Sequential Slotted Amplify-and-Forward with Partial Relay Isolation

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***Abstract*—In this paper, we propose a novel relay ordering and scheduling strategy for the sequential slotted amplify-and-forward (SAF) protocol and evaluate its performance in terms of diversity-multiplexing trade-off (DMT). The relays between the source and destination are grouped into two relay clusters based on their respective locations. The proposed strategy achieves partial relay isolation and decreases the decoding complexity at the destination. We show that the DMT upper bound of sequential-SAF with the proposed strategy outperforms other amplify and forward protocols and is more practical compared to the relay isolation assumption made in the original paper [1]. Simulation result shows that the sequential-SAF protocol with the proposed strategy has better outage performance compared to the existing AF and non-cooperative protocols in high SNR regime.**

I. Introduction

In cooperative communication systems, different terminals collaborate to form a virtual-multiple antenna array system and exploit the spatial diversity to achieve a better performance [2]–[4]. Cooperative communication protocols can be generally categorized into amplify-and-forward (AF) protocol and decode-and-forward (DF) protocol and are often compared using the trade-off measurement between the error probability and the data rate of a multiple-input multiple-output (MIMO) network known as diversity-multiplexing trade-off (DMT) [5].

In [1], the authors pointed out the fact that both nonorthogonal amplify-and-forward (NAF) [3], [4] and dynamic decode-and-forward (DDF) [4] protocols fail to achieve high diversity gain ($d(r) > 1$) in the high multiplexing gain regime ($r > 0.5$), due to the fact that half of the source signal is not forwarded by the relays. To mitigate this problem, the authors proposed a new AF protocol called the slotted amplify-and-forward (SAF) protocol and showed that it achieves the multiple-input single-output (MISO) bound when the number of transmission slots goes to infinity. The authors subsequently proposed a new AF protocol called the sequential slotted amplify-and-forward (SAF) protocol and showed that it achieves the multiple-input single-output (MISO) bound when the number of transmission slots goes to infinity, with the assumption of isolated relays. Recently in [6], the authors proposed opportunistic DF (ODF) protocol to achieve the DMT upper bound of N-relay $(N+1)$-slot sequential-SAF without relay isolation model. However the protocol requires the n-th relay to listen for n slots to decode the n-th signal, for $n \in \{1, \cdots, N\}$.

On the other hand, the authors in [1] proposed the two-relay three-slots sequential-SAF protocol and demonstrated that it has the best performance among all the cooperative communication protocols for the case of two relays, without the assumption of relay isolation. The two-relay M-slot protocol, also known as two-path relaying protocol or diamond relay channel, was first introduced in [7] and extended from two-relay to N-relay with relay selection and limited-feedback [8]. It has been well studied and analysed to demonstrate the potential benefits of this promising approach. In [9], the successive decoding with partial or full inter-relay interference cancellation at the destination was proposed with the assumption of weak inter-relay interference. While in [10], assume strong inter-relay interference (IRI), the relay decode the source signal using the decoded interference signal. However the above-mentioned protocols are restricted to decode-and-forward protocol. Recently in [11], the authors showed that without the condition of relay isolation, the two-path relaying achieves the DMT upper-bound of the two-relay sequential-SAF with IRI cancellation.

The two-path relaying protocol in [11] demonstrated a superior performance for the case of two relays. However, the protocol cannot extend from two-relay to N-relay protocol, due to the fact that the protocol requires self-transmitted information to perform inter-relay interference cancellation. In this paper, we propose a novel relay ordering and scheduling strategy for the sequential-SAF and analyse it in term of DMT. With the proposed relay ordering and scheduling strategy, the sequential-SAF achieves partial relay isolation model which is more practical compared to the isolated relay model. We consider perfect partial relay isolation model, where the three hop signals is ignored and compute the DMT upper bound, which can be expressed as

$$d_{\text{UB}}(r) = (N+1)\left(1 - \frac{M+1}{M} r\right)^{+}. \tag{1}$$

We show that the DMT upper bound of sequential-SAF with the proposed relay ordering and scheduling strategy has the best DMT performance compare to the other AF protocol without the assumption of isolated relays.

The rest of the paper is organized as follows: The problem definition and background are presented in Section II. In Section III we propose a novel relay ordering and scheduling strategy for the sequential-SAF protocol and compute the DMT in Section IV. The numerical result is presented in Section V, whereas Section VI concludes this paper.

II. Problem Definition and Background

A. System Model

Consider a cooperative relaying network where the source information bits are modulated, pre-coded, divided into M symbols and are to be transmitted from the source s to the destination d with the assistance of the N relays r_i. Without loss of generality, we presume one pre-coded symbol is transmitted during each transmission slot, x_m, for $m = 1, \cdots, M$, and M is even. The total number of transmission slot is $M+1$, where one additional slot is added at the end of the transmission sequence for the relay to forward the last pre-coded symbol and thus a multiplexing gain of $\frac{M}{M+1}$ is achieved. The power allocation of s and r for mth slot are denoted by π_m and $\bar{\pi}_m$ and satisfies $\sum_{m=1}^{M+1}(\pi_m + \bar{\pi}_m) = M$.

The physical links between terminals are assumed to be slowly faded and are modelled as independent quasi-static fading channels, i.e., the channel gains do not change during the transmission of a cooperative frame. The channel gain between s and d is denoted by $h_{\mathsf{s},\mathsf{d}}$. Similarly, $h_{\mathsf{s},n}$ and $h_{n,\mathsf{d}}$, respectively denote the channel gain between s and r_n and the ones between r_n and d, for $n \in \{1, \cdots, N\}$. $\gamma_{i,j}$ denotes the channel gain between r_i and r_j. The channel gains between all the terminals are modelled as i.i.d complex Gaussian random variable with zero mean and variance $\sigma^2_{i,j}$, for $i, j \in \{\mathsf{s}, \mathsf{d}, 1, \cdots, N\}$, $i \neq j$. The variance of the channel gain is parameterized by the distance between terminals and can be expressed as

$$\sigma^2_{i,j} = \left(\frac{D_0}{D_{i,j}}\right)^{\alpha},$$

where $D_{i,j}$ is the normalized distance between terminals. $D_0 > 0$ denotes the reference distance, where we restrict the distance between terminals, $D_{i,j} \geq D_0$ to ensure that the received power is always bounded. α denotes the path loss exponent.

For the case of exposition and practical relevance, we assume that each terminal is installed with half duplex single antenna that can either transmit or receive signals at any given time. However, the result presented in this paper can be easily extended to the case where terminals have multiple and/or full duplex antennas. Throughout the paper, we assume perfect synchronization between all the transmitting terminals during each transmission slot.

B. Problems Definition

The NAF protocol was first proposed in [3] and extended from single relay protocol to N relay protocol in [4]. The DMT upper-bound of the NAF protocol can be written as

$$d^*_{\text{NAF}}(r) = (1-r) + N\,(1-2r)^+, \tag{2}$$

where $d^*(r)$ denotes the optimal DMT curve and $x^+ \triangleq \max\{x, 0\}$. It achieves maximum diversity gain at multiplexing gain of $r = 0$ and has the best DMT performance for single half duplex relay scenario. However the protocol fails to achieve high diversity gain ($d(r) > 1$) in the high multiplexing gain regime ($r > 0.5$) for multiple relays scenario, due to the fact that only half of the source signal is *protected* by the relays.

The sequential-SAF protocol proposed in [1] demonstrated a superior diversity gain performance for the multiplexing-gain limits of $0.5 \leq r \leq 1$ compared to the NAF protocol. The DMT upper bound of the sequential-SAF protocol can be expressed as

$$d^*_{\text{SAF}}(r) = (1-r) + N\left(1 - \frac{M}{M-1}r\right)^+. \tag{3}$$

It achieves the MISO transmit diversity upper bound, with large number of transmission slots. However the perfect relay isolation model assumption is made in [1] for achieving the DMT upper bound of the sequential-SAF is impractical. On the other hand, the authors in [11] showed that the DMT upper bound of the sequential-SAF is achievable without the assumption of isolated relays, since the main and first sub-diagonal elements in the channel matrix remain the same as the case with the isolated relays. However the decoding complexity at d is too prohibitive and thus it is impractical.

The authors in [11] on the other hand, proposed a two-path relaying protocol, where the first relay performs IRI cancellation on every received signal before forward it to the destination. The first relay is able to perform IRI cancellation since it has the perfect knowledge of the self-transmitted signal, under the assumption that the relay has the knowledge of the inter-relay channel gain, $\gamma_{2,1}$, as well as the scaling factor of the second relay. It achieves the DMT upper bound of the 2-relays $(M+2)$-slots sequential-SAF without isolated relay assumption. However the two-path relaying protocol cannot be extended to multiple relay case, since the IRI cancellation process requires the perfect knowledge of the self-transmitted signals. On the other hand, the two-path relaying protocol requires accurate channel estimation to perform IRI cancellation which is too prohibitive at the relay. The DMT of the two path relaying protocol can be expressed as

$$d_{\text{2-PATH}}(r) = 3\left(1 - \frac{M+2}{M}r\right)^+. \tag{4}$$

III. Proposed Strategy for Sequential-SAF

A. Relay Ordering and Scheduling Strategy

Without loss of generality, we assume that there is a global scheduler that has the knowledge of the channel-state informations between all terminals. Instead of having relays with random location between s and d, the global scheduler groups the relays into two relay clusters according to their respective locations[1]. The relays that are closed to s are grouped into the "odd cluster" and are ordered to forward the odd-slot symbols, x_m, for $m = 1, 3, \cdots, M-1$, while the relays that are closed to d are grouped into the "even cluster" and are ordered to forward the even-slot symbols, $x_{m'}$, for $m' = 2, 4, \cdots, M$ in

[1]The relay grouping process can be easily done with the aid of the Global Positioning System (GPS) or any other location tracking system which are widely used in wireless networks.

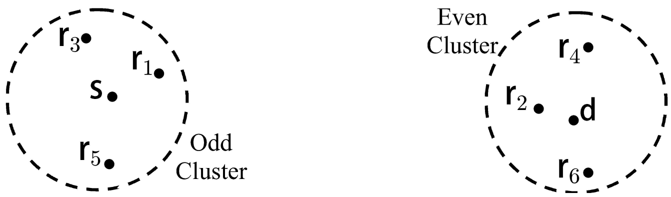

Fig. 1. System model for sequential-SAF with 6 partial isolated relays after the process of relay ordering. The orders of the relays are indicated by numbers.

round-robin manner. On top of that, we assume that both of the relay clusters contain $\lceil N/2 \rceil$ number of relays. Fig. 1 shows an example of the proposed cooperative relaying network after the relay grouping process.

With the basic ideas, we have developed a general algorithm for scheduling strategy. For $i, j \in \{1, \cdots, N\}$, $i \neq j$, denote C_i and $C_{i,j}$, respectively, the cost function of two hop signal and three hop signal [1], are defined by:

$$C_i \triangleq \frac{\mathsf{SNR}^2 |h_{\mathsf{s},i} h_{i,\mathsf{d}}|^2}{1 + \mathsf{SNR}|h_{i,\mathsf{d}}|^2},$$
$$C_{i,j} \triangleq \frac{\mathsf{SNR}^3 |h_{\mathsf{s},i} \gamma_{i,j} h_{j,\mathsf{d}}|^2}{1 + \mathsf{SNR}^2 |\gamma_{i,j} h_{j,\mathsf{d}}|^2}.$$

Then, the scheduler applies the scheduling strategy as follows:

1) r_1 is selected from "odd cluster" to forward the symbol x_1 based on the cost function C_1, where $C_1 \geq \max\{C_3, \cdots, C_{N-1}\}$.
2) r_2 is selected from "even cluster" to forward the symbol x_2 based on the cost function $C_{1,2}$ and C_2, where $C_{1,2} + C_2 \geq \max\{C_{1,4} + C_4, \cdots, C_{1,N} + C_N\}$
3) r_3 is selected from "odd cluster" to forward the symbol x_3 based on the cost function $C_{2,3}$, where $C_{2,3} \leq \min\{C_{2,5}, \cdots, C_{2,N-1}\}$.
4) Repeat step 2 and 3 for the rest of the relays.

Note that starting from step 3, we choose r_m, for $m = 3, \cdots, M-1$, based on the weakest inter-relay interference compared to step 1, since the main focus of the proposed relay ordering and scheduling strategy is to achieve partial relay isolation condition.

B. Equivalent Channel Models

With the above-mentioned relay ordering and scheduling strategy, we can now conveniently characterize the baseband-received signal models using a time-division notation.

At the first time slot, s transmits the first pre-coded symbol, x_1 to r_1 and d. The received signals can be expressed as

$$y_{\mathsf{r},1} = \sqrt{\pi_1 \,\mathsf{SNR}} h_{\mathsf{s},1} x_1 + n_{\mathsf{r},1},$$
$$y_{\mathsf{d},1} = \sqrt{\pi_1 \,\mathsf{SNR}} h_{\mathsf{s},\mathsf{d}} x_1 + n_{\mathsf{d},1},$$

where $y_{\mathsf{r},1}$ and $y_{\mathsf{d},1}$ denote the received signal at r_1 and d at the first time slot, respectively. $n_{\mathsf{r},1}$ and $n_{\mathsf{d},1}$ denote independent additive white Gaussian noise (AWGN), respectively, with zero mean and unit variance. SNR denotes the signal to noise ratio.

After receiving the signal from s, r_1 performs linear processing at the received signal and transmits it during the second time slot. On the other hand, s transmits the second pre-coded symbol, x_2 to both r_2 and d. The received signals can be expressed as

$$y_{\mathsf{r},2} = \sqrt{\pi_2 \,\mathsf{SNR}} h_{\mathsf{s},2} x_2 + \sqrt{\bar{\pi}_2 \,\mathsf{SNR}} \gamma_{1,2} b_1 y_{\mathsf{r},1} + n_{\mathsf{r},2},$$
$$y_{\mathsf{d},2} = \sqrt{\pi_2 \,\mathsf{SNR}} h_{\mathsf{s},\mathsf{d}} x_2 + \sqrt{\bar{\pi}_2 \,\mathsf{SNR}} h_{1,\mathsf{d}} b_1 y_{\mathsf{r},1} + n_{\mathsf{d},2},$$

where b_1 denotes the scaling factor at r_1 with the power constraint of $\mathbb{E}\{|b_1 y_{\mathsf{r},1}|^2\} \leq 1$, and $\mathbb{E}\{\cdot\}$ denotes the statistical expectation operator.

At the third time slot, both r_3 and d receive a linear combination of the pre-coded symbol and the linearly processed signal from s and r_2, respectively, and the received signals can be expressed as

$$y_{\mathsf{r},3} = \sqrt{\pi_3 \,\mathsf{SNR}} h_{\mathsf{s},3} x_3 + \sqrt{\bar{\pi}_3 \,\mathsf{SNR}} \gamma_{2,3} b_2 y_{\mathsf{r},2} + n_{\mathsf{r},3},$$
$$y_{\mathsf{d},3} = \sqrt{\pi_3 \,\mathsf{SNR}} h_{\mathsf{s},\mathsf{d}} x_3 + \sqrt{\bar{\pi}_3 \,\mathsf{SNR}} h_{2,\mathsf{d}} b_2 y_{\mathsf{r},2} + n_{\mathsf{d},3},$$

These transmission steps are continuously repeated until M pre-coded symbols have been transmitted from s.

Lemma 1: The partial relay isolation model is achievable with the proposed relay ordering and scheduling strategy shown above.

Proof: Since s is closed to the relays from "odd cluster" and the relays from "even cluster" are closed to d, let us assume that the power allocation at s and r_{m-1} fulfil the condition below:

$$\pi_m \geq \bar{\pi}_m, \quad \text{for } m = 3, 5, \cdots, M-1.$$

Note that during the relay scheduling, we choose the r_m to forward the symbol x_m based on the cost function of three hop signal, $C_{m,m+1}$, which has the weakest IRI among all the other relays from "odd cluster". With the consideration of the channel quality s, r_{m-1} and r_m, which is parameterized by the distance between terminals, it is easy to find that

$$\mathbb{E}\left\{\pi_m |h_{\mathsf{s},\mathsf{d}}|^2\right\} \gg \mathbb{E}\left\{\bar{\pi}_m |\gamma_{m-1,m}|^2\right\}.$$

Since the received power from s is much larger compared to the received power from r_{m-1}, the IRI can be treated as noise and the partial relay isolation model is achieved. This complete the proof of Lemma 1. ∎

From Lemma 1, we consider the following perfect partial relay isolation model, where the three hop signal from "even cluster" to the "odd cluster" is ignored. The received signal at r_m, with the assumption of perfect partial relay isolation model, can be rewritten as

$$y_{\mathsf{r},m} = \sqrt{\pi_m \,\mathsf{SNR}} h_{\mathsf{s},m} x_i + n_{\mathsf{r},m}, \text{ for } m = 3, 5, \cdots, M-1.$$

Fig. 2 shows the transmission sequence of the sequential-SAF protocol with perfect partial relay isolation model. Solid box denotes transmitted symbols and dashed box denotes received symbols.

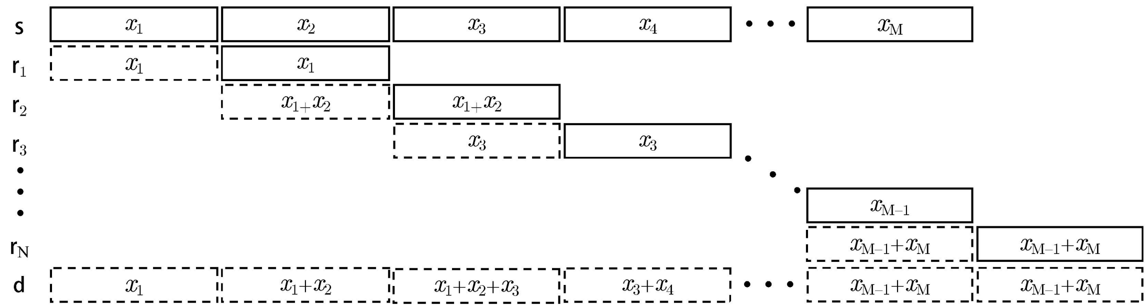

Fig. 2. Transmission sequence of the sequential-SAF protocol with perfect partial relay isolation model. Solid box denotes transmitted symbols and dashed box denotes received symbols.

IV. Diversity Multiplexing Trade-off

The DMT introduced in [5] is a performance measurement between the error probability and the data rate of a multiple-input multiple-output (MIMO) network for quasi-static Rayleigh fading channels in high SNR regime. A family of a codes $\{\mathcal{C}(\mathsf{SNR})\}$ is said to achieve the multiplexing gain, r and the diversity gain, d if

$$\lim_{\mathsf{SNR}\to\infty} \frac{R(\mathsf{SNR})}{\log \mathsf{SNR}} = r, \quad \lim_{\mathsf{SNR}\to\infty} \frac{\log P_e(\mathsf{SNR})}{\log \mathsf{SNR}} = -d,$$

where $R(\mathsf{SNR})$ is the data rate measured by bits per channel use (BPCU) and is increased with SNR, $R(\mathsf{SNR}) = r\log(\mathsf{SNR})$. $P_e(\mathsf{SNR})$ denotes the average error probability using the maximum-likelihood (ML) detector which can be derived based on the outage probability, $P_e \approx P_{\text{out}}$, in high SNR regime with an arbitrary long code length.

Using the received signals at d and r_i from Section III, the equivalent received signal model can be expressed as

$$\mathbf{y} = \sqrt{\mathsf{SNR}}\mathbf{H}\mathbf{x} + \mathbf{B}'\mathbf{n}. \tag{5}$$

where $\mathbf{y} \in \mathbb{C}^{M+1}$ is the received signal vector at d. $\mathbb{C}^{M\times N}$ denotes a $M \times N$ complex matrix, while $\mathbb{C}^N$ denotes a size N complex row matrix. $\mathbf{x} \in \mathbb{C}^M$ denotes the transmitted signal vector. $\mathbf{H} \in \mathbb{C}^{(M+1)\times M}$ denotes the equivalent channel matrix. $\mathbf{B}'\mathbf{n} \in \mathbb{C}^{M+1}$ denote the received noise matrix at d, where $\mathbf{B}' = [\mathbf{B}\ \mathbf{I}_{M+1}]$ and $\mathbf{n} = \left[\mathbf{n}_r^{\mathrm{T}}\ \mathbf{n}_d^{\mathrm{T}}\right]^{\mathrm{T}}$. $\mathbf{n}_r \in \mathbb{C}^M$ and $\mathbf{n}_d \in \mathbb{C}^{M+1}$ denote the additive white Gaussian noise vectors at r and d, respectively. $\mathbf{B} \in \mathbb{C}^{(M+1)\times M}$ denotes the forwarded noise matrix. Using the equivalent received signal model in (5), we obtain the mutual information of the sequential-SAF with perfect partial relay isolation model:

$$\begin{aligned} I(\mathbf{x};\mathbf{y}) &= \log\det\left[\mathbf{I}_M + \mathsf{SNR}\,\mathbf{H}^\dagger\Sigma_{\mathbf{x}}\mathbf{H}\left(\mathbf{B}'^\dagger\Sigma_{\mathbf{n}}\mathbf{B}'\right)^{-1}\right] \\ &= \log\det\left[\mathbf{I}_{M+1} + \mathsf{SNR}\,\mathbf{H}\mathbf{H}^\dagger\left(\mathbf{B}\mathbf{B}^\dagger + \mathbf{I}_{M+1}\right)^{-1}\right] \end{aligned} \tag{6}$$

where $\Sigma_{\mathbf{x}} = \mathbb{E}\{\mathbf{x}^\dagger\mathbf{x}\} = \mathbf{I}_M$ and $\Sigma_{\mathbf{n}} = \mathbb{E}\{\mathbf{n}^\dagger\mathbf{n}\} = \mathbf{I}_{2M+1}$. $\log(\cdot)$ denotes the logarithm based 2. Note that the noise matrix, $\mathbf{B}'\mathbf{n}$ is not white in general and does not affect the DMT computation [1].

The DMT of the partial isolated relay model can be obtained using equation (5) and (6). Let

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & 0 & \cdots & 0 & 0 \\ h_1 & h_0 & 0 & \ddots & 0 & 0 \\ h_{1,2} & h_2 & h_0 & \ddots & \ddots & \vdots \\ 0 & 0 & \ddots & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & h_{M-2} & h_0 & 0 \\ 0 & \dots & 0 & 0 & h_{M-1} & h_0 \\ 0 & \dots & 0 & 0 & h_{M-1,M} & h_M \end{bmatrix}, \tag{7}$$

where $h_0 \triangleq h_{\mathsf{s,d}}$ and

$$\begin{aligned} h_i &\triangleq h_{\mathsf{s},i}h_{i,\mathsf{d}}, && \text{for } i = 1,\cdots,M, \\ h_{i,j} &\triangleq h_{\mathsf{s},i}\gamma_{i,j}h_{j,\mathsf{d}}, && \text{for } i = 1,3,\cdots,M-1, j = i+1. \end{aligned}$$

Note that the terms b_i, π_i and $\bar{\pi}_i$ have been ignored since they do not impact the DMT computation [1, Lemma 1]. Using channel matrix (7), the outage probability of the perfect partial relay isolation model can be expressed as

$$\begin{aligned} &P_{\text{out}}(R(\mathsf{SNR})) \\ &\quad = \Pr\left[I(\mathbf{x};\mathbf{y}) \le (M+1)R(\mathsf{SNR})\right] \\ &\quad = \Pr\Big[\log\det\Big(\mathbf{I}_{M+1} + \mathsf{SNR}\,\mathbf{H}\mathbf{H}^\dagger\left(\mathbf{B}\mathbf{B}^\dagger + \mathbf{I}_{M+1}\right)^{-1}\Big) \\ &\qquad\qquad \le (M+1)R(\mathsf{SNR})\Big] \\ &\quad \doteq \Pr\left[\log\det\left(\mathbf{I}_{M+1} + \mathsf{SNR}\,\mathbf{H}\mathbf{H}^\dagger\right) \le (M+1)R(\mathsf{SNR})\right] \\ &\quad \doteq \Pr\left[\log\det\left(\mathsf{SNR}\,\mathbf{H}\mathbf{H}^\dagger\right) \le (M+1)R(\mathsf{SNR})\right]. \end{aligned}$$

The term $(\mathbf{B}\mathbf{B}^\dagger + \mathbf{I}_{M+1})^{-1}$ has been eliminated since we are only interested in the exponential function of $P_{\text{out}}(R(\mathsf{SNR}))$. The factor $M+1$ is to account for the fact that the proposed strategy takes $M+1$ transmission slots to transmit M symbols. $\doteq$ denotes asymptotic equality in the high SNR regime, i.e, $p_1 \doteq p_2$ denotes

$$\lim_{\mathsf{SNR}\to\infty} \frac{\log p_1(\mathsf{SNR})}{\log \mathsf{SNR}} = \lim_{\mathsf{SNR}\to\infty} \frac{\log p_2(\mathsf{SNR})}{\log \mathsf{SNR}}.$$

Lemma 2: Given a positive semidefinite matrix $\mathbf{H}\mathbf{H}^{\dagger}$, we have

$$\det\left(\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right) \leq \mathsf{SNR}^{M}\left(M|h_0|^2+\sum_{m=1}^{M}|h_m|^2+\sum_{m\in\mathcal{S}}|h_{m,m+1}|^2\right)^{M}$$

where $\mathcal{S}$ being a set of positive odd integer with the limits of $1 \leq \mathcal{S} \leq M$.

Proof: Using eigenvalue decomposite, the semidefinite matrix $\mathbf{H}\mathbf{H}^{\dagger}$ can be factored as

$$\det\left(\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right) = \det(\mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{T}) = \prod_{m=1}^{M}\lambda_m,$$

where $\mathbf{Q}$ and $\boldsymbol{\Lambda}$ is the $M \times M$ matrix with each column is the eigenvector and the diagonal matrix with the diagonal elements are the corresponding eigenvalues, λ_m of the matrix $\mathbf{H}\mathbf{H}^{\dagger}$, respectively. $(\cdot)^{T}$ denotes the transpose of a matrix. Using the inequality of arithmetic and geometric means, we have

$$\prod_{m=1}^{M}\lambda_i \leq \left(\frac{\sum_{m=1}^{M}\lambda_m}{M}\right)^{M},$$

which lead us to the following expression

$$\det\left(\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right) \leq \left(\frac{\sum_{m=1}^{M}\lambda_m}{M}\right)^{M} \dot{\leq} \operatorname{Tr}\left(\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right)^{M},$$

since $\mathbf{H}^{\dagger}\mathbf{H}$ is a square $M \times M$ matrix. From the equation (7), it is not hard to show that

$$\operatorname{Tr}\left(\mathsf{SNR}\,\mathbf{H}\mathbf{H}^{\dagger}\right)^{M} = \mathsf{SNR}^{M}\left(M|h_0|^2+\sum_{m=1}^{M}|h_m|^2+\sum_{m\in\mathcal{S}}|h_{m,m+1}|^2\right)^{M}.$$

This complete the proof of the lemma. ■

Let $v_{h_i} = \lim_{\mathsf{SNR}\to\infty}\frac{\log|h_i|^2}{\log\mathsf{SNR}}$ being the exponential order of $1/|h_i|^2$. Since h_i's are independent Gaussian variables, with the aid of Lemma 2, the DMT of the sequential-SAF with perfect partial relay isolation model can be obtained using

$$d_{\mathrm{UB}}(r) = \inf_{\mathcal{O}^{+}}\left\{v_{h_{\mathsf{s},\mathsf{d}}}+\sum_{n=1}^{N}\left(v_{h_{\mathsf{s},n}}+v_{h_{n,\mathsf{d}}}\right)+\sum_{n\in\mathcal{S}}v_{\gamma_{n,n+1}}\right\}.$$

where $d_{\mathrm{UB}}(r)$ is the DMT upper bound of the sequential-SAF with perfect partial relay isolation model.

Since solving the optimization problem with different value of exponential order being prohibitive in general, let us assume

$$v_1 \triangleq v_{h_{\mathsf{s},1}} = v_{h_{\mathsf{s},2}} = \cdots = v_{h_{\mathsf{s},N}}$$
$$v_2 \triangleq v_{h_{1,\mathsf{d}}} = v_{h_{2,\mathsf{d}}} = \cdots = v_{h_{N,\mathsf{d}}}$$
$$v_3 \triangleq v_{\gamma_{1,2}} = v_{\gamma_{3,4}} = \cdots = v_{\gamma_{M-1,M}},$$

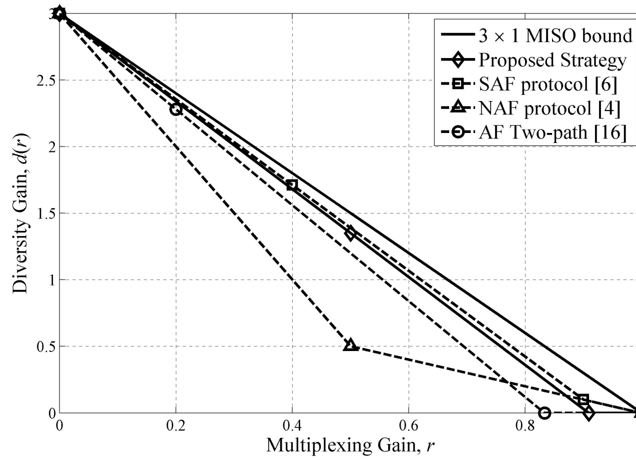

Fig. 3. Diversity-multiplexing trade-off comparison between the sequential-SAF with proposed strategy and the existing AF protocol with two relays. Here, the total transmitted symbols are $M = 10$

and the optimization problem is now reduced to

$$d_{\mathrm{UB}}(r) = \inf_{\mathcal{O}^{+}}\left\{v_{h_{\mathsf{s},\mathsf{d}}}+N(v_1+v_2)+\frac{N}{2}v_3\right\}.$$

Let $\mathbf{v} \triangleq [v_{h_{\mathsf{s},\mathsf{d}}}, v_1, v_2, v_3]$, for large SNR, the outage events is dominated by

$$\mathcal{O}^{+} = \left\{\mathbf{v}\,\middle|\,\max\begin{pmatrix}(1-v_{h_{\mathsf{s},\mathsf{d}}})^{+}\\(1-v_1-v_2)^{+}\\(1-v_1-v_2-v_3)^{+}\end{pmatrix} \leq \frac{M+1}{M}r\right\}.$$

By solving the optimization problem, we have the DMT upper bound of the sequential-SAF with perfect partial relay isolation model:

$$d_{\mathrm{UB}}(r) = (N+1)\left(1-\frac{M+1}{M}r\right)^{+}. \tag{8}$$

Note that from the DMT expressions in (8), the sequential-SAF with the proposed relay ordering and scheduling strategy achieves the $(N+1)\times 1$ MISO transmit bound, with sufficiently large number of transmission slots. On the other hand, the proposed strategy achieves partial relay isolation which is more practical and is easier to realise compared to the sequential-SAF with relay isolation model shown in [1].

The authors in [11] proved that the DMT upper bound of the SAF is achievable without the assumption of the isolated relay. However, the decoding complexity is prohibitive since it required the destination to decode a full $M \times M$ non-zero matrix. The proposed strategy on the other hand, eliminate the third hop signals with the relay ordering and scheduling strategy and the destination is required to decode a $M \times M$ penta-diagonal matrix which has lower decoding complexity.

Fig. 3 compares the DMT of the sequential-SAF with the proposed strategy with other AF cooperative protocols for two relays scenario. It can be seen that the DMT of the sequential-SAF with the proposed strategy dominates both NAF and the two-path relaying protocols. On the other hand, DMT upper

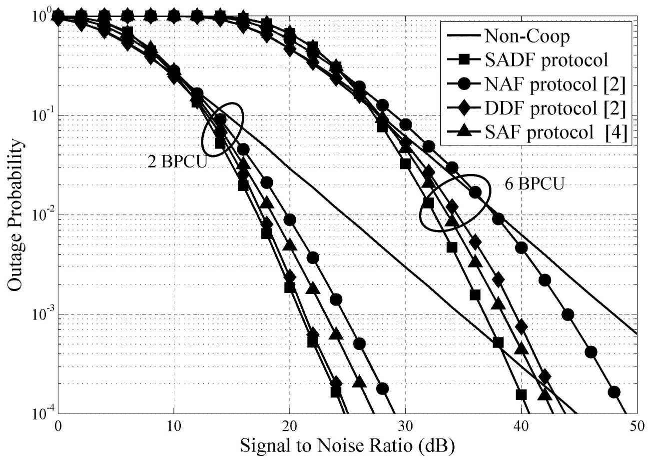

Fig. 4. Outage probabilities of the non-cooperative, NAF, sequential-SAF with isolated relays and sequential-SAF with partial isolated relays in a two relays network ($N = 2$) and information rates of 2 BPCU and 6 BPCU. The total transmitted symbols for both sequential-SAF protocols are $M = 10$.

bound of the sequential-SAF with isolated relays has slightly better performance compared to the proposed strategy, due to the fact that the proposed strategy require one additional slot to forward the last pre-coded symbol to d for achieving maximum diversity order.

V. Numerical Results

In this section, we present the numerical result of the sequential-SAF with proposed strategy and compare with different cooperative and non-cooperative protocols with quasi-static Rayleigh fading channels. The power allocation for all the cooperative protocols are $\pi_m = \bar{\pi}_m = 0.5$, while the information rate is measured in BPCU. The sequential-SAF with isolated relays proposed in [1] is labelled as “S-SAF (Ori)” while the one with the proposed relay ordering and scheduling strategy is labelled as “S-SAF (Prop)”.

Fig. 4 compares the sequential-SAF with proposed strategy with the non-cooperative protocol and different cooperative protocols in a two relays network. The total transmitted symbols for both of the sequential-SAF protocols are $M = 10$. It can be seen that with both low information rate (2 BPCU) and high information rate (6 BPCU), the sequential-SAF with proposed strategy has slightly better outage performance compared to the sequential-SAF with isolated relays assumption. On the other hand, the sequential-SAF protocols have better outage probabilities at high SNR regime compared to the NAF and the non-cooperative protocols, i.e., both of the protocols are achieving about 6 dB gain when compared with the NAF protocols at outage probability of 10^{-4} for high spectral efficiency case.

VI. Conclusion

We propose a novel relay ordering and scheduling strategy for the sequential slotted amplify-and-forward protocol (SAF) for the cooperative communication network with half duplex, single antenna relays and evaluate its performance in terms of diversity-multiplexing trade-off (DMT). The relays between the source and the destination are grouped into two relay clusters based on their respective locations. The relays from “odd cluster”, which are closed to the source forward the odd number symbols while the relays from “even cluster” which are closed to the destination forward the even number symbols. With the proposed relay ordering and scheduling strategy, the sequential-SAF protocol achieves partial relay isolation and decreases the decoding complexity at the destination. We then consider perfect partial relay isolation model, where the three hop signals from the “even cluster” to the “odd cluster” is ignored and compute the DMT upper bound. We show that the DMT upper bound of sequential-SAF with the proposed relay ordering and scheduling strategy has the best DMT performance compared to the other AF protocol without the assumption of isolated relays. Simulation result demonstrates that the sequential-SAF protocol with the proposed strategy has the same outage performance compared to the sequential-SAF with isolated relays and outperforms the NAF and non-cooperative protocols in high SNR regime.

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