students from what I had expected as a teacher. As such, it is likely that some of the students' insights for remediation may be resisted by teachers as contrary to conventional views that have become part of their repertoires for good teaching practice.

For example, as a central guiding principle of teaching, teachers focus on understanding, whereas the students' focus was observed to be on remembering and immediate gains of success. While the need for scaffolding was a shared view, students' thoughts about how scaffolding should occur was different. Students were in interested in *what works* rather than what is more conventional (section 6.3). In contrast, teachers' interests are based on building and establishing strategies, using materials to develop concepts and a structured sequence of learning.

In the opinion of students, building beliefs about self-efficacy, confidence and success in remembering was initially more important than understanding, while understanding and strategy building emerged with increased motivation after experiencing success (section 6.5).

The students' approach to remediation was well received by peers having difficulty and was effective. Effective peer tutors were found to be able to evaluate where their peers were in their understanding. They then chose appropriate task approaches and teaching strategies such as chunking, scaffolding with increasing levels of difficulty, and jotting and connecting knowledge.

In reflection, these findings present a challenge to teachers to consult their own values, interests, and beliefs in defining their own teaching priorities, at the same time marshalling accessible student insights for guidance towards how learning can be achieved. This theme is further discussed in section 7.4.

6.3 RESISTANCE TO LEARNING

This section discusses the need to understand the role of resistance in decimal knowledge acquisition. Resistance is seen to result from (a) affects/emotions that are provoked by difficulties experienced; and (b) incongruence between teacher and student goals and the value of teacher-selected materials for learning. Teacher reaction to students' resistance to learning from teacher-selected resources is also discussed. This is followed by an analysis of differences in student and teacher use of materials. In the summary, a model for resistance is proposed.

6.3.1 Affect-related resistance

In deeper reflection of students' mathematical difficulties (section 6.2), it is evident that cognitive processes (to know) are necessary but insufficient for decimal knowledge acquisition. Consideration of affective (to feel) and conative (to react) elements is also required (Alsop & Watts, 2003; Clore & Storbeck, 2006). Yet, little is known of the coordination between cognitive, metacognitive, affective, motivational processes and interpersonal patterns of relations shaping students' transactions when attempting the acquisition of decimal numbers.

Cognition analysis in most research studies (in decimal learning contexts) ignores issues of students' affect and conative action. As shown in section 6.2, schemas are stable cognitive structures that organise students' thinking and perceptions (Demetriou & Wilson, 2009). This not only links students' resistance to mathematical cognition but also affects influence cognition (Izard, 2011).

Feelings and reactions to the difficulty of decimals

Students' reaction to mention of decimals was at first one of 'eyes glazing over'. The mention of a test on decimals (Chapter 4) was met with sighs of dismay. Students who normally acted with generosity and goodwill became competitive and their learning became rigid with defence. The students' goal was getting the lesson on decimals over and done with, as fast and painlessly as possible—*like a Bandaid!* This was the reality of the classroom. As the teacher, it was clear that my detailed lesson plans based on strong theoretical knowledge were inadequate without some synergy between my intentions and those of the students. Students were not simply resisting learning; they were resisting the whole context in which they were being taught. A student summed up how she felt in a short sentence—*I feel sick in the inside.* Sparks (20l1) says that resistance to maths is not uncommon, and in fact disliking maths is considered perfectly socially acceptable.

In the opinion of students, the reason for resistance was that it was too hard and too confusing. Most students agreed that it was possible for them to learn if they were given more time and exposure to experiences. However, in their minds, failure, fear, and lack of trust in their own ability was given currency by their immediate indicators of difficulty with acquiring decimal knowledge.

Reasons for compliance were essentially extrinsically motivated—*to get good marks so that I can get a good job in the future* or punishment related. Students' comments suggested that students and teachers were navigating participation in learning through unconnected paths with different agendas. While teachers are seeking to engage students in meaningful learning (ACARA, 2012) of decimal knowledge, students' participation was externally driven to meet expectation of others—*I only try to learn maths because I don't have a choice*. During whole-class discussions, such as when proposing the concept of ecologies of active participation (Gest & Rodkin, 2011), students were able to articulate and interweave complex ideas to legitimatise practices that promote and advocate active participation when difficulty in learning is experienced. Yet, in practice, students resisted participation when faced with difficulty. Most common forms of resistance were avoidance, self-attack—attributing helplessness to lack of ability—and withdrawal/disengagement in order to build their own protective space (Wentzel, 1999).

Fear of not meeting expectations

Deeper analysis suggested that fear of not meeting perceived expectations was one of the most dominant overriding emotions that had an incapacitating effect on complicated knowledge processing and acquisition. During the implementation of the pre-test, most students came with an initial mindset of high expectations. They displayed great distress when they felt they could not meet their own expectations. As a result of the observed decrease in motivation and high levels of anxiety, the pre-test (Chapter 4) was stopped and the students settled before restarting the test.

Cognition researchers (e.g., Lyons & Beilock, 2011; Sparks, 2011) explain that during initial problem solving, a student processes information through the amygdala. The amygdala then prioritises information going to the prefrontal cortex, the part responsible for the brain's working memory and critical thinking. During stress such as that experienced during a test, there is more activity in the amygdala than the prefrontal cortex. This can literally disconnect the working memory needed to learn and solve problems. Studies have mapped brain activity during emotion regulation to the lateral prefrontal cortex, the medial prefrontal cortex, the medial orbitofrontal cortex and the amygdala (Forgas, 2008; Moss & Damasio, 2001).

Students' anxiety was still evident at the end of the test. During collection of test papers, some students were reluctant to hand in their papers claiming they had not finished and needed more time. Others wanted to hand in their test papers after a short attempt, in acceptance of defeat.

The characterised body language indicating shame (Sparks, 2011) such as averted gaze and facial blush, eyes and head downcast, was evident in students when marked pre-test results were given back to them for review. Several studies (Hahn, 2000; McGregor & Elliot, 2005) show that participants experiencing shame display similar feelings to those reported by my students—such as the intense desire to hide or shrink into the floor or to disappear or escape from the shame-eliciting situation. Overall, the extant data collected during entry and teaching experiment (Chapters 4 and 5) indicated that students with a high fear of failure were socialised by school discourse in a way that oriented them to the possibility of failure, exerted pressure on them to succeed beyond their capacity, and exacted relational costs (Wentzel, 1999) should failure occur.

Fear of failure and shame

Avoidance and resistance to learning was noticed to be used as the defence mechanism to cope with the over sensitised fear of shame—*I don't want to even look, think or talk about that test ever again!* Atkinson (1957) portrays fear of failure as "the capacity or propensity to experience shame upon failure" (p. 360). Hahn (2000) explains shame as a complex combination of emotions, physiological responses, and imagery that involves an intense feeling of guilt of not being able to meet expectations, profound sense of inadequacy and perceiving others as critical and condemning. Research literature (Hahn, 2000; Johnson, 2012), on the developmental progression of shame gives psychological insight into how the reaction of shame could have developed through repeated experiences in school contexts.

Students appeared to have learnt to define failure as an unacceptable event that carries negative implications for their self-worth and status, and relations with peers, teachers and their parents, which led them to vigilantly orient to and seek to avoid failure in achievement situations. As such, it is argued that it is difficult for teachers to address resistance due to shame easily in school contexts, especially when the students' experiences resonate with many of the education practices such as unintended punishment through assessment, competition and comparison that contributes to a sense of unworthiness, inadequacy, and defectiveness that introjects shame (Johnson, 2012).

Research into the management of shame (Benson, 2006; Hahn, 2000; Morrison, 1990) focuses mainly on parents as caregivers. However, the research can be applied within school contexts. The management of shame is reliant on the creation and maintenance of a safe supportive environment wherein the disclosure of shamefulness (in this case not knowing or having difficulty grasping decimal knowledge) is accepted by others and oneself where self-acceptance can unfold. While such theories are logical, studies explain why in practice, management of shame has been so resistant to classroom learning—as observed during this research. For example, in a national sample of 148,189 students, Benson (2006) found that only 29%–45% of those surveyed reported they had social competencies such as empathy, decision making, and conflict resolution skills, and only 29% indicated that their school provided a caring, encouraging environment.

6.3.2 Resistance to teacher choice of materials

Results during the teaching experiment showed (section 5.2.5) that resistance and loss of interest occurs when there is incongruence between student and teacher beliefs about the value and interest in teacher-selected resources for learning.

The mention of Multi-base Arithmetic Blocks (MAB) was met with protests (section 5.2.5). In acknowledgement of students' resistance, there was minimal use of such materials in the initial knowledge building session. Therefore, at first glance, many actions discussed in Chapter 5 would appear to be weak in relation to recommendations of good practice espoused in literature (Stacey, 2005).

Literature recommendations for material use

Research literature (English & Halford, 1995; Stacey, 2005) emphasises the need for conceptual understanding to avoid simply teaching mathematical rules and procedures. To do this, the use of concrete materials such as Base 10 MAB, Linear Arithmetic Blocks (LAB) or Area Cards is recommended (Roche, 2010). These representations are argued to provide students with a visual reference point, allowing associations such as hundredths and thousandths and how they fit into the place value system. Therefore they are useful for activities such as naming, ordering by size, estimation, addition, subtraction, multiplication and division (as repeated addition and subtraction). Materials such as MAB are argued to be used to mirror the base 10 features of our numeration system and serve as an analogy to the structure of decimals (English & Halford, 1995).

Students were familiar with the use of these common teacher-selected materials, and it would make sense to utilise this prior knowledge to facilitate understanding of decimal concepts. However, it was decided that it would be more productive to take into account students' resistance to using specific resources and come to a collaborative agreement for alternatives.

6.3.3 Teacher reaction to students' resistance

Students' main argument for their resistance to the use of MAB was that:

> *I can't really see the point of it, it's a waste of time, every year we had to work with the MAB in groups, and we just made bridges and stuff when the teacher wasn't looking.*

This statement told me that, first, the students were already in a mindset of a predetermined idea that the resource was not something they would learn from or wanted to learn from. Secondly, it also told me that if the students were forced to work with the resources, this would create a problem where the students and teacher would not be working together as a team to achieve a shared goal. To address students' resistance, I asked them to offer their input in finding an alternative path to meeting the desired mathematical objectives. The following actions were taken:

- Acknowledged their input as important and valuable.
- Listed mathematical objectives.
- Asked students to offer suggestions to create new paths to reaching desired goals. This included the role and use of material to illustrate and reinforce concepts.
- Discussed probable obstacles and predictions of effectiveness of actions proposed.

From this discussion, a rationale for using concrete materials was established and a set of actions supported by research literature was determined.

It was decided that students would be given the option to select their own resources during peer teaching. To ensure their appropriateness and practicality of use, the selected resources were to be teacher checked so that together the effectiveness of resources could be enhanced or modified if needed. Students decided they would create a document in their e-portfolio to list resources under specific categories with a short description. They could then share ideas and resources with those who were having difficulty finding resources.

6.3.4 Materials as a teaching tool rather than a learning tool

Despite students' initial protest and obvious dislike for MAB, it was interesting to note that they included MAB among other resources when they were peer teaching. Students claimed that the use of concrete resources became better understood when they were used in peer teaching. They said that they became more aware that resources such as concrete models were simply assistive tools that were meant to make understanding easier, rather than another task to add onto a complex topic—*I didn't have a problem with understanding the maths, it was the blocks and the colouring of the grid that I never got. I found it very distracting!*

Students argued that when resources were used as illustrative tools to explain an idea to address a question, then it made sense, but often teachers create entire activities and have test questions that rely on the understanding of the resource. Students stated that they found this distressing:

> *I don't know why, I can't stop my mind from going blank whenever I see those block kind of stuff. When I have to do a test, and it has them in it, I think I should have listened! I get the real maths work right and the colour in blocks wrong, because I can't work out what they want. Do they want me to colour the row, one block or count all the squares? A part of something could be part of one square or one or more rows or the whole grid. It is like a lame puzzle you have to work out because different teachers ask you to do different things.*

6.3.5 Difference in students' use of learning materials

Students said the reason why they used MAB blocks when they were peer teaching was that it was different from how most teachers use it. It wasn't

> *…here are the blocks, and go do all these exercises. We weren't learning about the resource from what we know about maths, but learning maths from resources. Resource should be something that we like and can relate to, not something more complicated that wastes time and puts you off maths.*

Students claimed that the reason why they were successful with using MAB during peer teaching was that it was a one-on-one situation. The resource was being used as a conceptual tool to teach an idea and to meet the immediate need of the student seeking meaning. Therefore the responsibility of learning was shared. If the resource was not effective, immediate feedback could be given and alternatives sought:

> *The difference is that the teacher was not standing up the front explaining to everyone what we have to do with the materials. Instead, we used it when we saw a need for it to help us work out something.*

Other visual tools such as the number line which was extensively used by teachers showed similar problems to those of MAB. When the number line was drawn and students were asked: *How many numbers exist between zero and one?* Answers included: *ten, hundred* and *none.* There were no students, including those categorised in the proficient category, who could provide the correct answer for this question. This clearly showed that, despite years of practice with the number line, its purpose as advocated in literature (Durkin & Rittle-Johnson, 2012) was never really understood.

From the comments made by students, it is clear that exposure to resources for understanding concepts does not always guarantee that students see the connections between resources and concepts. These findings direct our attention to assumptions that are made when students and teachers draw on their own expertise and experiences. Hence it highlights the need to find common ground by problematising practices as sound or unsound based on where the students are in terms of existing knowledge and perceptions.

6.3.6 Individual differences in material use

Observations of peer tutors adjusting and modifying resources to address missing schemas highlighted how standardised materials can be a source of extraneous cognitive load (Slava, 2011) and be counterproductive for students. Further analysis of students' arguments suggested that mismatches of cultural background, disposition, learning style, prior knowledge and proficiency levels were contributors to the observed frustrations and resistance to materials for learning.

Proficient students who already possessed the required schemas in their long-term memory argued that they resisted the use of materials because its need was redundant. They said that their interest in material for learning was renewed when the task demand was altered from learning to organising and integrating materials to teach peers.

However, not all students who showed high levels of understanding were able to relate to materials commonly used by teachers such as the MAB. Some were found to have more difficulties with such materials than students at lower levels of proficiency. Deeper analysis of results adds to the understanding that this effect results from certain teaching materials losing their effectiveness as learner proficiency increases.

Research literature on the use of materials acknowledges such differentiated needs of students in that it focuses on the ways that materials can promote thinking and conceptual understanding (Moody, 2011; Moyer & Jones, 2004; Shaughnessy, 2011). In suggesting that materials have the potential to represent explicitly and concretely abstract mathematical ideas to develop students' understanding, literature (Moyer & Jones, 2004) identifies the differences in how individual students connect to learning experiences.

Yet, in practice, students in this research, as also argued by other researchers (Marshall & Swan, 2008; Moyer & Jones, 2004), claim that teachers often use materials without questioning the appropriateness or value of the material they use. A possible reason given (Moyer & Jones, 2004) for this approach is that teachers believe that mathematics materials are inherently good and that lessons that make use of mathematics materials are pedagogically sound. Marshall and Swan (2008) noted that despite teachers' belief that materials are beneficial, less than 10% of the teachers participating in their study indicated they had engaged in professional development and only 19% indicated they would like further training in the use of materials. Marshall and Swan (2008) claimed that justifications given by teachers in their research for the use of materials were insufficient in that they were vague clichés about engaging students in hands-on maths or moving from the concrete to abstract or making mathematics fun.

These findings support the views of students in this research regarding the need for more critical judgements about the suitability of materials in terms of purpose of use and how the purpose fits in with the needs of the students. As discussed in the next section, students in this research demonstrated the effectiveness of materials when they were given a choice to co-select, create and make critical judgements about the materials that were most suited to their needs.

6.3.7 Effectiveness of students selecting resource

In analysing the effectiveness of having students select their own resources, several advantages were highlighted. First, it taught students how to learn and think about why and what role a resource plays in facilitating understanding:

> *I think I learnt more from thinking about creating resources than using it, because when you are making something, you have to think about everything that you need to get someone to learn from it.*

Second, it gave students a sense of agency. The process of having students identify peer needs, locate, select, critique and negotiate the use of resources and then test their effectiveness was found to affirm students' self-efficacy (Bandura, 1977). As students became more confident and familiar with concepts, they became more creative in developing their own resources. They also became more willing to contest objections with sound arguments and more accepting of frustrations involved in locating suitable resources. Most of all, they said they enjoyed the experience of testing their ideas.

The success and benefits gained from having students select their own resources made me realise that teaching mathematics is not just about guiding students through a series of theoretically based and sequenced mathematical experiences that gives the illusion of a metacognitive approach. Rather, for metacognition to occur, teaching for learning requires both the teacher and students having an understanding of each other's practices so that true collaboration can occur.

Relating to issues of such an approach, Vygotsky's (1978) work bears relevance, most significantly his notion of the zone of proximal development, or the difference between one's actual and potential levels of cognitive development, a difference that can be overcome through collaboration.

6.3.8 Summary

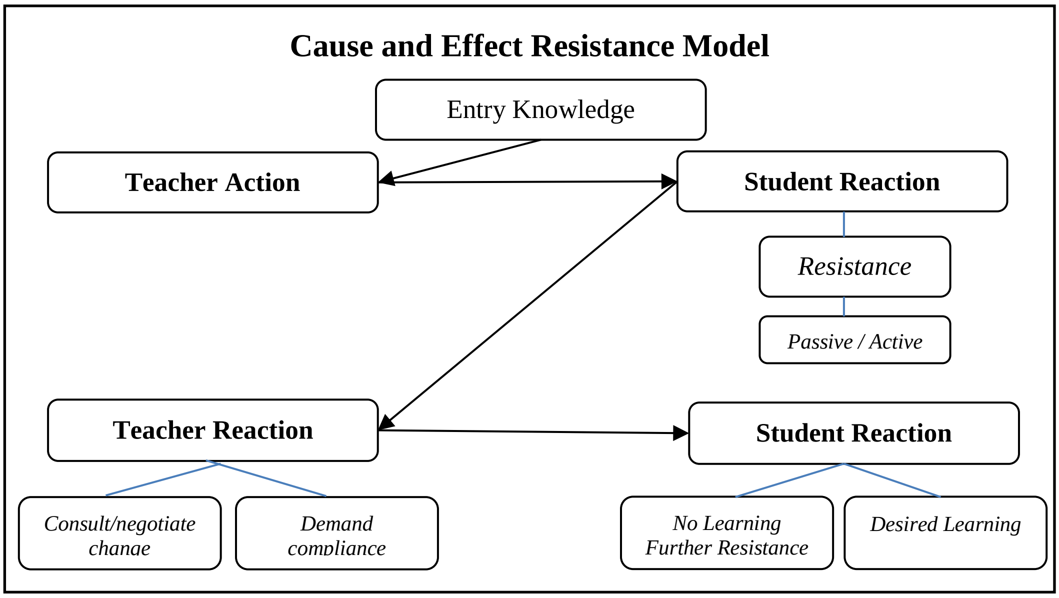


Figure 6.3 Cause and effect resistance model.

Figure 6.3 illustrates the zigzag resistance model that defines this section. In this model, resistance is understood as an accepted cognitive and emotional reaction to students' experiences of difficulties with decimal concepts.

Incongruence between teacher and student goals and beliefs about learning and the value of resources used to learn are suggested to create further dissonance and loss in students' interest. As such, teacher reaction to students is argued to determine desired action or further reaction.

6.4 METACOGNITION AND LEARNING

This section presents a rationale for metacognition in decimal knowledge acquisition processes. Metacognitive teaching strategies (Appendix B) used in this research to train students and motivate metacognition are analysed to determine their effectiveness in improving students' decimal knowledge and practices for learning.

6.4.1 Rationale for metacognition

As suggested by all other sections in this chapter and in consensus with research literature, evidence supports the need for metacognition in relation to decimal knowledge and practices (Brown, 1987; Flavell, 1976; Steinle & Stacey, 2004). Research literature highlights that teachers' difficulty in teaching decimals calls for metacognition for both the teacher and students (Brousseau, 2008; Irwin, 2001; Schoenfeld, 1987).

Making students aware of discrepancies between their prior and new knowledge, which strategies can be used for different tasks and letting students try out what works best for them provided a framework for metacognition based on learning and teaching. Importantly, it encouraged students to take ownership of their own learning. Bandura's (1977) work shows that taking control of learning is directly related to self-efficacy which is argued to play an essential role in students' motivation to learn.

Metacognitive theorists claim that metacognition leads to a consciousness of learning (Brown, 1987; Flavell, 1979). The learning process is then guided by the response/feedback on two main types of knowledge. The first is knowledge about abilities, strategies and resources that are necessary to complete a task. The second is knowledge after completion of a task—that the knowledge is understood and retained and how to move on from this point (Boström & Lassen, 2006).

The resistance model (section 6.3.8) shows that metacognition may be either facilitated or obstructed by different types of emotions that individual students experience when faced with difficulties. There are two main metacognitive actions that are important for counteracting students' resistance to learning with purposeful engagement. They are: (a) development of conscious awareness, and (b) development of self-regulatory mechanisms.

The development of students' reflective conscious awareness (Brown, 1987) refers to both their cognition and affects. This involves being aware of sources and occurrences of misconceptions and gaps in knowledge so that students can select, evaluate, revise, and abandon cognitive tasks, goals, and strategies (Flavell, 1979).

The development of self-regulatory mechanisms is seen as important for conation (Hofer & Sinatra, 2010). As pointed out in section 6.2, it was not enough for students to have metacognitive knowledge and skills. They also needed to have the ability to regulate its use when confronted with stress, competing avoidance options and fatigue.

Students needed to be able to distinguish between metacognitive processes (s), asking awareness questions such as: *What am I about to do? How do I feel about the* *decimal test?* And regulatory questions such as: *What am I going to do now? How effective is my strategy?* It was found that the students' own interpretation of their state could not be accounted for by consideration of only one of these factors. Rather, through the interplay between awareness and regulation, interpretations were constructed.

6.4.2 Practices obstructing metacognition

While metacognition is clearly needed for understanding decimal concepts (section 6.4.1), the need to become consciously aware of automated practices that inhibit metacognition was also required (Dresel & Haugwitz, 2005).

Automated teacher practices

Automated practices as a result of familiarity often mask the need for monitoring. This is not simply students' practices but also includes my own practices as a teacher. For example, it was only when I started to mentally map and connect interpretations from literature against my analysis of students' errors in the pre-test items, that my awareness of the implicit aspects of the structure and links between decimal number concepts was activated. Many of these understandings had become automated in my own practice due to familiarity with decimals. As was also found by Peled and Shahbari (2009), "decimals are straightforward for the teacher who understands their structure and can use them flexibly and efficiently" (p. 73).

At this point, I wanted to share my findings with my students so that they too could see the cascading effect of how misunderstanding of one concept led to the errors in connecting prior understanding of other ideas. I wanted to step back and recreate in them the wonder of revealing misconceptions and seeing what solutions my students could come up with to elicit deeper understanding of the mathematics in decimals.

To co-explore misconceptions with my students, varied teaching strategies (e.g., error identifying, error listing, error analysis, peer teaching and argumentation) were found effective in stimulating metacognition from different lines of thought. Although many of the strategies used were found to exist in literature (e.g., Baturo, 1998; Borasi, 1994; Brousseau, 2008; Lave & Wenger, 1991; Stacey, 2005) their implementation was not predetermined. Rather, they emerged and evolved from interactions, while the students and I were having fun in working out ways to learn and teach.

It is unlikely that I would have known or considered the depth of my students' misconceptions if I wasn't conducting this research. I fear that this experience would be all too common in classrooms, since pragmatically, teachers cannot normally undertake such in-depth analysis of students' understanding. These findings suggest that teachers' attempts with metacognitive approaches to teaching can be rendered superficial when they don't fully understand the connections that students make in understanding the decimal system.

In reflection it seems that the more a teacher understands and is familiar with the work, the less they are able to empathise with the students' perspective. They therefore lack a critical part of teaching, which is to have a firm grasp of what may be required to prompt learning. This highlights a glitch in teaching practices that can be caused by familiarity, where a teacher feels increasingly confident about having improved as a teacher, whereas the students feel the opposite about their knowledge and ability to learn.

Automated student practices

Similarly, students experienced automated thinking due to familiarity creating blind spots in their knowledge and understanding. These were found particularly detrimental in decimal knowledge acquisition because prior knowledge of memorised procedures did not support conflicting new decimal knowledge (Crossland, 2010; Merenluoto & Lehtinen, 2004; Peled, & Shahbari, 2009). For example:

> *I don't know how I did it, I don't have to think about it anymore and I can just do it in my head.*

Students were not always accurate in describing their own ability and thinking processes. Students who experienced some difficulties but sought understanding often underestimated their mathematical ability. On the other hand, some students who had found success in well-practised sets of procedures (without any understanding of the procedures) overestimated their ability and felt no need for further learning.

Students claimed that familiar reflective questionnaires given by teachers to prompt metacognitive thought did not always make them think about their thinking. Students said that these checklists often consisted of questions such as: What new ideas and skills have I learnt? What did I find difficult? Which activity did I enjoy or dislike?

> *I don't like doing it but it makes the time go! To be honest I never really thought what it was for, just did it because it is something we always have to do in every subject at the start and end.*

> *I couldn't really say I disliked it at all because the teacher will take it personally and give you a bad mark!*

These comments suggested that students did not always understand or share the same goals in reflective tasks. In fact students resisted honest participation in fear of social consequences that may result from reflection. Co-analysis with students indicated that we often complete some tasks and resist others according to our perception of the threats and benefits where fear of threats often overrides benefits. Therefore metacognition becomes central in developing a conscious awareness of practices (Flavell, 1976) and in weakening resistance.

Metacognition in weakening resistance

This research points out obstacles to metacognition that result from habitual/automated practices and strong emotions. It also suggests that through collaborative endeavours (Larkin, 2006), awareness can be sharpened and emotions regulated to enable students and teachers to correct distortions in their perceptions, interpretations and beliefs (Brookfield, 2006; Mezirow, 1998; Procee, 2006). From this point of view, the research proposes that metacognition can be used to weaken resistance to improving learning and practices. It also suggests that cognitive, affective and conative dimensions of learning awareness and regulatory functions of metacognition (Brown, 1987; Flavell, 1976; Larkin, 2006) are convolutedly interwoven.

6.4.3 Identifying cognitive errors

The overall objective for directing students' awareness to cognitive errors was to make thinking transparent to the students, and elicit motivation for reorganisation or expansion of knowledge (Zhao, 2011). Listing and analysing errors was found to guide students in co-identifying and isolating key decimal concepts involved in each test item. This process revealed common errors and initiated students' awareness of their own conceptions of decimal knowledge. It drew students' attention to the strategy or heuristics they used in responding to the test item. From this awareness, errors became an instrument for learning by providing a stimulus for contextualised error analysis during peer teaching that had personal relevance.

Error analysis was used as "springboards for learning" (Borasi, 1994, p. 166) during peer teaching. Findings and student opinions indicated several benefits of using error analysis. These included:

- Motivating students to explore and seek answers to questions.
- Giving students the trust of responsibility and ownership, hence changing the motivation for learning.
- Giving a reason for students to verbalise their mathematical ideas.
- Showing students that mathematics is more than right or wrong answers and procedures.
- Teaching students, through experience, the importance of careful reasoning and disciplined understanding.
- Providing evidence that mathematics is alive and exciting.

Analysis of own and others' errors directed students to think within alternative systems of thought, therefore assessing their assumptions. The potential of using errors to help students learn mathematics and identify misconceptions has been documented in research literature (e.g., Borasi, 1994; Isotani et al., 2011; Resnick et al., 1989; Steinle & Stacey, 2004). Isotani et al. (2011) also offer reasons that may deter teachers from encouraging students to see errors in a positive light. For example, the behaviourist perspective (Skinner, 1968) takes the view that without a penalty for making errors, errors are likely to get further reinforced and repeated. Hence, the use of errors is still controversial.

Students' initial view of errors

Analysis of students' initial reaction to the task of reviewing errors indicated: (a) a fear of errors; (b) not knowing how to check for errors; and (c) error analysis as a teacher responsibility.

First, analysis of students' reactions during the pre-test (Chapter 4) indicated that errors are an emotional event. Students felt negative emotions about their own errors and felt that this affected their performance ability and motivation in the test. Similarly, when first introduced to using errors for learning (Chapter 5) students showed signs of anxiety and employed a range of avoidance strategies when asked to review errors. *I don't ever want to look at that test; I know I did really badly.* Research literature explains that emotion drives us to perceive, think, or act in certain ways (Izard, 2011; Zhao, 2011).

Secondly, most students had little interest and did not know how to check their work for errors. Despite commonly known models such as Polya's (1945) four-step problem-solving strategy, where the fourth step highlights the importance of checking and looking back to reflect on what worked and what didn't, most students avoided checking. *You just want to get rid of it fast.*

Students said that most teachers asked them to check their work. However students' comments suggested that neither teachers nor textbooks actually paid much attention on how this could be done:

> *Teachers always say check your work because it is just something they always say. Probably to make us try our hardest. Actually when they say that, you don't even think they are talking to you directly, because you know you tried your hardest. Everyone usually looks around the room to see if it's safe to hand in their test and once it is out of our hand then you think yay, it's over.*

Students also seemed to have little understanding of strategies such as using estimation for checking. Students claimed that estimation was *confusing and harder than actually doing the questions.* Some students' comments suggested that estimation was often taught and learnt in isolation as a topic on its own, but its use in checking for reasonableness of answers was often ignored. Students said that unless stated by the question to estimate, it was avoided because it increased the margin for error in comparison *to doing the whole thing again properly.*

Thirdly, from the students' view, error analysis was a responsibility and task undertaken only by expert teachers or even computer programs to determine the sources of errors. However, in my experience, the sheer workload limits teachers' ability to spend time on in-depth error analysis. Hence, as shown in most research literature (Gersten, Jordan, & Flojo, 2005), error analysis is generally undertaken by teachers from a deficit model to account for students' difficulties rather than to help students co-analyse and learn from errors.

The implications of these findings were that students had to unlearn how to be a student. To do this, when planning for learning from errors, considerations of cognitive and affect regulative factors were made. These included:

- Building students' knowledge about the accuracy and effectiveness of their own cognitive ability and the cognitive strategies they used (Flavell, 1979).
- Regulation (Zimmerman, 2000) through emotion and cognitive monitoring (e.g., error detection, source monitoring in memory retrieval).
- Cognitive control (e.g., conflict resolution, error correction).

Changes in view about using errors for learning

The discomfort or embarrassment felt initially about publicly listing errors was observed to reduce after the first experience. It was observed that by listing errors on the board as a class, students became more desensitised to the fear of errors and errors became less personalised (Zhao, 2011). Knowing that others were also having difficulties was noted to have a large influence on students' affective states.

Concept maps were drawn from the error listing to identify key concepts associated with the test item being reviewed. Over time, the collection of concept maps drawn from all the test items provided a useful visual map for identifying the mathematical concepts that were involved in each test item. Three main benefits were observed from concept mapping:

- Students were able to view the identified concepts separately, as well as in connection to related test items to provide a more complete understanding of decimal concepts.
- Students were also able to identify that the same concepts were replicated in different test items even though the question was framed differently. Therefore they were starting to focus more on mathematical concepts instead of relying on familiarity of wording of questions.
- Students were able to make comparative analysis of their own thinking in relation to others which provoked metacognition and further exploration.

The use of errors for learning was found to be a powerful stimulant for metacognition. Some recent researchers (e.g., Isotani et al., 2011) found that using erroneous examples was helpful in relation to building students' knowledge in decimal numbers, but was not as effective as they had expected. In contrast, using examples of errors for learning in this research was found effective in terms of motivation and understanding. Students claimed that they learnt from analysing errors and that it redirected their thinking from inabilities to analytic thinking about possibilities of acquiring ability.

6.4.4 Strategic learning

Knowledge building

Whole-class knowledge building was found effective in preparing students for peer teaching and pinpointing overall difficulties. It was used to construct and deconstruct conceptions and refine skills related to decimals. Through this co-deconstruction, teaching and learning objectives were made explicit and meaningful to students. Students' mathematical knowledge and learning dispositions became evident during knowledge building. This was found useful for determining the level of scaffolds students needed and the level of monitoring needed during peer teaching.

Analysis of transcripts of the teaching experiment highlighted that the working backwards strategy, as used during knowledge building, was useful in building students' sense of self-efficacy (Bandura, 1977). Research literature shows that students' belief in their capacity to achieve specific tasks has a strong influence on their levels of persistence and their choices with regard to which activities to pursue (Bandura, 1977; Hahn, 2000; Hetherington & Parke, 1999).

At the start of knowledge building, students were asked to independently re-attempt the test item that was intended to be explored in the session. Given that students were observed to be anxious with low motivation to engage in the review of concepts, Polya's (1945) strategy of using working backwards was found useful in generating curiosity and establishing relevance. Although the use of this backwards strategy ran some risks of students using inaccurate steps to get to the right answer, it was found effective in getting students started and motivated.

Students' body language and comments indicated interest and engagement in the task; such as *ah, I remember…don't tell me the answer for the next one. Can you* *give us another one?* These comments also suggested that when students found success it gave them a renewed sense of confidence in their own ability.

In students' analysis of the knowledge building strategy, *it didn't really feel like we were just learning about maths from the teacher but that is when we also learnt what we should know and how we going to organise everything to learn ourselves and teach others.*

Peer teaching

Drawing on Vygotsky's (1978) concept of the zone of proximal development and guidance with more capable peers, the flexible use of a combination of peer teaching and reciprocal peer learning to match the situation was found to stimulate a more balanced learning experience.

Although there were some obvious benefits for tutees, peer tutors were found to take a more proactive role in planning, questioning, and sharing knowledge (Larkin, 2006). To avoid dependency and give the tutee a sense of accomplishment and confidence, reciprocal learning where roles were reversed with the same peer (when suited) or becoming a tutor for another student was found motivationally effective.

Analysing of errors and problem posing were the two main foci of peer teaching. As found in other studies (Corder, 1981; Durkin & Rittle-Johnson, 2012; Isotani et al., 2011; Zhao, 2011) students' errors were not random. They contained an element of logic, even though they were inaccurate. Analysis of students' errors suggested that not all students took the same route to the same error. Observations also showed that a student at different times may produce the same error for different reasons. Importantly, as shown in section 6.4.3, despite the potential of learning from errors (Borasi, 1994), errors were feared and avoided.

Peer teaching was found to provide the means for students to draw attention to the nature of errors and motivate co-exploration. One of the main drivers of students' fear of errors was concern about what others would think of their failure (Izard, 2011; Johnson, 2012; Runesson, 2005). Therefore working with others seemed to have redirected students' focus from personal error to the error itself.

In this sense peer teaching was found effective in exploring the basis of students' conceptions or misconceptions of decimal-specific knowledge and skills.

At the same time it was also effective in the management of social-emotional competencies that affect students' self-efficacy (Bandura, 1977) and persistence in confronting errors.

Problem posing

Problem posing (Cai & Hwang, 2003; English, 1998) was found to provide a means for self-regulatory critical monitoring of metacognitive judgements. The success and difficulties experienced by students during problem posing indicated that students were becoming more aware of the explicit and implicit aspects of decimal concepts that were needed to construct and deconstruct questions.

Problem posing was found to be a valuable assessment and learning tool (English, 2003; Toluk-Ucar, 2009). When problem posing, most students were observed to have a preference for posing questions by using the structure of test item questions as examples. It was found that only when students were comforted with consistent success, they became more confident to experiment with applying their knowledge to other contexts.

Proficiency levels became blurred during the generation of new problems and reformulation of a solution to a given problem. This was because formulating questions required students to be able to deconstruct, construct and defend their answers or questions when queried (English, 1998).

Observation of student interactions during problem posing revealed that high performance in a test was not always an indication of what the students knew. Rather, it showed how well they had rehearsed and remembered what they were taught. Students who had succeeded in the test due to familiarity of questions were unsuccessful when this structure was taken away. The questions they posed replicated the ones they already knew. Similarly they used resources that were safely familiar to peer teach. Students' proficiency therefore became better defined by their ability to apply understanding to novel contexts and evident in the questions they posed and resources they created.

Post-test results supported the blurring of proficiency findings highlighted during problem posing. For example, students U4 (low in proficiency) and S1 (semi-proficient) were initially categorised according to the pre-test results and interviews.

However, post-test results indicated that both these students performed at the same level or better than their peers who were categorised as proficient students.

During problem posing S1 and U4's proficiency level was noticed to be equal or above those who had performed better than them on the pre-test. The nature of problems posed by these students and the questions asked about solutions given suggested that these students were seeking understanding.

A possible reason why S1 and U4's ability was not easily recognisable during the interviews was that these students were distressed about their result and had "shut down":

> *If you don't get something, it's really frustrating when you trying to work it out and then you seem to be the only one having trouble because it doesn't make sense. You kind of know at the back of your mind that they are just copying things and they probably don't get it but at the time you feel really stupid.*

> *You never ever get time like this to work out things. You don't get this chance to talk about it. If you don't get it as fast as everyone then you are just marked as fail. You have to accept it and move to next topic.*

6.4.5 Reflective strategy

The community of inquiry was found to be one of the most powerful strategies in provoking students into metacognitive reflective thinking, not just in mathematics, but all their practices in learning (Dumitru, 2012). Students claimed they felt important and trusted. Therefore, it was a space where the argumentation of often perceived expert (Burgh & Yorshansky, 2011) opinion (teacher, research literature, educational authorities) by non-experts (students) was openly acknowledged and considered valuable in informing practice and knowledge.

Findings in this research fit well with Kreber and Castleden's (2009) findings. They emphasise the role of reflective thinking in promoting deeper ways of meaning-making of one's experience that is instrumental, communicative and emancipatory in nature (Kreber & Cranton, 2000; Mezirow 1998). Instrumental learning is essentially based on students validating their knowledge claims by posing them as a hypothesis which we then test during the inquiry. In communicative learning, students validate their knowledge as they engage in dialogue within a community to achieve a shared interpretation on our assumptions. Emancipatory learning involves reflection on premises and the questioning of core beliefs that define how students interpret their practice (Kreber & Cranton, 2000; Mezirow, 1998).

Idealistic thoughts of expectations could be understood at a pragmatic level. For example, during a philosophical inquiry most students claimed they knew what they should do to improve their learning practices. Through argumentation they were able to identify that *knowing and feeling were two different things.* For example, students knew they could learn from their errors but they found it difficult to regulate their emotions accordingly:

> *I know that my fear makes no sense but knowing that there is nothing to fear and feeling is like two different things. I still feel worry even when I know that there is no need to.*

Such illuminations suggest that students have the capacity to engage in philosophical metacognitive thought that allows them to understand themselves and their actions. Students claimed that through this understanding, they were able to regulate emotions. This regulation was identified by students as important in *helping us from shutting down* when faced with failure or confronted with dissonance. Furthermore, it assisted in developing a milieu (Brousseau, 2008) that was conducive for learning:

> *I don't have to worry about what others will say, because I have already told the whole class what I think. Talking it through with everyone makes us understand and trust each other.*

This form of inquiry was found to help students think beyond the obvious correctness of answers related to mathematical questions and ideas about learning or teaching and researching. Knowledge construction through argumentation (Reznitskaya, 2007) was understood as a new form of classroom discourse (Kennedy, 2004), not as a debate or a competitive test of ability; rather, as communal scaffolding through free deliberation of opinions and concepts and used as arguments or counter arguments to come to a collectively agreed conclusion.

Students' analysis of the community of inquiry suggested that argumentation during the inquiry ignited students' curiosity by provoking them to question underlying assumptions such as practices, and interpretations of pedagogical jargon and intent (Cam, 2008; Dumitru, 2012; Knight & Collins, 2010).

For example, on entering into an inquiry about the concept of metacognition, students were able to identify the advantages of metacognition in relation to a deeper understanding of decimals. Students were in agreement that metacognitive processes such as reasoning and explicit justification of reasoning were crucial to mathematical learning culture if true learning was to occur and decimal misconceptions were to be confronted.

However, in counter arguments students also highlighted the feelings of discomfort/comfort before, during and after the learning process. In this sense metacognition was seen as being deeply interwoven in the conscious consideration of intellectual experiences that accompanies success or failures in the process of learning decimals. Students claimed that in the case of failure, the discomfort often overrode the desire to question or think about their thinking:

> *I know it makes sense to think about your thinking to fix any bad ideas, but it isn't so simple. When the work looks too hard, you can't stop feeling bad and stressed. So, the normal thing to do is you try and shut down and not think about it.*

Another inhibiting factor for metacognition, according to the students, was that they were not always aware of what they were thinking. When they were aware, they either had no time or did not know what to do about it. The notion of metacognition as thinking about your thinking in the situation of difficulty was contested:

> *I am usually just trying to remember things when I am doing a task. I kind of think about it, but not really because you don't know if you are thinking the right thing or not. That is why you having problems to start with, it's because you don't know. To search and get help so that you can think about it is not always practical. You don't have the time to think, if you don't know, you just have to accept it. The rest of the class can't sit and wait till you figure it out. Or you fall behind with the other work or the bell rings and you have to do something else.*

Students' comments suggest that providing them with opportunities to engage in metacognition does not always guarantee that students will engage in metacognition. Rather, metacognition is often resisted and requires focused provocation and convincing.

These examples of rich dialogue during the community of inquiry highlight the value of providing a space for students to explore their own thought with others. Argumentation used in community of inquiry was found to provoke metacognition, revealing individual capabilities sometimes surprisingly sophisticated and sometimes unexpectedly poor. However a well-focused metacognitive analysis through the collaborative approach of a community of inquiry was found to clarify the relationship between meanings and sharpen awareness.

6.4.6 Summary

The findings analysed in this section highlight the significance of metacognition in decimal knowledge. They also point to the complexity of internal and external processes that influence, facilitate and obstruct metacognition. Figure 6.4 illustrates some of the main concepts that emerged from the analysis of the findings in this section.

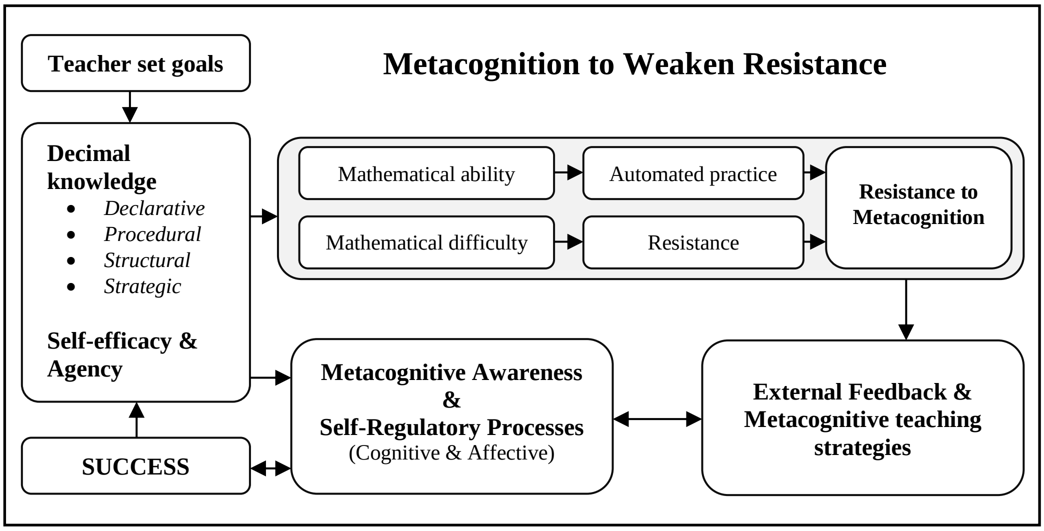


Figure 6.4 Outline of influences on metacognition.

Figure 6.4 illustrates that teacher goals for developing students' decimal domain and strategy knowledge can lead to two paths. The first is where the students are able to engage in the task metacognitively, select and execute the appropriate strategies and obtain feedback for assessment to achieve the desired outcome of success in learning.

The second path shows that metacognition may be obstructed or resisted when students who are having no difficulties with generating correct answers have become automated in their thinking and stop seeing the need for metacognition to understand the process they are using.

Metacognition may also be obstructed when students are having mathematical difficulties. Findings in this research indicate that different types of strong emotions are prompted by the experience of difficulty which can deter the desire to reflect on students' own thinking. This resistance is argued to be weakened through metacognitive teacher reaction and collaborative feedback to allow for success in learning.

6.5 SUCCESS BEFORE UNDERSTANDING

This section discusses students' arguments that the feeling of success was needed before understanding was desired. To this end the students' approach to a task is described and risks of success before understanding are analysed.

6.5.1 Success as a common theme for motivation

At first glance, the approach of success through rote teaching of steps before understanding taken by peer tutors may be seen as bad practice in light of research recommendations (ACARA, 2012; NCTM, 2012). Research literature emphasises building on students' conceptual knowledge first, then leading students into procedural understanding to avoid procedural knowledge without understanding (Hiebert & Wearne, 1986; Rittle-Johnson, Siegler, & Alibali, 2001; Steinle & Stacey, 2004). Teaching strategies such as using a variety of representations including concrete models and real-world problems to help students visualise or relate, rather than rote learning of procedural steps, are recommended (Irwin, 2001; Lee et al., 2011; Moody, 2011).

The students' argument for their approach was based on the idea that conceptual knowledge has to be used with some practical application before concepts can be formed about it. *Otherwise you won't bother listening if you have nothing to relate it to.* They claimed that real-world applications are often complex and messy whereas procedures can be learnt step by step providing a context for potentially successful positive experience.

The significance of success in motivating and prompting learning was noticed as a common theme in all the previous sections in this chapter. For example, in section 6.2, resistance was attached to the fear of not succeeding. In section 6.3, students opposed the use of materials claiming that teacher-selected materials did not always guarantee success due to the extraneous cognitive load placed by materials. In section 6.4.4, the success experienced by students from the use of Polya's (1945) backwards strategy was found effective in fostering interest and motivation. Students were found to lead their peers into understanding procedures by first building the confidence of peers who were struggling with learning through experiences of success.

The failure cycle

Co-analysis of the failure cycle supported students' advocacy for considering success before learning as a motivator to break the failure cycle. The failure cycle involved a combination of various elements as described below.

Acquiring decimal knowledge was observed to involve a wide range of cognitive skills and thorough understanding of several mathematical components that constitute decimal concepts (Resnick, et al., 1989). It required knowledge of mathematical procedures (syntactic) as well as the purpose and meaning of the procedures (semantic). This implied the potential of a wide spectrum of difficulties and misconceptions.

Difficulties and misconceptions observed (Chapter 5) included: cognitive conflict (Liu, 2005; Moody, 2011) that resulted from the incompatibility between students' prior knowledge of whole numbers and the new decimal knowledge; weak structural understanding of the decimal number; difficulties with implementation of arithmetic procedures; difficulties in keeping track of the sequence of steps in calculations; semantic memory difficulties such as number fact retrieval and several other subgroups of skills and concepts that provide relational links to decimals.

Therefore in this context of decimal knowledge acquisition, metacognitive skilfulness (Keith & Frese, 2005) and regulation was observed to be the most important predictor of learning success. As research literature (Brown, 1987; Flavell, 1976) suggests, awareness of own thinking, and awareness of content of conceptions as well as active monitoring of one's cognitive processes helps regulate cognitive processes to further learning.

However, as mentioned earlier (section 6.4.2), motivating students to engage in metacognitive thought was not straightforward. Students, particularly those who had long histories of repeated school failure in mathematics, were observed to have a common sense of helplessness and a distorted perception of reasons for the failure. As explained by attribution theory (Hetherington & Parke, 1999; Wadsworth, 2004), these students found little relevance in metacognition and held strong expectations of continued failure. The students stated they believed that learning mathematics was related to ability rather than effort.

This behaviour was not only seen in students who were accustomed to failure. Other students who had histories of generally performing well in mathematics also showed similar negative attitudes when faced with low achievement in the diagnostic test. They too seemed to be set on a default of wanting to move away from the evidence of failure as fast as possible.

6.5.2 Student approach to teaching

In exploring the patterns and practices preferred by students during peer teaching, students seemed to deviate from the recommended flow of developing conceptual understanding before, or in tandem with, procedural understanding.

Peer tutors seemed to have intuitively recognised there was a need to combine learning strategies with a teaching approach that directly addressed their peers' attributions about successes and failures in mathematics. To do this, many students empathetically reacted to their peers' difficulties by teaching procedures step by step together with rote learning before moving into understanding of procedures. *If I teach the steps then they can get it straightway. Later when they feel like they can do it, we can teach them how the steps work.*

Their argument was that it was easier for their peers to experience success using a series of prescribed manageable steps. This success was needed to motivate students to want to understand the concepts behind the procedures:

> *You need to have something to show them that they know to explain how it works. If you teach them all this other stuff and then try to put them together to show them how they all fit in together it doesn't work. They find the steps hard enough so they not going to think of, hmm, let's see how it all relates.*

As Starko (2001) concurs, the greater a student's intrinsic motivation, the greater the likelihood of creative mathematical applications and discoveries.

Student peer teachers organised and generated procedural steps for their peers to learn, practise and memorise. The chunking of steps and the amount of information given to explain reasoning behind steps varied depending on the tutor's judgement of their peer's ability to manage the amount of information.

An example of these steps written during practice of writing numbers in words was: *(a) draw the place value chart; (b) place the digits in the correct positions; (c) group numbers to write them in words; (d) use and for the decimal point; and (e) look at the last digit and position to decide the name.*

Students in the low-proficiency levels were observed to enjoy the success of getting right answers from the use of this strategy. Most of these students were interested in maintaining this success. *Tell me slowly so I can write it down step by step.* They were reluctant to alter the steps or experiment with new strategies. For example, they drew place value charts and systematically placed each number in its respective place carefully with consistent checking. At times this careful checking led to situations where the low-proficiency students had more correct answers than students at higher levels of proficiency because they made less mistakes.

When leading on to building understanding of procedures, peers realised that some students (particularly those in the low-proficiency group) found concept building very difficult. Explaining and practising with concrete materials was inadequate. Students had difficulty in conceptualising place values. Peer tutors found that using representative objects for illustrative purposes and then getting peers to draw the connections between place values, rather than asking their peers to replicate the process using materials, worked well for some students:

> *It seems to make it less distracting and they find it easier to see it all as one thing drawn on the paper instead of having to look at two different things. It looks easy because you have real things like a place value chart and blocks and stuff to explain things but is actually harder.*

Success before understanding relied on moderating the difficulty of tasks to ensure that the task remained appropriately challenging for the individual student with experiences of positive outcomes. Therefore students were motivated to develop autonomy in learning by being given choices of the sequence of learning with scaffolds to guide learning.

6.5.3 Risks of understanding before learning

During reflective argumentation about using success before understanding, risks were collaboratively recognised and identified to include:

- Adaptation rather than assimilation of decimal knowledge.
- Excessive focus on success leading to over simplification of knowledge and avoidance of progressing to more cognitively challenging tasks needed for deep understanding.
- Peer tutee controlling peer tutor to adapt perfectly, but without the peer tutee learning.
- False sense of accomplishment by copying complicated steps resulting in correct answers, but the peer tutee really does not understand the reasoning behind the steps.

Students claimed that by simply talking about the listed risks (through identification and awareness), many of these risks were effectively reduced. However, students were not keen to use the list of risks as reflective checklists after each peer teaching session. They claimed that the effectiveness of the checklists was limited when used as routine:

> *After a while you start just answering the questions in checklists because you have to, and then it doesn't mean anything anymore.*

Students found it more effective to use these ideas of risks in supporting or refuting arguments during a community of inquiry where it was used to remind them or talked about in context of a heartfelt opinion.

Students came to the conclusion that to avoid the potential pitfalls in risks identified with using success before understanding, the concept of accomplishments had to be redefined. It was decided that accomplishment would be determined during collaborative learning in terms of accomplishment being relative to students' own perceptions of how difficult they found the work. Therefore elation from the feeling of success came from mathematical discovery.

6.5.4 Summary

Findings analysed in this section indicate that the students' argument for success before understanding was supported by their success in breaking the failure cycle. Their argument was based on the assumption that it is not the order of teaching that matters, but what works for the individual student.

Figure 6.5 illustrates the failure cycle where mathematical difficulties were identified to lead to undesired paths. One path shows where students avoided metacognition due to the emotions of fear and shame that deterred them from wanting to think about their thinking. The other path shows where students accepted their difficulties and attributed them to uncontrollable external and internal factors, therefore seeing no need for metacognition. This creates the ongoing cycle of failure.

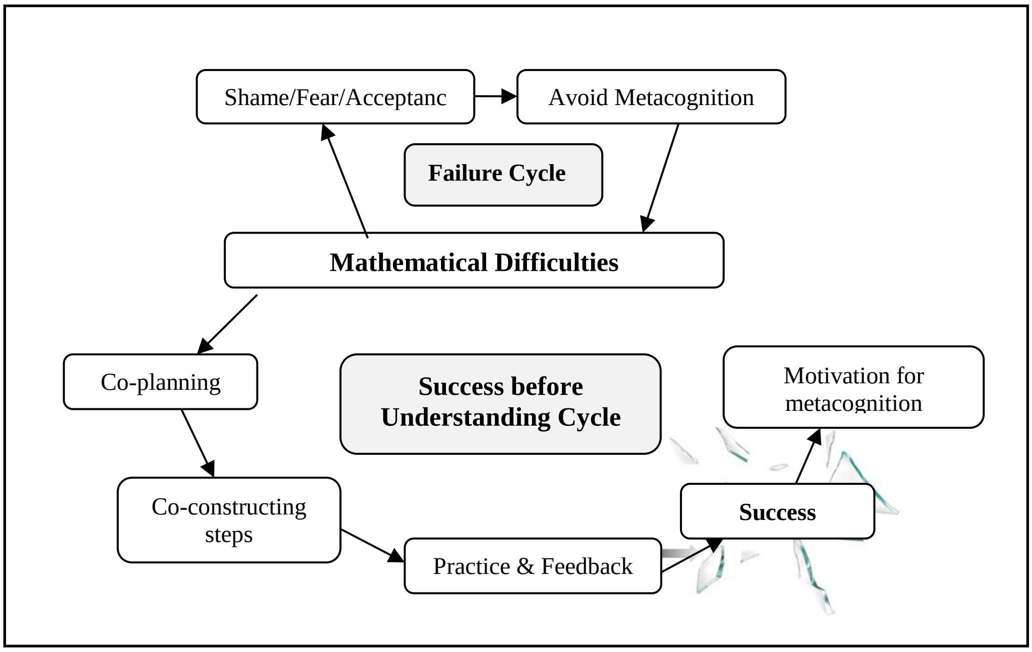


Figure 6.5 Students' success before understanding approach to break the cycle of failure.

Students' approach to breaking the failure cycle during peer teaching was observed to be focused on success. This success was found by chunking procedures into small steps in consultation with their peers. The steps were memorised and practised (not necessarily with understanding). The experience of success was found to increase students' sense of self-efficacy, hence their motivation to learn:

Let me do this one by myself, give me a harder question.

Understanding was sought after success. Students argued that without the feeling of success they were more likely to fall into a failure cycle.

Chapter 7: Conclusion

7.1 OVERVIEW

In this concluding chapter, a brief overview of overall findings in relation to the objectives is presented. The emergence, use and forces acting on SPARC are discussed (section 7.2). Limitations of the research are considered (section 7.3) and finally implications from this research are proposed (section 7.4).

Overall findings in relation to research objectives

The overall purpose of this study was to co-develop, with students, teaching and learning strategies to address their difficulties with decimal knowledge acquisition. Particular objectives were to: (a) co-improve students' knowledge and understanding of decimal numbers; and (b) co-improve own practices for decimal knowledge acquisition.

This research highlighted that despite initial *dislike* of mathematics related to decimals, and fear of the *hardness* of decimal concepts, students' conceptions are not fixed; rather, their conceptions of learning mathematics appear to develop with new experiences as students encounter new learning environments, and different ways of teaching and learning.

Substantial improvements in students' decimal knowledge were evidenced in the comparative analysis of students' decimal knowledge in pre- and post-test results (sections 4.8.3 and 5.8). Student actions, reactions and reflections (Chapters 4 and 5) show the progressive changes that occurred in students' beliefs, attitudes and desire to learn during this research. These changes were observed to influence and effectively improve students' practices for decimal knowledge acquisition.

7.2 STUDENTS PARTICIPATING AS RESEARCH CO-CONSTRUCTORS (SPARC)

SPARC was the cohesive element underlying the processes used for this research. Essentially SPARC is about listening to students (Bland & Atweh, 2007) and co-creating new pedagogical knowledge for improving students' decimal knowledge and improving practice to bring about change, not only as a consequence of it. As argued throughout the analysis (Chapter 6) and evidenced in students' success in the post-test, students did not resist change but resisted being changed if they were not complicit in the processes of bringing about change.

7.2.1 SPARC as an emergent approach

SPARC emerged as a data gathering approach from a need to reveal incongruence in interpretation of students' cognitive and affective states during research, and to reduce blind spots created from over-representations of one perspective.

Traditionally, authority has been associated with expertise of knowledge and recognition of status (Atweh & Brady, 2009; Gee, 2005; Trent, 2003). While teachers and researchers have authority by virtue of their academic training, this research highlights several important disparities that exist between students' and teachers' perceptions. An example of the incongruences between teacher and students was the use and value of materials (section 6.3.2). This example elucidated how perceptions of what is valuable, interesting and engaging differ between teacher and individual students. Further it illustrates how such differences can distance students from the intended outcomes for learning.

The collaborative approach (SPARC) was found particularly effective for mediating differences and aligning objectives. Students' conceptions and ideas could be accessed which then helped with co-designing activities and modes of delivery that, in turn, shaped and developed their conceptions in ways that were beneficial to their academic and personal progress. Students claimed that changes in conceptions evoked through the SPARC approach rather than learning per se, had a strong influence on their approach to learning and, ultimately, their learning outcomes.

7.2.2 Starting at where the students are at

The importance of knowing and starting from where the students were at cognitively and affectively at the start of a learning experience was made clearly evident when working with students. Empirical evidence during this research corroborates with findings in literature that students arrive at school with preset ideas and beliefs about learning mathematics which have been influenced and shaped by their prior learning experiences (Alexander, 2009; Bloom, 1981; Estes, 2004; Irwin, 2001; Moss & Case, 2002).

Students' ideas and beliefs were shown to be critical in determining how they perceive learning, with consequent impacts on the qualitatively different ways in which students experienced learning (Bandura, 1977; Boström & Lassen, 2006; Gee, 2005). This includes students' conceptions of learning, their approaches to learning (e.g., deep, surface, and strategic), and how academic achievement positions them in the classroom milieu *(I am dumb, he is the smart one)*.

In total, findings indicated that the differences in conceptions about decimal and the purpose of practices used for learning and teaching are diverse among students. This is further compounded when students' expectations of their learning overall do not align with those of the teacher. Hence, even meticulous planning for exposure of mathematical concepts may fall short if students do not have a shared understanding of the purpose and value of learning outcomes (Claxton, 1991; Gee, 2005). Understanding this conception gap was found crucial in drawing assumptions about effective teaching and learning strategies, and the types of knowledge valued by students.

The strength of SPARC emerged with development of a culture of willingness to make oneself vulnerable to another person despite uncertainty regarding motives, intentions, and prospective actions. Students claimed that the culture underlying SPARC hinged on an appraisal of expectations of others and emotional rapport with others. This confidence in others was argued to have developed along with actions that expressed care and concern for the relationship, rather than just actions that focus on task outcomes:

> *It is not easy to open up to others about what you don't know. It is embarrassing and humiliating normally, but if you really think about it you can say that SPARC developed SPARC to change us.*

7.2.3 Forces acting on SPARC

As shown in Figure 7.1, the triangle of forces acting on SPARC were analysed from the three linked psychological domains of functioning: the affective, the cognitive, and the conative (Hilgard, 1980). Analysis of students' responses revealed a strong correlation between their cognitive difficulties in the decimal domain and their affective state. Deeper analysis confirmed the assumption that affect, cognition, and conation could not be studied in exclusion of each other; each of these mental states affected the other when attempting to understand students' decimal knowledge acquisition.

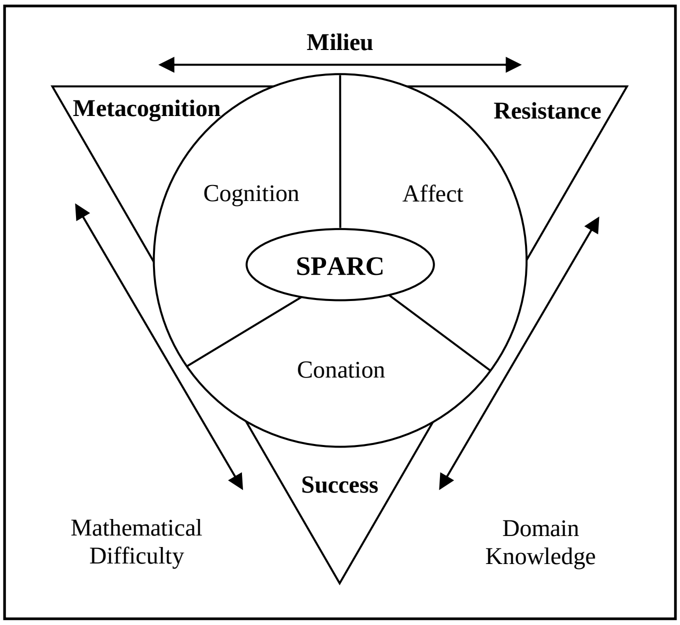


Figure 7.1 Influences acting on SPARC.

Cognitive difficulties in the decimal domain

In terms of students' cognitive difficulties in the decimal domain, misconceptions in decimal knowledge were characterised by heterogeneity and gaps in mathematical knowledge among students. One of the greatest challenges faced by students in learning decimal concepts was that their prior knowledge did not always support decimal ideas. Further, many students were found to not simply lack structural understanding of decimal numbers (Baturo, 1998; Resnick et al., 1989), but also to have a superficial understanding of whole numbers (section 6.2). Conceptions such as zero were found confusing by students because even when talking about numbers, zero is never mentioned in a number, hence zero was considered unimportant and could be left out. Without sufficient and appropriate prior knowledge, students were unable to make correct connections between knowledge (section 6.2).

Table 7.1 presents a summary and illustrates the wide range of difficulties students experienced with decimal concepts. In view of this range of students' difficulties with decimal number concepts, it is understandable that learning generates a range of cognitive and affective responses that are unique to individual students in the way in which students learn, as well as why and how they learn.

Table 7.1

Examples of Students' Difficulties in Knowledge at Entry Stage

Knowledge	Examples of mathematical difficulties (entry knowledge)
Retention	Difficulty with recall of fundamental mathematical concepts (such as number facts).
Prior knowledge	Gaps in knowledge and mismatch between new and existing knowledge (such as whole-number and decimal concepts).
Comprehension of the problem	Not knowing what the question was asking as a result of experiences with alternate written or spoken representations.
Language and symbol knowledge	Confusion resulting from inconsistent use of language. For example, numbers often spoken using individual digits (such as *two point one* rather than the conventional *two and one tenth*).
Relational knowledge	Unable to see relationship between knowledge (such as the various symbolic representations of the same fractional value).
Context knowledge	Difficulties with transferring knowledge gained from practised situations to less familiar situations or differently structured problems.
Structural knowledge	Unaware of multiplicative and additive structure of decimal numbers. Unaware of the differences between whole and decimal numbers.
Conceptual knowledge	Weak conceptual understanding (such as the place value of each digit and relationship between digits to represent the value of the entire number).
Role-of-zero knowledge	Not identifying the role of zero as a place value holder due to the belief that zero meant nothing.
Computation knowledge	Knowing the procedures of various computational strategies without knowing how they were related or could be used flexibly to effectively suit the situation.

Affective states

In terms of students' affective states, it is surprising that affects in relation to difficulties in decimals have rarely been mentioned in literature. The centrality of affect in decimal knowledge acquisition was made evident from the initial stages of implementation of this research. Emotions (such as excitement, happiness, moderate anxiety, thrill and hope) were observed when students became absorbed and intrigued by the mathematics of decimal concepts. Beliefs about their own capacity to learn mathematics (Tanner & Jones, 2003) and the discursive expectations of milieu influenced students' ability to manage feelings of inadequacy felt when faced with the difficulty of learning decimal concepts (Chapter 4).

Alsop and Watts (2003) state that affective states unchecked can inadvertently overwhelm thinking, deaden curiosity and shut down memory and concentration, such that learners' efforts can become swamped and rendered wholly ineffective.

Conative action

Knowing is different from being able to do it. I know and can tell you all the right things I should do to learn. But, when it comes to actually doing it, I can't stop the feeling scared that takes over and stops me doing it, even when I know that the fear does not make sense (irrational)!

In terms of conation, the intricate interplay between how students consciously think and reason (cognition) and experience specific subjective feelings (affects) was found to predict students' resistance or perseverance (conation) with learning.

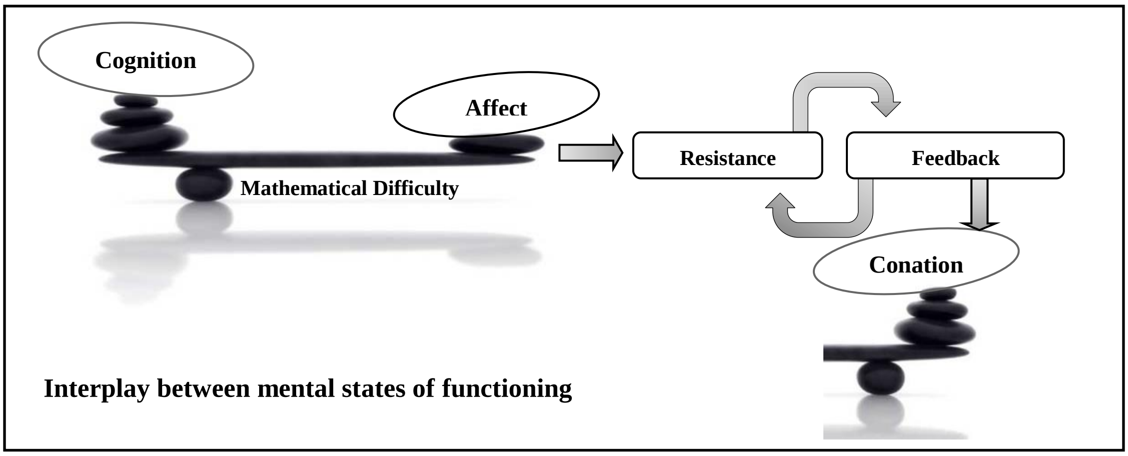


Figure 7.2 Interplay between the tri-mental states of functioning.

This interplay between conation, cognition and affect is further illustrated in Figure 7.2 which shows that cognition co-evolved with affect (Demetriou & Wilson, 2009) to deal with students' mathematical difficulty. When students experienced cognitive difficulty (e.g., during the pre-test in Chapter 4) they reacted emotionally. Through their own subjective appraisal of the situation and personal information, students determined what the difficulty meant to them. Students cognitively scanned and evaluated the difficulty experienced to determine whether their goals were being achieved or how they would best cope with their feelings of inadequacy or weakness.

To reduce the affective discomfort and cognitive dissonance (Limon, 2001) experienced, students adaptively resisted participation in the task (section 6.3), retreating to avoidance strategies and dismissing cognitive engagement in the topic as being *too difficult* and *too boring*.

The amplification affects of impending social humiliation served as a warning to avoid the learning/ teaching experience. Instead students' conative ability directed their goals to self-preservation. The result of this was resistance and ascribing of blame to inability and other factors. Within this frame of thinking, students' fear of the risk of further exposure and the potential for humiliation appeared to override any desire to seek improvement in knowledge.

To break this mental state that was non-conducive to learning required an understanding of how resistance, metacognition and success (Figure 7.1) could be coordinated by the milieu of SPARC to redirect students' conation to desired outcomes. Instead of viewing resistance as a negative factor, resistance was understood as a necessary and valued element in instigating metacognition. It was through resistance that redundant routinised practices were brought to the fore (section 6.4.2). Students' expectations about how a teacher would respond to their actions (*they will get angry if I don't agree or do it differently*), fear of others' critical view *(I am dumb),* and perceptions self *(that's not me, I can't do maths)* were examples of students' inner thoughts that are not always considered as part of mathematics learning but arguably influence mathematics learning outcomes.

The nature of feedback provided to students by others (teacher and peers) in relation to their resistance to learning was found to determine students' path of conation, to persevere with resolving the mathematical difficulty experienced or further resist engagement in the task. The notion of failure and success and its social consequences was found (section 6.3.1) to be of great significance to students. Observations during the implementation of the pre-test indicated that students' beliefs about the importance of improving decimal knowledge were counterbalanced by strong negative feelings of fear and discomfort. Many students were found to attribute their experience of difficulty to lack of cognitive ability or external sources to justify their reason for disengagement.

For some students, many of the attributions formed the underlying basis of students' definitions of social reality and were subsequently reaffirmed through classroom cultural consensus. Those who had accepted their inability to keep up with others claimed cognitive disability. *Do we have to do the test? We normally don't have to because we are in the dumb group.*

Hence feedback did not simply mean providing students with advice, rather it involved providing students with alternative options for actions to mediate students' resistance and direct the path of conation. An example for alternative options for actions was the management of students' resistance to using materials (such as MAB blocks) for developing conceptual decimal knowledge. In this situation, feedback was given in terms of acknowledging the differences in students' and teacher beliefs about the purpose and value of materials.

Using the SPARC approach, communications were opened, the intended learning objectives were made explicit and students were given the option to use alternative materials to illustrate their ideas which they found more suited to their needs. Interestingly, several positive outcomes ensued: *we learnt more from planning and teaching than just learning about it because we had to really think about what and why the materials would work or not*. A positive correlation between students' expectations of other's cooperation and their own levels of cooperation was observed. Some students who had initially protested strongly against the use of specific materials were found to be using the materials and using them as teaching tools.

The SPARC approach was found effective in guiding students' metacognition in re-assigning beliefs and distancing measures of importance to various fears about failure and errors (section 6.3) from a personal problem to a task problem. As such, success was reconceptualised as the development of mutual trust and respect for student potential rather than task performance. Students attributed this form of success as one of the main contributors to growth in self-confidence.

7.3 LIMITATIONS OF THE RESEARCH

The main limitations of this research study were the sample size and the focus on a group of students in one year level. The generalisability of the research was further limited by the focus on the individual. Learning plans and their implementation were neither linear nor rigidly structured. In fact the success of students was often measured by their ability to deconstruct and reconstruct plans so that they better fitted their individual need. Learning was focused on where the individual student was at and their progress towards improvement in knowledge and practice. Such approaches generated varied individual rather than group data.

The novelty of the study for students to undertake the role of co-constructors of knowledge was welcomed by students. In turn, this may have affected their motivation and subsequent performance as described by the Hawthorne effect (Macefield, 2007).

Another limitation was in the writing of events that occurred during the research. Many of the descriptions within this research have ineffable qualities that can only be fully understood or interpreted by experience.

7.4 IMPLICATIONS

Research that focuses on including students is important because it supports genuine interdependence in co-creating teaching, learning and researching ideas for decimal knowledge acquisition. It offers students a form of education that engages in critical examination and analysis of their own mathematical learning experiences to enable emancipation through personal and social transformation. Therefore co-research not only increases active participation, but also develops a metacognitive understanding of the psychological, social, and cultural dimensions of learning to bring about personal change.

7.4.1 Implications for teaching practice

This research provides unique insights for educational practice and advances the field in research on decimal knowledge. Insights from this research open new ways of thinking for educators about why students resist attempts to promote understanding.

Student comments reveal how students translate and interpret teacher actions/teaching approaches, and serve to remind us of the importance of reflecting upon teaching methods and how students may view such approaches. Students' arguments expose contradictions, showing how teacher practices and judgement can become clouded by ideological ideas.

As discussed in Chapter 2, theorists and researchers of education in general have long advocated specific pedagogies in relation to the teaching of decimals. This includes recommendations such as: student-centred learning instead of teacher-centred; use of materials to develop conceptual knowledge instead of mnemonic steps of procedures; and a developmental scope and sequence of experiences (ACARA, 2012). In principle, few of us would argue against these compelling ideologies of teaching; however, students' reflections of their experiences as described in Chapter 5 suggest that the merging of practical realities and theoretical complexities has the potential to collapse the binary process of theory-based intent and actual practice within the constraints of the classroom.

Practical considerations, such as students' experiences, class size, required grading criteria and, in my situation, expected consistency in planning and delivery by co-teachers for the same year levels across the three campuses of our school, challenge the very idea of flexibility in practices. Further, it cannot be assumed that students understand or share the same enthusiasm for these experiences. In a sense, the students' reflections in Chapter 5 suggest that teachers' teaching often mirrors students' learning in terms of following a set of procedures rather than a metacognitively interactive platform for teaching and learning.

This research embraces the messiness of human and institutional factors of practice and theoretical ideologies. Ideologies that underlie both pedagogical strategies and mathematical knowledge construction are assumed by this research to only serve as convenient constructs for categorising activities and responses. A key element for balancing constraints and theories was using a give-and-take approach with students. I asked my students questions about their purposes for learning and the learning and teaching strategies they used or disputed. Focusing on planning, accessing prior knowledge, organising ideas and selecting resources became a major component of learning. This process of co-planning for balancing responsibilities, expectations, and achievement goals with our students is a step often overlooked as we rush to teach a group of mathematical concepts within an allocated time.

Use of strategies such as error identification, error analysis, community of inquiry, peer co-planning and problem posing was found to shift students' attention from undesired actions to an interest in the mathematical problem. Examples given in Chapters 4 and 5 such as resistance to learning, students' emphasis on easy and instant success, avoidance of difficult work, and assigning of work beyond comfort level as boring, illustrate the complexities that teachers face in aligning theory with practice. Reactions such as *I belong to the dumb group and do not have to do this work* warn that even concepts such as differentiated teaching can become misunderstood as teaching to students' comfort levels where students see it as the norm and begin to expect it from learning.

The positive impact, however, became more evident with each consecutive session where students displayed a deeper understanding of decimal concepts and connections between concepts. Deeper understanding made the learning of decimal concepts time efficient with less time needed in consecutive sessions. Moreover, students' reflection revealed an awareness of cognitive processing rather than a concern for the routine of giving right answers to a set of decimal-related problems. Paying attention to metacognition encouraged students to become active in their own learning and invited self-discovery and development of skills with others. Paying attention to students' affects prompted responses to provide the right amount of affective conflict to instigate desired conation.

7.4.2 Implications for further research

This research reveals and challenges new ways of thinking about the pedagogical dilemma of students' difficulties with decimal knowledge acquisition. Implications for future research include exploration of: (a) differences in information on students from tests verses interviews; (b) what students find important when they are peer teaching other students; (c) students' attitudes to mathematical materials (e.g., MAB); (d) the effect of teaching about learning/thinking and teaching about research on student performance; (e) ways of finding cognitive and affective balance needed for coordinating desired conation; (f) the role of affect in inhibiting and promoting metacognition; (g) exploring the role of resistance in learning mathematics; and (h) effect of developing a metacognitive awareness of affects during learning and strategies for regulation of affects.

This research stresses the socially situated nature of learning, positioning learners and teachers as collaborating in the co-creation of new knowledge about how cognitive and affective aspects of teaching and learning can be coordinated to align and deepen theoretical understanding of the processes that underlie decimal knowledge acquisition.

7.4.3 Summary of personal reflections

In reflection of my participation in this research, the writing of this thesis and my experiences of working with students as co-researchers has given me the opportunity to undertake extraordinary personal growth. The task of writing a thesis has enabled me to think clearly and articulate the strategies that I believe can make a difference to the academic outcomes and life chances for students learning how to learn mathematics.

Co-participation in research has also made me aware of my own assumptions about my practices where over-familiarity with mathematical knowledge and teaching practices had created blind spots in my awareness. Teachers can become trapped in the comfort of their own vision of assuming what is good practice. As Estes (2004) also concluded, while teachers claim to value student-centred learning, these values, as evidenced in practice, are often teacher centred.

This research does not offer a set of prescriptive practices. Rather, it highlights that there is great potential in combining both teacher and student expertise (SPARC). Much can be learnt from co-construction and co-reflection about the congruency (or lack thereof) between students' and teachers' knowledge and practices for teaching and learning of decimal knowledge.

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Appendices

Appendix A
Diagnostic decimal knowledge test instrument

Adapted from Baturo (1998)

Number Identification

1. Write these numbers in words.
   a. 3.826…………………………….
   b. 607.012…………………………

2. Write these numbers in digits

a) Five and three hundred and six thousandths -------------

b) Eleven and seven thousandths -------------

Place Value

1. Write the number that has [position]

a) 5 tenths, 2 hundredths, 9 ones, 3 thousandths ………………………

b) 7 tens, 7 tenths ………………………

c) 2 thousandths, 8 tenths ………………………

2. Ring the number in which the 7 is worth the most. [Value]

94 376   70.523   1762   1.762

3. In 625.078 : a) the 2 is worth? ……………………………………………

b) the 8 is worth? …………………………………………….

c) the 50 is worth? ……………………………………………

4. Write the missing numbers [Multiplicative relationships]

a) 0.7 × 10 =…………. …………….   f) 58.7 ÷ 10 = ……………………..

b) 0.02 × 100= …………………….   g) 0.075 × 10 = ……………………..

c) 5.346 × 10 = …………………….   h) 7.03 ÷ 10 = ……………………..

d) 8 ÷ 10 = …………………………..   i) 481 × 100 = ……………………..

e) 0.2 × 100 = ……………………..   j) 62 ÷ 1000 = ………………….

5. Write yes or no to each of the following. [Role of zero]

a) Does 7.2 have the same value as 7.020? …………….

b) Does 7.2 have the same value as 07.2? …………….

c) Does 7.2 have the same value as 7.200? ……………..

d) Does 7.2 have the same value as 0.72? ………………..

Regrouping

1. Write what's missing in each of the following

2.614 ⇨

	ones		tenths		Thousandths

70.365 ⇨
………….. ⇨

703		65	
71	tenths	4	thousandths

2.007 ⇨

200		7	

2. Write the missing numbers

a) 5.017= ……………thousandths   b) 6.2 = ……………….thousandths

c) 74 tenths 53 thousandths = …………. d) 3 tenths 6 hundredths 17 thousandths = …………….

Counting

Write the number that is 1 thousandth more than:

a) 3.563…………….. b) 5.269……………. c) 0.09 ……………… d) 4.591 …………….. e) 6 ……………………

Complete the counting sequences.

8.527, 8.528, 8.529, ……………, ………………, …………………

2.197, 2.198, 2.199, ……………, ………………, ………………..

2.803, 2.802, 2.801 …………….., ………………, ……………….

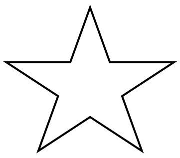
Approximating & Estimating

1. Round each number to the nearest whole number.

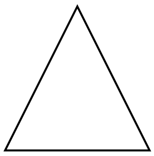
   a) 8.623……………… b) 0.347………………… c) 1.096……………….

2. Do the following

a) Colour 0.935 of this shape

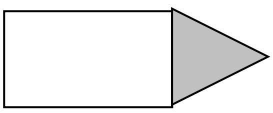


b) Colour 0.457 of this shape



This is 0.521 of a shape. Draw the whole shape

3. Circle the number below that shows about how much of the shape has been shaded.



0.250 0.535 0.924

Appendix B
Metacognitive teaching experiment strategies

Table B.1

Metacognitive Teaching Strategies Used in the Teaching Experiment

	Teaching experiment strategy	Learning experience objective
(1)	Error listing	To become metacognitively aware of types of errors made by the students. Additional errors from literature were also listed to expand scope of analytic exploration.
(2)	Knowledge building	To strengthen students' knowledge base so that they are able to critique prior knowledge and refine their mathematical understanding.
(3)	Peer teaching *Co-planning* *Resource selecting* *Teaching*	To promote self-regulated learning and engage students in a collaborative inquiry with peers in an environment that allows them to be able to respond to the affect and cognition demands of knowledge acquisition. It was anticipated that through the process of co-planning, co-designing and co-teaching, students will: • Develop an awareness of specific cognitive demands in varied decimal-related tasks. • Gain an insight into their own and other thinking such as differences in approach, weaknesses and strengths in their explanations. • Understand reasoning in procedural knowledge of when and where to use acquired strategies.
(4)	Error analysis	
(5)	Problem posing	To reinforce, test and refine students' understanding through writing questions and answering new questions.
(6)	Community of inquiry	To engage in reflective inquiry about the effectiveness of teaching, learning and SPARC action research processes to make informed suggestions for enhancing knowledge acquisition.

Appendix C
Community of inquiry

Table C.1

Co-Constructed Guidelines for Participation in a Community of Inquiry

1. Respect (for all participants)
2. Ways to participate
   - Posing critical questions
   - Expressing an opinion and giving reasons (agree/disagree because…)
   - Listening attentively
   - Initiating dialogue/ideas
   - Speaking freely and feeling safe to express thoughts
   - Clarifying and elaborating on ideas
   - Suggesting alternatives for actions
   - Signalling to contribute to discussion only after the speaker has finished speaking
3. Opportunities to participate
   - Contributing to an equitable participation
   - Showing willingness to participate
   - Making invitations to participate

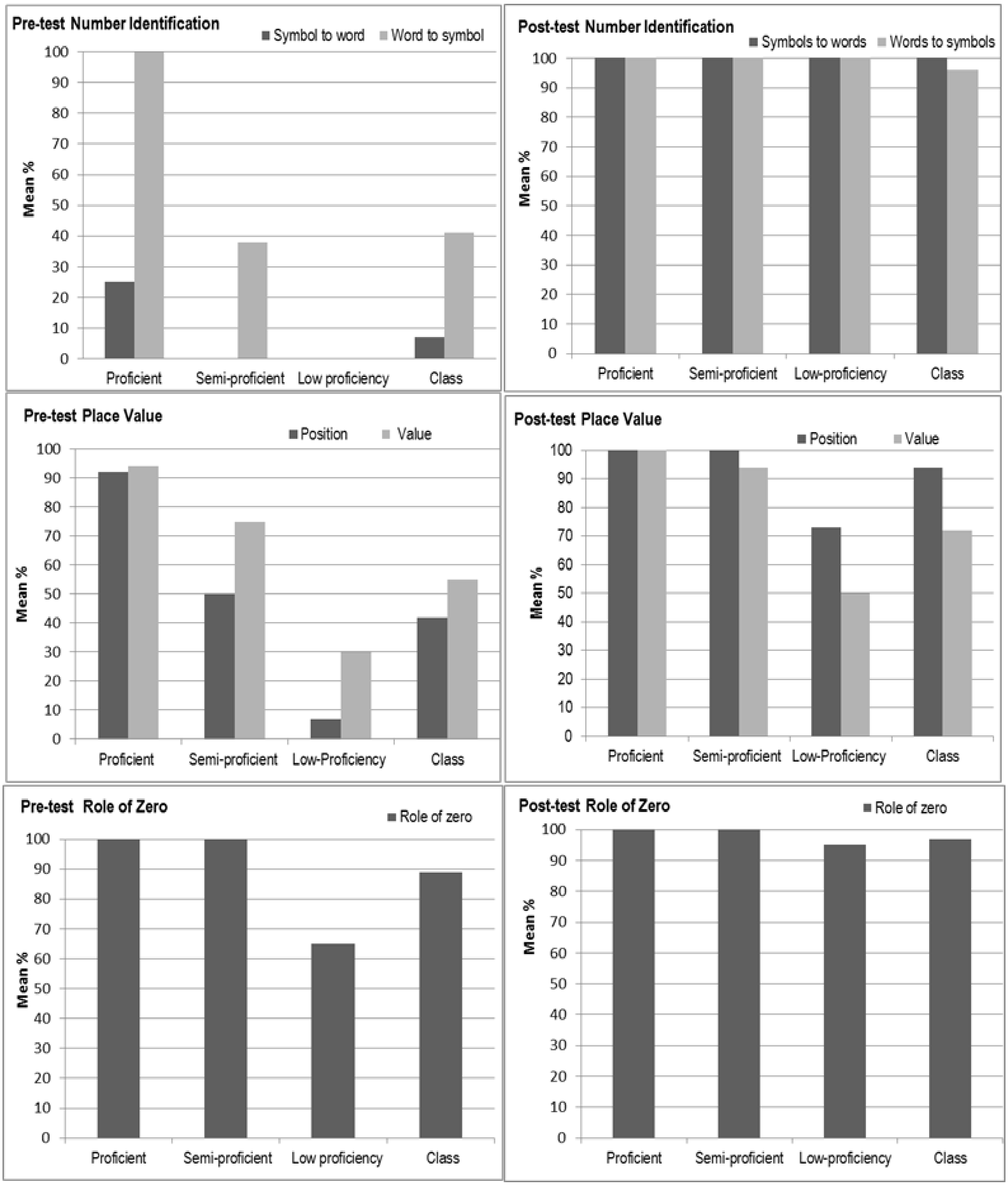
Appendix D
Comparison between whole and decimal numbers

Table D.1

Student Guide for Own Analysis of Own Errors (adapted from Resnick et al., 1989)

Decimal	Whole number	
Value increases (right to left)	Value increases (right to left)	**Student listed errors**
Each column is 10 times greater than column to right	Each column is 10 times greater than column to right	
Zero serves as a place holder	Zero serves as a place holder	
Zero added to left most column does change total value	Zero added to left most column does not change total value	
Values decrease as move occurs away from decimal point	Values increase as move occurs away from decimal point	
Naming		
End in -ths	End in -s	
Start with tenths	Start with ones	
Naming sequence is tenths, hundredths, thousandths	Naming sequence is ones, tens, hundreds, thousands	
Reading sequence is tenths, hundredths, thousandths	Reading sequence is thousands, hundreds, tens, one	

Appendix E
Pre-test and post-test results



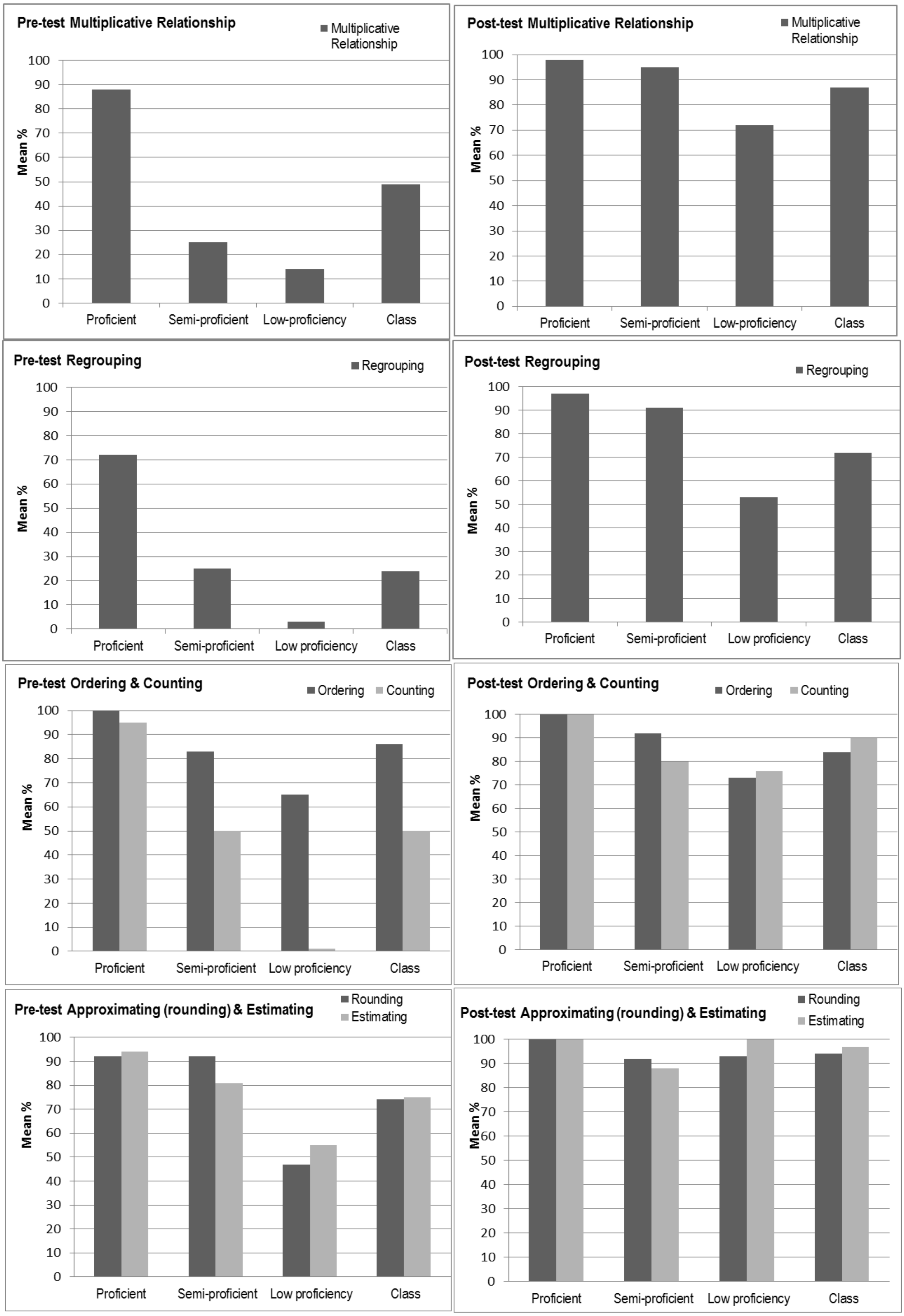


Figure E.1 Comparison of students' performance in test items.

Table E.1

Pre-Test Results of Students (within the three proficiency categories)

Pre-test Results in Categories of Proficiency																	
Proficiency Level		Proficient					Semi-Proficient					Low Proficiency					
Test Item	Concept	H1	H2	H3	H4	TOTAL	S1	S2	S3	S4	TOTAL	U1	U2	U3	U4	U5	TOTAL
1a	NI-W	0	0	1	0	25%	0	0	0	0	0%	0	0	0	0	0	0%
1b		0	0	1	0		0	0	0	0		0	0	0	0	0	
2a	NI-S	1	1	1	1	100%	0	0	0	0	38%	0	0	0	0	0	0%
2b		1	1	1	1		1	1	0	1		0	0	0	0	0	
1a	PV-P	1	1	1	1	92%	1	1	0	0	50%	0	1	0	0	0	7%
1b		1	1	0	1		1	1	0	0		0	0	0	0	0	
1c		1	1	1	1		1	1	0	0		0	0	0	0	0	
2	PV-V	1	0	1	1	94%	1	1	1	1	75%	1	0	1	0	0	30%
3a		1	1	1	1		1	1	1	1		1	0	1	0	0	
3b		1	1	1	1		1	1	0	1		0	1	1	0	0	
3c		1	1	1	1		1	0	0	0		0	0	0	0	0	
4a	PV-M	1	1	1	1	88%	0	0	0	0	25%	1	1	0	0	0	14%
4b		1	1	1	1		0	0	1	0		0	0	0	0	0	
4c		1	1	1	1		1	0	0	0		0	1	0	0	0	
4d		0	1	0	1		1	0	0	0		0	0	0	0	0	
4e		1	1	1	1		0	0	1	0		1	0	0	0	0	
4f		1	1	0	1		1	0	0	0		0	1	0	0	0	
4g		1	1	1	1		0	0	0	1		0	0	0	1	0	
4h		1	1	0	1		1	0	0	0		0	0	0	0	0	
4i		1	1	1	1		1	0	0	1		1	0	0	0	0	
4j		1	1	0	1		1	0	0	0		0	0	0	0	0	
5a	PV-Z	1	1	1	1	100%	1	1	1	1	100%	1	1	1	0	0	65%
5b		1	1	1	1		1	1	1	1		1	1	1	0	0	
5c		1	1	1	1		1	1	1	1		1	1	0	1	0	
5d		1	1	1	1		1	1	1	1		1	1	1	1	0	
1a	RG	1	1	1	0	72%	0	0	1	0	25%	0	0	1	0	0	3%
1b		1	1	1	1		1	1	1	0		0	0	0	0	0	
1c		1	1	1	0		0	0	0	0		0	0	0	0	0	
1d		1	1	1	1		1	0	1	0		0	0	0	0	0	
2a		1	1	0	0		0	0	1	0		0	0	0	0	0	
2b		1	1	1	0		0	0	0	0		0	0	0	0	0	
2c		1	1	1	0		0	1	0	0		0	0	0	0	0	
2d		1	0	0	0		0	0	0	0		0	0	0	0	0	
1a	C	1	1	1	1	95%	1	1	1	0	50%	0	0	0	0	0	0%
1b		1	1	1	1		1	0	1	0		0	0	0	0	0	
1c		1	1	0	1		1	0	0	0		0	0	0	0	0	
1d		1	1	1	1		1	1	0	0		0	0	0	0	0	
1e		1	1	1	1		0	1	1	0		0	0	0	0	0	
2a	OD	1	1	1	1	100%	1	1	1	1	83%	1	1	1	1	1	67%
2b		1	1	1	1		1	1	1	1		0	0	1	1	1	
2c		1	1	1	1		0	1	0	1		0	0	1	0	1	
1a	A&E	1	1	1	1	92%	1	1	0	1	92%	1	1	0	0	1	47%
1b		1	0	1	1		1	1	1	1		1	1	0	0	0	
1c		1	1	1	1		1	1	1	1		1	1	0	0	0	
2a	A&E-V	1	1	1	1	94%	1	0	1	1	81%	1	1	1	1	0	55%
2b		1	1	1	1		1	1	1	1		1	1	0	1	0	
2c		1	1	1	0		1	0	1	1		1	0	1	0	0	
2d		1	1	1	1		1	1	0	1		0	1	0	1	0	
TOTAL		**45/48** 94%	**43/48** 90%	**40/48** 83%	**39/48** 81%		**33/48** 69%	**24/48** 50%	**22/48** 46%	**20/48** 42%		**16/48** 33%	**16/48** 33%	**12/48** 25%	**8/48** 17%	**4/48** 8%	

Table E.2

Post-Test Results of Students (within the three proficiency categories)

Post-test Results in Categories of Proficiency

Test item	Concept	Proficient H1	H2	H3	H4	TOTAL	Semi-Proficient S1	S2	S3	S4	TOTAL	Low Proficiency U1	U2	U3	U4	U5	TOTAL
1a	NI-W	1	1	1	1	100%	1	1	1	1	100%	1	1	1	1	1	100%
1b		1	1	1	1		1	1	1	1		1	1	1	1	1	
2a	NI-S	1	1	1	1	100%	1	1	1	1	100%	1	1	1	1	1	100%
2b		1	1	1	1		1	1	1	1		1	1	1	1	1	
1a	PV-P	1	1	1	1	100%	1	1	1	1	100%	1	1	1	1	1	73%
1b		1	1	1	1		1	1	1	1		0	0	0	1	1	
1c		1	1	1	1		1	1	1	1		1	1	1	1	0	
2	PV-V	1	1	1	1	100%	1	1	1	1	94%	1	0	1	1	0	50%
3a		1	1	1	1		1	1	1	1		1	0	1	1	0	
3b		1	1	1	1		1	1	1	1		0	1	1	1	0	
3c		1	1	1	1		1	0	1	1		0	0	0	1	0	
4a	PV-M	1	1	1	1	98%	1	1	1	1	95%	1	1	1	1	1	72%
4b		1	1	1	1		1	1	1	1		1	0	0	1	0	
4c		1	1	1	1		1	1	1	1		1	1	1	1	1	
4d		1	1	1	1		1	0	1	1		0	1	0	1	0	
4e		1	1	1	1		1	1	1	1		1	1	0	1	0	
4f		1	1	1	1		1	1	1	1		1	1	1	1	0	
4g		1	1	0	1		1	1	1	1		1	1	0	1	1	
4h		1	1	1	1		1	1	1	1		1	1	1	1	1	
4i		1	1	1	1		1	1	1	1		1	1	1	1	1	
4j		1	1	1	1		1	1	0	1		0	0	0	1	0	
5a	PV-Z	1	1	1	1	100%	1	1	1	1	100%	1	1	1	1	1	95%
5b		1	1	1	1		1	1	1	1		1	1	1	1	1	
5c		1	1	1	1		1	1	1	1		1	1	1	1	0	
5d		1	1	1	1		1	1	1	1		1	1	1	1	1	
1a	RG	1	1	1	1	97%	1	1	1	1	91%	0	0	1	1	1	53%
1b		1	1	1	1		1	1	1	1		1	1	1	1	1	
1c		1	1	1	1		1	1	1	1		1	0	0	1	0	
1d		1	1	1	1		1	1	1	1		0	1	0	1	1	
2a		1	1	1	1		1	1	1	1		1	0	1	1	1	
2b		1	1	1	1		1	0	1	1		0	0	0	1	0	
2c		1	1	1	1		1	1	1	1		0	0	1	1	0	
2d		1	1	1	0		1	0	0	1		0	0	0	1	0	
1a	C	1	1	1	1	100%	1	1	1	1	80%	1	1	1	1	1	76%
1b		1	1	1	1		1	0	1	1		1	0	0	1	0	
1c		1	1	1	1		1	0	0	1		1	1	1	1	0	
1d		1	1	1	1		1	1	0	1		1	0	1	1	1	
1e		1	1	1	1		1	1	1	1		1	1	1	1	0	
2a	OD	1	1	1	1	100%	1	1	1	1	92%	1	1	1	1	1	73%
2b		1	1	1	1		1	1	1	1		0	0	1	1	1	
2c		1	1	1	1		1	1	0	1		0	0	1	1	1	
1a	A&E	1	1	1	1	100%	1	1	0	1	92%	1	1	0	1	1	93%
1b		1	1	1	1		1	1	1	1		1	1	1	1	1	
1c		1	1	1	1		1	1	1	1		1	1	1	1	1	
2a	A&E-V	1	1	1	1	100%	1	0	1	1	88%	1	1	1	1	1	100%
2b		1	1	1	1		1	1	1	1		1	1	1	1	1	
2c		1	1	1	1		1	0	1	1		1	1	1	1	1	
2d		1	1	1	1		1	1	1	1		1	1	1	1	1	
TOTAL		48/48	48/48	47/48	47/48		48/48	40/48	42/48	48/48		36/48	32/48	35/48	48/48	30/48	
		100%	100%	98%	98%		100%	83%	88%	100%		75%	67%	73%	100%	63%	

Table E.3

Whole-Class Pre-Test Results

Pretest- Decimal Results Class 6L

Proficiency Level		Proficient				Semi-Proficient				Low Proficiency					C	D	B	B	B	B	C	D	C	D			
Test Item	Concept	H1	H2	H3	H4	S1	S2	S3	S4	U1	U2	U3	U4	U5	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Result	%	Test item
1a	NI-W	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	7%	NI-W
1b		0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1		
2a	NI-S	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	7	41%	NI-S
2b		1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	1	1	0	0	1	0	1	1	12		
1a	PV-P	1	1	1	1	1	1	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0	10	42%	PV-P
1b		1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	1	1	11		
1c		1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	8		
2	PV-V	1	0	1	1	1	1	1	1	1	0	1	0	0	1	0	1	1	0	0	1	1	1	1	16	55%	PV-V
3a		1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	1	0	0	0	1	0	1	0	14		
3b		1	1	1	1	1	1	0	1	0	1	1	0	0	0	1	1	0	0	0	1	1	1	0	14		
3c		1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	o	0	0	7		
4a	PV-M	1	1	1	1	0	0	0	0	1	1	0	0	0	1	1	1	0	1	1	0	0	1	1	13	49%	PV-M
4b		1	1	1	1	0	0	1	0	0	0	0	0	0	0	1	1	1	1	1	0	1	0	1	12		
4c		1	1	1	1	1	0	0	0	0	1	0	0	0	1	0	1	1	0	1	1	0	0	0	11		
4d		0	1	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	1	1	**9**		
4e		1	1	1	1	0	0	1	0	1	0	0	0	0	0	1	1	1	0	1	0	0	0	0	10		
4f		1	1	0	1	1	0	0	0	0	1	0	0	0	1	0	1	1	1	1	0	0	1	1	**12**		
4g		1	1	1	1	0	0	0	1	0	0	0	1	0	1	0	1	1	1	1	1	0	1	1	14		
4h		1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	**8**		
4i		1	1	1	1	1	0	0	1	1	0	0	0	0	1	1	1	0	1	1	0	0	1	1	14		
4j		1	1	0	1	1	0	0	0	0	0	0	0	0	1	0	1	1	0	1	0	0	1	1	**10**		
5a	PV-Z	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	21	89%	PV-Z
5b		1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	21		
5c		1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0	0	1	19		
5d		1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	21		
1a	RG	1	1	1	0	0	0	1	0	0	0	1	0	0	0	0	0	1	0	1	0	0	0	0	7	24%	RG
1b		1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	9		
1c		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	5		
1d		1	1	1	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	7		
2a		1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	4		
2b		1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3		
2c		1	1	1	0	0	1	0	0	0	0	0	0	0	1	0	1	1	1	1	0	0	0	0	9		
2d		1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1		
1a	C	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	0	1	0	15	62%	C
1b		1	1	1	1	1	0	1	0	0	0	0	0	0	1	0	1	1	1	0	1	0	1	0	12		
1c		1	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	7		
1d		1	1	1	1	1	1	0	0	0	0	0	0	0	1	0	1	1	1	1	1	0	1	0	13		
1e		1	1	1	1	0	1	1	0	0	0	0	0	0	0	0	1	1	1	0	1	0	0	0	10		
2a	OD	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23	86%	OD
2b		1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	1	1	1	1	1	1	1	20		
2c		1	1	1	1	0	1	0	1	0	0	1	0	1	0	1	1	1	1	1	1	1	0	1	16		
1a	A&E	1	1	1	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1	0	1	0	18	74%	A&E
1b		1	0	1	1	1	1	1	1	1	1	0	0	0	1	0	1	1	1	1	1	0	0	0	15		
1c		1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	1	0	18		
2a	A&E-V	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	20	75%	A&E-V
2b		1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	1	0	0	1	1	18		
2c		1	1	1	0	1	0	1	1	1	0	1	0	0	1	1	0	0	1	1	1	0	0	0	13		
2d		1	1	1	1	1	1	0	1	0	1	0	1	0	1	1	1	1	1	1	1	0	1	1	18		
	TOTAL	**45/48**	**43/48**	**40/48**	**39/48**	**33/48**	**24/48**	**22/48**	**20/48**	**16/48**	**16/48**	**12/48**	**8/48**	**4/48**	**25/48**	**21/48**	**35/48**	**33/48**	**29/48**	**31/48**	**24/48**	**10/48**	**27/48**	**21/48**			
		94%	90%	83%	81%	69%	50%	46%	42%	33%	33%	25%	17%	8%	52%	44%	73%	69%	60%	65%	50%	21%	56%	44%			

Table E.4

Whole-Class Post-Test Results

Post Test -Decimal Results Class 6L

Proficiency Level		Proficient				Semi-Proficient				Low Proficiency					C	D	B	B	B	B	C	D	C	D			
Test Item	Concept	H1	H2	H3	H4	S1	S2	S3	S4	U1	U2	U3	U4	U5	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Result	%	Test item
1a	NI-W	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23	100%	NI-W
1b		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23		
2a	NI-S	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23	96%	NI-S
2b		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	21		
1a	PV-P	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23	94%	PV-P
1b		1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	20		
1c		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	22		
2	PV-V	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	0	1	1	1	1	20	72%	PV-V
3a		1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	1	1	1	0	1	0	1	0	16		
3b		1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	1	1	1	0	1	1	1	0	17		
3c		1	1	1	1	1	0	1	1	0	0	0	1	0	0	0	1	1	1	1	0	1	0	0	13		
4a	PV-M	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23	87%	PV-M
4b		1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	20		
4c		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	22		
4d		1	1	1	1	1	0	1	1	0	1	0	1	0	1	0	1	1	1	1	1	0	1	1	**17**		
4e		1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1	20		
4f		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	1	1	1	0	1	1	**19**		
4g		1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	0	1	1	20		
4h		1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	0	1	0	1	1	**19**		
4i		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	22		
4j		1	1	1	1	1	1	0	1	0	0	0	1	0	1	1	1	1	1	1	1	0	1	1	**17**		
5a	PV-Z	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23	97%	PV-Z
5b		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23		
5c		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	1	20		
5d		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23		
1a	RG	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	0	0	1	0	0	17	72%	RG
1b		1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	0	20		
1c		1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	0	1	1	0	18		
1d		1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	0	0	18		
2a		1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0	19		
2b		1	1	1	1	1	0	1	1	0	0	0	1	0	1	1	1	1	1	0	1	0	0	0	14		
2c		1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	1	1	0	0	0	17		
2d		1	1	1	0	1	0	0	1	0	0	0	1	0	1	1	1	1	0	0	0	0	0	0	10		
1a	C	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	22	90%	C
1b		1	1	1	1	1	0	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	19		
1c		1	1	1	1	1	0	0	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	19		
1d		1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	21		
1e		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	22		
2a	OD	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	21	84%	OD
2b		1	1	1	1	1	1	1	1	0	0	1	1	1	0	0	1	1	1	1	1	1	1	1	19		
2c		1	1	1	1	1	1	0	1	0	0	1	1	1	0	1	1	1	1	1	1	1	0	1	18		
1a	A&E	1	1	1	1	1	1	0	1	1	1	0	1	1	0	1	1	1	1	1	1	1	1	1	20	94%	A&E
1b		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	22		
1c		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23		
2a	A&E-V	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	21	97%	A&E-V
2b		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23		
2c		1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	22		
2d		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	23		
	TOTAL	**48/48**	**48/48**	**47/48**	**47/48**	**48/48**	**40/48**	**42/48**	**48/48**	**36/48**	**32/48**	**35/48**	**48/48**	**30/48**	**38/48**	**41/48**	**47/48**	**47/48**	**47/48**	**40/48**	**43/48**	**32/48**	**37/48**	**36/48**			
		100%	100%	98%	98%	100%	83%	88%	100%	75%	67%	73%	100%	63%	79%	85%	98%	98%	98%	83%	90%	67%	77%	75%			