



Queensland University of Technology
Brisbane Australia

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Supplemental Materials

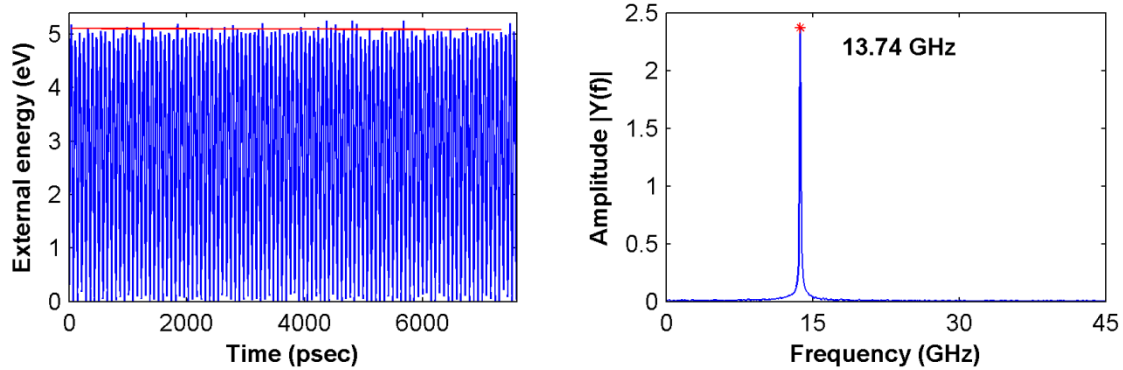
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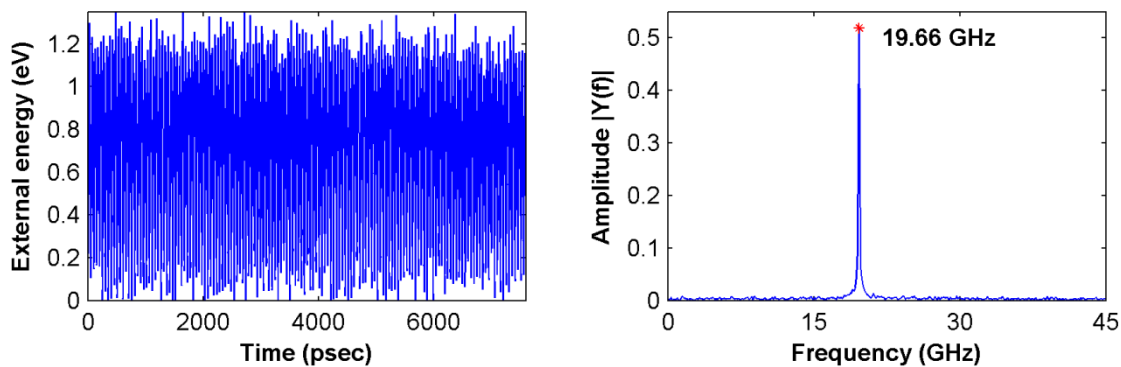
1. Simulation results

Following MD results are for the $\langle 110 \rangle$ NW with rhombic, truncated rhombic and triangular cross-sections, with a side length b equals 3 nm. The vibration testing is carried out at the temperature of 10 K for a total of 7600 picoseconds. For each case, two figures are presented, including the time history of the external energy and its corresponding frequency spectrum (truncated at the frequency of 45 GHz).

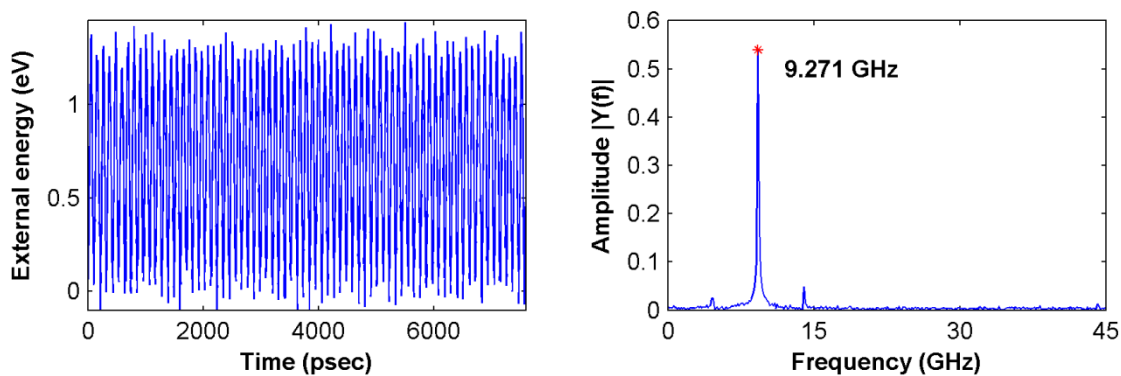
1.1. NW with rhombic cross-section: actuation along y -axis



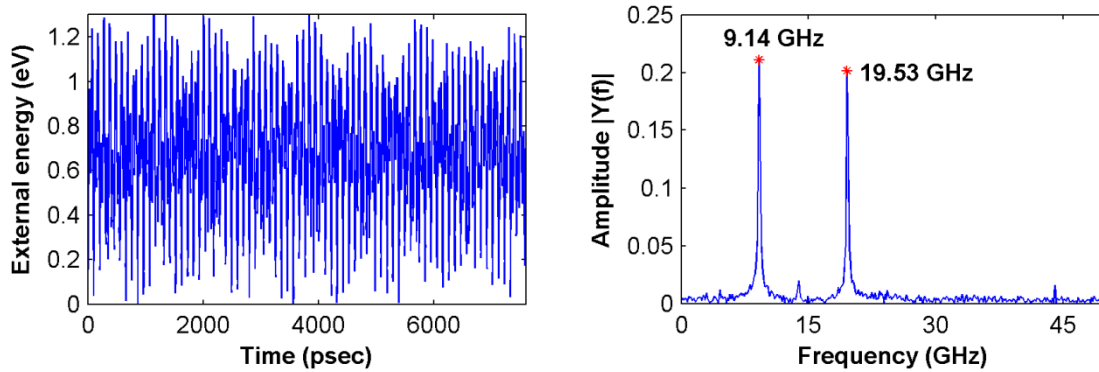
1.2. NW with triangular cross-section (α): actuation along x -axis



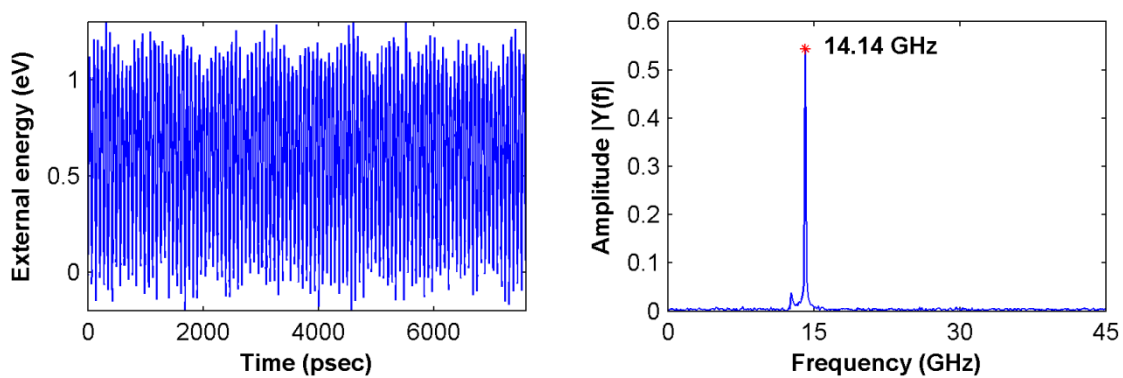
1.3. NW with triangular cross-section (α): actuation along y -axis



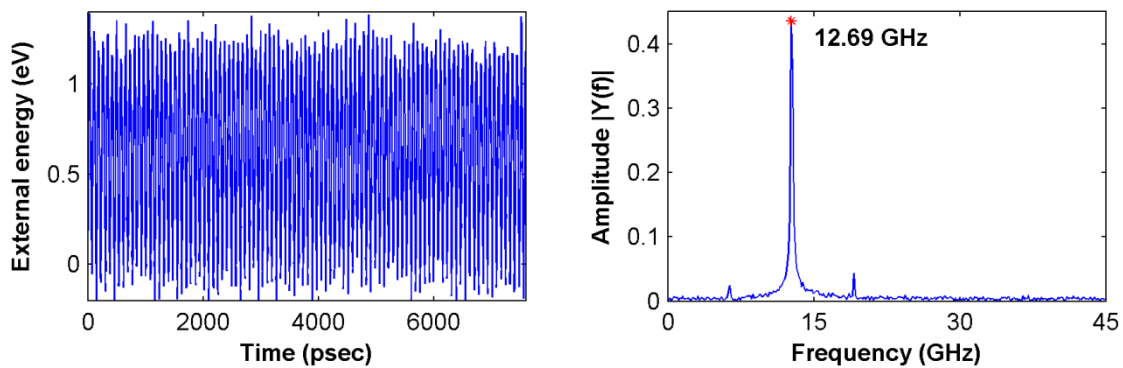
1.4. NW with triangular cross-section (α): actuation along x' -axis



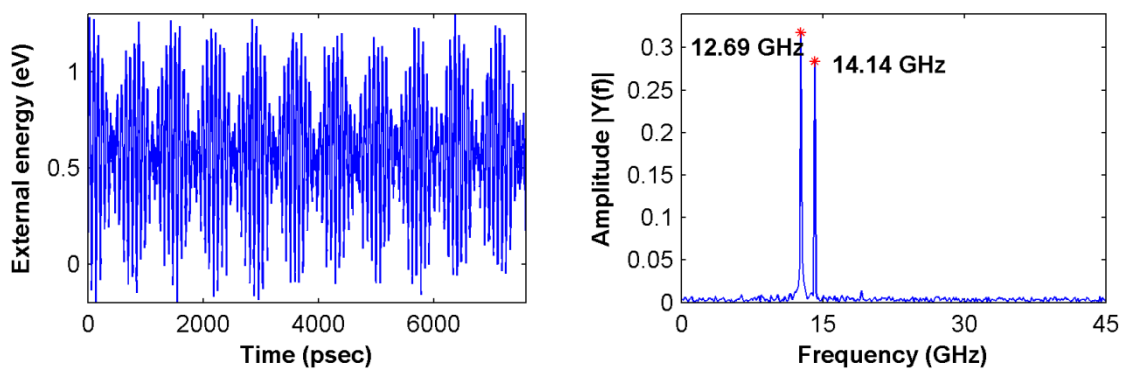
1.5. NW with triangular cross-section (β): actuation along x -axis



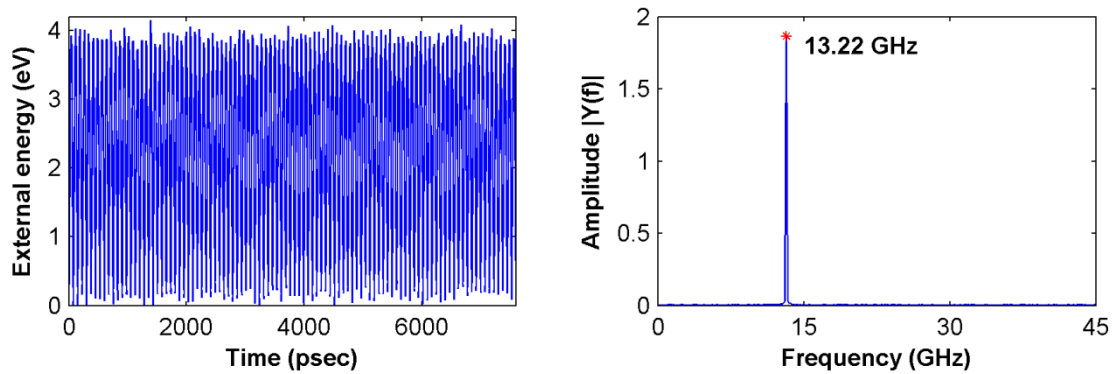
1.6. NW with triangular cross-section (β): actuation along y -axis



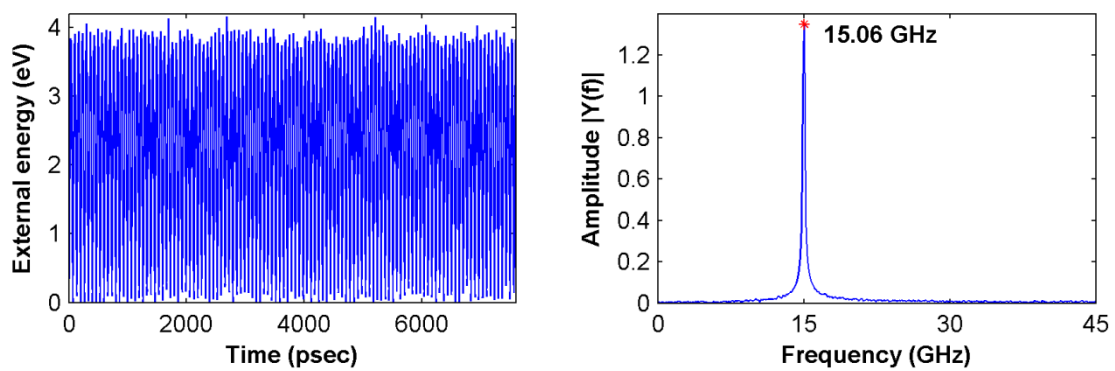
1.7. NW with triangular cross-section (β): actuation along x' -axis



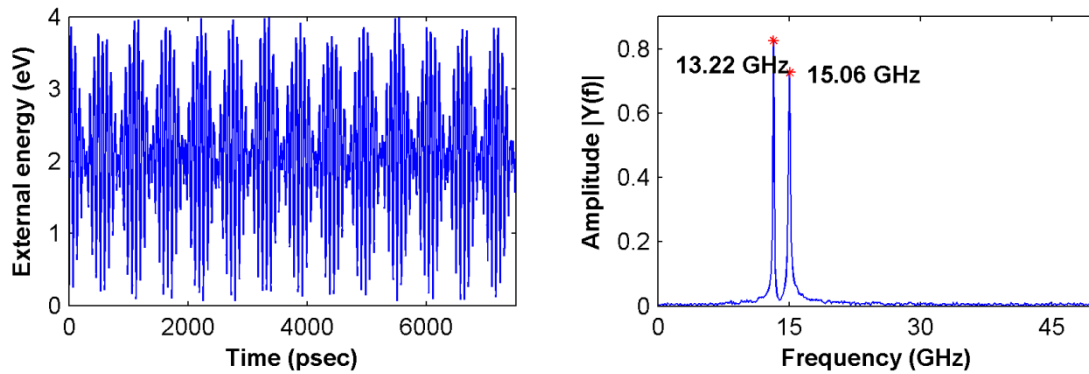
1.8. NW with truncated rhombic cross-section: actuation along x -axis



1.9. NW with truncated rhombic cross-section: actuation along y -axis



1.10. NW with truncated rhombic cross-section: actuation along x' -axis



2. Principal moments of inertia under discrete model

2.1. Rhombic Cross-section

Figure 1 shows the atomic layer of the rhombic cross-section of the $\langle 110 \rangle$ Ag NW, which is enclosed by four $\{111\}$ surfaces.

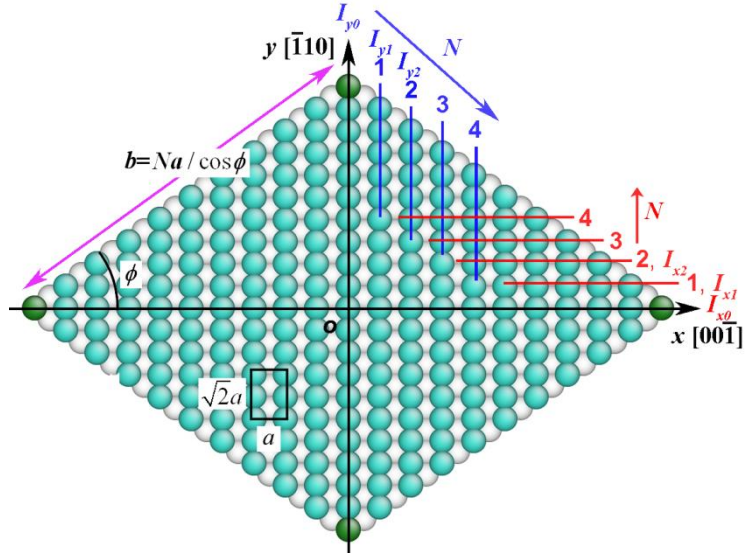


FIG. 1 Atomic layer of the rhombic cross-section, the grey edge schematically refers the surface layers.

For the discrete model, the principal moments of inertia are deduced as below. As seen in Fig. 1, the atoms column number in the x-axis and y-axis are the same (denoted as N). Thus, for I_x , we have,

$$I_x = I_{x0} + I_{xn}$$

$$I_{x0} = (2N + 1)I_a$$

$$I_{xn} = 2(I_{x1} + I_{x2} + \dots + I_{xi} + \dots + I_{xN}), \text{ and } I_{xi} = [2(N - i) + 1] \left[A_a \left(ia\sqrt{2} / 2 \right)^2 + I_a \right]$$

Hence

$$I_{xn} = \sum_{i=1}^N [2(N - i) + 1] \left[A_a a^2 i^2 + 2I_a \right]$$

$$I_x = (2N + 1)I_a + \sum_{i=1}^N [2(N - i) + 1] \left[A_a a^2 i^2 + 2I_a \right]$$

For I_y , we have,

$$I_y = I_{y0} + I_{yn}$$

$$I_{y0} = (2N + 1)I_a$$

$$I_{yn} = 2(I_{y1} + I_{y2} + \dots + I_{yi} + \dots + I_{yN}), \text{ and } I_{yi} = [2(N - i) + 1] \left[A_a (ia)^2 + I_a \right]$$

Hence

$$I_{yn} = 2 \sum_{i=1}^N [2(N - i) + 1] \left[A_a a^2 i^2 + I_a \right]$$

$$I_y = (2N+1)I_a + 2\sum_{i=1}^N [2(N-i)+1] [A_a a^2 i^2 + I_a]$$

where N is related with the side length b by $b = Na / \cos \phi$, ϕ is the angle as specified in Fig. 1. The cross-section area is given as $A = (2N^2 + 2N + 1)A_a$.

2.2. Triangular Cross-section (α)

Figure 2 shows the atomic layer of the triangular cross-section (α) of the $\langle 110 \rangle$ Ag NW, which is enclosed by two $\{111\}$ surfaces and one $\{110\}$ surface.

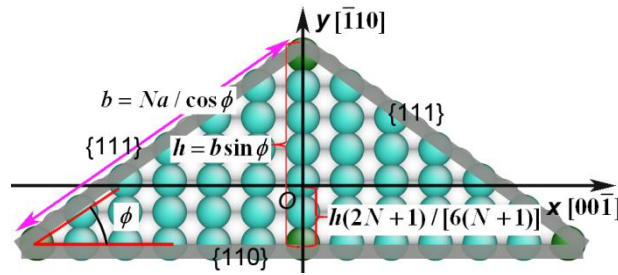


Fig. 2 Atomic layer of the triangular cross-section (α), the grey edge schematically refers the surface layers.

As is known, for the continuum model, the centroid of the above triangular is at $h/3$. According to the definition of centroid, i.e.,

$$x_C = \sum_{i=1}^n A_i x_{Ci} / A, \text{ and } y_C = \sum_{i=1}^n A_i y_{Ci} / A, \text{ where } A \text{ is the total area, } A_i \text{ is the area segment with } x_{Ci}$$

and y_{Ci} as its corresponding centroid coordinates, n is the number of area segments. Obviously,

under the discrete model, the centroid has moved from $h/3$ to $h \frac{2N+1}{6(N+1)}$.

Upon this fact, the principal moments of inertia are derived as below. For I_x , we have,

$$I_x = I_{x1} + I_{x2} + \dots + I_{xi} + \dots + I_{x(N+1)}, \text{ and } I_{xi} = [2(N+1-i)+1] \left\{ A_a \left[(i-1)a\sqrt{2}/2 - h \frac{2N+1}{6(N+1)} \right]^2 + I_a \right\}$$

Hence

$$I_x = \sum_{i=1}^N [2N-2i+3] \left\{ A_a \left[(i-1)a\sqrt{2}/2 - h \frac{2N+1}{6(N+1)} \right]^2 + I_a \right\}$$

For I_y , we have,

$$I_y = I_{y0} + I_{ym}$$

$$I_{y0} = (N+1)I_a$$

$$I_{ym} = 2(I_{y1} + I_{y2} + \dots + I_{yi} + \dots + I_{yN}), \text{ and } I_{yi} = [(N-i)+1][A_a(ia)^2 + I_a]$$

Hence

$$I_{ym} = 2 \sum_{i=1}^N [(N-i)+1][A_a a^2 i^2 + I_a]$$

$$I_y = (N+1)I_a + 2 \sum_{i=1}^N [(N-i)+1][A_a a^2 i^2 + I_a]$$

where N is related with the side length b by $b = Na / \cos \phi$. The cross-section area is given as

$$A = (N^2 + 2N + 1)A_a.$$

2.3. Triangular Cross-section (β)

Figure 3 shows the atomic layer of the triangular cross-section (β) of the $\langle 110 \rangle$ Ag NW, which is enclosed by two $\{111\}$ surfaces and one $\{100\}$ surface.

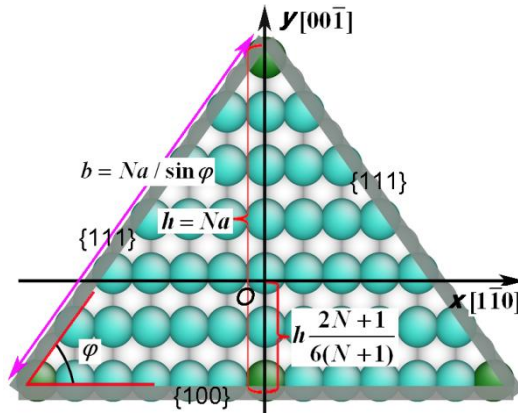


FIG. 3 Atomic arrangement of the triangular cross-section (β), the grey edge schematically refers the surface layers.

Similar as the triangular cross-section (α), the principal moments of inertia are deduced as below.

For I_x , we have,

$$I_x = I_{x1} + I_{x2} + \dots + I_{xi} + \dots + I_{x(N+1)}, \text{ and } I_{xi} = [2(N+1-i)+1] \left\{ A_a \left[(i-1)a - h \frac{2N+1}{6(N+1)} \right]^2 + I_a \right\}$$

Hence

$$I_x = [2N - 2i + 3] \left\{ A_a \left[(i-1)a - h \frac{2N+1}{6(N+1)} \right]^2 + I_a \right\}$$

For I_y , we have,

$$I_y = I_{y0} + I_{yn}$$

$$I_{y0} = (N+1)I_a$$

$$I_{yn} = 2(I_{y1} + I_{y2} + \dots + I_{yi} + \dots + I_{yN}), \text{ and } I_{yi} = [(N-i)+1] \left[A_a \left(\frac{i\sqrt{2}a}{2} \right)^2 + I_a \right]$$

Hence

$$I_{yn} = \sum_{i=1}^N [(N-i)+1] [A_a a^2 i^2 + 2I_a]$$

$$I_y = (N+1)I_a + \sum_{i=1}^N [(N-i)+1] [A_a a^2 i^2 + 2I_a],$$

where N is related with the side length b by $b = Na / \sin \phi$, ϕ is the angle as specified in Fig. 3. The cross-section area is given as $A = (N^2 + 2N + 1)A_a$.

2.4. Truncated rhombic Cross-section

Figure 4 shows the atomic layer of the truncated rhombic cross-section of the $\langle 110 \rangle$ Ag NW, which is enclosed by four $\{111\}$ surfaces and two $\{100\}$ surface.

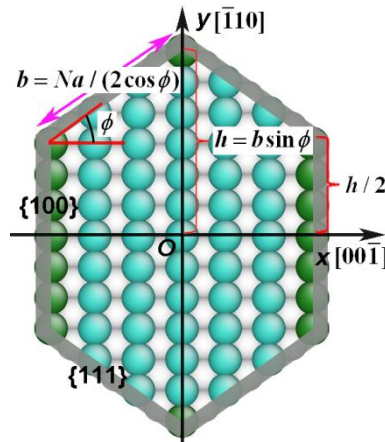


FIG. 4 Atomic arrangement of the truncated rhombic cross-section, the grey edge schematically refers the surface layers.

Under the discrete model, the principal moments of inertia are derived as below.

For I_x , we have,

$$I_x = I_{x0} + I_{xn}$$

$$I_{x0} = (N+1)I_a$$

$$I_{xn} = 2(I_{x1} + I_{x2} + \dots + I_{xi} + \dots + I_{xN/2} + I_{x(N/2+1)} + \dots + I_{xj} + \dots + I_{xN}), \text{ and}$$

$$I_{xi} = (N+1) \left[A_a (ia\sqrt{2}/2)^2 + I_a \right] \text{ and } I_{xj} = [2(N-j)+1] \left[A_a (ja\sqrt{2}/2)^2 + I_a \right]$$

Hence

$$I_{xn} = (N+1) \sum_{i=1}^{N/2} (A_a a^2 i^2 + 2I_a) + \sum_{j=N/2+1}^N [2(N-j)+1] [A_a a^2 j^2 + 2I_a]$$

$$I_x = (N+1)I_a + (N+1) \sum_{i=1}^{N/2} (A_a a^2 i^2 + 2I_a) + \sum_{j=N/2+1}^N [2(N-j)+1] [A_a a^2 j^2 + 2I_a]$$

For I_y , we have,

$$I_y = I_{y0} + I_{yn}$$

$$I_{y0} = (2N+1)I_a$$

$$I_{yn} = 2(I_{y1} + I_{y2} + \dots + I_{yi} + \dots + I_{y(N/2)}), \text{ and } I_{yi} = [2(N-i)+1] [A_a (ia)^2 + I_a]$$

Hence

$$I_{yn} = 2 \sum_{i=1}^{N/2} [2(N-i)+1] [A_a a^2 i^2 + I_a]$$

$$I_y = (2N+1)I_a + 2 \sum_{i=1}^{N/2} [2(N-i)+1] [A_a a^2 i^2 + I_a]$$

where N is related with the side length b by $b = Na / \cos \phi$. The cross-section area is given as

$$A = (3N^2 / 2 + 2N + 1)A_a .$$