Slotted Distributed Space-Time Coding

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***Abstract*—Distributed space-time coding (DSTC) exploits the concept of cooperative diversity and space-time coding to offer a powerful bandwidth efficient solution with improved diversity. In this paper, we evaluate the performance of DSTC with slotted amplify-and-forward protocol (SAF). Relay nodes between the source and the destination nodes are grouped into two relay clusters based on their respective locations and these relay clusters cooperate to transmit the space-time coded signal to the destination node in different time frames. We further extend the proposed Slotted-DSTC to Slotted DSTC with redundant code (Slotted-DSTC-R) protocol where the relay nodes in both relay clusters forward the same space-time coded signal to the destination node to achieve a higher diversity order.**

I. Introduction

In cooperative communication systems, different nodes collaborate to form a virtual multiple antennas array system and exploit the spatial diversity to achieve a better performance. In [1], the authors considered a class of amplify-and-forward protocol (AF) known as Slotted AF (SAF) and demonstrated that it has the best performance among all the AF protocols for the case of two relay nodes. Under the assumption of relay isolation, the diversity-multiplexing trade-off (DMT) upper-bound of SAF asymptotically achieves the multiple-input-single-output (MISO) bound when the number of transmission slots, M goes to infinity. The authors subsequently extended SAF to sequential SAF (SSAF), where only one relay is permitted to forward an amplified signal to the destination node during each transmission slot. By doing this, the SSAF achieved the DMT upper-bound of SAF with arbitrary number of transmission slots, M.

Distributed space-time coding (DSTC) combines the concepts of cooperative communications and space-time coding (STC) to improve the bandwidth efficiency and the diversity [2], [3]. Particularly, in a traditional relay network, relay nodes simply amplify or decode the received signal before forwarding to the destination node. DSTC on the other hand, allows relay nodes to cooperate with each other to achieve a better performance. It has been well studied and analysed to demonstrate the potential benefits of this promising approach [2]–[7].

In [4], the authors introduced a DSTC protocols in which the relay nodes perform linear transformation of the received signal according to the orthogonal space-time block code (OSTBC) design matrices. The DSTC protocol shows a lower error rate and achieves higher diversity order than the random codes DSTC [3]. However for a large number of relay nodes, the ML decoding complexity of the DSTC protocol becomes overly prohibitive at the destination node. Recently in [5]–[7], the single symbol and multigroup ML decodable DSTC were introduced to address the decoding complexity problem. However, the abovementioned DSTC protocols do not take into account certain practical issues in designing DSTC, such as increasing the achievable diversity order without significantly affecting the symbol rate and the minimum coherence interval required by the STC. Thus we propose a DSTC protocol to address these issues.

In this paper, we present a SAF based two-hop DSTC protocol and evaluate its performance in terms of pair-wise error probability (PEP) and bit-error rate (BER). For simplicity, we have limited our analysis to OSTBC based DSTC protocols which requires a simple ML detection to decode the received signal at the destination node. However, the proposed Slotted-DSTC protocol can be easily extended to other types of STC and pre-coding techniques. We first propose a Slotted-DSTC protocol where the relay nodes in the cooperative wireless network are grouped into two different relay clusters to forward the signals to the destination nodes alternately to achieve a higher symbol rate. We have subsequently propose a Slotted-DSTC with redundant codes (Slotted-DSTC-R) protocol, where the relay clusters will forward the same signals alternately and achieve a higher diversity order, without trading off the symbol rate and the ML decoding complexity at the destination node significantly.

The rest of the paper is organized as follows: The proposed Slotted-DSTC and Slotted-DSTC-R are presented in Section II and Section III. The discussion and simulation results are presented in Section IV and V, respectively, whereas Section VI concludes this paper.

II. Proposed Slotted-DSTC

Consider a cooperative relaying network consisting of a source node, a destination node and 2 sets of relay clusters, represented by $\mathcal{S}$, $\mathcal{D}$ and $\mathcal{C}_k$, $k \in \{1, 2\}$, respectively. Each relay cluster is formed by N number of relay nodes and the relay node in $\mathcal{C}_k$ is defined as $\mathcal{R}_{k,i}$, $i \in \{1, 2, \cdots, N\}$. For the case of exposition and practical relevance, we assume that each node is installed with half duplex single antenna that can either transmit or receive signals at any given time. However, the result presented in this paper can be easily extended to the case where nodes have multiple and/or full duplex antennas. All the nodes have the same power constraint and operate synchronously. Throughout the paper, we assume that there is no direct communication link between $\mathcal{S}$ and $\mathcal{D}$ and between $\mathcal{C}_1$ and $\mathcal{C}_2$. The channel link between the nodes are quasi-static flat Rayleigh-fading, i.e., the channel

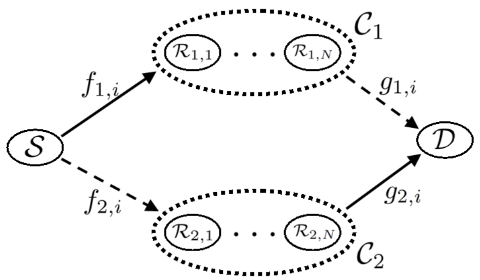

Fig. 1. System Model for Slotted-DSTC, with the assumption of no direct communication link between the source node, $\mathcal{S}$ and the destination node, $\mathcal{D}$, and between the relay clusters, $\mathcal{C}_1$ and $\mathcal{C}_2$. Solid arrows denote odd time frame transmission and dashed arrows denote even time frame transmission.

gains remain constant during the coherence interval, T_c, and change independently from one coherence interval to another. Furthermore, we restrict our attention on the network 1 in [4], where no channel knowledge at $\mathcal{S}$ and $\mathcal{R}_{k,i}$, perfect state channel information at $\mathcal{D}$ and no feedback to the transmitting node is permitted.

The goal is to send information from $\mathcal{S}$ to $\mathcal{D}$. The source transmission bits are modulated and divided into L frames, each frame containing T_1 number of symbols. The l-th frame is denoted as $\mathbf{x}^{(l)} = [x(1)\ x(2)\ \cdots\ x(T_1)]^{\mathrm{T}}$ with the normalization $\mathbb{E}\left\{\left[\mathbf{x}^{(l)}\right]^{\dagger}\mathbf{x}^{(l)}\right\} = \mathbf{I}_{T_1}$. T_1 denotes the total number of symbols transmit in a $T_2 \times N$ OSTBC matrix. For the sake of simplicity, we assume that the OSTBC matrix deployed in the DSTC satisfies all the properties in [5, Lemma 1]. Fig. 1 shows the transmission protocol of the Slotted-DSTC, where solid arrows denote odd time frame transmissions and dashed arrows denote even time frame transmissions.

A. Transmission Sequence

At the first time frame, $t = 1$, $\mathcal{S}$ sends $\mathbf{x}^{(1)}$ to all the relay nodes in $\mathcal{C}_1$. The received signal at the relay node $\mathcal{R}_{1,i}$ is:

$$\mathbf{y}_{1,i}^{(1)} = \sqrt{P_1 T_1} f_{1,i}\mathbf{x}^{(1)} + \mathbf{v}_{1,i}^{(1)}, \tag{1}$$

where $f_{1,i} \sim \mathcal{CN}(0,1)$ denotes the channel coefficient from $\mathcal{S}$ to the relay node $\mathcal{R}_{1,i}$. $\mathbf{v}_{1,i}^{(1)} \sim \mathcal{CN}(0,1)$ is the AWGN at the relay node $\mathcal{R}_{1,i}$. P_1 denote the average power at $\mathcal{S}$.

After receiving the signal from $\mathcal{S}$, $\mathcal{R}_{1,i}$ performs a linear transformation on the received signal $\mathbf{y}_{1,i}^{(1)}$ and waits until next time frame to transmit the space-time coded signal to $\mathcal{D}$. The transmitted space-time coded signal of $\mathcal{R}_{1,i}$, can be written as

$$\mathbf{x}_{1,i}^{(2)} = \sqrt{\frac{P_2 T_2}{(P_1+1)T_1}}\left[\mathbf{A}_i\mathbf{y}_{1,i}^{(1)} + \mathbf{B}_i\left(\mathbf{y}_{1,i}^{(1)}\right)^*\right]. \tag{2}$$

where $\mathbf{A}_i$ and $\mathbf{B}_i$ are $T_1 \times T_2$ design matrices for $T_2 \times N$ OSTBC matrix. P_2 denotes the average transmit power of the relay nodes.

At the second time frame, $t = 2$, the received signal at $\mathcal{D}$ can be expressed as

$$\begin{aligned}\mathbf{y}_{\mathcal{D}}^{(2)} &= \sum_{i=1}^{N} g_{1,i}\mathbf{x}_{1,i}^{(2)} + \mathbf{v}_{\mathcal{D}}^{(2)} \\ &= \sqrt{\rho}\mathbf{C}_1^{(2)}\mathbf{h}_1 + \mathbf{w}^{(2)},\end{aligned} \tag{3}$$

where $\rho = \frac{P_1 P_2 T_2}{P_1+1}$. $g_{1,i} \sim \mathcal{CN}(0,1)$ denotes the channel coefficient between $\mathcal{R}_{1,i}$ and $\mathcal{D}$. $\mathbf{C}_1^{(2)}$ denotes the $T_2 \times N$ OSTBC matrix from N number of relay nodes in the relay cluster $\mathcal{C}_1$, generated using the design matrices $\mathbf{A}_i$ and $\mathbf{B}_i$, at time frame, $t = 2$. $\mathbf{h}_1 = [g_{1,1}\hat{f}_{1,1}\ g_{1,2}\hat{f}_{1,2}\ \cdots\ g_{1,N}\hat{f}_{1,N}]^{\mathrm{T}}$ denotes the equivalent channel vector, where $\hat{f}_{k,i}$ can be define as

$$\hat{f}_i = \begin{cases} f_i^*, & \text{if } \mathbf{A}_i = \mathbf{0} \\ f_i, & \text{if } \mathbf{B}_i = \mathbf{0} \end{cases},$$

and $\mathbf{0}$ denotes the zero matrix. At the same time frame, $\mathcal{S}$ sends $\mathbf{x}^{(2)}$ to all the relay nodes in $\mathcal{C}_2$, and the received signal at the relay node $\mathcal{R}_{2,i}$ can be expressed as

$$\mathbf{y}_{2,i}^{(2)} = \sqrt{P_1 T_1} f_{2,i}\mathbf{x}^{(2)} + \mathbf{v}_{2,i}^{(2)}, \tag{4}$$

where $f_{2,i} \sim \mathcal{CN}(0,1)$ denotes the channel coefficient from $\mathcal{S}$ to the relay node $\mathcal{R}_{2,i}$. $\mathbf{v}_{2,i}^{(2)} \sim \mathcal{CN}(0,1)$ is the AWGN at the relay node $\mathcal{R}_{2,i}$. Here we assume that the distance between two relay clusters are far apart and thus the received signal at the relay nodes in the second relay cluster, $\mathbf{y}_{2,i}^{(2)}$, in (4) is not interfered by the transmitted signal from the relay nodes in first relay cluster and vice versa.

Similarly, relay nodes in $\mathcal{C}_2$ performs a linear transformation on the received signal and wait for the next time frame. The transmitted space-time coded signal of $\mathcal{R}_{2,i}$ is

$$\mathbf{x}_{2,i}^{(3)} = \sqrt{\frac{P_2 T_2}{(P_1+1)T_1}}\left[\mathbf{A}_j\mathbf{y}_{2,i}^{(2)} + \mathbf{B}_i\left(\mathbf{y}_{2,i}^{(2)}\right)^*\right]. \tag{5}$$

At the time frame, $t = 3$, the space-time coded signal, $\mathbf{x}_{2,i}^{(3)}$ is transmitted from $\mathcal{R}_{2,i}$, the received signal of $\mathcal{D}$ can be written as

$$\begin{aligned}\mathbf{y}_{\mathcal{D}}^{(3)} &= \sum_{i=1}^{N} g_{2,i}\mathbf{x}_{2,i}^{(3)} + \mathbf{v}_{\mathcal{D}}^{(3)} \\ &= \sqrt{\rho}\mathbf{C}_2^{(3)}\mathbf{h}_2 + \mathbf{w}^{(3)},\end{aligned} \tag{6}$$

$g_{2,i} \sim \mathcal{CN}(0,1)$ denotes the channel coefficient between $\mathcal{R}_{2,i}$ and $\mathcal{D}$. $\mathbf{C}_2^{(3)}$ denotes the $T_2 \times N$ OSTBC matrix from relay cluster $\mathcal{C}_2$ generated using the design matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ at the time frame, $t = 3$. $\mathbf{h}_2 = [g_{2,1}\hat{f}_{2,1}\ g_{2,2}\hat{f}_{2,2}\ \cdots\ g_{2,N}\hat{f}_{2,N}]^{\mathrm{T}}$ denotes the equivalent channel vector. At the same time frame, $\mathcal{S}$ transmits $\mathbf{x}^{(3)}$ to all the relay nodes in $\mathcal{C}_1$. $\mathbf{w}^{(t)}$ denotes the received noise at $\mathcal{D}$ which can be expressed as

$$\begin{aligned}\mathbf{w}^{(t)} =& \sqrt{\frac{P_2 T_2}{(P_1+1)T_1}}\sum_{i=1}^{N} g_{2,i}\left[\mathbf{A}_i\mathbf{v}_{2,i}^{(t-1)} + \mathbf{B}_i\left(\mathbf{v}_{2,i}^{(t-1)}\right)^*\right] \\ &+ \mathbf{v}_{\mathcal{D}}^{(t)}.\end{aligned} \tag{7}$$

These transmission steps are then continuously repeated until LT_1 symbols have been transmitted from $\mathcal{S}$.

B. ML Decoding and Achievable Diversity Order

In this subsection, we discuss the ML decoding and the achievable diversity order of the proposed Slotted-DSTC protocol. Assuming $\mathcal{D}$ has perfect channel state information for all links, the ML decoding of the proposed Slotted-DSTC protocol can be expressed as

$$\begin{aligned} &\arg\min_{\mathbf{C}_1^{(t)}} \left\| \mathbf{y}_{\mathcal{D}}^{(t)} - \sqrt{\rho}\mathbf{C}_1^{(t)}\mathbf{h}_1 \right\|_{\mathrm{F}}^2, \qquad \text{for } t = 2,4,6,\cdots \\ &\arg\min_{\mathbf{C}_2^{(t)}} \left\| \mathbf{y}_{\mathcal{D}}^{(t)} - \sqrt{\rho}\mathbf{C}_2^{(t)}\mathbf{h}_2 \right\|_{\mathrm{F}}^2, \qquad \text{for } t = 3,5,7,\cdots \end{aligned} \tag{8}$$

Since the relay nodes in both of the relay clusters used the same OSTBC design matrices, $\mathbf{A}_i$ and $\mathbf{B}_i$, the received signal at the destination node $\mathcal{D}$ in different time frame can be decoded based on one codebook. Using the ML decoding expression in (8), we can derive the PEP of the proposed Slotted-DSTC protocol, given the equivalent channel vector, $\mathbf{h}_k$, of choosing $\tilde{\mathbf{C}}_k$ when $\mathbf{C}_k$ is transmitted, can be expressed as

$$P\left(\mathbf{C}_k \to \tilde{\mathbf{C}}_k \middle| \mathbf{h}_k\right) = Q\left(\sqrt{\frac{\rho}{2\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^k\right)} \left\|\Delta\mathbf{C}_k\mathbf{h}_k\right\|_{\mathrm{F}}^2}\right),$$

where $\Delta\mathbf{C}_k$ denotes $\left(\mathbf{C}_k - \tilde{\mathbf{C}}_k\right)$. $\mathbf{C}_k$ and $\tilde{\mathbf{C}}_k$ ($\mathbf{C}_k \neq \tilde{\mathbf{C}}_k$) are the two possible codeword of the $T_2 \times N$ OSTBC matrix for relay cluster $\mathcal{C}_k$, $k \in \{1,2\}$. $\Sigma_{\mathbf{w}}^k$ denotes the covariance of the noise vector $\mathbf{w}$ at the destination node, when receiving the space-time coded signal from the relay nodes of k-th relay cluster.

From (8), it can be seen that the following expression is needed to decode the symbols from $\mathcal{C}_1$, at even time frames, $t = \{2,4,6,\cdots\}$:

$$\arg\min_{\mathbf{C}_1^{(t)}} \left\| \mathbf{y}_{\mathcal{D}}^{(t)} - \sqrt{\rho}\mathbf{C}_1^{(t)}\mathbf{h}_1 \right\|_{\mathrm{F}}^2, \tag{9}$$

where the minimization is performed over all possible codeword matrices $\mathbf{C}_1^{(t)}$. Using the inequality, $(\mathbf{A}_i\mathbf{A}_i^\dagger + \mathbf{B}_i\mathbf{B}_i^\dagger) \leq \mathbf{I}_{T_2}$, we have

$$\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^1\right) \leq T_2 + \frac{P_2 T_2^2}{(P_1+1)T_1}\sum_{i=1}^{N}|g_{1,i}|^2, \tag{10}$$

where $\Sigma_{\mathbf{w}}^1$ denotes the covariance matrix of the noise component from $\mathcal{C}_1$. We omitted the superscript (t), since the decoding scheme is the same for all the t-th time frames. The PEP of the proposed Slotted-DSTC protocol, given the equivalent channel vector, $\mathbf{h}_1$, of mistaking $\mathbf{C}_1$ by $\tilde{\mathbf{C}}_1$, can be expressed as

$$\begin{aligned} P\left(\mathbf{C}_1 \to \tilde{\mathbf{C}}_1 \middle| \mathbf{h}_1\right) &= Q\left(\sqrt{\frac{\rho}{2\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^1\right)} \left\|\Delta\mathbf{C}_1\mathbf{h}_1\right\|_{\mathrm{F}}^2}\right) \\ &\leq \frac{1}{2}\exp\left[-\frac{\rho\mathbf{h}_1^\dagger\Delta\mathbf{C}_1^\dagger\Delta\mathbf{C}_1\mathbf{h}_1}{4\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^1\right)}\right] \end{aligned}$$

where $Q(x) \leq \frac{1}{2}e^{-x^2/2}$ is the Chernoff bound of Q-function. We can further rewrite $\mathbf{h}_1$ as $\mathbf{h}_1 = \mathbf{F}_1\mathbf{g}_1$, where

$$\begin{aligned} \mathbf{F}_1 &= \mathrm{diag}\left(\hat{f}_{1,1}, \hat{f}_{1,2}, \cdots, \hat{f}_{1,N}\right), \\ \mathbf{g}_1 &= \left[g_{1,1}\ g_{1,2}\ \cdots\ g_{1,N}\right]^{\mathrm{T}}. \end{aligned}$$

Consequently, the average PEP is given by

$$\begin{aligned} P\left(\mathbf{C}_1 \to \tilde{\mathbf{C}}_1\right) &\leq \mathbb{E}_{\mathbf{h}_1}\left\{\frac{1}{2}\exp\left[-\frac{\rho\mathbf{h}_1^\dagger\Delta\mathbf{C}_1^\dagger\Delta\mathbf{C}_1\mathbf{h}_1}{4\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^1\right)}\right]\right\} \\ &\leq \mathbb{E}_{\mathbf{F}_1}\left\{\det{}^{-1}\left[\mathbf{I}_N + \frac{\rho\mathbf{F}_1^\dagger\Delta\mathbf{C}_1^\dagger\Delta\mathbf{C}_1\mathbf{F}_1}{4\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^1\right)}\right]\right\} \\ &\leq \mathbb{E}_{\mathbf{F}_1}\left\{\left[\prod_{i=1}^{N}\left(1 + \frac{\rho\lambda_{1,i}\left|f_{1,i}\right|^2}{4\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^1\right)}\right)\right]^{-1}\right\} \end{aligned}$$

where $\lambda_{1,i}$ is the i-th eigenvalue of $\Delta\mathbf{C}_1^\dagger\Delta\mathbf{C}_1$.

Similar approach can be done to the odd time frames and the PEP of both relay cluster can be generalized into

$$P\left(\mathbf{C}_k \to \tilde{\mathbf{C}}_k\right) \leq \mathbb{E}_{\mathbf{F}_k}\left\{\left[\prod_{i=1}^{N}\left(1 + \frac{\rho\lambda_{k,i}\left|f_{k,i}\right|^2}{4\mathrm{Tr}\left(\Sigma_{\mathbf{w}}^k\right)}\right)\right]^{-1}\right\}.$$

Note that the STC matrix of the proposed Slotted-DSTC protocol, $\mathbf{C}_k$ remain the properties of orthogonality due to the assumption of no direct communication link between the relay clusters, $\mathcal{C}_k$. For any OSTBC matrix deployed in Slotted-DSTC protocol, $\mathbf{C}_k$, the eigenvalue of $\Delta\mathbf{C}_k^\dagger\Delta\mathbf{C}_k$, $\lambda_{k,i} > 0$, $i \in \{1,2,\cdots,N\}$. Thus the relay cluster with N relay nodes of the proposed Slotted-DSTC protocol achieves a diversity order of $D = N$ at any time frame, when the SNR is high.

III. Proposed Slotted-DSTC with Redundant Code

The proposed Slotted-DSTC protocol discussed in Section II achieves full diversity in each relay cluster for any time frame, due to the fact that the STC of the proposed protocol has non-zero eigenvalue of $\Delta\mathbf{C}_k^\dagger\Delta\mathbf{C}_k$. However, the overall system achieves a diversity order of N using a total of $2N$ number of relay nodes, with the assumption of each relay cluster is formed by N number of relay nodes. This is because each of the relay clusters of the proposed protocol performs linear transformation on the symbol from different frame, and forwards the space-time coded signal to the destination node during different time frame to achieve a higher symbol rate.

In this section, we consider the same cooperative relaying network and transmission sequence shown in Section II, while $\mathcal{S}$ is now transmitting the same signal over 2 consecutive time frames to both of the relay clusters, $\mathcal{C}_k$, $k \in \{1,2\}$. The transmission frames of $\mathcal{S}$ of the proposed Slotted DSTC with redundant code (Slotted-DSTC-R) protocol can be expressed as $\mathbf{x}^{(t)} = \mathbf{x}^{(t+1)}$, for $t = 1,3,5,\cdots,2L-1$. Since both of the relay clusters perform linear transformation on the same frame symbol, the proposed Slotted-DSTC protocol is able to achieve a diversity order of $2N$, with the assumption that each relay cluster consists of N number of relay nodes.

A. ML Decoding and Achievable Diversity Order

Since the relays from both of the relay clusters performs the linear transformation on the same frame symbol, the ML decoding can be done by combining the two consecutive time frames. Let us assume the following equivalent channel model:

$$\mathbf{y}^{(m)} = \mathbf{C}_R^{(m)}\mathbf{h} + \mathbf{z}^{(m)} \tag{11}$$

where

$$\mathbf{y}^{(m)} = \begin{pmatrix} \mathbf{y}_{\mathcal{D}}^{(t)} \\ \mathbf{y}_{\mathcal{D}}^{(t')} \end{pmatrix}, \quad \mathbf{z}^{(m)} = \begin{pmatrix} \mathbf{w}^{(t)} \\ \mathbf{w}^{(t')} \end{pmatrix}, \quad \mathbf{h} = \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{pmatrix}. \tag{12}$$

$\mathbf{y}^{(m)}$ is the received signal vector with size $2T_2$ at $\mathcal{D}$, with $m \in \{1, 2, \cdots, L\}$. $\mathbf{z}^{(m)}$ denotes the size $2T_2$ combined received noise vector at $\mathcal{D}$. t denotes the odd time frames, $t \in \{1, 3, 5, \cdots 2L-1\}$ and t' denotes the even time frames, $t' \in \{2, 4, 6, \cdots 2L\}$. Using the combined vectors in equation (12), to decode the m-th frame symbol, the ML decoding can be written as

$$\arg \min_{\mathbf{C}_R^{(m)}} \left\| \mathbf{y}^{(m)} - \sqrt{\rho}\mathbf{C}_R^{(m)}\mathbf{h} \right\|_{\mathrm{F}}^2 \tag{13}$$

where $\mathbf{C}_R^{(m)}$ is the $2T_2 \times 2N$ combined STC matrix with the assumption of no direct link between two relay clusters and can be expressed as

$$\mathbf{C}_R^{(m)} = \begin{pmatrix} \mathbf{C}_1^{(t)} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2^{(t')} \end{pmatrix} \tag{14}$$

Using the ML decoding expression in (13), the PEP of the proposed Slotted-DSTC-R protocol, given the equivalent channel vector, $\mathbf{h}$, of mistaking $\mathbf{C}_R$ by $\tilde{\mathbf{C}}_R$, can be expressed as

$$P\left(\mathbf{C}_R \to \tilde{\mathbf{C}}_R \middle| \mathbf{h}\right) = Q\left(\sqrt{\frac{\rho}{2\mathrm{Tr}\left(\Sigma_{\mathbf{z}}\right)} \left\|\Delta\mathbf{C}_R\mathbf{h}\right\|_{\mathrm{F}}^2}\right),$$

where $\mathbf{C}_R$ and $\tilde{\mathbf{C}}_R$ ($\mathbf{C}_R \neq \tilde{\mathbf{C}}_R$) are the two possible codeword of the $2T_2 \times 2N$ combined STC matrix for the proposed Slotted-DSTC-R protocol. $\Sigma_{\mathbf{z}}$ denotes the covariance of the combined noise vector $\mathbf{z}$ at the destination node.

For simplicity, we will omit the superscript, (m), since the analysis is the same for any m-th frame. We can check the orthogonality of the combined STC using the following equation:

$$\begin{aligned} \mathbf{C}_R^\dagger\mathbf{C}_R &= \begin{pmatrix} \mathbf{C}_1^\dagger & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2^\dagger \end{pmatrix} \begin{pmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{C}_1^\dagger\mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2^\dagger\mathbf{C}_2 \end{pmatrix} \end{aligned} \tag{15}$$

Note that both $\mathbf{C}_1$ and $\mathbf{C}_2$ are generalized OSTBC. Using equation (15), the coding gain of the combined STC can be expressed as

$$\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R = \begin{pmatrix} \boldsymbol{\lambda}_1 & \mathbf{0}_N \\ \mathbf{0}_N & \boldsymbol{\lambda}_2 \end{pmatrix} \tag{16}$$

where $\boldsymbol{\lambda}_k = \mathrm{diag}(\lambda_{k,1}, \cdots \lambda_{k,N})$, denote the eigenvalue of $\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R$, for $k \in \{1, 2\}$.

Next, we need to prove that the diversity performance of the Slotted-DSTC-R protocol is evaluated based on the eigenvalue of $\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R$. Let $\mathbf{f} = [\mathbf{f}_1\ \mathbf{f}_2]^{\mathrm{T}}$, where $\mathbf{f}_k = [\hat{f}_{k,1}\ \hat{f}_{k,2}\ \cdots\ \hat{f}_{k,N}]^{\mathrm{T}}$, for $k \in \{1, 2\}$. Using the DOSTBC properties in [5, Lemma 1], we have

$$\mathbf{f}^\dagger\mathbf{C}_R^\dagger\Sigma_{\mathbf{z}}\mathbf{C}_R\mathbf{f} = \mathbf{f}_1^\dagger\mathbf{C}_1^\dagger\Sigma_{\mathbf{w}}^1\mathbf{C}_1\mathbf{f}_1 + \mathbf{f}_2^\dagger\mathbf{C}_2^\dagger\Sigma_{\mathbf{w}}^2\mathbf{C}_2\mathbf{f}_2. \tag{17}$$

and we can conclude that the combined STC of the Slotted-DSTC-R protocol satisfy the DOSTBC properties if the STC deployed in both of the relay clusters satisfy the DOSTBC properties.

The PEP of the proposed Slotted-DSTC-R protocol, given the equivalent channel vector, $\mathbf{h}$, of mistaking $\mathbf{C}_R$ by $\hat{\mathbf{C}}_R$, can be expressed as

$$\begin{aligned} P\left(\mathbf{C}_R \to \tilde{\mathbf{C}}_R \middle| \mathbf{h}\right) &= Q\left(\sqrt{\frac{\rho}{2\mathrm{Tr}\left(\Sigma_{\mathbf{z}}\right)} \left\|\Delta\mathbf{C}_R\mathbf{h}\right\|_{\mathrm{F}}^2}\right) \\ &\leq \frac{1}{2}\exp\left[-\frac{\rho\mathbf{h}^\dagger\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R\mathbf{h}}{4\mathrm{Tr}\left(\Sigma_{\mathbf{z}}\right)}\right] \end{aligned} \tag{18}$$

Let $\mathbf{h} = \mathbf{F}\mathbf{g}$, where $\mathbf{F} = \mathrm{diag}(\mathbf{f}_1, \mathbf{f}_2)$ and $\mathbf{g} = [\mathbf{g}_1\ \mathbf{g}_2]^{\mathrm{T}}$, the average PEP can be expressed as

$$\begin{aligned} P\left(\mathbf{C}_R \to \tilde{\mathbf{C}}_R\right) &\leq \mathbb{E}_{\mathbf{h}}\left\{\frac{1}{2}\exp\left[-\frac{\rho\mathbf{h}^\dagger\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R\mathbf{h}}{4\mathrm{Tr}\left(\Sigma_{\mathbf{z}}\right)}\right]\right\} \\ &\leq \mathbb{E}_{\mathbf{F}}\left\{\det{}^{-1}\left[\mathbf{I}_N + \frac{\rho\mathbf{F}^\dagger\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R\mathbf{F}}{4\mathrm{Tr}\left(\Sigma_{\mathbf{z}}\right)}\right]\right\} \\ &\leq \mathbb{E}_{\mathbf{F}}\left\{\left[\prod_{k=1}^{2}\prod_{i=1}^{N}\left(1 + \frac{\rho\lambda_{k,i}\left|f_{k,i}\right|^2}{4\mathrm{Tr}\left(\Sigma_{\mathbf{z}}\right)}\right)\right]^{-1}\right\} \end{aligned}$$

where $\lambda_{k,i}$ is the i-th eigenvalue of $\Delta\mathbf{C}_R^\dagger\Delta\mathbf{C}_R$. From (16), the eigenvalue of $\Delta\mathbf{C}_R$, $\lambda_{k,i} > 0$, $i \in \{1, 2, \cdots, N\}$, $k \in \{1, 2\}$ and the proposed Slotted-DSTC-R protocol achieves a diversity order of $D = 2N$.

IV. Discussion

In this section, we compare the performance of the proposed Slotted-DSTC and Slotted-DSTC-R protocols with the performance of standard DSTC protocols in terms of the symbol rate, the minimum coherence time interval requirement and the decoding complexity.

A. Symbol Rate

The OSTBC based DSTC protocol presented in [4] transmits T_1 number of symbols in T_1 time slots, during the listening phase and N number of relay nodes perform linear transformation of the received signal to a OSTBC with the dimension of $T_2 \times N$ to forward the T_1 number of symbols to $\mathcal{D}$. It achieves diversity order of N with the symbol rate of $R = \frac{T_1}{T_1+T_2}$. For example, the symbol rate of a DSTC protocol using the rate-1 2×2 Alamouti code [8] is $R = 1/2$.

Proposed OSTBC based Slotted-DSTC protocol presented in Section II, $\mathcal{S}$ transmits T_1 symbols in T_1 time slots during the listening phase of $\mathcal{C}_1$. Simultaneously, relay nodes in $\mathcal{C}_2$ cooperate to transmit orthogonal space-time coded T_1 data signal to the destination $\mathcal{D}$ during the transmit phase of $\mathcal{C}_2$. In this case the dimension of the distributed OSTBC matrix is $T_2 \times N$. Thus the proposed Slotted-DSTC protocol transmits LT_1 number of symbols to $\mathcal{D}$ in $LT_2 + T_1$ time slots. It achieves a diversity order of N with the symbol rate of $R = \frac{LT_1}{LT_2+T_1}$. The symbol rate, R of the proposed Slotted-DSTC is upper-bounded by $R \leq \frac{T_1}{T_2}$, when the size of the data frame is large i.e. $L \to \infty$.

In the case of Slotted-DSTC-R protocol, the relay nodes in both relay clusters $\{\mathcal{C}_k\}_{i=1,2}$ use the same OSTBC matrix with the dimension of $T_2 \times N$ to forward T_1 symbols to $\mathcal{D}$ in two different time frames. It transmits LT_1 number of symbols to $\mathcal{D}$ in $2LT_2 + T_1$ time slots. The proposed Slotted-DSTC-R achieves a diversity order of $2N$ with the symbol rate of $R = \frac{LT_1}{2LT_2+T_1}$. Assuming that the size of the data frame, L is large, upper-bound of the symbol rate in this case can be express as $R \leq \frac{T_1}{2T_2}$.

B. Minimum Coherence Interval Requirement

Most of the existing cooperative relaying schemes assume the channels between nodes are slow fading channels, where the channels are static over the transmission sequence. However, this assumption is not realistic since the positions of all the nodes are not stationary and the channel between the nodes may not be static.

Generally, a $T_2 \times N$ OSTBC transmits T_1 number of symbols, and requires the minimum coherence interval $T_c \geq T_2 T_s$ to achieve a diversity order of N, where T_s is the symbol interval. High minimum coherence interval OSTBC, such as the systematic design complex OSTBC proposed in [9], achieves a diversity order of six and seven with the minimum coherence interval of $30T_s$ and $56T_s$, which could be impossible to realise in high mobility cooperative communication systems, if T_s is large. For example, the period of the OFDM symbol is assumed to be high enough to mitigate the effect of the multipath fading channel.

Slotted-DSTC-R protocol is proposed to present a DSTC protocol that is practical to be deployed in high mobility wireless relaying network. It increases the achievable diversity order from N to $2N$, but keeps the minimum coherence interval to $T_2 T_s$. For example, to achieve a diversity order of four, one can use a 4×4 OSTBC matrix [10] with a minimum coherence interval of $4T_s$. Slotted-DSTC-R protocol on the other hand, uses the 2×2 Alamouti code [8] in each relay cluster, $\{\mathcal{C}_k\}_{k=1,2}$, and achieves a diversity order of four with a minimum coherence interval of $2T_s$.

C. Decoding Complexity

OSTBC is able to achieve full diversity order and has a very simple ML decoding algorithm based only on linear processing at the receiver [8]. Thus OSTBC has been adopted in many DSTC protocols. However, the ML decoding complexity at the destination node increases significantly when the number of relay nodes increases. In [11], the authors proposed a measurement of computational complexity of the optimum decoding of an OSTBC that does not depend on the constellation size of the transmitted signal. The computational complexity, (C_A, C_M) is measured by the number of real additions, C_A, and real multiplications, C_M. Thus the decoding complexity of a $T_2 \times N$ OSTBC can be expressed as

$$\begin{aligned} C_A &= 4T_1T_2 + 2N - 2T_1 - 1 \\ C_M &= 4T_1T_2 + 2N + 2T_1 + 4, \end{aligned} \tag{19}$$

assuming that only one receive antenna at $\mathcal{D}$. It can be seen that the computational complexity increases significantly when size of the OSTBC matrix increases.

Slotted-DSTC-R protocol is proposed to address the issue of significant increase in the ML decoding complexity while achieving a higher diversity order. Using the computational complexity measurement presented in (19), the (C_A, C_M) for decoding 4×4 and 8×8 OSTBCs are $(49, 66)$ and $(135, 156)$ respectively. The proposed Slotted-DSTC-R protocol achieves diversity order of $2N$ by repeating the $T_2 \times N$ OSTBC matrix in two different time frames. Thus, it achieves a diversity order of four and eight with the computational complexity of $(35, 48)$ and $(105, 122)$ respectively. It can be seen that the proposed Slotted-DSTC-R protocol can reduce the computational complexity by 30% and 20% while keeping the diversity order unchanged. On the other hand, the decoding complexity can be further reduced by applying the pre-coding techniques proposed in [7].

Table I compares the achievable diversity order, the symbol rate, the minimum coherence interval of the STC and the decoding computational complexity between the OSTBC based DSTC protocol [4] and the proposed Slotted-DSTC and Slotted-DSTC-R protocols.

TABLE I
COMPARISON BETWEEN THE PROPOSED SLOTTED-DSTC AND THE DSTC USING OSTBC IN [4]

Diversity Order	Protocol	Symbol Rate	Min Coherence	Decoding Complexity
2	Slotted-DSTC	1	$2T_s$	$(15, 28)$
	DSTC [4]	$1/2$	$2T_s$	$(15, 28)$
4	Slotted-DSTC-R	$1/2$	$2T_s$	$(35, 48)$
	Slotted-DSTC	$3/4$	$4T_s$	$(49, 66)$
	DSTC [4]	$3/7$	$4T_s$	$(49, 66)$

V. SIMULATION RESULTS

In this section, we present the simulated bit-error rate of the proposed Slotted-DSTC protocols and compare it with the OSTBC based DSTC protocol presented in [4]. The total power consumed in the whole network for transmission of LT_1 symbols is set to be PLT_1.

Fig. 2 compares the BER between the OSTBC based DSTC protocl presented in [4] and the proposed Slotted-DSTC in Section II with BPSK modulation. The OSTBC based DSTC protocol is simulated with two and four relay nodes ($N = 2$ and $N = 4$), while the Slotted-DSTC protocol is simulated with two relay clusters and each relay cluster contains two

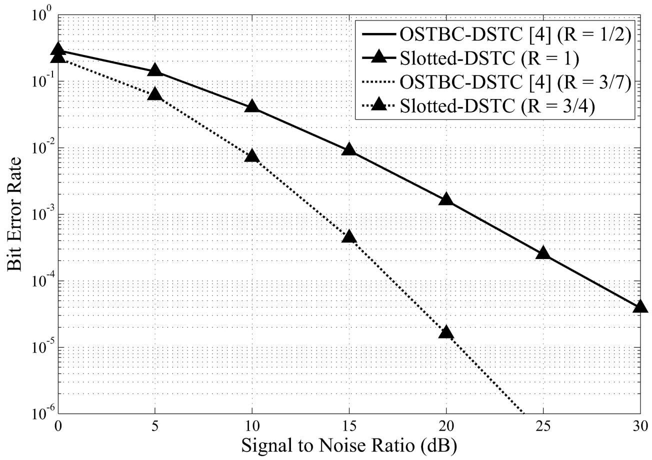

Fig. 2. Bit Error Rate comparison between the DSTC protocol [4] and Slotted-DSTC protocol using OSTBC with BPSK Modulation. Solid and dashed lines denote the DSTC protocol with 2 and 4 relays, respectively, and R denotes symbol rate.

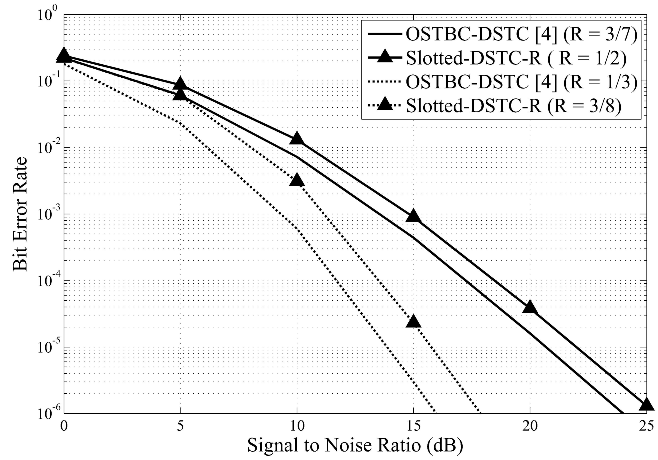

Fig. 3. Bit Error Rate comparison between the DSTC protocol [4] and Slotted-DSTC-R protocol using OSTBC with BPSK Modulation. Solid and dashed lines denote the DSTC protocol with 4 and 8 relays, respectively, and R denotes symbol rate.

and four relay nodes. Both two and four relay OSTBC based DSTC protocls are simulated in [4] and the author showed that they achieve a diversity order of two and four at high SNR regime. It can be seen that both Slotted-DSTC protocol with two and four relay nodes in each relay cluster, has exactly the same BER performance and can achieve a diversity order of two and four at high SNR regime. However, the proposed Slotted-DSTC protocol achieves higher symbol rate compared to the OSTBC based DSTC protocol.

Fig. 3 shows the BER performance between the OSTBC based DSTC and the proposed Slotted-DSTC-R protocol in Section III with BPSK modulation. The OSTBC based DSTC protocol is simulated with four and eight relay nodes ($N = 4$ and $N = 8$), while the Slotted-DSTC-R protocol is simulated with two relay clusters and each relay cluster contains two and four relay nodes ($N = 2$ and $N = 4$). Note that the BER performance of the proposed Slotted-DSTC-R protocol is 1 dB and 2 dB less than the corresponding OSTBC based DSTC protocol for the case of achievable diversity order of four ($D = 4$) and eight ($D = 8$), respectively. However, the proposed Slotted-DSTC-R protocol has a lower decoding complexity and achieves a diversity order of $2N$ at high SNR regime.

VI. Conclusion

In this paper, we present a SAF based two-hop DSTC protocol and evaluate its performance. Here, the relay nodes between the source and the destination nodes are grouped into two relay clusters according to their respective location and used to forward the space-time coded signal to the destination node for two consecutive time frames. We further extend the proposed Slotted-DSTC to Slotted-DSTC with redundant code (Slotted-DSTC-R) protocol, where the relay nodes in both of the relay clusters perform linear transformation on the same signal and forward the space-time coded signal to the destination node to achieve higher diversity order and a lower decoding complexity while maintaining the minimum coherence interval required by the space-time codes. The simulation results show that with the same BER performance, the proposed Slotted-DSTC protocol achieves a higher symbol rate compare to the OSTBC based DSTC protocol. Finally, we emphasize that the proposed Slotted-DSTC and Slotted-DSTC-R protocols are able to work with any STC or precoding techniques and achieve better performance.

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