# *Degrees of Metastability in Gauge Mediated Supersymmetry Breaking*

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A Thesis presented for the degree of
Doctor of Philosophy

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*To Katherine*

*whose continuing support shows no sign of metastability*

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## Abstract

This thesis consists of an investigation into supersymmetry and its breaking. Emphasis is placed on the question of metastability and the role of non-topological solitons in the hidden sector.

A desirable feature in models employing direct gauge mediation is that of tree level metastability, in order to generate large enough gaugino masses. An explicit realisation of this idea is constructed via a simple deformation of SQCD that is well motivated and needs no fine tuning [1]. Any viable metastable supersymmetry breaking vacuum must also be stable enough to survive until the present day. Non-topological solitons, or Q-balls, are supported in all such vacua where there is a conserved, global U(1) symmetry and no massless, charged scalars. It is shown that for a broad class of models Q-balls are extremely influential on the vacuum lifetime and make seemingly viable vacua catastrophically short lived [2]. Even when there is no effect on vacuum stability flat directions charged under an R-symmetry are a ubiquitous feature of O'Raifeartaigh models. Non-topological solitons associated with this symmetry, R-balls, are likely to form through the fragmentation of a condensate. Their cosmology is studied and it is found that they can have significant observable effects, from acting as a good dark matter candidate to providing the primary source of reheating in the early universe [3].

In addition to the body of work presented here, refs. [4,5] were completed during the writing of this thesis.

# Declaration

The work in this thesis is based on research carried out at the Centre for Particle Theory, Department of Mathematical Sciences, University of Durham, England. No part of this thesis has been submitted elsewhere for any other degree or qualification and it is all my own work unless referenced to the contrary in the text.

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# Contents

# Chapter 1

# Introduction and background

As scenarios of physics beyond the Standard Model go supersymmetry (SUSY) is one of, if not *the* most popular contender. It is interesting in its own right, as the only possible way of extending the spacetime symmetry of the universe within the framework of quantum field theory, but the real reason behind its popularity is likely to be the plethora of phenomenological benefits it bestows. Chief among them are a solution to the hierarchy problem, improved gauge coupling unification and a natural dark matter candidate. Even ignoring these properties there are strong theoretical reasons for studying SUSY. Our current best effort at a quantum theory of gravity, string theory, is naturally supersymmetric for example.

Of course we are yet to observe any elementary scalars in nature, leading to the inevitable conclusion that, should SUSY exist, it must be broken. Most of the phenomenological benefits require SUSY breaking effects to start appearing in the Standard Model around the TeV scale. This is precisely the experimental regime currently being scrutinised by both the Tevatron and the LHC, so we might start to see evidence in the near future. If we do not, other ideas must be found to understand the hierarchy problem and so forth. However, supersymmetric field theories remain important as the primary way of extracting phenomenological consequences from stringy models. Regardless of why one studies SUSY the question of how it is eventually broken is an important one, with the solution having major, observable repercussions in the behaviour of the low energy theory.

In this chapter we shall consult the literature to establish a general understanding of SUSY using superfield notation. The basic principles of SUSY breaking are well understood in this language and we will be immediately able to place some powerful constraints on prospective models. One must also consider how the symmetry breaking is communicated to the visible sector. There is no unique way of doing this, but an appealing option is to utilise gauge interactions. Finally we will see how these ideas naturally lead to the idea of metastable SUSY breaking and investigate some example models. A recent review of much of what is said here can be found in ref. [6].

## 1.1 A crash course in $\mathcal{N} = 1$ supersymmetry

Throughout this work we will be concerned only with the minimal example of $\mathcal{N} = 1$ global SUSY so as to maintain close contact with phenomenology. Even so there is a huge volume of theory underpinning the idea. This section will concentrate on the results pertinent to later observations and serve to establish notation. There are many excellent and more thorough discussions in the literature; refs. [7–9] to name but a few.

### 1.1.1 Superfields

$\mathcal{N} = 1$ SUSY corresponds to adding a single set of fermionic generators, $Q$ and $\bar{Q}$, to the Poincaré algebra describing the spacetime symmetry of the universe. These generators are Weyl spinors and obey anticommutation rather than commutation relations:

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu \quad \{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0 \quad [Q_\alpha, P^\mu] = [\bar{Q}_{\dot{\alpha}}, P^\mu] = 0 \quad (1.1)$$

where $P$ is the usual four-momentum generator of spacetime translations and the $\sigma$'s denote Pauli matrices (with spinor indices). The most natural language to work in is an irreducible representation of the resulting algebra. For this purpose the notion of *superspace* proves useful, although this is really just a neat book keeping device to keep track of various components. One extends the standard spacetime coordinates by adding some extra, anticommuting (or Grassmann) coordinates $\theta$

and $\bar{\theta}$ then considers the action of the generators on functions $S(x,\theta,\bar{\theta})$, known as *superfields*. The resulting representation is

$$P_\mu = i\partial_\mu \qquad iQ_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \qquad i\bar{Q}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu. \tag{1.2}$$

Note that the algebra demands $Q$ and $\bar{Q}$ have mass dimension $1/2$ so $\theta$ and $\bar{\theta}$ must have dimension $-1/2$. Generic functions of the superspace coordinates form a reducible representation of the algebra so we must apply covariant constraints to find an irreducible one. An obvious choice is to impose the reality condition $S^\dagger = S$: this defines a *vector superfield*. Another choice is to note that the fermionic derivative operators

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu \tag{1.3}$$

anticommute with $Q$ and $\bar{Q}$. The constraints

$$D_\alpha S = 0 \qquad \bar{D}_{\dot{\alpha}} S = 0 \tag{1.4}$$

are thus covariant and define a right and left handed *chiral superfield* respectively. Amongst other properties, one can show that products of exclusively left/right handed chiral superfields are themselves left/right handed chiral superfields, and that a product of a left handed chiral superfield with right handed one is a vector superfield.

Denoting a left handed chiral superfield by $\Phi$ we can expand in component fields

$$\Phi(x,\theta,\bar{\theta}) = \varphi + \sqrt{2}\theta\psi + \theta^2 F + i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\bar{\theta} - \frac{1}{4}\theta^2\bar{\theta}^2\partial^2\varphi \tag{1.5}$$

for complex scalars $\varphi(x)$ and $F(x)$ and a left handed Weyl spinor $\psi(x)$ (the spinor indices have been suppressed). We can then use the expressions (1.2) to explicitly find the transformation properties of $\Phi$ under an infinitesimal SUSY transformation $\Phi \to \Phi + \delta\Phi$. Parameterising with some anticommuting variables $\xi$, $\bar{\xi}$ gives

$$\delta\Phi = i(\xi Q + \bar{\xi}\bar{Q})\Phi \qquad \Longrightarrow \qquad \begin{aligned} \delta\varphi &= \sqrt{2}\xi\psi \\ \delta\psi &= \sqrt{2}\xi F + i\sqrt{2}\partial_\mu\varphi\sigma^\mu\bar{\xi} \\ \delta F &= \frac{i}{\sqrt{2}}\partial_\mu\psi\sigma^\mu\bar{\xi}. \end{aligned} \tag{1.6}$$

The important thing to note here is that the F-term of any chiral superfield transforms as a total derivative. Equivalent results clearly hold for a right handed chiral superfield. Meanwhile, a vector superfield $V$ is given by

$$V(x,\theta,\bar{\theta}) = \theta\sigma^\mu\bar{\theta}A_\mu + i\theta^2\bar{\theta}\bar{\lambda} - i\bar{\theta}^2\theta\lambda + \frac{1}{2}\theta^2\bar{\theta}^2 D + i(\chi - \chi^\dagger) \tag{1.7}$$

for a real vector $A(x)$, Weyl spinors $\lambda(x)$ and $\bar{\lambda}(x)$, a real scalar $D(x)$ and a chiral superfield $\chi$. There could be other terms but the components given here explicitly transform among themselves under SUSY transformations, as long as we consider the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ rather than the vector field itself:

$$\delta V = i(\xi Q + \bar{\xi}\bar{Q})V \qquad \Longrightarrow \qquad \begin{aligned} \delta F^{\mu\nu} &= i\partial_\mu(\xi\sigma^\nu\bar{\lambda} - \lambda\sigma^\nu\bar{\xi}) - i\partial_\nu(\xi\sigma^\mu\bar{\lambda} - \lambda\sigma^\mu\bar{\xi}) \\ \delta\lambda_\alpha &= -iD\xi_\alpha - \frac{1}{2}(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta\xi_\beta F_{\mu\nu} \\ \delta D &= \partial_\mu(\lambda\sigma^\mu\bar{\xi} - \xi\sigma^\mu\bar{\lambda}). \end{aligned} \tag{1.8}$$

Again, the important thing to note is that the D-term of any vector superfield transforms as a total derivative.

Note that a standard abuse of notation, and one which we shall use frequently, is to denote both a chiral superfield and its scalar component by the same symbol. The context in which the symbol is used leaves the meaning clear in most cases.

### 1.1.2 Supersymmetric Lagrangians

The previous section suggests a straightforward way of constructing supersymmetric Lagrangians. Any linear combination of chiral superfield F-terms and vector superfield D-terms transforms as a total derivative under SUSY transformations, so a Lagrangian built out of them yields an action that is invariant in the absence of boundary terms. For example, given a set of chiral superfields $\Phi_i$ the most general, supersymmetric, renormalisable Lagrangian is

$$\mathcal{L} = \int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2\, \Phi_i^\dagger\Phi_i + \left(\int \mathrm{d}^2\theta\, W(\Phi) + \text{h.c.}\right) \tag{1.9}$$

for some arbitrary, holomorphic, function $W(\Phi)$ known as the *superpotential* and where the sum over indices is understood. The superspace integrations pick out the appropriate components of the superfields: the first picks out the D-term of the vector superfield $\Phi_i^\dagger \Phi_i$ and the second the F-terms of the chiral superfields $W(\Phi)$ and $W^\dagger(\Phi^\dagger)$ (recall that a product of chiral superfields is itself a chiral superfield). The measures $\mathrm{d}\theta$ and $\mathrm{d}\bar{\theta}$ have dimension 1/2 so the $\Phi_i$ are dimension 1 and the superpotential dimension 3, i.e. it is at most cubic in $\Phi$ for a renormalisable theory.

Expanding this out in components, one finds that the D-terms contribute the kinetic part of the Lagrangian and the F-terms the interactions. However, it is clear from eq. (1.5) there are no derivative terms for the $F_i$ appearing anywhere in the Lagrangian. Hence the $F_i$ are auxiliary fields. Indeed, one can readily solve the field equations to find

$$F_i^\dagger = -\frac{\partial W(\varphi)}{\partial \varphi_i} \tag{1.10}$$

where $\varphi_i$ is the scalar component of $\Phi_i$. Substituting back in and expanding the rest of the Lagrangian we find

$$\mathcal{L} = \partial_\mu \varphi_i^\dagger \partial^\mu \varphi_i + i\bar{\psi}_i \bar{\sigma}^\mu \partial_\mu \psi_i - F_i^\dagger F_i - \frac{1}{2}\left(\frac{\partial^2 W(\varphi)}{\partial \varphi_i \varphi_j}\psi_i \psi_j + \text{h.c.}\right). \tag{1.11}$$

In other words, we have arrived at a theory of interacting complex scalars and Weyl fermions with a scalar potential $U(\varphi) = F_i^\dagger F_i$.

A more interesting class of theories, and one we will use heavily, is that of supersymmetric gauge theories. Consider a non-supersymmetric U(1) gauge theory of complex scalars and Weyl fermions with charge $\pm q$. One includes a vector field $A$ and defines gauge transformation via

$$\begin{aligned} &\varphi_1 \to e^{iq\chi}\varphi_1 \qquad &&\psi_1 \to e^{iq\chi}\psi_1 \qquad &&A_\mu \to A_\mu - \partial_\mu \chi \\ &\varphi_2 \to e^{-iq\chi}\varphi_2 \qquad &&\psi_2 \to e^{-iq\chi}\psi_2 && \end{aligned} \tag{1.12}$$

for an arbitrary scalar function $\chi(x)$. Our chiral superfield contains a complex scalar and a Weyl fermion, whereas our vector superfield contains a vector field. It therefore makes sense to collect the scalars and fermions into a chiral superfields whilst putting the vector field into a vector superfield. The supersymmetric version of the gauge transformation is then

$$\begin{aligned} &\Phi_1 \to e^{iq\chi}\Phi_1 \qquad &&V \to V + \frac{i}{2}(\chi - \chi^\dagger) \\ &\Phi_2 \to e^{-iq\chi}\Phi_2 && \end{aligned} \tag{1.13}$$

for an arbitrary chiral superfield $\chi$. Expanding in components and setting $\chi$ to be the scalar component of its parent chiral superfield reproduces the non-supersymmetric case. Note that the chiral superfield component of eq. (1.7) can now be associated with a gauge transformation and set to zero. This choice of gauge is known as the *Wess-Zumino gauge* and, while not manifestly supersymmetric (a generic SUSY transformation takes one out of this gauge), it is typically the most physically transparent gauge to work in so will be implemented henceforth.

The equivalent of the field strength tensor is the gauge invariant chiral superfield

$$W_\alpha = \bar{D}^2 D_\alpha V = 4i\lambda_\alpha - 4\theta_\alpha D - 2i(\sigma^\mu\bar{\sigma}^\nu)_\alpha{}^\beta\theta_\beta F_{\mu\nu} + 4\theta^2\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\bar{\lambda}^{\dot{\alpha}}. \tag{1.14}$$

Contracting the spinor indices and taking the F-term yields the supersymmetric pure gauge Lagrangian

$$\mathcal{L}_{\text{gauge}} = \frac{1}{32}\int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \text{h.c.} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{2}D^2 + \text{h.c.} \tag{1.15}$$

up to total derivative terms. This is the usual pure gauge Lagrangian with the addition of some free, massless fermions known as *gauginos*. To couple the gauge and matter sectors observe that the vector superfield $\Phi_1^\dagger e^{-2qV}\Phi_1$ is gauge invariant, so we can use it as a replacement for the D-term of eq. (1.9) (and similarly for $\Phi_2$). A supersymmetric Lagrangian for a U(1) gauge theory with matter of charge $\pm q$ can thus be written

$$\mathcal{L} = \int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2\left(\Phi_1^\dagger e^{-2qV}\Phi_1 + \Phi_2^\dagger e^{2qV}\Phi_2\right) + \frac{1}{32}\left(\int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \text{h.c.}\right). \tag{1.16}$$

As in the theory of chiral superfields discussed above there are no kinetic terms for the $F$ components of the $\Phi$'s and, from eq. (1.14), it is clear that there are no kinetic terms for the $D$ component of $V$ either. The vector superfield D-term is therefore another auxiliary field and we can solve the field equations to find

$$D = q(\varphi_1^\dagger\varphi_1 - \varphi_2^\dagger\varphi_2). \tag{1.17}$$

Plugging back in and expanding in components:

$$\begin{aligned}\mathcal{L} = &(D_\mu\varphi_1)^\dagger(D^\mu\varphi_1) + (D_\mu\varphi_2)^\dagger(D^\mu\varphi_2) + i\psi_1\sigma^\mu D_\mu\bar{\psi}_1 + i\psi_2\sigma^\mu D_\mu\bar{\psi}_2 + \\ &i\sqrt{2}q(\varphi_2^\dagger\psi_2 - \varphi_1^\dagger\psi_1)\lambda + i\sqrt{2}q(\varphi_1^\dagger\bar{\psi}_1 - \varphi_2^\dagger\bar{\psi}_2)\bar{\lambda} - \frac{1}{2}D^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + i\lambda\sigma^\mu\partial_\mu\bar{\lambda}\end{aligned} \tag{1.18}$$

where $D_\mu = \partial_\mu \pm iqA_\mu$ is the usual gauge covariant derivative. The result is a U(1) gauge theory of equally charged scalars and Weyl fermions with the addition of some extra Weyl fermions; the gauginos. These interact with matter through Yukawa couplings whose strengths are fixed by the gauge coupling. Furthermore there is another contribution to the scalar potential in the form of $U(\varphi) = \frac{1}{2}D^2$.

We can generalise further by considering non-Abelian gauge theories. This is achieved by elevating $V$ to a matrix valued vector superfield, i.e. $V = V^a T^a$ where the $T^a$ are generators of the chosen gauge group. The field strength superfield (1.14) becomes

$$W_\alpha = -\frac{1}{4}\bar{D}^2 e^{-V} D_\alpha e^V \tag{1.19}$$

whereas the general form of the matter couplings is unchanged from eq. (1.16). In each term the matrices $T^a$ appearing in $V$ belong to the representation appropriate for $\Phi$. Putting it all together and allowing for a superpotential term, the Lagrangian for a general, renormalisable, supersymmetric gauge theory with chiral matter $\Phi_i$ is

$$\mathcal{L} = \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2\, \Phi_i^\dagger e^V \Phi_i + \frac{1}{4g^2}\left(\int \mathrm{d}^2\theta\, W^\alpha W_\alpha + \text{h.c.}\right) + \left(\int \mathrm{d}^2\theta\, W(\Phi) + \text{h.c.}\right) \tag{1.20}$$

for a gauge coupling $g$. In component form there are three main contributions: $\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{yuk}} - U(\varphi)$. First is the kinetic term

$$\mathcal{L}_{\text{kin}} = (D_\mu\varphi_i)^\dagger(D^\mu\varphi_i) + i\psi_i\sigma^\mu D_\mu\bar{\psi}_i - \frac{1}{4g^2}F^{a\mu\nu}F^a_{\mu\nu} + i\lambda^a\sigma^\mu D_\mu\bar{\lambda}^a \tag{1.21}$$

for a gauge covariant derivative $D_\mu = \partial_\mu + igA^a_\mu T^a$, second are the Yukawa couplings

$$\mathcal{L}_{\text{yuk}} = -i\sqrt{2}g\phi_i^\dagger T^a\lambda^a\psi_i - \frac{1}{2}\frac{\partial^2 W}{\partial\varphi_i\varphi_j}\psi_i\psi_j + \text{h.c.} \tag{1.22}$$

and finally comes the scalar potential

$$U(\varphi) = F_i^\dagger F_i + \frac{1}{2}D^a D^a = \sum_i \left|\frac{\partial W}{\partial\varphi_i}\right|^2 + \frac{1}{2}g^2\sum_{i,a}\left(\varphi_i^\dagger T^a\varphi_i\right)^2. \tag{1.23}$$

There may be times when we want to look beyond renormalisable theories and treat our gauge theory only as an effective theory. For example, later on we will consider strongly coupled supersymmetric gauge theories with low energy effective descriptions. Or we might want to consider the effects of gravity on our models. Extra terms in the Lagrangian (1.20) are then permitted. Of particular importance will be the lifting of the restriction for the superpotential to be at most cubic in the chiral superfields, and the generalisation of the D-term

$$\int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2\, \Phi_i^\dagger \Phi_i \longrightarrow \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2\, K(\Phi^\dagger, \Phi) \tag{1.24}$$

for some arbitrary, real function $K$, known as the *Kähler potential.* Unlike the superpotential, the Kähler potential need not be holomorphic. Since it arises from a high energy regime of the theory of which we have no knowledge its effects must be treated carefully. However, we do know that any non-canonical contributions should be suppressed by the scale at which our effective theory breaks down. This could be the Planck scale or, as we shall see later, the scale at which the effective description becomes strongly coupled.

### 1.1.3 R-symmetry

To close this section we shall look briefly at two other important properties of supersymmetric theories. The first is the idea of R-symmetry. In $\mathcal{N} = 1$ SUSY, R-symmetries are at most U(1) symmetries. An intuitive way of thinking about them is as global rotations in superspace, i.e. $\theta \to e^{i\alpha}\theta$, where $\theta$ has charge +1. Looking at the definition (1.5), we see that the components of a chiral superfield $\Phi$ with R-charge $R_\Phi$ must therefore have charges

$$R(\varphi) = R_\Phi \qquad R(\psi) = R_\Phi - 1 \qquad R(F) = R_\Phi - 2 \tag{1.25}$$

and similarly for a vector superfield $V$ (1.7) with R-charge $R_V$

$$R(A) = R_V \qquad R(\lambda) = R_V + 1 \qquad R(D) = R_V. \tag{1.26}$$

In a gauge theory with Lagrangian given by eq. (1.20) we see that the superpotential must have R-charge +2 (recall that $\mathrm{d}\theta$ has charge $-1$: $\int \mathrm{d}\theta \equiv \partial/\partial\theta$ due to the Grassmann nature of the superspace coordinates) and also that $R_V = 0$ such that the Lagrangian is neutral.

The fact that vector superfields always have $R = 0$ implies that gauginos have R-charge +1. A Majorana gaugino mass term then has $R = +2$; it is forbidden in R-symmetric theories and can be difficult to generate. Shortly, we will see that there is in fact a deep relationship between R-symmetries and globally stable SUSY breaking. This leads to a conundrum, in that we want a model that is R-symmetric so SUSY can be broken, yet we also want to generate gaugino masses. A possible solution is to instead consider Dirac gauginos [10–28]. Such models allow for R-symmetric gaugino masses, but often at the price of increased complexity and pushing the theory towards strong coupling in the UV.

### 1.1.4 Non-renormalisation theorems

One of the most powerful features of SUSY is the existence of non-renormalisation theorems. It is possible to show that the superpotential is not directly renormalised at the level of perturbation theory, but only through wave function renormalisation. We can therefore write down the most general superpotential consistent with the symmetries of a model and know that no other terms can be generated perturbatively. This stems from the fact that radiative corrections can always be written as D-terms whereas the superpotential consists only of F-terms. An intuitive way to understand this result was provided by Seiberg [29]. Consider a theory of a single chiral superfield $\Phi$ with tree level superpotential

$$W_{\text{tree}} = m\Phi^2 + \kappa\Phi^3. \tag{1.27}$$

If we think of $\kappa$ as the SUSY preserving expectation of a chiral superfield, rather than a coupling constant, the model has an R-symmetry with $R(\Phi) = 1$ and $R(\kappa) = -1$. In conjunction with holomorphy, this greatly restricts the perturbatively renormalised superpotential to be of the form

$$W_{\text{eff}} = m\Phi^2 f\left(\frac{\kappa\Phi}{m}\right) \tag{1.28}$$

for some polynomial $f$. In the limit $m \to 0$, $\kappa \to 0$ and $m/\kappa = \text{constant}$ it must be that $W_{\text{eff}} = W_{\text{tree}}$, therefore $f(\kappa\Phi/m) = 1 + \kappa\Phi/m$. However, $\kappa\Phi/m$ is arbitrary in this limit so $f$ must evaluate to $f(\kappa\Phi/m) = 1+\kappa\Phi/m$ for *all* values of its argument, i.e. the superpotential is not renormalised.

A similar result holds in supersymmetric gauge theories. Strictly speaking, the Lagrangian of eq. (1.15) contains an additional term involving the dual field strength tensor

$$\mathcal{L}_{\text{gauge}} \supset \frac{\omega}{32\pi^2} F^{\mu\nu}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}. \tag{1.29}$$

We ignored it earlier as it is a topological surface term with no bearing on the perturbative theory. Indeed, at this level one is free to change the parameter $\omega$ at will. However, the term can be included in eq. (1.20) by replacing the gauge kinetic part of the Lagrangian with

$$\frac{1}{4g^2}\int \mathrm{d}^2\theta\, W^\alpha W_\alpha \longrightarrow \frac{1}{16\pi^2}\int \mathrm{d}^2\theta\, \tau W^\alpha W_\alpha \quad \text{where} \quad \tau = \frac{4\pi^2}{g^2} + i\omega \tag{1.30}$$

and again considering $\tau$ as the expectation of a chiral superfield. Since perturbation theory (ergo the tree level superpotential) is invariant under translations in $\omega$ and the superpotential can only include $\tau$ holomorphically, it must be that the superpotential is independent of $\tau$. In other words, it is not directly renormalised by gauge effects.

Beyond perturbation theory this argument no longer holds. Non-perturbative effects (gaugino condensation, for example) often break the continuous translation symmetry in $\omega$ to a discrete one. Suppose the translation symmetry is broken down to $Z_N$, i.e. $\omega \to \omega + 2\pi N$. We would then expect corrections to the superpotential to be of the form

$$\Delta W \sim e^{-n\tau/N} w_n(\Phi) \tag{1.31}$$

for some integer $n$ and a corresponding holomorphic function $w_n$. This is actually a remarkable result, as originally noticed by Witten [30]. If we take a theory that does not break SUSY at tree level, non-renormalisation theorems stop it breaking SUSY at all orders in perturbation theory. They do not, however, prevent it from being broken dynamically. If it is broken dynamically, the above results show there is an automatic exponential suppression as long as Re[$\tau$] is large, i.e. $g$ is small. Hence an exponential hierarchy between the Planck scale and the SUSY breaking scale is perfectly natural in theories of dynamical SUSY breaking.

An important exception to these non-renormalisation theorems is the Kähler potential, which is not holomorphic so need not obey them. Precisely these kinds of terms are responsible for the wave function renormalisation still experienced by supersymmetric theories and, subsequently, for corrections to physically observable quantities. Indeed, a chiral superfield is renormalised as

$$\int d\theta^2 d\bar{\theta}^2 \, \Phi^\dagger \Phi \longrightarrow \int d\theta^2 d\bar{\theta}^2 \, Z(t) \Phi^\dagger \Phi \tag{1.32}$$

for some function of renormalisation group (RG) scale $Z(t)$ (where $t = \ln \mu$), whereas vector superfields are renormalised through the field strength superfield by replacing $\tau \to \tau(t)$ in eq. (1.30).

The renormalisation of the field strength term actually suggests a subtlety in the above reasoning. These are F-terms so we do not expect them to be renormalised by radiative corrections. The solution lies in our choice of action. All of the previous statements apply to the Wilsonian effective action $S_W(\mu)$, in which one discards low virtual momentum ($p < \mu$) contributions to vacuum loops. Clearly this removes any IR divergences, but it also changes the physics in theories with massless, interacting particles, such as gauge theories. Instead we should use the one particle effective action to determine the running of the gauge coupling. This is exposed to holomorphic anomalies in the IR and results in perturbative renormalisation of $\tau$ upon converting back to the Wilsonian action. The phenomenon was studied in detail in ref. [31], where the authors also provided a proof that the holomorphic gauge coupling is only renormalised at one loop.

## 1.2 Breaking supersymmetry

The problem of how exactly to break SUSY is a persistent one and, despite great leaps in understanding, there is yet to be a conclusive solution. As we will see, the specifics of the SUSY breaking and how it is communicated to the visible sector have a profound effect on the resulting phenomenology. Conversely, current phenomenological observations and expectations can tell us a lot about the fundamental physics responsible for SUSY breaking.

The basics of spontaneous SUSY breaking follow immediately from the SUSY algebra (1.1) which gives an expression for the Hamiltonian

$$H = P^0 = \frac{1}{4}(Q_1 \bar{Q}_{\dot{1}} + \bar{Q}_{\dot{1}} Q_1 + Q_2 \bar{Q}_{\dot{2}} + \bar{Q}_{\dot{2}} Q_2) \tag{1.33}$$

where $\bar{Q}_{\dot{\alpha}}$ is the Hermitian adjoint of $Q_\alpha$. It follows that the Hamiltonian is positive semi-definite. For SUSY to be broken spontaneously the vacuum cannot be invariant under SUSY transformations, i.e. $Q_\alpha|\text{vac}\rangle \neq 0$ or $\bar{Q}_{\dot{\alpha}}|\text{vac}\rangle \neq 0$, which is true if and only if $\langle H\rangle \neq 0$ and the vacuum has non-zero energy. To investigate whether a given model breaks SUSY one therefore calculates the scalar potential $U(\varphi)$ and finds its minima. If the minimum is at $U(\varphi) \neq 0$ SUSY is broken. A supersymmetric generalisation of Goldstone's theorem predicts a massless fermion, the *Goldstino*, associated with SUSY breaking as the broken generator is fermionic. If SUSY is gauged (to include gravity) this Goldstino is eaten by the gravitino and the gravitino acquires a mass via an analogue of the Higgs mechanism.

We found the scalar potential for a supersymmetric gauge theory in eq. (1.23). There are F-term and D-term contributions, both of which are positive. Either one of these could take a non-zero value in the vacuum and result in SUSY breaking. *D-term SUSY breaking* can occur via the Fayet-Iliopoulos mechanism [32] in U(1) gauge theories (or gauge theories with a U(1) subfactor) by adding the gauge invariant term

$$\mathcal{L}_{\text{FI}} = d\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2\, V = dD \tag{1.34}$$

to eq. (1.16). The scalar potential is minimised at $U = \frac{1}{2}d^2 \neq 0$ and so breaks SUSY. However, terms like this are not important when considering dynamical SUSY breaking for the following reason. Any dynamical terms depend on the complex gauge coupling $\tau$, which can be considered as the expectation of a chiral superfield so appears holomorphically at tree level. Dynamical D-terms are generated at one loop so are proportional to the sum of charges propagating around the loop

$$d \propto \sum_i q_i. \tag{1.35}$$

We can only generate a non-zero D-term if this sum does not vanish, but then the gauge theory has a gravitational anomaly and we cannot hope to find a consistent UV completion. As such, we shall consider this type of SUSY breaking no further. Instead, we will focus on *F-term SUSY breaking* where

$$U(\varphi) = F_i^\dagger F_i = \sum_i \left|\frac{\partial W}{\partial \varphi_i}\right|^2 \neq 0. \tag{1.36}$$

Since the F-term contribution of each chiral field is independently positive it suffices to find a single $F_i \neq 0$.

### 1.2.1 Witten's index theorem

Regardless of the type of SUSY breaking, an important quantity to consider is that of the *Witten index* [33]. Generically, one can write

$$Q_\alpha |\varphi\rangle = \sqrt{E}|\psi\rangle \tag{1.37}$$

for a bosonic and fermionic states $|\varphi\rangle$ and $|\psi\rangle$ with energy $E$, a function of the parameters of the theory. Hence any bosonic state with non-zero energy can be paired up with a fermionic state of the same energy. Now consider the quantity

$$\Delta = \mathrm{Tr}\left[(-1)^F e^{-\beta H}\right] \tag{1.38}$$

known as the Witten Index, where $F$ is the fermion number of a state and $\beta$ is the inverse temperature. States of non-zero energy do not contribute as they come in fermionic-bosonic pairs, hence this parameter counts the number of supersymmetric (zero energy) bosonic vacua minus the number of supersymmetric fermionic vacua. It is clear that no supersymmetric vacua exist and SUSY is globally broken only if $\Delta = 0$ but the real power in the Witten index comes from the fact that it is invariant under continuous deformations of all parameters of the theory. To see this, note that varying a parameter such that the energy of a bosonic state goes to zero inevitably brings its fermionic partner along for the ride, in which case $\Delta$ is unchanged. Conversely if a similar variation lifts a zero energy bosonic state, it must be accompanied by a fermionic one that originally had zero energy so $\Delta$ is again unchanged. A subtlety arises when a variation changes the asymptotic behaviour of the theory. It is then possible to bring in/push out a zero energy state from/to infinity without a partner, changing the value of the Witten index. One can think of this as somehow changing the topology of the model's moduli space.

An important example of these ideas can be found in massive, supersymmetric QCD (SQCD), i.e. a model with gauge group SU($N$), 'quark' chiral superfields $Q$ and $\tilde{Q}$ and a superpotential $W = m\tilde{Q}Q$. One can increase the mass parameter

$m$ until the quarks become heavy enough to be integrated out leaving an SU($N$) super Yang-Mills theory. The Witten index of this theory is known to be $N$ [33], hence massive SQCD must have at least $N$ supersymmetric vacua. Indeed, the same reasoning suggests that *any* supersymmetric gauge theory with massive, vector-like matter has a supersymmetric vacuum state. We can therefore conclude that, in order to dynamically break SUSY globally, we need to consider theories with either chiral or massless matter.

### 1.2.2 The return of R-symmetry

As alluded to earlier R-symmetries have an important role to play in the story of SUSY breaking. To summarise, the Nelson-Seiberg theorem [34] states that the existence of an exact R-symmetry is a necessary condition for a generic, calculable model of globally stable SUSY breaking. "Generic" means that the model's superpotential contains all operators permitted by its symmetry group and "calculable" (in this case) means that, at low energies, the model is described by an effective theory without gauge fields and the Kähler potential is under control. We are typically interested in generic theories for reasons of naturalness, whereas incalculable theories are of limited use due to their inability to make concrete predictions: the theory should either be weakly coupled in the IR or any strong coupling effects should be accounted for by, for example, some kind of duality. The net result is a theory of $n$ chiral superfields $\Phi_i$ with Lagrangian

$$\mathcal{L} = \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2\, K(\Phi_i^\dagger, \Phi_i) + \left( \int \mathrm{d}^2\theta\, W(\Phi) + \text{h.c.} \right) \tag{1.39}$$

where the superpotential is generic but not necessarily renormalisable.

In order to break SUSY globally it must be that the set of $n$ equations

$$F_i = \frac{\partial W}{\partial \Phi_i} = 0 \tag{1.40}$$

has no consistent solution. We now consider three different cases.

- **No global symmetries**: There are $n$ unknowns $\Phi_i$ and $n$ equations (1.40). We can generically find a solution and a supersymmetric vacuum exists.

- **Normal global symmetry**: Take a global U(1) symmetry under which $\Phi_i$ has charge $q_i$ and $q_n \neq 0$. We can redefine the fields in terms of some neutral fields $\chi_r$:

$$\{\Phi_i\} \to \{\Phi_n, \chi_r\} \quad \text{where} \quad \chi_r = \frac{\Phi_r}{\Phi_n^{q_r/q_n}} \quad \text{and} \quad r = 1, \ldots, n-1. \tag{1.41}$$

  The superpotential is holomorphic and neutral under the U(1) symmetry so it can be written in terms of the $\chi$'s only: $W(\Phi) = w(\chi)$. $F_n$ is now automatically zero, but the remaining $n-1$ F-term equations corresponding to the $n-1$ neutral $\chi$'s still generically have a solution. For non-abelian symmetry groups a similar argument holds. We simply factor out one charged superfield for each generator.

- **Global R-symmetry**: We take the same approach as before, recasting the fields as

$$\{\Phi_i\} \to \{\Phi_n, \chi_r\} \quad \text{where} \quad \chi_r = \frac{\Phi_r}{\Phi_n^{R_r/R_n}} \quad \text{and} \quad r = 1, \ldots, n-1 \tag{1.42}$$

  but now the superpotential must have charge +2 so $W(\Phi) = \Phi_n^{2/R_n} w(\chi)$. Eq. (1.40) becomes

$$w(\chi) = 0 \qquad\qquad \frac{\partial w}{\partial \chi_r} = 0 \tag{1.43}$$

  as long as $\Phi_n$ is finite and non-zero. There are now $n$ equations but there are only $n-1$ unknowns. Generically there is no consistent solution and SUSY is broken.

It follows that SUSY can only be broken globally in the presence of an exact R-symmetry. An immediate corollary is that a spontaneously broken R-symmetry ($\Phi_n$ is finite and non-zero) is a sufficient condition for SUSY breaking.

This actually leads to a problematic massless Goldstone boson: the *R-axion*, $a_R$. The axion acquires a mass and couplings to the visible sector. Both are inversely related to the scale of spontaneous R-symmetry breaking $f_R$ because R-symmetry is (usually) anomalous with respect to the visible sector gauge group. Unless $f_R \gtrsim 10^7$ GeV the axion couplings

$$\mathcal{L} \supset \frac{a_R}{f_R}\frac{\alpha}{8\pi} F^{\mu\nu}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \tag{1.44}$$

permit stellar cooling at a rate faster than allowed by astrophysical observations [35]. A more restrictive bound of $f_R \gtrsim 10^9$ GeV is derived through the non-observation of a gamma ray burst in association with the supernova SN 1987A [36,37]. Conversely, if $f_R \gtrsim 10^{12}$ GeV the classical oscillations of the axion field dissipate too slowly and their energy density overcloses the universe [38]. Only a small window is left for the scale of R-symmetry breaking, greatly undermining model building flexibility.

The symmetry could be made anomalous under some other gauge group to alleviate these constraints by increasing the mass of the axion. However, dynamical R-breaking terms in the effective superpotential can then restore SUSY (indeed, we will see an explicit example of this idea later). Alternatively we could ensure R-symmetry is not spontaneously broken, but this would forbid gaugino masses. It would therefore seem that the tendency for models of globally stable SUSY breaking to come with R-axions presents a fundamental model building dilemma.

### 1.2.3 Pseudo-moduli and the Coleman-Weinberg potential

*Pseudo-moduli*, or classical flat directions, also have a large effect on the viability of SUSY breaking models. Simply put these are scalar fields that are not Goldstone bosons, but whose values have no bearing on the classical value of the potential in the ground state, so remain undetermined at tree level. They parameterise a set of classically degenerate vacuum states, the *pseudo-moduli space*. It has been shown [39, 40] that pseudo-moduli are a generic feature of globally stable, SUSY breaking vacua in calculable, renormalisable models. Such models are known as *Wess-Zumino* models and a sketch of the proof is as follows.

In the absence of gauge fields the scalar potential is given by the F-terms only

$$U(\varphi) = F_i^\dagger F_i = W_i^\dagger W_i \tag{1.45}$$

where $W_i$ denotes $\partial W/\partial\Phi_i$ and the superpotential is at most cubic in the $\Phi$'s. For a locally stable SUSY breaking vacuum to exist we must be able to find a solution $\varphi_i = \langle\varphi_i\rangle$ to the equations

$$\frac{\partial U}{\partial \varphi_i} = W_{ij} W_j^\dagger = 0 \qquad\qquad W_i \neq 0 \quad \text{for at least one value of } i. \tag{1.46}$$

Note from eq. (1.11) that the first equation contains the fermion mass matrix $\mathcal{M}_{1/2} = W_{ij}$. These two statements thus confirm that there is at least one massless fermion, the Goldstino, corresponding to the fermionic component of the chiral superfield whose F-term gets a VEV. The scalar mass matrix can also be read off from eq. (1.11) to be

$$\mathcal{M}_0^2 = \begin{pmatrix} \mathcal{M}_{1/2}^\dagger \mathcal{M}_{1/2} & \mathcal{F}^\dagger \\ \mathcal{F} & \mathcal{M}_{1/2} \mathcal{M}_{1/2}^\dagger \end{pmatrix} \quad \text{where} \quad \mathcal{F}_{ij} = W_{ijk} W_k^\dagger \tag{1.47}$$

evaluated at $\varphi_i = \langle \varphi_i \rangle$. Unless $\mathcal{M}_0^2$ is positive semi-definite the vacuum is not locally stable.

Now take the direction $v_i = W_i^\dagger|_{\varphi_i = \langle \varphi_i \rangle}$ (corresponding to the scalar component of the Goldstino superfield, or indeed any other combination satisfying $\mathcal{M}_{1/2} v = 0$) and consider the norm

$$\begin{pmatrix} v^\dagger & v^T \end{pmatrix} \mathcal{M}_0^2 \begin{pmatrix} v \\ v^* \end{pmatrix} = v^T \mathcal{F} v + \text{h.c.} \tag{1.48}$$

This quantity is necessarily positive if the vacuum is globally stable, ergo it must be that $\mathcal{F} v = 0$ and $(v, v^*)$ corresponds to a massless scalar. Otherwise we could always rotate the phase of $v$ to make the right hand side negative. In fact, it is better than that for renormalisable models. In terms of the superpotential, $\mathcal{F} v = 0$ is written

$$(\mathcal{F} v)_i = \Big[ W_{ijk} W_j^\dagger W_k^\dagger \Big]_{\varphi_i = \langle \varphi_i \rangle} = 0. \tag{1.49}$$

Moving along the direction $v_i = W_i^\dagger|_{\varphi_i = \langle \varphi_i \rangle}$ we thus have

$$U(\langle \varphi \rangle + \alpha v) = U(\langle \varphi \rangle) + \delta W_i^\dagger \delta W_i \tag{1.50}$$

for some complex parameter $\alpha$ where

$$\delta W_i = \left[ \alpha W_{ij} W_j^\dagger + \frac{1}{2} \alpha^2 W_{ijk} W_j^\dagger W_k^\dagger \right]_{\varphi_i = \langle \varphi_i \rangle} = 0 \tag{1.51}$$

up to fourth derivatives of $W$ (i.e. for a cubic superpotential). The right hand side follows from eqs. (1.46) and (1.49). In other words the scalar potential, and the F-terms themselves, are invariant under complex translations along the scalar component of the Goldstino superfield (or any other superfield containing a massless fermion): it is a pseudo-modulus. For non-renormalisable models the scalar partner of the Goldstino is still massless, but the above expansion of the potential is no longer exact so we do not necessarily find a pseudo-modulus. Quartic terms in the superpotential, for example, could be present and lift the erstwhile flat direction.

Although the potential is classically flat along pseudo-moduli, the physical spectrum of the model can change significantly. It is therefore important that there is a mechanism to stabilise the VEVs of the pseudo-moduli. When quantum corrections are taken into account precisely such a mechanism appears in the *Coleman-Weinberg* potential [41], which is generated at one loop by SUSY breaking effects and is given by

$$U_{\mathrm{CW}} = \frac{1}{64\pi^2}\mathrm{STr}\mathcal{M}^4 \ln\left(\frac{\mathcal{M}^2}{\Lambda^2}\right) = \frac{1}{64\pi^2}\sum\left[\mathcal{M}_0^4 \ln\left(\frac{\mathcal{M}_0^2}{\Lambda^2}\right) - \mathcal{M}_{1/2}^4 \ln\left(\frac{\mathcal{M}_{1/2}^2}{\Lambda^2}\right)\right]. \tag{1.52}$$

We take the *supertrace* (the bosonic component minus the fermionic component) of the operator $\mathcal{M}^4 \ln(\mathcal{M}^2/\Lambda^2)$, where $\mathcal{M}$ is the mass matrix evaluated in the vacuum and $\Lambda$ is the model's UV cutoff scale. Since the expectations of the pseudo-moduli are arbitrary the mass matrix, and subsequently the Coleman-Weinberg potential, are typically functions of their VEVs. Minimising it determines the actual values.

To see where the potential comes from, consider the vacuum energy of a theory of bosons and fermions

$$U = \sum_{\text{bosons}} \frac{1}{2}\bar{h}\omega_0 - \sum_{\text{fermions}} \frac{1}{2}\bar{h}\omega_{1/2} = \sum_i (-1)^F \int \frac{\mathrm{d}^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2+m_i^2}. \tag{1.53}$$

The quartic divergence (proportional to $\sum(-1)^F\Lambda^4$) of this integral cancels for supersymmetric theories, as does the quadratic divergence (proportional to $\sum(-1)^F m_i^2$ – one can explicitly calculate the mass matrix for a general supersymmetric gauge theory from eq. (1.20) and show that this sum vanishes for any non-anomalous gauge theory) leaving only the logarithmic divergence of eq. (1.52). Note that this potential vanishes in a supersymmetric vacuum where the boson and fermion masses are equal.

The general course of action is therefore as follows. Having found a candidate SUSY breaking vacuum and identified the pseudo-moduli we should find and attempt to minimise the Coleman-Weinberg potential. There are three general possibilities, all of which were extensively studied in ref. [42].

- **Good**: The Coleman-Weinberg potential has a stable minimum, fixing the VEVs of the pseudo-moduli. The SUSY breaking vacuum is well defined and we have a viable model (dashed line in figure 1.1).
- **Bad**: The Coleman-Weinberg potential does not have a stable minimum and the VEVs of the pseudo-moduli runaway towards the UV cutoff scale. The SUSY breaking vacuum is not well defined (dotted line in figure 1.1).
- **Incalculable**: Contributions from the Coleman-Weinberg potential are subdominant relative to those coming from the Kähler potential. Strong coupling or gravitational effects coming from above the cutoff scale are thus important. These are incalculable so we cannot say whether the SUSY breaking vacuum is stable or not.

Case three can be identified by comparing the pseudo-modulus mass generated by the Coleman-Weinberg potential with that generated by the Kähler potential. For a pseudo-modulus $X$ the Kähler potential will generate a mass from operators

$$K \supset \frac{(X^\dagger X)^2}{\Lambda^2} \qquad \Longrightarrow \qquad \mathcal{L} \supset \frac{F^2}{\Lambda^2} X^\dagger X \tag{1.54}$$

where $F$ denotes the F-term that breaks SUSY. If the mass from the Coleman-Weinberg potential is $m_X$, the pseudo-modulus is incalculable if $m_X^2 < F^2/\Lambda^2$.

Even when all pseudo-moduli in the SUSY breaking sector are under control they may yet have an important role to play. The flatness of their potential can lead to some interesting cosmological consequences [3].

## 1.3 Gauge mediation

In an ideal world SUSY would be automatically broken within the MSSM. Sadly, it is not. A quick way to see this is via the sum rule used in the previous section

$$\sum_i (-1)^F m_i^2 = 0 \tag{1.55}$$

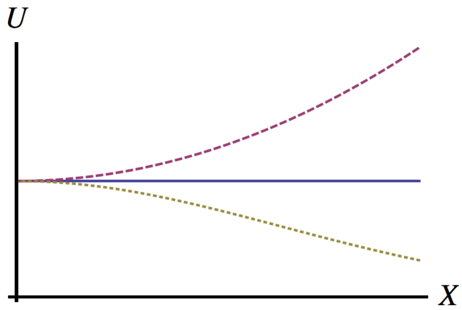


Figure 1.1: Quantum corrections to a pseudo-modulus $X$ in a SUSY breaking vacuum. At tree level (solid line) the potential is flat in the $X$ direction. At one loop the Coleman-Weinberg potential lifts the vacuum degeneracy. Good pseudo-moduli are stabilised (dashed line) but bad ones are tachyonic and run away, destabilising the vacuum (dotted line).

where the right hand side is evaluated to zero by explicitly calculating the mass matrix derived from eq. (1.20). This result is valid at tree level whether SUSY is broken or not. Unless some of the sfermions are lighter than their Standard Model counterparts, which observations tell us cannot be the case, the expression does not hold[1]. Besides which, the MSSM lacks a suitable gauge singlet whose F-term can acquire a SUSY breaking VEV and a D-term for the hypercharge symmetry does not produce an acceptable spectrum. Instead, we are led to the situation outlined in figure 1.2, where some hitherto unobserved sector breaks SUSY and this breaking is somehow communicated to the MSSM in the visible sector at one loop or beyond.

Perhaps the obvious choice of mediation sector is gravity. We know gravity exists and that it couples to all sources of energy, so will necessarily facilitate communication between hidden and visible sectors [43–45]. A simple dimensional argument (which is confirmed by a more rigorous approach) predicts soft terms of order the gravitino mass $m_{3/2} \sim F/M_{\rm Pl}$. Taking a Planck scale of $M_{\rm Pl} \sim 10^{19}$ GeV we thus require the SUSY breaking scale to be of order $\sqrt{F} \sim 10^{11}$ GeV in order to generate

[1]Extra states with $\sum_i (-1)^F m_i^2 < 0$ *could* be added to counter this, but to avoid observation they would all need masses above the TeV scale. We would then integrate them out to derive the MSSM as a low energy effective theory, where non-renormalisable terms in the Kähler potential violate the sum rule. These states can thus be considered as the mediation sector.

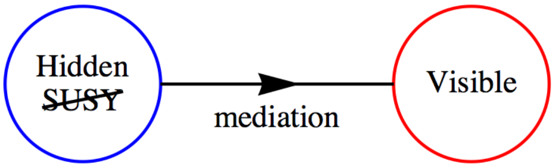


Figure 1.2: The general structure required for SUSY breaking. SUSY is broken in a hitherto unobserved hidden sector then mediated to the MSSM in the visible sector.

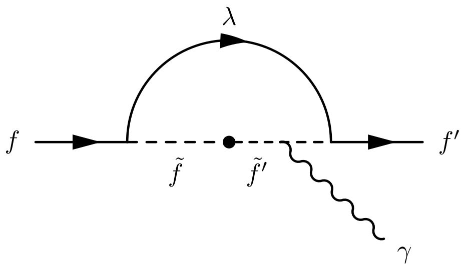


Figure 1.3: Flavour changing interactions like this are common to models of gravity mediation but are very tightly constrained by experiment. The sfermion mass matrix (the blob on the diagram) is off-diagonal, resulting in decays of the form $f \to f'\gamma$ via a sfermion/gaugino loop.

TeV scale soft masses. Not only is this scale very high so experimentally inaccessible, but the reliance on gravitational interactions is technically unappealing while we lack a satisfactory model of quantum gravity. Most troubling, however, are the flavour changing processes associated with gravity mediation. Supergravity corrections to the scalar potential contain soft terms of the form

$$U_{\text{sugra}} \supset e^{K/M_P^2} U(\varphi) \supset \frac{F^2}{M_P^2} y_{ij} \tilde{f}_i^\dagger \tilde{f}_j \tag{1.56}$$

for sfermions $\tilde{f}$. Since these terms are generated at the Planck scale, necessarily above the scale the Standard Model flavour structure arises, there is no immediate reason to believe they should respect it, i.e. the sfermion mass matrix need not be diagonal. Diagrams such as those in figure 1.3 are consequently responsible for flavour changing decays and in most models these are well in excess of tight experimental constraints.

An alternative is to communicate SUSY breaking through Standard Model gauge interactions [46–55]. These have the advantage of (often) being fully calculable and allow for lower scale mediation which is of great phenomenological interest. This is due to the fact that the scale appearing in the soft masses is now some messenger scale that is much smaller than the Planck mass. They also automatically respect the flavour structure of the Standard Model as soft terms are typically generated after the dynamics responsible for flavour structure has frozen out (gauge interactions are flavour blind). The simplest example is that of minimal gauge mediation [55]. SUSY is broken by the F-term of a chiral superfield $X$ (the Goldstino superfield) acquiring a VEV

$$X = X_+ + \theta^2 F \tag{1.57}$$

then transmitted to the visible sector by $N_\varphi$ pairs of *messenger* chiral superfields $\varphi$ and $\tilde{\varphi}$ charged under the Standard Model gauge group (e.g. in the $\mathbf{5} + \overline{\mathbf{5}}$ representation of SU(5)). Communication is via the superpotential interaction

$$W = X\tilde{\varphi}\varphi. \tag{1.58}$$

where the coupling constant has been absorbed into $X$. The F-terms for the messengers are

$$F_\varphi = X\tilde{\varphi} \qquad\qquad F_{\tilde{\varphi}} = X\varphi \tag{1.59}$$

and are set to zero by choosing $\varphi = \tilde{\varphi} = 0$, whereas the precise value of $X_+$ is determined by the details of the SUSY breaking sector. It is straightforward to check that the tree level messenger masses are

$$m_0^2 = X_+^2 \pm F \qquad\qquad m_{1/2}^2 = X_+^2 \tag{1.60}$$

with the requirement $X_+^2 > |F|$ imposed to avoid tachyons in the messenger sector. In fact one usually takes the limit of small SUSY breaking, $X_+^2 \gg |F|$, such that the mass splitting between messenger scalars and fermions is much less than their overall mass. This allows for greater perturbative control which, after all, is one of the main aims of gauge mediation. It is worth pointing out that in many models of SUSY breaking (O'Raifeartaigh models for example) the superpotential contains a term linear in $X$, implying that $X$ has R-charge +2. A scalar VEV $X_+ \neq 0$, which is necessary for a stable vacuum, thus breaks R-symmetry.

Gaugino masses are generated at one loop in this model via the diagram in figure 1.4. Sfermion masses, on the other hand, are generated at two loops and receive contributions from several different diagrams; an example is also given in figure 1.4 (see e.g. ref. [55] for a complete list). The simplest way to actually calculate the soft masses [56] is to look at the terms

$$\mathcal{L}_{\text{soft}} \supset -\frac{1}{2}(m_\lambda \lambda\lambda + \text{h.c.}) - m_{\tilde{f}}^2 \tilde{f}^\dagger \tilde{f} \tag{1.61}$$

and consider their origins. As long as the SUSY breaking is small we can work in a manifestly supersymmetric framework. Owing to the non-renormalisation theorems protecting the superpotential the only way to generate dependence on the scale $X_+$ is through renormalisation of the first two terms in eq. (1.20), the field strength F-term and the matter D-term, described by eqs. (1.30) and (1.32) respectively. Thus gaugino masses originate from

$$\mathcal{L} \supset \frac{1}{16\pi^2} \int \mathrm{d}^2\theta \, \tau(X,t) W^\alpha W_\alpha \tag{1.62}$$

where $t$ is the logarithm of the RG scale. This is an F-term so at one loop $\tau$ must be a holomorphic function of $X$. Its expansion is

$$\tau(X,t) = \tau(t) + X \frac{\partial \tau}{\partial X}(t) + \mathcal{O}(X^2). \tag{1.63}$$

Replacing $X$ with its VEV (1.57), canonically renormalising $\sqrt{\tau(t)} W_\alpha \to W_\alpha$ then extracting the F-term of the Lagrangian yields

$$m_\lambda = -\frac{1}{2} \frac{F}{X_+} \left. \frac{\partial \ln \tau(X,t)}{\partial \ln X} \right|_{X=X_+} . \tag{1.64}$$

Similarly, the sfermion masses arise from

$$\mathcal{L} \supset \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2 \, Z(X^\dagger, X, t) \Phi^\dagger \Phi \tag{1.65}$$

where $\Phi$ is a chiral superfield containing MSSM matter. This is a D-term so $Z$ depends on both $X$ and $X^\dagger$

$$Z(X,X^\dagger,t) = Z(t) + X^\dagger \frac{\partial Z}{\partial X^\dagger}(t) + X \frac{\partial Z}{\partial X}(t) + X^\dagger X \frac{\partial^2 Z}{\partial X^\dagger \partial X}(t) + \mathcal{O}(X^3). \tag{1.66}$$

We take the same approach as before: replace $X$ with its VEV (1.57), canonically renormalise

$$\sqrt{Z(t)} \left( 1 + \theta^2 F \frac{1}{Z(t)} \frac{\partial Z}{\partial X} \right) \Phi \to \Phi \tag{1.67}$$

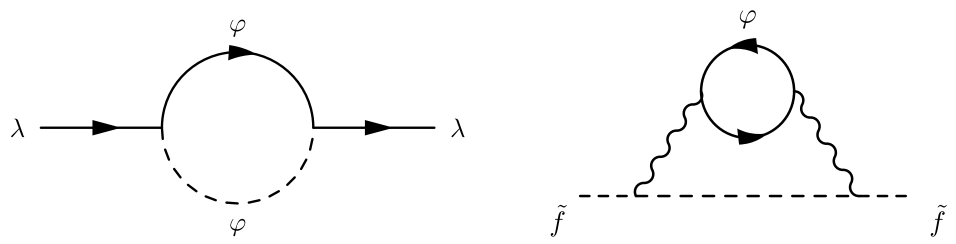


Figure 1.4: Gaugino masses (left) are generated at one loop and go like $g^2F/16\pi^2X_+$. Sfermion masses (e.g. right) occur at two loops and are of a similar order.

then extract the D-term to find

$$m_{\tilde{f}}^2 = -\frac{F^\dagger F}{X_+^\dagger X_+} \frac{\partial^2 \ln Z(X^\dagger, X, t)}{\partial \ln X^\dagger \partial \ln X}\bigg|_{X=X_+}. \tag{1.68}$$

All that remains is to insert the precise forms of $\tau$ and $Z$. Fortunately both are known to one loop order. They are found by integrating the supersymmetric RG equations

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{g^2}\right) = \frac{b(t)}{8\pi^2} \qquad\qquad \frac{\mathrm{d}Z}{\mathrm{d}t} = \frac{Cg^2}{4\pi^2} \tag{1.69}$$

over the messenger mass scale, then using holomorphy and chiral symmetry to relate $g$ to $\tau$ and to replace the messenger mass with $X$ (see e.g. ref. [56] for a more detailed discussion). The coefficient $b$ satisfies $b(t < \ln X) - b(t > \ln X) = N_\varphi$ (and will be discussed shortly), whereas the quadratic Casimir $C$ is given by $C = (N^2-1)/2N$ for fundamentals of SU($N$) and $C = Y^2$ for the U(1) hypercharge factor. For $\tau$ the explicit result is

$$\tau(X,t) = \tau(\Lambda) + \frac{b(t > \ln X)}{32\pi^2}\ln\left(\frac{X}{\Lambda}\right) + \frac{b(t < \ln X)}{32\pi^2}\ln\left(\frac{e^t}{X}\right) \tag{1.70}$$

from which a little more work yields an expression for $Z$.

Substituting into the soft mass formulae and evaluating at $X = X_+$ one finds

$$m_{\lambda_r} = k_r\Lambda_G\left(\frac{g_r}{4\pi}\right)^2 \qquad\qquad m_{\tilde{f}}^2 = 2\sum_{r=1}^{3} C_r(\tilde{f})k_r\left(\frac{g_r}{4\pi}\right)^4\Lambda_S^2. \tag{1.71}$$

In these expressions $r$ denotes the Standard Model gauge group, $k_1 = 5/3$, $k_2 = k_3 = 1$ and the gauge couplings are normalised to unify at the GUT scale. The scales $\Lambda_G$ and $\Lambda_S$ are evaluated in this minimal case to be

$$\Lambda_G = N_\varphi\frac{F}{X_+}\left[1 + \mathcal{O}\left(\frac{F^2}{X_+^4}\right)\right] \qquad\qquad \Lambda_S^2 = N_\varphi\frac{F^2}{X_+^2}\left[1 + \mathcal{O}\left(\frac{F^2}{X_+^4}\right)\right] \tag{1.72}$$

and so the ratio $\Lambda_G^2/\Lambda_S^2$ is just the number of messenger generations $N_\varphi$. In more general models the situation is more complicated and it is simpler to treat $\Lambda_G$ and $\Lambda_S$ as the free parameters of the model as opposed to $F/X_+$ and $N_\varphi$. The ratio $\Lambda_G^2/\Lambda_S^2$ then gives the *effective number of messengers*. All of the other parameters in eq. (1.71) are order one at the messenger scale $X_+$ so it is clear that, if the effective number of messengers is small (much less than one), the gauginos are much lighter than the sfermions.

A non-minimal example of particular interest is that of extra ordinary gauge mediation [57]. Here, the superpotential (1.58) is generalised to include an explicit messenger mass term and off-diagonal couplings to the Goldstino superfield

$$W = \lambda_{ij} X \tilde{\varphi}_i \varphi_j + M_{ij} \tilde{\varphi}_i \varphi_j. \tag{1.73}$$

Messenger masses are now given by the eigenvalues $\bar{M}_i$ of the matrix $\lambda X + M$ so, putting them in ascending order, we can again integrate the RG equations over all mass scales to find

$$\tau(X,t) = \tau(\Lambda) + \frac{b(t > \ln \bar{M}_1)}{32\pi^2} \ln\left(\frac{\bar{M}_1}{\Lambda}\right) + \frac{b(\ln \bar{M}_1 > t > \ln \bar{M}_2)}{32\pi^2} \ln\left(\frac{\bar{M}_2}{\bar{M}_1}\right) + \ldots \tag{1.74}$$

At each scale a single vector like pair of messengers is integrated out so $b$ satisfies

$$b(\ln \bar{M}_i > t > \ln \bar{M}_{i+1}) - b(\ln \bar{M}_{i-1} > t > \ln \bar{M}_i) = 1. \tag{1.75}$$

Summing up all terms, substituting into the soft mass formula then comparing with eq. (1.71) the above formulae for $\Lambda_G$ and $\Lambda_S$ are generalised to read

$$\Lambda_G = F \frac{\partial}{\partial X} \sum \ln(\lambda X + M) \qquad \Lambda_S^2 = \frac{1}{2} F^2 \frac{\partial^2}{\partial X^\dagger \partial X} \sum \left[\ln(\lambda X + M)^2\right]^2 \tag{1.76}$$

to leading order in $F/X_+^2$, where the sum is over the eigenvalues of the messenger mass matrix and similar techniques have been used to evaluate $Z$.

### 1.3.1 Landau poles and the $\mu$-$B_\mu$ problem

An unavoidable consequence of gauge mediation is the addition of matter charged under the Standard Model gauge group. This affects the running of the gauge couplings and can be troublesome. As long as messengers are added in complete

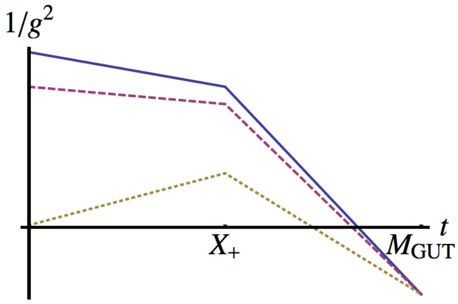


Figure 1.5: Messengers deflect the RG flow of the gauge couplings and can lead to strong coupling before the GUT scale.

Standard Model multiplets the couplings still unify but the RG flow is deflected. At leading order, the gauge coupling $\beta$-function for an SU($N$) gauge theory is given by [58]

$$\beta_{1/g^2} = \frac{b}{8\pi^2} = \frac{3N - \frac{1}{2}\sum_i n_i}{8\pi^2} \tag{1.77}$$

where $n_i$ is the Dynkin index of matter representation $i$ (the exact one loop expression contains additional contributions from the anomalous dimensions of the matter fields). These indices are typically positive so adding messengers makes the $\beta$-function more negative. As such they may push the theory towards strong coupling (i.e. it hits a *Landau pole*) before reaching the GUT scale, rendering any perturbative GUT useless. The situation is illustrated in figure 1.5. As the RG scale reaches the messenger scale $X_+$ the messengers deflect the RG flow. If deflected enough the coupling $1/g^2$ dips below zero, therefore $g$ itself diverges, before the GUT scale.

To avoid this problem one can either ensure the number of messengers is small or the messenger mass is large. Unfortunately this is not always so simple. The structure of the SUSY breaking sector often constrains the number of messengers (particularly in the models of direct mediation we will see later) and heavy messengers can lead to cosmological problems. In the case that the lightest messenger is stable, it's relic abundance is expected to overclose the universe if heavier than a few TeV [59]. Alternatively the Landau pole may not be a problem at all. One could view it as a sign that our proposed GUT is actually an IR free dual description of some asymptotically free UV theory [60, 61]. Gauge coupling unification is expected to survive across such a duality [62].

The second main problem with models of gauge mediation is the $\mu$-$B_\mu$ problem. For successful electroweak symmetry breaking the MSSM scalar potential must contain the two Higgs mass terms

$$U_{\text{MSSM}} \supset |\mu|^2(|h_u|^2 + |h_d|^2) + B_\mu(h_u^\dagger h_d + h_d^\dagger h_u) \tag{1.78}$$

alongside the diagonal soft mass terms $m_u^2|h_u|^2$ (often negative) and $m_d^2|h_d|^2$. Otherwise the potential is not bounded from below ($\mu = 0$) or has a minimum at $h_d = 0$ so does not result in down quark masses ($B_\mu = 0$). The $\mu$-term comes from a supersymmetric mass term $W \supset \mu\tilde{H}H$ whereas the $B_\mu$-term is a soft SUSY breaking mass. Both terms should be of the same order: the electroweak symmetry breaking scale. The natural scale for $\mu$ is high, the GUT or Planck scale, so we would like a mechanism to explain why it is much lower. Perhaps the most realistic way of achieving this in gauge mediation is through radiative corrections. For example, take a model where the $\mu$-term is forbidden by some symmetry in the limit of exact SUSY, but suppose there are couplings

$$W \supset (\lambda_1\tilde{\varphi}_1\varphi_1 + \lambda_2\tilde{\varphi}_2\varphi_2)X + \lambda H\varphi_1\varphi_2 + \tilde{\lambda}\tilde{H}\tilde{\varphi}_1\tilde{\varphi}_2 \tag{1.79}$$

in the messenger sector [63]. At one loop these generate D-terms (see figure 1.6) that, in turn, generate the desired $\mu$ and $B_\mu$-terms when $X$ gets its SUSY breaking VEV:

$$\begin{aligned} \frac{1}{X_+}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2\, \tilde{H}HX^\dagger \quad &\longrightarrow \quad \frac{\tilde{\lambda}\lambda}{16\pi^2} f\left(\frac{\lambda_1}{\lambda_2}\right)\frac{F}{X_+}\int \mathrm{d}\theta^2\, \tilde{H}H \\ \frac{1}{X_+^2}\int \mathrm{d}\theta^2\mathrm{d}\bar{\theta}^2\, \tilde{H}HX^\dagger X \quad &\longrightarrow \quad \frac{\tilde{\lambda}\lambda}{16\pi^2} f\left(\frac{\lambda_1}{\lambda_2}\right)\frac{F^2}{X_+^2}\tilde{h}h. \end{aligned} \tag{1.80}$$

The function $f$ is known but we can read off the ratios of the scales regardless. Denoting the loop factor by $l$ one finds $\mu^2 = l^2F^2/X_+^2$ and $B_\mu = lF^2/X_+^2$. Hence the ratio is given by $\mu^2/B_\mu = l \sim 10^{-2}$. This leads to a conundrum. Either $B_\mu$ lies at the weak scale and $\mu$ is an order of magnitude too small, or $\mu$ lies at the weak scale and $B_\mu$ is an order of magnitude too large. The problem originates from both terms, which are of different dimensionality, being generated at the same loop order.

An attractive solution is to consider some of the Higgses as pseudo-Goldstone bosons of a spontaneously broken global symmetry [63]. If at least one of the Higgses

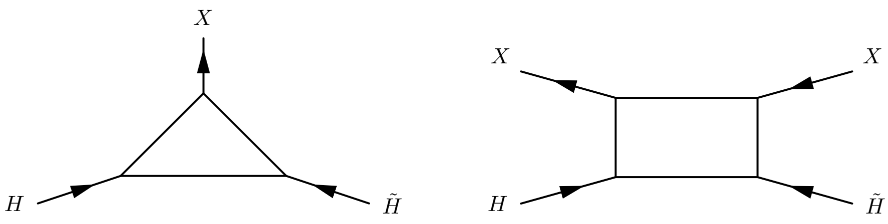


Figure 1.6: Diagrams responsible for radiatively generating the $\mu$ and $B_\mu$-terms in a typical model of gauge mediation. Messengers propagate around the loop. The appropriate terms are generated when $X$ gets its SUSY breaking VEV.

is massless the determinant of their mass matrix must vanish, i.e. $\mu^2 = B_\mu$. Gauge and Yukawa couplings break any such symmetry but only contribute to the Higgs masses at one loop, or two loops in the case of the neutral combination $H + \tilde{H}^\dagger$. We thus find $\mu^2 = B_\mu$ at the one loop level and retain $\mu^2 \sim B_\mu$ at two loops. An explicit example would be to consider the superpotential

$$W \supset S(\tilde{H}H + \tilde{N}N - M_N^2) \tag{1.81}$$

for singlets $S$, $N$ and $\tilde{N}$. Triplets $(H, N)$ and $(\tilde{H}, \tilde{N})$ transform under a global U(3) which is spontaneously broken to U(2) in the vacuum $\tilde{N}N = M_N^2$. The Higgs doublets are the associated Goldstone bosons and one indeed finds $\mu^2 \sim B_\mu$.

An alternative description of this mechanism is to consider the $\mu$-term as arising from the higher derivative operator

$$\int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2 \tilde{H}HD^2(X^\dagger X) \tag{1.82}$$

where $D$ is the supersymmetric covariant derivative. The quantity $D^2(X^\dagger X)$ is a right handed chiral superfield so this operator does not generate a corresponding $B_\mu$-term. Instead $B_\mu$ is generated by higher order operators such as

$$\int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2 \tilde{H}HX^\dagger X\bar{D}^2 D^2(X^\dagger X) \tag{1.83}$$

so picks up the required extra loop factor. Numerous other solutions to the $\mu$-$B_\mu$ problem have been investigated in the literature [64–72]. These utilise a variety of techniques including constraining couplings via discrete symmetries, strong coupling effects in the hidden/messenger sector, direct couplings of the Higgs fields to the messenger sector and even using the Higgses themselves as messengers.

### 1.3.2 Dark matter

No discussion of SUSY would be complete without a section on dark matter. We will not dwell on this subject for long here but will nonetheless mention some of the phenomenological consequences of gauge mediation. In any model of SUSY breaking the gravitino mass goes like $m_{3/2} \sim F/M_{\text{Pl}}$. Since $F$ is much lower in models of gauge mediation than in their gravity mediation counterparts the gravitino is much lighter. Indeed, it is necessarily the lightest supersymmetric particle in the MSSM if the gravity mediated soft masses (proportional to $m_{3/2}$) are to be subdominant, thereby avoiding the problematic flavour changing decays of figure 1.3. For a SUSY breaking scale above about 100 TeV one can expect anything from a few eV upwards. With a mass of a few keV, the gravitino serves as a good warm dark matter candidate [73].

A second option comes from the messenger sector. The superpotential (1.58) and its many generalisations support a global U(1) messenger number symmetry. Hence the lightest messenger is stable. As long as it is not charged under the Standard Model gauge group it could fulfill the role of cold dark matter. However, unless this messenger is lighter than a few TeV it is generically expected to overclose the universe [59]. In fact we will see later that the existence of an exact messenger number symmetry is anyway problematic in models of metastable SUSY breaking, as it tends to result in a rapid phase transition to the supersymmetric vacuum mediated by non-topological solitons.

A related proposal [3] which we will discuss in detail later is to use non-topological solitons themselves as dark matter. In fact, the presence of both a flat direction and an R-symmetry in the SUSY breaking sector lend themselves well to this idea. We will see that R-balls, non-topological solitons associated with this R-symmetry, can be a good candidate in gauge mediated SUSY breaking. Even if not dark matter, they can lead to a variety of interesting cosmological consequences.

## 1.4 Metastability

Mediation mechanism aside, section 1.2 laid out some concrete constraints a model must satisfy if it is to globally break SUSY. Of particular importance are those

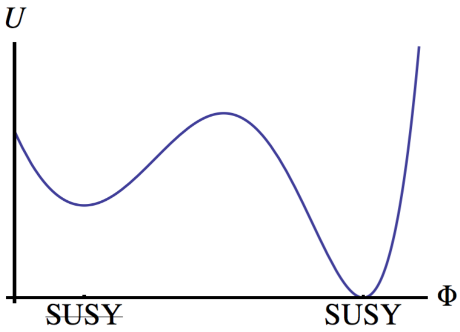


Figure 1.7: Metastable SUSY breaking. Although a locally stable SUSY breaking vacuum exists, one allows the existence of other, supersymmetric vacua elsewhere in field space.

imposed by the Witten index and R-symmetry. To recap, we found that any supersymmetric gauge theory that spontaneously and globally breaks SUSY must contain either chiral or massless matter for a vanishing Witten index. Furthermore, if our model is to be generic and calculable it must possess an anomaly free R-symmetry. We could choose to abandon the notion of dynamical SUSY breaking, making it easier to avoid the Witten index constraint, but would then have to explain somehow the large hierarchy between the Planck and SUSY breaking scales. We could also write down a non-generic model in order to avoid the R-symmetry constraint, but would then have to come up with a reason for omitting certain couplings consistent with the symmetries of the model. It thus seems that for a truly satisfactory model of SUSY breaking we have our work cut out. Examples of models satisfying these constraints are known (see e.g. ref. [55]) but they are complicated, even before an attempt is made to implement them in a phenomenologically viable way. Even then one may still need a technique to deal with the cosmologically troublesome R-axion.

However, all violating these constraints really tells us is that there exists a supersymmetric vacuum somewhere in field space. It does not preclude the possibility of there being another, metastable, SUSY breaking vacuum elsewhere. For example, suppose we take a model with superpotential

$$W = W_{\text{R-sym}} + \varepsilon W_{\text{R-breaking}} \tag{1.84}$$

for $\varepsilon \ll 1$. Thanks to the second term the R-axion can be given a suitable mass, yet

R-symmetry remains an approximate symmetry of the model. For $\varepsilon = 0$ we assume there is no supersymmetric vacuum. For $\varepsilon \neq 0$ we break R-symmetry so expect to introduce one. The supersymmetric vacuum must be brought in from infinity, such that the vacuum structure is continuous as $\varepsilon \to 0$, hence the two vacua are separated by a large distance in field space. Due to the huge tunnelling action involved in a transition to the supersymmetric vacuum, the SUSY breaking vacuum can be extremely stable. The situation is summarised in figure 1.7. Now that we have allowed the appearance of a supersymmetric vacuum there is no particular need for the Witten index to vanish either. In the landmark paper by Intriligator, Seiberg and Shih (ISS) [74] many of these ideas were demonstrated in the surprisingly simple model of massive SQCD. As such, we will use their model as an illustrative example of metastable SUSY breaking. However, we must first discuss one of the remarkable properties of SQCD: *Seiberg duality*.

### 1.4.1 Seiberg duality

SQCD is a supersymmetric gauge theory with gauge group $\mathrm{SU}(\bar{N})$ and $N_f$ flavours of quark chiral superfield $Q$ and $\tilde{Q}$. The anomaly free global symmetry group of the theory is $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f) \times \mathrm{U}(1) \times \mathrm{U}(1)_R$. The quarks transform under this group as shown in table 1.1. Depending on the number of colours and flavours the theory has several different phases [75]. For $N_f > 3\bar{N}$ it is IR free, when $3\bar{N}/2 < N_f < 3\bar{N}$ it flows to a non-trivial conformal fixed point and when $\bar{N} + 1 \leq N_f < 3N/2$ it is asymptotically free. Seiberg postulated [76, 77] that, in each of these windows, the long distance behaviour can equivalently be modelled by a dual theory. Typically one refers to the original theory as the *electric theory* and its dual as the *magnetic theory*. The magnetic theory is of a very similar form: SQCD with $N = N_f - \bar{N}$ colours, $N_f$ flavours of quark $q$ and $\tilde{q}$ and an extra chiral superfield $\Phi$ transforming as a meson. It necessarily has the same global symmetry group but now also possesses a superpotential

$$W_{\mathrm{mg}} = h\tilde{q}\Phi q \tag{1.85}$$

for some perturbative coupling constant $h$. It is summarised in table 1.1.

Though the duality is yet to be proven, there is a good deal of evidence suggesting

| | SU($\bar{N}$) | SU($N_f$) | SU($N_f$) | U(1)$_B$ | U(1)$_R$ |
|---|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $\mathbf{1}$ | $+\frac{1}{\bar{N}}$ | $1-\frac{\bar{N}}{N_f}$ |
| $\tilde{Q}$ | $\tilde{\square}$ | $\mathbf{1}$ | $\square$ | $-\frac{1}{\bar{N}}$ | $1-\frac{\bar{N}}{N_f}$ |

| | SU($N$) | SU($N_f$) | SU($N_f$) | U(1)$_B$ | U(1)$_R$ |
|---|---|---|---|---|---|
| $q$ | $\square$ | $\tilde{\square}$ | $\mathbf{1}$ | $+\frac{1}{N}$ | $1-\frac{N}{N_f}$ |
| $\tilde{q}$ | $\tilde{\square}$ | $\mathbf{1}$ | $\tilde{\square}$ | $-\frac{1}{N}$ | $1-\frac{N}{N_f}$ |
| $\Phi$ | $\mathbf{1}$ | $\square$ | $\square$ | $0$ | $2\frac{N}{N_f}$ |

Table 1.1: The matter content of the electric theory (top) and the magnetic theory (bottom): $N = N_f - \bar{N}$.

it is true. We shall take some time to review the three main pieces here, all of which arise from RG invariant properties of the theory. First of all the moduli of the theories should match. Since the electric theory has no superpotential its moduli are simply the gauge invariant (and therefore D-flat) directions

$$M = \tilde{Q}Q \qquad\qquad B = \epsilon^{(\bar{N})}Q^{\bar{N}} \qquad\qquad \tilde{B} = \epsilon_{(\bar{N})}\tilde{Q}^{\bar{N}} \tag{1.86}$$

i.e. the mesons and baryons respectively. Contractions are taken over colour indices in these expressions, with $\epsilon^{(\bar{N})}$ denoting the rank $\bar{N}$ alternating tensor, but the flavour indices are left free. It is clear that the mapping

$$M \longrightarrow \Lambda\Phi \qquad B \longrightarrow \Lambda^{N_f-2N}\epsilon_{(N_f)}\epsilon^{(N)}q^N \qquad \tilde{B} \longrightarrow \Lambda^{N_f-2N}\epsilon_{(N_f)}\epsilon_{(N)}\tilde{q}^N \tag{1.87}$$

between electric and magnetic theories, for some scale $\Lambda$, respects all global symmetries and preserves the classical dimensions of the operators. One may wonder what happens to $\tilde{q}q$ in the magnetic theory, as it would appear to be another modulus. However, the $\Phi$ F-terms from the magnetic superpotential set this operator to zero so it not an independent degree of freedom after all.

A second check arises from considering deformations of the theory. Suppose we add a mass term for one flavour of quark to the electric theory

$$W_{\rm el} = \mu\tilde{Q}_1Q_1. \tag{1.88}$$

This removes a flavour from the low energy theory, i.e. $\mathrm{SU}(N_f) \to \mathrm{SU}(N_f-1)$. If we ran down to the IR then moved to the dual theory, we would therefore find that the number of colours and flavours had both decreased by one. Conversely, eq. (1.87) tell us that a mass term in the electric theory corresponds to deforming the magnetic superpotential to

$$W_{\mathrm{mg}} = h\tilde{q}\Phi q + h\mu\Lambda\Phi_{11}. \tag{1.89}$$

The $\Phi_{11}$ F-term is thus modified to

$$\tilde{q}^1 q^1 + \mu\Lambda = 0 \tag{1.90}$$

resulting in $q^1$ and $\tilde{q}^1$ picking up VEVs and Higgsing the gauge group down to $\mathrm{SU}(N-1)$. In addition these quarks and the meson components $\Phi_{i1}$ and $\Phi_{1i}$ get tree level masses so are integrated out in the IR. We are left with an equivalent theory with one less colour and one less flavour, precisely as we expect.

The third check on the duality we will discuss is that of ‘t Hooft anomaly matching [78]. Here one considers what would happen if the global symmetry group was to be weakly gauged with coupling $g'$. Most of the global symmetries would not remain symmetries of the quantum theory as they become anomalous at one loop. However, one could add some spectator fields charged only under the global symmetry group to exactly cancel these anomalies. Subsequently taking the limit $g' \to 0$ decouples the gauge bosons and spectator fields but leaves the anomalies unchanged. Suppose we now calculate the anomaly in the electric theory to be $A_{\mathrm{el}}$. Then the spectator fields contribute $-A_{\mathrm{el}}$. Moving to the magnetic description does not affect the spectator fields so the anomaly becomes $A_{\mathrm{mg}} - A_{\mathrm{el}}$. If the gauged global symmetry is a consistent symmetry of the electric description it must remain so in the magnetic description so we require $A_{\mathrm{mg}} - A_{\mathrm{el}} = 0$. In other words the anomalies must be the same in both theories. Due to the large number of mixed anomalies derived from table 1.1 this test is highly non-trivial.

An important observation about the nature of the duality can be made by looking at the one loop $\beta$-function of the gauge coupling (1.77), which takes the specific form

$$\beta_{1/g^2} = \frac{3N - N_f}{8\pi^2} \tag{1.91}$$

for SQCD with $N$ colours and $N_f$ flavours. Adding a mass term to the electric theory only reduces the number of flavours by one so increases the $\beta$-function. This means the theory becomes more strongly coupled in the IR. On the other hand, both the number of colours and the number of flavours are decreased by one in the magnetic theory so its $\beta$-function is decreased and the theory becomes more weakly coupled in the IR. Seiberg duality is therefore a strong/weak duality, which is of great use in phenomenological applications.

### 1.4.2 Extensions of Seiberg duality

As well as SU gauge groups, Seiberg's original work [76, 77] found similar results for both SO and Sp groups with matter in the fundamental representation. Since then there has been a great deal of activity in extending the idea of $\mathcal{N} = 1$ gauge-gauge duality to include theories with matter in representations other than the fundamental [4, 79–87]. Unlike the previous case, many of these dualities rely on a specific form for the superpotential. A simple way to see why this is necessary is as follows.

Consider adding an SU($N$) adjoint $X$ to the electric theory in table 1.1. It is now possible to form a new meson by sandwiching this new chiral superfield between two quarks, i.e.

$$M_i = \tilde{Q} X^i Q. \tag{1.92}$$

Clearly we can add as many powers of the adjoint as we wish and generate arbitrarily many new mesons. Since the rule in Seiberg duality is typically to include an elementary superfield in the magnetic theory for each meson of the electric theory, this would lead to infinitely many degrees of freedom and a nonsensical dual description. The solution is to *truncate the chiral ring*; in other words we try to add a superpotential so as to cutoff this tower of mesons. For the case at hand a good choice [79, 80, 83] is

$$W_{\text{el}} = X^{k+1} \tag{1.93}$$

whereupon the equations of motion fix $X^k = 0$ and thus leave $M_{k-1}$ as the highest dimension meson. The dual description then turns out to be an SU($kN_f - N$) gauge theory, also with an adjoint $x$ and a superpotential of the form

$$W_{\text{mg}} = x^{k+1} + \sum_{i=0}^{k-1} x^{k-1-i}\tilde{q}\Phi_i q. \tag{1.94}$$

In ref. [4] we built on this idea to realise Seiberg duality in theories closer to those we (perhaps) observe in nature, namely SU($N$) gauge groups with more than one generation of antisymmetric tensor. Truncating all directions in the chiral ring for this kind of theory is difficult and success entailed the introduction of a gauge singlet superfield with appropriate superpotential couplings. However, one is then able to find dual descriptions of SU($N$) gauge theories with an arbitrary number of adjoints, antisymmetrics or symmetrics using the same techniques.

### 1.4.3 The ISS model

Bringing together many of the above results we can now construct a strikingly simple model of dynamical SUSY breaking. The ISS model [74] of metastable SUSY breaking consists of massive SQCD in the free magnetic phase, i.e. the number of colours and flavours is chosen to satisfy $\bar{N}+1 \leq N_f < 3\bar{N}/2$ such that the electric theory is asymptotically free. The superpotential is

$$W_{\text{UV}} = -h\mu\tilde{Q}Q \tag{1.95}$$

for a perturbative coupling constant $h$ and mass matrix $\mu$. Sums over the flavour and colour indices are understood and we will assume the simplest case where the mass matrix has the form $\mu\mathbb{1}_{N_f}$ and both coupling constants are real and positive. The remaining global symmetry group is

$$\text{SU}(N_f) \times \text{U}(1)_B \times \text{U}(1)_{R'} \tag{1.96}$$

with the matter content transforming as in table 1.2. Note that the superpotential is generic but the R-symmetry is anomalous so, by the Nelson-Seiberg theorem [34] we expect the theory to have a supersymmetric vacuum. In fact, the Witten index of this theory is just that of SU($\bar{N}$) super Yang-Mills theory so it should have at least $\bar{N}$ supersymmetric vacua.

| | $SU(\bar{N})$ | $SU(N_f)$ | $U(1)_B$ | $U(1)_{R'}$ |
|---|---|---|---|---|
| $Q$ | $\square$ | $\square$ | $+\frac{1}{\bar{N}}$ | 1 |
| $\tilde{Q}$ | $\tilde{\square}$ | $\tilde{\square}$ | $-\frac{1}{\bar{N}}$ | 1 |

| | $SU(N)$ | $SU(N_f)$ | $U(1)_B$ | $U(1)_{R'}$ |
|---|---|---|---|---|
| $q$ | $\square$ | $\tilde{\square}$ | $+\frac{1}{N}$ | 0 |
| $\tilde{q}$ | $\tilde{\square}$ | $\square$ | $-\frac{1}{N}$ | 0 |
| $\Phi$ | $\mathbf{1}$ | $(\square, \tilde{\square})$ | 0 | 2 |

Table 1.2: The matter content of the ISS model's electric theory (top) and its magnetic theory (bottom).

Since the electric theory is asymptotically free it hits a Landau pole at some scale $\Lambda$ as it flows down to the IR. Fortunately (as we have just shown) Seiberg duality enables us to use an alternative description of the low energy physics here that is weakly coupled. This theory has $N = N_f - \bar{N}$ colours implying that $N_f > 3N$ and the dual theory is IR free. It is therefore under complete control at low energy and can be applied to investigate the vacuum structure of the model. The global symmetry group remains the same with matter content as given in table 1.2 and tree level superpotential

$$W_{\text{mg}} = h\tilde{q}\Phi q - hm^2 \,\text{Tr}\,[\Phi] \tag{1.97}$$

where we have defined the parameter $m^2 = \mu\Lambda$. Of course, the dual description only makes sense if $\mu \ll \Lambda$ (and subsequently $m^2 \ll \Lambda^2$). Otherwise one should integrate out the electric quarks before moving to the magnetic description and the $\text{Tr}\,[\Phi]$ term is not generated.

Now consider the $\Phi$ F-terms arising from the magnetic superpotential

$$F_\Phi = h\tilde{q}q - hm^2 \mathbb{1}_{N_f}. \tag{1.98}$$

Both terms are $N_f \times N_f$ matrices. The first comes from the contraction over colour indices of the quarks (each an $N \times N_f$ matrix) so is at most rank $N$, whereas the second is the rank $N_f$ identity matrix. $N_f > N$, indeed $N_f > 3N$, so there is no way all F-terms can be set to zero. The best we can do is set $N$ of them to zero leaving $N_f - N$ of them to break SUSY. Setting the quark F-terms to zero and retaining as much of the global symmetry as possible, one finds a candidate SUSY breaking vacuum $|\text{vac}\rangle_{\text{ISS}}$ at

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & X \end{pmatrix} \qquad q = \begin{pmatrix} m\mathbb{1}_N \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} m\mathbb{1}_N \\ 0 \end{pmatrix} \tag{1.99}$$

for $X$ an undetermined $(N_f - N) \times (N_f - N)$ matrix, i.e. a pseudo-modulus. In fact we should really include a phase on the quark VEVs providing another pseudo-modulus, which will emerge shortly. The vacuum energy is $U_{\text{ISS}} = (N_f - N)h^2 m^4$ and the symmetry group is broken down to

$$\text{SU}(N)_D \times \text{SU}(N_f - N) \times \text{U}(1)_{B'} \times \text{U}(1)_{R'}. \tag{1.100}$$

The $\text{SU}(N)_D$ is a diagonal combination of the flavour symmetry and the gauge group, which is completely Higgsed.

Before concluding anything we must check whether the pseudo-moduli are properly stabilised. To do so one expands around the vacuum using degrees of freedom

$$\Phi = \begin{pmatrix} Y & \tilde{Z} \\ Z & X \end{pmatrix} \qquad q = \begin{pmatrix} m\mathbb{1}_N + \chi \\ \rho \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} m\mathbb{1}_N + \tilde{\chi} \\ \tilde{\rho} \end{pmatrix} \tag{1.101}$$

where $Y$, $\chi$, $\tilde{\chi}$ are $N \times N$ matrices and $Z$, $\rho$, $\tilde{\rho}$ are $(N_f - N) \times N$ matrices. Substituting back into the superpotential (1.97) all components get tree level masses of order $hm$ other than

$$\text{Re}[\rho + \tilde{\rho}] \qquad \text{Im}[\rho - \tilde{\rho}] \qquad \text{Im}[\chi - \tilde{\chi}] \tag{1.102}$$

which are Goldstone bosons of the symmetry breaking and

$$\text{Re}[\chi - \tilde{\chi}] \qquad X \tag{1.103}$$

which contain the pseudo-moduli. Some of these modes, $\text{Im}[\chi - \tilde{\chi}]$ and the traceless part of $\text{Re}[\chi - \tilde{\chi}]$, are eaten by the super-Higgs mechanism and acquire masses of order $gm$ with the gauge bosons. This still leaves two pseudo-moduli, $\text{Tr}\,\text{Re}[\chi - \tilde{\chi}]$

and $X$, classically massless. Calculating the precise spectrum as function of the pseudo-moduli is straightforward in principle but fairly involved in practise. Full details can be found in ref. [74] and we will see an explicit example of this type of calculation later. Once completed, one substitutes the expressions for the scalar and fermion masses into the expression (1.52) for the one loop Coleman-Weinberg potential to find

$$U_{\rm CW} = \frac{h^4 m^2 (\ln 4 - 1)}{8\pi^2} \left( \frac{N_f - N}{2} \mathrm{Tr}\,\mathrm{Re}[\chi - \tilde{\chi}]^2 + N\,\mathrm{Tr}\left[X^\dagger X\right] \right) + \ldots . \qquad (1.104)$$

where the dots denote terms of higher order in the pseudo-moduli. Both pseudo-moduli thus get loop suppressed masses of order $h^2 m$. Effects from the strongly coupled electric theory manifest as terms in the Kähler potential (1.54) suppressed by the strong coupling scale $\Lambda$, which also serves as the cutoff scale for the IR free magnetic theory. Since $m \ll h\Lambda$ this incalculable contribution is subdominant and we conclude that the vacuum $|\mathrm{vac}\rangle_{\rm ISS}$ is stabilised at $\chi = \tilde{\chi} = X = 0$.

Recalling that the Witten index for the electric theory was $\bar{N}$, and that this quantity is RG invariant, one may wonder what has happen to the $\bar{N}$ supersymmetric vacua we were expecting. These appear when we consider the dynamical superpotential of SQCD, which is generated by a gaugino condensation. Moving along the direction $\Phi = |\Phi| \mathbb{1}_{N_f}$ the magnetic quarks all get a mass $h|\Phi|$. Below this scale they are integrated out. Matching the holomorphic gauge coupling at $h|\Phi|$ we thus find

$$e^{-2\tau} = \left( \frac{\Lambda'}{h|\Phi|} \right)^{3N} = \left( \frac{\Lambda}{h|\Phi|} \right)^{3N - N_f} \qquad (1.105)$$

where $\Lambda'$ is the effective scale of the low energy super Yang-Mills theory. Gaugino condensation contributes a term $N\Lambda'^3$ (confirmed by an explicit instanton calculation in ref. [88]) yielding the full magnetic superpotential

$$W_{\rm mg} = h\tilde{q}\Phi q - hm^2\,\mathrm{Tr}\,[\Phi] + N \left( h^{N_f} \Lambda^{3N - N_f} \det \Phi \right)^{1/N} . \qquad (1.106)$$

Taking the determinant of $\Phi$ generalises to cases where its VEV is not diagonal. Note that this superpotential does not respect the anomalous R-symmetry of table 1.2, hence we expect it to restore SUSY. Indeed, the extra term allows all F-terms to be solved simultaneously leading to the supersymmetric vacuum $|\mathrm{vac}\rangle_0$ defined by

$$q = \tilde{q} = 0 \qquad\qquad \Phi = \frac{1}{h}\Lambda\left(\frac{m}{\Lambda}\right)^{2N/(N_f-N)}\mathbb{1}_{N_f}. \tag{1.107}$$

As $\Phi$ is a complex scalar, this provides $N_f - N = \bar{N}$ different roots in agreement with the Witten index of SQCD.

Of upmost importance is the lifetime of the metastable vacuum. Even though it is locally stable one can still tunnel to the supersymmetric vacuum. The tunnelling rate is proportional to $e^{-S}$ where $S$ is the Euclidean action of the classical bounce solution. The height of the potential barrier goes like $N_f h^2 m^4$ from eq. (1.98), whereas the difference in energy between the two vacua is of the same order at $(N_f - N)h^2m^4$. This means the thin wall limit [89] is not applicable for estimating the bounce action and one should instead use the triangle barrier estimate of ref. [90]. We find

$$S \sim \frac{(\Delta\Phi)^4}{U_{\rm ISS}} \sim \frac{1}{h^6}\left(\frac{\Lambda}{m}\right)^{4(N_f-3N)/(N_f-N)} \gg 1 \tag{1.108}$$

using eq. 1.107 to evaluate $\Delta\Phi$. Since $N_f > 3N$ such that the magnetic theory is IR free, $m \ll \Lambda$ so the IR description is valid and the pseudo-moduli calculable, and $h \lesssim 1$ due to it being a perturbative coupling constant, the bounce action can be made arbitrarily large. It thus seems as though the metastable vacuum can be made arbitrarily stable. However, we we see later on that this is not necessarily the case. When one considers the effects of non-topological solitons associated with the global symmetries of the vacuum it transpires that this vacuum can be destabilised over a significant region of the allowed parameter space [2].

### 1.4.4 Gauge mediation in the ISS model

Postponing the discussion of vacuum stability for now, we can investigate how the ISS model can be implemented in a model of gauge mediation. The most obvious choice [91] is to add in some messengers and couple them to the SUSY breaking meson superfield $\Phi$:

$$W_{\rm mg} = h\tilde{q}\Phi q - hm^2\,{\rm Tr}\,[\Phi] + \lambda\,{\rm Tr}\,[\Phi]\,\tilde{\varphi}\varphi + M\tilde{\varphi}\varphi. \tag{1.109}$$

Note that the explicit messenger mass $M$ is necessary to avoid tachyons (1.60) in the metastable vacuum, where the meson VEV is zero. It is immediately apparent that this mass term breaks the R-symmetry of the model. Consequently we find a tree level supersymmetric vacuum at

$$q = \tilde{q} = 0 \qquad \mathrm{Tr}\,[\Phi] = -\frac{M}{\lambda} \qquad \tilde{\varphi}\varphi = \frac{hm^2}{\lambda}. \tag{1.110}$$

If $\lambda \ll 1$ this new vacuum is very far away from the metastable SUSY breaking vacuum and does not appear to compromise its stability (we will see shortly that is actually does, again due to non-topological solitons) and the model remains viable. Indeed, ref. [91] finds large range of acceptable parameter space for model building. One of the key observations is an explanation for the smallness of $\lambda$ that does not require an appeal to fine tuning. Tracing the origin of the $\lambda\Phi\tilde{\varphi}\varphi$ back to the electric theory, which acts as a UV completion via eq. (1.87), we find

$$W_{\mathrm{el}} \supset \frac{\bar{\lambda}}{M_{\mathrm{Pl}}}\tilde{Q}Q\tilde{\varphi}\varphi \tag{1.111}$$

i.e. the term picks up a suppression from some high scale (e.g. the Planck scale) due to it having classical dimension 4. Hence we expect $\lambda \sim \bar{\lambda}\Lambda/M_{\mathrm{Pl}}$ which is generally small even for $\bar{\lambda}$ of order one.

A more elegant approach is that of *direct mediation* [1, 62, 65, 92–107]. The magnetic theory in the ISS model already has a large flavour group so it is tempting to gauge it and associate it with the Standard Model gauge group. In the minimal example one would choose an electric theory with $\bar{N} = 5$ and $N_f = 7$. The magnetic theory then has $N = 2$ colours and a flavour symmetry SU(7) that is broken to SU(5) in the metastable vacuum, ergo easily mapped to the Standard Model via an SU(5) GUT. This approach eliminates the need for an independent messenger sector, but comes with another problem. The SUSY breaking vacuum of the ISS model retains an anomalous R-symmetry, forbidding perturbative, Majorana gaugino masses. Such models therefore turn out to have anomalously light gauginos. We will now see how this fact is actually deeply connected to the vacuum structure of the theory and not just its R-symmetries.

### 1.4.5 Tree level metastability

In section 1.2.3 we reviewed an important result of refs. [39, 40]. Namely that, in a locally stable vacuum of any Wess-Zumino model, the scalar partner of a massless fermionic direction is itself massless. The most general superpotential for this type of model can be written

$$W = FX + (\lambda_{ij}X + M_{ij})\varphi_i\varphi_j + \kappa_{ijk}\varphi_i\varphi_j\varphi_k. \tag{1.112}$$

where $X$ is the Goldstino superfield whose F-term gets a SUSY breaking VEV. All other superfields are normalised so they have vanishing VEVs. The components of the scalar mass matrix (1.47) are easily read off to be

$$\mathcal{M}_{1/2} = \lambda X + M \qquad\qquad \mathcal{F} = \lambda F. \tag{1.113}$$

We can always perform a unitary rotation on the superfields to put the two superpotential terms into the form

$$\lambda = \begin{pmatrix} \bar{\lambda} & 0 \\ 0 & 0 \end{pmatrix} \qquad\qquad M = \begin{pmatrix} \bar{M} & 0 \\ 0 & 0 \end{pmatrix} \tag{1.114}$$

such that $\det(\bar{\lambda}X + \bar{M})$ is non-zero for generic $X$. In other words we only consider the superfields that do not contain massless fermions everywhere in the pseudo-moduli space. Now consider the quantity

$$\det(\bar{\lambda}X + \bar{M}) = \sum_r c_r(\bar{\lambda}, \bar{M})X^r \tag{1.115}$$

which is a polynomial in $X$. Accordingly it is either a constant or has at least one root somewhere in the complex plane. Around a root $X = X_0$ we can find a direction $v$ such that

$$(\bar{\lambda}X_0 + \bar{M})v = 0 \tag{1.116}$$

in which case the fermionic component of the corresponding superfield is massless. We know the scalar partner of $v$ is also massless, so $\mathcal{F}v = \bar{\lambda}v = 0$ from eqs. (1.47) and (1.113), and consequently $\bar{M}v = 0$ too. This contradicts our assertion that $\det(\bar{\lambda}X + \bar{M})$ is non-zero for generic $X$. It therefore cannot be that $\det(\bar{\lambda}X + \bar{M})$ has any roots, i.e. it is a constant

$$\det(\bar{\lambda}X + \bar{M}) = \det \bar{M}. \tag{1.117}$$

This result has some immediate phenomenological repercussions. On inspection, we can see that renormalisable models of gauge mediation fall into this category if some of the $\varphi$'s are associated with messenger superfields. Furthermore eqs. (1.71) and (1.76) tell us the gaugino masses are given by

$$m_{\lambda_r} = k_r \left(\frac{g_r}{4\pi}\right)^2 F \frac{\partial}{\partial X} \ln \det (\lambda X + M). \quad (1.118)$$

We have just shown that the determinant is a constant in such models, therefore gaugino masses vanish at the lowest order in the SUSY breaking. Even though many models of SUSY breaking are not immediately of the Wess-Zumino form considered here it is true that their low energy description often is. The ISS model, for example, has the correct low energy form so models of direct mediation built around it typically suffer from anomalously light gauginos [62, 65, 95–101].

One may wonder why this is a problem. After all, we have forbidden tachyonic directions everywhere in the pseudo-moduli space in the reasoning above. If we permit them, rendering the vacuum metastable as we plan to do anyway, the result quoted at the start of this section does not hold throughout the pseudo-moduli space and larger gaugino masses can be generated. However, tachyonic directions emerging beyond tree level are insufficient. This is the case in the ISS model where the supersymmetric vacuum appears dynamically. Any extra contributions to gaugino masses are thus due to non-perturbative effects which are sub-dominant in calculable models. Instead we must consider models of gauge mediation where metastability already exists at tree level. For example, adding messengers to the ISS model as in ref. [91] solves the problem as it introduces a tree level supersymmetric vacuum [103] but abandons the elegance of direct mediation.

The sought after situation is illustrated in figure 1.8. Somewhere along the pseudo-moduli space a tree level tachyonic direction develops, allowing gaugino masses but at the same time causing the theory to flow into a lower vacuum. Of course, once loop corrections have been accounted for it should be that the uplifted vacuum is rendered sufficiently long lived to be phenomenologically viable. All of these factors present a serious model building challenge. Examples have, however, been found [1, 102–108] and it is one of these we will now discuss in detail.

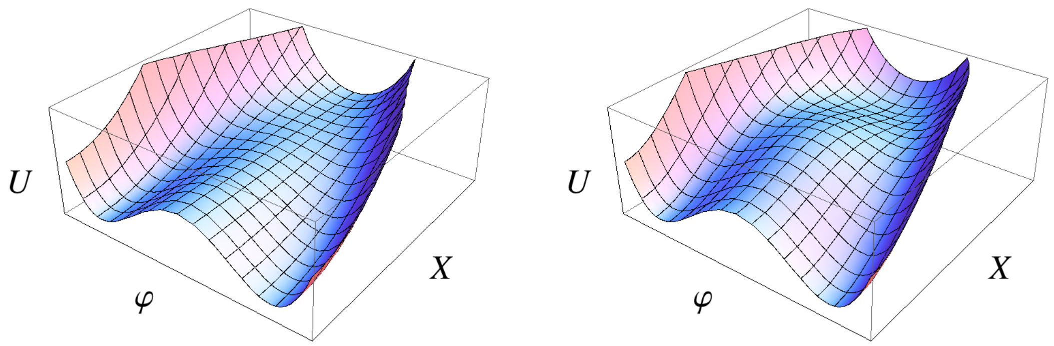


Figure 1.8: Tree level metastability. At tree level (left) some direction $\varphi$ becomes tachyonic along a pseudo-modulus direction $X$, causing the theory to run to a lower vacuum. After loop effects are considered (right) the uplifted vacuum is stabilised.

# Chapter 2

# Tree level metastability in baryon deformed SQCD

Since the importance of tree level metastability was realised [40] there have been several examples of SUSY breaking models possessing it [1, 102–108]. Actually, at the time of ref. [40], models of gauge mediation with tree level metastability already existed (see ref. [91] for example) but *not* in the context of direct mediation. The challenge is often to construct a vacuum that is sufficiently stable, i.e. the tachyonic directions do not rear their heads too soon and lead to a vacuum that is unacceptably short lived. Even if one finds such a model it is desirable for it to be well motivated, rather than a collection of arbitrary scales and superpotential deformations. We will thus begin this chapter with a brief review of the first explicit such construction [102] before going onto to discuss ref. [1], in which an attempt is made to realise the idea in a more natural framework. Both models are based around the ISS model [74] reviewed in section 1.4.3.

## 2.1 Uplifted vacua in massive SQCD

Ref. [102] investigates some of the other vacua that exist in massive SQCD. The ISS vacuum (1.99) allows the matrix $\tilde{q}q$ to fulfil its maximal rank but, by reducing the rank, one can find other stationary points at higher energies. Specifically, in

$|\text{vac}\rangle_{\text{ISS}}$ we have

$$\tilde{q}q = m \begin{pmatrix} \mathbb{1}_N & 0_{N\times(N_f-N)} \\ 0_{(N_f-N)\times n} & 0_{(N_f-N)\times(N_f-N)} \end{pmatrix} \tag{2.1}$$

but we could have

$$\tilde{q}q = m \begin{pmatrix} \mathbb{1}_{N-k} & 0_{(N-k)\times(N_f+k-N)} \\ 0_{(N_f+k-N)\times(N-k)} & 0_{(N_f+k-N)\times(N_f+k-N)} \end{pmatrix} \tag{2.2}$$

for any positive integer $k \leq N$. Such states remain stationary points of the tree level potential (provided the VEVs of the quarks are chosen to minimise the D-terms) but have a higher energy

$$U = (N_f + k - N)\, h^2 m^4 \tag{2.3}$$

than the ISS vacuum. If the states can be made locally stable, they are precisely what we are looking for.

In ref. [102] stabilisation takes place at one loop via the inclusion of an operator $\tilde{Q}Q\tilde{Q}Q$ (with non-trivial index contraction) in the electric superpotential. In addition, the parameter $m$ is split into two different mass scales by including two different quark masses in the electric superpotential. As expected from section 1.4.5 the new vacua do indeed bestow similar masses to both gauginos and sfermions in a direct mediation context. However, although successful in its aims the model has several unwelcome features. It relies on fine tuning between the two new mass scales and the $\tilde{Q}Q\tilde{Q}Q$ coupling constants. Furthermore the relative sizes of the scales are somewhat mysterious, with large ratios appearing for unexplained reasons.

Suppose we focus on the state $q = \tilde{q}^T = 0$ in the undeformed ISS model; the extremal case of eq. (2.2) with $k = N$. The VEV of the tree level potential in this state is $N_f h^2 m^4$. Unlike the ISS vacuum *all* components of $\Phi$ are massless at tree level so should be considered as pseudo-moduli, but we can always use the remaining $\mathrm{SU}(N_f)$ flavour symmetry to make $\Phi$ diagonal. To investigate the stability of the stationary point one must calculate the corresponding mass spectrum. Using the superpotential (1.97), the scalar quark eigenstates and eigenvalues around $q = \tilde{q}^T = 0$ are

$$q \pm \tilde{q}^\dagger : \quad m_0^2 = h^2 \left(|\Phi|^2 \mp m^2\right) \tag{2.4}$$

with equivalent relations for their conjugates. We see that $q + \tilde{q}^\dagger$ becomes tachyonic when $|\Phi| < m$, in which case the theory flows back to the ISS vacuum. As a result, $|\Phi|$ must be stabilised above $m$ for the stationary point to become a vacuum of the theory.

The Coleman-Weinberg potential (1.52) around this point is approximated by

$$U_{\rm CW} \approx \frac{N h^4 m^4}{32\pi^2}\left[3 + 2\ln\left(\frac{h^2|\Phi|^2}{\Lambda^2}\right)\right] \tag{2.5}$$

if $|\Phi| \gg m$. This function has no minimum so we conclude that $\Phi$ is not stabilised with a VEV much greater than $m$. Actually, the approximation becomes valid even when $|\Phi|$ is only a little bigger than $m$, ergo $\Phi$ is not stabilised at a suitable value at all. This is exactly the problem encountered in ref. [102]. The spirit of the solution there was to add a second quark mass term to the electric theory. A stable vacuum can then be found when $\Phi$ is between the two. Ref. [1] takes an alternative approach that is more natural from the electric point of view: adding a baryonic deformation.

## 2.2 Baryon deformations in SU(2)

If we want to stabilise the diagonal components of the pseudo-modulus $\Phi$ by adding an operator to the superpotential there are two simple options with a clear interpretation in the electric theory. The first is to add a meson deformation to the superpotential:

$$W_{\rm mg} \longrightarrow W_{\rm mg} + f(\Phi) \tag{2.6}$$

for a polynomial $f(\Phi)$. This deformation also leads to several new supersymmetric vacua and is the one taken in ref. [102].

The second option is to deform the superpotential with baryonic deformations, which are easily generated in the electric theory via the baryon map (1.87)[1]. Baryonic deformations do not include $\Phi$ explicitly so can only stabilise it via the Coleman-Weinberg potential. As such, we restrict ourselves to the magnetic gauge group

[1]While baryonic deformations were discussed in a similar context in [97] the vacua studied here are not the same; they are uplifted and hence have very different phenomenological properties.

SU(2) where baryonic deformation look like mass terms and will clearly contribute. The magnetic superpotential is deformed to

$$\frac{1}{h}W_{\text{mg}} \longrightarrow \tilde{q}\Phi q - m^2\Phi + m_q\epsilon^{(2)}q_1q_2 + \tilde{m}_q\epsilon_{(2)}\tilde{q}^1\tilde{q}^2 \tag{2.7}$$

where $m_q$ and $\tilde{m}_q$ are dimension 1 coupling constants and $\epsilon^{(2)}$ represents a rank 2 alternating tensor. The subscripts on the quarks (superscripts on the antiquarks) denote flavour indices so we see that this deformation explicitly breaks the global symmetry group, from $\mathrm{SU}(N_f) \times \mathrm{U}(1)_B$ down to $\mathrm{SU}(2) \times \mathrm{SU}(N_f - 2)$. The other flavour and colour indices have been suppressed.

The baryonic deformation retains the classical stationary point at $q = \tilde{q}^T = 0$ and does not change the pseudo-modulus status of $\Phi$. Breaking the flavour symmetry suggests a more natural component expansion

$$q = \begin{pmatrix} x \\ y \end{pmatrix} \qquad \tilde{q} = \begin{pmatrix} \tilde{x} & \tilde{y} \end{pmatrix} \qquad \Phi = \begin{pmatrix} \phi & \tilde{\rho} \\ \rho & \chi \end{pmatrix} \tag{2.8}$$

around the stationary point, where $x$ and $\phi$ are $2\times2$ matrices, $y$ and $\rho$ are $(N_f-2)\times2$ matrices and $\chi$ is a $(N_f-2)\times(N_f-2)$ matrix (with transposed relations for tilded components). The baryon deformation only affects the SU(2) flavour sector (we will discuss the remainder of the theory later) and we can again use the residual flavour symmetry to restrict our attention to the diagonal components of $\Phi$. The relevant superpotential terms are therefore

$$\frac{1}{h}W_{\text{mg}} \supset \tilde{x}\phi x - m^2\phi + m_q\epsilon^{(2)}x_1x_2 + \tilde{m}_q\epsilon_{(2)}\tilde{x}^1\tilde{x}^2 \tag{2.9}$$

with the $\phi$'s inheriting pseudo-moduli status from $\Phi$. For this superpotential, the mass squared eigenvalues for the $x$'s are

$$\begin{aligned} m_0^2 =& \frac{h^2}{2}\left(m_+^2 + m_-^2 + |\phi_+|^2 + |\phi_-|^2\right) \pm \\ & h^2\sqrt{\left(m_+^2 + |\phi_-|^2\right)\left(m_-^2 + |\phi_+|^2\right) + m^4 \pm 2m^2|\phi_+|\sqrt{m_+^2 + |\phi_-|^2}} \\ m_{1/2}^2 =& \frac{h^2}{2}\left(m_+^2 + m_-^2 + |\phi_+|^2 + |\phi_-|^2 \pm 2\sqrt{\left(m_+^2 + |\phi_-|^2\right)\left(m_-^2 + |\phi_+|^2\right)}\right) \end{aligned} \tag{2.10}$$

where the $\pm$'s are all independent and we have defined

$$\phi_\pm = \frac{1}{\sqrt{2}}\left(\phi_1^1 \pm \phi_2^2\right) \qquad m_\pm = \frac{1}{\sqrt{2}}\left(m_q \pm \tilde{m}_q\right). \tag{2.11}$$

Evidently the stationary point $x = \tilde{x} = \phi = 0$ has no tree level tachyons only if

$$m_+^2 + m_-^2 > 2\sqrt{m_+^2 m_-^2 + m^4}. \tag{2.12}$$

The simplified cases, where either $m_+$ or $m_-$ vanish, can be investigated analytically without too much trouble. It is then possible to numerically interpolate between these two extremes. First, consider the case $\tilde{m}_q = -m_q$, i.e. $m_+ = 0$. The masses acquired by the pseudo-moduli at one loop are

$$m_{\phi_+}^2 = \frac{h^4 m^2}{8\pi^2}\left[\left(\frac{m_-^2}{m^2}+2\right)\ln\left(\frac{m_-^2}{m^2}+2\right)+\left(\frac{m_-^2}{m^2}-2\right)\ln\left(\frac{m_-^2}{m^2}-2\right)- \frac{m_-^2}{m^2}\ln\left(\frac{m_-^4}{m^4}\right)\right] \tag{2.13}$$

and

$$m_{\phi_-}^2 = \frac{h^4 m^2}{8\pi^2}\left[\left(\frac{3m_-^2}{m^2}-\frac{m_-^4}{m^4}-2\right)\ln\left(\frac{m_-^2}{m^2}-2\right)-\frac{4m_-^2}{m^2}\left(1-\ln\left(\frac{m_-^3}{m^3}\right)\right)+ \left(\frac{3m_-^2}{m^2}+\frac{m_-^4}{m^4}+2\right)\ln\left(\frac{m_-^2}{m^2}+2\right)\right]. \tag{2.14}$$

One immediately sees that these masses are well defined only for $m_-^2 > 2m^2$, equivalent to eq. (2.12) evaluated at $m_+ = 0$. Their behaviour is shown in figure 2.1. Both pseudo-moduli acquire non-tachyonic masses and are stabilised at the origin. Consequently a stable vacuum exists there with the mass eigenvalues in the $x$-sector evaluating to

$$m_0^2 = h^2\left(\frac{1}{2}m_-^2 \pm m^2\right) \qquad\qquad m_{1/2}^2 = \frac{1}{2}h^2 m_-^2\,. \tag{2.15}$$

Alternatively, for $m_- = 0$, the masses for the pseudo-moduli are

$$m_{\phi_+}^2 = \frac{h^4 m^2}{8\pi^2}\left[\left(\frac{3m_+^2}{m^2}+2\right)\ln\left(\frac{m_+^2}{m^2}+2\right)+\left(\frac{3m_+^2}{m^2}-2\right)\ln\left(\frac{m_+^2}{m^2}-2\right)- \frac{4m_+^2}{m^2}\ln\left(\frac{m_+^3}{m^3}\right)\right] \tag{2.16}$$

and

$$m_{\phi_-}^2 = \frac{h^4 m^2}{8\pi^2}\left[\left(\frac{m_+^2}{m^2}+2\right)\ln\left(\frac{m_+^2}{m^2}+2\right)+\left(\frac{m_+^2}{m^2}-2\right)\ln\left(\frac{m_+^2}{m^2}-2\right)- \frac{m_+^2}{m^2}\ln\left(\frac{m_+^4}{m^4}\right)\right]. \tag{2.17}$$

Now we need $m_+^2 > 2m^2$ (again, eq. (2.12) evaluated at $m_- = 0$) for well defined masses. However, this time $\phi_+$ acquires a tachyonic mass so the origin is unstable. This is also shown in figure 2.1.

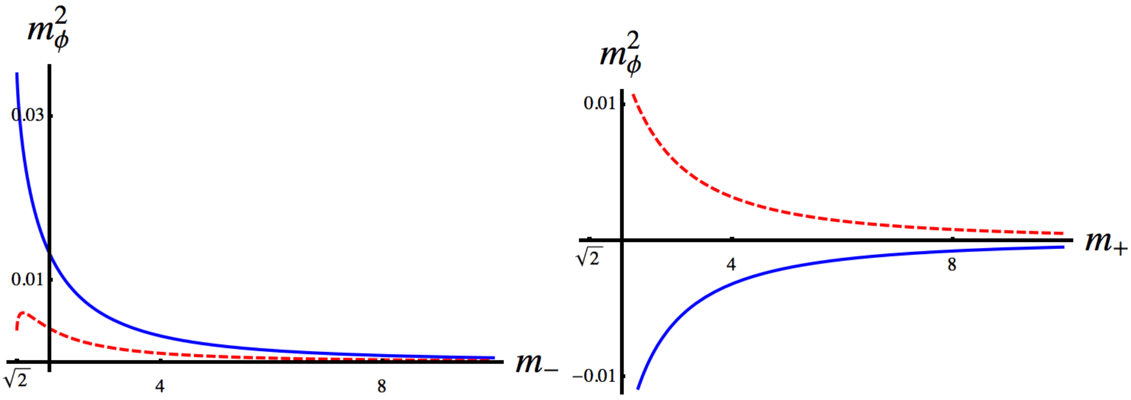


Figure 2.1: The masses of the pseudo-moduli as functions of $m_-$ when $m_+ = 0$ (left) and $m_+$ when $m_- = 0$ (right). The solid line shows the mass-squared of $\phi_+$ and the dashed line $\phi_-$. Masses $m_\pm$ are in units of $m$ and $m_\phi^2$ is in units of $h^4 m^2$.

A surface plot of the full one loop potential is shown in figure 2.2. Both cases contain tree level tachyons somewhere in the pseudo-moduli space (where the one loop potential cannot be defined) but when $m_+ = 0$ both $\phi_+$ and $\phi_-$ are stabilised around zero and the SU(2) flavour symmetry remains unbroken. To get a handle on where the tachyons appear one can set $\phi_+ = m_+ = 0$ in eq. (2.10) to find

$$m_0^2 = \frac{h^2}{2}\left(m_-^2 + |\phi_-|^2 \pm 2\sqrt{m_-^2|\phi_-|^2 + m^4}\right) \tag{2.18}$$

which becomes negative when

$$m_-^2 - 2m^2 < |\phi_-|^2 < m_-^2 + 2m^2. \tag{2.19}$$

Increasing $m_-$ therefore moves the tachyon containing regions further away from $|\phi_-| = 0$, whereas increasing $m$ makes them wider. On the other hand, when $m_- = 0$, $\phi_+$ runs away from the origin to a region containing tachyons among the $x$ fields so this stationary point is unstable. In either case the theory runs to a vacuum similar to the one discussed in [97] once it strays into a region containing tachyons.

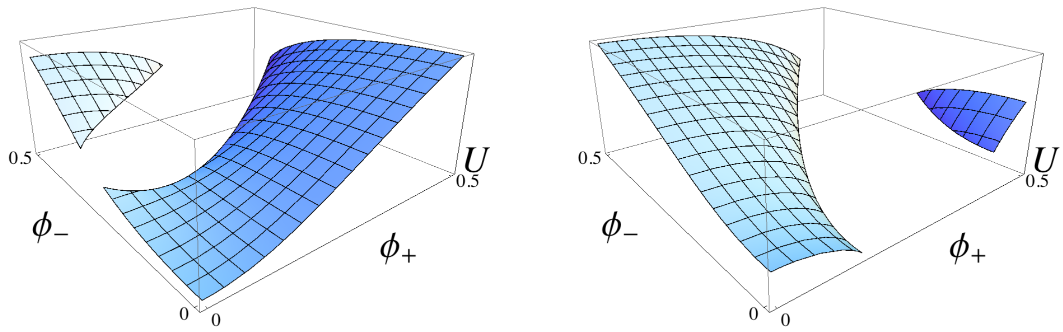


Figure 2.2: The one loop potential as a function of the pseudo-moduli $\phi_\pm$ (assumed real) when $m_+ = 0$ (left) and $m_- = 0$ (right). It is symmetric about the origin in $\phi_\pm$ for both cases. Gaps in the surface show regions containing tree level tachyons among the $x$ fields, where the theory presumably runs to the vacuum similar to that of [97]. Parameters have been set to $\Lambda = 1$, $h = 1$, $m = 1/10$ and $m_\pm = \sqrt{2}/5$ when not equal to zero.

## 2.3 Gauging the flavour group

To use this model as a SUSY breaking sector in a direct mediation scenario the flavour group should be gauged. Then $\phi_-$ is no longer a pseudo-modulus as it is not a D-flat direction; D-terms automatically stabilise it at $\phi_- = 0$. This opens up a new possibility for the deformed superpotential (2.7) that can yield a stable, uplifted vacuum: setting $\tilde{m}_q = 0$ (or equivalently $m_q = 0$).

One can go through the process of finding the mass eigenstates and calculating the Coleman-Weinberg potential as before. This is now a function of the one remaining pseudo-modulus $\phi_+$ and is easily deduced from eq. (2.10) by setting $m_+ = m_- = m_q/\sqrt{2}$, $\phi_- = 0$ and redefining $\phi_+ = \sqrt{2}\hat{\phi}$ (where $\phi = \hat{\phi}\mathbb{1}_2$). For $m^2 \ll m_q^2$ we find

$$U_{\mathrm{CW}} \approx \frac{h^4 m^4}{2\pi^2\left(m_q^2 + 4|\hat{\phi}|^2\right)}\left[m_q^2 + 12|\hat{\phi}|^2 + \left(m_q^2 + 4|\hat{\phi}|^2\right)\ln\left(\frac{h^4|\hat{\phi}|^4}{\Lambda^4}\right) + m_q\frac{m_q^2 + 2|\hat{\phi}|^2}{\sqrt{m_q^2 + 4|\hat{\phi}|^2}}\ln\left(\frac{m_q^2 + 2|\hat{\phi}|^2 + m_q\sqrt{m_q^2 + 4|\hat{\phi}|^2}}{m_q^2 + 2|\hat{\phi}|^2 - m_q\sqrt{m_q^2 + 4|\hat{\phi}|^2}}\right)\right]. \tag{2.20}$$

This function can be reduced to a function of $|\hat{\phi}|/m_q$ then shown numerically to have a minimum at $|\hat{\phi}| \approx m_q/4$ for any values of $h$, $\Lambda$ and $m^2 \ll m_q^2$. The minimum provides a new uplifted vacuum for the theory. In it one finds

$$U_{\mathrm{CW}} \approx \frac{h^4 m^4}{\pi^2}\left[3 + 2\ln\left(\frac{hm_q}{4\Lambda}\right)\right] \tag{2.21}$$

and the mass of $\hat{\phi}$ goes like $0.63h^4m^4/m_q^2$. Eq. (2.10) demonstrates that there are no tachyons in the $x$-sector as long as $|\hat{\phi}|^3 \gtrsim 4m_q m^2$. For the minimum at $|\hat{\phi}| \approx m_q/4$ to fall in this regime we require $m_q \gtrsim 16m$. A typical example of the potential is plotted in figure 2.3. Despite the fact that $\phi$ acquires a VEV in this vacuum it is only the trace component that is non-zero. Consequently, the SU(2) symmetry remains unbroken.

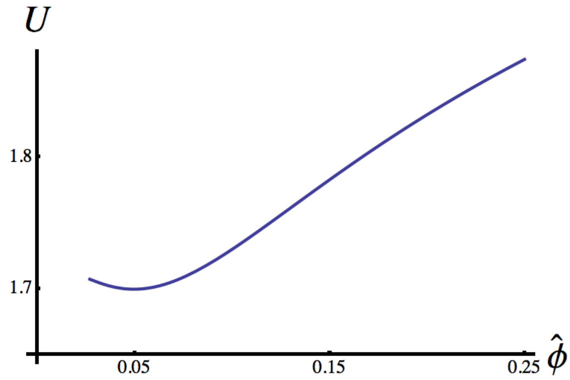


Figure 2.3: The one loop potential as a function of the pseudo-modulus $\hat{\phi}$ (assumed real) when $\tilde{m}_q = 0$ and the flavour symmetry is gauged. Parameters have been set to $\Lambda = 1$, $h = 1$, $m = 1/200$ and $m_q = 1/5$. The potential is given in units of $m^4$. Note the minimum at $\hat{\phi} \approx m_q/4$. Increasing $m_q$ moves the minimum to the right, whereas increasing $m$ moves the tachyon containing region (where the potential is not defined) further right.

## 2.4 Direct mediation

In the setup discussed above there are always tachyonic directions at tree level somewhere in the pseudo-moduli space. We will now see how this solves the problem of anomalously small gaugino masses common to theories of direct mediation. Three possible approaches for using this model in a direct mediation scenario will be discussed. We set $\tilde{m}_q = 0$ in each so the results from section 2.3 apply.

### 2.4.1 SU($N_f - 2$) mediation

The most obvious choice is to gauge the entire flavour group and embed the visible sector gauge group in the SU($N_f-2$) part of the symmetry. Unfortunately this sector is decoupled from the SUSY breaking in the SU(2) flavour sector. To rectify this we must include a coupling from $\phi$ to either the $y$'s or the $\rho$'s of (2.8). Generating couplings to the quarks in the magnetic theory using operators in the electric theory is difficult so we will use the $\rho$'s instead. An appropriate coupling can then be generated by a cubic meson operator

$$\frac{1}{h}W_{\text{mg}} = \tilde{q}\Phi q - m^2\Phi + m_q\epsilon^{(2)}x_1x_2 + \eta\,\text{Tr}\left[\rho\phi\tilde{\rho}\right] + \frac{1}{2}m_\chi\,\text{Tr}\left[\chi^2\right]. \tag{2.22}$$

We have also added a mass term for $\chi$ which stabilises it at $m^2/m_\chi$; without this term, $\chi$ would be unstable for the same reasons as $\Phi$ in section 2.1.

Both deformations correspond to non-renormalisable operators in the electric theory but their presence will be motivated in section 2.5 using arguments similar to [91]. In brief, the magnetic deformations considered here correspond to *all* generic superpotential deformations up to dimension six in the electric theory that are compatible with the symmetries. Higher dimension operators (and, as we shall shortly see, similar terms from the Kähler potential) are parametrically suppressed in the magnetic theory so can be safely ignored. Even so, one could in principle add other operators arising from $\Phi^2$ or $\Phi^3$ deformations without breaking the residual flavour symmetry. Those not involving $\phi$ do not change our results significantly (in fact, they only improve the situation by pushing the masses of the $\rho$'s higher so that Landau poles pose less of a problem) but those that do could have an adverse effect.

Specifically we want to avoid the operators $\phi^2$, $\phi\chi$, $\phi^3$, $\phi^2\chi$ and $\phi\chi^2$. The mixed $\phi$-$\chi$ operators correspond to multitrace deformations in the electric theory. Stringy UV completions, in which the gauge theory arises as the low energy description of a model of intersecting branes (ref. [109] for example), can naturally motivate an absence of such operators. We will assume something like this is at play here so will consider only single trace terms. To see why the remaining terms are not allowed, imagine restoring SUSY by setting $m = 0$. An anomalous R-symmetry is also restored with

$$\begin{array}{llll} R(x) = 1 & R(y) = R_\rho & R(\rho) = R_\rho & R(\phi) = \frac{1}{2} \\ R(\tilde{x}) = \frac{1}{2} & R(\tilde{y}) = 1 - R_\rho & R(\tilde{\rho}) = \frac{3}{2} - R_\rho & R(\chi) = 1 \end{array} \tag{2.23}$$

forbidding both $\phi^2$ and $\phi^3$ at tree level (this R-symmetry explicitly breaks the flavour symmetry of the undeformed model, but the baryon deformation does this anyway; the unified meson superfield $\Phi$ is used in (2.22) merely for notational brevity). Since the superpotential is otherwise generic, R-symmetry can be explicitly broken to render the vacuum metastable (see ref. [34] or section 1.2.2). One can therefore imagine a situation where the above R-symmetry is initially preserved (at least classically), forbidding the unwanted couplings, but is spontaneously broken with order parameter $m$. Precisely the same order parameter describes the SUSY breaking.

The tree level vacuum resulting from (2.22) is

$$|\text{vac}\rangle_{N_f-2}: \quad q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \tilde{q} = \begin{pmatrix} 0 & 0 \end{pmatrix} \quad \Phi = \begin{pmatrix} \hat{\phi}\mathbb{1}_2 & 0 \\ 0 & m^2/m_\chi \end{pmatrix} \quad V_{\text{tree}} = 2h^2 m^4. \tag{2.24}$$

with the fields expanded as in (2.8). The $\rho$'s act as messengers, coupling the SUSY breaking in the SU(2) flavour sector to the SU$(N_f - 2)$ flavour sector. The tree level masses in the $x$-sector are unchanged from section 2.3. Meanwhile fluctuations of the remaining scalar fields acquire tree level masses

$$\begin{aligned} y, \tilde{y}: &\quad m_0^2 = h^2 m^4 / m_\chi^2 \\ \chi: &\quad m_0^2 = h^2 m_\chi^2 \\ \rho \pm \tilde{\rho}^\dagger: &\quad m_0^2 = h^2 \left( \eta^2 |\hat{\phi}|^2 \mp 2\eta m^2 \right). \end{aligned} \tag{2.25}$$

One thus requires $|\hat{\phi}|^2 > 2m^2/\eta$ for the $\rho$'s to be non-tachyonic. The effect on the Coleman-Weinberg potential is to add an extra term to (2.20)

$$\Delta U_{\text{CW}} = \frac{h^4 \eta^2 m^4}{4\pi^2} \left[ 3 + 4 \ln \left( \frac{h^4 \eta^4 |\hat{\phi}|^4}{\Lambda^4} \right) \right]. \tag{2.26}$$

Provided $\eta \lesssim 1/3$, $\Delta U_{\rm CW} \lesssim U_{\rm CW}/10$ so the perturbation does not destroy the minimum at $|\hat{\phi}| \approx m_q/4$. Combined with the tachyon constraint $|\hat{\phi}|^2 > 2m^2/\eta$ evaluated at $|\hat{\phi}| = m_q/4$ minimum we find

$$\frac{32m^2}{m_q^2} \lesssim \eta \lesssim \frac{1}{3}. \tag{2.27}$$

As $m_q \gtrsim 16m$ the left hand side can be at most $1/8$ so this range of $\eta$ is quite reasonable. Plots of the one loop potential for various values of $\eta$ are given in figure 2.4.

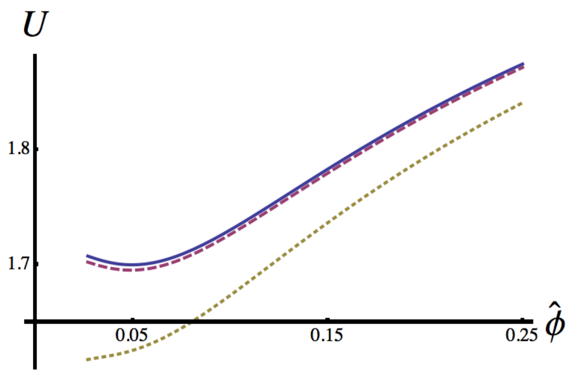


Figure 2.4: The one loop potential as a function of the pseudo-modulus $\hat{\phi}$ (assumed real) in the SU$(N_f - 2)$ direct mediation scenario. Parameters have been set to $\Lambda = 1$, $h = 1$, $m = 1/200$, $m_q = 1/5$. The potential is given in units of $m^4$. The solid line shows $\eta = 0$, the dashed line $\eta = 1/10$ and the dotted line $\eta = 1/2$. The minimum at $\hat{\phi} \approx m_q/4$ is robust to the inclusion of $\eta$, for $\eta \lesssim 1/3$.

Gaugino masses [57] can be calculated to be

$$m_\lambda \sim \Lambda_G = F_{\hat{\phi}} \frac{\partial}{\partial \hat{\phi}} \ln \det (m_{\rm mess}) = \frac{2hm^2}{|\hat{\phi}|} \tag{2.28}$$

and the sfermion masses are

$$m_{\tilde{f}}^2 \sim \Lambda_S^2 = \frac{1}{2} F_{\hat{\phi}}^2 \frac{\partial^2}{\partial \hat{\phi} \partial \hat{\phi}^*} \sum_{\rm eigenvalues} \left[ \ln \left( m_{\rm mess}^2 \right) \right]^2 = \frac{32h^2m^4}{|\hat{\phi}|^2} \tag{2.29}$$

where $m_{\rm mess} = \eta \hat{\phi} \mathbb{1}_2$ is the messenger mass matrix with eigenvalues $\eta\hat{\phi}$. The ratio of the gaugino masses to sfermion masses is conveniently parameterised by the effective number of messengers

$$\mathcal{N}_{\rm eff} = \frac{\Lambda_G^2}{\Lambda_S^2} = \frac{1}{8} \tag{2.30}$$

which is less than one but still sufficiently large. Around the vacuum at $|\hat{\phi}| \approx m_q/4$ for visible sector gauge coupling $\alpha_{\rm vis}$ we have

$$m_\lambda \sim \frac{2\alpha_{\rm vis} h m^2}{\pi m_q} \qquad\qquad m_{\tilde{f}} \sim \frac{4\sqrt{2}\alpha_{\rm vis} h m^2}{\pi m_q}. \tag{2.31}$$

### 2.4.2 SU(2) × SU(3) mediation

A second possibility is to choose $N_f = 5$. The SUSY breaking sector then has an SU(2) × SU(3) flavour symmetry that can neatly be identified with the visible sector gauge group. We can thus get away with the simpler superpotential

$$\frac{1}{h} W_{\rm mg} = \tilde{q}\Phi q - m^2\Phi + m_q \epsilon^{(2)} x_1 x_2 + \frac{1}{2} m_\chi \,{\rm Tr}\left[\chi^2\right]. \tag{2.32}$$

SUSY breaking occurs in the SU(2) sector and is now mediated to the visible sector by the $x$'s. Mediation to the SU(3) sector occurs via visible sector interactions so is suppressed by an extra loop. This approach leads to an interesting signature: SU(3) soft masses would be less than SU(2) soft masses by a factor of one loop. However, it would likely upset gauge coupling unification in the visible sector as SU(2) messenger masses go like $m_q$ whereas SU(3) messenger masses go like $m^2/m_\chi$. As we will see shortly there is generally a substantial hierarchy between these two scales. Experimental limits on gluino masses (the lower bound currently sits around 500 GeV [110]) also force SU(2) gauginos to be heavy, compromising SUSY's ability to solve the hierarchy problem if their mass is much larger than a few TeV.

### 2.4.3 Sp($N_f$) mediation

It is worth briefly mentioning a third possibility for direct mediation available in this model. Rather than using a baryon deformation of the form

$$\frac{1}{h} W_{\rm mg} \supset m_q \epsilon^{(2)} q_1 q_2 \tag{2.33}$$

that singles out two flavours of quark, we could generalise and use a deformation

$$\frac{1}{h} W_{\rm mg} \supset m_q^{ij} \epsilon^{(2)} q_i q_j \tag{2.34}$$

where $m_q$ is now an antisymmetric matrix in flavour space. This deformation explicitly breaks the flavour group from SU($N_f$) to Sp($N_f$), which we could then gauge.

We no longer have to expand the quarks as in (2.8); instead, the whole model behaves as the SU(2) flavour sector in section 2.2, only with a rescaled potential $U \to N_f U/2$. The only pseudo-modulus behaves like $\hat{\phi}$ so is safely stabilised around $m_q/4$. Unfortunately, it is difficult to embed the visible sector into Sp($N_f$) (which does not accommodate complex representations) so using this method for direct mediation is of limited practical use.

## 2.5 Phenomenological viability

To complete the example we will discuss the phenomenological aspects of the SU($N_f-2$) direct mediation scenario of section 2.4.1. For simplicity we will analyse the minimal case: take $N_f = 7$ and embed the visible sector in an SU(5) GUT. Regarding parameters in the superpotential (2.22), we have five: $h$, $m$, $m_q$, $\eta$ and $m_\chi$. All have dimension 1 except $h$ and $\eta$, which are dimensionless. The parameter $h$ is simply an order 1 coupling constant that translates directly from the electric theory. The magnitudes of the other parameters can be estimated by looking at where they come from in the UV [91].

Using the baryon map (1.87) we have (schematically)

$$\begin{aligned} m^2\Phi &\leftrightarrow \mu \,\mathrm{Tr}\left[\tilde{Q}Q\right] \\ m_q\epsilon^{(2)}q_1q_2 &\leftrightarrow \frac{1}{M^2}\epsilon^{(5)}Q^5 \\ \eta\phi\tilde{\rho}\rho &\subset \frac{1}{M^3}\,\mathrm{Tr}\left[(\tilde{Q}Q)^3\right] \\ m_\chi\chi^2 &\subset \frac{1}{M}\,\mathrm{Tr}\left[(\tilde{Q}Q)^2\right] \end{aligned} \quad (2.35)$$

where $M$ is some high scale. These operators are the only single trace, perturbative operators in the electric theory which are consistent with its symmetries (including the R-symmetry discussed in section 2.4.1) and of dimension six or lower. Higher dimension operators may exist but will be further suppressed so can be discarded. From dimensional arguments one expects

$$m \sim \sqrt{\mu\Lambda} \qquad m_q \sim \Lambda\left(\frac{\Lambda}{M}\right)^2 \qquad \eta \sim \left(\frac{\Lambda}{M}\right)^3 \qquad m_\chi \sim \Lambda\left(\frac{\Lambda}{M}\right) \quad (2.36)$$

and then, from (2.25)[2]

$$m_y \sim \mu \left(\frac{M}{\Lambda}\right) \qquad\qquad m_\rho \sim \sqrt{\Lambda^2 \left(\frac{\Lambda}{M}\right)^8 \pm \mu\Lambda \left(\frac{\Lambda}{M}\right)^3}. \tag{2.37}$$

The fermionic components of these superfields acquire the same masses except for the $\rho$'s whose fermions get masses $\sim \Lambda(\Lambda/M)^4$.

In sections 2.3 and 2.4.1 we required

$$m_q \gtrsim 16m \quad \text{and} \quad \frac{32m^2}{m_q^2} \lesssim \eta \lesssim \frac{1}{3} \tag{2.38}$$

for vacuum stability, which corresponds to

$$\mu \lesssim 10^{-9}\Lambda \quad \text{and} \quad \frac{\Lambda}{M} \lesssim 10^{-1}. \tag{2.39}$$

The scale $\mu$ is the only unnaturally small parameter (as in the ISS model) and we shall see shortly that it can be associated with the physically meaningful weak scale. For the gaugino mass scale (2.31) to be around the weak scale (assuming $h \sim 1$) we further require

$$m_\lambda \sim \frac{2\alpha_{\rm vis} h m^2}{\pi m_q} \sim 10^{-2}\mu \left(\frac{M}{\Lambda}\right)^2 \sim 100 \text{ GeV} \qquad \Longrightarrow \qquad \mu \sim \left(\frac{\Lambda}{M}\right)^2 10^4 \text{ GeV}. \tag{2.40}$$

Finally, for the theory to remain calculable, mass contributions from the Kähler potential (which go like $F_{\hat\phi}^2/\Lambda^2 = 4h^2m^4/\Lambda^2$ [42]) must be smaller than one loop masses (which go like $h^4m^4/m_q^2$). This last constraint reduces to

$$\frac{\Lambda}{M} \lesssim \sqrt{h} \tag{2.41}$$

so is already satisfied by eq. (2.39) when $h \sim 1$.

Approximating the minimum in figure 2.4 with a triangular potential barrier one can make a rough estimate of the bounce action [90], and therefore the lifetime of the uplifted minimum. The result is

$$S \sim \frac{\Delta\hat\phi^4}{U_{\rm min}} \sim \frac{\Lambda^2}{10^{10}\text{ GeV}^2} \left(\frac{\Lambda}{M}\right)^4 \left(7 + \frac{1}{\pi^2}\left[3 + 4\ln\left(\frac{\Lambda}{4M}\right)\right]\right)^{-1} \tag{2.42}$$

[2]Note that the VEV of $\chi$, therefore $m_y$, increases with $M$. Ensuring that the vacuum falls within the remit of the magnetic theory prevents $m_y$ from being too large. This is automatic in models with $\Lambda/M \gtrsim 10^{-9}$ due to other constraints.

where $U_{\rm min}$ denotes total potential in the minimum and is given by $7h^2m^4 + U_{\rm CW}$, using eq. (2.21) to evaluate $U_{\rm CW}$. The width of the barrier is $\Delta\hat{\phi} \sim m_q/4$. For $\Lambda/M \gg 10^{-7}$ the bounce action is well approximated by $S \sim \Lambda^6/M^4 10^{11}$ GeV$^2$ and is easily compatible with the observed lifetime of the universe for a suitable choice of mass scales.

All of these constraints are consistent with one another and leave a wide range of choices for the three fundamental parameters input into the electric theory; $\mu$, $\Lambda$ and $M$ (assuming all dimensionless couplings are of order 1). If we minimise the number of scales by saturating (2.39) and choosing $\Lambda \sim 10^{-1}M$, we will always require $\mu \sim 100$ GeV and will achieve a stable vacuum for any high scale $M \gtrsim 10^{12}$ GeV. This is quite an attractive scenario as we effectively only require two scales: a high scale $M$ (such as the Planck scale or the GUT scale) and the weak scale of 100 GeV. It also results in a very large bounce action and consequently a long lived vacuum.

For the messengers $\rho$ and additional matter charged under the visible sector ($y$ and $\chi$) we find[3]

$$m_y \sim 10\mu \sim 10^3 \text{ GeV} \quad m_\rho \sim 10^{-5}M \gtrsim 10^7 \text{ GeV} \quad m_\chi \sim 10^{-2}M \gtrsim 10^{10} \text{ GeV}. \tag{2.43}$$

The splitting of the masses in the $\rho$ sector is much smaller than their central mass scale so $m_\rho$ is the mass of both the fermionic and bosonic components of the $\rho$'s. We see an important prediction in eq. (2.43): the existence of two pairs of new particles ($y$ and $\tilde{y}$) charged under the visible sector gauge group with masses independent of $M$ at about 1 TeV. These particles should be visible at the LHC as they would couple to the visible sector through Standard Model gauge interactions.

When running up in energy the first visible sector gauge coupling to hit a Landau pole will always be $\alpha_{\rm QCD}$ (see figure 2.5). The $\beta$-function coefficient for the QCD

[3]A brief note on the decays of these particles: $\chi$ decays rapidly to $\tilde{y}y$ through couplings in the superpotential. At first glance the fields $y$ and $\rho$ can only decay through mutual annihilation so we expect them to be fairly stable. The mass of the $y$'s is less than 10 TeV which is insufficient for their relic density to overclose the universe [59], but the mass of the $\rho$'s is large enough to cause cosmological problems. Gravitational interactions can, however, induce extra superpotential terms that allow decays to $y$'s or visible sector particles [111].

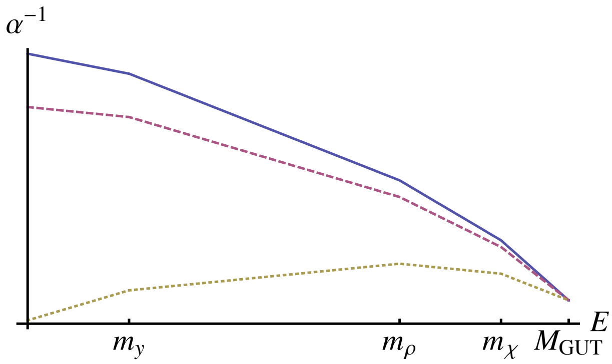


Figure 2.5: The running of the gauge couplings in a typical SU(5) GUT. The dotted line is for the strong force, the dashed for the weak and the solid for hypercharge. As the RG scale reaches certain values new particles from the SUSY breaking sector deflect the running. As long as $M \gtrsim 10^{16}$ GeV the flow reaches $M_{\rm GUT} \sim 10^{16}$ GeV before $\alpha^{-1}_{\rm QCD}$ reaches zero, avoiding the Landau pole problem.

gauge coupling constant is

$$b_{\rm QCD} = 3 - \Delta_{\rm mess}(E) \tag{2.44}$$

where $\Delta_{\rm mess}(E)$ is the contribution from the SUSY breaking sector and is a function of RG scale. It is

$$\Delta_{\rm mess}(E) = \begin{cases} 0 & E < m_y \\ 2 & m_y \leq E < m_\rho \\ 4 & m_\rho \leq E < m_\chi \\ 7 & E \geq m_\chi \end{cases} \tag{2.45}$$

where the masses are defined in eq. (2.43). Taking $\alpha_{\rm QCD} \sim 10^{-1}$ at $m_y$ we find a Landau pole in the visible sector at

$$E \sim 10^{-4} \left( \frac{M}{1 \text{ GeV}} \right)^{5/4} \text{GeV}. \tag{2.46}$$

The scale is large enough to avoid the Landau pole problem for any high scale[4] $M \gtrsim 10^{16}$ GeV. For $M$ larger still the Landau pole is moved further and further

[4]The choice $M \sim 10^{16}$ GeV is actually quite interesting. We would then have the Landau pole of the visible sector coinciding with both the Landau pole in the SUSY breaking sector and the GUT scale. This would give no shortage of strong dynamical effects to generate the SUSY breaking terms. Of course, reaching a Landau pole in the SUSY breaking sector suggests we should actually be working in the electric theory instead, so this analysis is not fully reliable.

above the GUT scale. Note that when $M$ approaches the Planck mass this model exhibits the gauge-gravity hybrid behaviour discussed in [112, 113].

## 2.6 Summary

In this section an alternative method for stabilising the uplifted vacua of SQCD has been discussed. By restricting the magnetic gauge group to SU(2) one can stabilise all pseudo-moduli with a baryon deformation to the superpotential. The baryon deformation appears as a mass term in the magnetic superpotential so stabilises the pseudo-moduli via the Coleman-Weinberg potential. There remain tachyonic directions at tree level elsewhere in the pseudo-moduli space so the model is able to produce gaugino masses comparable to sfermion masses when implicated in a direct mediation scenario.

The method is theoretically economical and all relevant quantities can be calculated and understood by simple, analytical expressions. In addition, there are some strong phenomenological motivations for using this kind of model as a SUSY breaking sector. Landau poles do not occur in the visible sector gauge couplings due to the smallness of the gauge group and the emergent mass hierarchy in the SUSY breaking sector. Only two fundamental scales are required: the weak scale of 100 GeV and some high scale $M \gtrsim 10^{16}$ GeV (which could easily be taken to be the GUT scale or the Planck scale) and are understood through considerations of the UV theory. Furthermore the deformations required to couple the SUSY breaking to the visible sector can be motivated by symmetry arguments if we presume the theory to possess an R-symmetry (which is broken explicitly by the same term responsible for SUSY breaking) and allow single trace operators of up to dimension six in the electric theory.

The model predicts an effective number of messengers of precisely 1/8 and the existence of several new particles around 1 TeV. These particles would couple to the visible sector through Standard Model gauge interactions so could be seen at the LHC. As well as the direct mediation scenario considered in depth here, other novel possibilities exist for this model. The vacuum discussed in section 2.2 preserves an unbroken flavour group $\mathrm{SU}(2) \times \mathrm{SU}(N_f - 2)$ which, for $N_f = 5$, could be identified with the visible sector gauge group.

# Chapter 3

# Solitonic supersymmetry restoration

One aspect of metastability that has thus far been unexplored is the effect of non-topological solitons, such as Q-balls [114], on the lifetime of the metastable vacuum. Q-balls exist in many models with scalar fields charged under an unbroken, global U(1) symmetry, and it has long been known [115, 116] that such objects can induce phase transitions. An important difference between phase transitions precipitated by Q-balls and those arising through more conventional means is that sub-critical vacuum bubbles can build up gradually in the former scenario: charge conservation ensures stability at any given stage [116–118]. The timescale for these decays is therefore much less than one would estimate using the usual tunnelling action.

Many models of metastable SUSY breaking include an unbroken U(1) symmetry in the metastable SUSY breaking vacuum and, of course, support scalar excitations. It is therefore important to ask whether Q-balls can speed up the decay of this vacuum relative to the usual tunnelling estimate. Perhaps of greatest phenomenological significance is the question of whether Q-balls can induce a decay to the true, supersymmetric vacuum, although decays to lower lying SUSY breaking vacua are also important in models that utilise them such as the one we have just discussed. As we shall soon see Q-balls do indeed have a major effect on the vacuum lifetime and often make seemingly viable metastable vacua decay on a cosmologically negligible timescale.

## 3.1 Q-balls in metastable vacua

Non-topological solitons are common in models whose vacua possess unbroken symmetries. Global [114, 119, 120], local [121], abelian and non-abelian [122, 123] symmetries have been considered, but only the simplest case will be investigated here: a global U(1). Specifically, consider a model of a complex scalar field $\varphi$ charged under such a symmetry and moving in a potential $U(\varphi)$. For a vacuum with unbroken U(1) to exist the potential must have a minimum at $\varphi = 0$ where, without loss of generality, we can fix $U(\varphi) = 0$. The physical argument for the existence of Q-balls then goes as follows. Given a total charge $Q$, which is a conserved quantity due to the unbroken U(1) symmetry, we must find the most energy efficient way of distributing it. If it is more economical to store the charge in a 'blob' of non-zero field VEV than in free scalar particles there are stable Q-ball solutions to the equations of motion.

Mathematically, a field configuration $\varphi(x, t)$ with total charge $Q$ should minimise the energy

$$E = \int \mathrm{d}^3x \left(\frac{1}{2}|\dot{\varphi}|^2 + \frac{1}{2}|\nabla\varphi|^2 + U(\varphi)\right) + \omega\left(Q - \frac{1}{2i}\int \mathrm{d}^3x\, \varphi^* \overleftrightarrow{\partial}_t \varphi\right) \tag{3.1}$$

where the second term ensures charge conservation via the Lagrange multiplier $\omega$ (which is ultimately eliminated in favour of $Q$). Rearranging gives

$$E = \int \mathrm{d}^3x \left(\frac{1}{2}|\dot{\varphi} - i\omega\varphi|^2\right) + \int \mathrm{d}^3x \left(\frac{1}{2}|\nabla\varphi|^2 + U_\omega(\varphi)\right) + \omega Q \tag{3.2}$$

where an effective potential

$$U_\omega(\varphi) \equiv U(\varphi) - \frac{1}{2}\omega^2\varphi^2 \tag{3.3}$$

has been defined. All time dependence occurs in the first term from which we can see that classical solutions take the form $\varphi(x, t) = \varphi(x)e^{i\omega t}$ for some real parameter $\omega$ and a real function $\varphi(x)$ that minimises

$$\int \mathrm{d}^3x \left(\frac{1}{2}|\nabla\varphi|^2 + U_\omega(\varphi)\right) \tag{3.4}$$

i.e. the problem reduces to finding the bounce solution associated with tunnelling in three Euclidean dimensions [114]. Solutions to this problem, and therefore Q-balls, generically exist if there are two minima of $U_\omega(\varphi)$ for a finite range of $\omega$; one at $\varphi = 0$ and a second *lower* minimum at $\varphi = \varphi_0 \neq 0$. When the original potential is everywhere positive this is equivalent to demanding the function $\mu^2(\varphi) = U(\varphi)/\varphi^2$ is minimised away from the origin.

If $U(\varphi)$ is allowed to take negative values the situation changes. Of interest here are models where the vacuum $\varphi = 0$ is only metastable due to the model having a second minimum, at $\varphi = \varphi_0 \neq 0$, where the potential $U(\varphi_0) = U_0$ is negative; the true vacuum. The effective potential (3.3) then automatically has a minimum at non-zero $\varphi$ for any choice of $\omega$. This is clearly true for $\omega = 0$ where said minimum coincides with the true vacuum. As $\omega$ is increased the effective potential becomes more negative and the second minimum is pushed to larger values of $\varphi$ until $\omega = m_\varphi$. At this point the minimum at the origin is destroyed but, as long as the allowed range of $\omega$ is non-trivial (in other words $m_\varphi > 0$) Q-ball solutions will always be supported. Increasing $\omega$ can never destroy the second minimum as higher order terms protect it. If they do not the original potential cannot be bounded from below.

To get a handle on what Q-balls look like we can use their spherical symmetry [114] to show that they satisfy

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} = -\frac{2}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} + \frac{\mathrm{d}U_\omega}{\mathrm{d}\varphi} \tag{3.5}$$

for radial coordinate $r$. The problem can be visualised as the damped Newtonian motion of a particle in a potential $-U_\omega(\varphi)$ with respect to 'time' $r$. Q-ball solutions start from rest at non-zero $\varphi$ and come to rest again after infinite time at $\varphi = 0$. In the absence of the damping term the particle would have to start between $\varphi_0$ and zero where $U_\omega(\varphi) = 0$, hence the full solution should lie somewhere a little further out than this where $U_\omega(\varphi) < 0$. Since $U_\omega(\varphi) < U(\varphi)$ this alone does not suggest that the actual potential of the Q-ball is negative, but for large enough Q-balls it is indeed the case.

To see why note that decreasing $\omega$ both moves the starting point of the motion away from the origin and decreases the curvature of the effective potential (see figure 3.1). The particle thus starts off more slowly and has further to travel so inevitably takes longer to reach $\varphi = 0$. In terms of the Q-ball this means that $\varphi$ is larger for a

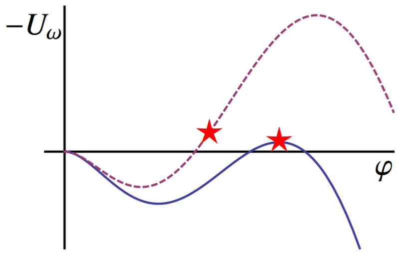


Figure 3.1: When finding Q-ball solutions one can consider damped Newtonian motion of a particle in a potential $-U_\omega(\varphi)$. The particle comes to rest after infinite time at the origin so it starts just beyond the point where $U_\omega(\varphi) = 0$ (stars). As $\omega$ increases (dashed) the starting point moves closer to the origin and the potential becomes steeper.

greater range of $r$, or small $\omega$ corresponds to a large Q-ball (see e.g. refs. [114, 124] for a more rigourous argument). Conversely, when a Q-ball is large enough $\omega$ is sufficiently small that the interior potential itself is negative, not just the effective potential.

As a function of charge the total Q-ball energy [114] can be expressed

$$E(Q) = \frac{Q^2}{2\int \mathrm{d}^3x\, \varphi(x)^2} + E_S + E_V \tag{3.6}$$

where $E_S$ and $E_V$ are the surface and potential (or volume) energies, and the first term can be thought of as kinetic energy. The surface energy is always positive but suppose the potential becomes negative and consider varying the radius of the Q-ball by a scaling factor $\alpha$ [115, 116]; the energy goes like

$$E_\alpha(Q) = \frac{1}{\alpha^3}\frac{Q^2}{2\int \mathrm{d}^3x\, \varphi(x)^2} + \alpha^2 E_S - \alpha^3 |E_V|. \tag{3.7}$$

For small values of $Q$ there are two stationary points with respect to $\alpha$: a local miniumum at $\alpha = 1$ (this is the Q-ball solution so exists by assumption) and a local maximum at $\alpha > 1$. However, as the charge is increased the two solutions move closer together until some critical charge $Q_c$ where there is only one, unstable stationary point. Above this critical charge $\alpha$ diverges, i.e. the Q-ball expands to fill the universe with its own internal state, precipitating a phase transition. The situation is summarised in figure 3.2.

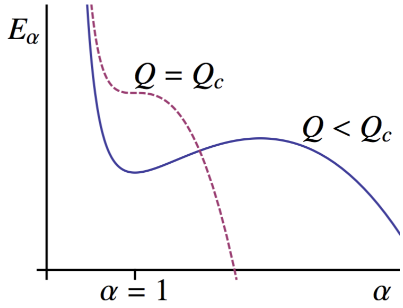


Figure 3.2: Q-ball energy in a metastable vacuum as a function of size $\alpha$. For charges less than the critical charge $Q_c$ there is a stable solution at $\alpha = 1$. For charges greater than or equal to $Q_c$ there are no stable solutions and the Q-ball expands to fill the universe, initiating a phase transition.

To examine the properties of critical Q-balls one typically needs to make some kind of approximation. First, suppose the Q-ball has radius $R$ and can be approximated by taking $\varphi(r) = \bar{\varphi} =$ constant inside the Q-ball and zero everywhere else. The energy can be rewritten

$$E(Q) = \frac{3Q^2}{8\pi R^3 \bar{\varphi}^2} + 4\pi R^2 S - \frac{4}{3}\pi R^3 |U(\bar{\varphi})|. \tag{3.8}$$

for the surface factor $S$, approximated by [114, 116]

$$S = \int_0^{\bar{\varphi}} \mathrm{d}\varphi \sqrt{2U(\varphi)}. \tag{3.9}$$

This is the thin wall limit. It can be derived by ignoring the damping term in the equation of motion then integrating to show $\varphi' = \sqrt{2U_\omega(\varphi)}$ on the surface, and is valid for large Q-balls when the two vacua of the original potential are nearly degenerate. Specifically, the energy barrier $\Delta U$ is much greater than the depth of the global minimum $|U_0|$ (see the solid curve in figure 3.1). We can then take $\bar{\varphi} \approx \varphi_0$ so the Q-ball interior coincides with the true vacuum, upon which the critical charge is defined explicitly by finding the value of $Q$ for which both the first and second derivatives of the energy vanish:

$$\frac{\mathrm{d}E}{\mathrm{d}R} = \frac{\mathrm{d}^2 E}{\mathrm{d}R^2} = 0 \quad \Longrightarrow \quad Q_c = \frac{100\sqrt{10}\pi S^3 \varphi_0}{81|U_0|^{5/2}}. \tag{3.10}$$

In addition, one can deduce the value of $R$ for which this occurs and write down the volume and energy of a critical Q-ball

$$V_c = \frac{500\pi S^3}{81|U_0|^3} \qquad\qquad E(Q_c) = V_c|U_0| \tag{3.11}$$

which will be of use later.

Alternatively the original potential could be such that $\Delta U \lesssim |U_0|$ and the two vacua are highly non-degenerate (the dashed curve in figure 3.1). Now the thick wall approximation [124] is more suitable. As mentioned earlier a large Q-ball corresponds to small $\omega$ so the limit $\omega \to 0$ can be taken for a critical Q-ball. The results of ref. [124] then yield a critical volume and energy[1]

$$V_c \sim \frac{1}{m_\varphi^3} \qquad\qquad E(Q_c) \sim V_c \Delta U \tag{3.12}$$

where $\Delta U$ is the height of the potential barrier. In both thin and thick wall cases the energy approaches a constant value when the charge becomes large, i.e. $\mathrm{d}E/\mathrm{d}Q \to 0$. This is manifest for the thick wall approximation; for the thin wall analogue it can be shown by substituting the above equations into the initial expression for the energy and noting that $|U_0|$ is small. Since the critical charge is the point at which the surface and potential energies balance it is perhaps not surprising that an extra unit of charge merits an almost equal and opposite contribution from each.

### 3.1.1 Solitosynthesis

The key difference between a Q-ball induced phase transition and tunnelling directly to the true vacuum is that a critical Q-ball does *not* have to form spontaneously. Instead it can grow gradually by accreting charge from its surroundings until it reaches the critical size. Stability is ensured at any stage in this process by charge conservation and, in models where Q-balls exist, there are classical solutions for arbitrarily small charges [124]. These initial Q-balls can be formed through particle

[1] Strictly speaking, ref. [124] only applies to Q-balls with a small enough charge (or large enough $\omega$). When $Q$ is too large the energy does not have a stationary point with respect to $\omega$. However, when the potential has a sufficiently deep global minimum the thick wall approximation remains valid even for $\omega \to 0$. The energy is minimised (albeit is not stationary) at this extremal value of $\omega$.

interactions, but it is more likely they are remnants of a previous phase transition (we will see an example of this in the next chapter) or were generated through field fluctuations when the universe was still very hot. Small charge solutions are expected to be resilient to quantum fluctuations above a charge of about seven [125].

In refs. [116–118] it was shown that one can start from a small Q-ball and gradually build up the charge through "solitosynthesis", where the accretion is facilitated by a chain of reactions in thermal equilibrium

$$\varphi_i + B(Q) \longleftrightarrow B(Q+q_i) \tag{3.13}$$

and $B(Q)$ denotes a Q-ball of charge $Q$. Other reactions like $2B(Q) \leftrightarrow B(2Q)$ can contribute, but are unimportant for large charge Q-balls as collisions between such objects are rare. Instead Q-balls accrete the vast majority of their charge from the sea of $\varphi$'s. Since the reactions are all in thermal equilibrium we can equate chemical potentials at each stage to find $\mu[B(Q)] = Q\mu[\varphi]$. From these expressions the number density relationship

$$n_{B(Q)} = \frac{g_{B(Q)}}{g_\varphi}\left(\frac{E(Q)}{m_\varphi}\right)^{3/2}\left(\frac{2\pi}{m_\varphi T}\right)^{3(Q-1)/2} e^{[m_\varphi Q - E(Q)]/T} \tag{3.14}$$

follows, where the $g$'s denote the number of degrees of freedom associated with each species. Taking $Q \gg 1$ and $g_{B(Q+1)} \approx g_{B(Q)}$ then gives

$$\frac{\mathrm{d}}{\mathrm{d}Q}\ln n_{B(Q)} \approx \frac{n_{B(Q+1)} - n_{B(Q)}}{n_{B(Q)}} \approx \left(\frac{n_\varphi}{g_\varphi}\right)\left(\frac{2\pi}{m_\varphi T}\right)^{3/2} e^{b_Q/T} - 1 \tag{3.15}$$

where $b_Q = m_\varphi - [E(Q+1) - E(Q)] \approx m_\varphi - \mathrm{d}E/\mathrm{d}Q$ is the binding energy. Furthermore, charge conservation implies that $\eta n_\gamma = n_\varphi + \sum Q n_{B(Q)}$ for a net charge asymmetry $\eta$ (the charge per photon). Ignoring the back reaction of the Q-balls this suggests $n_\varphi = \eta n_\gamma \sim \eta T^3$. Putting it all together we find that the right hand side of eq. (3.15) become positive, ergo the critical Q-ball population grows exponentially, when

$$\begin{aligned} T_c &= \left(m_\varphi - \left.\frac{\mathrm{d}E}{\mathrm{d}Q}\right|_{Q=Q_c}\right)\left(|\ln\eta| + \frac{3}{2}\ln\frac{m_\varphi}{T_c} - \ln g_\varphi\right)^{-1} \\ &\approx \frac{m_\varphi}{|\ln\eta|}. \end{aligned} \tag{3.16}$$

The charge asymmetry is expected to be small so the $|\ln \eta|$ term dominates the denominator whereas the derivative term has been set to zero owing to the reasons presented earlier.

Evidently a vital requirement for solitosynthesis is the size of the chemical potential; charge conservation means it is large, counteracting the Q-ball Boltzmann factor. Even though their formation is energetically favourable, Q-balls inevitably have a relatively large energy and thus a suppression $e^{-E(Q)/T}$ in their number density. However, when $\mu[B(Q)] = Q\mu[\varphi]$ is included this decreases to

$$e^{(Q\mu[\varphi]-E(Q))/T} > e^{Q(\mu[\varphi]-m_\varphi)/T} \sim \left(\frac{T}{m_\varphi}\right)^{3Q/2} \tag{3.17}$$

using $n_\varphi = \eta n_\gamma \sim \eta T^3$ for the last relation. Consequently the suppression is much smaller than one may originally expect. A second thermodynamic consideration is the entropy, which is clearly reduced by consolidating charge into Q-balls. The absorption of a single $\varphi$ changes the Helmholtz free energy by

$$\Delta F = -b_Q - T(s_{B(Q+1)} - s_{B(Q)} - s_\varphi) \tag{3.18}$$

for entropy per particle $s$, and only favours Q-ball production if negative. For non-relativistic particles one anticipates each $s$ to be of order one, hence critical Q-ball formation is allowed below $T \sim m_\varphi$.

To reach the critical charge unhindered the freeze out temperature of the accretion reactions, $T_f$, must be less than $T_c$. Links in the chain involving Q-balls typically have a large cross section (about the physical size of the Q-ball) so the process is limited by the reactions keeping the $\varphi$'s in equilibrium. Hence $T_f$ is the freeze out temperature of the $\varphi$'s, whereupon general thermodynamical arguments lead to

$$T_f \approx \frac{m_\varphi}{\ln\left(M_P m_\varphi \sigma\right)} \tag{3.19}$$

with $\sigma$ the cross section for the annihilation of $\varphi$'s to light particles. Note that this temperature is less than $m_\varphi$ for reasonable choices of $\sigma$ so there is no conflict with the constraint arising from the free energy. Ergo we will generally find a window of temperature $T_f < T < m_\varphi$ in which critical Q-balls are able to form.

Enforcing the inequality $T_c > T_f$ now collapses to a bound on the charge asymmetry

$$\eta > \frac{1}{M_P m_\varphi \sigma}. \tag{3.20}$$

Even if the overall charge of the universe is zero, there will always be a statistical contribution to $\eta$ over a finite region of space. In a comoving volume $V$, the relative excess of charge goes like $1/\sqrt{n_\varphi V}$ ($n_\varphi V$ being the total number of $\varphi$'s) so the charge asymmetry is given by

$$\eta_{\text{stat}}(V) = \frac{n_\varphi - n_{\varphi^*}}{n_\gamma} = \left(\frac{n_\varphi - n_{\varphi^*}}{n_\varphi}\right)\frac{n_\varphi}{n_\gamma} \sim \frac{1}{\sqrt{n_\varphi V}}\frac{n_\varphi}{n_\gamma} \tag{3.21}$$

for messenger and photon number densities $n_\varphi$ and $n_\gamma$. From eq. (3.20) this is large enough to support critical Q-balls all the way down to the freeze out temperature in any volume

$$V < \frac{n_\varphi}{n_\gamma^2}(M_P m_\varphi \sigma)^2 \sim \frac{M_P \sigma}{m_\varphi^2}\left[\ln\left(M_P m_\varphi \sigma\right)\right]^{9/2}. \tag{3.22}$$

If a critical Q-ball comfortably fits into this volume, i.e.

$$V_c \ll \frac{M_P \sigma}{m_\varphi^2}\left[\ln\left(M_P m_\varphi \sigma\right)\right]^{9/2} \tag{3.23}$$

statistical fluctuations alone are enough to seed their formation. Physically, we expect to form equal numbers of critical Q-balls carrying both positive and negative charge by this approach. As they destabilise and expand, their boundaries inevitably collide and the net charge is annihilated when the system relaxes to the true vacuum.

The final obstacle to solitosynthesis is the presence of light, charged fermions or vector bosons. If these exist, they will absorb any charge instead of the Q-balls and solitosynthesis will not take place. Since all charged scalars are automatically massive enough not to encroach on critical Q-ball formation, it is sufficient to check that none of the charged fermions and vector bosons in equilibrium at $T \sim m_\varphi$ are lighter than the lightest charged scalar. Assuming this and all other conditions are met, the time at which the phase transition happens can be estimated via the Hubble time scale at the temperature $m_\varphi$:

$$t_c \sim \frac{M_P}{m_\varphi^2}. \tag{3.24}$$

This demands a scalar mass less than $10^{-11}$ GeV for $t_c$ to be greater than the age of the universe, $10^{10}$ years. As such, the time taken for critical Q-balls to form imposes no additional constraints on whether the metastable vacuum decays or not.

In principle the global U(1) symmetry could be broken in the UV completion of the theory and emerge only as an approximate symmetry in the IR. This would be the case in stringy models, for example, where global symmetries are not respected by gravity. Any effects will be highly suppressed but could provide a decay channel for large Q-balls, where a large internal field value counters the suppression. An approximation of the decay rate in the thin wall limit (a parametrically similar argument applies to thick wall Q-balls) can be made by considering Planck suppressed operators inside the Q-ball of the schematic form

$$\mathcal{L} \supset \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2 \, \frac{\varphi^{2+n}}{M_P^n} \supset \frac{\omega^2 \bar{\varphi}^{n-1}}{M_P^n} \varphi^3 \tag{3.25}$$

for some positive integer $n$. Such terms allow U(1) violating decays, leading to evaporation with

$$\dot{Q} \sim -\int \mathrm{d}V \, \omega\varphi^2 \Gamma \tag{3.26}$$

where $\omega\varphi^2$ gives the charge density and $\Gamma$ is the decay rate. Setting $\varphi \sim \varphi_0$ and taking the decay rate to be constant at $\Gamma \approx \omega^3 \varphi_0^{2n-2}/4\pi M_P^{2n}$ we find

$$\dot{Q} \sim -\frac{\omega^3 \varphi_0^{2n-2}}{4\pi M_P^{2n}} Q \quad \Longrightarrow \quad \tau \sim \frac{4\pi M_P^{2n}}{\omega^3 \varphi_0^{2n-2}}. \tag{3.27}$$

As long as $\tau \gg M_P/m_\varphi^2$, or equivalently

$$\frac{4\pi m_\varphi^2 M_P^{2n-1}}{\omega^3 \varphi_0^{2n-2}} \gg 1 \tag{3.28}$$

there is no effect on critical Q-ball formation. Since $\varphi_0 \ll M_P$ in any reasonable model and $\omega < m_\varphi$ in all Q-ball solutions this inequality always holds.

When they do occur, phase transitions can occur in one of two ways as illustrated in figure 3.3. First is *direct* decay: the metastable vacuum decays, via Q-balls, directly to the true vacuum. In this approach the interior of the Q-ball is stable in all directions and roughly coincides with the true vacuum, where the corresponding U(1) symmetry is necessarily broken. A more subtle variant that can occur when the model contains multiple fields is *indirect* decay. Even if the U(1) symmetry persists

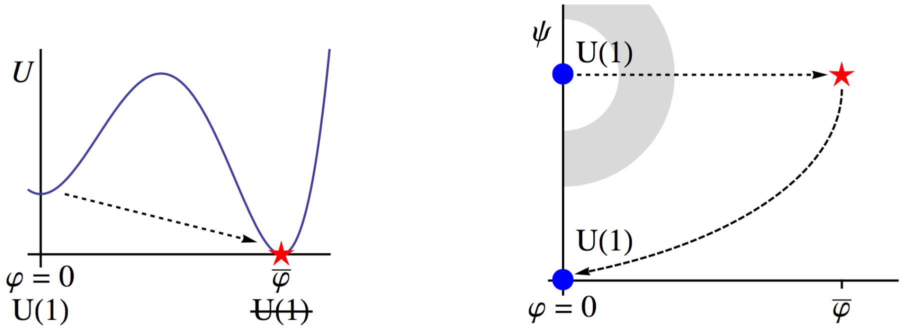


Figure 3.3: Left: Direct decay. The interior state of the Q-ball (star) coincides with the true vacuum with broken U(1). Decay from the metastable vacuum proceeds directly through synthesis of critically charged Q-balls. Right: Indirect decay. The metastable vacuum (top circle) decays through a potential barrier (shaded) through synthesis of critically charged Q-balls involving the field $\psi$. After this decay, the model flows to the true vacuum (bottom circle) where U(1) is restored.

in the true vacuum there is always a finite region around it where the symmetry is broken yet the potential remains negative. Such a region can support Q-ball solutions that quickly decay to the true vacuum once a phase transition has taken place. In other words, Q-balls can do the hard work of tunnelling through the potential barrier after which the model quickly completes the transition of its own accord. We will see examples of both variants shortly.

In summary, Q-balls are generically able to induce vacuum decay in models of massive scalar fields where the metastable vacuum supports an unbroken, global U(1) symmetry. The main barriers to decay are cosmological: either the reactions responsible freeze out before critical Q-balls become important or light fermions/vector bosons absorb charge in place of Q-balls. The former does not occur if the charge asymmetry satisfies eq. (3.20), the latter if any charged fermions or vector bosons are heavier than the lightest charged scalar. Furthermore if eq. (3.23) is true, statistical fluctuations in the charge asymmetry can seed critical Q-balls. When a decay does take place the lifetime of the metastable vacuum is given by eq. (3.24) and the temperature of the universe at decay lies between $T_f < T < m_\varphi$. These results are readily generalised to models with multiple scalar fields using ref. [119].

## 3.2 Messenger number and gauge mediation

It is now possible to apply these ideas to some popular models of metastable SUSY breaking and search for instances of solitonic SUSY restoration (SSR). A simple example of direct SSR can be found in the minimal model of gauge mediation [55] discussed in section 1.3. SUSY is broken by the $F$-term of a chiral superfield $X$ acquiring a VEV, then transmitted to the visible sector by a single pair of messenger chiral superfields $\varphi$ and $\tilde{\varphi}$ charged under the Standard Model gauge group (e.g. in the $\mathbf{5}+\overline{\mathbf{5}}$ representation of SU(5)).

Communication is via superpotential interactions

$$W = W_{\text{SB}}(X, \psi) + X\tilde{\varphi}\varphi + M\tilde{\varphi}\varphi. \tag{3.29}$$

$W_{\text{SB}}$ denotes the (unspecified) superpotential of the SUSY breaking sector – it is a function of $X$ and some other chiral superfields $\psi$ – and an explicit messenger mass $M$ has been included (the coupling constant in the second term has been absorbed into the field $X$). This model clearly admits a global U(1) messenger number symmetry, under which $\varphi$ and $\tilde{\varphi}$ have charges $+1$ and $-1$. Messenger number is in fact necessarily conserved in any vacuum that does not break the Standard Model gauge group. Note that the explicit mass term precludes the possibility of the model having an R-symmetry. This is not strictly necessary, but allows for large gaugino masses due to the model having a supersymmetric vacuum [34].

The $F$-terms derived from eq. (3.29) are

$$\begin{aligned} F_X &= F(X, \psi) + \tilde{\varphi}\varphi \\ F_\varphi &= (M + X)\tilde{\varphi} \\ F_{\tilde{\varphi}} &= (M + X)\varphi \end{aligned} \tag{3.30}$$

with $F(X, \psi) = \partial W_{\text{SB}}/\partial X$ responsible for breaking SUSY in the metastable vacuum, where it takes the value $F$. The remaining $F$-terms are set to zero by choosing

$$|\text{vac}\rangle_+ : \qquad \varphi_+ = 0 \qquad \tilde{\varphi}_+ = 0 \tag{3.31}$$

whereas the VEV of $X$, fixed at some value $X_+$, is determined by the details of the SUSY breaking sector. It is straightforward to check that the tree level messenger masses are

$$m_0^2 = \bar{M}^2 \pm F\,, \quad m_{1/2}^2 = \bar{M}^2 \quad \text{where} \quad \bar{M} = |M + X_+| \tag{3.32}$$

with the usual requirement $\bar{M}^2 > |F|$ imposed by vacuum stability. $X$ is the goldstino superfield (models where the goldstino is a linear combination involving other fields will be discussed later) so its scalar component is a classical flat direction, or pseudo-modulus [39, 40]. Hence the VEV of $X$ can be chosen at will without affecting the tree level potential and one finds an additional, supersymmetric minimum at

$$|\text{vac}\rangle_0 : \qquad \tilde{\varphi}_0\varphi_0 = -F \qquad X_0 = -M \qquad U_0 = -F^2. \tag{3.33}$$

Of course, for the SUSY breaking vacuum to be locally stable the scalar component of $X$ must be stabilised by a mass term arising from loop corrections. This mass term disappears in the supersymmetric vacuum where the loop corrections vanish.

The above model has all the ingredients for direct SSR; a metastable vacuum with an unbroken U(1) symmetry, charged scalar fields and a supersymmetric vacuum where the symmetry is broken. However, before reaching any conclusion it must be checked that Q-balls can be built up via solitosynthesis (i.e. there are no light charged fermions or vector bosons and the messengers remain in thermal equilibrium) and that the critical Q-ball formation temperature $T_c$ is greater than the freeze out temperature of the messengers $T_f$ (i.e. eq. (3.20) is satisfied). The only fields carrying messenger number are the messengers themselves and their masses were calculated in eq. (3.32): there is always a charged scalar lighter than the lightest charged fermion so solitosynthesis proceeds unhindered. Meanwhile messengers are kept in thermal equilibrium by Standard Model gauge interactions, at the messenger scale $\bar{M}$, so the appropriate freeze out cross section is $\sigma \sim g_{\text{SM}}^2/\bar{M}^2 \sim 1/\bar{M}^2$. We thus finds that any charge asymmetry

$$\eta \gtrsim \frac{\bar{M}}{M_P} \tag{3.34}$$

is enough to build critical Q-balls.

Statistical fluctuations of $\eta$ seed critical Q-ball formation if eq. (3.23) is satisfied, but there are two cases we must consider. If the SUSY breaking is small then the messenger mass $\bar{M}$ is much greater than the supersymmetric mass splitting $\sqrt{F}$, therefore the potential barrier is much larger than the difference in energy between the metastable and supersymmetric vacua. This is the thin wall limit so the critical volume is given by eq. (3.11):

$$V_c \approx 50 \left(\frac{\bar{M}}{F}\right)^3 \tag{3.35}$$

where the surface factor (3.9) has been approximated in the limit $F \ll \bar{M}^2$ by

$$S \approx \sqrt{(X_0 - X_+)^2 + 2(\varphi_0 - \varphi_+)^2}\sqrt{2U_0} \approx \sqrt{2}\bar{M}F. \tag{3.36}$$

Eq. (3.23) is satisfied unless

$$\bar{M} > \left(\frac{F}{\bar{M}}\right)^{3/4} \left(\frac{M_P}{50}\right)^{1/4} \left[\ln\left(\frac{M_P}{\bar{M}}\right)\right]^{9/8}. \tag{3.37}$$

leaving open a substantial region in parameter space for which all constraints for SSR are satisfied and the metastable vacuum decays. Conversely, if SUSY breaking is large $F$ and $\bar{M}^2$ are of a similar order and one should use the thick wall limit (3.12) for the critical volume instead. The result is a modified bound

$$\bar{M} > M_P \left[\ln\left(\frac{M_P}{\bar{M}}\right)\right]^{9/2} \tag{3.38}$$

for a viable vacuum. This cannot hold for any messenger mass below the Planck scale. In other words, SSR always takes place in the case of large SUSY breaking.

Using the fact that the gaugino mass is given by $F/16\pi^2\bar{M} \sim 1$ TeV, we find an absolute bound

$$\bar{M} > 10^8 \text{ GeV} \tag{3.39}$$

on the messenger mass in the case of small SUSY breaking. When SSR does occur, is does so at time (3.24)

$$t_c \sim \frac{M_P}{\bar{M}^2} \tag{3.40}$$

which is much less than the age of the universe, $10^{10}$ years, for all realistic choices of messenger mass (e.g. $t_c \sim 10^{-8}$ s for $\bar{M} \sim 1$ TeV and decreases as $\bar{M}$ gets larger).

### 3.2.1 Beyond minimal gauge mediation

In the above only a stripped down version of gauge mediation was considered. The most obvious way to go beyond the minimal model is to add more messenger fields. This barely changes the conclusions reached above. Indeed, we could extend the superpotential (3.29) to

$$W = W_{\rm SB}(X,\psi) + \lambda_{ij} X \tilde{\varphi}_i \varphi_j + M_{ij} \tilde{\varphi}_i \varphi_j \tag{3.41}$$

for some coupling constants $\lambda_{ij}$. The messenger mass matrix $\lambda X_+ + M$ can always diagonalised, upon which the constraints (3.39) must be satisfied with $\bar{M}$ replaced by the smallest eigenvalue of $\lambda X_+ + M$.

Alternatively we could generalise the SUSY breaking sector so $X$ does not coincide exactly with the goldstino superfield, hence is not necessarily a pseudo-modulus. We must then consider the details of the SUSY breaking sector to see if and where a supersymmetric vacuum occurs. Assuming there is one, SSR proceeds much as before. Light charged fermions and vector bosons remain absent and the temperatures $T_f$ and $T_c$ depend only on the messenger sector so are unchanged.

Even if messenger number persists in the new supersymmetric vacuum there will be a finite region around it where the messenger VEV is non-zero and the relative potential is negative, allowing for indirect SSR. However, the final constraints (3.39) will be different; they depend on how fields from the SUSY breaking sector affect the Q-ball configuration. For example, new field VEVs appear in the surface factor (3.9) and subsequently the critical volume. This in turn is vital for figuring out whether statistical charge fluctuations can seed critical Q-balls, which must be considered on a case by case basis.

## 3.3 Baryon number and the ISS model

One may also ask whether Q-balls can destabilise metastable vacua in the absence of an explicit messenger sector. A popular example of metastable SUSY breaking we have already seen is the ISS model [74]. This permits a global symmetry group

$SU(N_f) \times U(1)_B \times U(1)_R$ under which

$$q \in (\square, +1, 0) \qquad \tilde{q} \in (\tilde{\square}, -1, 0) \qquad \Phi \in (\mathbf{Adj} + \mathbf{1}, 0, 2). \tag{3.42}$$

For $N_f > 3N$ the model is infrared free and supersymmetry is broken due to the rank condition in the metastable vacuum

$$|\text{vac}\rangle_+ : \qquad \tilde{q}_+ q_+ = m^2 \text{diag}(\mathbb{1}_N, 0) \qquad \Phi_+ = 0 \tag{3.43}$$

the vacuum energy is $(N_f - N)h^2 m^4$ and there is a residual global symmetry group of $SU(N) \times SU(N_f - N) \times U(1)_{B'} \times U(1)_R$. As discussed in section 1.4.3 there is another, supersymmetric minimum at

$$|\text{vac}\rangle_0 : \qquad \tilde{q}_0 = 0 \qquad q_0 = 0 \qquad \Phi_0 = \frac{1}{h}\Lambda \left(\frac{m}{\Lambda}\right)^{2N/(N_f - N)} \mathbb{1}_{N_f}. \tag{3.44}$$

As long as $m \ll \Lambda$ all calculations are under control and the metastable vacuum is seemingly long lived.

The U(1) symmetries in the SUSY breaking vacuum comprise a baryon number and an R-symmetry, however the R-symmetry is anomalous[2] leaving baryon number as the prime candidate for SSR. As before, it is convenient to expand around the metastable vacuum using degrees of freedom

$$\Phi = \begin{pmatrix} Y & \tilde{Z} \\ Z & X \end{pmatrix} \qquad q = \begin{pmatrix} m\mathbb{1}_N + \chi \\ \rho \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} m\mathbb{1}_N + \tilde{\chi} \\ \tilde{\rho} \end{pmatrix} \tag{3.45}$$

($Y$ and $X$ are $N \times N$ and $(N_f - N) \times (N_f - N)$ matrices and the dimensions of the other components follow suit) with baryon numbers

$$\begin{aligned} B'(Y) = B'(X) = B'(\chi) = B'(\tilde{\chi}) &= 0 \\ B'(\rho) = B'(\tilde{Z}) &= +1 \\ B'(\tilde{\rho}) = B'(Z) &= -1. \end{aligned} \tag{3.46}$$

[2]Besides, spontaneously broken R-symmetries come with an exactly massless, charged fermion in SUSY breaking vacua – the goldstino – so do not allow for solitosynthesis. This renders them defunct from an SSR point of view.

The bosonic and fermionic components of all charged fields acquire masses of order $hm$, with the exception of the scalar combinations $\mathrm{Re}[\rho+\tilde{\rho}]$ and $\mathrm{Im}[\rho-\tilde{\rho}]$ which are massless Goldstone bosons of the various broken symmetries.

The presence of these massless, charged scalars prevents the formation of Q-balls. To see why, note that the effective potential (3.3) has no minimum at the origin for any non-zero value of $\omega$: the addition of a mass term $-1/2\omega^2\varphi^2$ to all charged scalars renders any massless ones tachyonic. However, when the ISS sector is employed in a direct mediation scenario (as in often the case [1, 62, 65, 95, 97–102, 104–107]) the flavour group is gauged; the prospective Goldstone bosons are eaten by gauge fields of the broken symmetry which gain a mass $g_{\mathrm{SM}}m$ via the super Higgs mechanism.

Another subtlety arises from the fact that baryon number remains unbroken in the supersymmetric vacuum. This is not actually a problem as there must exist a finite region in field space around the supersymmetric vacuum where baryon number *is* broken but the potential remains negative. Indirect SSR remains possible. Consider, for example, starting from the supersymmetric vacuum and giving all components of the meson a VEV $\hat{\Phi}$, but leaving the quark VEVs fixed at zero. The relative scalar potential is calculated from the dynamical ISS superpotential and goes like

$$U(\hat{\Phi}) \sim Nh^2m^4 - 2N_f h^{(N_f+N)/N} m^2 \Lambda^{(3N-N_f)/N} \hat{\Phi}^{(N_f-N)/N} + N_f h^{2N_f/N} \Lambda^{2(3N-N_f)/N} \hat{\Phi}^{2(N_f-N)/N}. \tag{3.47}$$

It is negative for a range of $\hat{\Phi}$ with similar magnitudes to $\Phi_0$, and baryon number is broken whenever the $Z$'s are non-zero. Hence Q-ball solutions will occur along $Z$ directions with interior field values of order $\Phi_0$.

Solitosynthesis is unimpeded by the charged gauge bosons if $h \leq g_{\mathrm{SM}}$ such that they are heavier than their scalar counterparts. All charged fermions already satisfy this constraint as their masses are equal to those of the scalars at $hm$. The only thing left to check is whether eq. (3.23) holds, such that statistical fluctuations in the charge asymmetry are capable of seeding critical Q-balls. We must first decide whether to work in the thin or thick wall limit for the purposes of calculating the critical volume. Since the charged scalars have masses $hm$ and the metastable vacuum energy is order $h^2m^4 > (hm)^4$ (recall, $h$ is a perturbative coupling constant) the vacua are highly non-degenerate and the thick wall approximation is most suitable. Once again the scalars are kept in equilibrium through Standard Model gauge interactions so $\sigma \sim g_{\rm SM}^2/h^2m^2 \sim 1/h^2m^2$ and, using the critical volume given in eq. (3.12) with $m_\varphi = hm$, eq. (3.23) is satisfied unless

$$hm > M_P \left[\ln\left(\frac{M_P}{hm}\right)\right]^{9/2}. \tag{3.48}$$

The result is, in fact, identical to that found for large SUSY breaking in minimal gauge mediation but with a messenger mass $hm$. This is of course due to the fact that direct mediation in the ISS model is a specific example of gauge mediation; the $\rho$'s and $Z$'s act as messengers and their masses are $hm$. The conclusions reached in the previous section thus hold here; any realistic choice of $hm$ (i.e. less than about $M_P$) results in SSR on a timescale much less than the age of the universe.

In summary, SSR occurs in models of direct mediation using an ISS SUSY breaking sector, unless $h > g_{\rm SM}$ where $g_{\rm SM}$ is the Standard Model gauge coupling and $h$ a perturbative coupling constant. It does not occur when the Goldstone bosons in the metastable vacuum corresponding to broken flavour symmetries remain massless, but any mechanism or deformation that gives them a mass would yield similar results. The phase transition takes place at time (3.24)

$$t_c \sim \frac{M_P}{h^2m^2} \tag{3.49}$$

which is again much less than the age of the universe unless $hm$ is extremely small (and well into the observable range).

## 3.4 Summary

Q-balls induce vacuum decay in a large class of models, and those of metastable SUSY breaking are no exception. So long as the metastable vacuum has an unbroken, global U(1) symmetry and no massless, charged scalars Q-balls always exist, with negative interior potential for a sufficiently large charge. After reaching a critical charge, they precipitate a phase transition to the supersymmetric vacuum regardless of whether or not the U(1) symmetry is preserved there. In the absence of light charged fermions or vector bosons, and if the scalars remain in thermal equilibrium, these critical Q-balls are able to build up gradually via solitosynthesis and are stable throughout the process due to charge conservation.

Furthermore, there is a range of parameter space where critical Q-balls are seeded by statistical charge fluctuations so no overall charge asymmetry is required. In order to calculate this range one must decide whether to work in the thick or thin wall approximation, depending on whether the scale of SUSY breaking is comparable to or much less than the charged scalar masses respectively. Having done so, the timescale of vacuum decay can easily be estimated and is almost always much less than the age of the universe.

These ideas have been applied to models of gauge mediation, where messenger number plays the role of the global U(1) symmetry, and direct mediation in the ISS model, where one can use baryon number. In both cases solitonic SUSY restoration occurs. Any messenger mass less than $10^8$ GeV results in vacuum decay with no net charge asymmetry in the minimal model of gauge mediation, with non-minimal models yielding similar (but model dependent) constraints. On the other hand messengers heavier than about 3 TeV are expected to overclose the universe, presenting a serious model building challenge. In the ISS model, SSR is dependent on the Goldstone bosons charged under baryon number getting a large enough mass. Direct mediation results in them being eaten by gauge bosons, that acquire a mass proportional to the Standard Model gauge couplings. Unless the gauge couplings are smaller than the perturbative coupling in the superpotential ($h > g_{\rm SM}$) solitosynthesis, ergo SSR, are unimpeded.

The simplest way to build long lived models of metastable SUSY breaking is to ensure that the spectrum contains massless charged scalars or charged fermions lighter than the lightest charged scalar. R-symmetries, for example, always meet this condition due to the massless Goldstino that is associated with SUSY breaking. Meanwhile the vanilla ISS model is also safe, as there are massless scalars with non-zero baryon number in the metastable vacuum which prevent the formation of Q-balls. Alternatively one could search for models of SUSY breaking with no unbroken U(1) symmetries at all, although even then one would have to check whether non-topological solitons corresponding to non-abelian symmetries resulted in SSR. Otherwise SSR is avoided by ensuring that there is no net charge asymmetry in the universe, and that statistical charge fluctuations are insufficient to build critical Q-balls.

It should be noted that this analysis assumes a reheat temperature greater than $T_f$. Another way to evade these conclusions is to not reheat to this temperature, but then it may no longer be possible for thermal effects to drive the theory to the metastable minimum in the first place [126–128]. One should also note that the same thermal effects, which lift the vacuum containing the fewest light degrees of freedom, could themselves prevent SSR. If the supersymmetric vacuum is lifted above its SUSY breaking counterpart, Q-balls no longer have negative internal potential and do not cause a phase transition (but may still form). Ref. [126] estimates the two vacua to be degenerate at $T \sim m_\varphi$ for the ISS model. Since solitosynthesis is active below this temperature the conclusions remain unchanged. For minimal gauge mediation we require knowledge of the SUSY breaking sector to find the detailed spectrum in both vacua, without which it is unclear which vacuum is lower at a temperature of the messenger scale. Although the visible sector gauge fields become massive in the supersymmetric vacuum, the messengers are much lighter. Hence for $\sqrt{F} < T < \bar{M}$ the SUSY breaking vacuum appears to have fewer light degrees of freedom, but for $T < \sqrt{F}$ the supersymmetric one does.

Ultimately the task of building a viable model of metastable SUSY breaking now seems even more daunting. Indeed, many existing models that appear to have long lived vacua are likely to be destabilised once Q-balls have been taken into account. Fortunately all is not lost. It is clear how SSR can be avoided, providing a concrete guide for future model building endeavours.

# Chapter 4

# Condensate cosmology in O'Raifeartaigh models

Having just seen that Q-balls pose a serious threat to metastable SUSY breaking vacua, we may wish to ask whether they have any other cosmological consequences. Probably the most well known example of their use in the visible sector is in Affleck-Dine baryogenesis [129]. Here, high scale SUSY breaking associated with inflation can result in a condensate with large expectation value forming along an MSSM flat direction charged under baryon number. If the model does not preserve it at high energies, this condensate carries significant fractional baryon number that survives to the present day. Originally it was thought that the condensate would evaporate into light fermions (imparting its baryon number to Standard Model degrees of freedom) but later studies [130–137] instead suggest that the condensate fragments into localised lumps, which coalesce into Q-balls [114].

Given that O'Raifeartaigh models have flat directions of their own, one can ask whether something similar happens in the hidden sector. Recent work [138–142] has certainly suggested that other, non-MSSM flat directions can have important consequences. Rather than baryon number, however, it is R-charge that is carried by flat directions in O'Raifeartaigh models, thus leading to *R-balls*: non-topological solitons stabilised by a global U(1) R-symmetry. Motivated by this observation, we will thus investigate the general, cosmological evolution of flat directions in O'Raifeartaigh models.

## 4.1 Lifting the flat direction

Any O'Raifeartaigh model can be recast in terms of a goldstino superfield $X$, whose F-term VEV is responsible for SUSY breaking, and some other superfields $\{\varphi_i\}$ [40]. In this basis we will allow the most general, renormalisable form for the superpotential

$$W = fX + (\mu_{ij} + \lambda_{ij}X)\varphi_i\varphi_j + \kappa_{ijk}\varphi_i\varphi_j\varphi_k \tag{4.1}$$

for coupling constants $f$, $\mu$, $\lambda$ and $\kappa$, where $f$ is assumed real and positive without loss of generality. Note that these symbols will frequently be used without their indices to denote the generic size of the couplings. The superfields $\varphi_i$ are defined so as to have vanishing VEVs whereas the scalar component of $X$ is our classical flat direction. Transforming into this basis may not respect the global symmetry group of the model but, in a model with an R-symmetric vacuum, $X$ clearly has R-charge +2. Any renormalisable O'Raifeartaigh model with an R-symmetric vacuum therefore possesses a flat direction with non-zero R-charge.

For the vacuum to be well defined $X$ cannot remain flat and must be stabilised. At low energy this is accomplished by quantum effects, typically at one loop via the Coleman-Weinberg potential [41] evaluated with respect to a UV cutoff scale $M$. Around this scale we may start to see non-renormalisable effects from an underlying microscopic theory, which could also lift the flat direction, but are highly suppressed at low energy so have little impact. Exactly where the Coleman-Weinberg potential is minimised depends on the details of the model but one can deduce its approximate form in two limits. Close to its minimum at $X = \langle X\rangle$ the effective potential goes like

$$U_{\text{eff}}(X) = U_0 + \frac{1}{2}m^2|X - \langle X\rangle|^2 + \mathcal{O}(|X - \langle X\rangle|^3). \tag{4.2}$$

If R-symmetry is preserved this minimum must be at $\langle X\rangle = 0$. The mass term $m^2$ depends on the couplings $f$, $\mu$ and $\lambda$ and one can deduce its value either by explicitly calculating the Coleman-Weinberg potential for a given model, or by using the more elegant methods developed in ref. [143]. Schematically it should go like

$$m^2 \sim \frac{\lambda^4 f^2}{16\pi^2\mu^2}. \tag{4.3}$$

At the other end of the scale when $X$ is large, specifically $|\mu + \lambda X|^2 \gg \lambda f$, the SUSY breaking seen by the rest of the fields is small. In fact an appeal to naturalness suggests $f$ and $\mu^2$ should not be too dissimilar so small SUSY breaking often implies $|\lambda X| \gg |\mu|$. In this limit we can find an alternative formulation for the effective potential by integrating out the massive $\varphi$'s. The process contributes an extra term to the Kähler potential [74]

$$K_{\text{eff}}(X, X^\dagger) = |X|^2 - \frac{1}{32\pi^2} \operatorname{Tr}\left[\mathcal{M}^\dagger\mathcal{M} \ln\left(\frac{\mathcal{M}^\dagger\mathcal{M}}{M^2}\right)\right] \tag{4.4}$$

and consequently leads to a one loop effective potential

$$U_{\text{eff}}(X) = \frac{|F_X|^2}{\partial_X \partial_{X^\dagger} K_{\text{eff}}} = f^2\left(1 + \frac{1}{16\pi^2}\sum_i |\lambda_i|^2 \left[1 + \ln\left(\frac{|\lambda_i X|}{M}\right)\right] + \mathcal{O}(\lambda^4)\right). \tag{4.5}$$

The mass matrix $\mathcal{M} = \lambda X$ has been taken from the superpotential (4.1) (neglecting $\mu$ in the limit of small SUSY breaking) where $\{\lambda_i\}$ are the eigenvalues of $\lambda$. If $\mu^2 \gg f$ for some reason, the Kähler potential is modified by the possible appearance of additional light states at certain points in the pseudo-moduli space, which cannot be integrated out. These lead to singular behaviour and our approximation must be reassessed. Regardless, all conclusions reached apply wherever $|\mu| \ll |\lambda X| \ll M$. Note also that eq. (4.5) is only valid up to second order in $f$ and $\lambda$. If the $\lambda$'s are large, or there are a vast number of non-goldstino superfields, higher loop corrections become important and invalidate the Coleman-Weinberg formula.

In the following we will be interested only in the potential relative to the SUSY breaking vacuum so will omit the constant term of eq. (4.2), which goes like $f^2$ up to loop effects. Normalising both potentials thus gives the final form

$$\begin{aligned} U_{\text{eff}}(X) &\approx \frac{1}{2}m^2|X - \langle X\rangle|^2 && \text{for } X^2 \ll f \\ U_{\text{eff}}(X) &\approx \frac{f^2}{16\pi^2}\sum_i |\lambda_i|^2\left[\zeta + \ln\left(\frac{|\lambda_i X|}{M}\right)\right] && \text{for } f \ll X^2 \ll M^2 \end{aligned} \tag{4.6}$$

for some order one parameter $\zeta$. For a model that is perturbatively well behaved and whose couplings satisfy $f \sim \mu^2$ this effective potential is illustrated in figure 4.1.

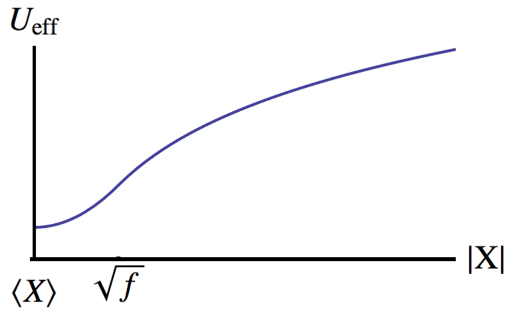


Figure 4.1: A typical low energy effective potential for the scalar component of the goldstino superfield $X$. At tree level $X$ is a flat direction but is lifted by loop effects. For $X^2 \ll f$ these are polynomial in nature and stabilise $X$ at some value $\langle X\rangle$ (equal to zero in an R-symmetric vacuum). For $X^2 \gg f$ the quantum corrections become logarithmic.

## 4.2 Cosmological evolution

Flat directions in the MSSM have been extensively studied (see e.g. ref. [120] and the references therein) and many of the techniques used are directly applicable here. In particular, a condensate forms along flat directions in a wide variety of models due to the Affleck-Dine mechanism [129]. The idea is that quantum fluctuations are spread out during an inflationary period of the universe's evolution, with only the long wavelength modes surviving to form a spatially constant condensate. Initially, the expectation value of the condensate field is set by high scale, SUSY breaking couplings to the field driving inflation: the inflaton. As the universe cools down, the expectation value of the condensate decreases until inflaton effects become subdominant and the condensate field begins to move in the low energy effective potential. At this point the condensate becomes unstable to spatial perturbations and begins to fragment [130–137], the final state being non-topological solitons stabilised by some conserved charge: Q-balls [114].

In O'Raifeartaigh models the overall picture is similar. We will keep with standard nomenclature and refer to the flat direction $X$ as the *condensate field*. A summary of the key points is then as follows.

- **Inflation:** A tachyonic soft mass, originating from the high scale SUSY breaking driving inflation, can drive the condensate field away from its low energy VEV. The condensate field is stabilised by higher order soft terms originating from a microscopic theory. For an accidental R-symmetry the condensate is stabilised at a scale $\Lambda$, parametrically between $M$ and $\sqrt{f}$, whereas for an exact R-symmetry it is stabilised around the Planck scale.

- **Rotation:** The Hubble parameter decreases until the inflaton induced soft terms are comparable in scale to the low energy effective potential. The low energy vacuum is restored and the condensate field begins to rotate in its potential well with magnitude $\Lambda$ and frequency $\lambda f/4\pi\Lambda$.
- **Fragmentation:** Shortly after beginning rotation the condensate is rendered unstable to spatial perturbations and fragments into localised lumps of size $2\pi\sqrt{2}\Lambda/\lambda f$. The fragments coalesce into extended, classical objects with large charge: R-balls.
- **Decay:** In R-symmetric models R-balls evaporate to gravitinos at tree level and other light fermions at one loop. In models with spontaneously broken R-symmetry R-balls can decay more quickly into other light fermions at tree level and also into light bosons.

We shall now discuss each stage of the evolution in more detail.

### 4.2.1 Inflation

At such early epochs one cannot consider the low energy O'Raifeartaigh model in isolation. Inflation induces extra soft terms in the potential related to the Hubble parameter [144]. Non-renormalisable operators arising from the microscopic theory also have a profound effect on the early universe dynamics, especially if they do not respect the R-symmetry of the low energy theory.

Cosmologically the scalar component of $X$ obeys the equation of motion

$$\ddot{X} + 3H\dot{X} - \frac{1}{a^2}\nabla^2 X + \frac{\partial V_{\text{eff}}}{\partial X^\dagger} = 0 \tag{4.7}$$

for a high energy effective potential $V_{\text{eff}}(X)$. $H$ is the Hubble parameter, $a$ is the scale factor of the universe and, for a homogeneous condensate, the gradient term obviously disappears. In the absence of thermal effects (the discussion of which is postponed until section 4.2.4) the general form of $V_{\text{eff}}(X)$ is known [144] to be

$$V_{\text{eff}}(X) = -cH^2|X|^2 + \frac{H}{M^{n-3}}\left(A\eta X^n + \text{h.c.}\right) + \frac{1}{M^{2n-6}}|\eta|^2|X|^{2n-2} \tag{4.8}$$

for a microscopic superpotential coupling $\eta$, order one constants $c$ and $A$, and where $n \geq 4$. The first term is a soft mass induced through inflaton couplings and is always present. The remaining terms are generated by non-renormalisable superpotential operators originating from the microscopic theory, which have been allowed to break R-symmetry (i.e. the R-symmetry of the low energy theory is accidental) and lift the flat direction.

If no non-renormalisable superpotential operators lift the flat direction, one would instead have

$$V_{\text{eff}}(X) = -cH^2|X|^2 + \frac{H^2}{M^{n-3}}AX^{n-1} + \frac{H^2}{M^{2n-2}}B|X|^{2n-4} \tag{4.9}$$

but the conclusions of the subsequent discussion are unaffected, other than replacing the scale $\Lambda$ (defined shortly) with the cutoff scale $M$. Even if R-symmetry is valid up to the Planck scale it must be broken to cancel the cosmological constant, so there are always Planck suppressed R-violating contributions. Actually, for D-term inflation one expects $A = 0$. This has little effect for an accidental R-symmetry but, when R-symmetry is exact, D-term inflation precludes the possibility of R-violating operators and the condensate always has vanishing charge.

The soft mass in eq. (4.8) is extremely important in the early universe when $H$ is large. For the minimal Kähler potential $K = X^\dagger X$ it is generated by supergravity corrections of the form

$$V_{\text{eff}}(X) = e^{K/M_P^2}V_{\text{inf}}(\chi) \tag{4.10}$$

where $\chi$ is the inflaton superfield. During inflation the inflaton vacuum energy dominates the universe so $V_{\text{inf}}(\chi) \sim H^2 M_P^2$, leading to a soft mass term with negative $c$. Therefore the effective potential is minimised at the origin and the dynamics are uninteresting. However, for a non-minimal Kähler potential it is quite possible that $c$ is positive. Consider, for example, the term $K \supset (\chi^\dagger\chi)(X^\dagger X)/M_P^2$ which is allowed by all possible symmetries of the model. In fact terms of this form are inevitable for superpotentials like (4.1): they arise as counterterms for Yukawa couplings [144–146]. Their contribution to the soft mass is

$$\delta\mathcal{L} = \int \mathrm{d}^4\theta \, \frac{(\chi^\dagger\chi)(X^\dagger X)}{M_P^2} = \frac{|F_\chi|^2}{M_P^2} X^\dagger X \sim H^2 X^\dagger X \tag{4.11}$$

so a *positive* Kähler potential coefficient results in a *negative* coefficient for the soft mass term, potentially winning out over the previous contribution. Since this soft mass is generated by the high scale SUSY breaking associated with inflation rather than the SUSY breaking associated with the MSSM, the result is independent of the scale appearing in the O'Raifeartaigh model itself.

Assuming the soft mass term *is* tachyonic, the effective potential is initially unstable around the origin and is only stabilised at

$$X \sim \left(HM^{n-3}\right)^{1/(n-2)} \qquad \Longrightarrow \qquad V_{\text{eff}}(X) \sim \left(H^{n-1}M^{n-3}\right)^{2/(n-2)} \tag{4.12}$$

with a choice of $n-2$ distinct minima corresponding to different choices of phase. The condensate field quickly settles into one of these minima and remains there throughout inflation due to the large, Hubble induced damping term in eq. (4.7) [144]. Immediately after inflation the Hubble parameter evolves as $1/t$. The minimum thus moves closer to the origin over time until $V_{\text{eff}}(X) \sim \lambda^2 f^2/16\pi^2$. At this stage the low energy effective potential (4.6) takes over, the corresponding Hubble parameter and condensate expectation value being

$$H \sim \left[\frac{1}{M^{n-3}}\left(\frac{\lambda f}{4\pi}\right)^{n-2}\right]^{1/(n-1)} \qquad \Longrightarrow \qquad X \sim \Lambda \equiv \left(\frac{\lambda f M^{n-3}}{4\pi}\right)^{1/(n-1)} . \tag{4.13}$$

Here, we have defined the parameter $\Lambda$ which will be important in all that follows. This scale is parametrically between the cutoff scale of the O'Raifeartaigh model $M$ and the loop suppressed SUSY breaking scale $\sqrt{\lambda f/4\pi}$. It actually turns out that the case $\Lambda^2 < f$ is uninteresting so we will henceforth assume $f^2 \ll \Lambda \ll M$. It will also be convenient to recast the above value of the Hubble parameter in terms of $\Lambda$

$$H \sim \frac{\lambda f}{4\pi\Lambda}. \tag{4.14}$$

We shall assume there are no independent minima at large $X$, i.e. ones that do not require a negative mass term centred around the origin to be stable. If such minima did exist, the condensate expectation value could remain near the cutoff scale and the theory would never flow into its low energy O'Raifeartaigh model description. This effect is not of interest here, but could provide a novel mechanism for models with uplifted vacua to find themselves in a higher energy vacuum.

### 4.2.2 Rotation

Below $H \sim \lambda f/4\pi\Lambda$ the condensate performs rotations about the minimum of the low energy effective potential (4.6). Whether R-symmetry is spontaneously broken or not, the logarithmic regime of eq. (4.6) is independent of the condensate's phase. An effective R-charge is thus conserved for the most part, resulting in approximately circular or elliptical motion. The charge stored in the condensate is determined by the interplay between the various R-violating operators in the high energy effective potential (4.8) when rotation begins. It starts out in a minimum of the high energy effective potential, with a phase evolving according to the relative sizes of these operators. At the transition point between high and low energy regimes (when the potential flattens in the angular direction) these are of comparable size to the R-preserving terms so can impart a sizeable 'torque' on the condensate, bestowing it with a large fractional charge. The only subsequent source of R-symmetry violation is the small $X$ regime, hence we can think of the trajectory as being smooth and elliptical, but possibly getting a kick if it gets too close to the bottom of the potential well. As an aside, relating R-charge to baryon number could lead to some interesting asymmetric dark matter scenarios such as those in refs. [141, 142].

At first $X^2 \sim \Lambda^2 \gg f$ so the large $X$ limit of eq. (4.6) applies. At any given time one could ignore the damping term in eq. (4.7) and find a circular solution $X = X_c e^{i\nu t}$, for constant amplitude $X_c$ and frequency

$$\nu^2 = \frac{1}{X_c}\frac{\partial U_{\text{eff}}}{\partial X} = \frac{\lambda^2 f^2}{16\pi^2 X_c^2}. \tag{4.15}$$

Elliptical variants will be mentioned in section 4.3.1. Initially $X_c \sim \Lambda$ and $H \sim \lambda f/4\pi\Lambda$ so the damping coefficient is similar to the frequency and the motion is critically damped. As the Hubble parameter continues to decrease, the damping follows suit and the motion becomes underdamped. We thus expect the effect of the damping to be small, the condensate expectation value and frequency of rotation remaining around $\Lambda$ and $\lambda f/4\pi\Lambda$ respectively.

### 4.2.3 Fragmentation

Left to its own devices the damping would, eventually, force the condensate into the small $X$ regime of eq. (4.6) and it would continue to perform underdamped oscillations about $X = \langle X \rangle$ with frequency $m$ and magnitude $\sqrt{f}$. However, over the domain $f \ll X^2 < \Lambda^2$ the effective potential is logarithmic so increases slower than quadratically. A condensate oscillating in this kind of potential behaves as matter with a negative pressure, i.e. it is unstable with respect to spatial perturbations [130–137].

For circular rotations it is possible to see this explicitly and estimate the typical size of the fragments [131,135]. Working in the underdamped regime discussed above one can write down the approximate solution

$$X \approx \Lambda e^{i(\lambda f/4\pi\Lambda)t}. \tag{4.16}$$

Now consider fluctuations in the magnitude and phase of $X$, of the forms $\delta\xi = \delta\xi_0 e^{\alpha t+ikx}$ and $\delta\theta = \delta\theta_0 e^{\alpha t+ikx}$ respectively. Unstable modes have $\alpha > 0$. Substituting into the equations of motion a non-trivial solution for $\delta\xi_0$ and $\delta\theta_0$ exists only if

$$\alpha^4 + 2\left(\frac{k^2}{a^2} + \frac{\lambda^2 f^2}{16\pi^2\Lambda^2}\right)\alpha^2 + \left(\frac{k^2}{a^2} - \frac{\lambda^2 f^2}{8\pi^2\Lambda^2}\right)k^2 = 0 \tag{4.17}$$

which is satisfied for real, positive $\alpha$ and $k$ only if

$$\left(\frac{k^2}{a^2} - \frac{\lambda^2 f^2}{8\pi^2\Lambda^2}\right) < 0 \quad \Longrightarrow \quad 0 < \frac{k}{a} < \frac{\lambda f}{2\pi\sqrt{2}\Lambda}. \tag{4.18}$$

The most amplified mode (largest $\alpha$) has a scale set by the upper limit of this inequality, so the fragments have typical size

$$r \sim \frac{a}{k} \sim \frac{2\pi\sqrt{2}\Lambda}{\lambda f}. \tag{4.19}$$

Moving away from circular trajectories requires a numerical approach, but for a logarithmic effective potential it only changes the result by factors of order unity [135]. Regardless, fragmentation occurs after the horizon size $H^{-1}$ surpasses the typical size of the fragments. This has already happened when the condensate begins to rotate at $H \sim \lambda f/4\pi\Lambda$ so the process can take place immediately.

### 4.2.4 Thermal effects

So far we have ignored the fact that all of these processes take place in a thermal bath. Below the reheat temperature $T_h$ the inflaton decays into light degrees of freedom. This reheats the universe, at which point it can be considered a cooling plasma with temperature $T \sim \sqrt{HM_P}$. Even beforehand the universe contains a dilute plasma with a lower temperature $T \sim (HM_PT_h^2)^{1/4}$ [147]. Note that this temperature should *not* be associated with the temperature of the R-balls as they are *not* generally in thermal equilibrium. The permitted values for the reheat temperature are constrained by gravitino cosmology. If reheating occurs too early gravitino decay products disrupt nucleosynthesis or, if the gravitino is stable, its relic density may be too high [148–152]. For an unstable gravitino (e.g. gravity mediation) one requires $T_h \lesssim 10^6 m_{3/2}$ whereas for a stable gravitino (e.g. gauge mediation) the bound is $T_h \lesssim 10^{14}(m_{3/2}/\mathrm{GeV})^{-2}$ GeV. Additionally, there is a lower bound of 10 MeV placed on the reheat temperature to ensure that reheating happens before nucleosynthesis.

Any particle coupling to the condensate field at tree level acquires a mass of order $|\lambda X|$. If this mass is greater than the plasma temperature, i.e. $|\lambda X| > T$, the state decouples from the condensate. Otherwise the condensate is eroded. The condensate expectation value goes like $\Lambda$ during fragmentation so one must have

$$\Lambda \gtrsim \frac{T_f}{\lambda} \tag{4.20}$$

for the process to proceed unhindered, where $T_f$ is the temperature of the universe at this time. If the inequality is satisfied there remain thermally sensitive loop corrections to the the low energy effective potential (4.6) to take into account [135]. Couplings to heavy states yield the extra term

$$U_{\mathrm{eff}}(X,T) = \frac{\lambda^2 T^4}{16\pi^2} \ln\left(\frac{|X|^2}{T^2}\right). \tag{4.21}$$

Unless $f > T^2$ this contribution dominates and the above expression should be used in place of eq. (4.6). However, since the form of the effective potential (which arises from loop corrections anyway) is not changed by the thermal contribution, its effects can be absorbed into the parameter $f$

$$f \longrightarrow f_T = \begin{cases} f & \text{for } T^2 < f \\ T^2 & \text{for } T^2 > f. \end{cases} \tag{4.22}$$

In addition one should include a temperature dependence in $\lambda$. We shall assume this is small enough to omit here so as to avoid further complicating the discussion. Other thermal also effects exist but are expected to be small for the large R-balls we will be discussing [131, 153] so will not be considered.

According to section 4.2.3 the condensate fragments at $H \sim \lambda f_T/4\pi\Lambda$ so, using the expressions given at the start of this section to relate the Hubble parameter to the plasma temperature, this equation breaks up into four possible domains depending on the relative sizes of the scales

$$\begin{aligned}
D_1: &\quad T_f < T_h\,,\, T_f^2 < f &\implies&\quad T_f^2/M_P \sim \lambda f/4\pi\Lambda \\
D_2: &\quad T_f > T_h\,,\, T_f^2 < f &\implies&\quad T_f^4/M_P T_h^2 \sim \lambda f/4\pi\Lambda \\
D_3: &\quad T_f > T_h\,,\, T_f^2 > f &\implies&\quad T_f^4/M_P T_h^2 \sim \lambda T_f^2/4\pi\Lambda \\
D_4: &\quad T_f < T_h\,,\, T_f^2 > f &\implies&\quad T_f^2/M_P \sim \lambda T_f^2/4\pi\Lambda.
\end{aligned} \tag{4.23}$$

There are then three different solutions for the fragmentation temperature; the fourth solution is just the boundary between two other domains. In each case the important parameters can be summarised as follows

| Domain | | $T_f$ | $f_T$ | $\Lambda_{\min}$ |
|---|---|---|---|---|
| $D_1:$ | $\Lambda > \max\left[\frac{\lambda f M_P}{4\pi T_h^2}\,,\,\frac{\lambda M_P}{4\pi}\right]$ | $\left(\frac{\lambda f M_P}{4\pi\Lambda}\right)^{1/2}$ | $f$ | $\left(\frac{f M_P}{4\pi\lambda}\right)^{1/3}$ |
| $D_2:$ | $\frac{\lambda f M_P}{4\pi T_h^2} > \Lambda > \frac{\lambda M_P T_h^2}{4\pi f}$ | $\left(\frac{\lambda f M_P T_h^2}{4\pi\Lambda}\right)^{1/4}$ | $f$ | $\left(\frac{f M_P T_h^2}{4\pi\lambda^3}\right)^{1/5}$ |
| $D_3:$ | $\Lambda < \min\left[\frac{\lambda M_P T_h^2}{4\pi f}\,,\,\frac{\lambda M_P}{4\pi}\right]$ | $\left(\frac{\lambda M_P T_h^2}{4\pi\Lambda}\right)^{1/2}$ | $\frac{\lambda M_P T_h^2}{4\pi\Lambda}$ | $\left(\frac{M_P T_h^2}{4\pi\lambda}\right)^{1/3}$ |

(4.24)

where $\Lambda_{\min}$ is the value demanded by the inequality (4.20).

## 4.3 R-balls

Much as MSSM condensates end up as B-balls (or L-balls), our condensate collapses into extended, classical configurations with large charge, R-balls, which are non-topological solitons formed under the influence of the U(1) R-symmetry [114, 119].

Recall that the low energy effective potential (4.6) preserves R-symmetry in the large $X$ regime whether or not it is spontaneously broken in the vacuum, hence R-balls form in *both* cases but have different decay properties depending on the eventual status of the symmetry.

Like Q-balls, R-ball solutions take the form $X(x,t) = X(x)e^{i\omega t}$, for a real parameter $\omega$ and a real function $X(x)$ that minimises

$$\int \mathrm{d}^3x \left( \frac{1}{2}|\nabla X|^2 + U_\omega(X) \right) \tag{4.25}$$

for the potential $U_\omega(X)$ defined in eq. (3.3). Assuming spherical symmetry the associated equations of motion are again

$$\frac{\mathrm{d}^2 X}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}X}{\mathrm{d}r} - \frac{\mathrm{d}U_\omega}{\mathrm{d}X} = 0 \tag{4.26}$$

for a radial coordinate $r$. One can always find a solution if $U_\omega(X)$ meets two criteria: it retains a stable minimum at $X = \langle X \rangle$ and there exists a non-zero $X$ such that $U_\omega(X) < U_\omega(\langle X \rangle)$. As long as $\omega < m$ the potential defined by eq. (4.6) satisfies both; $\omega^2 X^2$ always beats the logarithm for large $X$.

For the case at hand we can be more precise owing to the effective potential being so flat [154, 155]. Substituting eq. (4.6) into the equation of motion (4.26) yields two limits:

$$X'' + \frac{2}{r}X' + \omega^2 X = \begin{cases} m^2 X & \text{for } X^2 \ll f \\ \zeta\lambda^2 f_T^2/16\pi^2 X & \text{for } X^2 \gg f. \end{cases} \tag{4.27}$$

In the first, the equation is exactly soluble. In the second the right hand side can be ignored as $X$ is large, in which case the equation is again exactly soluble. Choosing the solution that is finite at $r = 0$, decays to zero as $r \to \infty$ and is continuous between the two regimes we find

$$X = \frac{X_R}{\omega r}\sin\omega r \quad \text{for } r < r_R, \quad X = \frac{X_R}{\omega r}\sin\omega r_R \, e^{(r_R - r)\sqrt{m^2-\omega^2}} \quad \text{for } r > r_R. \tag{4.28}$$

The R-ball's width is approximated by $r_R \approx \pi/\omega$, such that the cross over occurs near the first zero of $\sin\omega r$ where $X$ is arbitrarily small. Note that a cross over to large $X$ is inevitable because the small $X$ solution diverges as $r \to 0$. When $\omega \ll m$

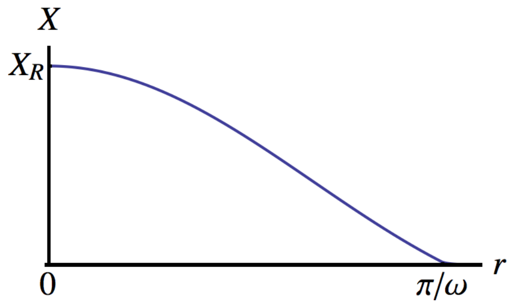


Figure 4.2: A typical R-ball solution. At small and large $r$ it has the forms $\sin\omega r/\omega r$ and $e^{-mr}$ respectively, with a width of about $\pi/\omega$. The parameter $\omega$ scales like $R^{-1/4}$, the height like $R^{1/4}$ and the energy $R^{3/4}$.

the R-ball is very large and the solution can be further simplified to find

$$X = \frac{X_R}{\omega r}\sin\omega r \quad \text{for } r < r_R, \qquad X = \frac{X_R}{\omega r_R}\sin\omega r_R\, e^{m(r_R - r)} \quad \text{for } r > r_R. \tag{4.29}$$

Evaluating the energy of this field configuration and choosing $\omega r_R \approx \pi$ gives

$$E = \frac{\zeta\pi^2\lambda^2 f_T^2}{12\omega^3} + \omega R + \mathcal{O}(\pi - \omega r_R). \tag{4.30}$$

Minimising with respect to $\omega$ and explicitly evaluating the charge finally yields expressions for the R-ball parameters

$$\omega_R \sim R^{-1/4}\sqrt{\lambda f_T} \quad r_R \sim \frac{R^{1/4}}{\sqrt{\lambda f_T}} \quad X_R \sim R^{1/4}\sqrt{\lambda f_T} \quad E_R \sim R^{3/4}\sqrt{\lambda f_T} \tag{4.31}$$

up to order one coefficients. Putting it all together, a typical R-ball solution is illustrated in figure 4.2.

### 4.3.1 Formation

In section 4.2.3 it was shown that, for an initially circular trajectory, the condensate forms fragments of size $r \sim 2\pi\sqrt{2}\Lambda/\lambda f_T$ shortly after it starts to rotate with magnitude $X_c \sim \Lambda$ and frequency $\nu_c \sim \lambda f_T/4\pi\Lambda$. We can use these details to estimate the typical charge [135]. The charge density during fragmentation is given by $2\nu_c X_c^2$ so the total charge per fragment is

$$R \sim \frac{100\Lambda^4}{\lambda^2 f_T^2}. \tag{4.32}$$

If each fragment collapsed into a single R-ball this would be their charge. Actually, numerical simulations [135] suggest that R-balls form slightly after fragmentation in a logarithmic potential, leading to a reduced charge of

$$R_c \sim \frac{\Lambda^4}{10\lambda^2 f_T^2}. \tag{4.33}$$

Each fragment therefore contains about $10^3$ R-balls.

Using the energy (4.31) the energy density stored in R-balls just after formation, at $H \sim \lambda f_T / 4\pi\Lambda$, can now be determined to be

$$\rho_R(T_f) \sim \frac{\lambda^2 f_T^2}{10} \qquad \Longrightarrow \qquad \Omega_R(T_f) = \frac{\rho_R(t_f)}{3H^2 M_P^2} \sim \frac{10\Lambda^2}{M_P^2} \tag{4.34}$$

unless $\Lambda \sim M_P$ whereupon $\Omega_R \sim 1$ and R-balls dominate the universe until they decay. Of course, this result scales with the expansion of the universe. If R-balls form above a temperature of $\sqrt{f}$ there is also lowering in density due to the decreasing value of $f_T$ (4.22). Treating R-balls as non-relativistic matter the density can be related to the scale factor via $\rho_R \sim a^{-3}$, implying that

$$\Omega_R(T) \propto \begin{cases} \sqrt{f_T} & \text{for matter domination} \\ \sqrt{f_T}/T & \text{for radiation domination.} \end{cases} \tag{4.35}$$

It should be noted that there are no other processes to form R-balls once the condensate has fragmented so they cannot maintain thermal equilibirum. The idea of solitosynthesis [116–118] does not apply here as the goldstino carries R-charge and is lighter than the condensate field.

To generalise to more eccentric condensate trajectories one can define the parameter $\epsilon = \nu/\nu_c = 4\pi\nu\Lambda/\lambda f_T$: the ratio of the angular velocity of the condensate to its maximal, circular value. Equivalently it can be thought of as the fraction of the maximum possible charge that is stored in the condensate. Clearly $\epsilon$ can lie anywhere in the range zero to one, zero corresponding to pure radial oscillations with vanishing net charge. Naively one might expect the charge of the resultant R-balls to be given by $\epsilon R_c$. This is mainly true. However, once $\epsilon$ drops below about 0.06, both positive and negatively charged R-balls are formed in comparable quantities and the typical charge becomes constant [133–135].

### 4.3.2 Decay

Bosonic decay modes do not exist when the vacuum of the low energy theory preserves R-symmetry. The R-ball is already the lowest energy scalar field configuration for a given charge and gauge bosons have R-charge zero. However, several other decay modes *do* exist in most models; basically any light fermion with non-zero R-charge. Even so decay is slower than one might think due to the Pauli exclusion principle. Within an R-ball a Fermi pressure opposes the creation of fermions so, effectively, R-balls only evaporate from their surface.

Consider first the couplings in the superpotential (4.1). In the absence of fine tuning one expects all particles to have tree level masses of order $\mu$, or zero if imposed by symmetry. Since the condensate field mass $m$ is generated at one loop it is comparatively suppressed and only decays to the massless fermions are kinematically allowed. Any massless fermion must be a null eigenvector of the fermionic mass matrix, given by $\mu$ in an R-symmetric model when $\langle X \rangle = 0$. Consequently its scalar partner is massless (see ref. [40] or section 1.2.3) and the superfield must be a null eigenvector of $\lambda$ as well. Therefore massless fermions are forbidden from coupling to the condensate field at tree level. The condensate can only decay to the fermionic components of the $\varphi$'s at tree level if R-symmetry is spontaneously broken.

A second option for R-ball decay is the gravitino. The goldstino has R-charge +1 and is originally exactly massless, but gets eaten by the gravitino which picks up a mass $m_{3/2} \sim f_T/M_P$. Decay then proceeds via goldstino interaction terms [156, 157]

$$\mathcal{L} \supset \frac{m^2}{f_T} X^\dagger \tilde{G}\tilde{G} \tag{4.36}$$

with $\tilde{G}$ denoting the gravitino. As the condensate field mass is generated at one loop the coupling is effectively loop suppressed. Loop decays to other light fermions (that may be external to the hidden sector) should therefore be considered too. One may also expect R-violating modes induced either by gravity, which is not excepted to respect any global symmetries, or the microscopic theory, which may break R-symmetry too. These decay channels will be discussed in the following section.

Rigorously estimating the lifetime of an R-ball in the thick wall limit is a difficult task [158] so here we shall take a simpler approach. The thin wall limit [159] is not appropriate for the R-ball solution given in (4.29) but the principles applied to bosonic decay modes in ref. [160] are. First consider tree level decays such as those to gravitinos. For a fermion coupling directly to the condensate field with strength $g$ (equal to $m^2/f_T$ for gravitinos) the penetration width is about $1/gX$. The fermion is produced inside the R-ball at some radius $r$ so one must have

$$\frac{1}{gX} > r_R - r \tag{4.37}$$

if it is to escape. Otherwise the Fermi pressure prevents decay. Since we are dealing with the interior of the R-ball, the large $X$ component of eq. (4.29) is appropriate and the inequality becomes

$$\frac{\pi - \omega_R r}{\omega_R r} \sin \omega_R r < \frac{\omega_R}{g X_R} \sim \frac{1}{g\sqrt{R}} \tag{4.38}$$

using $\omega_R r_R \approx \pi$ and, for the last term, eq. (4.31). When the charge is large (specifically $R \gg g^{-2}$)[1] this inequality can only satisfied around the surface of the R-ball, $r = r_R$. One thus finds

$$\dot{R} \approx -4\pi\omega_R \int_{r_R}^{\infty} \mathrm{d}r\, r^2 \Gamma X^2 \tag{4.39}$$

where $\Gamma$ is the decay rate at a given point inside the R-ball and $\omega_R X^2$ is the local charge density.

Decreasing the charge of the R-ball by two (a single condensate quanta) liberates energy of order $\omega_R$, yet fermions coupling to the condensate field at tree level gain local masses of order $gX$. Unless $gX < \omega_R$ a local, tree level decay is kinematically forbidden and decay can only occur via heavy particle loops. On the surface $X = \sqrt{f_T}$ by definition so the inequality is already satisfied there as long as $g^2 < \lambda R^{-1/2}$. If not, the crossover occurs at a radius

$$\gamma r_R \quad \text{for} \quad \gamma \sim 1 + \frac{1}{4 m r_R} \ln\left(\frac{R g^4}{\lambda^2}\right) \tag{4.40}$$

where the expressions for the small $X$ component of eq. (4.29) and the R-ball parameters (4.31) have been utilised. Large R-balls have $r_R \approx \pi/\omega_R \gg 1/m$ so the second

[1]For gravitinos this reduces to $\Lambda \gtrsim 10\mu\lambda^{-3/2}$ and is expected to be true for most models. It certainly is for the models we will examine shortly.

term is subdominant and we can approximate $\gamma$ by one, meaning that $gX < \omega_R$ everywhere from the surface outwards. The decay rate is therefore constant at its tree level value $\Gamma \approx g^2\omega_R/4\pi$. Essentially the R-ball solution decays very rapidly beyond the surface, so if $gX < \omega_R$ is not already satisfied there it soon will be. Putting it all together we find

$$\dot{R} \sim -\frac{g^2 f_T \omega_R^2}{4m^3}\left(1 + 2mr_R + 2m^2 r_R^2\right) \sim -\frac{\pi^2 g^2 f_T}{2m} \tag{4.41}$$

where the last equality again follows from the fact that $mr_R \approx \pi m/\omega_R \gg 1$ for large charge. In more detail, the decay rate starts off constant but, as the charge of the R-ball decreases, the other two terms grow and decay speeds up. Consequently R-balls decay more quickly with decreasing size, as was seen numerically for Q-balls in ref. [158].

Loop decays tell a similar story. As long as the mass of the heavy particle propagating around the loop is greater than the mass it acquires from its tree level coupling to the condensate, the result $\Gamma \approx g^2\omega_R/4\pi$ is unchanged. One simply replaces $g$ with the appropriate effective coupling. This will always be the case just beyond the surface for the reasons mentioned above. Coupling fields in the loop to light fermions with strength $h$ thus gives a decay rate $\Gamma \approx g^2 h^4 \omega_R^3/16\pi^2\mu^2$ and, subsequently

$$\dot{R} \sim -\frac{\pi\lambda g^2 h^4 f_T^2}{8m\mu^2} R^{-1/2}. \tag{4.42}$$

When $T_f^2 < f$ the tree level decay rate is easily integrated to find an approximate lifetime

$$\tau_{\mathrm{RS}} \sim \frac{2m}{\pi^2 g^2 f} R \sim \frac{\Lambda^4}{10^3 g^2 f^2 \mu} \tag{4.43}$$

using eq. (4.33) to set the initial charge and eq. (4.3) to eliminate $m$. If loop decays dominate this result becomes

$$\tau_{\mathrm{RS}} \sim \frac{16m\mu^2}{3\pi\lambda^3 h^4 f_T^2} R^{3/2} \sim \frac{\Lambda^6 \mu}{10^3 \lambda^2 g^2 h^4 f^4}. \tag{4.44}$$

Otherwise the lifetime depends on the thermal history of the universe. Decay is quicker and the initial charge is smaller when the temperature is greater than $\sqrt{f}$, due to eq. (4.22). A lower bound is thus found by fixing $f_T$ at $T_f^2$, whereupon one can simply replace $f$ with $T_f^2$ in the above expressions.

### 4.3.3 Low energy theories without R-symmetry

In O'Raifeartaigh models that spontaneously break their R-symmetry the initial situation remains broadly similar. Immediately after fragmentation the condensate moves in the logarithmic regime of the effective potential (4.6). This conserves R-charge so supports the formation of R-balls as before. The main difference is that there may now be R-violating bosonic decay modes available, which are not stifled by a Fermi pressure so can shorten the lifetime. Expressions for the fermionic decay rate are unaffected (we assumed nothing about charge conservation in the previous section short of establishing what decay modes were allowed) and the effect of bosonic modes can be estimated in a similar manner [160].

The key difference is that the lower limit of the integral (4.39) is zero in the absence of Fermi pressure, so tree level decays acquire an extra contribution from the interior of the R-ball

$$\Delta\dot{R} \approx -4\pi\omega_R \int_0^{r_R} \mathrm{d}r\, r^2 \Gamma X^2 \approx -g^2\omega_R^2 \left( \frac{g^2\omega_R^2}{4\pi} \int_0^{\gamma r_R} \mathrm{d}r\, r^2 + \int_{\gamma r_R}^{r_R} \mathrm{d}r\, r^2 X^2 \right). \quad (4.45)$$

Between $r = 0$ and $\gamma r_R$ decay is at one loop via particles of mass $gX$, giving $\Gamma \approx g^4\omega_R^3/16\pi^2 X^2$, whereas between $\gamma r_R$ and the surface we instead have $\Gamma \approx g^2\omega_R/4\pi$. Due to the arguments associated with eq. (4.40) one expects $\gamma \leq 1$, with equality when $g^2 > \lambda R^{-1/2}$. For $g^2 < \lambda R^{-1/2}$ the crossover occurs inside the R-ball where the solution changes more slowly so one expects the deviation from $\gamma \approx 1$ to be significant. Substituting in the large $X$ component of eq. (4.29) and $r_R \approx \pi/\omega_R$ the increase is given by

$$\begin{aligned} \Delta\dot{R} &\sim -\frac{\pi^2\gamma^3 g^4\omega_R}{12} - \frac{g^2 X_R^2}{4\omega_R}(2\pi(1-\gamma) + \sin 2\pi\gamma) \\ &\sim -\frac{g^2\sqrt{\lambda f_T}}{4} R^{3/4}(2\pi(1-\gamma) + \sin 2\pi\gamma). \end{aligned} \quad (4.46)$$

Large $R$ means small $\omega_R$ (specifically $\omega_R \ll m$, implying that $\omega_R \ll f_T/m$) whereupon the first term on the first line can be neglected. The second term, however, is extremely significant for large R-balls as it dominates the overall decay rate.

Loop decays to bosons are similarly enhanced, contributing

$$\Delta\dot{R} \sim -\frac{g^2 h^4 (\lambda f_T)^{3/2}}{16\pi\mu^2} R^{1/4}(2\pi(1-\gamma) + \sin 2\pi\gamma). \quad (4.47)$$

The positive exponent attached to $R$ means that even one loop bosonic channels can be more important than tree level fermionic ones if the charge is large enough. However, note that if $g^2 > \lambda R^{-1/2}$ (or $g > \mu/\sqrt{f_T}$ for loop decays) $\gamma \approx 1$ as before. Decay to bosons is then suppressed everywhere inside the R-ball due to the mass they acquire from condensate field couplings. Evaporation is only from the surface as per the R-symmetric case and there is no longer a significant enhancement.

Assuming that $g^2 < \lambda R^{-1/2}$ and $T_f^2 < f$ the revised lifetime for tree level decay modes is easily evaluated to be

$$\tau_{\mathrm{RB}} \sim \frac{4}{g^2\sqrt{\lambda f}} R^{1/4} \sim \frac{\Lambda}{\lambda g^2 f} \tag{4.48}$$

using eq. (4.33) to set the initial charge and assuming that $(2\pi(1-\gamma)+\sin 2\pi\gamma)/4 \sim 1$. For loop decays one finds

$$\tau_{\mathrm{RB}} \sim \frac{64\pi\mu^2}{3\lambda^2 h^4 (\lambda f)^{3/2}} R^{3/4} \sim \frac{10\Lambda^3\mu^2}{\lambda^3 g^2 h^4 f^3}. \tag{4.49}$$

Just as in the R-symmetric case one can find a lower bound on the lifetime when the formation temperature is greater than $\sqrt{f}$ by replacing $f$ with $T_f^2$ in these expressions. Of course, all of this assumes the existence of light bosons that the condensate is able to decay. It could be that all bosons are too heavy. Or the model parameters could conspire to enable the fermionic decay rate to overtake the bosonic one. In either case the lifetime reverts to that given in the previous section.

We are also able to estimate the effects of R-violating operators from the microscopic theory. Inside the R-ball two types of effective operator are important. R-violation in the Kähler potential of the low energy effective theory induces the first type:

$$\mathcal{L} \supset \int \mathrm{d}\theta^2 \mathrm{d}\bar{\theta}^2 \frac{X^{n-1}}{M^{n-3}} \supset \frac{\omega_R^2 X_R^{n-4}}{M^{n-3}} X^3 \tag{4.50}$$

and R-violation by non-renormalisable superpotential operators induces the second type:

$$W \supset \frac{X^n}{M^{n-3}} \quad \Longrightarrow \quad \mathcal{L} \supset \frac{f_T X_R^{n-4}}{M^{n-3}} X^3. \tag{4.51}$$

for the same $n \geq 4$ and $M$ used in section 4.2.1. Producing free $X$ particles is kinematically forbidden but bound particles with lower effective mass can be produced. This effective mass increases as the R-ball decays until the total charge is reduced to $E_R/m$, whereupon it breaks up into so many free particles. It is thus more appropriate to assume a constant decay rate over the entire R-ball when finding the lifetime.

For the first type of operator the decay rate is $\Gamma \approx \omega_R^3 X_R^{2n-8}/4\pi M^{2n-6}$, yielding

$$\dot{R} \sim -\frac{(\lambda f_T)^{(2n-5)/2}}{4\pi\Lambda^{2n-6}} R^{(2n-7)/4} \tag{4.52}$$

upon replacing $M$ with $\Lambda$ and using eq. (4.31) for $\omega_R$ and $X_R$. Integrating from the initial charge to $E_R/m \sim R_c^{3/4}\sqrt{\lambda f_T}/m \ll R_c$ provides an estimate of the lifetime that depends on $n$. For $n = 4, 5$ it is

$$\tau_{\text{RV}} \sim \frac{10^{(2n-3)/4}\Lambda^5}{\lambda^3 f^3} \tag{4.53}$$

and for larger values

$$\tau_{\text{RV}} \sim \frac{10^{(43-2n)/16}\lambda^{(2n-23)/4}\Lambda^{(2n+9)/4}}{f^3\mu^{(2n-11)/4}}. \tag{4.54}$$

For the second type of operator $\Gamma \approx f_T^2 X_R^{2n-8}/4\pi\omega_R M^{2n-6}$ and similar reasoning, now using eq. (4.13) to eliminate $M$, leads to

$$\tau_{\text{RV}} \sim \frac{10^{(71-2n)/16}\lambda^{(2n-11)/4}\Lambda^{(2n+13)/4}}{f^3\mu^{(2n-7)/4}}. \tag{4.55}$$

In either case R-violating decays can be important, the details being strongly dependent on the dimension of the operators lifting the erstwhile flat direction. Superpotential operators always dominate when present dominant because $\omega_R^2 \ll f_T$.

## 4.4 R-ball phenomenology

The cosmological behaviour of R-balls is wide ranging. To give a general overview of the features various models can exhibit, this section will focus mainly on models obeying the following criteria. The strength of all couplings in the superpotential (4.1) will be taken to be of order $\lambda$, whereas 'naturalness' suggests that the two scales in the superpotential (4.1) should be similar, i.e. $\mu^2 \sim f_T$, accounting for thermal corrections with eq. (4.22). Increasing $\mu$ above this scale increases the tree level decay rate but decreases the one loop decay rate. We will further assume that all degrees of freedom in the hidden sector are heavy with masses $\mu$, other than the condensate field.

R-ball formation takes place in O'Raifeartaigh models whenever the condensate field (the scalar partner of the Goldstino) picks up a tachyonic soft mass due to couplings to the inflaton. When it does the characteristic R-ball scale $\Lambda$, defined in eq. (4.13), is parametrically between the cutoff scale of the model and the SUSY breaking scale. The temperature of the universe at formation (4.24) and typical charge (4.33) of large R-balls are then given in the following table

| Domain | $D_1$ | $D_2$ | $D_3$ |
|---|---|---|---|
| $T_f$ | $\left(\frac{\lambda f M_P}{4\pi\Lambda}\right)^{1/2}$ | $\left(\frac{\lambda f M_P T_h^2}{4\pi\Lambda}\right)^{1/4}$ | $\left(\frac{\lambda M_P T_h^2}{4\pi\Lambda}\right)^{1/2}$ |
| $R$ | $\frac{\Lambda^4}{10^5\lambda^2 f^2}$ | $\frac{\Lambda^4}{10^5\lambda^2 f^2}$ | $\frac{\Lambda^6}{10^3\lambda^4 M_P^2 T_h^4}$ |

. (4.56)

Domain boundaries are determined by when R-ball formation takes place relative to the decay of the inflation, and whether thermal effects dominate the effective potential during this process. They are defined by

$$\begin{aligned}
D_1: &\quad T_f < T_h\,,\, T_f^2 < f \quad \Longrightarrow \quad \Lambda > \max\left[\frac{\lambda f M_P}{4\pi T_h^2}\,,\,\frac{\lambda M_P}{4\pi}\,,\,\left(\frac{f M_P}{4\pi\lambda}\right)^{1/3}\right] \\
D_2: &\quad T_f > T_h\,,\, T_f^2 < f \quad \Longrightarrow \quad \frac{\lambda f M_P}{4\pi T_h^2} > \Lambda > \max\left[\frac{\lambda M_P T_h^2}{4\pi f}\,,\,\left(\frac{f M_P T_h^2}{4\pi\lambda^3}\right)^{1/5}\right] \\
D_3: &\quad T_f > T_h\,,\, T_f^2 > f \quad \Longrightarrow \quad \min\left[\frac{\lambda M_P T_h^2}{4\pi f}\,,\,\frac{\lambda M_P}{4\pi}\right] > \Lambda > \left(\frac{M_P T_h^2}{4\pi\lambda}\right)^{1/3}.
\end{aligned} \tag{4.57}$$

where $T_h$ denotes the reheat temperature of the universe after inflaton decay. Outside of these domains the condensate is eroded before R-balls are able to form. For gravity mediated SUSY breaking the reheat temperature must lie in the range 10 MeV $\lesssim T_h \lesssim 10^6 m_{3/2}$ whereas for gauge mediation the bound is 10 MeV $\lesssim T_h \lesssim 10^{14}(m_{3/2}/\text{GeV})^{-2}$ GeV.

The initial energy density stored in R-balls (4.34) is given by

$$\Omega_R(T_f) \sim \frac{10\Lambda^2}{M_P^2} \tag{4.58}$$

(or one if $\Lambda \sim M_P$) and scales with the subsequent expansion of the universe (4.35) as

$$\Omega_R(T) \sim \begin{cases} 10\Lambda^2/M_P^2 & \text{for } T_f > T > T_h \\ (T_h/T)(10\Lambda^2/M_P^2) & \text{for } T_h > T > T_e \\ (T_h/T_e)(10\Lambda^2/M_P^2) & \text{for } T_e > T \end{cases} \tag{4.59}$$

where $T_e \sim 1$ eV is the usual temperature at which the universe becomes matter dominated. If $T_f < T_h$ one replaces $T_h$ with $T_f$ in the second equation and, for $\Lambda \in D_3$, the density is multiplied by $\sqrt{f_T}/T_f$ due to the temperature dependence (4.22) of the R-ball energy (4.31).

From here on the analysis becomes strongly dependent on the particulars of the model and the available decay modes. These in turn are specific to the mechanism chosen to mediate SUSY breaking, and on the form of the operators in the microscopic theory lifting the flat direction. The main examples we will consider are gravity and gauge mediated SUSY breaking, for preserved and spontaneously broken R-symmetry, both for strongly and weakly coupled superpotentials. To maximise the effect of R-violation in the microscopic theory we will assume that the flat direction is lifted by R-violalting, dimension 4 operators. Some general features that crop in these models are as follows.

Perhaps of most immediate interest is the idea that R-balls are long lived and still exist today, contributing to the dark matter density of the universe. According to eqs. (4.43), (4.44), (4.48), (4.49) and (4.55) long lifetimes correspond to a high cutoff scale, a small SUSY breaking scale (i.e. a small gravitino mass) and/or a weakly coupled condensate field. If sufficiently long lived, non-relativistic R-balls behave as cold dark matter with a density given by eq. (4.59). One must find $\Omega_R(T_e) \leq 0.22$ if they are not to exceed the total observed value. Note that light gravitinos, one of the requirements for R-ball dark matter, are specific to gauge mediated SUSY breaking, hence one does not expect this kind of dark matter in models of gravity mediation.

If not sufficiently long lived to survive until the present day R-balls must, obviously, decay. The effect this has on the visible sector varies and could in principle result in either heating or cooling of the universe. Each quanta of condensate that decays carries away energy

$$\Delta E \sim \begin{cases} \lambda f/\Lambda & \text{for } \Lambda \in D_1, D_2 \\ \lambda f_T^{1/2} T_f/\Lambda & \text{for } \Lambda \in D_3 \end{cases} \tag{4.60}$$

using eqs. (4.31) and (4.33). Regardless of what the decay products actually are, this sets their maximum mass and characteristic temperature.

Since R-balls are not in thermal equilibrium with the rest of the universe their decay products will initially be out of equilibrium as well. Large R-ball density (4.59) during decay thus results in a secondary reheating, or cooling of the universe. An immediate consequence is that an R-ball dominated epoch decouples the generation of visible sector matter and radiation from the dynamics of the inflaton. R-ball, rather than inflaton, decay can be responsible for the present contents of the universe, with the new reheat temperature $\Delta E$ obeying the constraints applied to the original one $T_h$. If, on the other hand, the R-ball density is small the temperature of the universe is unchanged. The decay products are brought into equilibrium with everything else, unless they are incapable of maintaining thermal equilibrium in which case they simply update the relevant relic abundance by a small amount. Either way the latter scenario is of limited interest.

Ignoring R-violating operators for now, R-symmetric models permit decays to light fermions alone. The only such fermion that can couple to the condensate field at tree level is the gravitino but, owing to the loop suppressed mass of the condensate field, decays at one loop may also be important. This is particularly so in the absence of light hidden sector states whereupon these decays are to visible sector gauginos (which have R-charge +1). When the O'Raifeartaigh model spontaneously breaks R-symmetry R-balls can also decay at one loop to visible sector gauge bosons, sfermions or even fermions. Alternatively, if light hidden sector degrees of freedom are included, one generally finds an increased decay rate as R-balls can decay directly to said fields rather than via loops or gravitinos. However, the observable effects in this scenario are highly model specific and depend on the exact properties of these new degrees of freedom so will not be discussed here.

R-violating operators induced by the microscopic theory permit decays to condensate field bosons. Eq. (4.60) does not apply to them, rather they are produced at a higher temperature $\lambda^2\sqrt{f_T}/4\pi$ in accordance with their mass. As such they can only remain in thermal equilibrium for a brief time before freezing out, potentially providing a non-equilibrium production mechanism for the pseudo-moduli dark matter scenarios of refs. [138–140]. Here we restrict to the case of $n = 4$, R-violating superpotential operators so the associated lifetime (4.55) is

$$\tau_{\mathrm{RV}} \sim \begin{cases} 10^4\Lambda^{21/4}/\lambda^{3/4}(m_{3/2}M_P)^{25/8} & \text{for } \Lambda \in D_1, D_2 \\ 10^4\Lambda^{21/4}/\lambda^{3/4}T_f^{25/4} & \text{for } \Lambda \in D_3 \end{cases} \tag{4.61}$$

for both gravity and gauge mediation. Increasing $n$ or restricting to R-violating Kähler potential operators generically increases this lifetime and disfavours decay to condensate field bosons.

### 4.4.1 Gravity mediation

In pure gravity mediation the O'Raifeartaigh model communicates with the visible sector only through gravitational interactions. Gravitinos couple to the condensate at tree level with strength $g \sim m^2/f_T \sim \lambda^4/16\pi^2$ so lead to an associated R-ball lifetime (4.43)

$$\tau_{\mathrm{RS}} \sim \begin{cases} 10\Lambda^4/\lambda^8(m_{3/2}M_P)^{5/2} & \text{for } \Lambda \in D_1, D_2 \\ 10\Lambda^4/\lambda^8 T_f^5 & \text{for } \Lambda \in D_3. \end{cases} \tag{4.62}$$

The only light states accessible to the condensate at one loop are those in the visible sector and it must be gravitinos propagating around the loop. Loop decay rates are therefore negligible as they contain higher orders of the already small gravitino couplings. Spontaneously broken R-symmetry opens up the possibility of decay to visible sector gauge bosons via gravitino loops. This decay rate is only significant if the R-ball interior contributes, i.e. $g < 1$ (see section 4.3.3), which is always true for the small gravitino coupling. Gravitino couplings to gauge bosons go like $h \sim m_{3/2}/M_P$ leading to the lifetime (4.49)

$$\tau_{\mathrm{RB}} \sim \begin{cases} 10^5\Lambda^3 M_P^2/\lambda^{11}m_{3/2}^6 & \text{for } \Lambda \in D_1, D_2 \\ 10^5\Lambda^3 T_f^4/\lambda^{11}m_{3/2}^8 & \text{for } \Lambda \in D_3. \end{cases} \tag{4.63}$$

We can use these lifetimes, in conjunction with eq. (4.61), to slice up the O'Raifeartaigh model parameter space ($m_{3/2}$, $\Lambda$, $\lambda$) into different regions of interest according to

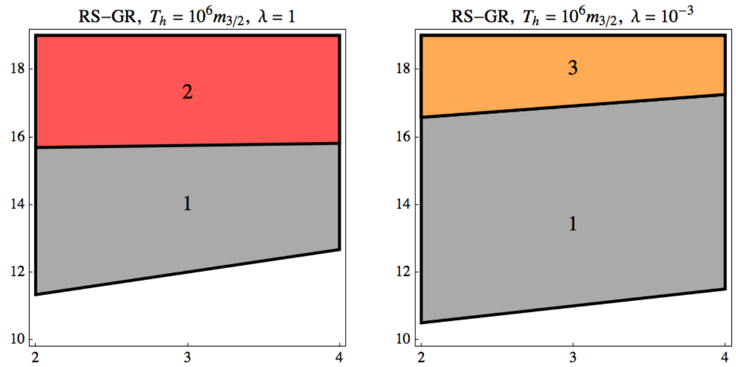


Figure 4.3: R-balls in R-symmetric, gravity mediated SUSY breaking as a function of $\log(m_{3/2}/\mathrm{GeV})$ (horizontal axis) and $\log(\Lambda/\mathrm{GeV})$ (vertical axis) for various values of $\lambda$ and $T_h$. In region 1 their density is small and they have no significant effects. In region 2/3 they reheat the universe by decaying to gravitinos/condensate field bosons. Outside the shaded region R-balls do not form due to thermal effects, or they contribute more than the observed cold dark matter density.

which of the expressions (4.61), (4.62) and (4.63) is smallest. It actually turns out that R-balls in gravity mediated SUSY breaking do not have a wide range of phenomenological consequences. They can only result in secondary reheating of the universe; by gravitinos for a strongly coupled condensate and by condensate field bosons for a weakly coupled condensate. This is because loop couplings remain sufficiently suppressed over the entire parameter space to favour tree level decays, and the high SUSY breaking scale tends to keep the lifetime short. Decay to condensate field bosons is favoured in weakly coupled models as self interactions are then relatively enhanced. Some R-symmetric examples are given in figure 4.3. R-breaking O'Raifeartaigh models still result in R-ball formation, but their density is typically too low to have any interesting consequences.

### 4.4.2 Gauge mediation

A combination of messenger loops and a low gravitino mass in gauge mediation lead to a much more interesting phenomenology. Although gravitinos are still the only light fermion coupling to the condensate field at tree level, there are now one loop couplings via messengers to visible sector gauge fields. These loops are much more significant than gravitino loops as messengers couple to the condensate field with strength $g \sim \lambda$, then through visible sector gauge couplings with strength $h \sim 1$. If gauginos are kinematically accessible, i.e. $\Delta E \gtrsim 1$ TeV (4.60), and decay to them is faster than to gravitinos, the associated R-ball lifetime (4.44) is

$$\tau_{\rm RS} \sim \begin{cases} \Lambda^6/10^3\lambda^4(m_{3/2}M_P)^{7/2} & \text{for } \Lambda \in D_1, D_2 \\ \Lambda^6/10^3\lambda^4 T_f^7 & \text{for } \Lambda \in D_3. \end{cases} \tag{4.64}$$

R-breaking O'Raifeartaigh models are even more sensitive to loop decays. Now visible sector gauge bosons are always accessible and come with an associated lifetime

$$\tau_{\rm RB} \sim \begin{cases} 10\Lambda^3/\lambda^5 m_{3/2}^2 M_P^2 & \text{for } \Lambda \in D_1, D_2 \\ 10\Lambda^3/\lambda^5 T_f^4 & \text{for } \Lambda \in D_3. \end{cases} \tag{4.65}$$

(using eq. (4.49)) or that of eq. (4.64) if smaller. In either case R-balls preferentially evaporate to gauge bosons rather than fermions due to the extra contribution from the interior.

The parameter space ($m_{3/2}$, $\Lambda$, $\lambda$) can be split as for gravity mediation but we now find more variation in the results. R-symmetric O'Raifeartaigh models demonstrate both gravitino reheating and cooling, condensate field boson reheating and R-ball dark matter. Dark matter corresponds to a small original reheat temperature so as to minimise the boost given to the density (4.59). Otherwise the regions of parameter space coincident with long lived R-balls tend to be associated with too high a density. Furthermore the gravitino mass is small in these regions so as to maximise the lifetime.

For all of the range in which R-balls live long enough the gravitino mass is less than a few keV. Its contribution to the overall dark matter density is therefore small [73], but R-balls can easily account for the entire $\Omega_{\rm DM} \approx 0.22$. If they do (the top of region 5 in figure 4.4) their charge, size and energy (4.31) are in the ranges

$$10^{35} \lesssim R \lesssim 10^{49}, \quad 10^{-12}\ \text{m} \lesssim r_R \lesssim 10^{-4}\ \text{m}, \quad 10^{31}\ \text{GeV} \lesssim E_R \lesssim 10^{38}\ \text{GeV}. \tag{4.66}$$

Individual R-balls can therefore be anything from fermi to micro scale and, regardless of size, are very dense objects. Outside of the dark matter region gravitino or

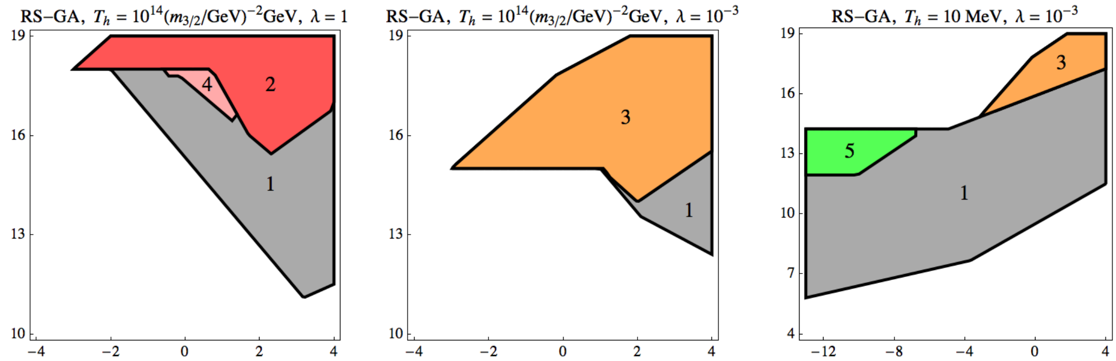


Figure 4.4: R-balls in R-symmetric, gauge mediated SUSY breaking as a function of $\log(m_{3/2}/\mathrm{GeV})$ (horizontal axis) and $\log(\Lambda/\mathrm{GeV})$ (vertical axis) for various values of $\lambda$ and $T_h$. In region 1 their density is small and they have no significant effects. In region 2/4 they reheat/cool the universe by decaying to gravitinos. Region 3 corresponds to reheating by condensate field bosons and region 5 to R-ball dark matter. Outside the shaded region R-balls do not form due to thermal effects, contribute more than the observed cold dark matter density or their decay violates the bounds on the reheat temperature.

condensate boson decay is the norm for R-symmetric models (depending on whether the condensate is strongly or weakly coupled) due to the kinematic constraints on forming gauginos. Three examples of R-balls in R-symmetric, gauge mediated SUSY breaking are provided in figure 4.4.

O'Raifeartaigh models with spontaneously broken R-symmetry have a somewhat different phenomenology. In all regions where R-ball decay has a significant impact on the evolution of the universe decay is to visible sector gauge bosons, and can result in either heating or cooling. Cooling takes place if the original reheat temperature is high and $\Lambda \lesssim 10^{18}$ GeV (the decreased lifetime wins out against the increased energy), whereas heating occurs elsewhere. There also remains a small region supporting R-ball dark matter, with parameters skewed towards the larger R-balls of eq. (4.66). Some example of R-balls in R-breaking, gauge mediated SUSY breaking are shown in figure 4.5.

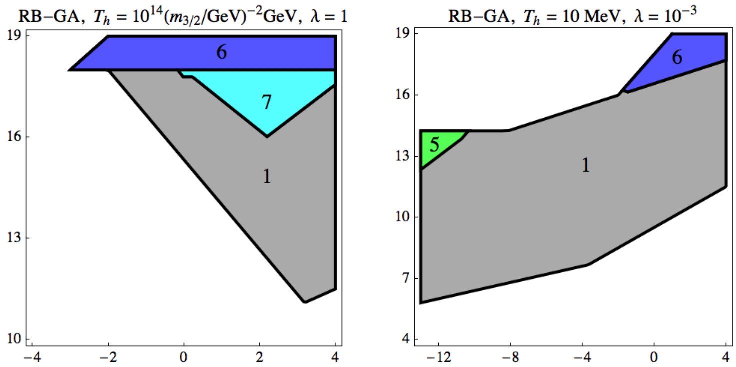


Figure 4.5: R-balls in R-breaking, gauge mediated SUSY breaking as a function of $\log(m_{3/2}/\text{GeV})$ (horizontal axis) and $\log(\Lambda/\text{GeV})$ (vertical axis) for various values of $\lambda$ and $T_h$. In region 1 their density is small and they have no significant effects. Region 5 corresponds to R-ball dark matter. In region 6/7 they reheat/cool the universe by decaying to visible sector gauge bosons. Outside the shaded region R-balls do not form due to thermal effects, contribute more than the observed cold dark matter density or their decay violates the bounds on the reheat temperature.

### 4.4.3 Detecting R-balls

Experimentally, dark matter R-balls would be challenging to observe. They cannot be produced in colliders, but one might hope for a dominant decay mode to visible sector particles that can be observed in some other type of experiment. Gravitino decays and any decays to the hidden sector are thus ruled out, but decays at one loop via the messengers of gauge mediation are a possible candidate. If R-symmetry is preserved the energy released in the decay of a single condensate quanta (4.60) must be sufficient to produce a pair of gauginos, but if R-symmetry is spontaneously broken R-balls can always decay to massless visible sector gauge bosons. From eq. (4.66) the expected energy range for R-ball dark matter is

$$10^{-8}\ \text{GeV} \lesssim \Delta E \lesssim 10^{-1}\ \text{GeV} \tag{4.67}$$

which is well below the gaugino mass. Ergo dark matter R-balls can only be observed through decay to visible sector particles if R-symmetry is spontaneously broken. In this case the energy of the decay products is well defined, potentially resulting in an observable spike in the photon spectrum somewhere in the above energy range.

Q-ball detection techniques, as studied in refs. [154, 161–167], do not apply to R-balls as the condensate field is not charged under the visible sector gauge group. One could perhaps search for them using direct detection experiments though. Visible sector matter will scatter elastically off R-balls at one loop, through penguin diagrams containing messengers, for example. Owing to the classical nature of R-balls this process is likely to be somewhat non-standard and could produce a distinctive signature. The details of both this and the possible photon spectrum are left for future work.

Evidence for R-balls decaying before the present day would surely necessitate the inception of a test with a more cosmological nature. Their formation and decay are potentially significant events in the evolution of the universe so may well have left an imprint on some large scale, cosmological observable. However, the details of such a test are beyond the scope of this work.

## 4.5 Summary

Condensates forming along flat directions of O'Raifeartaigh models can have a significant impact on the evolution of the universe. They are somewhat generic, emerging in any model where the flat direction acquires a tachyonic soft mass through couplings to the inflaton. When a condensate does form it eventually fragments into non-topological solitons with conserved R-charge, known as R-balls. These objects are large, classical configurations and allow an approximate, analytical description. Formation is insensitive to whether or not R-symmetry is spontaneously broken, but decay is not.

Depending on the scale up to which the O'Raifeartaigh model is valid, the scale of SUSY breaking and the strength of tree level couplings in the superpotential R-balls result in a variety of phenomena (figures 4.3, 4.4 and 4.5). In gravity mediated SUSY breaking they can reheat the universe through decays to gravitinos or condensate field bosons. In gauge mediation they provide a good dark matter candidate if stable, or decay to gravitinos, condensate field bosons or visible sector gauge bosons, either reheating or cooling the universe. Both mediation mechanisms enable one to decouple the generation of visible sector matter from inflaton dynamics, instead using R-balls to reheat the universe.

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