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Natural convection of nanofluids in open rectangular cavity

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Abstract

An investigation of the effect of nano particles on natural convection of water based nanofluids contained in an open rectangular cavity is carried out numerically. The flow pattern and heat transfer characteristics are studied for different values of volume fraction in the range $0 \le \phi \le 0.2$, Rayleigh number in the range $1 \le Ra \le 10^9$ and the nano particles with different thermo physical properties. It was found that for low Rayleigh numbers, heat transfer exhibits a decreasing trend for increasing values of volume fraction of oxide nanofluids, whereas for higher values of Rayleigh numbers, an increasing trend of heat transfer was observed due to increase in the volume fraction of nanofluids.

Keywords: Natural convection; Nano Fluids; Open Cavity.

Nomenclature

A	aspect ratio	x, y	non dimensional coordinate axis
C_p	specific heat at constant pressure (JK^{-1})		*Greek symbols*
g	acceleration due to gravity (ms^{-2})	α	thermal diffusivity $(k/\rho C_p)$
H	height of the enclosure	β	thermal expansion coefficient (K^{-1})
h	mesh spacing	θ	non dimensional temperature
(i, j)	nodal locations of (x, y) on grid	μ	dynamic viscosity $(m^{-1}s^{-1})$
I, J	maximum grids along coordinate axis	ν	kinematic viscosity $m^2 s^{-1} Kg^{-1}$
k	thermal conductivity $(Wm^{-1}K^{-1})$	ρ	density of fluid (Kgm^{-3})
L	length of the cavity	ϕ	nano particles volume fraction
Nu	local Nusselt number	$\overline{\psi}$	stream function $(m^2 s^{-1} Kg^{-1})$
$\overline{Nu}$	average Nusselt number	ψ	non dimensional stream function
$\overline{p}$	fluid pressure (Pa)	$\overline{\omega}$	dimensional vorticity function (s^{-1})
Pr	Prandtl number of fluid	ω	non dimensional vorticity function
Ra	Raleigh number		*Subscripts*
$\overline{T}$	dimensional temperature (K)	s	solid particles properties
$\overline{T}_H, \overline{T}_C$	maximum and minimum temperature (K)	f	fluid properties
$\overline{T}_0$	average/reference temperature (K)	nf	nanofluid properties
$\bar{t}$	dimensional time (s)	eff	effective property
t	non dimensional time	in	incoming

$\bar{u},\bar{v}$	velocity components (ms^{-1})	out	outgoing
u,v	non dimensional velocity components	0	reference state
$\bar{x},\bar{y}$	dimensional coordinate axis (m)		

1. Introduction

Nano fluid is a remarkable advancement in heat transfer engineering, as it has numerous applications in the practical situations where enhanced heat transfer is required. Heat transfer enhancement can be acheived by suspending nano size solid particles of relatively larger thermal conductivity in pure fluids. This idea of nanofluids was first proposed by Choi [1]. Khanafer et al. [2] numerically investigated the natural convection of nanofluids in a two dimensional enclosure. They considered different models of nanofluids and showed that the heat transfer of nanofluids is greater than that of pure fluid for all values of Grashof number. Hwang et al. [3] conducted the buoyancy-driven heat transfer study of water-based Al_2O_3 nanofluids in a rectangular cavity. Ho et al. [4] considered natural convection of nanofluid in a square enclosure. They used water $-Al_2O_3$ nano fluid to compare the heat transfer for four different models of effective dynamic viscosity and effective thermal conductivity that were found in literature. Oztop and Abu-Nada [5] considered natural convection of water based nanofluids in partially heated rectangular enclosures. They considered three different types of nano particles namely Copper (Cu), Al_2O_3 and TiO_3. They considered the volume fraction in the range $0 \le \phi \le 0.2$. The implementation of boundary conditions for open ended rectangular domains differ a lot from the usual solid wall boundary conditions. A detailed description can be seen in Roache[6], whereas recent work on open ended domains is considered by Saleem et al. [7,8]. We consider the natural convection of water based nanofluid in an open rectangular domain, and focus on investigating the heat transfer of nanofluids in open ended domain.

2. Mathematical Formulation

Consider two-dimensional flow of a nanofluid confined in an open rectangular cavity of length L and height H. The left wall is assumed to be at temperature $\bar{T}_H$. The temperature of the fluid that enters the cavity region from the right end is supposed $\bar{T}_C$ at $\bar{t}=0$ (where $\bar{T}_H > \bar{T}_C$). We use the subscripts 'in' for incoming, and 'out' for outgoing fluid respectively. Moreover the subscripts 's' and 'f' are respectively taken for physical properties of solid particles and pure fluid, whereas the subscript 'nf' stands for the physical properties of nanofluid. Flow configuration and physical boundary conditions are shown in Figure 1.

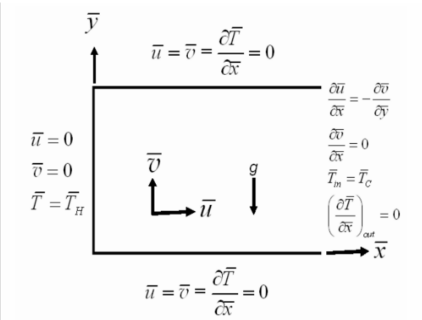

Fig.1. Flow configuration in coordinate system

Let ϕ be the volume fraction of nanoparticles with in the fluid, and ρ_s, ρ_f be the densities of the solid particles and the fluid respectively, then the effective density of the nanofluid is given by (see Khanafer et al. [2], Oztop and Abu-Nada [5])

$$\rho_{nf} = (1-\phi)\phi_f + \phi\rho_s \tag{1}$$

Like wise the effective heat capacity for nanofluid is expressed as

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \tag{2}$$

Further we approximate the effective thermal conductivity of the nanofluid by the relation

$$k_{eff} = \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \tag{3}$$

Moreover the viscosity of nanofluid is given by

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{4}$$

Here the effective thermal conductivity and the effective dynamic viscosity are chosen in view of the maximum heat transfer enhancement (see also [3-5]). Finally the thermal diffusivity of nanofluid is given by

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{5}$$

Further we introduce the non dimensional variables by using the following transformations for stream vorticity form

$$\begin{aligned} &x = \frac{\bar{x}}{H}, \quad y = \frac{\bar{y}}{H}, \quad t = \bar{t}\,\frac{\alpha_f}{H^2}, \quad u = \bar{u}\,\frac{H}{\alpha_f}, \quad v = \bar{v}\,\frac{H}{\alpha_f}, \\ &\psi = \frac{\bar{\psi}}{\alpha_f}, \quad \omega = \bar{\omega}\,\frac{H^2}{\alpha_f}, \qquad \theta = \frac{\bar{T} - \bar{T}_C}{T_H - \bar{T}_C}, \quad A = \frac{L}{H} \end{aligned} \tag{6}$$

where x, y are the non dimensional coordinate axis, u, v are the non dimensional velocity components, ψ and ω are the non dimensional stream and vorticity functions, t is the non dimensional time, θ is the non dimensional temperature, whereas A is the aspect ratio of the cavity. By making use of these dimensionless parameters, for the unsteady motion of nanofluid, the Navier Stokes equations in stream-vorticity form, along with the energy equation in dimensionless rectangular coordinate system are given by (see [3-5])

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \tag{7}$$

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = \left[\frac{\Pr}{(1-\phi)^{0.25}\left\{(1-\phi) + \phi\frac{\rho_s}{\phi_f}\right\}}\right]\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) + Ra\Pr\left[\frac{1}{\frac{(1-\phi)}{\phi}\frac{\rho_f}{\rho_s} + 1}\frac{\beta_s}{\beta_f} + \frac{1}{\frac{\phi}{(1-\phi)}\frac{\rho_f}{\rho_s} + 1}\right]\frac{\partial \theta}{\partial x} \tag{8}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \lambda\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{9}$$

with the following boundary conditions (see [6-8])

$$\begin{array}{l} t < 0 \;\; u = v = \psi = \omega = \theta = 0 \quad 0 \le y \le 1 \quad 0 \le x \le A \\ t \ge 0 \;\; u = v = \psi = 0, \quad \omega = -\frac{\partial u}{\partial y}, \quad \frac{\partial \theta}{\partial y} = 0, \quad y = 0 \quad 0 \le x \le A \\ u = v = \psi = 0, \quad \omega = -\frac{\partial u}{\partial y}, \quad \frac{\partial \theta}{\partial y} = 0, \quad y = 1 \quad 0 \le x \le A \\ u = v = \psi = 0, \qquad \omega = \frac{\partial v}{\partial x}, \qquad \theta = 1 \quad x = 0 \quad 0 \le y \le 1 \\ \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}, \qquad \omega_{in} = \theta_{in} = 0 \qquad x = A \qquad 0 \le y \le 1 \\ \frac{\partial v}{\partial x} = \left(\frac{\partial \omega}{\partial x}\right)_{out} = \left(\frac{\partial \theta}{\partial x}\right)_{out} = 0 \qquad x = A \qquad 0 \le y \le 1 \end{array} \tag{10}$$

where $$\Pr = \frac{\nu_f}{\alpha_f}, \quad Ra = \frac{g\beta_f (T_H - \bar{T}_C) H^3}{\alpha_f \nu_f}, \quad \lambda = \frac{\frac{k_{nf}}{k_f}}{(1-\phi) + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}} \tag{11}$$

are respectively the Prandtl number, the Rayleigh number and the diffusion coefficient for the energy equation of nanofluid. We finally define the dimensionless local and average heat transfer rate of the left wall for nanofluid the following relations (see also [5]).

$$Nu = -k_{eff}\left(\frac{\partial \theta}{\partial x}\right)_{x=0} \qquad \bar{N}u = \int_0^1 Nu\,dy \tag{12}$$

where dy is the element of length y along the wall. In order to compare the heat transfer characteristics pure fluid and the fluid with nano-particles, we consider three different water based nanofluid combinations. The physical properties of the fluid and nano-particles are given in table 1. (see[5])

Table 1. Thermo physical properties of pure fluid (water) and nano particles under consideration

Physical properties	Pure fluid Water (H_2O)	Copper Cu	Aluminum Oxide Al_2O_3	Telinium Oxide TiO_3
C_p	4179	385	765	686.2
ρ	997.1	8933	3970	4250
k	0.613	400	40	8.9538
α	1.47×10^{-7}	1.1631×10^{-4}	1.317×10^{-5}	3.07×10^{-6}
β	2.1×10^{6}	1.67×10^{5}	8.5×10^{4}	9×10^{4}

3. Method of Solution

The dimensionless form of the governing equations given by (8) and (9) are descretized using the Alternate Direct Implicit method. For non linear terms of these equations, we used the second upwind difference technique. The elliptic partial differential equation (8), is solved using the Successive Over Relaxation technique with residual tolerance of the order of 10^{-5}. The solution is complemented with the descretized form of boundary conditions given in equations (10). With H as the reference height of the cavity, we have considered a uniform grid of size $h = H/(J-1)$, where J is the maximum number of grids along coordinate axes. Throughout the computation we take $H = 1$. In order to meet the convergence to the steady state, we have considered the tolerance of order 10^{-8}. Further details of the employed method can also be found in [6-8]. All computations are performed for $A = 2$. Intel 1.83 GHz C machine is used for the entire computation.

4. Results and Discussion

We have considered the natural convection of water based nanofluid in an open rectangular cavity. The left wall is considered at a higher temperature than the one of the opening. The effect of volume fraction of nano particles, and Rayleigh number for three different nanofluids is studied. The result are graphically represented in terms of streamlines, isotherms, and heat transfer rate for different values of these governing parameters.

4.1 *The Effect of Volume Fraction*

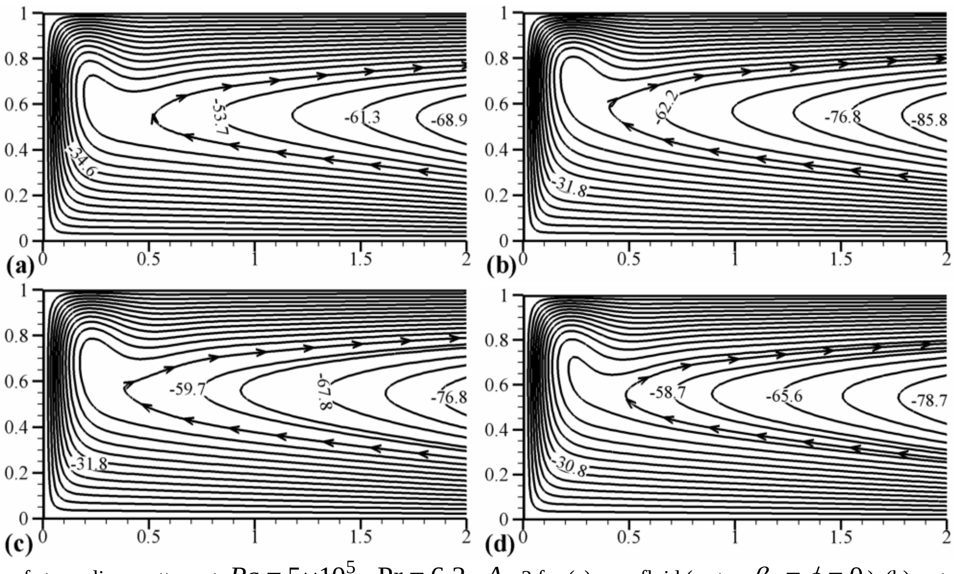

Fig.2. Comparison of streamlines pattern at $Ra = 5\times10^5$, $\Pr = 6.2$, A =2 for (a) pure fluid (water $\beta_s = \phi = 0$), (b) water-copper nanofluid ($\beta_s = 1.67\times10^5, \phi = 0.05$) (c) water- Al_2O_3 nanofluid ($\beta_s = 0.85\times10^5, \phi = 0.05$) (d) water- TiO_3 nanofluid ($\beta_s = 0.9\times10^5, \phi = 0.05$)

Figure 2 represents the comparison of streamlines between pure fluid (water) and water based nanofluids while $Ra = 5\times10^5$, $\Pr = 6.2$. Figure 2 (a) represents the streamline pattern for pure fluid. Comparing Figure 2 (a) and 2 (b), we see that the magnitude of the strength of flow close to the opening in case of water is 68.9, whereas it is 85.8 in case of water-Cu nanofluid. Thus the flow strength of nanofluid is greater than that of the pure fluid in this case. Figure 2 (c) and 2 (d)show the stream line pattern for water- Al_2O_3 and water- TiO_3 nano fluids, whereas their strength is numerically 76.8 and 78.7 respectively. From Figure 2, it can be discerned that the flow in case of nanofluid increases. Moreover, comparing Figure 2 (b)-2 (d) one can see that the strength of metal based nanofluid is maximum. Although the nanofluids of oxides have greater strength than that of the pure fluid, yet the flow is not more than that of the water-Cu nanofluid.

Let us now consider the effect of volume fraction on these nanofluids separately. Figure 3 shows the average heat transfer rate as a function of volume fraction for (a) water-Cu, (b) water- Al_2O_3 and (c) water- TiO_3 nanofluids while Pr=6.2 at four different values of Rayleigh number. Let us first consider Figure 3 (a). It can be seen that heat transfer increases with the increase in volume fraction of solid particles. Even at $Ra = 10^3$, heat transfer increases from 0.775 to 1.3, which does not seem significant due to large scale of the figure along $y - axis$. Now comparing 3 (a) with 3 (b) and 3 (c), we observe that this increasing trend is more prominent in water-Cu nanofluid for $Ra = 10^4, 10^5, 10^6$. For instance at $Ra = 10^6$, in Figure 3 (a), heat transfer increases from 22.6 to 38.3 between $0 \le \phi \le 0.2$ for water-Cu nanofluid, whereas it reaches up to 32.6 for water- Al_2O_3 as in Figure 3 (b), and has a maximum value of 31.6 in case of water- TiO_3 nanofluid at $\phi = 0.2$ as shown in Figure 3 (c). However the behavior of heat transfer in Figure 3 (b) and 3 (c) at $Ra = 10^3$ require some attention. There is a very slight decline in the heat transfer rate at $Ra = 10^3$ in Figure 3 (b) and 3 (c), which is not significantly visible due to scale of the Figure. This deviating behavior for water- Al_2O_3 in Figure 3 (b) is such that the heat transfer very slightly increases from 0.775 to 0.779 between $0 \le \phi \le 0.05$, and then decreases up to 0.602 in the range $0.05 \le \phi \le 0.2$. This variation of heat transfer water- Al_2O_3 nanofluid was so slight in comparison to the other values in Figure 3 (b) that it was not visible. Now for water- TiO_3 nanofluid, first the heat transfer rate increases from 0.775 to 0.797 in the range $0 \le \phi \le 0.07$, then it decreases up to 0.704 between $0.07 \le \phi \le 0.2$. Thus we can say that for water-Cu (metallic) nanofluid, Heat transfer rate increases with volume fraction for all Rayleigh numbers in the Range $10^3 \le Ra \le 10^6$. However heat transfer does not monotonically increase with volume fraction at $Ra = 10^3$, for Al_2O_3 and water- TiO_3 (oxide) nanofluids at $Ra = 10^3$.

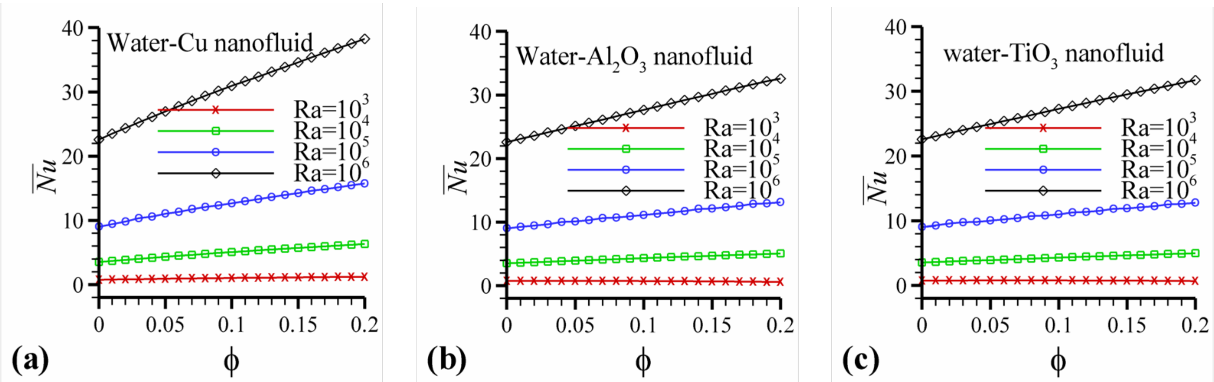

Fig.3. Average heat transfer rates as a function of volume fraction of nanofluids at different values of Rayleigh number for (a) water-Cu nanofluid (b) water- Al_2O_3 nanofluid (c) water- TiO_3 nanofluid combinations

4.2 *The Effect of Rayleigh number*

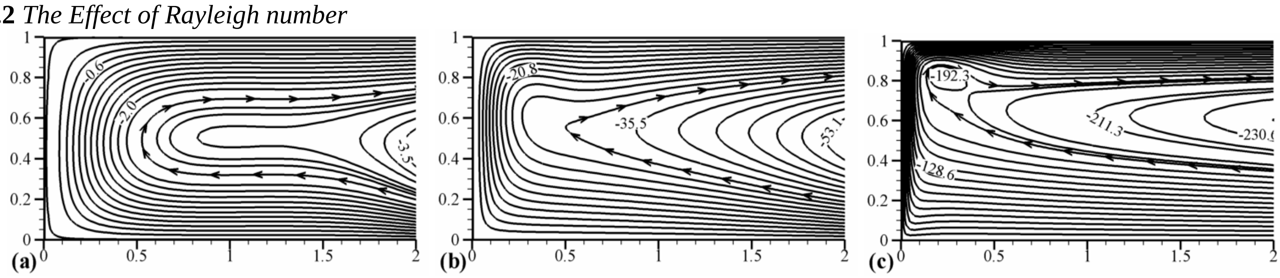

Fig.4. Streamlines of water- TiO_3 nanofluid at $\Pr = 6.2$, A =2, $\phi = 0.04$, $t = 0.5$ for (a) $Ra = 10^3$ (b) $Ra = 10^5$ (c) $Ra = 10^7$

Figure 4 shows the pattern of streamlines of water- TiO_3 nanofluid while $\Pr = 6.2$, A =2, $\phi = 0.04$, $t = 0.5$ for (a) $Ra = 10^3$ (b) $Ra = 10^5$ (c) $Ra = 10^7$ respectively. It can be seen from Figure 4 (a) that at $Ra = 10^3$, the strength of flow is very weak. Comparing it with Figure 4 (b) and (c) we see that the strength of the flow increases and the recirculation starts to grow. As can be seen from Figure 4 (c), the strength of flow close to the opening is 230 for $Ra = 10^7$, whereas it was merely 3.5 for $Ra = 10^3$ shown in Figure 4 (a). Finally, Figure 5 shows a comparison of the average heat transfer rate as a function of Rayleigh number for different nanofluids. This Figure also depicts that visible increase in heat transfer for all combinations takes place beyond $Ra > 10^3$. By comparison it can be seen that heat transfer for all nanofluids is greater than the base fluid (water), at any given value of Rayleigh number. However by carefully observing the graph we see that heat transfer in case of water-Cu nanofluid is maximum. For instance at $Ra = 10^8$, Heat transfer rate of pure water is 51.4, whereas its value is 59.0, 58.5, and 57.4 for water-Cu, water-Al_2O_3 nanofluid and water-TiO_3 nanofluid combinations respectively. It can now again be seen that heat transfer of water-Cu nanofluid is maximum among these three at any given value of Rayleigh number.

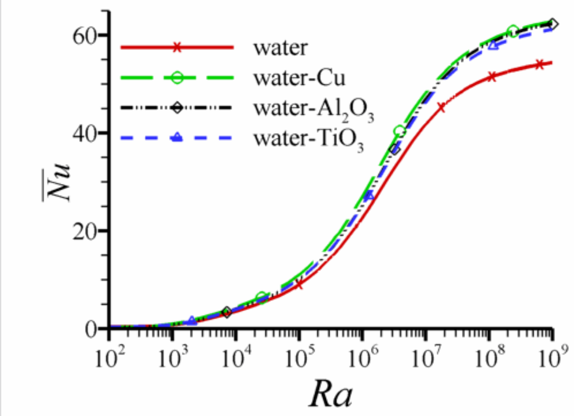

Fig.5. Comparison of the average Nusselt number of heated wall as a function of Rayleigh number, between pure fluid (water $\phi = \beta_s = 0$, $\Pr = 6.2$, A =2), and different water based nanofluid combinations at $\phi = 0.05$, $\Pr = 6.2$, A =2.

5. Conclusions

Natural convection of three different types of water based nanofluids in an open rectangular cavity is numerically studied. It was found that for low Rayleigh numbers ($Ra = 10^3$), heat transfer exhibits a decreasing trend for increasing values of volume fraction of oxide nanofluids in the range $0.05 \le \phi \le 0.2$, whereas for higher values of Rayleigh numbers where the dominant convection takes place ($10^4 \le Ra \le 10^6$, an increasing trend of heat transfer was observed due to increase in the volume fraction of nanofluids. However, water-Cu nanofluid showed an increasing trend for all values of Rayleigh number considered. In comparison to the other two oxide nanofluids considered, water-Cu nano fluid proved to be a better nanofluid in the sense that it has greater heat transfer efficacy and shows an increasing trend for range of Rayleigh number considered. A heat transfer comparison of the nanofluids with that of pure fluid is also made for different values of Raleigh numbers in the range $1 \le Ra \le 10^9$, and it is seen that heat transfer in case of nanofluid is greater than that of the base fluid in the region of dominant convection ($10^3 \le Ra \le 10^9$.

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