Sequential Slotted Amplify-Decode-and-Forward

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***Abstract*—In this paper, we propose a novel slotted hybrid cooperative protocol named the sequential slotted amplify-decode-and-forward (SADF) protocol and evaluate its performance in terms of diversity-multiplexing trade-off (DMT). The relays between the source and destination are divided into two different groups and each relay either amplifies or decodes the received signal. We first compute the optimal DMT of the proposed protocol with the assumption of perfect decoding at the DF relays. We then derive the DMT closed-form expression of the proposed sequential-SADF and obtain the proximity gain bound for achieving the optimal DMT. With the proximity gain bound, we then found the distance ratio to achieve the optimal DMT performance. Simulation result shows that the proposed protocol with high proximity gain outperforms other cooperative communication protocols in high SNR regime.**

I. Introduction

In cooperative communication systems, different terminals collaborate to form a virtual-multiple antenna array system and exploit the spatial diversity to achieve a better performance [1], [2]. Cooperative communication protocols can be generally categorized into amplify-and-forward (AF) protocol and decode-and-forward (DF) protocol and are often compared using the trade-off measurement between the error probability and the data rate of a multiple-input multiple-output (MIMO) network known as diversity-multiplexing trade-off (DMT) [3].

In [4], the authors pointed out the fact that both nonorthogonal amplify-and-forward (NAF) [1], [2] and dynamic decode-and-forward (DDF) [2] protocols fail to achieve high diversity gain ($d(r) > 1$) in the high multiplexing gain regime ($r > 0.5$), due to the fact that half of the source signal is not forwarded by the relays. The authors subsequently proposed a new AF protocol known as the sequential slotted amplify-and-forward (SAF) protocol and demonstrated that it achieves the multiple-input single-output (MISO) bound when the number of transmission slots, M goes to infinity, with the assumption of isolated relays. Recently in [5], the authors proposed opportunistic DF (ODF) protocol to achieve the DMT upper bound of N-relay $(N+1)$-slot SAF without relay isolation model. However the protocol requires the nth relay to listen for n slots to decode the nth signal, for $n \in \{1, \cdots, N\}$.

In this paper, we propose a slotted hybrid cooperative protocol named the sequential slotted amplify-decode-and-forward (SADF) protocol for half duplex single antenna multiple relay channels. The source information is modulated and divided into M symbols and transmitted from the source to the destination with the assistance of N relays. The relays between the source and the destination are divided into DF relays and AF relays. A DF relay decodes the received signal if it is not an outage event and a AF relay performs linear processing on the received signal and forwards it to the destination in a round robin manner. We first derive the optimal DMT of the proposed sequential-SADF, on the assumption that all the DF relays are able to decode the received signal perfectly. The optimal DMT of the sequential-SADF can be expressed as

$$d^*(r) = (N+1)\left(1 - \frac{M+1}{M}r\right)^+ . \qquad (1)$$

where $(x)^+$ denotes $\max\{x, 0\}$. We then derive the closed-form DMT expression of the proposed sequential-SADF by considering the effect of inter relay interference (IRI) and the outage event using the proximity gain η. We found that the optimal DMT of the sequential-SADF is achievable if $\eta > 1 + \frac{N_\mathsf{A} M}{2N_\mathsf{D}(M+1)}$, where N_D and N_A denote the number of DF and AF relays respectively. With the aid of the proximity gain bound, the optimal DMT of the sequential-SADF is achievable using the relay ordering and scheduling strategy proposed in [6].

II. The Sequential-SADF Protocol

A. System Model

Consider a cooperative relaying network where the source information bits are modulated and divided into M symbols which are to be transmitted from the source s to the destination d with the assistance of the N relays r_i. Without loss of generality, we presume that one symbol is transmitted during each transmission slot, x_m, for $m = 1, \cdots, M$, and M is even. One additional slot is added at the end of the transmission sequence for the relay to *protect* the last symbol and thus a multiplexing gain of $\frac{M}{M+1}$ is achieved. The power allocation of s and r for mth slot are denoted by π_m and $\bar{\pi}_m$ and satisfy $\sum_{m=1}^{M+1}(\pi_m + \bar{\pi}_m) = M$.

Let us assume that there are $N = N_\mathsf{A} + N_\mathsf{D}$ relays, where N_A and N_D denote the the number of AF and DF relays, respectively. Denote $\mathcal{S}$ a set of positive even integer with the limits of $\mathcal{S} = \{2, 4, \cdots, M\}$ and $L = \frac{M}{2N_\mathsf{A}}$, the AF relay r_m, $m \in \mathcal{S}$ amplifies and forward x_m to the destination in round robin manner, $\mathsf{r}_m = \mathsf{r}_{2lN_\mathsf{A}+m}$, where $l \in [1, L-1]$. Denote $\mathcal{S}'$ a set of positive odd integer with the limits of $\mathcal{S}' = \{1, 3, \cdots, M-1\}$ and $L' = \frac{M}{2N_\mathsf{D}}$, the DF relay $\mathsf{r}_{m'}$, $m' \in \mathcal{S}$ decodes and forward $x_{m'}$ to the destination in round robin manner, $\mathsf{r}_{m'} = \mathsf{r}_{2l'N_\mathsf{D}+m'}$, where $l' \in [1, L'-1]$. Note that the values of M, N_A and N_D are chosen carefully to ensure that both L and L' are integer numbers.

The channel gains between terminals are modelled as independent quasi-static fading channels, i.e., the channel gains do not change during the transmission of a cooperative frame. The quality of the channel gain is parameterized by the distance between terminals $D_{i,j}$, for $i, j \in \{\mathsf{s}, \mathsf{d}, 1, \cdots, M\}$, $i \neq j$. Let $h_{i,j}$ be the i.i.d complex Gaussian random variable with zero mean and unit variance. The channel gain between s and d is modelled as $g_0 \triangleq \sqrt{D_{\mathsf{s},\mathsf{d}}^{-\alpha}} h_{\mathsf{s},\mathsf{d}}$. Similarly, $f_i \triangleq \sqrt{D_{\mathsf{s},i}^{-\alpha}} h_{\mathsf{s},i}$ and $g_i \triangleq \sqrt{D_{i,\mathsf{d}}^{-\alpha}} h_{i,\mathsf{d}}$, respectively denote the channel gain between s and r_i and the ones between r_i and d. $\gamma_{i,j} \triangleq \sqrt{D_{i,j}^{-\alpha}} h_{i,j}$ is used to denote the channel gain between r_i and r_j. α denotes the path loss exponent.

For the case of exposition and practical relevance, we assume that each terminal is installed with a half duplex single antenna that can either transmit or receive signals at any given time. However, the result presented in this paper can be easily extended to the case where terminals have multiple and/or full duplex antennas. Throughout the paper, we assume perfect synchronization between all the transmitting terminals during each transmission slot.

B. Equivalent Channel Model

At the first time slot, s transmits the first symbol, x_1 to r_1 and d. The received signals can be expressed as

$$\begin{aligned} y_{\mathsf{r},1} &= \sqrt{\pi_1 \, \mathsf{SNR}} f_1 x_1 + n_{\mathsf{r},1}, \\ y_{\mathsf{d},1} &= \sqrt{\pi_1 \, \mathsf{SNR}} g_0 x_1 + n_{\mathsf{d},1}, \end{aligned}$$

where $y_{\mathsf{r},1}$ and $y_{\mathsf{d},1}$ denote the received signals at r_1 and d during the first time slot, respectively. $n_{\mathsf{r},1}$ and $n_{\mathsf{d},1}$ denote independent additive white Gaussian noise (AWGN), with zero mean and unit variance. SNR denotes the signal to noise ratio.

After receiving the signal from s, r_1 attempts to decode the received signal only if it is not an outage event, i.e. $\log(1+\mathsf{SNR}\, \pi_1 |f_1|^2) > r$, where $\log(\cdot)$ denotes logarithm base 2 and r denotes the data rate in bits per channel use (BPCU). For non-outage scenario, r_1 encodes the decoded signal and transmit during the second time slot. At the same time slot, s transmits the second symbol, x_2 to both r_2 and d. The received signals can be expressed as

$$\begin{aligned} y_{\mathsf{r},2} &= \sqrt{\pi_2 \, \mathsf{SNR}} f_2 x_2 + \sqrt{\bar{\pi}_2 \, \mathsf{SNR}} \gamma_{1,2} x_1 + n_{\mathsf{r},2}, \\ y_{\mathsf{d},2} &= \sqrt{\pi_2 \, \mathsf{SNR}} g_0 x_2 + \sqrt{\bar{\pi}_2 \, \mathsf{SNR}} g_1 x_1 + n_{\mathsf{d},2}. \end{aligned}$$

At the third time slot, r_2 performs linear processing on the received signal and transmits to r_3 and d. Both r_3 and d receive a linear combination of the transmitted symbol and the linearly processed signal from s and r_2 respectively, and can be expressed as

$$\begin{aligned} y_{\mathsf{r},3} &= \sqrt{\pi_3 \, \mathsf{SNR}} f_3 x_3 + \sqrt{\bar{\pi}_3 \, \mathsf{SNR}} \gamma_{2,3} b_2 y_{\mathsf{r},2} + n_{\mathsf{r},3}, \\ y_{\mathsf{d},3} &= \sqrt{\pi_3 \, \mathsf{SNR}} g_0 x_3 + \sqrt{\bar{\pi}_3 \, \mathsf{SNR}} g_2 b_2 y_{\mathsf{r},2} + n_{\mathsf{d},3}, \end{aligned}$$

where b_2 is the processing gain at r_2 subject to the power constraint $\mathbb{E}\{b_2 y_{\mathsf{r},2}\} \leq 1$ and $\mathbb{E}\{\cdot\}$ denotes the statistical expectation operator. During the fourth time slot, r_3 decodes the received signal only if it is not an outage, i.e.

$$\log \left[1 + \frac{\pi_3 \, \mathsf{SNR} \, |f_3|^2}{1 + \bar{\pi}_3 \, \mathsf{SNR} \, |\gamma_{2,3}|^2} \right] > r,$$

and transmits the encoded signal to r_4 and d. Source node on the other hand, transmit the fourth symbol x_4 and the received signal of r_4 and d can be expressed as

$$\begin{aligned} y_{\mathsf{r},4} &= \sqrt{\pi_4 \, \mathsf{SNR}} f_4 x_4 + \sqrt{\bar{\pi}_4 \, \mathsf{SNR}} \gamma_{3,4} x_3 + n_{\mathsf{r},4}, \\ y_{\mathsf{d},4} &= \sqrt{\pi_4 \, \mathsf{SNR}} g_0 x_4 + \sqrt{\bar{\pi}_4 \, \mathsf{SNR}} g_3 x_3 + n_{\mathsf{d},4}. \end{aligned}$$

These transmission steps are continuously repeated until M symbols have been transmitted from s.

III. Diversity Multiplexing Trade-off

The DMT introduced in [3] is a trade-off measurement between the reliability and the throughput of a multiple-input multiple-output (MIMO) network for quasi-static Rayleigh fading channels in the high SNR regime. A family of a codes $\{\mathcal{C}(\mathsf{SNR})\}$ is said to achieve the multiplexing gain, r and the diversity gain, d if

$$\lim_{\mathsf{SNR} \to \infty} \frac{R(\mathsf{SNR})}{\log \mathsf{SNR}} = r, \quad \lim_{\mathsf{SNR} \to \infty} \frac{\log P_e(\mathsf{SNR})}{\log \mathsf{SNR}} = -d,$$

where $R(\mathsf{SNR})$ is the data rate measured by BPCU and is increased with SNR, $R(\mathsf{SNR}) = r \log(\mathsf{SNR})$. $P_e(\mathsf{SNR})$ denotes the average error probability using the maximum-likelihood (ML) detector which can be derived based on the outage probability, $P_e \approx P_{\text{out}}$, in high SNR regime with an arbitrarily long code length.

A. Optimal DMT of the sequential-SADF

We first assume a genie-aided model, where all the DF relays are able to decode the received signal perfectly. Using the received signals at d and r_i from above, the received signal model at the destination can be expressed as

$$\mathbf{y} = \sqrt{\mathsf{SNR}} \mathbf{H} \mathbf{x} + \mathbf{B}' \mathbf{n}. \tag{2}$$

Here $\mathbf{y} \in \mathbb{C}^{M+1}$ is the received signal vector at d, where $\mathbb{C}^L$ denotes a set of complex L-tuples. $\mathbf{x} \in \mathbb{C}^M$ denotes the transmitted signal vector. $\mathbf{H} \in \mathbb{C}^{(M+1) \times M}$ denotes the equivalent channel matrix. $\mathbf{B}'\mathbf{n} \in \mathbb{C}^{M+1}$ denotes the received noise matrix at d, where $\mathbf{B}' = [\mathbf{B} \ \mathbf{I}_{M+1}]$ and $\mathbf{n} = \left[\mathbf{n}_{\mathsf{r}}^{\mathrm{T}} \ \mathbf{n}_{\mathsf{d}}^{\mathrm{T}}\right]^{\mathrm{T}}$. $\mathbf{n}_{\mathsf{r}} \in \mathbb{C}^M$ and $\mathbf{n}_{\mathsf{d}} \in \mathbb{C}^{M+1}$ denote the additive white Gaussian noise vectors at r and d, respectively. Using the received signal model in (2), we obtain the mutual information

$$\begin{aligned} I(\mathbf{x}; \mathbf{y}) &= \log \det \left[\mathbf{I}_M + \mathsf{SNR} \, \mathbf{H}^\dagger \Sigma_{\mathbf{x}} \mathbf{H} \left(\mathbf{B}'^\dagger \Sigma_{\mathbf{n}} \mathbf{B}' \right)^{-1} \right] \\ &= \log \det \left[\mathbf{I}_{M+1} + \mathsf{SNR} \, \mathbf{H} \mathbf{H}^\dagger \left(\mathbf{B} \mathbf{B}^\dagger + \mathbf{I}_{M+1} \right)^{-1} \right] \end{aligned} \tag{3}$$

where $\Sigma_{\mathbf{x}} = \mathbb{E}\{\mathbf{x}\mathbf{x}^\dagger\} = \mathbf{I}_M$ and $\Sigma_{\mathbf{n}} = \mathbb{E}\{\mathbf{n}\mathbf{n}^\dagger\} = \mathbf{I}_{2M+1}$. Note that the noise matrix, $\mathbf{B}'\mathbf{n}$ is not white in general and does not affect the DMT computation [4].

The optimal DMT of the proposed sequential-SADF protocol can be obtained using equation (3). From the transmission sequence, we have the channel matrix:

$$\mathbf{H}=\begin{bmatrix} g_0 & 0 & 0 & & \cdots & 0 \\ g_1 & g_0 & 0 & & & \\ \gamma_{1,2}g_2 & f_2 g_2 & g_0 & 0 & \ddots & \vdots \\ 0 & 0 & g_3 & g_0 & \ddots & \\ & & \ddots & \ddots & \ddots & 0 \\ \vdots & & & 0 & g_{M-1} & g_0 \\ 0 & \cdots & & 0 & \gamma_{M-1,M}g_M & f_M g_M \end{bmatrix}. \tag{4}$$

Note that the linear processing and the power allocation terms have been ignored since they do not impact the DMT computation [4, Lemma 1]. Using channel matrix (4), the outage probability of the proposed sequential-SADF with the assumption of perfect decoding at the DF relays can be expressed as

$$\begin{aligned} &P_{\text{out}}(R(\mathsf{SNR})) \\ &\quad= \Pr\left[I(\mathbf{x};\mathbf{y}) \le (M+1)R(\mathsf{SNR})\right] \\ &\quad\doteq \Pr\left[\log\det\left(\mathbf{I}_{M+1}+\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right) \le (M+1)R(\mathsf{SNR})\right] \end{aligned}$$

The term $(\mathbf{B}\mathbf{B}^{\dagger}+\mathbf{I}_{M+1})^{-1}$ has been eliminated since we are only interested in the exponential function of $P_{\text{out}}(R(\mathsf{SNR}))$. The factor $M+1$ is to account for the fact that the proposed strategy takes $M+1$ transmission slots to transmit M symbols. $\doteq$ denotes asymptotic equality in the high SNR regime.

We consider the following lemma to compute the optimal DMT of the proposed sequential-SADF protocol.

Lemma 1: Given a positive semidefinite $(M+1)\times(M+1)$ penta-diagonal matrix $\mathbf{H}\mathbf{H}^{\dagger}$, we have

$$\begin{aligned} \det\left(\mathbf{I}_{M+1}+\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right) &\le \left(1+\mathsf{SNR}\,|g_0|^2\right)^M \\ &+\prod_{i\in\mathcal{S}}\left[\left(1+\mathsf{SNR}\,|f_i g_i|^2\right)\ \left(1+\mathsf{SNR}\,|\alpha_{i-1}|^2\right)\right], \end{aligned} \tag{5}$$

where $\alpha_i = \sqrt{|g_i|^2+|\gamma_{i,i+1}g_{i+1}|^2}$ and $\mathcal{S}$ being a set of positive even integers with the limits of $\mathcal{S}\in[2,M]$. Equation (5) is expended into (6) for DMT computation.

Please refer to the appendix for the proof of Lemma 1.

Let $v_{g_i}=\lim_{\mathsf{SNR}\to\infty}\frac{\log|g_i|^2}{\log\mathsf{SNR}}$ be the exponential order of $1/|g_i|^2$, and v_{f_i}, $v_{\gamma_{i,j}}$ are similarly defined. Since the channel gains are independent Gaussian variables, the optimal DMT of the sequential-SADF can be obtained using

$$d^*(r)=\inf_{\mathcal{O}^+}\left\{v_{g_0}+\sum_{i\in\mathcal{S}}(v_{g_i}+v_{g_{i-1}}+v_{f_i}+v_{\gamma_{i-1,i}})\right\}, \tag{7}$$

where the outage events $\mathcal{O}^+$ are dominated by (8) for large SNR.

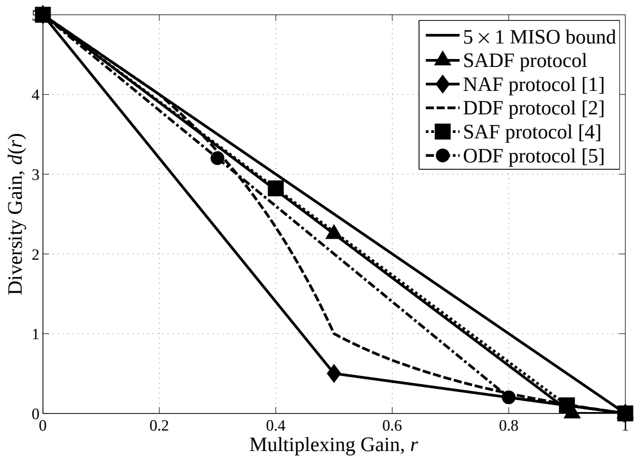

Fig. 1. Diversity-multiplexing trade-off comparison between the optimal sequential-SADF and the existing cooperative communication protocol with four relay (N = 4). Here, the total transmission slots are $M = 10$.

Since solving the optimization problem with different values of exponential order being prohibitive in general, let us assume

$$\begin{aligned} \bar{v}_g &= \min\{v_{g_i}\}_{i=1}^M, \\ \bar{v}_f &= \min\{v_{f_i}\}_{i=1}^M, \\ \bar{v}_\gamma &= \min\{v_{\gamma_{i,j}}\}_{i,j=1}^M. \end{aligned}$$

The optimization problem is now reduced to

$$d^*(r)=\inf_{\mathcal{O}^+}\left\{v_{g_0}+N_{\mathsf{A}}(\bar{v}_f+\bar{v}_g)+N_{\mathsf{D}}(\bar{v}_\gamma+\bar{v}_g)\right\}, \tag{9}$$

with

$$\mathcal{O}^+=\left\{\max\begin{pmatrix} M(1-v_{g_0})^+, \\ \frac{M}{2}(1-\bar{v}_g)^+, \\ \frac{M}{2}(1-\bar{v}_f-\bar{v}_g)^+, \\ \frac{M}{2}(1-\bar{v}_g-\bar{v}_\gamma)^+, \\ \frac{M}{2}(2-\bar{v}_f-2\bar{v}_g)^+, \\ \frac{M}{2}(2-\bar{v}_f-2\bar{v}_g-\bar{v}_\gamma)^+ \end{pmatrix}\le(M+1)r\right\}$$

By solving the optimization problem, we can obtain the optimal DMT of the proposed sequential-SADF protocol:

$$d^*(r)=(N_{\mathsf{A}}+N_{\mathsf{D}}+1)\left(1-\frac{M+1}{M}r\right)^+, \tag{10}$$

Note that for $N=N_{\mathsf{A}}+N_{\mathsf{D}}$, the sequential-SADF achieves the $(N+1)\times1$ MISO transmit bound, for sufficiently large M.

Fig. 1 compares the optimal DMT of the sequential-SADF with other cooperative communication protocols for a four relays scenario (N = 4) and transmission slot, $M = 10$. It can be seen that the DMT of the sequential-SADF dominates both NAF [2] and ODF [5] protocols. On the other hand, the DMT of the sequential-SADF dominates the DDF protocol in most of the multiplexing gain region without the assumption of an infinite block length. Note that SAF protocol has better a slightly better DMT performance due to the fact that the proposed protocol requires an extra time slot for the relay to forward the last symbol to the destination.

$$\det\left(\mathbf{I}_{M+1}+\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right)\leq\left(1+\mathsf{SNR}\,|g_0|^2\right)^M+\prod_{i\in\mathcal{S}}\left[\left(1+\mathsf{SNR}\,|f_ig_i|^2\right)\ \left(1+\mathsf{SNR}(|g_{i-1}|^2+|\gamma_{i-1,i}g_i|^2)\right)\right]$$

$$\leq\left(1+\mathsf{SNR}\,|g_0|^2\right)^M+\prod_{i\in\mathcal{S}}\left[1+\mathsf{SNR}\left(|f_ig_i|^2+|g_{i-1}|^2\ +|\gamma_{i-1,i}g_i|^2\right)+\mathsf{SNR}^2|f_ig_i|^2\left(|g_{i-1}|^2\ +|\gamma_{i-1,i}g_i|^2\right)\right] \quad (6)$$

$$\mathcal{O}^+=\left\{\max\begin{pmatrix} M(1-v_{g_0})^+,\\ \sum_{i\in\mathcal{S}}(1-v_{g_{i-1}})^+,\\ \sum_{i\in\mathcal{S}}(1-v_{f_i}-v_{g_i})^+,\\ \sum_{i\in\mathcal{S}}(1-v_{g_i}-v_{\gamma_{i-1,i}})^+,\\ \sum_{i\in\mathcal{S}}(2-v_{f_i}-v_{g_i}-v_{g_{i-1}})^+,\\ \sum_{i\in\mathcal{S}}(2-v_{f_i}-2v_{g_i}-v_{\gamma_{i-1,i}})^+\end{pmatrix}\leq(M+1)r\right\} \quad (8)$$

B. Closed-form DMT of the sequential-SADF

In practice, relays may not always decode the received signal perfectly. In this subsection, we analyse the closed-form DMT of the sequential-SADF with the effect of the inter-relay interference (IRI) on each DF relays, where we define the term η as the average SNR of the source to relay links with IRI in dB scale.

The outage probability of the sequential-SADF protocol without the assumption of perfect decode at the DF relays can be defined as

$$P_{\text{out}}=\sum_{K=0}^{N_{\mathsf{D}}}\Pr(E_K)P_{\text{out}\,|E_K},$$

where E_K denote the event that K number of DF relays are able to decode the source signal perfectly. $P_{\text{out}\,|E_K}$ and $\Pr(E_K)$ denote the outage probability and the probability of the event that K number of DF relays are able to decode the source signal perfectly, respectively.

The DF relay r_i decodes the received signal according to the mutual information $I(x_i;y_{\mathsf{r},i})$ and the outage probability of each individual DF relay can be defined as

$$\begin{aligned}P_{\text{out}\,|\mathsf{r}_i}&=\Pr\left[I(x_i;y_{\mathsf{r},i})<R(\mathsf{SNR})\right]\\&=\Pr\left[\log\left(1+\frac{\pi_i\,\mathsf{SNR}\,|f_i|^2}{1+\bar{\pi}_i\,\mathsf{SNR}\,|\gamma_{i-1,i}|^2}\right)<R(\mathsf{SNR})\right]\\&\doteq\ \Pr\left[\log\left(\frac{\mathsf{SNR}^{\eta_i}|h_{\mathsf{s},i}|^2}{|h_{i-1,i}|^2}\right)<r\log(\mathsf{SNR})\right]\\&\doteq\ \Pr\left[\frac{|h_{\mathsf{s},i}|^2}{|h_{i-1,i}|^2}<\mathsf{SNR}^{-(\eta_i-r)^+}\right],\end{aligned}$$

where

$$\mathsf{SNR}^{\eta_i}\triangleq\frac{\pi_i}{\bar{\pi}_i}\left(\frac{D_{i-1,i}}{D_{\mathsf{s},i}}\right)^{\alpha}. \quad (11)$$

The proximity gain, η_i is the scaling factor on the average SNR between the s to r_i link and r_{i-1} to r_i link in dB. Consider X and Y are i.i.d exponential distributed random variables with unit means, applying [7, Lemma 3.2], we have the cumulative distribution function of $Z=\frac{X}{Y}$ is given by

$$F_Z(z)=1-\frac{1}{1+z}$$

and the outage probability of each individual DF relay can be rewritten as

$$\begin{aligned}P_{\text{out}\,|\mathsf{r}_i}&\doteq\ \Pr\left[\frac{|h_{\mathsf{s},i}|^2}{|h_{i-1,i}|^2}<\mathsf{SNR}^{-(\eta_i-r)^+}\right]\\&\doteq 1-\frac{1}{1+\mathsf{SNR}^{-(\eta_i-r)^+}}\\&\doteq\mathsf{SNR}^{-(\eta_i-r)^+}\end{aligned} \quad (12)$$

If we assume that all the DF relays have the proximity gains with the order of $\eta\leq\eta_1\leq\cdots\leq\eta_{M-1}$, $\Pr(E_K)$ can be expressed as

$$\begin{aligned}\Pr(E_K)&=\left(P_{\text{out}\,|\mathsf{r}}\right)^{(N_{\mathsf{D}}-K)}\times\left(1-P_{\text{out}\,|\mathsf{r}}\right)^K\\&\doteq\ \left(\mathsf{SNR}^{-(\eta-r)^+}\right)^{(N_{\mathsf{D}}-K)}\times\left(1-\mathsf{SNR}^{-(\eta-r)^+}\right)^K\\&\doteq\mathsf{SNR}^{-(N_{\mathsf{D}}-K)(\eta-r)^+},\end{aligned}$$

since $\mathsf{SNR}^{-a}-\mathsf{SNR}^{-(a+b)}\doteq\mathsf{SNR}^{-a}$, for a, $b>0$.

The outage probability of the sequential-SADF protocol with K non-outage DF relays, $P_{\text{out}\,|E_K}$ can be expressed as

$$\begin{aligned}P_{\text{out}\,|E_K}&=\Pr\Big[\log\det\left(\mathbf{I}_{M+1}+\mathsf{SNR}\,\mathbf{H}_K\mathbf{H}_K^{\dagger}\right)\\&\qquad\qquad\leq(M+1)R(\mathsf{SNR})\Big]\\&\doteq\mathsf{SNR}^{-d_K(r)},\end{aligned}$$

where $\mathbf{H}_K$ and $d_K(r)$ denote the channel matrix and the DMT of the sequential-SADF with K non-outage DF relays, respectively.

Using Lemma 1, we have the DMT of the sequential-SADF protocol with K number of DF relays perfectly decoding the source signal can be written as

$$\begin{aligned}d_K(r)&=\left(1-\frac{M+1}{M}r\right)^+\\&\quad+(N_{\mathsf{A}}+K)\left(1-\frac{M+1}{M/2+2LK}r\right)^+.\end{aligned}$$

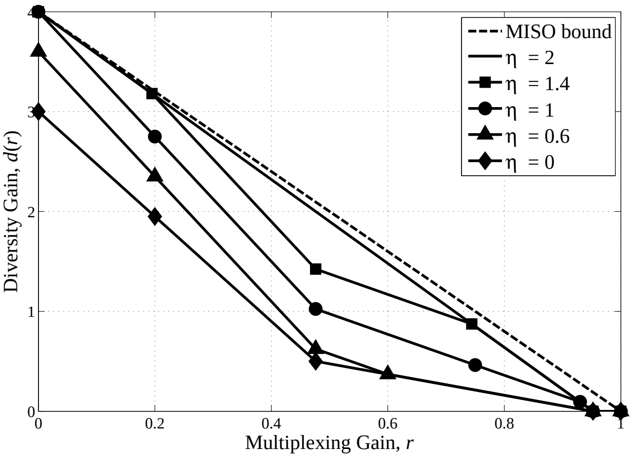

Fig. 2. DMT comparison of the proposed sequential-SADF for different proximity gain, η. Here, the number of DF and AF relays are $N_{\mathsf{A}} = 1$ and $N_{\mathsf{D}} = 3$, respectively. The total transmitted symbols are $M = 20$.

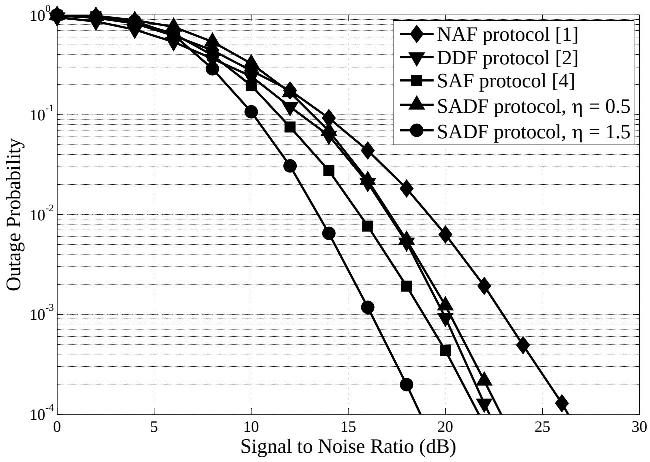

Fig. 3. Outage probabilities of the non-cooperative, NAF, DDF, sequential-SAF and the proposed sequential-SADF with proximity gain $\eta = 0.5, 1.5$ in a four relay network ($N = 4$) and data rates of 2 BPCU. The total transmission slots for both sequential-SAF and sequential-SADF protocols are $M = 20$.

Let us assume two events, where all the DF relays are either in outage (E_0) and non-outage ($E_{N_{\mathsf{D}}}$), which is the worst or optimal scenarios of the proposed sequential-SADF protocol respectively. The outage probability of the sequential-SADF protocol can be written as

$$\begin{aligned} P_{\text{out}} &= \Pr(E_0) P_{\text{out}\,|E_0} + \Pr(E_{N_{\mathsf{D}}}) P_{\text{out}\,|E_{N_{\mathsf{D}}}} \\ &\doteq \mathsf{SNR}^{-\left(1-\frac{M+1}{M}r\right)^+ - N_{\mathsf{A}}\left(1-\frac{M+1}{M}2r\right)^+} \mathsf{SNR}^{-N_{\mathsf{D}}(\eta-r)^+} \\ &\quad + \mathsf{SNR}^{-(N_{\mathsf{A}}+N_{\mathsf{D}}+1)\left(1-\frac{M+1}{M}r\right)^+} \\ &\doteq \mathsf{SNR}^{-d(r)}, \end{aligned} \tag{13}$$

where $d(r)$ is given by the absolute value of the largest exponent term. The closed-form DMT of the proposed sequential-SADF protocol can be obtained by using equation (13), which is shown in (14), where $d_{\min}(r)$ can be expressed as

$$d_{\min}(r) = \min \left\{ \begin{array}{c} (N_{\mathsf{A}} + N_{\mathsf{D}}) \left(1 - \frac{M+1}{M} r\right)^+, \\ N_{\mathsf{A}} \left(1 - \frac{M+1}{M} 2r\right)^+ + N_{\mathsf{D}} \left(\eta - r\right)^+ \end{array} \right\}.$$

From the closed-form DMT of the sequential-SADF protocol in (14), the optimal DMT (10) is achievable if the proximity gain, $\eta > 1 + \frac{N_{\mathsf{A}} M}{2N_{\mathsf{D}}(M+1)}$. Note that from equation (11), the proximity gain is characterized according to the distance ratio between the $D_{i-1,i}$ and $D_{\mathsf{s},i}$. Thus the optimal DMT of the sequential-SADF is achievable if we choose the DF relay, r_i that has a low $D_{\mathsf{s},i}$ and high $D_{i-1,i}$. Such distance ratio is achievable by applying the relay ordering and scheduling strategy proposed in [6] for achieving partial relay isolation condition. On the other hand, proximity gain bound for achieving optimal DMT of the proposed sequential SADF can be lowered if the number of DF relays, N_{D} is increased.

Fig. 2 shows the DMT curve of the proposed sequential-SADF protocol for different proximity gains. It can be seen that, for the proximity gain range of $\frac{M}{M+1} < \eta \leq 1 + \frac{N_{\mathsf{A}} M}{2N_{\mathsf{D}}(M+1)}$, the proposed sequential-SADF protocol achieves full diversity order of $d = N+1$, for multiplexing gain $r = 0$. In the interesting case of $\eta \to 0$, for sufficiently large number of transmission slots, the DMT performance of the proposed sequential-SADF would be equivalent to the NAF protocol with N_{A} number of relays. Thus we can conclude that NAF protocol with N_{A} relays is the worst case DMT of the proposed sequential-SADF protocol.

IV. Numerical Results

Fig. 3 shows the numerical result of the proposed sequential-SADF with different proximity gain with quasi-static Rayleigh fading channels. The power allocations for all the cooperative protocols are $\pi_i = \bar{\pi}_i = 0.5$, with the data rate of 2 BPCU. The proposed protocol is labelled as "SADF protocol". The proposed protocol and different cooperative protocols were simulated in a four relay network ($N = 4$). The total transmission slots for both of the proposed protocol and the sequential-SAF is $M = 20$. It can be seen that the sequential-SADF protocol with $\eta = 1.5$ outperforms other cooperative communication protocols at high SNR regime. The proposed protocol achieves 2 dB gain compared with the sequential-SAF protocols and about 4 dB and 8 dB gains, respectively, compared with the DDF and NAF protocols at the outage probability of 10^{-4}. For the case of the sequential-SADF protocol with $\eta = 0.5$, the proposed protocol achieves 4 dB gain compared with the NAF protocol at the outage probability of 10^{-4}.

V. Conclusion

We proposed a novel slotted hybrid cooperative network named sequential slotted amplify-decode-and-forward (SADF) protocol for half duplex single antenna multiple relay channels. The relays between the source and destination perform either decode or linear processing on the received signal and forward it to the destination in a round robin manner. We first derived the optimal DMT of the proposed sequential-SADF with the

$$d(r) = \begin{cases} \left(1-\frac{M+1}{M}r\right)^{+} + N_{\mathsf{A}}\left(1-\frac{M+1}{M}2r\right)^{+} + N_{\mathsf{D}}\left(\eta - r\right)^{+}, & \text{if } 0<\eta\le\frac{M}{M+1} \\ \left(1-\frac{M+1}{M}r\right)^{+} + d_{\min}(r), & \text{if } \frac{M}{M+1}<\eta\le 1+\frac{N_{\mathsf{A}}M}{2N_{\mathsf{D}}(M+1)} \\ \left(N_{\mathsf{A}}+N_{\mathsf{D}}+1\right)\left(1-\frac{M+1}{M}r\right)^{+}, & \text{if } \eta > 1+\frac{N_{\mathsf{A}}M}{2N_{\mathsf{D}}(M+1)} \end{cases} \quad (14)$$

assumption that all the DF relays are able to decode the received signal perfectly. We showed that the optimal DMT of the sequential-SADF can achieve the multiple-input single-output (MISO) DMT bound. We then derived the closed-form DMT expression of the proposed sequential-SADF by considering the effect of IRI and the outage event with the proximity gain. Using the proximity gain bound, we propose a distance bound for the sequential-SADF to achieve the optimal DMT bound. Simulation result shows that the sequential-SADF protocol with the high proximity gain strategy has the best outage performance compared to other cooperative protocols in high SNR regime.

VI. Appendix

A. Proof of Lemma 1

Let $\mathbf{G}$ be the class of hermitian matrices such that $\mathbf{A} = \mathbf{H}\mathbf{H}^{\dagger} \in \mathbf{G}$ where $\mathbf{H}$ is the channel matrix in (4). It is easy to verify that i.e. the probability $P(\mathbf{A})d\mathbf{A}$ that the matrix $\mathbf{G}$ is orthogonal invariant where the automorphism $\mathbf{A} \to \mathbf{U}\mathbf{A}\mathbf{U}^{\dagger}$ and belongs to

$$d\mathbf{A} = \prod_{k\le j} d\mathrm{Re}\left(\mathbf{A}_{k,j}\right)\prod_{k\le j} d\mathrm{Im}\left(\mathbf{A}_{k,j}\right)$$

where $\mathrm{Re}(\mathbf{A}_{k,j})$ and $\mathrm{Im}(\mathbf{A}_{k,j})$ are the real and imaginary part of $\mathbf{A}_{k,j}$. k and j denote the row and column of the matrix $\mathbf{A}$ and $\mathbf{U}$ is a unitary matrix. Here we can use the householder transformation without changing the spectrum of $\mathbf{A}$. Let $\mathbf{P}_{(i)}$ and $\mathbf{Q}_{(i)}$ be the householder matrices for $\mathbf{H}_{(i)}$, where the subscript (i) denotes the ith householder transformation, for $i = 1, \cdots, M-2$. Then $\mathbf{P}^{\dagger}_{(i)}$ and $\mathbf{Q}^{\dagger}_{(i)}$ are a pair of householder matrices for $\mathbf{H}^{\dagger}_{(i)}$. We see that according to the structure of householder matrices, the spectrum of $\mathbf{A}_{(i)}$ does not change under the transformation

$$\mathbf{H}_{(i-1)}\mathbf{H}^{\dagger}_{(i-1)} \to \mathbf{P}_{(i)}\mathbf{H}_{(i)}\mathbf{Q}_{(i)}\mathbf{Q}^{\dagger}_{(i)}\mathbf{H}^{\dagger}_{(i)}\mathbf{P}^{\dagger}_{(i)}. \quad (15)$$

At the same time, let us select $\mathbf{P}_{(i)}$ and $\mathbf{Q}_{(i)}$ such that $\mathbf{H}_{(M-2)}$ is bi-diagonalised. Using the householder procedure, we derive

$$\mathbf{P}_{(1)} = \begin{bmatrix} 1 & 0 & & \cdots & & 0 \\ 0 & \frac{g_1}{a_1} & \frac{f_2 g_2}{a_1} & 0 & \cdots & 0 \\ 0 & \frac{f_2 g_2}{a_1} & 1-\frac{|f_2 g_2|^2}{2a_1^2 b_1^2} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & & \cdots & 1 \end{bmatrix}$$

where $a_i = \sqrt{|g_i|^2 + |\gamma_{i,i+1} g_{i+1}|^2}$ and $b_i = \sqrt{\frac{1}{2}\left(1-\frac{g_i}{a_i}\right)}$, and $\mathbf{Q}_{(1)} = \mathbf{I}_M$ is an identity matrix.

We see that $\mathbf{H}_{(0)}$ is transformed into

$$\mathbf{H}_{(1)} = \left[\begin{array}{c|ccc} g_0 & 0 & \cdots & 0 \\ \hline a_1 & & & \\ \vdots & & \mathbf{B}_{(2)} & \\ 0 & & & \end{array}\right]$$

where $\mathbf{B}_{(2)}$ is obtained by operating $\mathbf{P}_{(2)}$ on $\mathbf{H}_{(1)}$. Similarly $\mathbf{H}^{\dagger}_{(0)}$ can be bi-diagonalised in the first two column by using $\mathbf{P}^{\dagger}_{(1)}$ and giving a sub-matrix $\mathbf{B}^{\dagger}_{(2)}$ as a result.

Since $\mathbf{A}_{(1)} = \mathbf{H}_{(1)}\mathbf{H}^{\dagger}_{(1)}$ is invariant under the transformation in (15), $\mathbf{B}_{(2)}\mathbf{B}^{\dagger}_{(2)}$ belongs to the same class of matrices as $\mathbf{G}$. Hence we may proceed inductively with the householder transformation and end up with the tridiagonal matrix, $\mathbf{H}_{(M-2)}\mathbf{H}^{\dagger}_{(M-2)}$

$$\begin{bmatrix} |g_0|^2 & g_0 a_1^* & \cdots & 0 \\ g_0^* a_1 & |g_0|^2 + |a_1|^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & g_0 f_M^* g_M^* \\ 0 & \cdots & g_0^* f_M g_M & |f_M g_M|^2 \end{bmatrix}$$

Using [4, Lemma 4] and [7, Lemma 3.1], we have

$$\det\left(\mathbf{I}_{M+1} + \mathsf{SNR}\,\mathbf{H}_{(M-2)}\mathbf{H}_{(M-2)^{\dagger}}\right) \ge \left(1+\mathsf{SNR}\,|g_0|^2\right)^M + \prod_{i\in\mathcal{S}}\left(1+\mathsf{SNR}\,|f_i g_i|^2\right)\left(1+\mathsf{SNR}\,|a_{i-1}|^2\right),$$

where $\mathcal{S}$ denotes a set of positive even integers with the limits of $\mathcal{S} \in [2, M]$. This completes the proof of the Lemma 1.

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