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This is the author's version of a work that was submitted/accepted for publication in the following source:

Bunker, Jonathan M. (2012) Passenger transmission and productiveness of transit lines with high loads. Transportation Research Record, 2274, pp. 144-153.

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http://dx.doi.org/10.3141/2274-16

## Passenger Transmission and Productiveness of Transit Lines with High Loads

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Submission date: 03/12/2012

Submitted for Publication to Transportation Research Record

| Word count |  |
| :--- | ---: |
| Abstract | 200 |
| Main text | 5391 |
| Figures and tables $(8 \times 250)$ | 2000 |
| Total | $\mathbf{7 5 9 1}$ |


#### Abstract

Deterministic transit capacity analysis applies to planning, design and operational management of urban transit systems. The Transit Capacity and Quality of Service Manual (1) and Vuchic $(2,3)$ enable transit performance to be quantified and assessed using transit capacity and productive capacity. This paper further defines important productive performance measures of an individual transit service and transit line. Transit work ( $\mathrm{p}-\mathrm{km}$ ) captures the transit task performed over distance. Passenger transmission (p$\mathrm{km} / \mathrm{h}$ ) captures the passenger task delivered by service at speed. Transit productiveness ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ) captures transit work performed over time. These measures are useful to operators in understanding their services' or systems' capabilities and passenger quality of service. This paper accounts for variability in utilized demand by passengers along a line and high passenger load conditions where passenger pass-up delay occurs. A hypothetical case study of an individual bus service's operation demonstrates the usefulness of passenger transmission in comparing existing and growth scenarios. A hypothetical case study of a bus line's operation during a peak hour window demonstrates the theory's usefulness in examining the contribution of individual services to line productive performance. Scenarios may be assessed using this theory to benchmark or compare lines and segments, conditions, or consider improvements.


## INTRODUCTION

The Transit Capacity and Quality of Service Manual (1) and Vuchic $(2,3)$ theoretically underpin deterministic transit capacity performance analysis, which is important in planning, design and operational management of urban transit systems. Measures describing productive performance of an individual transit service or a whole line, offered or utilized, are very useful to the operator as they quantify their resources' capabilities and passenger quality of service. This paper defines a number of useful productive performance measures. Individual transit services or transit lines experience variability in passenger demand, and high passenger load effects including pass-ups are becoming more commonplace on transit services and lines, both of which are considered.

## Definitions

Transit service defines an individual transit vehicle that traverses a line or route, for instance a bus, ferry, or train. A line includes a train line, Bus Rapid Transit (BRT) corridor, bus route or similar. A segment is a section of line between two discrete stops. A stop includes a train station, bus stop, ferry terminal or similar.

Passenger flow expresses passenger demand for transit travel along a line over time ( $\mathrm{p} / \mathrm{h}$ ), and can be computed when the pattern of boarding and alighting passengers along the entire line is known (2). The maximum load segment (MLS) incurs the highest passenger flow along the line.

Transit line service capacity (veh/h) is that achievable under stipulated repeatable, safe working conditions resulting in a maximum achievable frequency. TRB (1) defines it as "the maximum number of transit vehicles that can pass a given location during a given time period" based on a minimum headway.

Vehicle passenger carrying capacity normally reflects a maximum scheduled load (MSL) representing a repeatable, safe working maximum, with all seats and available standing spaces occupied. Depending on vehicle type, an individual transit service's capacity may be represented on a whole-vehicle basis, generally for bus or ferry, or linear passenger loading, generally for train (1).

Offered line passenger carrying capacity is the product of theoretical transit line service capacity and passenger carrying capacity of the vehicle used for service (2). TRB (1) defines this as "the maximum number of people that can be carried past a given location during a given time period under specified operating conditions; without unreasonable delay, hazard, or restriction; and with reasonable certainty". Typically the time period is a peak hour and the point of interest is the MLS.

Vuchic (2) introduces scheduled line capacity. Practically, scheduled transit line service capacity must be less than offered line service capacity. TRB (1) takes care of this by setting capacity as the minimum of these values.

Vuchic (3) defines transit work ( $\mathrm{p}-\mathrm{km} \mathrm{)} \mathrm{as} \mathrm{the} \mathrm{product} \mathrm{of} \mathrm{number} \mathrm{of} \mathrm{transported} \mathrm{objects}$, akin to force, and distance over which they are carried.

Vuchic (3) defines vehicle productivity as the transit work a vehicle performs per unit time during revenue service, being the product of spaces utilized and vehicle speed ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ). This quantity is akin to physical power delivered, so is appealing to the operator in describing active performance of an individual service along a line, or on average across a number of services traversing a line. However, its terminology is ambiguous in that it may be easily confused with the econometric term transit productivity, which is widely used in transit system effectiveness measurement (4, 5, 6).For instance, in describing the San Diego Metropolitan Transit Development Board's process for evaluating transit services, Cheung and Daney (7) list three measures used in an index score to evaluate productivity; passengers per revenue mile, passengers per revenue hour, and subsidy per passenger. In contrast, Conlon et al (8) use service productivity to define ridership (boarding $\mathrm{p} / \mathrm{veh}-\mathrm{h}$ ). To avoid confusion, vehicle productivity is redefined here as passenger transmission per service ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ).

This paper defines transit productiveness as transit work delivered over time, by an individual service along a line or a number of services traversing the line ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ). This quantity of utilization is appealing to the operator in describing how productive a service or line is over a time period of interest, from a more aggregate perspective.

Vuchic (2) defines a line's productive capacity as the product of line passenger carrying capacity and operating speed $\left(\mathrm{p}-\mathrm{km} / \mathrm{h}^{2}\right)$. This quantity is the minimum of offered and scheduled, rather than utilized. He argues that this measure is one of the most useful line performance measures, as it both incorporates capacity, which is important to the operator, and speed of moving passengers along the line, which is important to the operator and reflects passenger quality of service. This quantity is extremely valuable; however, has not been extended to a whole of line analysis and does not describe utilization. As such, this paper expands on the concepts of an individual service's and whole of line's utilized productive performance using passenger transmission and transit productiveness.

## THEORETICAL PROPOSITIONS

## Variation along Line

Passenger demand tends to be spread out over both space and time, which in turn prevents offered transit point capacity from being fully utilized throughout the peak period (1). Temporal variation is accommodated in capacity calculations through the use of a Peak Hour Factor, reflecting the most intense 15 minutes. Spatial diversity can be manifested in a number of ways, from boarding and alighting locations at the macro scale to the distribution of passengers within vehicles at the micro scale (1).

Vuchic (2) overcomes the point capacity limitation by evaluating a line by segment. Maximum flow can ordinarily be achieved only on the MLS, while the passenger demand pattern results in reduced flow on all other segments. Vuchic introduces the manner in which an entire line may be analysed in terms of utilized transit work (his 2005 definition). This is informative to the operator in providing a picture of total transit performance along the line during a time period. For this study, all individual services and passenger patterns at stops during the hour are considered accordingly.

## High Passenger Load Conditions

Pass-ups occur when passengers are left behind when a service departs under MSL, and are considered as unreasonable delays to passengers (1). However, in many large cities, transit lines are under increasing pressure as population grows and transit ridership grows due to modal shifts away from private car usage (10). Consequently, it is not uncommon that a transit service, particularly during peak hour, must leave passengers behind once it reaches MSL. This would normally occur at the stop prior to the MLS, but can give rise to multiple MLSs.

Peak spreading is a consequence, along with an increase in dwell times at stops, which increases vehicle travel times along the line. While this occurs, traffic growth, particularly for buses on a public road network or BRT corridor, tends to increase congestion, further increasing segment running times. This may result in disparity in schedule keeping between services on segments along the line. Vuchic (2) prescribes a means of estimating terminal time, however this does not address schedule keeping by segment, which influences transit productive performance. Ding and Chen (11) go some way towards this but with a focus on real time control rather than the outcomes of actual schedule keeping by segment. Chien (12) presents a method for disseminating real-time bus arrival information for pre-trip passengers, which can be adapted to account for stochasticity and improvement of reliability with real-time control.
van Ooort et al (13) describe a methodology to improve reliability in short headway transit services, including both schedule-based and headway-based holding strategies at points along a line, finding a preference for schedule-based holding in reducing additional travel time. Delgado et al (14) propose a model to minimize total times experienced by passengers in a bus corridor system using two control policies; headway based vehicle holding applicable at any stop, and boarding limits that constrain passenger boarding even when the vehicle is at less than MSL. Passenger delays imposed by boarding limits are accounted for in determining overall travel times. Use of both control policies together was found to be superior to vehicle holding.

Kurauchi et al (15) propose an approach to solving the transit network loading problem using an absorbing Markov chain analogy, which incorporates line capacity constraints through formulation of failure to board probabilities. Their research is relevant at the network and common-line level, with a
focus on route choice by way of adapting strategic transport modelling theory; however, it is not readily adaptable to this deterministic line productiveness approach.

## THEORY DEVELOPMENT

## Passengers On-board Service on a Segment

The number of passengers on board a given service $h$ on a given segment $i$ is given by (p):

$$
P_{O B, h, i}=P_{O B, h, i-1}-P_{A A, h, i}+P_{B A, h, i}
$$

## Equation 1

Where:

$$
\begin{align*}
& P_{O B, h, i-1}=\left\{\begin{array}{c}
0 ; \text { where } i=1 \\
\text { passengers on board service } h \text { on segment } i-1 ; \text { where } i>1
\end{array}\right. \\
& P_{A A, h, i}=\text { passengers actually alighting service } h \text { at stop before segment } i \tag{p}
\end{align*} P_{B A, h, i}=\text { passengers able to board service } h \text { at stop before segment } i \quad \text { (p) } \quad .
$$

This equation is the same formulation to estimate passenger volume on a given segment (2) and onboard a given vehicle (1); however, it allows for two conditions:

- where not all passengers $P_{B, h, i}$ who wish to board the vehicle used for service $h$ at the stop before segment $i$ are able to do so, due to the maximum scheduled load $P_{m s l, h}$ being reached either before boarding commences or during passenger boarding; and
- where, upstream of segment $i-1$, some additional passengers are onboard service $h$ above its latent demand, having boarded because they were passed up by service $h-1$, who then need to alight at a stop downstream of segment $i$.

Here, the number of passengers able to board service $h$ at the stop before segment $i$ is given by (p):

$$
P_{B A, h, i}=\min \left(P_{B, h, i}+P_{P U, h-1, i}, P_{m s l, h}-P_{O B, h, i-1}+P_{A A, h, i}\right)
$$

## Equation 2

Where:
$P_{B, h, i}=$ latent passenger boarding demand for service $h$ at stop before segment $i \quad$ (p)
$P_{P U, h-1, i}=$ passenger pass-ups by service immediately before $h$ at stop before segment $i$ (p)
Eq 2 presumes no boarding limit control when the vehicle used for service is at less than MSL.
The number of passengers passed up by a given service $h$ at the stop before segment $i$ is given by (p):

$$
P_{P U, h, i}=\left\{\begin{array}{c}
0 ; \text { where } h=0 \\
\max \left(P_{B, h, i}+P_{P U, h-1, i}+P_{O B, h, i-1}-P_{A A, h, i}-P_{m s l, h}, 0\right) ; \text { otherwise }
\end{array}\right.
$$

## Equation 3

Eq 3 presumes that no passengers have been passed up prior to the first service, hypothetically being $h=0$. In a practical setting, for instance a morning peak period, if the first service in the analysis period is not the line's first service of the day, it is necessary to ensure a buffer prior to the analysis period sufficient that no passengers have been passed up prior to a service under consideration on segment $i$.

Alternatively, if the number of passengers passed up by the service immediately prior to the first service of interest $h=1$ is known, then $P_{P U, 0, i}$ may be set to that value.

Passengers passed up by a given service is itself a useful passenger quality of service measure, as it may relate to an individual stop, or it may vary along a line, and in either case may vary with time. For each passenger passed up their extra delay beyond their expected wait time is equal to the headway until the subsequent service which they are able to board. Fan and Machemehl (16) developed a predictive linear model for expected passenger wait time for transport planning purposes as Wait $=2.28+0.29$ $B L H$; Bus Line Headway being the only independent variable. Chowdhury and Chien (17) extended the analysis of wait time to transfer passengers.

The number of passengers actually alighting service $h$ at the stop before segment $i$ is given by (p):

$$
P_{A A, h, i}=P_{A, h, i}+P_{P U A, h-1, i}
$$

Equation 4
Where:
$P_{A, h, i}=$ latent passenger alighting demand for service $h$ at stop before segment $i$
$P_{P U A, h-1, i}=$ passengers passed up by service $h-1$ at an upstream stop who board service $h$ and alight at stop before segment $i \quad$ (p)

The pattern $P_{P U A, h-1, i}$ needs to be determined for service $h-1$ for all segments downstream from where they were originally passed up.

## Scheduled Service Journey Time

Vuchic (2) defines the basic model of travel time between a transit service's departures from two adjacent stops as the sum of running time and stop standing time. Scheduled stop to stop journey time for service $h$ along segment $i$ is given by ( min ):

$$
t_{s, h, i}=t_{s r, h, i}+t_{s s, h, i}
$$

## Equation 5

Where:

$$
\begin{align*}
& t_{s r, h, i}=\text { scheduled running time for service } h \text { along segment } i \\
& t_{s s, h, i}=\text { scheduled stop standing time for service } h \text { at stop before segment } i \tag{min}
\end{align*}
$$

In the absence of a known schedule, scheduled stop standing time for a given transit service can be estimated as dwell time. Clearance time may be included as part of running time. TRB (1) discusses methods of estimating clearance time for various transit modes, which are a function of the quality of input data available and operating conditions.

According to TRB (1), for bus transit dwell time may be estimated by one of three methods; field measurements e.g. for existing operations, default values e.g. for future planning, and calculation. Calculation is based on utilization of the busiest door of the transit vehicle, normally front door of a bus, busiest gangway of a ferry, or busiest door along a train. Dwell time is the sum of: the product of passengers alighting through the busiest door and alighting time per passenger, the product of passengers boarding through the busiest door and boarding time per passenger, and door opening and closing time. TRB (1) provides extensive data for selection of appropriate values to estimate dwell time for various modes. Jaiswal et al $(18,19)$ provide guidance on estimating dwell time for buses serving a BRT station. In estimating scheduled stop to stop journey time, it may be wise to include an operating margin on dwell time, for which TRB (1) provides guidance.

Vuchic $(2,3)$ provide methodologies for estimating a service's scheduled running time, provided its dynamic operating characteristics are known. TRB (1) specifies methods for various transit modes which account for relevant line effects. Jong et al (20) estimates various time components during train line operation for capacity models for Taiwan.

Otherwise, for an existing line that generally obeys its schedule, field trial data under day to day operating conditions may be used, or for a proposed transit provision, simulated runs along the line.

Cumulative scheduled journey time for service $h$ to the end of segment $i$ is given by (min):

$$
T_{s, h, i}=\sum_{j=1}^{i} t_{s, h, j}
$$

Where:

$$
j=\text { segment increment from the starting terminus }
$$

## Actual Service Journey Time

This theory accounts for services deviating from their schedule due to high passenger load conditions and/or traffic congestion along the line. For an existing facility this requires field data of required running time and upstream stop standing time for each segment $i$. Alternatively, particularly for planning tasks, transport modelling may be used to estimate required running time.

Actual cumulative journey time for service $h$ to reach the end of segment $i$ is given by (min):

$$
T_{h, i}=\left\{\begin{array}{c}
0 ; \text { where } i=0 \\
\max \left(T_{h, i-1}+t_{s a, h, i}+t_{r a, h, i}, T_{s, h, i}\right) ; \text { otherwise }
\end{array}\right.
$$

Equation 7

Where:

$$
\begin{aligned}
& t_{r a, h, i}=\text { required running time for service } h \text { along segment } i \\
& t_{s a, h, i}=\text { required stopped time for service } h \text { at stop before segment } i
\end{aligned}
$$

Eq 7 presumes no cumulative journey time prior to the first segment along the line, hypothetically being $i=0$. Deadhead time between a depot and originating terminus is excluded as there is no transit productiveness. Further, this theory conservatively presumes 100 percent holding whereby a service will not depart any stop along the line prior to its scheduled departure time.

Application of field data to Eq 7 enables irregularity in required running time between services, and therefore irregularities in headways, to be realistically reflected, which in turn affects productiveness and quality of service.

## Transit Service Work

Transit work performed by an individual transit service $h$ along segment $i$ is given by ( $\mathrm{p}-\mathrm{km}$ ):

$$
W_{h, i}=P_{O B, h, i} s_{i}
$$

Equation 8
Where:
$s_{i}=$ length of segment $i$
This measure does not reflect passenger flow; rather, purely passenger numbers.
Total transit work performed by service $h$ along line $L$ is the sum of the transit work performed along all consecutive segments along that line, given by ( $\mathrm{p}-\mathrm{km}$ ):

$$
W_{h, L}=\sum_{i=1}^{n} W_{h, i}
$$

## Equation 9

Where:
$n=$ number of consecutive segments constituting line $L$ traversed by service $h$

## Transit Service Passenger Transmission and Productiveness

Passenger transmission by service $h$ along segment $i$ is given by (p-km/h):

$$
\begin{equation*}
\Theta_{h, i}=\frac{60}{\left(T_{h, i}-T_{h, i-1}\right)} W_{h, i} \tag{Equation 10}
\end{equation*}
$$

Overall passenger transmission by service $h$ in completing transit line $L$ is given by ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ):

$$
\Theta_{h, L}=\frac{60}{T_{h, n}} W_{h, L}
$$

Equation 11

A particular service's transit productiveness needs to be isolated within a defined time window, $Z$, for instance its route duration, or more generally a one hour peak period. Provided that during $Z$, the specific consecutive segments along which the service's progression can be identified, productiveness for service $h$ between segments $p$ and $q$ along transit line $L$ is given by ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ):

$$
\Pi_{h, Z}=\frac{60}{T_{h, Z}} \sum_{i=p}^{q} W_{h, i}
$$

Equation 12

Where:
$p=$ first segment along line traversed by service $h$ during time window $Z ; 1 \leq p \leq n$
$q=$ last segment along line traversed by service $h$ during time window $Z ; p \leq q \leq n$
$T_{h, Z}=$ duration of time window $Z$ pertaining to transit service $h$
(min)

## Transit Line Average Passenger Transmission and Total Productiveness

Transit line average passenger transmission per service needs to be calculated by segment $i$ equals 1 to $n$ for all services $k$ equals 1 to $m$ that complete that segment over a defined time window $Z$, for instance a one hour peak period. For line $L$ this is given by $(\mathrm{p}-\mathrm{km} / \mathrm{h})$ :

$$
\begin{equation*}
\Theta_{L, Z}=\frac{60 \sum_{i=1}^{n}\left(s_{i} \sum_{k=1}^{m} P_{O B, k, i}\right)}{\sum_{i=1}^{n}\left(\sum_{k=1}^{m}\left(T_{k, i}-T_{k, i-1}\right)\right)} \tag{Equation 13}
\end{equation*}
$$

Where:

$$
P_{O B, k, i}=\text { passengers on board } k^{t h} \text { service during time window } Z \text { on segment } i
$$

Transit line total productiveness needs to be calculated similarly, and for line $L$ during time window $Z$ is given by ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ):

$$
\Pi_{L, Z}=\frac{60}{T_{L, Z}} \sum_{i=1}^{n}\left(s_{i} \sum_{k=1}^{m} P_{O B, k, i}\right)
$$

Where:
$T_{L, Z}=$ duration of time window $Z$ pertaining to transit line $L$
(min)

To use Eqs 13 and 14 correctly, each service expected to traverse any part of line $L$ during time window $Z$ should first be analysed, so for each segment $i$ the set of consecutive services $k$ equals 1 to $m$ which traverse that segment during $Z$ may be windowed. Any instances when a service traverses a segment on the line which do not occur within $Z$ must be excluded. Eqs 13 and 14 are applicable to lines that carry multiple routes, provided that these routes' services are only assigned to the segments $s_{i}$ upon which they operate.

## CASE EXAMPLES

## Service Passenger Transmission and Productiveness

Table 1 presents an example of a hypothetical individual bus service $h$ which operates on a line with nine segments. The bus MSL is 65p. A pattern of passengers passed up by the service immediately prior to service $h$ is given by $P_{P U, h-1, i}$. The only occurrence was four passengers at the stop before segment 5 . Passengers on board prior to commencement of boarding at the originating terminus before segment 1 , $P_{O B, h, 0}$, was zero.

The series of calculations for $P_{B A, h, i}, P_{P U, h, i}, P_{O B, h, i}, T_{h, i}, W_{h, i}$, and $\Theta_{h, i}$ are performed for segments $i$ equals 1 to 9 . An important check is that the passengers on board after the terminating stop ought to be 0 p .

The transit work performed by service $h$ along the entire line length was $546 \mathrm{p}-\mathrm{km}$. Note that no passengers were left behind by $h$. Overall passenger transmission by service $h$ along the entire line $L$ was $745 \mathrm{p}-\mathrm{km} / \mathrm{h}$, and peaked at $1,500 \mathrm{p}-\mathrm{km} / \mathrm{h}$ on segment 5 .

Bus service $h$ completed its journey within 44 min , which is less than a one hour period that might be considered for, say, a peak hour analysis. For its time window duration $T_{h, Z}$ equal to 44 min , productiveness was consequently $745 \mathrm{p}-\mathrm{km} / \mathrm{h}$.

Before leaving this example the usefulness of actual transit service passenger transmission is demonstrated. For a planning study underway, a future projection of passenger transmission of service $h$, may be compared against the existing value. Consider a passenger growth rate of 10 percent and an increased stop time at each stop along the route of 9 percent, and an increase in segment running time due to additional congestion en route of 5 percent.

The procedure yields an actual cumulative journey time $T_{h^{\prime}, L}$ of 46.1 min . Therefore the 44 min schedule cannot be met. Consequently 1 min is added onto the schedule along each of segments 3,5 , and 9 , for a total cumulative schedule journey time of 47 min . Total transit work $W_{h^{\prime}, L}$ performed by service $h^{\prime}$ along the entire line is $560 \mathrm{p}-\mathrm{km}$, or 2.5 percent higher than by service $h$.

In this case 7 p were passed up at stop CRW before segment 5 compared with none by service $h$. This represents 6 percent of all passengers wishing to board service $h$ ' passed up and an additional passenger delay of 70 p -min assuming a 10 min headway, or an average pass-up delay of 4.2 min in addition to expected wait time.

Overall passenger transmission $\Theta_{h^{\prime}, L}$ by service $h^{\prime}$ is $715 \mathrm{p}-\mathrm{km} / \mathrm{h}$, or 4 percent lower than service $h$ because journey time increases more than work. Passenger transmission does not automatically increase
with passenger growth under high load conditions, here due to the vehicle MSL being exceeded at stop CRW and the need to lengthen the schedule by 3 min due to higher passenger loads and congestion en route.

## Transit Line Average Passenger Transmission and Total Productiveness

This hypothetical transit line example uses the same bus line with nine segments of lengths $s_{i}$ from Table 1. MSL of all buses is 65 p . Two routes service this common line at 20 minute alternating frequencies, for a combined 10 min scheduled frequency.

Figures 1 and 2 illustrate hypothetical distributions of latent boarding and alighting demands for each service $h$ at the stop before each segment $i$ around peak hour $Z$. Some boardings and alightings for some services may occur outside $Z$, but are shown as they are needed to calculate passengers onboard on each segment.

Figure 3 illustrates for each service the segments traversed during peak hour $Z$ as scheduled. For example, Service 1 was operating within the downstream segments of the line during $Z$, traversing only segments 8 and 9 . Conversely, service 9 was operating within the upstream segments during $Z$, traversing only segments 1,2 , and 3 .

Figure 4 illustrates for each service $h$ its required stopped time at the stop before each segment plus required segment running time, $t_{s a, h, i}+t_{r a, h, i}$, by segment $i$ around peak hour $Z$. Shown for comparison are scheduled segment times along the line applicable to all nine services. The schedule is conservative regarding required time for all services for segments 1 to 4 , with schedule finesse limited to the whole minute. Schedule time for segment 7 is higher than times required by all nine services, reflecting a time point at the segment end (stop CCR) for slack. Slack is also apparent for segment 9 (terminus RST) allowing for schedule recovery.

Figure 5 illustrates for each service $h$ its actual journey periods accumulated by segment $i$. For schedule planning this provides a means of highlighting segments or lines where schedule is not being met for consideration of improvements, and variation within segment times across services for consideration of treatments to improve reliability and consistency.

Given that no passengers were passed up at any stop prior to the first service operating during peak hour $Z$, Figure 6 illustrates the distributions of passengers on board by segment along the line by service during $Z$. Service 5 experienced its MSL along segment 5 leaving passenger/s behind at stop CRW, while service 6 experienced its MSL along segments 4 and 5 leaving passenger/s behind at both stops COO and CRW. The 1 p passed up at stop COO and 14 p passed up at stop CRW were recovered by service 7. Passenger pass-up delay was $10 \mathrm{p}-\mathrm{min}$ at COO and 140 p-min at CRW, totalling $2.5 \mathrm{p}-\mathrm{h}$ or an average pass-up delay of 0.27 min .

Figure 7 illustrates the distributions of productiveness contributions by service by segment for peak hour, $T_{L, Z}=60 \mathrm{~min}$. The most productive segment is segment 5 , being the longest at 3 km , and the most highly loaded, while services 3 and 4 operated at high loadings and services 5,6 and 7 operated at MSL. This was followed by segment 3, which carried the maximum of seven services, each with moderately high loading. Segment 9 was the least productive due to very few passengers on board services 1 to 6 before terminus RST.

Total transit work performed by the line during peak hour $Z$ was $2,995 \mathrm{p}-\mathrm{km}$. Total line productiveness was $2,995 \mathrm{p}-\mathrm{km} / \mathrm{h}$ accordingly.

Average passenger transmission per service along the line during peak hour $Z$ was $675 \mathrm{p}-\mathrm{km} / \mathrm{h}$. It is useful to compare this value with the highest overall passenger transmission of $857 \mathrm{p}-\mathrm{km} / \mathrm{h}$ by service 7 . A resultant peaking characteristic is defined as the quotient, here 79 percent, reflecting for this line example that peak passenger demand cannot be sustained consistently during the peak hour. This phenomenon is somewhat similar to that implied by the Peak Hour Factor used in transit capacity analysis (1).

This comprehensive understanding of line transit work performed, passenger transmission, and productiveness can be used to compare between lines/routes and, within a given line, various operating scenarios and time horizons. High patronage segments and/or services can be identified in planning
efforts to target improvements. For example, Figures 7 and 3 indicate that travel time improvements on segment 5 might enable service $8 \mathrm{and} /$ or 2 to also complete this segment during Z . Completion of segment 5 by service 8 during $Z$ would increase both average passenger transmission and total line productiveness by 5 percent.

## CONCLUDING REMARKS

This paper extends previous work (1, 2, 3) by theoretically elaborating deterministic productive performance measures of an individual transit service and a transit line. Utilized transit work ( $\mathrm{p}-\mathrm{km}$ ) provides a useful measure of the transit task performed. Passenger transmission ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ) captures transit task performed by service at speed. Transit productiveness ( $\mathrm{p}-\mathrm{km} / \mathrm{h}$ ) captures transit work performed over time. These terms are useful to the operator in understanding their services' or systems' capabilities and passenger quality of service.

Because individual services' passenger loading demands vary along the line during time, previous work $(2,3)$ was extended here to address variation in demand patterns along the line, rather than merely assessing a specific location such as a maximum load segment. Due to high passenger load conditions becoming more prevalent on urban transit systems, passenger pass-ups and pass-up delay were incorporated here.

A hypothetical case study of an individual bus transit service's operation along a line demonstrated the usefulness of passenger transmission in comparing an existing and growth scenario, showing that in this instance the limitations of the service's capacity can significantly impact upon any improvements in productive performance. A hypothetical case study of a bus transit line's operation during a peak hour, while a number of bus services were active, demonstrated the usefulness of examining the contribution of individual services to the transit line's total productiveness. This was evidenced in Figure 7, as well as individual services' journeys along the line compared to the schedule as evidenced by Figures 4 and 5, and onboard loading pattern variation along the line as evidenced by Figure 6. Further understanding of peaking was also demonstrated by way of a peaking characteristic ratio between all services' average passenger transmission and that of the highest performing service. Scenarios could be usefully assessed by quantifying transit work performed, passenger transmission, and/or line productiveness, along with services' running and onboard loading patterns, to benchmark or compare services, lines and segments, conditions, or consider improvements.

This paper focussed on deterministic productive performance measures of utilized service under a given passenger demand pattern along a line. However, it is also important to gain a better appreciation of offered service with respect to these measures. Future research will focus on capacity states of deliverable transit work, deliverable passenger transmission, and transit productive capacity. Understanding the patterns between utilized and offered values of the productive performance measures, by service by segment along a line, will provide further insight into transit service and lines capabilities with respect to achievable efficiency.

## ACKNOWLEDGEMENTS

The author wishes to acknowledge the contribution of Denise Redfern of QUT's Smart Transport Research Centre in providing editorial support in preparation of this paper.

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FIGURE 4 Services’ Required \& Scheduled Stopped Plus Running Periods by Segment around Peak Hour $Z$.

FIGURE 5 Actual Services' Segment Journey Periods along Line around Peak Hour Z.
FIGURE 6 Actual Passengers On Board Services by Segment during Peak Hour Z.
FIGURE 7 Services’ Productiveness Contributions by Segment during Peak Hour Z.

TABLE 1 Passenger Transmission by a Service

| Term | Units | Segment i |  |  |  |  |  |  |  |  | $\sum$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |  |
| $P_{m s l, h}$ | $P$ |  |  |  |  |  |  |  |  |  | 65 |
| Stop Before | Segment $i$ : | $C D L$ | CNA | CHL | COO | CRW | MHL | SBK | $C C R$ | KGS | RST |
| $s_{i}$ | km | 1.9 | 1.6 | 1.8 | 1.0 | 3 | 0.8 | 1.0 | 0.9 | 0.6 | 12.6 |
| $P_{B, h, i}$ | $p$ | 24 | 6 | 11 | 19 | 12 | 6 | 5 | 12 | 3 |  |
| $P_{P U, h-1, i}$ | $p$ | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 |  |
| $P_{A, h, i}$ | $p$ | 0 | 0 | 0 | 5 | 6 | 12 | 13 | 25 | 34 | 3 |
| $P_{A A, h, i}$ | $p$ | 0 | 0 | 0 | 5 | 6 | 13 | 14 | 26 | 35 | 3 |
| $t_{s, h, i}$ | min | 4 | 4 | 4 | 4 | 7 | 2 | 3 | 9 | 7 |  |
| $T_{s, h, i}$ | min | 4 | 8 | 12 | 16 | 23 | 25 | 28 | 37 | 44 |  |
| $t_{s a, h, i}$ | min | 1.0 | 0.5 | 0.67 | 1.0 | 0.75 | 0.75 | 0.5 | 1.0 | 1.0 |  |
| $t_{r a, h, i}$ | min | 3.0 | 3.5 | 3.0 | 3.0 | 7.0 | 1.5 | 1.5 | 8.5 | 5.5 |  |
| $P_{O B, h, i-1}$ | $p$ | 0 | 24 | 30 | 41 | 55 | 65 | 58 | 49 | 35 | 3 |
| $P_{B A, h, i}$ | $p$ | 24 | 6 | 11 | 19 | 16 | 6 | 5 | 12 | 3 | 0 |
| $P_{P U, h, i}$ | $p$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $P_{O B, h, i}$ | $p$ | 24 | 30 | 41 | 55 | 65 | 58 | 49 | 35 | 3 | 0 |
| $T_{h, i}$ | min | 4.0 | 8.0 | 12.0 | 16.0 | 23.8 | 26.0 | 28.0 | 37.5 | 44.0 |  |
| $W_{h, i}$ | $p-k m$ | 45.6 | 48.0 | 73.8 | 55.0 | 195 | 46.4 | 49.0 | 31.5 | 1.8 | 546 |
| $\Theta_{h, i}$ | $p-k m / h$ | 684 | 720 | 1,107 | 825 | 1,500 | 1,265 | 1,470 | 199 | 17 | 745 |
| $\Pi_{h, Z}$ | p-km/h |  |  |  |  |  |  |  |  |  | 745 |



FIGURE 1 Services' Latent Boarding Demand by Segment around Peak Hour Z.


FIGURE 2 Services' Latent Alighting Demand by Segment around Peak Hour Z.


FIGURE 3 Services Traversing Segments during Peak Hour Z.


FIGURE 4 Services' Required \& Scheduled Stopped Plus Running Periods by Segment around Peak Hour Z.


## Bus Service $h$

FIGURE 5 Actual Services' Segment Journey Periods along Line around Peak Hour Z.


FIGURE 6 Actual Passengers On Board Services by Segment during Peak Hour Z.


FIGURE 7 Services’ Productiveness Contributions by Segment during Peak Hour Z.

