INTEGRATED HEALTH PREDICTION OF BRIDGE SYSTEMS USING DYNAMIC OBJECT ORIENTED BAYESIAN NETWORKS (DOOBNS)

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ABSTRACT

The serviceability and safety of bridges are crucial to people's daily lives and to the national economy. Every effort should be taken to make sure that bridges function safely and properly as any damage or fault during the service life can lead to transport paralysis, catastrophic loss of property or even casualties. Nonetheless, aggressive environmental conditions, ever-increasing and changing traffic loads and aging can all contribute to bridge deterioration. With often constrained budget, it is of significance to identify bridges and bridge elements that should be given higher priority for maintenance, rehabilitation or replacement, and to select optimal strategy. Bridge health prediction is an essential underpinning science to bridge maintenance optimization, since the effectiveness of optimal maintenance decision is largely dependent on the forecasting accuracy of bridge health performance.

The current approaches for bridge health prediction can be categorised into two groups: condition ratings based and structural reliability based. A comprehensive literature review has revealed the following limitations of the current modelling approaches: (1) it is not evident in literature to date that any integrated approaches exist for modelling both serviceability and safety aspects so that both performance criteria can be evaluated coherently; (2) complex system modelling approaches have not been successfully applied to bridge deterioration modelling though a bridge is a complex system composed of many inter-related bridge elements; (3) multiple bridge deterioration factors, such as deterioration dependencies among different bridge elements, observed information, maintenance actions and environmental effects have not been considered jointly; (4) the existing approaches are lacking in Bayesian updating ability to incorporate a variety of event information; (5) the assumption of series and/or parallel relationship for bridge level reliability is always held in all structural reliability estimation of bridge systems.

To address the deficiencies listed above, this research proposes three novel models based on the Dynamic Object Oriented Bayesian Networks (DOOBNs) approach. Model I aims to address bridge deterioration in serviceability using condition ratings as the health index. The bridge deterioration is represented in a hierarchical relationship, in accordance with the physical structure, so that the contribution of each bridge element to bridge deterioration can be tracked. A discrete-time Markov process is employed to model

deterioration of bridge elements over time. In Model II, bridge deterioration in terms of safety is addressed. The structural reliability of bridge systems is estimated from bridge elements to the entire bridge. By means of conditional probability tables (CPTs), not only series-parallel relationship but also complex probabilistic relationship in bridge systems can be effectively modelled. The structural reliability of each bridge element is evaluated from its limit state functions, considering the probability distributions of resistance and applied load. Both Models I and II are designed in three steps: modelling consideration, DOOBN development and parameters estimation. Model III integrates Models I and II to address bridge health performance in both serviceability and safety aspects jointly. The modelling of bridge ratings is modified so that every basic modelling unit denotes one physical bridge element. According to the specific materials used, the integration of condition ratings and structural reliability is implemented through critical failure modes.

Three case studies have been conducted to validate the proposed models, respectively. Carefully selected data and knowledge from bridge experts, the National Bridge Inventory (NBI) and existing literature were utilised for model validation. In addition, event information was generated using simulation to demonstrate the Bayesian updating ability of the proposed models. The prediction results of condition ratings and structural reliability were presented and interpreted for basic bridge elements and the whole bridge system. The results obtained from Model II were compared with the ones obtained from traditional structural reliability methods. Overall, the prediction results demonstrate the feasibility of the proposed modelling approach for bridge health prediction and underpin the assertion that the three models can be used separately or integrated and are more effective than the current bridge deterioration modelling approaches.

The primary contribution of this work is to enhance the knowledge in the field of bridge health prediction, where more comprehensive health performance in both serviceability and safety aspects are addressed jointly. The proposed models, characterised by probabilistic representation of bridge deterioration in hierarchical ways, demonstrated the effectiveness and pledge of DOOBNs approach to bridge health management. Additionally, the proposed models have significant potential for bridge maintenance optimization. Working together with advanced monitoring and inspection techniques, and a comprehensive bridge inventory, the proposed models can be used by bridge practitioners to achieve increased serviceability and safety as well as maintenance cost effectiveness.

Keywords: Health prediction; dynamic object oriented Bayesian networks (DOOBNs); bridge deterioration model; condition states; structural reliability.

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LIST OF ABBREVIATIONS

AASHTO	American Association of State Highway Transportation Officials
AHP	Analytic Hierarchy Process
ANN	Artificial Neural Networks
BMS	Bridge management system
BNs	Bayesian Networks
BRP	Binary Recursive Partitioning
CBR	Case-based Reasoning
CDF	Cumulative Distribution Function
CPD	Conditional Probability Distribution
CPT	Conditional Probability Table
DBNs	Dynamic Bayesian Networks
DOOBNs	Dynamic Object Oriented Bayesian Networks
EM	Expectation-maximization
EU	Expected Utility
FE	Finite Elements
FHWA	Federal Highway Administration
FORM	First-order Reliability method
GA	Generic Algorithms
IDs	Influence Diagrams
LRFD	Load and Resistance Factor Design
MCS	Monte Carlo Simulation
MEU	Maximum Expected Utility
MLE	Maximum Likelihood Estimation
MR&R	Maintenance, Rehabilitation and Replacement

NBI	National Bridge Inventory
NDT	Non-Destructive Testing
OOBNs	Object Oriented Bayesian Networks
ORC	Overall Condition Rating
PDF	Probability Density Function
POD	Probability of Detection
RSM	Response Surface Method
SHM	Structural Health Monitoring
SORM	Second-order Reliability method

Statement of Original Authorship

The work contained in this thesis has not been previously submitted to meet requirements for an award at this or any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: ___________________________

Date: ___________________________

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Chapter 1: Introduction

1.1 Background

During the last several decades, a large number of infrastructure assets for transport networks have been built owing to fast urbanisation. Regarded as a critical component of a transport network, bridge has experienced a period of massive construction. Overall, there are several different types of bridge structures: reinforced concrete bridges, prestressed concrete bridges, steel bridges, arch bridges, cable stayed bridges, suspension bridges, movable bridges and footbridges [61]. Because of aggressive environmental conditions, ever-increasing and changing traffic loading effects and bridge aging, bridges are supposed to deteriorate over time. The bridge serviceability and safety are always the primary concerns for bridge owners and bridge maintenance engineers. Every effort should be taken to assure bridges function properly and safely as any damages or faults during the service life can lead to transport paralysis, catastrophic loss of property or even casualties.

In 2007, the I-35W Mississippi River Bridge (Figure 1-1) crossing the Mississippi River in Minneapolis, Minnesota, collapsed suddenly, which killed 13 people and injured 145 people [171]. Many people were stranded and in danger. There were vehicles on fire as well. Overall this catastrophe costed millions of dollars. Although this kind of disaster seldom happens in our daily life, we can see the consequence is extremely painful and long-lasting. Another instance, in the same year, is the Captain Cook Bridge over Brisbane River, which is the busiest bridge in Queensland. It was closed due to safety concern caused by a crack in structure [170]. The closure of Captain Cook Bridge affected people's daily life and national economy significantly.

There are approximately 33,500 road bridges in Australia. Most of them are critical bridges like Captain Cook Bridge. Especially, in some regional areas, bridge closures can bring even hundreds miles' detour, which really cause enormous inconvenience to people. To avoid any bridge collapse and unnecessary bridge closures, the importance of proper bridge maintenance activities cannot be over emphasized.

Figure 1-1. Scene of the collapse, the Interstate 35W Bridge over the Mississippi River in Minneapolis, Minnesota, 2007 [145]

Figure 1-2. The Captain Cook Bridge over Brisbane River (Courtesy of Tim Marsden)

In Australia, billions of dollars are spent on the activities related to bridge inspection and maintenance every year. However, there is always a significant potential for saving on current overspending due to ineffective maintenance, which is mainly owing to inaccurate prediction of bridge health performance. In addition, the budget funding for bridge preservation is always constrained. Hence, it is of particular importance to identify bridges that should be given higher priority for maintenance, rehabilitation or replacement, and to select the optimal strategy so that risk and life-cycle cost of those bridges can be reduced. In recent times, sustainable maintenance of bridges has been drawing increasing attention. Bridge management systems (BMS) are designed to consider decisions in design and selection of materials, and to optimize maintenance, rehabilitation and replacement (MR&R) decisions for bridge networks under financial constraints [112].

According to the guidelines and requirements outlined by the American Association of State Highway Transportation Officials (AASHTO), a BMS should include five basic components: a database, cost models, bridge deterioration models for health performance prediction, optimisation models for analysis and updating functions. In BMSs, bridge deterioration models are crucial as their ability to forecast bridge health performance largely determines the effectiveness of optimal strategy. Bridge health prediction has become an essential underpinning science to effective bridge maintenance optimization with the following reasons: (1) since a large number of bridges are identified as structurally deficient or functionally obsolete, and have been servicing beyond the design life, accurate health prediction is of significance to ensure these bridges are safe and reliable; (2) bridge health prediction provides the best information about bridge deterioration to assist decision-making about bridge maintenance; (3) economically responsible, safe, life-cycle management of bridges depends on accurate bridge health prediction over time. Currently, various bridge health prediction approaches have been developed. According to used bridge health indexes, current approaches can be classified into: that of condition ratings (condition states) based and structural reliability based. Among them the Markov chain model is the most commonly used by the existing BMSs. However, criticising the limitations, such as state space explosion and inconvenience for testing and monitoring data incorporation, an open research question arises as to on how to continuously improve bridge deterioration models.

Furthermore, the recent development of advanced sensing techniques and non-destructive testing (NDT) techniques has also offered an opportunity for enhancing bridge health prediction. To supplement the visual inspection, NDT techniques have been applied by BMSs to detect bridge elements of interest whose deterioration is not visible. Structural Health Monitoring (SHM) systems have been established as well to detect deterioration, track the real time vibration/dynamic response of a structure along with inputs and provide real time sensor values. Therefore, more objective and quantitative information about bridge deterioration becomes available. These objective data are supposed to include operational and environmental data as well as historical maintenance records. Compared with subjective data estimated from expert knowledge and visual inspection, objective data provide better insights into bridge real deterioration so that we can calibrate bridge deterioration models to mitigate uncertainties and to acquire more accurate results. Nonetheless, current bridge deterioration models have not been ready yet to utilise the available objective data to improve their prediction.

This research concentrates on bridge health prediction. Three bridge deterioration models based on Dynamic Object Oriented Bayesian Networks (DOOBNs) are developed. They can model bridge deterioration from both serviceability and safety aspects jointly. Cost-effective maintenance strategies require health prediction in these two performance criteria. However, because the existing approaches are segregated and mutually exclusive, their prediction results cannot be utilised cooperatively for bridge maintenance optimization. Therefore, the proposed models will be valuable on this matter. Additionally, the proposed models are able to incorporate different individual methods and a variety of subjective and objective data so as to maximise the advances of the current bridge health prediction.

1.2 Research gaps

At present, various bridge health prediction approaches are available for BMS. Although they have advanced the knowledge for bridge maintenance optimization, there are a number of identified deficiencies and gaps in current research based on a comprehensive literature review and they are listed as follows:

- **It is not evident in literature to date that any existing approaches have been yet proven to be generally sufficient and consistent to model bridge health prediction using both condition ratings (condition states) and structural reliability in an integrated manner so that both performance criteria can be evaluated coherently.**

Bridge serviceability and safety are two different concerns about its health performance. Bridge serviceability concerns bridge faults, such as pot holes in the concrete deck and spalling on the concrete beams, which will not trigger a bridge collapse but may result in a bridge repair. Bridge safety concentrates entirely on bridge load-carrying capacity. Normally, condition ratings and structural reliability are two commonly used health indexes to describe bridge health performance in these two concerned aspects. Condition ratings (condition states) mainly derived from visual inspection are estimated by bridge inspector with their subjective judgement. Structural reliability, defined through limit state functions, is an objective measure of probabilities that the demand applied to a structure may exceed its capacity. Cost-effective maintenance strategies require health prediction in these two performance criteria. However, because the existing approaches are segregated and mutually exclusive, their prediction results cannot be utilised cooperatively for bridge maintenance optimization. An integrated approach for bridge health prediction in terms of both condition ratings and structural reliability is highly desirable. This is elaborated as follows:

- Condition ratings (condition states) and structural reliability are implicitly correlated since they both reflect the deterioration processes of a bridge. A mechanism is needed to consider this implicit relationship for consistent and more accurate health prediction results.
- Bridge health prediction involves uncertainties. These uncertainties can be mitigated by adopting multiple approaches concurrently. However, the existing approaches are mutually exclusive and segregated.
- As the existing segregated approaches have limited ability to deal with uncertainties, they may lead to different tendencies in maintenance decisions. Moreover, maintenance decision-making based on only single health index is often not cost-effective.

➢ **Complex system modelling approaches have not been successfully applied for bridge deterioration modelling though a bridge is a complex system composed of many inter-related elements.**

A bridge structure is a complex system composed of many inter-related bridge elements. The deterioration of the bridge is largely dependent on the deterioration of each element. Additionally, each bridge element deteriorates with temporal uncertainties.

Therefore, a modelling approach that is able to facilitate the probabilistic representation of a complex problem domain in hierarchical ways is more appropriate for bridge deterioration modelling. Eventually, bridge maintenance optimization can benefit from this approach. Current research, however, has not investigated complex system modelling approaches for bridge health prediction effectively.

- **Multiple bridge deterioration factors, such as deterioration dependencies among different bridge elements, different types of observed information, maintenance actions and environmental conditions have not been considered jointly by the existing approaches.**

To achieve accurate health prediction results, a number of different deterioration factors should be considered jointly. For instance, because of deterioration dependencies, the deterioration of one bridge element can accelerate that of another. For example, the deterioration of a concrete deck accelerates when its bearings do not function properly. If the bearings freeze due to corrosion, the deck will be subjected to expansion and contraction stresses that cause cracking [142]. Therefore, it is vital to take into account the deterioration dependencies among bridge elements. Furthermore, during the service life of bridges, observed information reflecting bridge real deterioration may be available. These records need to be incorporated into modelling for results updating. Similarly, maintenance actions, environmental conditions, such as traffic load, wind load, temperature and humidity, also have effects on bridge deterioration. All these factors should be handled in an integrated manner. However, the current approaches have not done so.

- **The assumption of series and/or parallel relationship for bridge level reliability is always held in all structural reliability estimation of bridge systems, but this assumption needs to be challenged.**

Conventionally, structural reliability of bridge systems is evaluated through structural reliability methods with the representation of a bridge system as basic parallel and/or series bridge element sets. However, because a bridge system is a complex system being composed of many inter-related bridge elements, this representation is never verified favourably in practice. For accurate estimation, this assumption should be removed.

- **Incorporation of a variety of information, such as monitoring data, expert knowledge and physical laws can effectively mitigate the uncertainties in bridge deterioration modelling. However, such an incorporation encounters difficulties since the current bridge health prediction approaches cannot act as an integration platform and lack in the Bayesian updating ability.**

It is necessary to reduce uncertainties related to lack of full knowledge of bridge deterioration behaviours, and to deterioration models by which real-life behaviours of bridges may not be fully represented [26]. These uncertainties can be mitigated by integrating various types of information, such as inspection records, expert knowledge, physical law, monitoring data from NDT and SHM [26, 33, 45, 96, 123, 130] as well as operational/environmental condition [59, 64, 153]. However, the current approaches lack the ability to integrate all the information mentioned above. Additionally, updating efficiencies of the existing approaches also bring difficulties for the incorporation since Bayesian updating is implemented manually. Therefore, an effective platform for information integration is desired.

1.3 Research objectives and scopes

In this research, a complex system modelling approach, known as Dynamic Object Oriented Bayesian Networks (DOOBNs), is examined and adopted to deal with the identified research gaps so as to develop an integrated health prediction approach. The fundamental goal is to pioneer an effective system approach so as to provide comprehensive information about bridge future health performance in both serviceability and safety aspects for cost-effective bridge maintenance optimization. This research develops three novel bridge deterioration models based on DOOBNs. The relationships among these three models are shown in Figure 1-3. The research objectives are detailed as follows:

- Model I for bridge condition rating prediction with the ability to:
    - facilitate probabilistic representation of bridge systems in a hierarchical way from bridge elements to the whole bridge system;
    - handle multiple deterioration factors, such as deterioration dependencies among different bridge elements, inspection records, maintenance actions and environmental effects concurrently;

- perform Bayesian updating efficiently;
- operate as an effective platform to integrate a variety of information;
- address bridge deterioration in serviceability aspect.

➢ Model II for bridge structural reliability prediction with the ability to:

- calculate time-variant structural reliability of bridge elements based on limit state functions;
- evaluate time-variant structural reliability of bridge systems based on not only series and/or parallel relationship but also complex probabilistic relationship;
- implement Bayesian updating for structural reliability estimation without the requirement of special knowledge in reliability analysis;
- address bridge deterioration in safety aspect.

➢ Model III for integrated bridge health prediction with the ability to:

- model bridge essential failure modes, such as corrosion, crack and spalling;
- incorporate Models I and II by means of essential failure modes;
- predict bridge health performance in terms of both condition ratings and structural reliability.

➢ Validation of the proposed models for bridge health prediction.

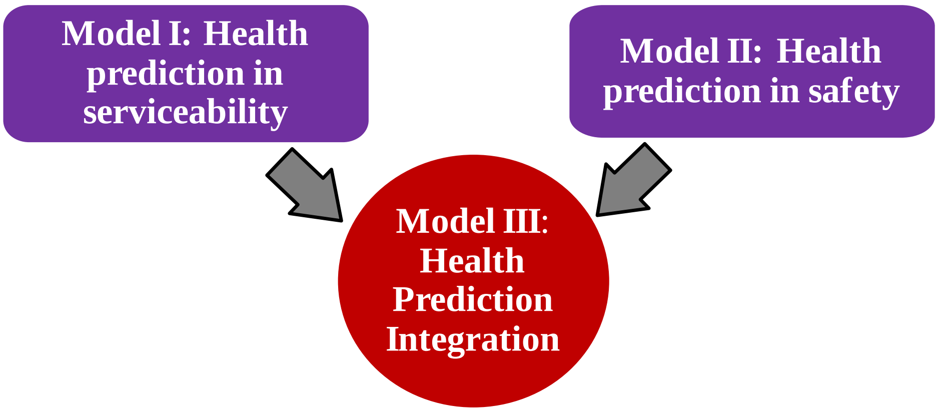

Figure 1-3. Relationships of the three developed models

There are various types of materials for bridges. This research will focus on the bridges made of reinforced concrete and steel associated with case studies since these two types of materials are the most commonly used. Other materials, such as timber and composite material, will not be considered in this research. For structural reliability, only ultimate limit state functions are considered. Deterioration mechanisms about corrosion, crack and spalling are presented so as to facilitate the health prediction integration. The assumption that deterioration process is stationary and follows first order Markov process is held in this research. The live load regarding structural reliability estimation is assumed to follow a time-invariant statistical distribution. As in practice the permitted weight of trucks for passing certain bridges can be controlled, so it is reasonable to hold this assumption. Further bridge maintenance optimization and detailed data acquisition from monitoring techniques, such as NDT and SHM, are beyond the scope of this research.

1.4 Originality and contribution

This research for the first time investigates DOOBNs in depth for integrated bridge health prediction. Three models based on DOOBNs are developed to address bridge health prediction using both condition ratings and structural reliability in an integrated manner. With more accurate and comprehensive prediction results, the proposed models are proved to be more effective than the existing bridge deterioration models. In addition, the proposed models can cope with versatility requirement for different BMSs and extensibility requirement for further maintenance optimization. This novel approach shows DOOBNs based bridge deterioration models have several unique features:

- Modelling of implicit correlation between condition ratings and structural reliability. Despite condition ratings and structural reliability are two different performance measures of bridge health they both reflect fundamental bridge deterioration processes. By means of essential failure modes, such as corrosion, crack and spalling, bridge deterioration in serviceability and safety aspects can be correlated to achieve integrated bridge health prediction.
- Hierarchical representation of bridge dynamic deterioration behaviours from bridge elements to the entire bridge. This representation facilitates integrated bridge management for the purpose of maintenance optimization. It also facilitates the implementation of “What-if” analysis to identify important bridge structural elements among a complex bridge system.

- Adaptive structural reliability estimation of the whole bridge systems. Limit state functions regarding bridge elements are modelled as the basis of bridge systems estimation. Considering not only series and/or parallel relationship among bridge elements but also complex probabilistic relationship, potential errors in bridge system estimation owing to inappropriate assumptions can be minimised. This adaptive ability facilitates modelling structural reliability of bridge systems under different types of relationships among bridge elements.
- Joint consideration of multiple bridge deterioration factors, such as deterioration dependency, observed information and environmental conditions as well as maintenance intervene. This ability generates more accurate health prediction results especially for bridge operation decisions.
- Incorporation of a variety of information for parameters estimation. Considering different data availabilities, detailed specifications to estimate conditional probability tables (CPTs) and priori probabilities based on bridge condition data, expert knowledge, combination of condition data and expert knowledge, theoretical deterioration equations and limit state functions as well as miscellaneous knowledge are all formulated. The inclusion of various types of data mitigates prediction uncertainties and data scarcity problems of current research.
- Bayesian updating ability for enhanced updating efficiency and prediction accuracy.

1.5 Thesis outline

Chapter 1 introduces the current development and significance of the research. In Section 1.2, several research gaps are identified from the research area. Targeting these limitations, research objectives are outlined in details in Section 1.3. Additionally, the scope of the research is described so that the study is constrained to a specific and tractable research area. Finally, the originality and knowledge contributions to the current research are discussed.

Chapter 2 presents a comprehensive literature review. In Section 2.2, a brief review about bridge management systems (BMS) is given. With the emphasis on bridge health prediction approaches, a critical review is conducted in Section 2.3 on bridge deterioration models. The review contains two key parts: condition rating based models and structural reliability based models. For each deterioration model, the limitations and merits are discussed. In Section 2.4, Structural Health Monitoring (SHM) and Non-destructive testing (NDT), which are closely related to bridge management, are introduced concisely. Finally, a number of identified research challenges in bridge health prediction are listed.

Chapter 3 aims to pave the roads for model development. The basic knowledge of Bayesian Networks (BNs) theory and bridge deterioration is introduced in Sections 3.1 and 3.2, respectively. In the first part, different classes of Bayesian Networks (BNs) are presented. In the second part, essential failure modes for steel and reinforced concrete as well as the corresponding physical equations are described in details. In Section 3.3, the issues about research strategy, data collection and modelling analysis process are described.

In Chapter 4, model I based on DOOBNs is developed for bridge condition ratings prediction. In Section 4.2, the proposed Model I is designed in three steps: modelling consideration, DOOBNs model development and parameters estimation. In the first step, bridge is decomposed into a number of bridge hierarchies. For each hierarchy, condition states definition, relative weight and involved deterioration dependencies are identified. In the second step, conceptual DOOBNs model is built up from the highest abstract level of the whole bridge system to the elementary level of bridge elements. In the last step, parameters estimation is addressed considering different types of data sources, such as expert knowledge and historical condition rating data. The feasibility of the proposed Model I is demonstrated on a steel truss bridge in Section 4.3, where expert knowledge is largely used to evaluate conditional probability tables (CPTs).

Chapter 5 focuses on bridge structural reliability prediction. Model II based on DOOBNs is proposed in section 5.2. Similarly, Model II is also composed of three steps: modelling consideration, DOOBN model development and parameters estimation. Section 5.2.1 recognizes several bridge structural hierarchies and develops limit state functions for each basic bridge structural element. Section 5.2.2 constructs the conceptual DOOBNs model, which includes hierarchical representation of the whole bridge system through several bridge hierarchies and time-variant structural reliability estimation for each basic bridge element. Section 5.2.3 parameterizes the conceptual DOOBNs model, where discretization is implemented on continuous variables to derive CPTs and prior probabilities based on discrete states. To validate the effectiveness of the proposed Model II, an application borrowed from an existing literature is conducted in Section 5.3.

Chapter 6 develops Model III based on DOOBNs for integrated health prediction. In Section 6.2, Model I for condition states prediction is modified to facilitate the modelling integration. Then, by means of essential failure modes, such as corrosion, crack and spalling, Models I and II are connected each other. At last, parameters estimation for the proposed Model III is given based on physical deterioration equations and condition ratings definition. In the pursuit of integrated health prediction, the practicability of the proposed Model III is evaluated in Section 6.3 using an application based on an open database and the existing literature.

Chapter 7 concludes the whole study. The capacity of the proposed models is clarified though only partial demonstration is implemented at the current stage due to time and data availability limitations. The possible future research directions are discussed as well. It is imperative to consider dynamic changing load and extend material variation.

Chapter 2: Literature review

2.1 Introduction

This chapter presents a thorough literature review of bridge health prediction. The prediction results are the base of bridge optimal maintenance practice. So far, owing to the increasing concern about economically sustainable maintenance practices, bridge management systems (BMS) have become more prevalent. Different BMS have been built up in many counties around the world. Moreover, the quality of decisions made by BMS largely depends on the accuracy of prediction results obtained from bridge deterioration models. Currently, various bridge deterioration models have been developed for BMS, most of which attempt to capture the uncertainties amongst bridge deterioration. This review begins with an introduction to BMS in Section 2.2. A comprehensive literature review about bridge health prediction approaches is carried out in Section 2.3. Other techniques closely related to bridges are reviewed in Section 2.4. The review is summarised in Section 2.5, where the open research areas are identified.

2.2 Bridge management systems (BMS)

2.2.1 Introduction of BMS

The concept of bridge management derives from the idea that decisions in design, construction, maintenance and repair can be made based on resource optimization [65]. Primarily, the existing bridges are considered by BMS to ensure that they achieve their design life, remain open to traffic continuously throughout their life and that their risk of failure is as low as possible [38, 39].

BMS have been developed to make decisions in design and material selection of materials, and to optimize maintenance, rehabilitation and replacement (MR&R) decisions for bridge networks under financial constraints [112]. Normally, BMS consider a wide range of activities that are commonly encountered in the day-to-day management of bridges such as inspection, assessment of load-carrying capacity and various types of testing. The essential parts of a BMS are bridge expected performance model, bridge expected demands model, and cost model for different options in structures and lost or lessened service[65]. The American Association of State Highway Transportation Officials

(AASHTO) outlined the guidelines and requirements for a BMS [1]. These guidelines recommend that a BMS should include five basic components: a database (data storage), cost models, deterioration models, optimisation models for analysis, and updating functions. Compared with other management systems, such as that of pavement-maintenance management, BMS have some unique characters [59]: (a) An extensive body of knowledge and data do not exist for bridges; (b) It is not meaningful to define a uniform unit for bridges similar to the lane-mile unit for roads; (c) There are more types and designs of bridges than pavements; (d) Various components deteriorate at differing rates; (e) Various bridge components of a bridge may be subjected to different environment factors; (f) The funding situation is more complex for bridges than for other entities; (g) Improvement activities are different from maintenance activities.

Currently, most of the existing BMS are based on bridge condition ratings, which are mainly estimated from visual inspection in the form of numerical ranking. From the definition of condition ratings, specifications for bridge damage related to serviceability are included. In condition ratings based BMS, structural safety is only directly mentioned in the worst condition rating, where a safety problem is suspected and an urgent intervention is anticipated [135]. Normally, these BMS are implemented in the project level and the network level. In the project level, BMS focus on individual bridges; whereas in the network level BMS deal with the management of bridge stocks [163]. Project level BMS mainly concentrate on aspects such as inspection, non-destructive tests, deciding maintenance requirements, appropriate prevention, remedial methods and monitoring strategies. Network level BMSs include the estimation of deterioration rate, prediction of future condition using Markov chain models, planning optimal maintenance programmes, prioritising maintenance and assessing the effectiveness of different maintenance strategies [163]. Network-level BMSs are more closely associated with the overall condition and serviceability of the stock and somewhat less concerned with the maintenance of individual bridges, however it is still important to note that most of the input information for a network-level algorithm is based on project-level inspection, assessments and test results [163]. Nonetheless, there are also other BMS designed based on structural reliability. Such reliability indicates the probabilities of failure which are formally defined through limit state functions. Therefore, the BMS are concerned more about bridge structure safety. Practical experience gained from UK Highways Agency’s bridge maintenance activities show that the main part of the work on an existing bridges depends on the load-carrying capacity (or structural reliability) of the bridge systems rather than the condition ratings of the bridge elements alone [54]. So far, a large amount of research has been conducted on structural reliability based bridge life-cycle management optimization [48, 53, 54, 83, 84]. Compared with the condition ratings based BMSs, the structural reliability based BMS mainly rely on quantitative and objective information rather than qualitative and subjective information [54]. By incorporating structural reliability, maintenance optimization and life-cycle costing, the new BMS overcome the limitations of the current BMS. For instance, maintenance actions are not necessarily related to condition ratings directly, and Markovian assumption is released as condition ratings are not used as the indicator of bridge damage. Although current BMS are mainly condition ratings oriented, some researchers are confident that future BMS will be shifted more towards structural reliability based approaches.

2.2.2 The existing BMS

To date, a number of BMSs have been developed in several countries, such as the Pennsylvania BMS in USA, the HiSMIS developed by High-Point Rendel in UK, the DANBRO developed by Danish, the BRISA owned by Sweden, Swiss bridge management system (KUBA-MS) [86] and so on. Among all the existing BMSs, two commonly used BMSs based on condition states are BRIDGIT [64] and Pontis [156].

BRIDGIT was developed from the National Cooperative Highway Research Project (NCHRP) with the aims to: (1) facilitate the organization of bridge data, the tracking of deterioration trends and repair performance; (2) provide clear, accurate and timely reporting; (3) rank bridge populations by a number of user-specified criteria; (4) allow the identification of critically deficient structures [3]. BRIDGIT assists in the establishment of bridge maintenance, rehabilitation and replacement program based on life-cycle costing and incremental benefit cost analysis. Markovian deterioration predictors provide necessary information for preservation considerations. A level-of-service (LOS) approach is employed for improvements, which also considers user costs associated with traffic accidents and detouring [64].

Another widely used BMS is Pontis that provides a systematic methodology for allocating funds, evaluating current and future needs of bridges and options to meet those needs, and recommending the optimal policy for each bridge in the context of overall network benefits, budgets and restrictions [59]. The essential parts of Pontis are a set of deterioration prediction and optimization models which derive their information from judgmental, engineering and economic models and various databases [59]. Pontis possesses the following key features [59]: (a) Pontis abandons Federal Highway Administration (FHWA) rating method in favour of requiring more detailed information on the conditions of all elements; (b) Maintenance optimization for dynamic process and static process (e.g. widening a bridge) are separated; (c) a set of bridge elements that can be used for building individual bridges are defined; (d) Predictive models start with elicited engineering judgement and become more accurate with time as the system is updated from real data; (e) Maintenance optimization is implemented by first considering the network of bridge elements and then combining the results to produce recommendations for individual bridges.

As a matter of fact, Pontis and BRIDGIT are often implemented in parallel. Since BRIDGIT can upload Pontis inspection data and can handle Pontis core elements, the system can offer a second independent analysis of their bridge networks and provide an independent set of recommended repair actions [64]. Additionally, both Pontis and BRIDGIT have considered effects of uncertainties associated with deterioration process as well as with maintenance interventions [123]. The failure dependencies among bridge deterioration are very common. For instance, the deterioration of a concrete deck accelerates when its bearings do not function properly. If the bearings freeze due to corrosion, the deck will be subjected to expansion and contraction stresses that cause cracking [142]. Pontis is able to take into account the failure dependencies determined by external environmental factors such as, traffic volumes, wind loads, and operating practice. In Pontis BMS, four standard environmental levels: benign, low, moderate and severe are defined [142]. In BRIDGIT, elements from paint and protective systems are treated separately for external environmental factors [64]. However, both Pontis and BRIDGIT can only account for interactions due to external environmental factors rather than internal factors, such as severe corrosion of one element. Therefore failure dependencies have not been adequately considered by the existing BMSs.

In Australia, although BMS are not well accepted, different BMS have been established by governments within different states. Compared with other countries, e.g. United States, the science of bridge management in Australasia has developed mainly on the initiative of the state road authority organisations [14]. For instance, in New South Wales, the PONTIS BMS has been adopted by Road and Traffic Authority New South

Wales (RTA NSW) who is one of the earlier BMSs users [94]. In Queensland, Department of Main Roads (DMR) has initiated a BMS called bridge asset management system (BAMS) which covers 2500 bridges and many thousands of major culverts [129]. In Victoria, VicRoads developed a management strategy to maintain Victoria's arterial bridges. The slow implementation of BMSs in Australia is due to that young bridge assets bring difficulty for the justification of a sophisticated BMS [14]. In addition, private organisations have been reluctant to commit funding where there is uncertainty about returns [14]. Nonetheless, with a boosted concern about bridge sustainable maintenance, it can be expected that BMS will be applied widely in Australia in the coming future.

2.3 Bridge health prediction approaches

In this section, a critical review about various types of bridge deterioration models is given. According to commonly accepted health indexes, the existing models can be classified into two groups: condition ratings based models and structural reliability based models. The former models concentrate on bridge serviceability and take advantage of the information resulting from visual inspection. Bridge inspectors estimate condition ratings based on their individual experience and judgements. Bridge deterioration in visible failure modes, such as corrosion, crack and spalling are included. Generally, condition ratings based models can be further categorised into three main groups, namely, deterministic models, stochastic process models and artificial intelligence models. Whereas the latter models focus on bridge structural safety that is normally defined through a number of limit state functions. Load carrying capacity of bridge structures in terms of strengths and stresses are considered. A detailed review for all the models is given in the rest of Section 2.3.

2.3.1 Models based on condition ratings

2.3.1.1 Definition of condition ratings

Condition ratings are usually quantified from good condition to failed condition and labelled with several numbers such as, 1, 2, 3, … , 9. As different BMS have their special requirements and concerns, there is no universal standard definition for bridge condition ratings. For instance, in National Bridge Inventory (NBI), bridge condition ratings shown in Table 2-1 are defined by Federal Highway Administration (FHWA) on a scale of **0-9**, in which **0** represents the worst condition rating while **9** represents the best. In Swiss BMS

(KUBA-MS) five condition ratings are defined with CS1 representing good condition rating and CS5 representing alarming condition rating [135]. Table 2-2 shows condition ratings definition used by Queensland Government, Department of Main Roads (DMR). The bridge condition ratings are defined with five condition ratings for the whole structure and with four condition ratings for the bridge elements, where CS1 denotes “Good condition”, CS4 denotes “Poor condition” and CS5 denotes “Unsafe condition” [129].

Table 2-1. Bridge condition ratings definition used in National Bridge Inventory (NBI) [51]

Condition States	Subjective Rating	Description
9	Excellent condition	—
8	Very good condition	No problem found
7	Good condition	Some minor problems
6	Satisfactory condition	Structural elements show some minor deterioration
5	Fair condition	All primary structural elements are sound but may have minor section loss, cracking, spalling or scour
4	Poor condition	Advanced section loss, deterioration, spalling or scour
3	Serious condition	Loss of section, deterioration, spalling or scour has seriously affected primary structural components. Local failures are possible. Fatigue cracks in steel or shear cracks in concrete may be present
2	Critical condition	Advanced deterioration of primary structural elements. Fatigue cracks in steel or shear cracks in concrete may be present or scour may have removed substructure support. Unless closely monitored it may be necessary to close the bridge until corrective action is taken
1	Imminent failure condition	Major deterioration or section loss present in critical structural components or obvious vertical or horizontal movement affecting structure stability. Bridge is closed to traffic but corrective action may put back in light service.
0	Failed condition	Out of service - beyond corrective action

Table 2-2. Bridge condition ratings definition used by Department of Main Roads, Queensland [129]

Condition States	Subjective Rating	Description
1	Good	Free of defects
2	Fair	Free of defects affecting structural performance, integrity and durability
3	Poor	Defects affecting the durability which require monitoring, detailed structural engineering inspection or maintenance
4	Very Poor	Defects affecting the performance and structural integrity of the structure which require urgent action as determined by a detailed structural engineering inspection
5 (whole structure rating only)	Unsafe	Bridge must be closed

2.3.1.2 Deterministic model

➢ Regression model

Deterministic models, the first applied bridge deterioration models in BMS, make the prediction by linking a number of relevant bridge deterioration factors to bridge condition ratings through a mathematical or a statistical formulation [112]. To estimate the parameters of deterministic models, normally, a large population of data records about condition ratings and affecting factors are needed. One typical deterministic model is regression model. The regression model used for bridge deterioration is statistical approach with the aims to find the relationship between condition rating and bridge age [72]. A third-order polynomial model was used to obtain the regression function of the relationship with the following formula [117]:

$$Y_i(t) = \beta_0 + \beta_1 t_i + \beta_2 t_i^2 + \beta_3 t_i^3 + \varepsilon_i \tag{2-1}$$

where $Y_i(t)$ is the condition rating of a bridge at age t, t_i is the bridge age, and ε_i is the error term. This formula was used by Jiang and Sinha [72] to predict average condition ratings of a number of bridges. The condition ratings of bridges are only dependent on bridge age.

Deterministic models are straightforward and can be easily used by bridge engineers and managers. However, they suffer from some critical limitations. Firstly, they neglect the uncertainties inherited with bridge deterioration. Instead, the deterioration process of a bridge is expressed in a deterministic way. Therefore, the prediction results cannot be accurate. Secondly, only the average condition ratings of a number of bridges can be derived rather than that of individual bridges, which has caused serious restriction to the application of deterministic models. Thirdly, deterministic models do not have the ability to incorporate newly observed condition data for prediction modification, which may eventually lead to unrealistic prediction results. Finally, deterioration dependencies among different bridge elements and effects of maintenance activities and environment effects cannot be taken into consideration by deterministic models.

2.3.1.3 Stochastic process models

- Markov chain

Stochastic process models capture time-varying uncertainties amongst bridge deterioration. They can be grouped into discrete time stochastic process models and continuous time stochastic process models. One of the most commonly used discrete time stochastic process models for bridge deterioration is the Markov chain model. A Markov chain can be seen as a special case of the Markov process which has a series of discrete random states. The assumptions of regular bridge inspection intervals and Markov property are held by a Markov chain. The Markov property assumes that the future condition ratings of a bridge or a bridge element do not depend on the history of its deterioration processes, but only depend on its last condition rating. Now Markov chain model has been largely applied in the state-of-art BMS, such as Pontis [156] and BRIDGT [64]. Based on transition probabilities matrix that indicates the probability deteriorating from one condition rating to another, Markov chain model predicts the probabilities of bridges in each condition state. Markov chain models can be divided into homogenous Markov chain model if transition matrix is not time-dependent or non-homogenous Markov chain model if transition matrix is time-dependent. Table 2-3 shows a typical transition matrix of order (5×5) for a deteriorating element without maintenance intervene. Given the initial condition vectors ($\mathbf{P}_0$) at time (T) and transition probability $\mathbf{P}$, the future condition vector ($\mathbf{P}_T$) at time (T) can be obtained as follows [124]:

$$\mathbf{P}_T=\mathbf{P}_0*\mathbf{P}^T \tag{2-2}$$

Table 2-3. Typical transition probability matrix without maintenance intervention

	1	2	3	4	5
1	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}
2	0	P_{22}	P_{23}	P_{24}	P_{25}
3	0	0	P_{33}	P_{34}	P_{35}
4	0	0	0	P_{44}	P_{45}
5	0	0	0	0	1

Transition matrix (transition probabilities) is normally estimated by using expert knowledge elicitation procedure, which requires the participation of experienced bridge engineers [155]. In addition, the Bayesian approach could be used to update the these probabilities [59].

Jiang and Sinha [72] applied Markov chain for bridge service life prediction. Morcous [109] used Markov chain to predict the condition performance of a bridge deck system. He also investigated the impact of regular inspection intervals and Markov property on the deterioration of bridge deck systems. The results indicated that various inspection periods may result in some errors in the prediction of bridge condition ratings, and Markov property (state independence) is acceptable. To consider the impact of environments on bridge deterioration, Morcous et al. [110] explicitly linked bridge elements with different environmental categories to different Markov chain models. Furthermore, a genetic algorithm (GA) was applied to determine the combinations of deterioration parameters that best fit each environmental category. Roselfstra et al. [135] proposed an alternative approach which took into account the physical phenomena when there were almost no inspection data for the worst and second worst condition states. In their paper, chloride-induced corrosion of steel reinforcement was modelled and simulated. The simulated results were mapped to condition ratings of Markov chain, and the transition matrices were calibrated as well.

Although the Markov chain model has been well accepted and has become so accepted and overcomes major shortcomings of deterministic models, it still attracts criticisms because of their limitations which affect accuracy of the prediction results [52, 99, 112]. First, Markov chain model assumes discrete condition ratings, discrete transition time intervals and time-independent transition probability, so the bridge deterioration cannot be modelled in a practical way. Second, because of Markov property, a Markov chain model cannot capture the history of bridge deterioration, though this may be acceptable in some cases. Third, bridge deterioration modelling is not implemented in an explicit way, and the latent nature of bridge deterioration is not recognized, either [99]. Fourth, deterioration dependencies among different bridge elements [142] cannot be effectively modelled by Markov chain. Fifth, observation data from visual inspection or condition monitoring cannot be incorporated by Markov chain model directly. Finally, Markov chain is not appropriate for modelling a complex system [168]. For the bridge system consisting of numerous elements, the total number of condition ratings for an adequate description of bridge system performance increases exponentially.

- Ordered probit model

Realising the latent nature of bridge deterioration, Madanat et al. [99] proposed an ordered probit model for Markovian transition probabilities estimation from condition data, which links the unobservable bridge deterioration to a vector of exogenous variables. The ordered probit model was originated from social sciences to deal with unobservable characteristics in the population [106]. For a bridge n in condition rating i, the continuous unobservable latent deterioration U_{in} is expressed as a linear function of a set of observable exogenous variables as follows [99]:

$$\log(U_{in}) = \boldsymbol{\beta}_i'\mathbf{X}_n + \varepsilon_{in} \tag{2-3}$$

where $\boldsymbol{\beta}_i'$ is a vector of parameters to be estimated; $\mathbf{X}_n$ is a vector of exogenous variables for bridge n; and ε_{in} is random error. With the assumption of the existence of an underlying continuous unobservable random variable, the ordered probit model is able to capture the latent nature of infrastructure health performance [99]. Linkage between bridge deterioration and relevant explanatory variables can be modelled explicitly. However, Bulusu and Sinha [24] argued that issues related to panel data should also be considered for transition probabilities estimation. Moreover, the ordered probit model always involves plenty of analytical manipulations.

➢ Binary probit model

In order to incorporate panel data, Bulusu and Sinha [32] proposed a binary probit model which considered the issue of state dependence and heterogeneity. During time period t, the continuous unobserved latent deterioration $U(i,t)$ for bridge i is presented as follows[24]:

$$U(i,t) = \mathbf{X}(i,t)\boldsymbol{\beta}' + \gamma Z(i,t-1) + \varepsilon(i,t)$$

$$Z(i,t) = \begin{cases} 1\ if\ U(i,t) > 0 \Rightarrow drop\ 1\ state \\ 0\ \ otherwise \Rightarrow stay\ in\ current\ state \end{cases} \quad (2\text{-}4)$$

Where $\mathbf{X}(i,t)$ is a vector of explanatory variables for bridge i; $\boldsymbol{\beta}'$ is a vector of parameters to be estimated; $\varepsilon(i,t)$ is random error term; γ is scalar coefficient for condition rating in previous time period; $Z(i,t-1)$ is transition indicator in previous time period. Like ordered probit model, plenty of analytical manipulations are involved in this approach.

➢ Bayesian approach

Additionally, Bulusu and Sinha proposed a Bayesian approach that combines expert data and observed data to update transition probabilities [24]. The priori transition probabilities are assumed to follow the Dirichlet distribution and estimated from expert knowledge. Observed data are assumed to follow a multinominal distribution. When newly observed data ε_i is available, the mean posterior transition probabilities $E(P_{i,j}|\varepsilon_i)$ are estimated as follows [24]:

$$E(P_{i,j}|\varepsilon_i) = c_i P_{i,j}^0 + (1-c_i)\frac{\varepsilon_{i,j}}{n_i} \quad (2\text{-}5)$$

$$c_i = \frac{\alpha_{i,0}}{\alpha_{i,0}+n_i} \quad (2\text{-}6)$$

$$\alpha_{i,0} = \sum_j \alpha_{i,j} \quad (2\text{-}7)$$

where $P_{i,j}^0$ is prior transition probabilities; $\varepsilon_{i,j}$ is the newly observed transitions from condition state i to condition state j; n_i is the total number in condition state i ; c_i is the proportion of weighted assigned to the priori mean transition probabilities; $\alpha_{i,0}$ is the priori total number in condition state i ; $\alpha_{i,j}$ is the priori transitions from condition state i to condition state j. Compared with the binary ordered model, the implementation of the Bayesian approach is more cost-effective.

- Semi-Markov model

However, one major drawback of Markov chain that has been questioned widely and yet to be solved is that the transition probability from an initial condition rating to the next condition rating does not relate to the resident time of the initial condition rating. To deal with this drawback, a more general stochastic model called semi-Markov process was investigated for bridge deterioration modelling [108, 140]. A semi-Markov process is a class of stochastic process which moves from one state to another with the successive states visited forming a Markov chain. The process stays in a particular state for a random length of time the distribution of which depends on the state and the next to be visited [136]. Semi-Markov model assumes the resident time of an initial condition rating follows a specified distribution. Thus, being dependent on the time spent on the initial condition rating, the transition probability to the next condition rating becomes more realistic. Moreover, semi-Markov model releases the assumption of discrete transition time interval in Markov chain. Some information related to semi-Markov can be found in the literature [99, 131].

- Continuous stochastic process models

Although semi-Markov model has released some assumptions of Markov chain, it still depends on discrete states. To avoid subjective discretization of condition ratings based only on engineering judgement, stochastic processes of continuous states such as Gamma process and Brownian motion with drift (Gaussian process) have been proposed as alternatives for modelling deterioration of infrastructures [159, 161, 162]. A gamma process is a continuous stochastic process $\{X(t); t \geq 0\}$ with independent non-negative increments $X(s+t) - X(s)$ having a gamma distribution. The increasing function $\eta(t)$ is the shape function, while $\xi > 0$ is the scale parameter. The monotonous property of a Gamma process makes it more attractive for modelling non-reversible deterioration process. On contrary, Brownian motion with drift is a continuous stochastic process $\{X(t); t \geq 0\}$ with the independent increment $X(s+t) - X(s)$ following a Gaussian distribution with mean ηt and variance$\sigma^2 t$, for all $s, t \geq 0$. The η and σ^2 are called the drift parameter and the diffusion parameter, respectively. A Gaussian process holds the characteristic that the structure resistance alternatively increases or decreases. Therefore, compared with the Gamma process, Brownian motion with drift (Gaussian process) is not appropriate for modelling deterioration process. Since the Gamma distribution and Gaussian distribution both belong to the class of infinitely divisible distributions, they are adopted as the distribution of independent increments of continuous stochastic process [25]. Samali et al. [139] investigated the feasibility of the Gamma process on bridge deterioration modelling. Based on simulated data, the Gamma process model showed its ability to capture the temporal uncertainties of the deterioration process effectively. Furthermore, the authors mentioned that other continuous states stochastic processes, such as lognormal diffusion process, could be the candidate as well [139]. Because inspection measurements generally consist of cumulative amounts of deterioration, the advantage of stochastic process for modelling the uncertainty in the cumulative amount of deterioration is evident [160]. Another advantage of stochastic deterioration processes is that the modelling of inspection is rather natural and realistic [160]. However, it is generally difficult to build up a bridge system model based on the stochastic process deterioration models of a number of bridge elements. In addition, the stochastic processes models cannot handle the deterioration dependencies amongst different bridge elements.

2.3.1.4 Artificial intelligence models

Artificial intelligence methods, such as, artificial neural networks (ANN), case-based reasoning (CBR), fault tree and Bayesian networks (BNs) have also been applied to bridge deterioration modelling.

- Artificial neural network

Artificial neural network (ANN) is a computational model that resembles some of the properties of brains: it consists of many simple units working in parallel with no central control. The connections between units have numeric weights that can be modified by the learning element[137]. During the past two decades, ANN has been comprehensively applied to bridges and other infrastructure components. Sobanjo [144] utilised ANN method for bridge deterioration modelling, in which the bridge age (in years) was chosen as the input while condition rating of the bridge superstructure was chosen as the output. Lee et al. [94] considered that there was insufficient historical condition ratings data of bridge elements for current bridge deterioration models. They proposed an ANN based prediction model which related the missing bridge condition ratings data to several non-bridge factors including local climates, number of vehicles and population growth in the area surrounding the bridge. The ANN method was also used to perform fuzzy inference for condition rating evaluation of concrete bridges by Kawamura and Miyamoto [78]. The ability to refine the knowledge base by means of back-propagation method was emphasised by the authors. Other literatures about application of ANN in bridge deterioration modelling can be found [105, 157, 175].

Although ANN has automated the process of finding the polynomial that best fits a set of data points, it still shares the problems of deterministic models [112]. One significant limitation of ANN is that the uncertainties associated with bridge deterioration cannot be captured. For training purpose, a large number of data are needed. Updating the model with newly observed data is rather difficult [112]. Additionally, human cannot construct or understand neural network representations because the calculations carried out by the network is not expressed in a semantically meaningful way [91].

- Knowledge-based system

The knowledge-based systems are also known as a rule-based system and expert system. To build a knowledge-based system, the elicitation of a wide range of experts is of importance since the knowledge-based system has to be established based on a large number of carefully crafted rules. While knowledge-based systems have succeeded in the area of medicine, the application of this type of method in bridge deterioration modelling is still constrained. Denmark developed knowledge-based systems for optimal reliability-based inspection and maintenance of reinforced concrete bridges [153]. Two modules, BRIDGE1 and BRIDGE2, were utilised to assist inspection and to analyse inspection results, respectively. For steel bridges, a knowledge-based system was proposed by Furuta et al. [68]. The case-based reasoning (CBR) method was employed to select repairing and retrofit methods for fatigue damage.

One disadvantage of a knowledge-based system is that the rules are only applicable to some specific systems. The designers of the model must have extensive knowledge of the subject system in order to achieve proper representation of the system and all of its uncertainty through the rules and certainty factors that incorporate a calculus of uncertainty [137].

- Case-based reasoning (CBR)

Case-based reasoning (CBR) is a kind of knowledge-based system which looks for previous cases (examples) that are similar to current problem and reuse them to solve problem[112]. These cases, which are stored in the so-called case library, are defined as instances that record problem definitions and their corresponding solutions [112]. Morcous et al. [111, 112] proposed a CBR based approach which can take advantage of inventory, inspection and maintenance data of BMS database to predict bridge deterioration. The architecture of the proposed CBR method is shown in Figure 2-1. This CBR approach enabled the representation of deterioration dependencies amongst different bridge elements. Nonetheless, the success of this CBR approach largely depends on the size and coverage of the case library, and correctness and availability of expert knowledge [112]. Furthermore, as a knowledge-based expert system, the CBR approach does not facilitate probabilistic representation of bridge deterioration with inherited uncertainty, but only represents bridge deterioration in the form of certain rules.

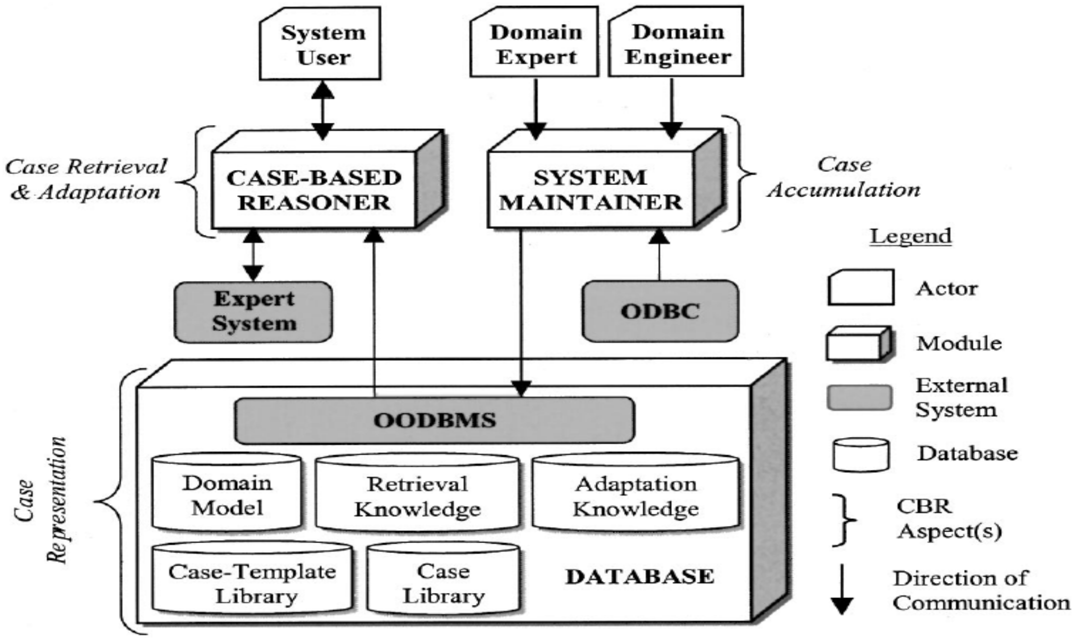

Figure 2-1. Architecture of CBR for modelling infrastructure deterioration [111]

- Fault tree

Fault tree, introduced in 1961 by Bell Telephone Laboratories, is a logic diagram consisting of a top event and a structure delineating the ways in which the top event can occur [173]. A fault tree diagram is also a systematic method of identifying faults and their interactions in a complex system [75]. The original purpose of fault tree analysis is to evaluate reliability of different designs. However, fault trees can also be used for the following:

1. to assess the probability of failure for the system (or top event), to compare design alternatives,

2. to identify critical events that will significantly contribute to the occurrence of the top event, and
3. to determine the sensitivity of the failure probability of the top event to various contributions of basic events [75].

Figure 2-2 presents one simple example of fault tree that is a combination of top event, basic events, intermediate events and logic gates.

As a systematic approach, the fault tree can be employed to derive system reliability based on the estimation of independent components [33]. So far, because most of the existing models failed to address the issue of element interactions, several researchers have resorted to fault tree for bridge deterioration modelling [75, 92, 93, 142]. Sianipar and Adams [142] applied the fault tree to model deterioration dependencies among bridge elements. The authors introduced deterioration dependencies phenomena in bridge deterioration and utilised the fault tree approach to represent and measure these dependencies. A case study about accelerated concrete bridge deck deterioration was given. LeBeau and Wadia-Fascetti [92, 93] argued that current BMS did not tackle deterioration dependencies appropriately. A fault tree model was built up based on hierarchical decomposition of a bridge. Probability of each basic event was acquired by interviewing seven bridge engineers and inspectors. The integration of fault tree analysis into BMS did provide the missing link between component condition and system performance [92]. Johnson [75] used fault tree to analyse the failure of a bridge due to scour and other geomorphic channel instability. Given three examples, the fault tree analysis showed the advantages, including no need of quantitative knowledge about deterioration dependencies, failure probability estimation of top event based on the probabilities of individual events, and no requirements of an exact value of probability[75].

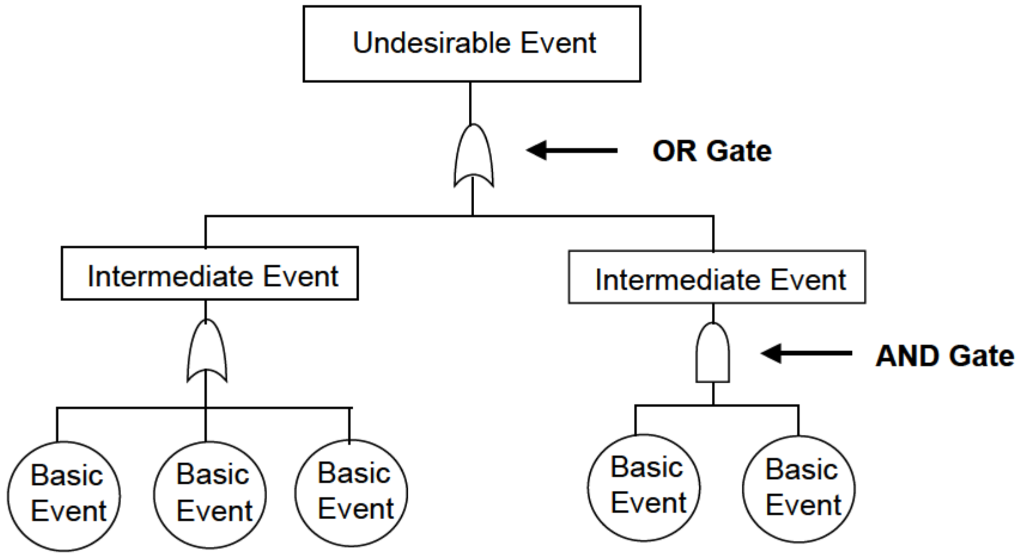

Figure 2-2. A simple example of fault tree [9]

One advantage of fault tree is its ability to unveil logical interrelationships of the bridge system both visually through the layout of the tree and mathematically through the Boolean algebra [91]. The bridge can be modelled by fault tree in its entirety including element interactions, redundancy, deterioration mechanisms, such as corrosion and fatigue, and environmental factors [91]. However, fault tree is also criticised for some drawbacks. For example, the construction of a fault tree can be laborious and time consuming. The basic events have to be independent which may not be practical. In addition, events related to Fault tree can only be modelled with binary states (0, 1) in different probabilities. Dependent failures, such as sequence failures are beyond fault tree. For common cause analysis, some nodes need to be duplicated. Dynamic behaviour of deterioration processes cannot be captured by fault tree, either.

- Binary recursive partitioning (BRP)

Similarly, binary recursive partitioning (BRP), which is actually a kind of classification tree, has been applied for deterioration modelling of a bridge deck by Pittou et al. [101]. The proposed method involved four basic modelling steps: tree building, stopping tree building, tree pruning, and optimal tree selection [87]. The author claimed that because BRP is a nonparametric method, it possesses several advantages, such as less stringent data requirements, no assumption for particular distribution, quick answer of explanatory variable selection, a practical means for data objectivity and smaller management of data from a smaller population. However, BRP suffers from the limitations of classification tree. It is difficult to obtain quantitative results from BRP.

- Bayesian Networks (BNs)

Bayesian networks (BNs) are directed acyclic graphs (DAG) formed by the variables (nodes) together with the directed edges, attached by a table of conditional probabilities of each variable on all its parents [71]. A BN encodes the probability density functions (PDFs) governing a set of conditional probability functions (CPFs) [89]. As powerful graphical models to describe conditional independence and to analyse probable casual influence between random variables, BNs have been widely used in areas, such as, marketing [15], industry [67, 169, 179], health [158], risk management [42, 43, 55, 82], reliability and maintenance [19, 20, 27, 89], ecosystem and environmental management [76].

Compared with other applications, BNs are not as common in bridge deterioration modelling. Currently, only a few researchers have applied BNs in the context of bridge deterioration modelling. Sloth et al. [143] proposed a Bayesian probabilistic network within which condition indicators work as a basis for bridge management decision making. The condition indicators for individual bridge component were formulated as time-variant condition probabilities which are based on their parents. The uncertain factors, such as concrete mix, exposure condition, and reinforcement in concrete bridge were shown by casual relationship under the Bayesian probabilistic framework. Attoh-Okine and Bowers [10] have investigated bridge deterioration modelling through BNs with the argument that fault tree is more suitable for immediate catastrophic failure rather than normal failure. For deterioration dependencies of bridge elements, a bridge was simply decomposed into deck, superstructure and substructure. The failure probability of the whole bridge system was based on the failure probabilities of bridge elements. However, their research is limited to a simply mapping of a fault tree model into a BN model. Therefore, their model does not seem to have greatly improved modelling performance as far as deterioration dependencies amongst different bridge elements are concerned. The advantages of BN could be utilized more in depth in their model. Lebeau [91] developed a novel load rating model for a prestressed concrete bridge beam element based on BNs, which can integrate bridge routine inspection into load-rating processes. The bi-directional ability through BNs has been emphasized to execute forward and backward evidence propagation.

Compared with fault tree and Markov chain, BNs show a number of advantages. BNs can deal with dependencies among elements of a complex system without holding deterministic and/or binary relationship (AND, OR) between nodes. Therefore, the constraints of fault tree, such as binary states and independent basic events can be removed. In addition, BNs can model a complex system in a compact representation of all the variables through localized network clusters, thereby avoiding the "state space explosion" problem of Markov chain. BNs have the updating ability when new information is available, and information can be updated from both system-level and element-level. The expert knowledge can be integrated into BNs as prior knowledge to tackle the situation when there are incomplete data. The partial observation can be handled by BNs with observable nodes. Furthermore, BNs are possible to combine different sources of information and individual methods to provide a global reliability assessment. In other words, BNs can act as a unifying tool to benefit maximally from the strengths of individual methods [55]. As an intuitive modelling tool, BNs allow users to learn casual relationship by offering a graphical data structure that captures the dependencies between variables. Therefore, BNs can be used to communicate with engineers and technicians. Besides, the extension of BNs (DBNs or DOOBNs) can be easily utilized to account for the temporal variability. To implement BNs, there has been a large number of commercial software available. Examples include Hugin (http://www.hugin.com/), BayesianLab (http://www.bayesian.com/) and Netica (http://www.norsys.com/).

Though BNs have presented many advantages, there are still a few limitations about BNs. The assumptions, such as the first-order Markov property, stationary structure and stationary transitions are all always held in most applications. Moreover, since BNs are directed acyclic graphs (DAG), the interactive relationship between two nodes never exists. BNs also do not allow continuous parent variables to have discrete child variables because of the limitations of current computational algorithms.

2.3.2 Model based on structural reliability

2.3.2.1 Introduction of structural reliability

Structural reliability theory focuses on whether the demand applied to a structure exceeds its capacity or not. For a bridge, demand refers to different types and magnitudes of applied loads including dead load, live load and wind load. Capacity refers to strength, fatigue, moment capacity and so on. To address uncertainties associated with material strength/stress, bridge geometry, mechanical loading, environmental stressors, all the variables related to demand and capacity should be represented as random variables rather than deterministic variables. Structural reliability theory provides formalized evaluations of the probability of failure when capacity is less than demand or equal to demand. However, since "failure" means different things to different people, a concept of a "limit state", which may represent any level of failure from collapse of all or part of a structure (ultimate safety) to disruption of normal use (serviceability), is adopted for failure definition [107].

A limit state is a boundary between desired and undesired performance of a structure and is often expressed mathematically by limit state functions or performance functions [121]. Generally, there are three types of limit states: ultimate limit state related to the load-carrying capacity for bending or shear, serviceability limit state related to gradual deterioration excess deflection and vibration or maintenance costs, and fatigue limit state related to the loss of strength and eventual damage under repeated loads [121]. Each type of limit state is associated with a set of limit state functions that are formulated for certain conditions. With different concerns of failure, the probabilities of failure are calculated based on different limit state functions. If R represents the total capacity of a structure and Q represents all demands, a basic limit state function can be expressed as

$$G(R, Q) = R - Q \tag{2-8}$$

If $G \geq 0$, then the structure is considered safe, otherwise the structure is failed. Based on this function, the basic structural reliability equation is given [107] where the probability of failure is the probability that Q are greater than or equal to R.

$$P_f = P(R - Q \leq 0) = P\big(G(R, Q)\big) \leq 0 = \int_{-\infty}^{+\infty} F_R\,(x) f_Q(x) dx \tag{2-9}$$

where

P_f = probability of failure

R = capacity (resistance)

Q = demand (type, magnitude)

G = limit state function

$F_R(x)$ = probability that the actual resistance R is less than some value x (representing time)

$f_Q(x)$ = probability that the load effect Q acting in the member has a value between x and $x = \Delta x$ in the limit as Δx approaches 0.

As the failure probability is very small, it is convenient to express it as a reliability index defined as [107, 121]

$$\beta \equiv -\Phi^{-1}(P_f) \tag{2-10}$$

where Φ is standardised normal distribution. Generally, if R and Q are uncorrelated random variables, the reliability index β can be quantified as [107, 121]

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{2-11}$$

where μ_R and μ_Q are the mean values of R and Q, and σ_R and σ_Q are the standard deviation of R and Q, respectively. Although Equation 2-9 appears to be to be uncomplicated, it is always difficult to evaluate those integrals [121]. This is because that integration requires special numerical techniques, and the accuracy of these techniques may not be adequate [121]. Therefore, some approximate methods are adopted in practice, such as First-order reliability method (FORM), Second-order reliability method (SORM), Monte Carlo simulation method and Response Surface Modes (RSM).

Nowadays, structural reliability theory has been comprehensively applied for bridge management. Thoft-Christensen [153] introduced the overall development of optimal structural reliability-based inspection and maintenance of reinforced concrete bridges in Denmark. A number of involved areas were listed which include reliability assessment of deteriorating bridge over whole life, bridge codes and design calibration, optimal maintenance strategies, expert bridge management systems (BMS) and decision tools. Das [40] presented structural reliability based bridge management procedures in UK. The new procedures were considered to be the pioneering technique for bridge assessment. It was expected that cooperation with bridge engineering community was necessary for optimal bridge management. Moreover, many structural design codes have been based on structural reliability, An example is Load and Resistance Factor Design (LRFD). Nowak et al [120] studied and compared three different design codes: Spanish Norma IAP-98, Eurocode and AASHTO code based on structural reliability. It was found that AASHTO tends to be the most tolerant code while Eurocode the most conservative code. Currently, load rating model is commonly used in the practice regarding load-carrying capacity in bridge safety evaluation [6]. Load rating is calculated from the ratio of reserve capacity to the applied live load. An investigation of the relationship between structural reliability and load rating was conducted by Akgul and Frangopol [5]. It was shown to be:

$$\beta = \frac{\mu_{(RF-1)Q_{LL+1}}}{\sigma_{(RF-1)Q_{LL+1}}} \tag{2-12}$$

where β means the structural reliability index; *RF* is the load rating factor; Q_{LL+1} represents the load effect including the impact; μ and σ are the mean values and standard deviations of product of *RF* and Q_{LL+1}, respectively.

2.3.2.2 Structural reliability methods

FORM is an approximate method in which the limit state function at any point is linearized through first-order Taylor series expansion at that point [107, 121]. After linearization, a straight line is generated which denotes the tangent to the limit state function at the point of interest. Normally, the mean value point is chosen as the expansion point of interest. With the newly obtained linear limit state function, reliability index is calculated based on an equation similar to equation 2-11. Compared with other methods, FORM possesses the best compromise between solution accuracy and computation economy. As a result, FORM has been widely used in bridge structural reliability problems [26, 31, 69]. However, when the limit state function has strong non-linearity and the estimation of FORM is not sufficiently accurate, SORM is chosen instead. SORM is also based on Taylor series expansion but uses the second order term as well, which makes it suitable for nonlinear limit state functions. Moreover, because SORM is more computationally intense, generally it yields better estimates of failure probability than FORM. As an alternative, Monte Carlo simulation (MCS) is such a special technique that can be used to generate some numeric results without actually doing any physical testing[121]. By largely sampling from the probability distribution of variables, the failure probability is simply estimated from the ratio of failure number to total sampling number. For example, regarding the basic limit state function in Equation 2-8, the failure probability P_{f} is estimated as follows:

$$P_{\mathrm{f}} = \frac{N_{\mathrm{f}}}{N} \tag{2-13}$$

where N is the total number of samples through simulation and N_{f} is the number of samples that satisfy $G(R,Q) \leq 0$. MCS is usually associated to some finite element (FE) analysis software such as ANSYS, ADINA and SAP for reliability estimation. Normally, MCS needs a huge number of samples especially when structural reliability problems involve lower probability of failure. Therefore, intensive computational efforts are needed.

As the number of samples increases, the standard deviation decreases. The required sampling size can refer to the two following rules[9, 63]:

$$A = 200\sqrt{\frac{(1-P_\mathrm{f})}{N(P_\mathrm{f})}} \tag{2-14}$$

$$N = 10/P_\mathrm{f} \tag{2-15}$$

where A is the acceptable percent error, N is sampling size and P_f is the failure probability. It has to be noticed that when it comes to structural reliability of bridge system, larger computational efforts are unavoidable. Therefore, MCS becomes less effective for structural reliability analysis of complex structures.

Response Surface Method (RSM) is a statistical regression analysis method developed by Box and Wilson [21]. To date, RSM has been successfully used in areas such as physics, engineering, medical science and sociology for the probabilistic evaluation of a system [119]. Nowak and Cho successfully applied RSM to an arch bridge for the analysis of its structural reliability [119]. Compared with Monte Carlo simulation and FORM/SORM, it was claimed that RSM can be practically applied to the reliability analysis of complex structures, and it is more appropriate to evaluate extremely smaller failure probability as the derivative terms of implicit limit state functions can be handled easily [119]. However, the accuracy of the estimated probability by RSM is largely dependent on the quality of selected parameters [62]. RSM also involves intensive computational efforts owing to a large number of related random variables.

2.3.2.3 Structural reliability of bridge systems

As a bridge is composed of a number of bridge elements, it is necessary to obtain the structural reliability of each bridge element before evaluation of structural reliability of bridge system occurs. Normally, a bridge element may suffer from multiple failure modes such as shear, moment and fatigue. Therefore, several limit state functions should be developed. With regard to a typical bridge girder, the limit state functions can be expressed from various aspects including bending moment capacity, shear capacity, buckling capacity deflection, vibration and accumulated damage condition [122]. Szerszen et al [150] established fatigue limit state functions for bridge girders in order to implement a fatigue reliability analysis for steel girder bridges. Tabsh and Nowak [151] developed a reliability evaluation procedure for noncomposite and composite steel girders, reinforced concrete T-beams, and prestressed concrete girders as well as the whole girder bridges based on moment limit state functions. Imai and Frangopol [69] formulated limit state functions for main bridge elements: main cable, hanger rope and stiffening girders, in order to estimate system reliability of suspension bridges. Considering wind-induced stability failure, Cheng and Li [31] carried out reliability analysis for long span steel arch bridges. With assumed wind loads, limit state functions were constructed based on overall estimated bucking load and minimum permissive bucking load.

Generally, bridge elements are constructed in series/parallel relationship to represent the whole bridge system [26, 47, 174]. In addition, statistical correlation among bridge elements was taken into account for structural reliability evaluation of bridge system. In practice, some bridge systems cannot be grouped as either series systems or parallel systems. Statistical correlation can help model a bridge system consisting of many interconnected bridge elements in a practical way. The descriptions about bridge system reliability with the consideration of statistical correlation in details can be found from [47, 121]. Furthermore, FE models are able to take into account load redistribution which can results in more practical reliability estimation. Generally, the whole calculation of structural reliability from bridge elements to the whole bridge was implemented with the help of finite element (FE) models [26, 31, 36, 96]. In these FE models, failure probabilities were often evaluated through the structural reliability methods of Monte Carlo simulation or RSM. With the development of a software interface strategy, Cheng and Li [31] attempted to integrate FORM/SORM methods into FE models.

2.3.2.4 Time-variant structural reliability

To facilitate bridge life-cycle management optimization, it is essential to model structural reliability of bridge systems in a time-dependent way. To date, a great deal of research has been conducted to model time-variant structural reliability. Thoft-Christensen [154] presented the time-variant structural reliability calculation for concrete bridges. Ultimate and serviceability limit state functions were established for reliability evaluation. To consider the bridge temporal deterioration, corrosion deterioration models under three deterioration levels were developed. Similarly, Czarnecki and Nowak [36] developed the calculation procedures for time-variant reliability estimation of steel girder bridges concerning corrosion. Considering three environmental levels and time-variant random variables, limit state functions for ultimate capacity and serviceability were established. Cheung and Kyle [32] used time-variant reliability as a measure of bridge performance for bridge maintenance decision making. For reinforced concrete slabs, five types of limit state functions considering flexural strength, punching shear, deflection, delamination and wearing surface were developed. Kim et al [80] displayed the evaluation procedures for fatigue reliability of an existing steel railroad bridge in both deterministic and probabilistic ways. According to different loading models, three procedures were developed: simplified, probabilistic and deterministic procedures. Comparisons among the results of fatigue reliability via the three procedures were also given.

In summary, the time-variant reliability of bridge system is generally evaluated through bridge elements reliability over time. The continuous deterioration of bridge resistance and dynamic changed load contribute to the decrease of bridge structural reliabilities eventually.

➢ Resistance

For bridge resistance deterioration, corrosion is always the main reason. Normally, only uniform corrosion is considered as the cause of reduction of bridge load carrying capacity. So far, there have been several papers addressing deterioration of bridge resistance as a result of corrosion [32, 36, 46, 47, 49, 104, 119, 130, 152, 154, 165]. Generally, two types of materials are considered: reinforced concrete and steel. For reinforced concrete, chloride-induced corrosion is the most frequent type and has a severe effect on the loss rebar cross-sectional area. As chloride ions penetrate deeper into concrete, it normally takes some time before the chloride-induced corrosion actually emerges on the surface of rebar.

The corrosion initiation time is a random variable depending on several factors such as environmental and geometrical factors. The most commonly used corrosion initiation model for concrete bridge deterioration is based on Fick's second law. As a partial differential equation, Fick's second law models the chloride diffusion process [146]. By using Crank's solution [34], the time to reach the critical level of chloride concentration can be estimated. This chloride diffusion model for corrosion initiation has been widely used [4, 44, 47, 91, 154]. Some improved chloride induced corrosion initiation models were also presented by Rafiq et al. [130], Vu and Stewart [165]. Moreover, the loss of rebar cross-sectional area is determined by the instantaneous corrosion rate, which is deemed to be a random variable. An improved corrosion rate model was given by Vu and Stewart [165]. The time-dependent rebar cross-sectional areas were used to update reliability results over time.

For steel material, corrosion is also one of the most important causes of deterioration [47]. Corrosion emerges immediately after the coatings lose their function. Cross-sectional areas of bridge elements decrease over time because of corrosion, which can significantly affect bridge safety. So far, an accurate prediction model for corrosion in steel has not existed. Instead, some empirical functions have been derived. One of the successfully applied models for structural safety evaluation purpose is an exponential function developed by Albrecht and Naeemi [8]. Based on the environment and the type of steel, corrosion penetrations can be predicted. These penetrations will lead to reduced cross-sectional areas which contribute to the decrease of bridge resistance. By updating the resistance of bridge elements each time, structural reliability of bridge elements and bridge system can be modelled temporally.

- Load

Dynamic load plays an important role in time-variant reliability of bridge system. Normally, loads include dead load, sustained/dynamic load, loads caused by winds, snow, earthquakes and tornadoes [121]. Among them, live load and dead load are two main load components which are usually modelled. Normally, dead load is seen as static load following a time-invariant distribution, for example, normal distribution. Live load is generally expected to increase annually and to follow a time-variant distribution. Quantifying live load is difficult due to plenty of uncertain parameters. Among them are the span length, axle loads, axle configuration, gross vehicle weight, position of the vehicle on the bridge (transversely and longitudinally), traffic volume, number of vehicles on the bridge (multiple presence), girder spacing, and mechanical properties of structural members [36]. So far, there are a few live load models available. Examples are Nowak live load model [118], Ghosn live load model [57] and AASHTO specifications [2]. AASHTO specifications address the modelling of live load in a time-invariant way. Nowak live load model is more common in current time-variant modelling of live load and it has been adopted by many researchers [36, 147, 165]. In that live load is formulated as a time-variant normal distribution. As the distribution approaches a Type I extreme value distribution, the mean and standard deviation of the live load in the future time are predicted based on measured traffic data.

The live load effects can also be estimated from SHM data [95, 123]. First, the data were used to derive the parameters of extreme value distributions, and then the future extreme values of SHM data were obtained. The ratio between future extreme values and current extreme values was used to qualify the increase of live load in the future. The time-variant live load and bridge resistance models were both utilised to generate time-dependent reliabilities of bridge elements. Bridge system reliability was calculated each time from reliabilities of bridge elements based on serial/parallel logical relationship.

➢ Observation

Nowadays, with the development of monitoring techniques such as SHM and NDT, more and more observed information correlated to bridge deterioration is available. This information reflects the actual bridge deterioration, and it is of significance to incorporate it into bridge time-variant structural reliability estimation.

Liu et al [96] incorporated SHM data into fatigue reliability evaluation of retrofitted steel bridge. An FE model was constructed to identify the critical location for potential fatigue cracking re-initiation. If the identified location was different from the sensors location, a spatial adjustment factor (SAF) can be applied to adjust the monitored data. At last, the modified data was used to derive the random variables related to fatigue reliability estimation. Marsh and Frangopol [104] investigated temporal and spatial variations of corrosion sensor data for the reduction of uncertainties in reliability estimation of reinforced concrete bridge deck. Based on empirical spatial and temporal relationships, corrosion sensor data were simulated for multiple critical sections all over the reinforced concrete bridge deck. An improved reliability model was developed to provide better estimation of bridge deck reliability. Catbas et al [26] estimated bridge reliability by incorporating long term environmental monitoring data. The monitoring data were utilised to calibrate an FE model of a long span truss bridge so that the uncertainties related to different environments can be minimised. Then the calibrated FE model was further used to estimate bridge system reliability based on the parallel and/or series relationship.

Bayesian updating was adopted by some researchers to improve prediction accuracy of bridge reliability [45, 130]. With inspection and monitoring data, Bayesian updating can effectively reduce the uncertainties associated to bridge deterioration modelling. It also facilitates the combination of observed data and expert judgement for more accurate prediction results. Monitoring data from SHM were used to estimate parameters of an extreme distribution for time-variant live load model [95, 123]. Zheng and Ellingwood [177] investigated the application of NDT in time-variant reliability estimation. Two types of uncertainties in NDT: flaw detection and flaw measurement error, were characterized by probability of detection (POD) and a linear regression function, respectively. Two instances based on magnetic particle and ultrasonic inspections were also used to illustrate the applications of NDT for time-dependent fatigue reliability.

Load testing techniques can be utilised to update bridge resistance. Stewart and Val[148] studied the role of proof loading in reliability-based decision analysis of aging bridges. The reliability of bridge can be updated whenever proof loading was carried out. Full-scale load (proof load) testing can not only evaluate the load carrying capacity of existing bridges, but also provide valuable information about structural behaviour that are related to validation of design assumption and construction quality [148] .

Structural reliability has been commonly applied to bridge performance deterioration. It is the basis of many structural design codes including Load and Resistance Factor Design (LRFD). It provides uniform and consistent estimation criteria for all types of bridges in terms of reliability index. Compared with condition ratings based models, structural reliability based models account for load-carrying capacity of bridges and failure probability is calculated objectively without any subjective condition assessment. In addition, failure modes related to strength and stress of bridge resistance are formulated in an explicit way. Although most of the current BMS are based on condition ratings, some researchers believe time-dependent reliability will be the direction of future BMS [84].

Nevertheless, structural reliability also shows some difficulties and disadvantages. For example, although several approximation methods are available, it is still a difficult task to evaluate structural reliability accurately since these methods might not reflect a realistic portrayal of the failure. Moreover, structural reliability mainly focuses on safety performance of a structure in term of strengths and stresses rather than serviceability in visual terms. In representation of a bridge system, a structural reliability model is mainly based on a combination of parallel and/or series bridge elements, which is not appropriate to address the deterioration dependencies among different bridge elements. Therefore, Frangopol et al. [52] pointed out that it was better to establish a generally acceptable and consistent methodology for probabilistic modelling of deterioration process of structural performance in terms of both condition ratings and reliability. Researchers have realised that BNs would be a proper candidate for integrated bridge health prediction in both serviceability and safety aspects [100, 132].

2.4 SHM and NDT

Besides the bridge health prediction approaches, there are other techniques closely related to bridge health management. Typically among them are structural health monitoring (SHM) and non-destructive testing (NDT). SHM is defined as an on-line system tracking the vibration/dynamic response of a structure along with inputs or monitoring of interest physical value, if possible, over a sufficiently long duration to determine anomalies, to detect deterioration and to identify damage for decision making [7]. To date, the application of SHM technology for surveillance, evaluation and assessment of existing or newly built bridges has attained some degree of maturity. On-structure long-term monitoring systems have been implemented on bridges in Europe [23], the United States [37], Canada [114], Japan [178], South Korea [176], China [172] and Colombia [127].

Generally, bridge SHM systems are envisaged to fulfil many assignments, such as, validation of design assumption, detection of anomalies, real-time monitoring of bridge safety and son on [81]. Measurements of SHM may include displacements, rotations, strains, temperature, force, vibrations and other environmental parameters, such as, humidity, rainfall and wind speed [22]. Currently, one of the most successful SHM systems is an integrated SHM system called Wind and Structural Health Monitoring System (WASHMS) conducted by Hong Kong SAR government onto three long-span bridges: Tsing Ma Bridge, Kap Shui Mun Bridge and Ting Kau Bridge since 1997, to monitor the structural health of them. Totally, more than 800 sensors are working on these bridges. Moreover, because of the development of sensor technology, many new techniques such as GPS, Video Cameras and Fibre Bragg Grating have been tested and included in WASHMS [28, 73, 172].

However, there are very few successful real-life examples on the integration of novel algorithms and SHM with advanced sensing technologies for objective evaluation of structural condition and reliability for decision making, which means more research is still highly demanded [26]. Catbas et al [26] used data obtained from SHM to calibrate a FE model for a long-span bridge. Then element reliability indices were calculated through the updated FE model. To evaluate system reliability, Monte Carlo simulations were carried out on the FE model. Orcesi et al [123] improved accuracy of the prediction by using SHM data to update existing limit state functions. The updated limit state functions were applied for the determination of the best maintenance strategies.

To date, NDT techniques have been largely used in bridge monitoring system [177]. NDT plays an essential role in time-dependent condition assessment and reliability analysis. NDT techniques, such as, ultrasonics, acoustic emission (AE), ground penetrating radar, impact-echo and infrared thermograph have been applied to detect and measure hidden flaws such as fatigue crack size. Because each NDT technique has its own limitations, often a combination of various NDT methods are needed to provide more effective information [105]. However, the data obtained from NDT methods cannot be utilised directly. Complex signal analysis and interpretation are usually needed first on the raw data. Artificial intelligence approaches such as artificial neural networks (ANN) and expert system are very useful in pattern recognition, classification and qualitative interpretation of data obtained from NDT methods[105]. Due to the uncertainties of NDT techniques, probabilistic methods are adopted to characterize these uncertainties and to update flaw sizes from stochastic fatigue crack growth analysis[177]. The applications of NDT for highway bridges in the USA have been described by Washer [167]. The author also discussed the potential impacts of NDT on bridge inspection and bridge health management [167].

2.5 Summary

Since the effectiveness of optimal strategies for bridge maintenance decision-making mainly depends on the ability to forecast bridge health performance, this review focuses on bridge health prediction approaches. In addition, the state-of-the-art of BMS and other relevant techniques, such as SHM and NDT are also presented briefly. This comprehensive review shows that there are various types of bridge health prediction approaches available for BMS. Overall, these approaches can be classified into two groups: condition ratings based and structural reliability based. The advantages and disadvantages of each approach are summarised in Table 2-4. Among them Markovian model is the most commonly used model and has been successfully applied to many BMS. However, it is also the most criticised model owing to its limitations. Overall, every bridge deterioration model possesses its own limitations. Thus, several areas to be researched are listed as follows:

- Bridge health performance in both serviceability and safety aspects is to be addressed in an integrated manner by the existing models

- Multiple bridge deterioration factors, such as, deterioration dependencies among different bridge elements, observed information, maintenance actions and environmental effects are to be considered jointly by the existing models.
- Approaches better than the currently used Series and/or parallel logical relationship for modelling complex relationship of bridge system are to be founded.
- A variety of information, such as, monitoring data, expert knowledge and physical laws, is to be integrated to mitigate the uncertainties in bridge deterioration modelling.
- Bayesian updating ability for dynamic prediction results updating is to be adopted by the existing models.

Table 2-4. A list of merits and limitations of different bridge deterioration models

Method	Merits	Limitations
Condition ratings based models		
Deterministic model		
Regression model	• Easily understood and used by bridge engineers	• Only average service life of bridges can be predicted • Uncertainties inherited with bridge deterioration are neglected
Stochastic process models		
Markov chains	• Used for any individual bridge or bridge element • Reflection of uncertainty from different sources • Future condition states are predicted based on current condition states • Computational efficiency and simplicity of use	• First-order Markov property • Subjective classification of condition states only based on engineering judgement • Discrete transition time intervals and time-independent transition probabilities • Inspection and monitoring data cannot be incorporated directly • Latent nature of deterioration is not recognized • State space explosion • Implicitly considered bridge deterioration • Stationary assumption of deterioration process
Ordered probit model	• Explicitly linkage between deterioration and relevant explanatory variables • Consideration of discrete condition states	• Panel data cannot be used • Too many analytical manipulation • Cannot deal with interactions among different bridge elements • Difficult to model hierarchically a whole bridge

Binary probit model	• Consideration of discrete condition states • Explicitly linkage between deterioration and relevant explanatory variables • Consideration of latent nature of bridge deterioration • Utilisation of panel data • Updated transition probabilities	• Too many analytical manipulation • Cannot deal with interactions between different bridge elements • Difficult to model hierarchically a whole bridge
Bayesian Approach	• Incorporation of expert knowledge and observation information • Updated transition probabilities • A cost-effective approach	• Still suffers from some limitations of Markov Chain • Cannot deal with interactions among different bridge elements
Semi-Markov model	• Release of the assumption that holding times of the Markov process is exponential or geometric • Time-dependent transition probabilities	• Discrete condition states
Continuous stochastic process model (Gamma process model, Gaussian process model)	• Natural and realistic modelling of bridge deterioration process • Continuous time intervals • Deterioration can be represented as the percentage of degradation	• Cannot deal with interactions among different bridge elements • Difficult to build a bridge system model based on bridge element models • Stochastic process, such as, Gamma process, cannot consider maintenance intervene
Artificial intelligence models		
Bayesian Network	• Can deal with dependencies among complex systems • Suitable for modelling of complex systems • Avoid state space explosion • Bi-directional updating ability • Incorporation of all formats of data • An unifying and intuitive modelling tool • Extension of BNs (DBN or DOOBN) can account for the temporal variability • Several commercial softwares are available	• Stationary structure and stationary transitions • No coupled relationship
Fault tree	• Visually and mathematically logical interrelationships of the bridge system • Modelling of bridge element interactions, redundancy, deterioration mechanisms in an entity • Qualitative and quantitative evaluation of bridge deterioration	• Construction can be laborious and time consuming • The basic events have to be independent • Binary states • Fails to model dependent failure and dynamic behaviours of deterioration processes
Binary Recursive Partitioning	• Less stringent data requirement • No assumption for particular distribution • Quick answer of explanatory variable selection • A practical means for data objectivity • Can handle the data from a smaller population	• Difficult to get quantitative results • Cannot deal with interactions between different bridge elements • A kind of classification tree
Case-based reasoning (CBR)	• Hierarchical decomposition of infrastructure facilities • Component interactions, condition states updating and temporal uncertainty of deterioration process can be all handled	• Largely depends on the size and coverage of the case library, and correctness and availability of expert knowledge • Modelling of bridge deterioration is not probabilistic • Uncertainty in bridge deterioration is not presented explicitly

Knowledge-based systems (including CBR)	• Expert knowledge is used	• Only responsible for some specific rules • Extensive knowledge of subjective system is required • Uncertainty within bridge deterioration is not represented in an explicit way
Artificial neural network	• Computational efficiency is high • Complex, multi-dimensional, non-linear relationship can be modelled	• Updating with new observation data is rather difficult • Prediction of bridge deterioration is not addressed in probabilistic way • A large number of data are needed for training • Calculations are not carried out in a semantically meaningful way
Structural reliability based model		
Structural reliability analysis model	• Basis of many structural design codes including Load and Resistance Factor Design (LRFD) • Concentrate on bridge load-carrying capacity and bridge safety • Bridge failure probabilities are calculated objectively based on limit state functions • Failure modes are expressed in an explicit way	• The interactions between bridge elements cannot be addressed explicitly • Visual deterioration information such as, corrosion and crack, cannot be incorporated directly • Hard to get accurate results • Representation of a bridge system as basic parallel and/or series bridge element sets

To achieve these goals, a more capable model for bridge health prediction is required. Dynamic Objective Oriented Bayesian Networks (DOOBNs), which are the extension of BNs, have shown the potential for bridge deterioration modelling and have been supported in a number of studies and applications with regards to deterioration prediction and decision making [42, 115, 149, 168, 169]. The DOOBNs are not only capable of overcoming the shortcomings of the current models, but also capable of incorporating various individual methods. In the future, the DOOBNs can be further extended for the purpose of bridge maintenance optimization. Influence diagrams (IDs), which are also extended from BNs by adding utility nodes and decision nodes, can be employed to optimize both inspection planning and maintenance actions.

Chapter 3: Overview of research basis

This chapter introduces the basic research knowledge for the proposed integrated health prediction. The aim is to pave the way for model development. Section 3.1 represents the different classes of BNs, including, Dynamic Bayesian Networks (DBNs), Object Oriented Bayesian Networks (OOBNs), Dynamic Object Oriented Bayesian Networks (DOOBNs) and Influence Diagram (IDs). Section 3.2 focuses on deterioration knowledge of bridges made of reinforced concrete and steel. For each material, the deterioration processes and corresponding physical equations descriptions are given in details. Section 3.3 addresses the issues regarding to research strategy, data collection and modelling analysis process.

3.1 Bayesian Network theory

3.1.1 Bayesian Networks (BNs)

According to Jensen and Nielsen [70], a BN is a probabilistic model in the form of directed acyclic graphs (DAG) with the directed edges and a table of conditional probabilities of each variable on all its parents. Fig 3-1 illustrates a simple example of a BN. Each node represents a probability distribution of a variable, which may in principle be continuous states or discrete states. Nodes X_2 and X_3 with arrows directed from other nodes are called child nodes. They have a common parent node X_1. Nodes without any arrows directed to them are called root nodes. An arrow between two nodes X_1 and X_2 indicates conditional dependence between the two variables. The dependence relationships are represented by a set of conditional probability distributions (CPDs) or conditional probability distributions (CPTs). For instance, the probability of a dependent variable X_2 being in a particular state given for each combination of the states of variable X_1 is expressed as $P(X_2|\ X_1)$. Prior probability tables or functions are held by root nodes.

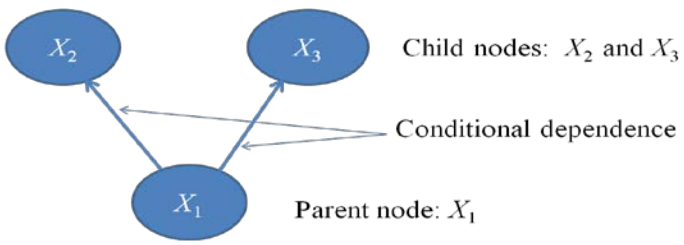

Figure 3-1. A simple BN consisting of three variables

As the probability of each variable is defined conditional on its parents, the joint probability of this network $P(X_1, X_2, X_3)$ is specified as a product of these conditional probabilities

$$P(X_1, X_2, X_3) = P(X_1)P(X_2|X_1)P(X_3|X_1) \tag{3-1}$$

where $P(X_2|X_1)$ and $P(X_3|X_1)$ are conditional probabilities given X_1, respectively, and $P(X_1)$ is prior probability. Moreover, with the assumptions of Markov property and conditional independence (d-separation [126]), the joint probability for any BNs is given as:

$$P(\boldsymbol{X}) = P(X_1, \cdots, X_n) = \prod_{i=1}^{n} P(X_i|Pa(X_i)) \tag{3-2}$$

where $Pa(X_i)$ is the set of the parents of node X_i. One distinctive advantage of BNs is the inference ability for calculation of beliefs of events based on new observed evidence. The beliefs (probabilities) are updated in accordance with observation using Bayesian updating. Assume an evidence e is observed, and then we have:

$$P(\boldsymbol{X}|e) = \frac{P(\boldsymbol{X},e)}{P(e)} = \frac{P(\boldsymbol{X},e)}{\sum_{\boldsymbol{X}} P(\boldsymbol{X},e)} \tag{3-3}$$

However, this computation is practical only if the network is small and each node has a few states. In practice, efficient algorithms have to be adopted. Now various inference algorithms are available for computing marginal probabilities for each unobserved node given a set of new observed evidence. The most commonly used algorithm is based on a tree-structure called junction trees [71]. Besides, there are also a number of exact and approximated inference algorithms available [116]. Without any observation information, the computation is based on a priori probabilities. When observation information is available, it is integrated into the network and all the probabilities are updated accordingly. Moreover, the observation information consists of hard evidence and soft evidence. Hard evidence indicates any particular state for a node directly (direct observation). Soft evidence only indicates any particular state for a node with probability (indirect observation).

In most engineering applications, the variables that refer to physical phenomena are continuous in nature. While BNs can handle both discrete and continuous variables, the formers are more typical since the associated algorithms are tailored to handle discrete variables effectively. Approximate inference algorithms such as Markov Chain Monte Carlo (MCMC) [17, 58] allow BNs to involve continuous random variables, yet this flexibility comes at a price. The drawback is that the rate of convergence can be extremely slow. As a result, any random variables that are defined in continuous states will be dsicretised and replaced by equivalent variables defined in a finite space. Furthermore, it is suggested to perform this discretization sequentially from parent nodes to child nodes. It is of importance to choose the discrete intervals for dividing a continuous variable into discrete states. One school of thought is to choose the discrete intervals that are suited to the characteristics of the variable (multivariate discretisation). Alternatively, the discrete intervals can be chosen uniformly for all variables (univariate discretisation), which is deemed to be suited for BNs [55]. To date, there are several discretisation algorithms available. For instance, a flexible way for multidimensional discretisation was proposed by Kozlov and Koller [85]. A detailed introduction about discretisation could be found in Friis-Hansen's PhD thesis [55].

The effectiveness of BNs is largely dependent on the accuracy of conditional probability tables (CPTs). Learning CPTs refers to the task of constructing a network model which best represents an underlying database or knowledge [55]. The CPTs can be estimated from four sources: statistic database, expert knowledge, physical laws and experiments data. In light of statistic database, some learning algorithms, such as, search- and scoring-based algorithms [66], and "Bayes Net Power Constructor" (BNPC) [30], are available. In addition, when some nodes are hidden or any data are missing, the expectation maximum (EM) algorithm, which is a two-step iterative algorithm, can be employed. However, it is a difficult task to collect enough statistics data for the estimation of CPTs. And statistics data found in the literature may not exactly represent the variables within the network. Expert knowledge is an alternative of statistics data. The questions are carefully administered to elicit knowledge from experts. The drawbacks of expert knowledge are the bias of questions and subjective judgement of experts. However, expert knowledge can simplify the estimation of CPTs for complex BNs. The CPTs estimation based on Physical laws is the best choice since physical laws are objective and can provide deterministic relationship between variables. Monte Carlo simulation is used to obtain statistics data based on Physical laws. As for the fourth source, data yielded from experiments can be used to estimate CPTs. Nonetheless, in real applications experiments may not always be realistic and affordable.

Validation of BNs can be done in three ways: sensitivity analysis, outcomes comparison and scenario testing. Sensitivity analysis is helpful in determining which basic input variables have the greatest influence on the output variables [91]. The prediction results of BNs can be compared with known results for model validation. Scenario testing is to model behaviours of BNs with different scenarios defined by experts and to assess whether the BNs behave as expected in term of past experience and in accordance with current credible research [18, 90]. As a whole, these three ways should be carried out together to validate BNs.

Compared with commonly used Markov chain, BNs show a number of advantages. BNs can model a complex system with plenty of variables in a compact representation through localized network clusters, thereby avoiding the “state space explosion” characteristic of Markov chain. Furthermore, with the Bayesian updating ability, BNs facilitate the integration of prior knowledge (expert knowledge) and new observations to tackle the situation when there are insufficient data. When new observations regarding to any variable are available, Bayesian updating can be implemented through the whole network. The latent nature of bridge deterioration can be modelled by BNs as well. As an intuitive modelling tool, BNs allow users to learn casual relationship by offering a graphical data structure that captures the dependencies between variables. Given its diagnostic and predictive capabilities, BNs can be used to diagnose root causes to specific output information or predict the outputs in the future.

3.1.2 Dynamic Bayesian Networks (DBNs)

Dynamic Bayesian Networks (DBNs) is a special class of BNs which includes a temporal dimension. A DBN is also referred to as state space models with two most common kinds of state-space models, namely Hidden Markov Models (HMMs) and Kalman Filter Models (KFMs) [116]. One simple example is shown in Figure 3-2. The DBN consists of a sequence of time slices i , each of which consists of one or more BN nodes. These slices are connected by direct links from nodes in slice i to corresponding nodes in slice i+1.The direct links between variables in different time slices represent temporal probabilistic dependence, which leads to the definition of CPTs. Normally, the Markov property is held by DBNs. If the model structure and the CPTs are identical all the time except for the initial time, DBNs are homogenous. A introduction about DBNs in details which includes representation, exact and approximate inference, and learning algorithms was provided by Murphy[116].

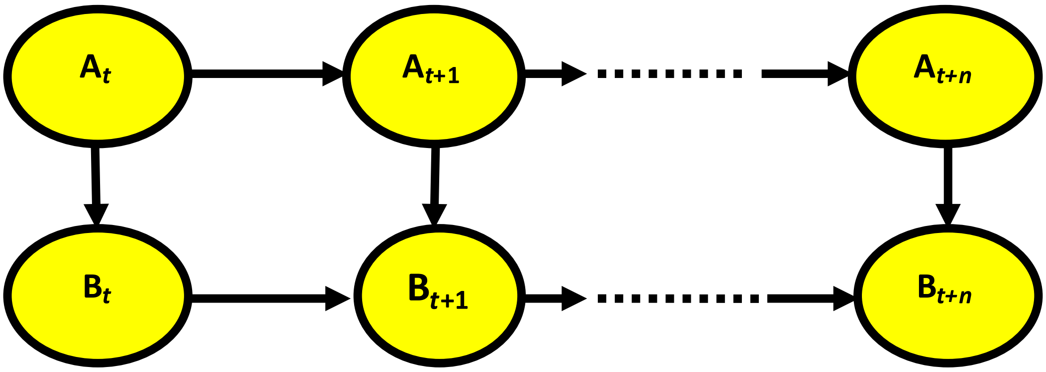

Figure 3-2. Simple example of dynamic BN

3.1.3 Object Oriented Bayesian Networks (OOBNs)

An Object Oriented Bayesian Network (OOBN) is a special class of BNs. In addition to usual nodes, an OOBN contains instance nodes [169]. In it, a physical or an abstract entity, or a relationship between two entities can all be an object. The object represents either a node or an instantiation of a network class (instance nodes). The definition of a network class enables OOBNs to be a more generic, reusable network, which facilitates hierarchical description of a problem domain. A network class is a named and self-contained representation of a network fragment with a set of interface and hidden nodes [169]. A class is generic network fragment. When this class is instantiated it is called an object [76]. A class may be instantiated many times [70]. And several classes can share common substructures. An example of a BN class is shown in Figure 3-3, where input nodes are ellipses with shadow dashed line borders and output nodes are ellipses with shadow bold line borders. An instantiation of this network class is also given in the Figure 3-3, which has one input C, and two outputs A and B.

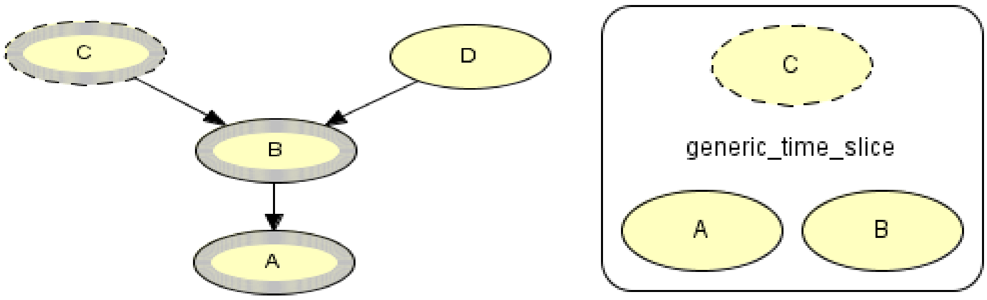

Figure 3-3. A simplified BN class and its instantiation

3.1.4 Dynamic Object Oriented Bayesian Networks (DOOBNs)

To address temporal behaviour of an OOBN, time slices are added to represent each period of interest. The resulting network, considering of several OOBNs time slices, is referred to as a dynamic OOBN (DOOBN) [76, 168]. Figure 3-4 shows a three-slice DOOBN. The inputs come from outputs in the previous time slice so that OOBNs in each time slice can be connected to address temporal behaviours.

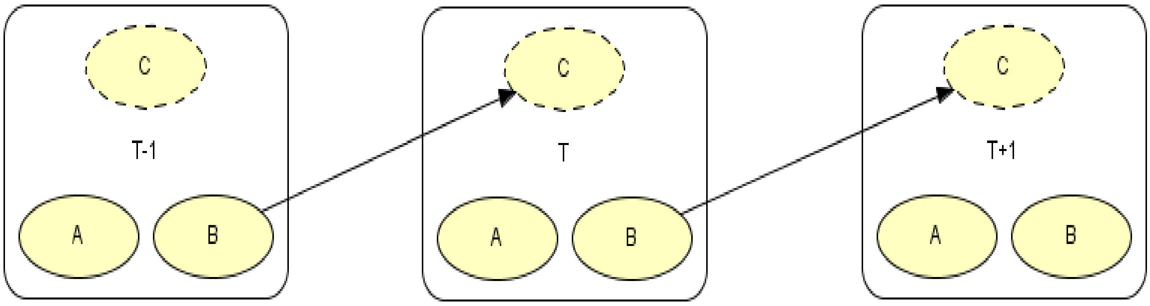

Figure 3-4. A simple three-slice DOOBN

3.1.5 Influence Diagrams (IDs)

Influence Diagrams (ID) are originally a representation of a fault tree for a symmetric decision scenario: one is faced with a specific sequence of decisions, and between each decision one observes a set of variables [42]. Nowadays, an ID is a Bayesian network, which is extended with utility nodes and decision nodes to solve decision problems [55]. Decision nodes define the action alternatives considered by the user. The available information about decisions instead of an expression of conditional probabilistic dependence is linked to the decision nodes as parents. Meanwhile, utility nodes are conditional on probabilistic and/or decision nodes but have no descendents. The utility nodes are the measures of decision nodes. To establish a rational basis for decision-making, one can compute the expected utility (EU) of each decision alternative (the global utility function is the sum of all the local utility functions). The alternative with the highest EU is chosen, which is known as the maximum expected utility (MEU) principle [42].

For instance, if there are a set **S** of possible configurations s_1, s_2, …, s_n, each associated with a probability $P(s_i)$, the expected utility under action a_j is represented as follows [55]:

$$EU(a_j) = \sum_S U(s_i)\, P(s_i|a_j) \tag{3-4}$$

According to the MEU principle, the best decision is chosen by performing the max-operation over the set of decision alternative D:

$$U(D) = \max_D (EU(a_j)) \tag{3-5}$$

By substituting Equation 3-4 into 3-5, the decision equation can be derived as:

$$U(D) = \max_D (\sum_S U(s_i)\, P(s_i|a_j)) \tag{3-6}$$

which is formulated as alternating sum- and max-operations.

3.2 Bridge deterioration description

Due to aggressive environment and steadily increasing traffic, bridges are supposed to deteriorate over time. The deterioration mechanisms differ for bridges. In this research, we focus on bridges made of reinforced concrete and steel. The bridge deterioration knowledge for steel bridges and reinforced concrete bridges is introduced respectively. The knowledge will be used for bridge deterioration modelling in the following chapters.

3.2.1 Steel bridges

3.2.1.1 Corrosion

For these bridges, the most common cause of deterioration is corrosion since all structural metals are prone to that. Corrosion can lead to cracking (fracture), yielding or bucking, bending or distortion, and slipping, which can result in stress concentration, change in geometric parameters, and a build-up of the corrosion products. Consequently, bridge serviceability and safety decrease over time. As far as steel bridge reliability is concerned, corrosion can cause a reduction in cross-section areas. The reduction of web area and plastic section modulus will result in shear capacity loss and moment capacity loss, respectively.

A number of factors can influence the propagation of corrosion. Temperature, amount of chloride, location environments and moisture are some of them. There are also different forms of corrosion, such as, pitting, crevice, galvanic and stress corrosion. In this study, the only general form of corrosion, uniform corrosion, is considered. Current available data are not sufficient to formulate analytical models for such corrosion. Therefore, it is only possible to use approximate empirical formulas. Normally, if effects of painting and coating are not considered, it is generally agreed to use a power function to describe corrosion propagation. An exponential function is given [8].

$$\boldsymbol{C} = \boldsymbol{A} \cdot \boldsymbol{t}^{\boldsymbol{B}} \tag{3-7}$$

where C is average corrosion penetration from corrosion loss after t years in micrometers (10^{-6} m), A is the corrosion loss after one year, and B is a regression coefficient numerically equal to the slope of Equation 3-7 in log-log plot. The values of A and B are dependent on the environment and steel type of bridge. For instance, in term of carbon steel and rural environment A=34 with coefficient of variation equal 0.09, and B= 0.65 with coefficient of variation equal 0.10 [8]. Based on Equation 3-7, new geometric parameters, such as, plastic section area and web area can be recalculated for structural reliability estimation of each bridge element.

3.2.2 Reinforced concrete bridges

For bridges made of reinforced concrete, the concrete itself is a relatively inert material. But if it is in contact with embedded steelwork or made from reactive aggregates, decay can ensue [98]. The concrete with reinforcing steel has distinctive deterioration processes. Concrete deteriorates because of internal pressures which are caused primarily by chemical reactions in the cement (sulphate attack), chemical reactions between the cement and aggregate (alkali-silica reaction), internal water movement owing to temperature gradients (freeze-thaw cycle attack), or expansion of corrosion products of reinforcing steel [44]. Reinforced steel deteriorates due to corrosion.

It is well accepted that chloride induced reinforcement corrosion is the primary cause of deteriorations of reinforced concrete bridges [166]. Normally, corrosion will not initiate until chloride penetrates into concrete and reaches a minimum concentration. The chloride contamination is initiated due to environmental exposure, such as deicing salts, salt spray of seawater and marine immersion environment. Chloride ions infiltrate through the porous concrete, and this chloride diffusion process is accelerated in the presence of cracks. Eventually, the chloride initiates corrosion of reinforcing steel, which further leads to other forms of severe deterioration, such as cracking, spalling and delamination. The sequential deterioration processes are described in details as follows. For bridge elements made of different concrete materials, the corresponding deterioration processes can be modelled by taking different parameter values.

3.2.2.1 Corrosion

The corrosion can cause a reduction of cross-section area of steel over its lifetime, which can reduce shear capacity and moment capacity of bridge. The reinforced concrete bridge structures suffer from two stage deterioration processes of corrosion: corrosion initiation and corrosion propagation. In the first stage, chloride is initiated through environmental exposure and penetrates into concrete. However, corrosion of reinforced steel has not actually happened. Fick's second law is commonly used to model the chloride penetration. Solved by Crank [34], $C_{\mathrm{x,t}}$ the chloride concentration at distance x from the surface at time t, with the assumption that chloride density on the surface is constant, can be described by

$$C_{\mathrm{x,t}} = C_0[1 - \mathrm{erf}\left(\frac{x}{2\sqrt{D_{\mathrm{c}}t}}\right)] \tag{3-8}$$

where C_0 is the chloride concentration on the concrete surface, D_{c} is the diffusion coefficient for chloride in concrete, and *erf* denotes the standard error function. Furthermore, in the second stage, corrosion initiates when the chloride concentration at the rebar surface reaches a minimum concentration. The corrosion initiation time when the critical chloride concentration C_{cr} is reached can be obtained by replacing $C_{\mathrm{x,t}}$ by critical chloride concentration C_{cr}, which is given by [154]:

$$T_{\mathrm{corr}} = \frac{x^2}{4D_{\mathrm{c}}[erf^{-1}(1-\frac{C_{\mathrm{cr}}}{C_0})]^2} \tag{3-9}$$

where T_{corr} is corrosion initiation time at any depth X from the surface. A limit state function for time to corrosion initiation at time t can be formulated as follows:

$$g_{corrosion}(t) = t - T_{\mathrm{corr}} \tag{3-10}$$

where $g_{corrosion}(t) > 0$ indicates the initiation of corrosion.

The diameter of reinforced steel bar at any time D_{t} is modelled as a function of time as follows [152]:

$$D_{\mathrm{t}} = D_0 - R_{\mathrm{corr}}\,(t - T_{\mathrm{corr}}) \tag{3-11}$$

$$D_{\mathrm{t}} = D_{\mathrm{t-1}} - R_{\mathrm{corr}} \tag{3-12}$$

where D_0 is the initial diameter of reinforcement steel bars, R_{corr} is the corrosion rate. The corrosion rate of a reinforced concrete bridge due to chloride induced reinforcement corrosion varies considerably depending on the environment around reinforced steel. If corrosion has been identified, then Equation 3-11 can be simply expressed as Equation 3-12. Moreover, the cross-section area of reinforced steel bar at any time A_t is given by [47] as:

$$A_t = \frac{n\pi {D_t}^2}{4} \tag{3-13}$$

where n is the number of bars experiencing active corrosion.

3.2.2.2 Crack

After corrosion initiation, further deterioration will bring crack to reinforced steel concrete cover. Prediction of the time from corrosion initiation to cracking is critical for modellings of the time to repair, rehabilitate and replace reinforced concrete structures in corrosive environment [97]. Based on experimental data obtained from simulated bridge deck slabs, Liu and Weyers have successfully developed a time to corrosion cracking model which considers the amounts of corrosion products needed to fill the interconnected void space around the reinforcing bar plus the amount of corrosion products needed to generate sufficient tensile stresses to crack the cover concrete [97]. The time from corrosion initiation to cracking $T_{\text{corr\_crack}}$ is predicted by the following equation [97]:

$$T_{\text{corr\_crack}} = \frac{W_{\text{crit}}^2}{2K_p} \tag{3-14}$$

where W_{crit} is the critical amount of corrosion products, K_{p} is the rate of rust production. W_{crit} and K_{p} are further expressed by Equation 3-15 and 3-16, respectively

$$W_{\text{crit}} = \rho_{rust}(\pi \left[\frac{Cf_t'}{E_{ef}}\left(\frac{a^2+b^2}{b^2-a^2} + \nu_c\right) + d_0\right] D + \frac{W_{st}}{\rho_{st}}) \tag{3-15}$$

where ρ_{rust} is the density of corrosion products; ρ_{st} is the density of steel; a is inner radius of a thick-wall concrete cylinder a= (D+2d_0)/2; b is outer radius of the thick-wall concrete cylinder b=C+(D+2d_0)/2; D is the diameter of reinforcement steel; d_0 is the thickness of the pore band around the steel/concrete interface; C is cover depth; ν_c is Poisson's ratio of the concrete; E_{ef} is an effective elastic modulus of the concrete where $E_{ef} = E_c/(1 + \varphi_{cr})$, E_c is elastic modulus of the concrete and φ_{cr} is the creep coefficient of the concrete; f_t' is the tensile strength of concrete; W_{st}, the amount of steel corroded, equals to αW_{crit}, in which α is represented as the molecular weight of steel weigh divided by the molecular weight of corrosion products

$$K_p = 0.098(1/\alpha)\pi D i_{\text{corr}} \quad (3\text{-}16)$$

where i_{corr} is the annual mean corrosion rate (mA/ft^2).

As the calculated $T_{\text{corr\_crack}}$ and T_{corr} are both probabilistic variables, a limit state function for time to crack at time *t* can be formulated as follows[103]:

$$g_{crack}(t) = t - (T_{\text{corr\_crack}} + T_{\text{corr}}) \quad (3\text{-}17)$$

where $g_{crack}(t) > 0$ indicates the initiation of crack.

3.2.2.3 Spalling

If crack is initiated, its width will grow. When a critical crack width is reached, spalling can be caused by severe cracking. The time to spalling is also critical for modelling of the time to repair, rehabilitate and replace reinforced concrete structures in a corrosive environment. An empirical model of time from crack initiation to spalling was derived from experimental results as follows [79]:

$$T_{crack\_spalling} = 0.0167 i_{\text{corr}}^{-1.1}[42.9\left(\frac{wc}{C}\right)^{-0.54} + ((w_{lim} - 0.3)/0.0062)^{1.5}] \quad (3\text{-}18)$$

$$0.3mm \leq w_{lim} \leq 1.0mm$$

where i_{corr} is corrosion rate (μA/cm^2); wc is water-cement ratio estimated from Bolomey's formula; *C* is concrete cover (mm). Similarly, the obtained $T_{crack\_spalling}$ is also probabilistic variable, and a limit state function for time to spalling at time *t* can be also formulated as follows[103]:

$$g_{spalling}(t) = t - (T_{\text{corr}_{\text{crack}}} + T_{\text{corr}} + T_{crack\_spalling}) \quad (3\text{-}19)$$

where $g_{spalling}(t) > 0$ indicates the initiation of spalling.

3.3 Research strategy and data specification

To adopt DOOBN approach to deal with the identified defects, three novel models will be proposed. Firstly, the Model I focuses on bridge deterioration in serviceability, which uses condition ratings as the health index. Secondly, the Model II concentrates on bridge deterioration in safety. Both Models I and II are designed in three steps: modelling consideration, DOOBN development and parameter estimation. Thirdly, Model III integrates Models I and II to address bridge deterioration in both serviceability and safety.

The integration of condition ratings and structural reliability is implemented through essential failure modes.

To validate the proposed three DOOBN based models, a large number of data are necessary. In this research, multiple data sources from bridge experts, the National Bridge Inventory (NBI) and the existing literature [47] will be utilised for model validation. An interview aiming to elicit expert knowledge was conducted. With carefully designed questions, bridge practitioners are able to provide their estimation about condition evolution for each bridge element over a certain period of time. To ensure the reliability and validity of their estimation, only bridge practitioners with excellent expertise and long-time working experience was selected. Since most of these engineers have poor understanding about the art of probability assessment, specialised training courses were given to them so that desired information can be provided. With proper designed questions and friendly presentation of them, the quality of the elicited data can be guaranteed. A highway bridge "E-17-AH" located in Denver, Colorado was selected from the existing literature [47] as a case study, where the data have been validated. For the NBI data, the selection criteria are to consider "Record Type", "Route Signing Prefix", "Kind of Material/Design" and "Type of Design/Construction". Considering the highway bridge "E-17-AH" in these aspects, relevant condition records were selected out.

Chapter 4: Bridge health prediction in serviceability aspect

4.1 Introduction

In this chapter, a DOOBN model for bridge condition ratings prediction is proposed. In Section 4.2, the proposed DOOBNs model I is developed to assist probabilistic modelling of bridge condition ratings deterioration in a hierarchical way. The model I and is outlined through three steps: modelling consideration, DOOBN development and parameters estimation. it provides BMS with more accurate prediction results by handling the multiple deterioration factors, such as, deterioration dependencies among different bridge elements, maintenance intervene and environmental effects jointly, and performing the Bayesian updating efficiently. Furthermore, the model I can incorporate different types of data, such as, expert knowledge and historical condition rating data, to deal with data insufficiency. To demonstrate the feasibility, an application of this model I to a steel truss railway bridge is given in Section 4.3.

4.2 Model I: using condition ratings

To fulfil versatility requirement for different BMSs and extensibility requirement for maintenance optimization, the DOOBNs model I for bridge condition ratings prediction is designed through three modelling steps: modelling consideration, DOOBNs development and parameters estimation. A bridge is modelled in a hierarchical way by the DOOBNs so that the deterioration contribution of each bridge element could be tracked. The merit of the proposed model lies in the propagation of temporal deterioration uncertainties from bridge elements to the whole bridge system. The following sections discuss the three modelling steps in details.

4.2.1 Modelling consideration

4.2.1.1 Bridge hierarchical decomposition

Systematic modelling of a bridge requires the identification of all bridge hierarchies related to the bridge deterioration. The condition ratings of bridge hierarchies contribute to overall condition ratings of the whole bridge. Therefore, bridge hierarchical decomposition is necessary for bridge deterioration modelling. In BMS, a bridge can be decomposed into a number of bridge hierarchies in several levels. According to the structures of BMS, different hierarchical decomposition strategies are taken, which results in different bridge hierarchies. For most of the currents BMS, the decomposition method used in Pontis is often adopted. A bridge is generally divided into three bridge components: deck, superstructure and substructure, and the bridge components are further divided into a quantity of bridge elements, such as, girder, expansion joints, pier and abutment. Here, the bridge elements mean basic units with primary inspection records. However, Morcous [113] criticised that this decomposition method does not categorise bridge elements based on their functions and locations, and therefore proposed a method that decompose a bridge into seven levels of granularity: root bridge, bridge massing, bridge system, bridge sub-system, bridge assembly, bridge sub-assembly and bridge element. No matter which decomposition method is used, the key objective is to identify all significant bridge hierarchies. Additionally, since almost all the existing BMSs use the Overall Condition Rating (OCR) method for overall evaluation of bridge or element condition, the location information of bridge hierarchies is normally not taken into account in the condition ratings evaluation of bridge deterioration. The same type of bridge hierarchies with different locations are usually treated as one bridge hierarchy. As a result, here basic bridge elements denote one entity of all the same type of bridge elements rather than any individual bridge elements.

4.2.1.2 Relative weights assignment for bridge hierarchies

Since different bridge hierarchies have different functions and roles, the impacts of each bridge hierarchy on bridge deterioration should be identified. Relative weights of bridge hierarchies have been adopted by the current BMS to represent their impacts on the whole bridge system and to evaluate the overall condition ratings. In different BMS, relative weights of the same bridge hierarchy may be different. Relative weights can be assigned by bridge partitioners directly with their fully knowledge about bridge deterioration, or estimated based on the methods, such as, pair wise comparison matrix method (AHP or Eigenvector method) [138] and Delphi method [29, 35].

4.2.1.3 Condition ratings definition

With the intention of bridge assessment, a number of exclusive condition ratings that describe bridge deterioration processes are essential to be defined. The condition ratings are usually defined from good condition to failed condition and labelled with numbers. According to the literature review, different definitions are adopted in the current BMS. Hence, the proposed model I is designed to be compatible with any kind of definitions.

4.2.1.4 Deterioration dependencies analysis

Since all the bridge elements are physically interconnected, the deterioration of one bridge element can influence the deterioration of another connected bridge element. Deterioration dependencies happen when the deterioration of a malfunctioning element accelerates the deterioration of another. In practice many deterioration dependencies have been observed by bridge inspectors. For example, the deterioration of a concrete deck accelerates if its bearings do not function properly [142]. When the bearing freezes because of corrosion, the deck is subjected to expansion and contraction stresses that cause cracking. Therefore, it is necessary to model bridge deterioration with the consideration of deterioration dependencies. Bridge maintenance engineers can provide their knowledge about deterioration dependencies. However, elicitation interviews have to be carried out. Alternatively, if there are sufficient condition data for bridge elements, statistical methods such as correlation analysis can be implemented to calculate the correlation between two bridge elements, which indicates if deterioration dependency exists between the two elements [142]. Moreover, effects from environmental condition, maintenance action and observed information are considered as deterioration dependencies, since the deterioration of bridge elements also depends on all the information.

4.2.2 DOOBNs model development

Based on the previous consideration, DOOBNs are developed from top level (the whole bridge system) to bottom level (bridge elements). Overall it consists of two major parts: bridge hierarchies modelling and bridge elements modelling. The first part focuses on probabilistic modelling of bridge system by means of bridge hierarchies. The second part focuses on modelling of bridge elements deterioration exclusively. In this section, a conceptual model for condition ratings prediction is formulated.

4.2.2.1 OOBNs model of bridge hierarchies

Consider that a bridge system is hierarchically decomposed into a number of bridge hierarchies in L ($L>2$) levels with the whole bridge system in the highest Level *1* and basic bridge elements in the lowest Level L. Additionally, condition ratings of each bridge hierarchy are defined over K exclusive assessment ratings denoted by $\mathbf{S}=\{S_{1,\ldots,}S_K\}$. And it is possible that different bridge hierarchies are defined over different condition ratings. Suppose a generic bridge hierarchy C in Level M ($M<L$) is further decomposed into N bridge sub-hierarchies A_i with relative weights W_i (i =1, …, N) in Level M+1. Particularly bridge hierarchies A_i denote primary bridge elements when M+1 equals to L. Because of the decomposition relationship between the bridge hierarchy C and the several bridge sub-hierarchies A_i, the deterioration of C is conditional on all her sub-hierarchies A_i. To model all the bridge hierarchies without being lost, the object oriented representation of BNs (OOBNs) are employed so that each time only one bridge hierarchy is focused on. All the nodes and links related to this bridge hierarchy can be encapsulated in one object. The modelling of this genetic bridge hierarchy C by means of OOBNs is given by Figure 4-1, where input nodes are ellipses with dashed line border and output nodes are ellipses with shadow bold line borders.

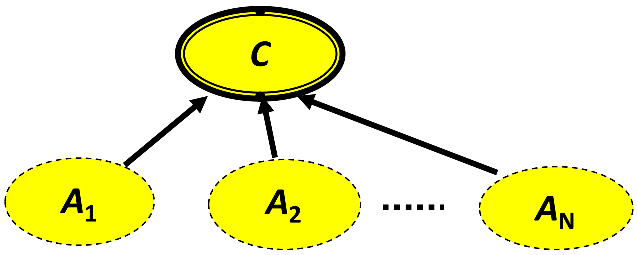

Figure 4-1. OOBNs model of a generic bridge hierarchy C for condition ratings prediction

In Figure 4-1, a large number of bridge hierarchies A_i can be directly linked to the bridge hierarchy C as its parent nodes. However, according to Langseth and Portinale [89], normally the maximum number of parent nodes for each node is suggested to be controlled under three or fewer, because too many parent nodes can affect the computational efficiency of BNs inference. Therefore, if the number of A_i is very large, the OOBNs modelling in Figure 4-1 will absolutely lead to slow computational efficiency and may be intractable. To overcome this problem, unnecessary bridge hierarchies, which have little impacts on bridge deterioration, can be eliminated. Nonetheless, this will obviously sacrifice the model accuracy. An alternative way is to introduce auxiliary nodes to bridge hierarchies modelling. Since auxiliary nodes enable indirect connection between parent nodes and children nodes, the number of each node's parent nodes can be effectively reduced. Taking this generic bridge hierarchy C as an example, if each bridge sub-hierarchy A_i has three condition ratings (K=3) and the number of bridge hierarchies A_i equals to 9 (i =1, … , 9), direct modelling like the one in Figure 4-1 will make inference computation rather time-consuming. In this case, three auxiliary nodes (B_1, B_2, B_3) can be are inserted between node C and its parent nodes A_i and become the new parent node. The OOBNs modelling of this bridge hierarchy C is represented in Figure 4-2, where input nodes are ellipses with dashed line borders and output nodes are ellipses with bold line borders. An instantiation of this network class is also given in the Figure 4-2, which has nine inputs A_i (i =1, … , 9) and one output C. By means of the auxiliary nodes, inference efficiency of the whole network can be improved dramatically. The auxiliary nodes (B_1, B_2, B_3) do not have any practical meaning, and each auxiliary node is defined with some numbered states according to the weighted sums of every condition ratings combination of all its parent nodes. This state definition of auxiliary nodes will be further addressed in the parameters estimation part. Finally, It should be noticed that the auxiliary nodes cannot relief the burden of CPTs estimation but only facilitate BNs inference.

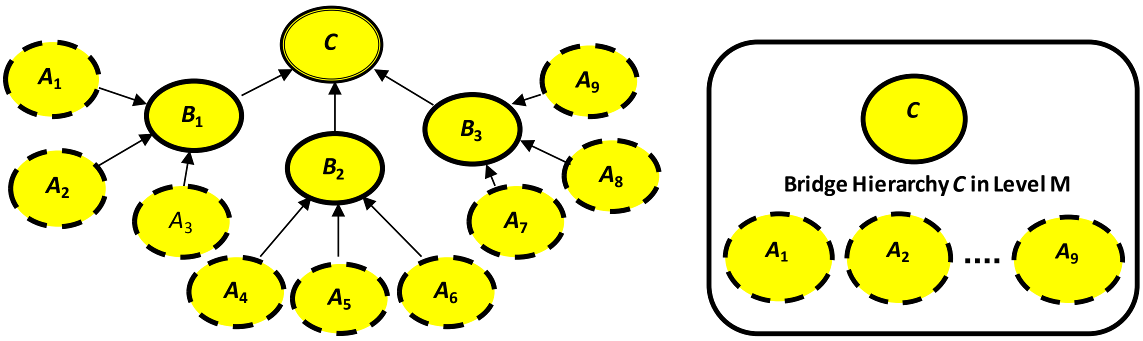

Figure 4-2. OOBNs model of a generic bridge hierarchy C with auxiliary nodes for condition ratings prediction

With OOBNs modelling of all the bridge hierarchies from Level 1 to Level M, the whole bridge system can be structured by simply connecting each bridge hierarchy in different levels. As one object has inputs and outputs, the logical relationships between different bridge hierarchies have been identified clearly. For instance, if a bridge is decomposed into three levels: a number of bridge components C_j and each bridge component is further decomposed into a number of bridge elements A_i, the system modelling of this bridge is represented in Figure 4-3, where input nodes are ellipses with dashed line borders and output nodes are ellipses with bold line borders. The whole bridge is in the highest level while bridge elements are in the lowest level.

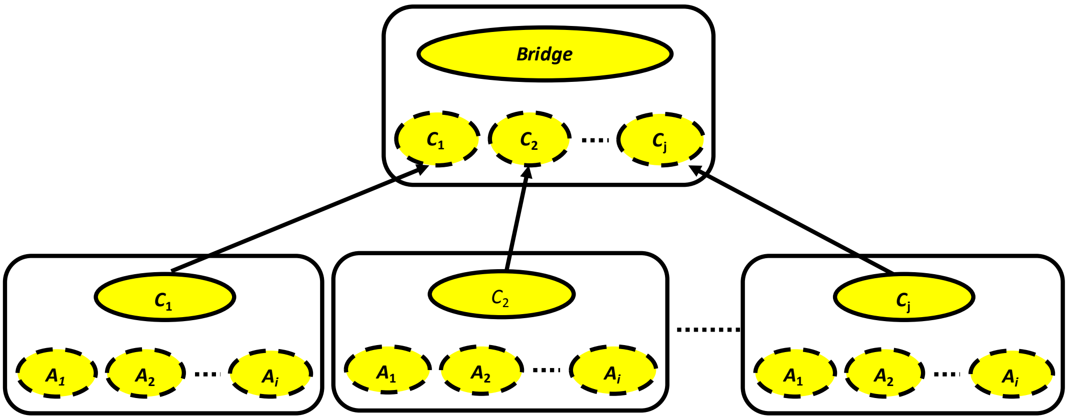

Figure 4-3. OOBNs model of a bridge system in three levels for condition ratings prediction

4.2.2.2 DOOBNs model of bridge elements

The deterioration processes of bridge elements are normally modelled by stochastic processes, such as, Markovian [72] or semi-Markovian [99] stochastic processes, Gamma process [139] and Gaussian process. In principle, both discrete-time and continuous-time stochastic processes are applicable to deterioration modelling of bridge elements. However, owing to the limitation of current inference algorithms and slow convergence rate, continuous variables cannot be dealt with efficiently. Therefore, discrete-time stochastic processes are preferred. For simplicity, discrete-time Markov process is employed to model the deterioration of bridge elements in this research. Additionally, the discrete-time Markov process can be homogeneous or non-homogeneous. If one bridge element E is defined with H exclusive condition ratings, Figure 4-4 describes an OOBN model representing the temporal deterioration of bridge element E between time t-1 and t by means of discrete-time Markov process defined by CPT of variable $E(t)$, where input nodes are ellipses with dashed line borders and output nodes are ellipses with shadow bold line borders.

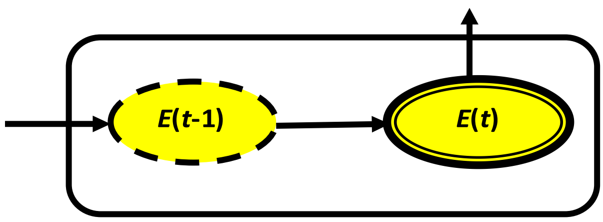

Figure 4-4. The OOBN model of a generic bridge element E for condition ratings prediction by means of discrete-time Markov process

To ensure the modelling consistency regarding actual deterioration of bridge elements, bridge deterioration factors related to maintenance actions, environmental effects and observed information should be considered jointly. As these factors are independent of the past, a set of variables $\mathbf{X}(t)$, $\mathbf{Y}(t)$ and $\mathbf{Z}(t)$ related to maintenance actions, environment levels and observation, respectively, can be individually added to the OOBN model in each time slice (Figure 4-5). The maintenance variables $\mathbf{X}(t)$ are defined for each bridge element with several states according to available maintenance actions. Different maintenance actions have different impacts on the deterioration of bridge elements. For instance, replacement and perfect repair will bring bridge elements into the new state. Minimal repair and no maintenance leave bridge elements in the unchanged state. Imperfect maintenance brings bridge elements into the state better than past state but worse than new state. For one bridge element, the probabilities over all the possible condition ratings can be used to express imperfect maintenance actions. The environmental variables $\mathbf{Y}(t)$ account for environmental effects, such as, traffic volumes, traffic loads, temperature, moisture and humidity. Four environmental states in the PONTIS BMS [60]: benign, low, moderate and severe are adopted. If common environmental variable is considered, the environmental variable of each bridge element can be connected to one common environmental variable. The observation variables $\mathbf{Z}(t)$ facilitate Bayesian updating when newly observed condition ratings data are available. Observations from visual inspection can directly reflect true condition ratings of bridge elements, while NDT and monitoring techniques only provide indirect information about bridge deterioration. This information can be characterised by a probability of detection (PoD) or measurement accuracy.

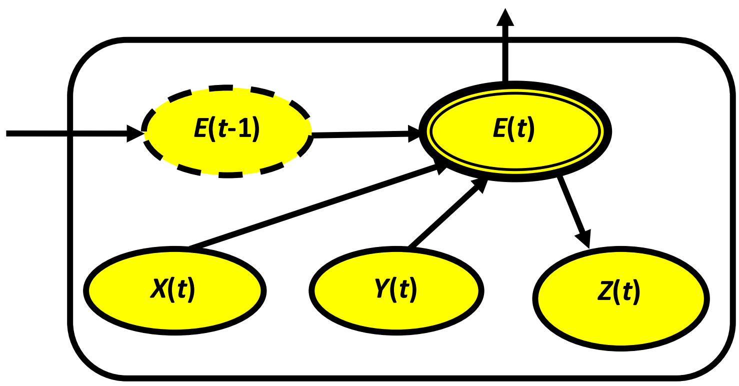

Figure 4-5. OOBNs model of a generic bridge element *E* for condition ratings prediction including maintenance intervene, environmental effects and observation

Besides, deterioration dependencies amongst different bridge elements should be considered as well. Suppose that the bridge element E is identified with the influence from another two bridge elements E_1 and E_2, this deterioration dependency can be modelled by the OOBN structure depicted by Figure 4-6, where input nodes are ellipses with dashed line borders and output nodes are ellipses with shadow bold line borders.

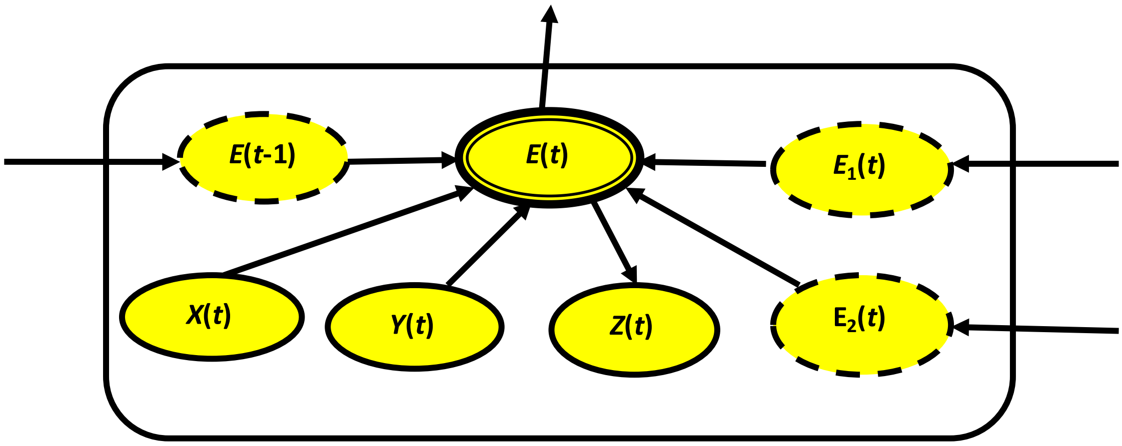

Figure 4-6. OOBNs model of a generic bridge element E for condition ratings prediction including deterioration dependency

So far, a generic OOBNs model has been proposed for bridge elements deterioration at any time slice. To address the temporal deterioration of bridge elements, the OOBNs models at different time slices are connected to formulate a DOOBNs model (Figure 4-7). The outputs of the DOOBNs model are further linked to the corresponding bridge elements modelled in the part of bridge hierarchies. Therefore, the whole conceptual model for bridge condition ratings prediction by means of DOOBNs is completed.

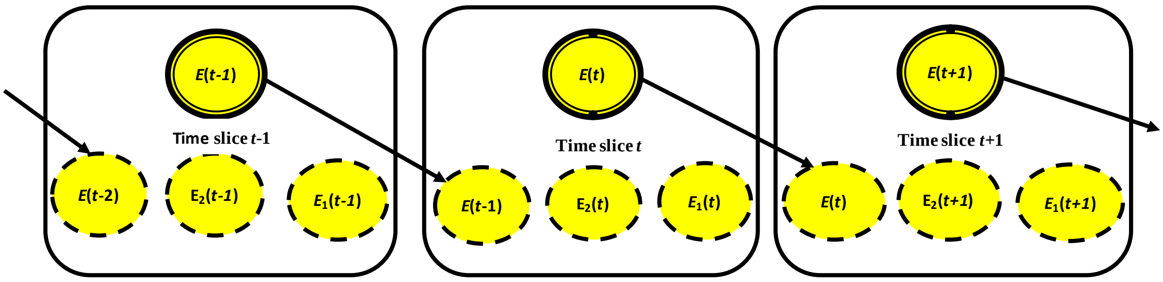

Figure 4-7. DOOBNs model of a generic bridge element for condition ratings prediction accounting for temporal deterioration

4.2.3 Parameters estimation

The final step of the proposed model is to estimate the CPTs and priori probabilities. Overall, the whole estimation is a complex task and needs combine a variety of data sources. No single method is versatile and is able to fulfil the CPTs estimation in all the circumstances. In this section, parameters estimation for bridge hierarchies and bridge elements is addressed, respectively.

4.2.3.1 Bridge hierarchies

In this part, the condition ratings distributions of bridge hierarchies are assumed to be a uniform distribution. As all the existing BMSs use the OCR method for overall evaluation of bridge hierarchies, relative weights are used to estimate CPTs of bridge hierarchies. Recall a generic bridge hierarchy C in the last section, which is further decomposed into N bridge sub-hierarchies A_i with relative weights W_i (i =1, …, N). The condition rating of bridge factor C is conditional on the condition ratings of bridge hierarchies A_i. For each combination of condition ratings of bridge sub-hierarchies A_i (i =1, …, N), a weighted sum of condition rating R is calculated as

$$R = \frac{\sum_{i=1}^{N}(r_i \times W_i)}{\sum_{i=1}^{N} W_i} \tag{4-1}$$

where r_i is the condition rating of each bridge sub-hierarchy A_i. If R is an integer, the corresponding condition rating conditional on this combination in CPT is filled with 1. Otherwise, two rounded condition ratings towards negative and positive infinity, R_f and R_c ($R_f < R < R_c$), conditional on this combination in CPT are assigned with R_c-R and R- R_f, respectively. Furthermore, when the condition grades of two adjacent levels are different, the weighted sum of condition rating R is modified as

$$R = \frac{\sum_{i=1}^{N}(r_i \times W_i)}{\sum_{i=1}^{N} W_i} \times \frac{P}{Q} \tag{4-2}$$

where Q is the condition ratings number of bridge sub-hierarchies A_i ; P is the condition ratings number of bridge hierarchy C. Then the CPTs are filled in exactly the same way as above. However, because condition grades of two adjacent levels are different, the resulting R is scaled up or down. Attention has to be paid to the combination that all the condition ratings of bridge sub-hierarchies A_i are denoted to be 1. The calculated R will not be an integer. In that situation, the CPT should be filled with 1. Moreover, the calculated

CPTs based on relative weights should be further examined. If some values are incorrect, the values directly obtained from expert knowledge are used instead.

Each auxiliary node will be defined with some numbered states which are calculated from the weighted sums of every condition ratings combination of all its parent nodes. Considering the case in Figure 4-2, the CPTs associated to A_i (i=1,2,3, or 4,5,6 or 7,8,9) all correspond to the identity operator. For instance, given one combination of the condition ratings of A_i (i=1, 2, 3, or 4, 5, 6 or 7, 8, 9), only the numbered state corresponding to the weighted sums of this combination is equal to 1 with others probabilities being equal to 0. To further estimate the CPT of bridge hierarchy *C*, each auxiliary node B_i ($i=1, 2, 3$) is assigned with relative weight being equal to the sum of relative weights of all its parent nodes. The CPT of bridge factor *C* will be filled out based on the Equation 4-2.

4.2.3.2 Bridge elements

- Bridge condition data

A BMS may have a bridge database including inventory data, condition ratings data (inspection data), appraisal data and maintenance data as well as monitoring data. By means of learning algorithms, all the CPTs related to bridge elements can be estimated based on the database. To date, a number of learning algorithms available have been listed by Murphy [116]. As for bridge elements modelled in Figure 4-6, parameters estimation always requires a large amount of data. As a result, available data are never sufficient compared with the number of evaluated parameters. In contrast, it is more realistic to estimate CPTs from condition ratings data for bridge elements modelled in Figure 4-4. Hence, the learning methods for this type of modelling are discussed in details.

Normally, reliable CPTs estimation demands as much as possible historical condition ratings data without maintenance intervenes and it is required that at least two consecutive historical condition ratings data without maintenance intervenes are available. In this study, for simplicity, bridge elements deterioration is assumed to follow discrete-time Markov process. Two commonly used methods are the non-linear least square optimization method and the maximum likelihood estimation (MLE) method. The non-linear least square optimization method minimizes the summation of squared difference between actual relative percentage from database and the expected percentage predicted from transition probabilities of all the condition ratings during a certain time. The transition probabilities are estimated by solving a non-linear optimization problem. The objective function and the constraints of this non-linear optimization problem can be written as follows [102]:

$$min \sum_{m=1}^{M} \sum_{n=1}^{N} \left(P_{m,n} - (P_0 \boldsymbol{T}^n)_m\right)^2 K(n) \quad (4\text{-}3)$$

Subject to $0 \leq T_{i,j} \leq 1$ $i, j = 1,2,\ldots,M$

$$\sum_{j=1}^{M} T_{i,j} = 1 \quad (4\text{-}4)$$

$$\boldsymbol{T} = \begin{bmatrix} T_{1,1} & \cdots & T_{1,M} \\ \vdots & \ddots & \vdots \\ T_{M,1} & \cdots & T_{M,M} \end{bmatrix} \quad (4\text{-}5)$$

where P_0 is a vector of the initial condition rating of a bridge element which is always assumed to be in good condition; $P_{m,n}$ is actual relative percentage in condition rating *m* at age *n*; $\boldsymbol{T}$ is the transition probabilities matrix defined over a certain transition period (Equation 4-5); *M* is the number of condition ratings; *N* is the number of years of condition data available; *K(n)* is the number of bridge elements at age *n* for weighting each term. It makes sense that all $T_{i,j}$ terms where *j* is larger than *i* are zero as the condition of any bridge element cannot improve without maintenance actions. In addition, generally, the assumption that condition of a bridge element will not jump down more than 1 condition in one transition period is often held. As a result, the estimated parameters are significantly reduced. The above non-linear problem can be solved easily by using the Optimization Toolbox in "MATLAB" Software.

As for MLE method, the parameter $T_{i,j}$ can be simply estimated from historical condition data based on Equation 4-6 [74].

$$T_{i,j} = \frac{n_{i,j}}{n_i} \quad (4\text{-}6)$$

where $n_{i.j}$ is the number of transitions from condition rating *i* to condition rating *j* during a given time period; n_i is the total number of bridge elements in condition rating *i* before the transition within the same time period. The relationship between $n_{i.j}$ and n_i is formulated as Equation 4-7.

$$\sum_{j=1}^{M} n_{i,j} = n_i \quad (4\text{-}7)$$

If $\varepsilon_{i,j}$ is the newly observed number of transitions from condition rating *i* to condition rating *j* during next time period and ε_i is the total newly observed number of bridge elements in condition rating i before the transition within the same time period, the parameter $T_{i,j}$ can be easily updated as follows:

$$T_{i,j} = m_i T_{i,j}^0 + (1 - m_i)\frac{\varepsilon_{i,j}}{\varepsilon_i} \tag{4-8}$$

where $T_{i,j}^0$ denotes the previous parameter, and

$$\mathcal{E}_i = \sum_{j=1}^{M} \mathcal{E}_{i,j} \tag{4-9}$$

$$m_i = \frac{n_i}{n_i + \mathcal{E}_i} \tag{4-10}$$

Comparing the two methods, the MLE method is more straightforward and can be easily used. Also, the transition probabilities matrix can be easily updated when newly observed condition state data are available. Nonetheless, if the historical condition state data are recorded in the form of relative percentages at different time units, the least square method is more appropriate because it is impossible to sort out transition numbers from this kind of data. Bridge experts can give their insights into bridge element deterioration based on their experience. The knowledge can be converted into relative percentages of different condition ratings at different time intervals. In this case, the least square method is preferred.

However, both the methods are affected by incomplete historical condition data. If historical condition data over a certain time period were not observed regularly, these data are called incomplete data. As for these data, although both the methods can still estimate transition probabilities matrix from incomplete historical condition data, there is no data-augmentation involved. Furthermore, it will be rather difficult for the MLE method to obtain results analytically from these incomplete data [141]. A feasible way is to use an iterative method, for instance, the Expectation-maximization (EM) algorithm, which is a data-augmentation method and an extension of the MLE method [141]. The flowchart of the EM algorithm is depicted in Figure 4-8. Overall, the whole process is mainly comprised of two steps, the Expectation (E) step and the Maximization (M) step. First, the observed incomplete condition data and an initial estimate of transition probabilities matrix is given, then the E step rebuilds all the possible sets of complete condition data and estimates the expected complete likelihood function of these complete data with a transition probabilities matrix. Next, by maximising the expected complete likelihood function, a new estimate of transition probabilities matrix is obtained in the M step. With this new estimate of transition probabilities matrix, the E step and M step are implemented again for another estimate of transition probabilities matrix. The both steps iterate until the estimated parameters converge. The EM algorithm for estimating transition probabilities of bridge elements is discussed in details as follows [141]:

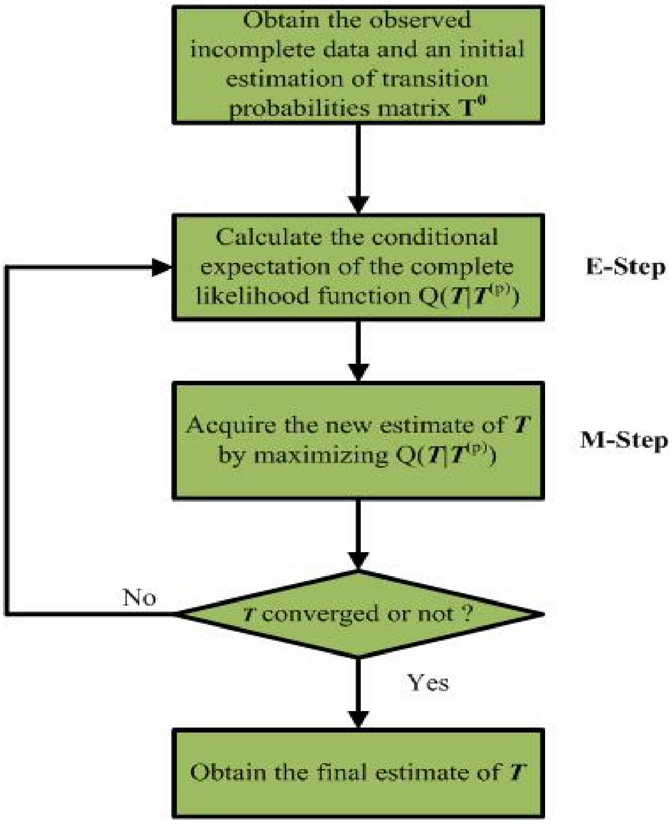

Figure 4-8.The flowchart of EM algorithm

Given the observed historical condition data Y and a transition probabilities matrix $\boldsymbol{T}$, several possible sets of 'complete' condition data X with different happening probabilities can be estimated. The likelihood of any set X conditional on the transition probabilities matrix $\boldsymbol{T}$ can be expressed by Equation 4-9

$$L(X|\boldsymbol{T}) = \prod_{i=1}^{S} \prod_{j=1}^{S} {T_{ij}}^{N_{ij}} \tag{4-11}$$

where N_{ij} is the total number of transitions from condition rating i to condition rating j within the 'complete' condition state data X ; S is the number of condition ratings. Therefore, the expectation of log-likelihood of X with a transition probabilities matrix $\boldsymbol{T}$ conditional on Y and a priori estimate of transition probabilities matrix $\boldsymbol{T}^{(P)}$ is described as follows:

$$Q\left(T\middle|T^{(P)}\right) = E\left[\log L(X|\boldsymbol{T})\middle|Y,T^{(P)}\right] = \sum_{i=1}^{S}\sum_{j=1}^{S} E[N_{ij}|Y,T^{(P)}]\log(T_{ij})$$
$$= \sum_{i=1}^{S}\sum_{j=1}^{S} n_{ij}^{(P)}\log(T_{ij})$$

(4-12)

where $n_{ij}^{(P)}$ is the expected number of transitions from condition rating *i* to condition rating *j* conditional on a priori estimate of transition probabilities matrix $\boldsymbol{T}^{(P)}$, and is further defined by Equation 4-13.

$$n_{ij}^{(P)} = E[N_{ij}|Y,T^{(P)}] \quad (4\text{-}13)$$

For the estimation of $n_{ij}^{(P)}$, consider a bridge element is observed in condition rating *f* at time t_0 and in condition rating *r* at time t_0+t. The probability that a transition from condition rating *i* to condition rating *j* happens at time t_0+k amid this observation, where $2\leq k\leq t-1$, is given by Equation 4-14

$$P_{ijk,mnt} = \frac{(\boldsymbol{T}^{k-1})_{mi}\boldsymbol{T}_{ij}(\boldsymbol{T}^{t-k})_{jn}}{(\boldsymbol{T}^{t})_{mn}} \quad (4\text{-}14)$$

where $(\boldsymbol{T}^{t})_{mn}$ denotes the probability of a bridge element being in condition rating *m* and being in condition rating *n* after *t* time units. Following Equation 4-14, the expected number of transitions from condition state *i* to condition state *j* within this observation can be obtained:

$$\sum_{k=1}^{t-1} P_{ijk,mnt} \quad (4\text{-}15)$$

In addition, the expected number of transitions from condition rating *i* to condition rating *j* for all such observations is shown by Equation 4-16.

$$O_{mnt}\sum_{k=1}^{t-1} P_{ijk,mnt} \quad (4\text{-}16)$$

where O_{mnt} denotes the number of such observed transitions being in condition rating *m* and being in condition rating *n* after *t* time units. Given all the observed condition rating data, expected number of transitions from condition raing *i* to condition rating $j, n_{ij}^{(P)}$, is estimated as follows:

$$\sum_{m}^{S}\sum_{n}^{S}\sum_{t}O_{mnt}\sum_{k=1}^{t-1}P_{ijk,mnt} \tag{4-17}$$

So far, the Q function is completely defined. By maximizing this function, a new estimate of the transition probabilities matrix $\boldsymbol{T}$ is given by:

$$T_{ij}=\frac{n_{ij}^{(P)}}{\sum_{q=1}^{S}n_{iq}^{(P)}} \tag{4-18}$$

This estimated transition probabilities matrix $\boldsymbol{T}$ is substituted into the Q function, and the E steps and M step stops when the Q function converges. The final estimate of transition probabilities matrix $\boldsymbol{T}$ will be the optimized solution given the observed incomplete condition data.

In this study, discrete-time Markov process is assumed with homogeneous transition probabilities matrix over time. But it may not be practical to hold this assumption. In fact, to meet homogeneity requirement, the condition data can be grouped at different time points so that it is reasonable to assume a homogenous transition probabilities matrix within each group. Then transition probabilities matrixes for different data groups can be estimated separately.

- Expert knowledge

Bridge practitioners with long-term working experience can acquire comprehensive bridge deterioration knowledge from the practice. The knowledge is referred to expert knowledge and deemed to be valuable for bridge deterioration modelling. Since the expert judgements have been verified in practice, it is straightforward to derive parameters based on them. Although subjective judgements may be involved, the newly obtained bridge condition data can mitigate the impacts of expert knowledge by means of periodic Bayesian updating of CPTs [27]. The elicitation process normally consists of five steps [134]: experts selection, experts training, questions preparation, expert judgement elicitation and results verification. First, several bridge maintenance engineers are selected according to their expertise and working experience. Since most of these engineers have no ideas about the art of probability assessment, training courses are necessary for them so that desired information can be provided. Additionally, the elicitation questions must be carefully designed to avoid subjective judgements. Questions can be designed as "What is the probability of a bridge element *E* being in condition rating *K* given all the information *X*?" or as "How likely is a bridge element *E* in condition rating *K* given all the information *X*?" Well designed questions facilitate the probability elicitation. Then the elicitor presents the bridge maintenance engineers the prepared questions friendly so that answers can be properly given. It is essential to ensure the bridge maintenance engineers understand the questions well and more explanations are necessary. Finally, the obtained answers should be checked carefully by the elicitor in order to exclude any incorrect answer. The obtained answers can be converted into conditional probabilities directly or indirectly. If the bridge maintenance engineers cannot indicate exact numbers but only a few words for his degree of believes on a scale, for instance, certain (100%-90%), probable (90%-75%), likely (75%-50%), unlikely (50%-25%), improbable (25%-10%), impossible (10%-0), the average number of each scale can be seen as the estimated probabilities. Other issues relating to probability elicitation from expert knowledge can be found from [134]. Overall, the efficiency and quality of this solution are totally dependent on the abilities of bridge practitioners. However, for a bridge system, elicitation work involves a formidable amount of conditional probabilities, which will definitely affect the coherence of bridge engineers' judgements. Also, the whole process tends to be quite time-consuming. Being aware of this disadvantage, all the efforts should be done to relieve the burden of elicitation before any actual probability elicitation work. In practical networks, some assumptions can ease the parameters elicitation from experts. For example, if it is reasonable to assume the influence of each parent node is independent, the Noisy-OR [125] or its extension Noisy-Max [41] can be applied. The joint CPTs are obtained from marginal conditional probability specified for each parent node by using the max function, so the number of parameters is reduced logarithmically.

Basically, some CPTs can be filled in by the developer based on miscellaneous knowledge. For example, maintenance variables have a dominant influence on the bridge elements deterioration compared with other variables. By defining the impacts of different maintenance activities, the CPT of a bridge element can be identified partially. Normally, replacement and perfect repair bring bridge elements into good condition. Minimal repair and no maintenance leave bridge elements in the same condition as before. Imperfect maintenance brings bridge elements into the condition better than past one but worse than good condition. In this research, imperfect maintenance is represented by the probabilities over possible condition ratings of a bridge element. The CPTs of observation nodes can be estimated based on the nature of inspection methods. For instance, if observed information is obtained through visual inspection, the CPT of this observation associated to a bridge element is set to be 1. Moreover, if observations are obtained from NDT or monitoring techniques, CPTs can be estimated from a probability of detection (PoD) model or measurement accuracy, respectively.

- Combination of limited bridge condition data and expert knowledge

Usually, when bridge condition data are limited or insufficient, CPTs estimation relies on expert knowledge. However, because of the subjective judgements from experts, the CPTs estimated from expert knowledge always tend to be biased. Meanwhile, the limited bridge data do provide some valuable information for CPTs estimation, which will improve the accuracy. So far, a formal method that assists the combination of limited environmental data and elicited expert knowledge in an ecological risk assessment has been presented by Pollino et al. [128]. In this study, a modified two-step method to combine the limited bridge data and expert knowledge is proposed as shown in Figure 4-9. The first step aims to formalise a criterion based on the limited bridge data and to regulate experts' estimation so that less subjective estimation of initial CPTs is obtained. The second step is an iteration process for combining the limited bridge data and different sizes of hypothetical data sampled from the initial CPTs.

It is of course that limited bridge data cannot be used to estimate the whole CPTs at all. Nonetheless, based on those limited bridge data, the marginal conditional probabilities that are conditional on only parts of parent variables can be obtained by using learning algorithms generally. For instance, bridge data that appear to be inadequate to estimate the CPTs in Figure 4-6 can still be utilised to parameterise the ones in Figure 4-4. Since the modelling in Figure 4-4 is a simplified case of the one in Figure 4-6, the parameterised CPT through limited bridge data can be considered as the marginal conditional probability specified as $p(E(\mathrm{t})|E(\mathrm{t}\text{-}1))$ in Figure 4-6. In addition, the obtained marginal conditional probabilities can function as a criterion so that the experts can adjust their estimation to reduce their subjective judgements. Here, a common criterion applicable for different BNs structures is given.

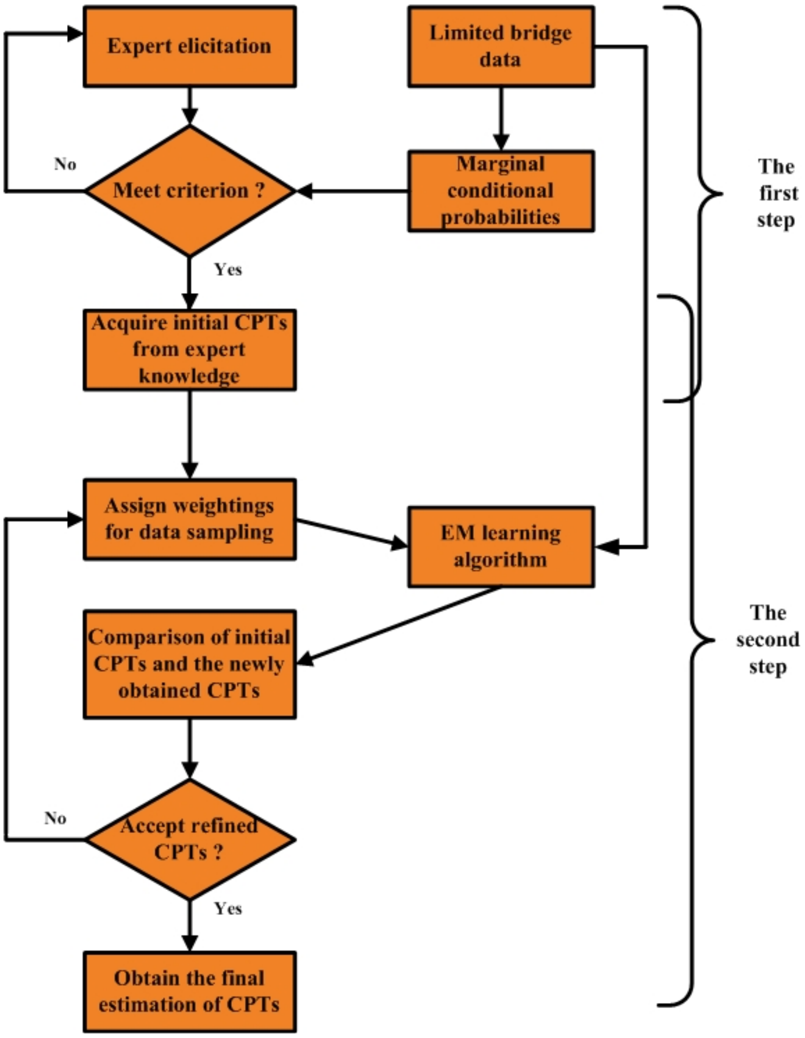

Figure 4-9. The proposed two-step method for combination of limited bridge data and expert knowledge

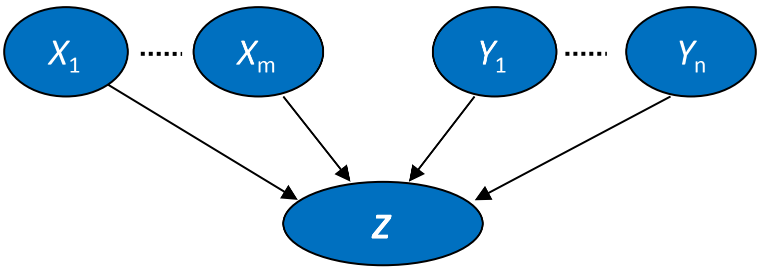

Figure 4-10. A general BN

In Figure 4-10, a general BN is illustrated with a random variable Z conditional on a set of random variables $\mathbf{X}=\{X_1,\ldots,X_m\}$ and a set of random variables $\mathbf{Y}=\{Y_1,\ldots,Y_n\}$. $\mathbf{X}$ denotes all the parent variables for which marginal conditional probabilities could be obtained based on available data. $\mathbf{Y}$ denotes all the other parent variables without data. On the one hand, by implementing learning algorithms, the marginal conditional probabilities $P(Z|\mathbf{X})$ are calculated based on available data first. On the other hand, $P(Z|\mathbf{X})$ can be estimated by using Bayes' theorem shown as Equation 4-19.

$$P(Z|\mathbf{X}) = \frac{P(Z,\mathbf{X})}{P(\mathbf{X})} = \frac{\sum_{\mathbf{Y}} P(Z,\mathbf{X},\mathbf{Y})}{P(\mathbf{X})} = \frac{\sum_{\mathbf{Y}} P(Z|\mathbf{X},\mathbf{Y})P(\mathbf{X},\mathbf{Y})}{P(\mathbf{X})} \tag{4-19}$$

where $P(Z|\mathbf{X},\mathbf{Y})$ is the complete CPTs to be estimated from expert knowledge; $P(Z,\mathbf{X},\mathbf{Y})$, $P(Z,\mathbf{X})$, $P(\mathbf{X},\mathbf{Y})$ and $P(\mathbf{X})$ are the joint probabilities of the corresponding variables. If $\mathbf{X}$ and $\mathbf{Y}$ are independent, the above equation can be simplified into Equation 4-20.

$$P(Z|\mathbf{X}) = \frac{\sum_{\mathbf{Y}} P(Z|\mathbf{X},\mathbf{Y})P(\mathbf{X},\mathbf{Y})}{P(\mathbf{X})} = \frac{\sum_{\mathbf{Y}} P(Z|\mathbf{X},\mathbf{Y})P(\mathbf{X})P(\mathbf{Y})}{P(\mathbf{X})} = \sum_{\mathbf{Y}} P(Z|\mathbf{X},\mathbf{Y})P(\mathbf{Y}) \tag{4-20}$$

where $P(\mathbf{Y})$ is the joint probabilities of all the parent variable without data. In addition, if each variable of $\mathbf{Y}$ is independent each other, Equation 4-20 can be further simplified into Equation 4-21

$$P(Z|\mathbf{X}) = \sum_{\mathbf{Y}} P(Z|\mathbf{X},\mathbf{Y})P(\mathbf{Y}) = \sum_{\mathbf{Y}} P(Z|\mathbf{X},\mathbf{Y})P(\mathrm{Y}_1)\cdots P(\mathrm{Y}_n) \tag{4-21}$$

where $P(\mathrm{Y}_1)\cdots P(\mathrm{Y}_n)$ are the marginal probabilities of each variable. Equation 4-19 is applicable for different BNs to help the experts regulate their estimation. With the marginal conditional probabilities $P(Z|\mathbf{X})$ estimated from available data, the experts should make their estimation based on Equation 4-19. Depending on the relationships among different parent variables, Equation 4-19 may be changed into Equation 4-20 or Equation 4-21. Regarding to the different joint probabilities, the probabilities can also be estimated from expert knowledge directly. Alternatively, if conditional relationships exist, the joint probabilities are calculated by further using Bayes' theorem.

In the first step, CPTs with the regulation from the criteria are estimated based on expert knowledge. During the expert elicitation process, the experts are required to assign a weight to each estimated parameter based on their confidence. If the expert is confident about his estimation, a high weight is assigned, and vice versa. Afterwards, these weightings are considered to be equivalent to the size of initial sampled data from the estimated CPTs. The scale of weightings is really dependent on the total size of data needed by learning algorithms and the size of available data. This issue will not be discussed in details.

In the second step, the hypothetical data are sampled from parent nodes to child nodes based on the CPTs estimated from expert knowledge. Then the sampled data and the available data are integrated into EM learning algorithm for parameters estimation. The newly obtained CPTs are compared with the original CPTs. Now there exist several methods for measuring the similarity between two probability distributions. Some of commonly used methods are Euclidean distance, Kullback-Leibler distance and Bhattacharyya distance. Higher distance between two CPTs indicates further improvements. Additionally, the newly obtained CPTs are also examined by the experts to see if each parameter is in the acceptable range. Any parameter treated as unrealistic is flagged for improvements and assigned with a new weighting value. Then the hypothetical data are generated again and the learning process is repeated. Finally, the experts take the responsibility to determine if the refined parameters are accepted or not. When further improvements are needed, the process is iterated. With regarding to weightings adjustment, an iterative algorithm [128] shown in Figure 4-11 can be applied. Again, the value of each weighting change is dependent on the scale of weighting. As for CPTs estimation of native fish BNs [128], an uplarge or downlarge was assigned with a weighting of five; upsmall/downsmall was assigned with a weighting of three; bounceup/bouncedown was assigned with a weighting of two; and a tweak was assigned with a weighting of one.

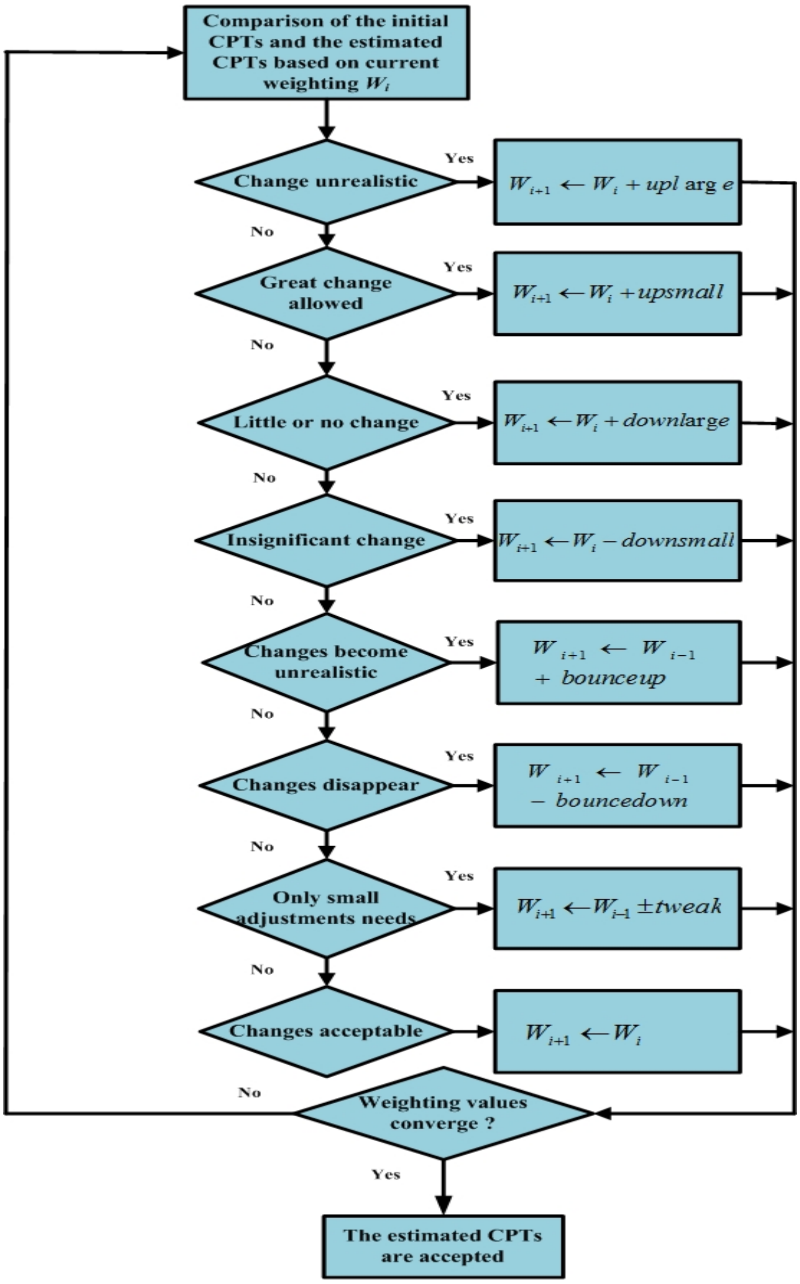

Figure 4-11.An iterative algorithm for weightings adjustment

➢ Other sources

There are also other sources available for parameters estimation, such as, experimental data and simulated data. Experimental data can be yielded from the experiments designed for bridge deterioration. However, it always involves a great deal of work but only acquires a small amount of data, which is not so cost-effective. Simulated data are generated from a theoretical deterioration model that is based on physical and chemical deterioration processes of bridge. By quantifying the parameters related to the bridge deterioration, the development of deterioration over time can be simulated in a quantitative manner based on Monte Carlo simulation. As a result, the deterioration can be mapped into a number of condition ratings that are used as simulated historical condition data for CPTs estimation. Nevertheless, it is often computationally intensive to simulate bridge deterioration since there are plenty of correlated parameters and each parameter is probabilistic rather than deterministic.

Overall, parameters estimation could be undertaken with different data sources. Proper methods should be chosen according to the data availability. It should be also noticed that the estimated CPTs need to be reviewed by bridge experts and engineers to determine if the CPTs really reflect the practical situation. Some modifications may be needed upon their comments.

4.3 Case study of a railway bridge: condition ratings prediction

The proposed DOOBNs model is applied to a railway bridge "Albert Bridge" (Figure 4-12) located in Brisbane, Queensland. The bridge functioning as a railway bridge is a two-span steel truss bridge built in 1893. A tailored Model I is developed for the bridge for condition ratings prediction in the next 100 years.

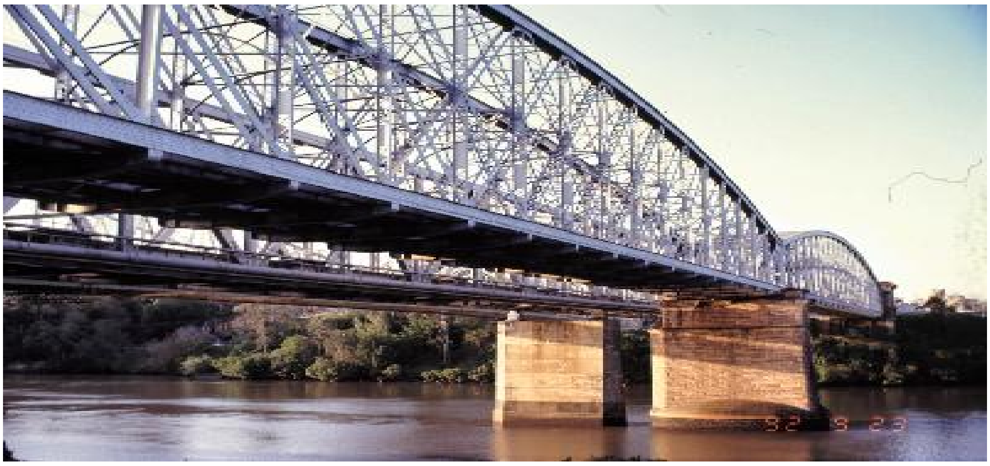

Figure 4-12. Picture of Albert Bridge in Brisbane, Queensland

4.3.1 Development of DOOBNs model for condition ratings

4.3.1.1 Bridge system analysis

To facilitate the development of DOOBNs model, a systematic analysis for Albert Bridge is implemented with the help of bridge maintenance engineers. The bridge is hierarchically decomposed into four levels with relative weights for each bridge hierarchy

(Table 4-1). The relative weights were directly assigned by bridge practitioners based on the importance of each bridge hierarchy to the deterioration of the whole bridge systems. Bridge condition ratings defined by Department of Main Roads, Queensland [129] are adopted in this case study. The whole bridge system, superstructure and substructure are defined with five condition ratings (Table 2-2), with CS1 denoting “Good condition”, CS4 denoting “Poor condition” and CS5 denoting “Unsafe condition”. Other bridge hierarchies (e.g. bridge elements) are defined with four condition ratings according to their materials and deterioration processes[129]. For steel bridge elements, four condition ratings are generally defined as CS1 (“Sound paint”), CS2 (“Paint distress”), CS3 (“Active corrosion”) and CS4 (“Strength loss”). For bridge elements made of stone masonry and red brick, four condition ratings are generally defined as CS1 (“Good condition”), CS2 (“Minor cracking”), CS3 (“Moderate cracking”) and CS4 (“Severe cracking”). In terms of detailed descriptions of all the condition states of bridge elements, please refer to the Bridge inspection manual [129]. Moreover, two deterioration dependencies phenomena among different bridge elements are identified. First, the riveted joints suffer from pack rusting and crevice corrosion. Because of tensioning, the pack rusting can cause elongation of rivets shank that finally reduces the shear capacity of rivets. In addition, when paint coating around rivets is failed, crevice corrosion happens to rivets shank, which can cause wasting of rivet shank. If one rivets joint fails, the load that it was carrying will be transferred to the adjacent joints. Therefore, with the increase of load, the adjacent joints will deteriorate quicker than before and affect the safety of the whole structure, finally. Second, owing to debris buildup in the bearing and corrosion, there is a minor effect on the bearing’s movement capabilities, which may cause cracking or spalling in the bearing support. However, in this case study, since rivet joints and bearing supports are not modelled individually but included in other bridge elements, only environmental effects and maintenance actions are considered. Four environmental levels [60]: Benign, Low, Moderate and Severe, are used, and the maintenance actions are assumed to be perfect.

Table 4-1. Decomposition of Albert Bridge with relative weights

Level 1 (Top level)	**Level 2**	**Level 3**	**Level 4 (Bottom level)**
The whole bridge	Bridge superstructure (3)	Truss member (2)	Main girder (3)
			Upper chord (2)
			Diagonals (1)
			Vertical (1)
			End post (1)
		Top + bottom wind bracing (1)	Diaphragm (1)
			Top wind bracing (1)
			Bottom wind bracing (1)
			Portal wind bracing (1)
			Top lateral bracing (1)
		Flooring system (2)	Longitudinal girder (2)
			Cross girder (3)
		Bearing (2)	
	Bridge substructure (3)	Pier (2)	Pier cap (1)
			Pier wall (1)
		Abutment (2)	Wing wall (1)
			Abutment wall (1)

4.3.1.2 DOOBNs model development of Albert Bridge

Based on the system analysis above, the conditional relationships among bridge hierarchies have been identified. The OOBNs models for bridge hierarchies in different levels are presented in Figures 4-13-4-20, where input nodes are ellipses with dashed line border and output nodes are ellipses with shadow bold line borders. By connecting these OOBNs models in different levels, the BNs model of the whole bridge system (Figure 4-21) can be derived. Then, the DOOBN models, accounting for temporal deterioration processes of each bridge element, are further constructed by means of discrete-time Markov process. Additionally, with the consideration of environmental effects, maintenance actions as well as the observations of bridge elements, three variables are introduced to the DOOBN model. For instance, Figures 4-22 and 4-23 present the BNs class corresponding to deterioration processes of a main girder and its DOOBN model for temporal behaviours, respectively. The outputs of this DOOBN model at each time slice are input into the bridge main girder modelled in Figure 4-21. Similarly, the outputs of other DOOBN models specified for other bridge elements at each time slice are also input into the corresponding bridge elements in Figure 4-21 so that condition ratings of the whole bridge are updated each time.

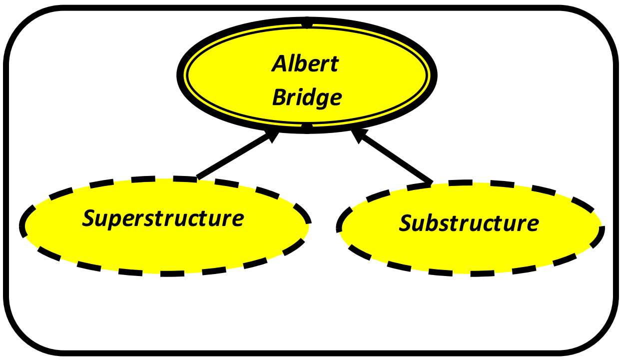

Figure 4-13. OOBN model of the whole bridge in Level1

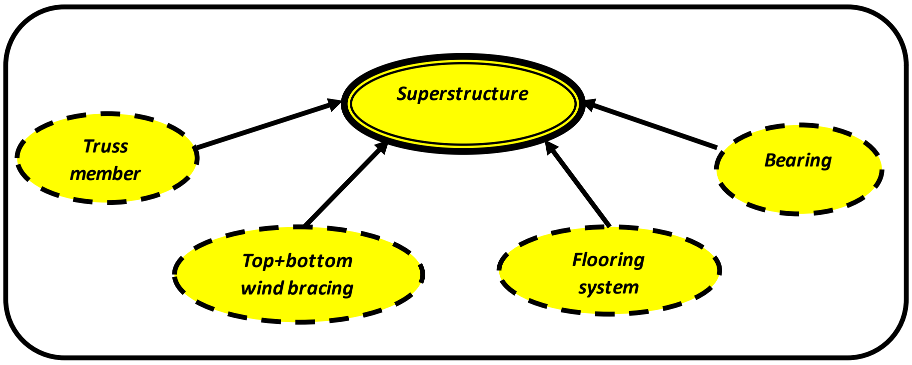

Figure 4-14. OOBN model of the superstructure in Level 2

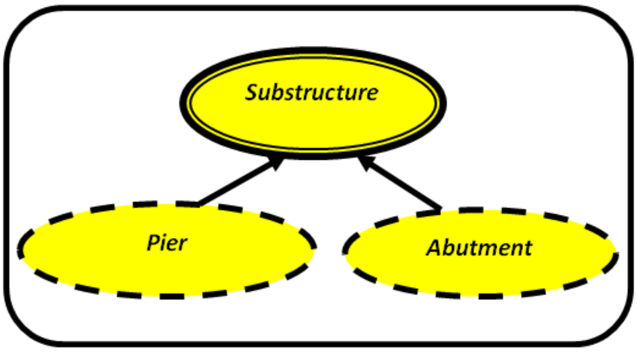

Figure 4-15. OOBN model of the substructure in Level 2

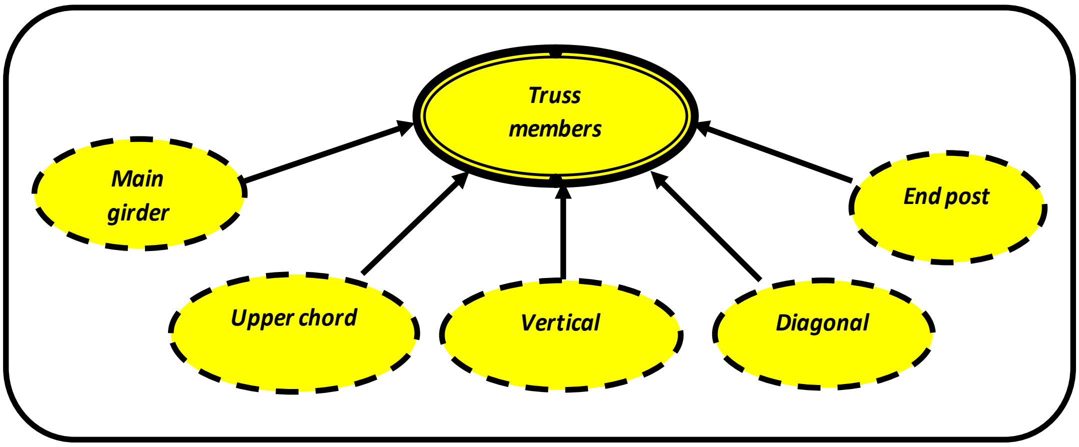

Figure 4-16. OOBN model of the Truss members in Level 3

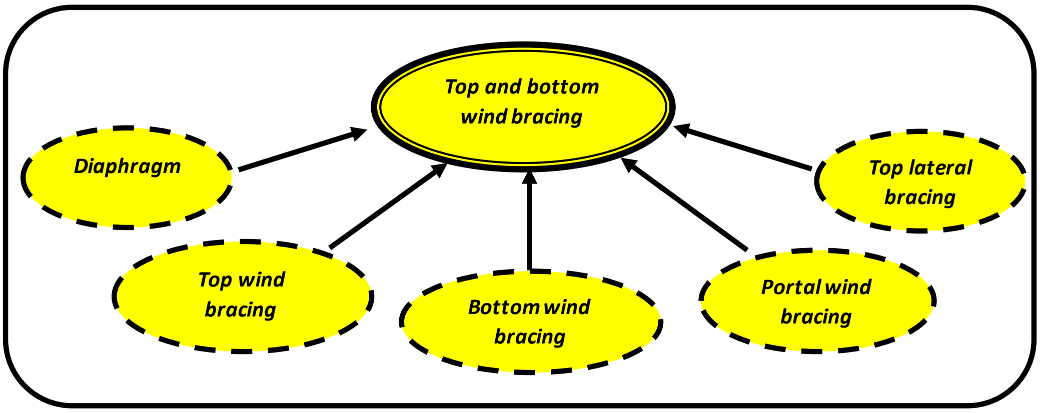

Figure 4-17. OOBN model of the Top + bottom wind bracing in Level 3

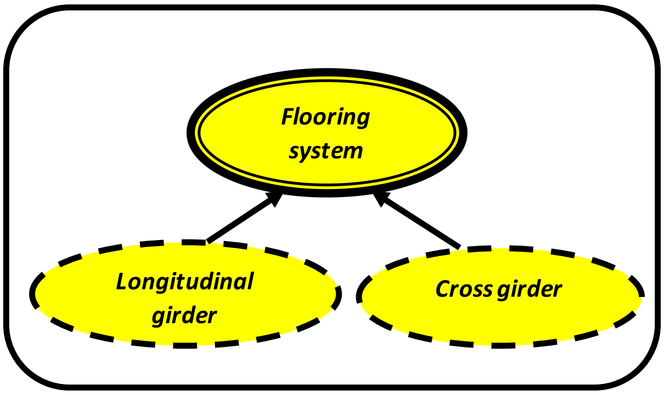

Figure 4-18. OOBN model of the Flooring system in Level 3

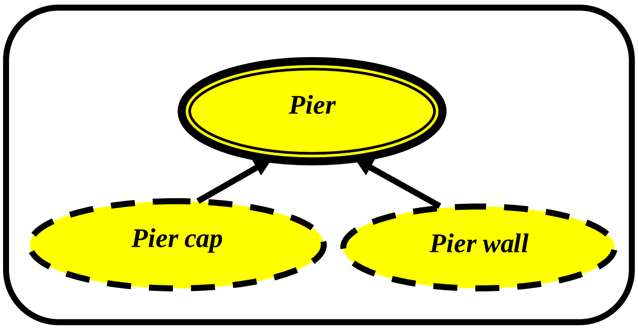

Figure 4-19. OOBN model of the Pier in Level 3

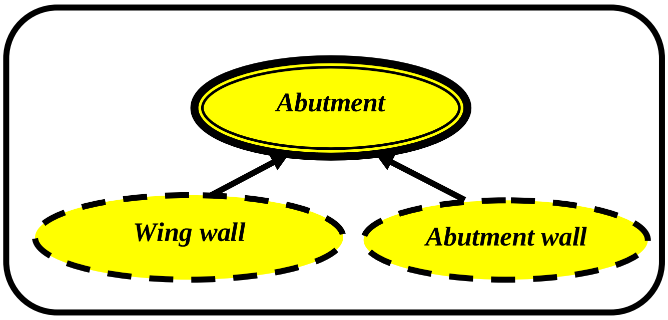

Figure 4-20. OOBN model of the Abutment in Level 3

Figure 4-21. The whole bridge system BNs model of Albert Bridge

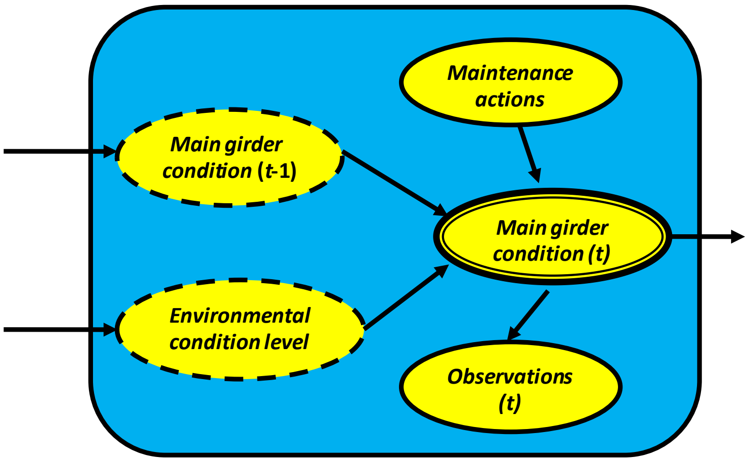

Figure 4-22. BN class for deterioration processes of a main girder for condition ratings prediction

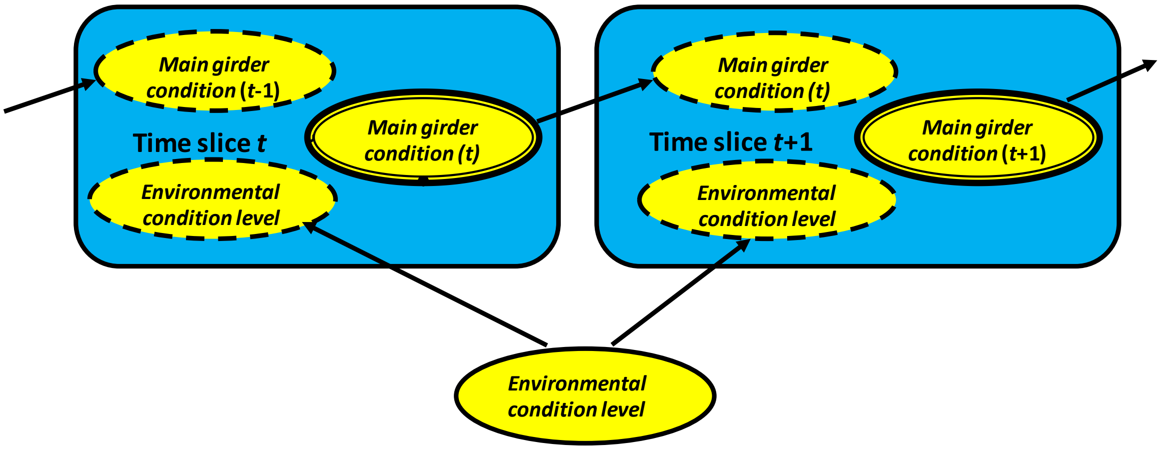

Figure 4-23. DOOBNs model of a bridge main girder for condition ratings prediction

4.3.1.3 CPTs estimation for the DOOBN model of Albert Bridge

As there are no historical condition rating data available for “Albert Bridge”, the CPTs are essentially estimated from the relative weights in Table 4-1 and expert knowledge. Regarding bridge hierarchies, the weighted sums of condition ratings are calculated based on Equation 4-2. CPTs are filled out according to the calculated results. Table 4-2 shows the estimated CPT of the flooring system of which condition rating is conditional on the cross girder and longitudinal girder.

Table 4-2. The CPT of flooring system

Cross girder	**Sound paint**				**Paint distress**			
Longitudinal girder	**Sound paint**	**Paint distress**	Longitudinal girder	**Strength loss**	**Sound paint**	**Paint distress**	**Active corrosion**	**Strength loss**
Sound paint	*1*	*0.6*	*0.4*	*0*	*0.4*	*0*	*0*	*0*
Paint distress	*0*	*0.4*	*0.6*	*0.8*	*0.6*	*1*	*0.6*	*0.2*
Active condition	*0*	*0*	*0*	*0.2*	*0*	*0*	*0.4*	*0.8*
Strength loss	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
Cross girder	**Active corrosion**				**Strength loss**			
Longitudinal girder	**Sound paint**	**Paint distress**	**Active corrosion**	**Strength loss**	**Sound paint**	**Paint distress**	**Active corrosion**	**Strength loss**
Sound paint	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
Paint distress	*0.8*	*0.4*	*0*	*0*	*0.2*	*0*	*0*	*0*
Active condition	*0.2*	*0.6*	*1*	*0.6*	*0.8*	*0.8*	*0.4*	*0*
Strength loss	*0*	*0*	*0*	*0.4*	*0*	*0.2*	*0.6*	*1*

For bridge elements, CPTs initially rely on expert knowledge. Based on comprehensive practical knowledge about bridge deterioration, the experts are able to provide their estimation about relative percentages of each condition rating under different environmental levels over a certain period of time. For instance, Table 4-3 illustrates the estimated condition percentages for the cross girder over 20 years, where the initial condition is assumed to be “Sound paint”. Based on these data, the least square method (Equation 4-3) is employed in order to minimise the differences between expert estimation and the expected percentages calculated from transition probabilities matrix. By using the Optimization Toolbox in MATLAB Software, all the CPTs associated to bridge elements can be estimated. Table 4-4 presents the estimated CPT associated to the bridge main girder under the environmental level of “low”. The CPT describes the discrete-time Markov process that models the bridge element deterioration with the considerations of environmental effects and maintenance actions.

Table 4-3. Relative condition percentages for the cross girder under the environmental level of “severe” over 20 years provided by bridge experts

Time (year)	**0**	**5**	**10**	**15**	**20**
Sound paint	*100%*	*10%*	*5%*	*0*	*0*
Paint distress	*0*	*15%*	*10%*	*5%*	*0*
Active corrosion	*0*	*75%*	*75%*	*45%*	*15%*
Strength loss	*0*	*0*	*10%*	*50%*	*85%*

Table 4-4. The CPT of a main girder under the environmental level of "low"

Environmental condition levels	Low							
Maintenance action	No maintenance				Maintenance			
Main girder self (t-1)	Sound paint	Paint distress	Active corrosion	Strength loss	Sound paint	Paint distress	Active corrosion	Strength loss
Sound paint	0.9802	0	0	0	1	1	1	1
Paint distress	0.0198	0.9019	0	0	0	0	0	0
Active condition	0	0.0981	0.9445	0	0	0	0	0
Strength loss	0	0	0.0555	1	0	0	0	0

4.3.2 Prediction results of condition ratings

With the accomplishment of CPTs estimation, the Model I predicts the condition evolution of the whole bridge as well as bridge hierarchies in the next 100 years. The operation is supported by the software GeNIe [56], which actually runs the inference algorithm for the condition ratings prediction. In this case study, because the current status of Albert Bridge shows no damage at all, condition ratings of all the bridge elements are assumed to be CS1 ("Sound paint" or "Good condition"). Two scenarios are conducted. First, a perfect maintenance action is simulated to bridge main girder at 50th year. This maintenance action renews the main girder into the condition "Sound paint", aiming to demonstrate its propagation through the DOOBN model. Second, to show the Bayesian updating ability, condition ratings data in Table 4-5 are simulated to bridge cross girder over 20 years. The simulation accords to normal inspection procedures. With a five-year inspection interval, visual inspection is implemented to rate the conditions of all the bridge main girders. The calculated percentages over different condition ratings are listed in Table 4-5. In addition, the effects of different environmental conditions are considered.

Table 4-5. Simulated condition rating percentages for bridge cross girder

Inspection time (year)	5	10	15	20
Sound paint (%)	100	50	0	0
Paint distress (%)	0	50	50	0
Active condition (%)	0	0	50	5
Strength loss (%)	0	0	0	95
Total (%)	100	100	100	100

By running the DOOBN model, the condition ratings probabilities of all the bridge hierarchies in next 100 years under different environmental conditions are acquired. In the first scenario, Figure 4-24 illustrates the evolution curve of the bridge main girder under the environmental level of “moderate” over 100 years. At 50th year, the condition rating of bridge main girder was renewed because of the maintenance activity. The condition ratings probabilities of the truss members and the flooring system under the environmental level of “moderate” over 100 years were presented by Figure 4-25 and Figure 4-26, respectively. According to the causal relationships modelled by DOOBN, the maintenance activity actually plays a role in the evolution of bridge truss members. However, owing to the deterioration of other bridge elements, bridge truss members are not renewed to be the condition of “Sound paint”. Conversely, bridge flooring system is immune from the maintenance activity because no conditional dependencies between the bridge main girder and the bridge flooring system are shown in Figure 4-21. The maintenance activity also has an impact on both bridge superstructure and the whole bridge. Figure 4-27 and Figure 4-28 exhibit the sudden changes at 50th years happening in both condition evolutions of superstructure and the whole bridge under the environmental level of “moderate”, respectively. Nonetheless, the sudden changes become less obvious as the modelling level turns to be higher. The same trend can also be found in Figure 4-29 and Figure 4-30 when the environmental condition comes to “Severe”.

In the second scenario, the simulated information in Table 4-5 was used to update the condition probabilities of the bridge cross girder. The original and updated evolution curves of the bride cross girder under the condition level of “Low” over 100 years are presented in Figure 31 and Figure 32, respectively. By comparison, we can see there are a large number of updates in condition probabilities of the bridge cross girder based on the observation. Moreover, because of causal relationships modelled by the DOOBN model, the effects of the observed information also propagate from the bridge elements to the whole bridge. Figure 33 and Figure 34 illustrate the original and updated condition probabilities of bridge flooring system under the environmental level of “Low” over 100 years, respectively. Obvious differences between these two curves have been found. However, as other bridge elements also deteriorate, the effects of the observed information become weaker and weaker when it propagates to higher levels. The original and updated condition evolutions of “Albert Bridge” are shown in Figure 35 and Figure 36, respectively. It only subjects to minor updates based on the simulated information.

Based on the two simulated scenarios, the Model I for "Albert Bridge" has approved the feasibility and merits of the proposed DOOBN model in bridge performance prediction. Not only the condition probabilities of the whole bridge but also the condition probabilities of other bridge hierarchies (bridge elements) were predicted. The DOOBN model is able to account for observed information and deterioration dependencies from maintenance actions and environmental effects so that more accurate prediction results are achieved. Although the long-term prediction results are not compared with the ones from the conventional methods due to the limited condition data, the Bayesian updating ability can secure the continuing improvement of the prediction results with more available condition data. In the future, the prediction results, which provide the insight into future performance, can be utilised for optimal planning of maintenance actions.

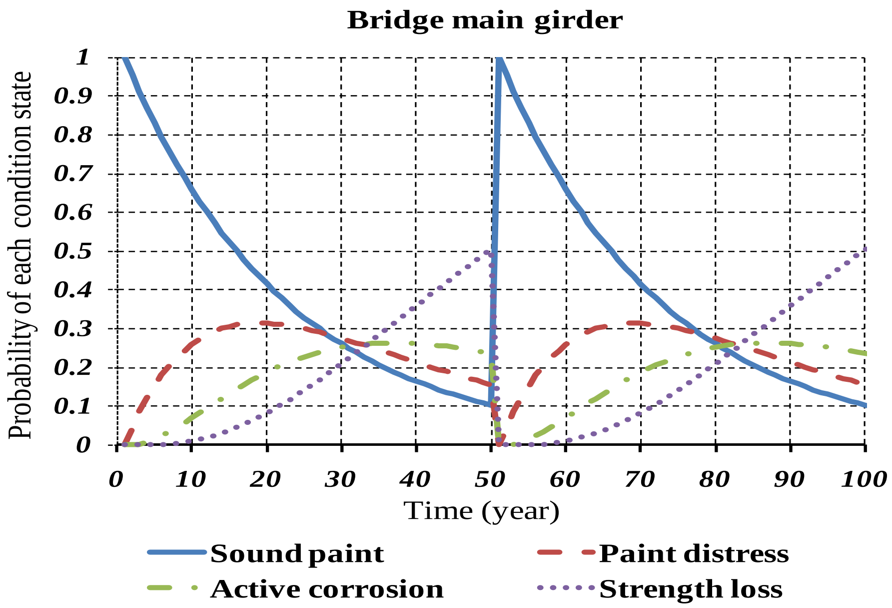

Figure 4-24. Condition states probabilities of bridge main girder over the next 100 years under the environmental level of "Moderate" and a perfect maintenance action at 50th year

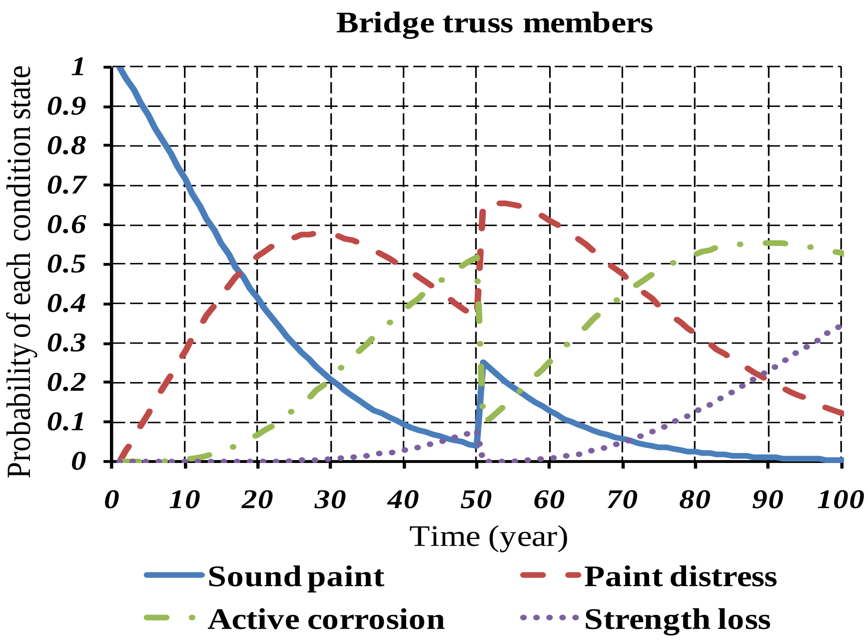

Figure 4-25. Condition states probabilities of bridge truss members over the next 100 years under the environmental level of “Moderate” and a perfect maintenance action on bridge main girder at 50th year

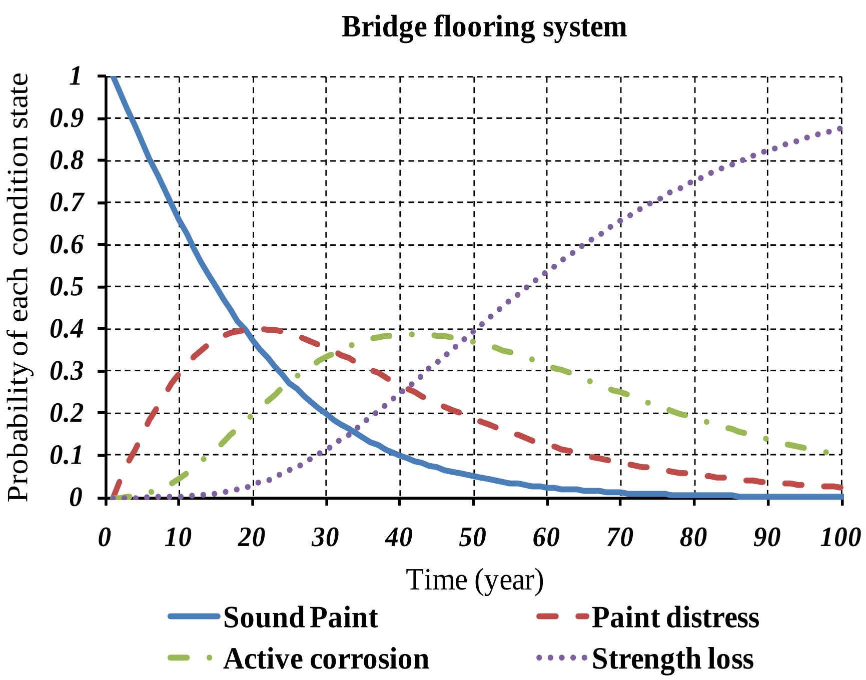

Figure 4-26. Condition states probabilities of bridge flooring system over the next 100 years under the environmental level of “Moderate” and a perfect maintenance action on bridge main girder at 50th year

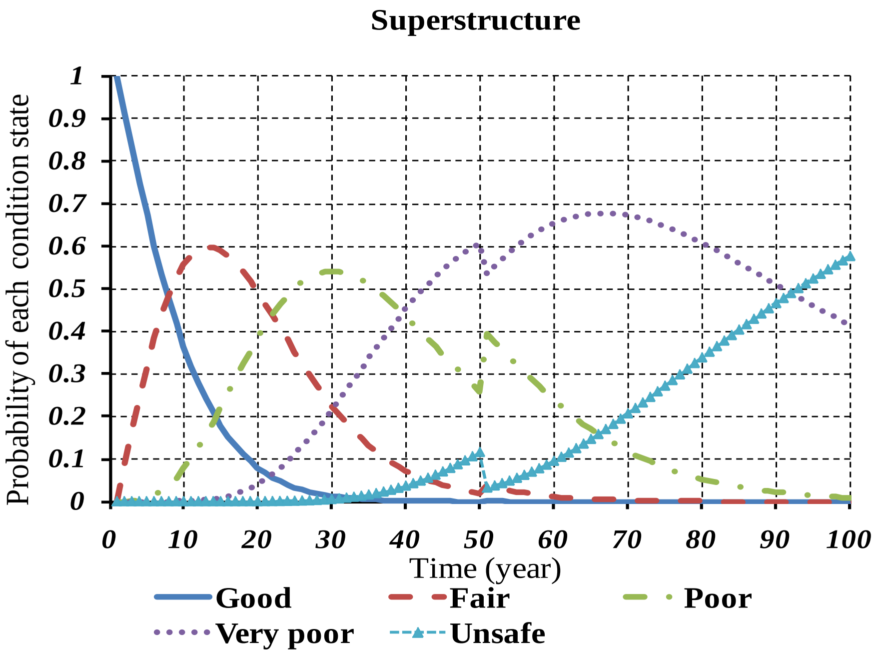

Figure 4-27. Condition states probabilities of bridge superstructure over the next 100 years under the environmental level of “Moderate” and a perfect maintenance action on bridge main girder at 50th year

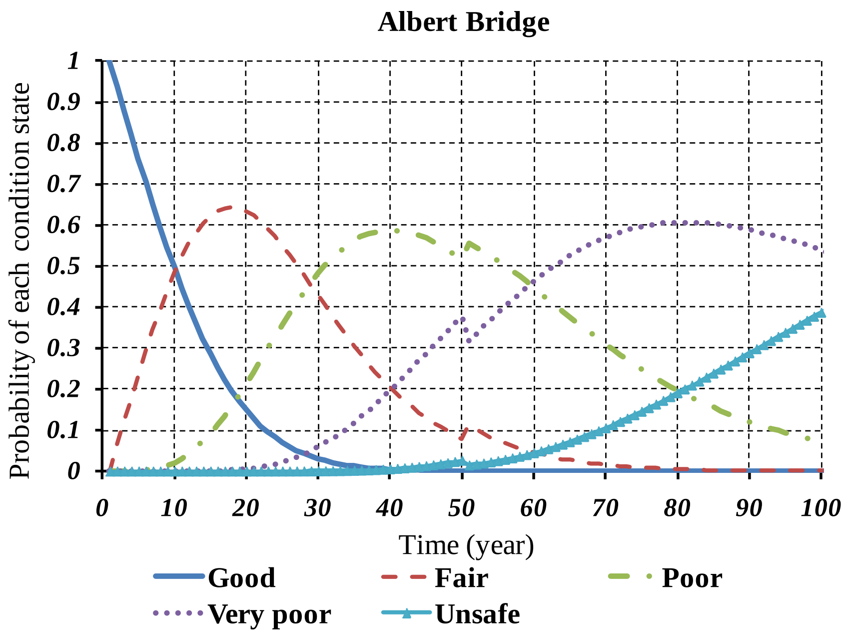

Figure 4-28. Condition states probabilities of Albert Bridge over the next100 years under the environmental level of “Moderate” and a perfect maintenance action on bridge main girder at 50th year

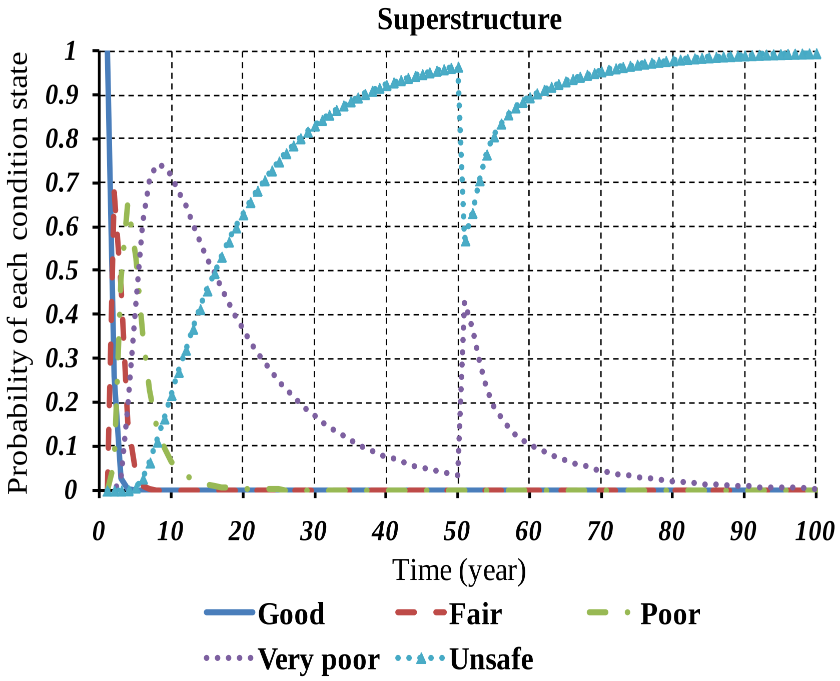

Figure 4-29. Condition states probabilities of bridge superstructure over the next 100 years under the environmental level of “Severe” and a perfect maintenance action on bridge main girder at 50th year

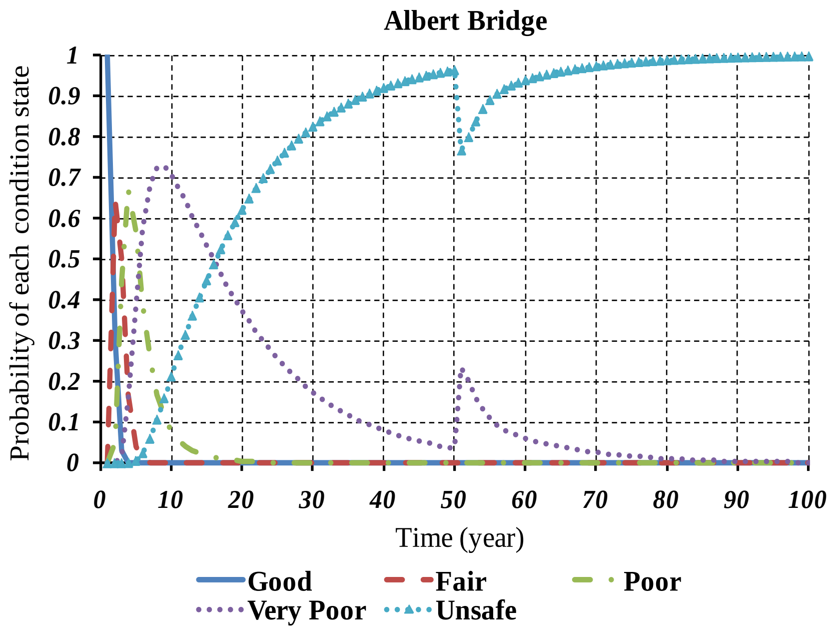

Figure 4-30. Condition states probabilities of Albert Bridge over the next100 years under the environmental level of “Severe” and a perfect maintenance action on bridge main girder at 50th year

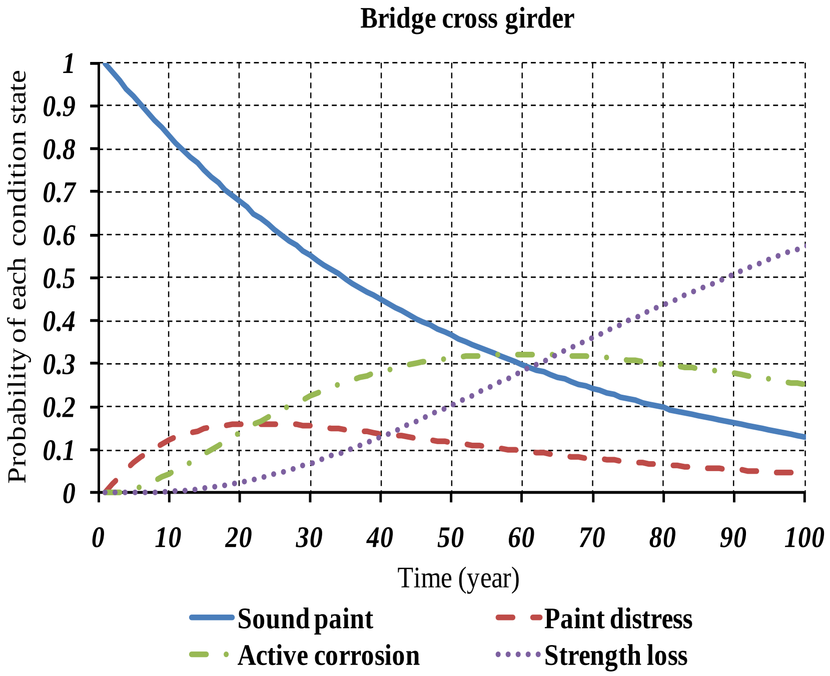

Figure 4-31. Original condition states probabilities of bridge cross girder over the next 100 years under the environmental level of “Low”

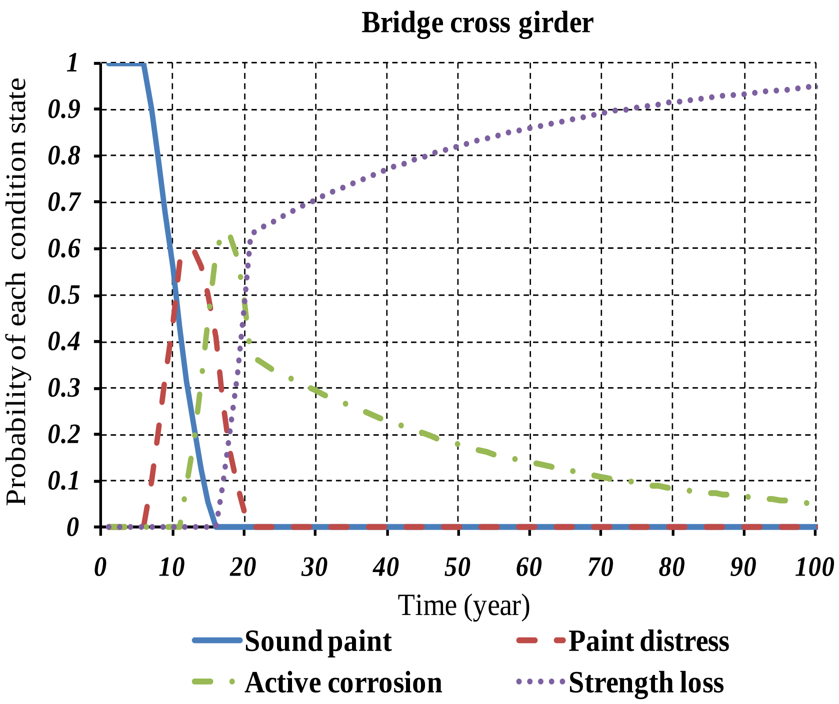

Figure 4-32. Updated condition states probabilities of bridge cross girder with observation

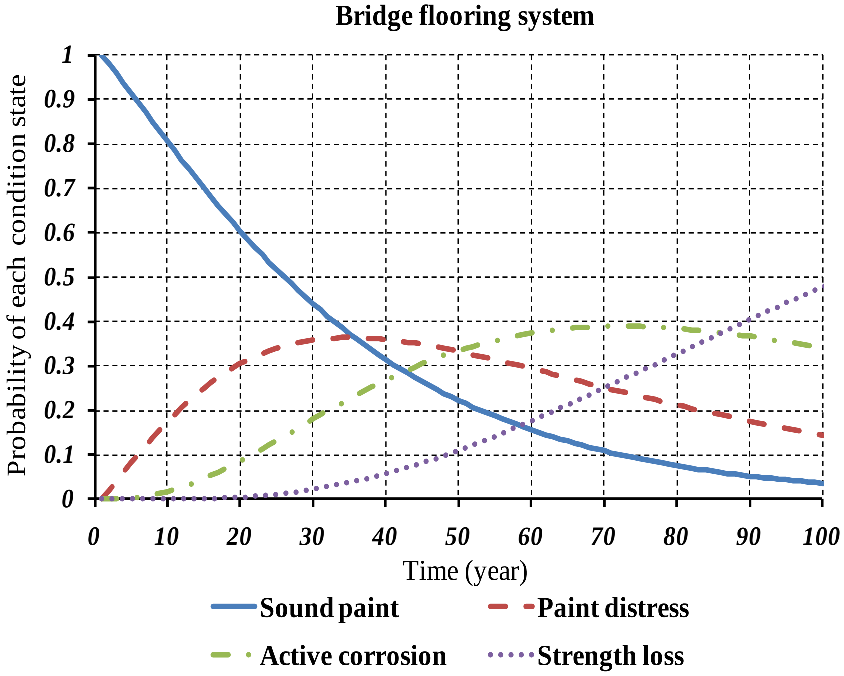

Figure 4-33. Original condition states probabilities of bridge flooring system over the next 100 years under the environmental level of “Low”

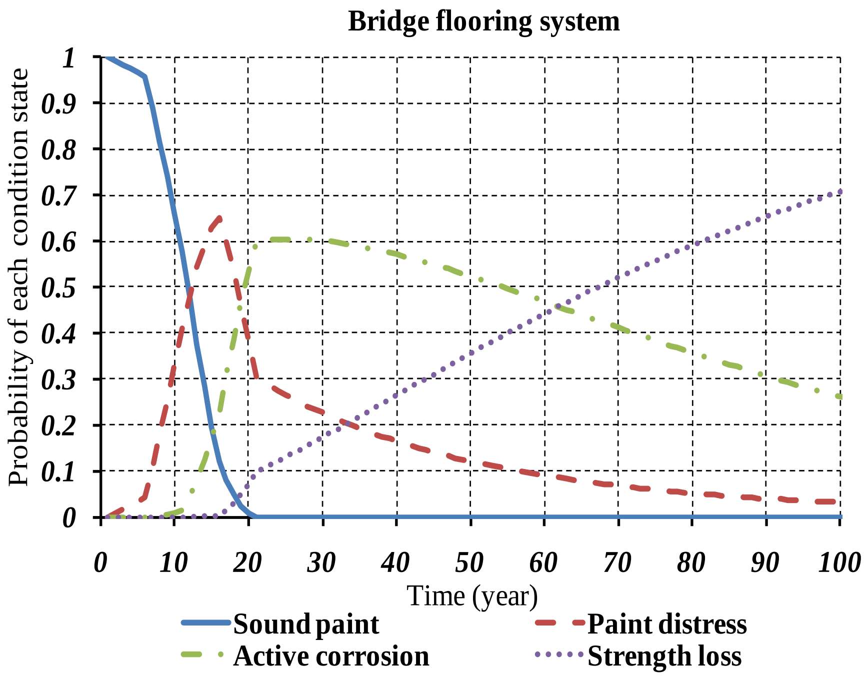

Figure 4-34. Updated condition states probabilities of bridge flooring system with observation

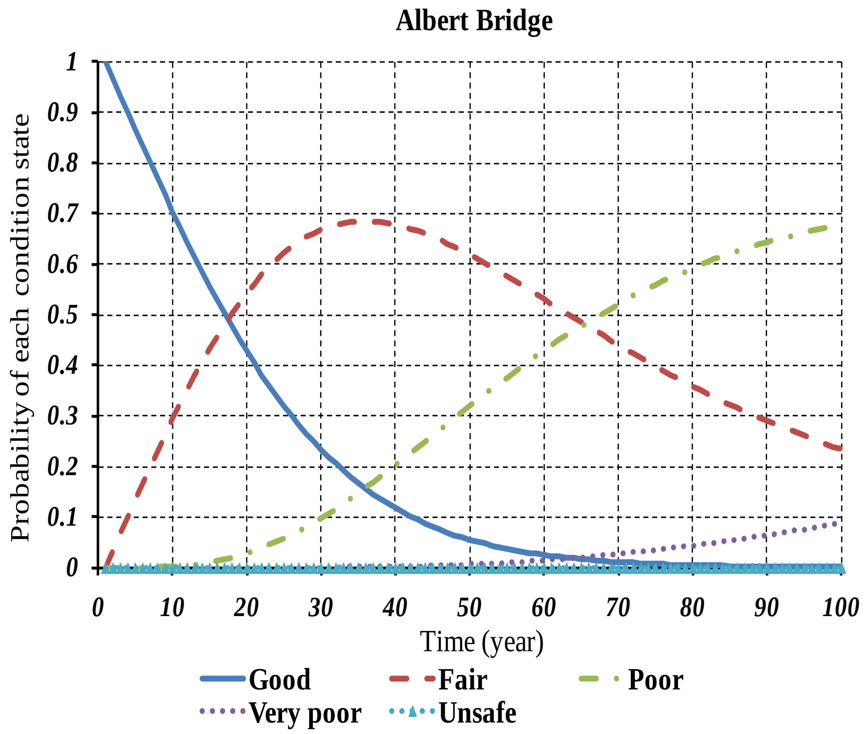

Figure 4-35. Original condition states probabilities of Albert Bridge over the next 100 years under the environmental level of “Low”

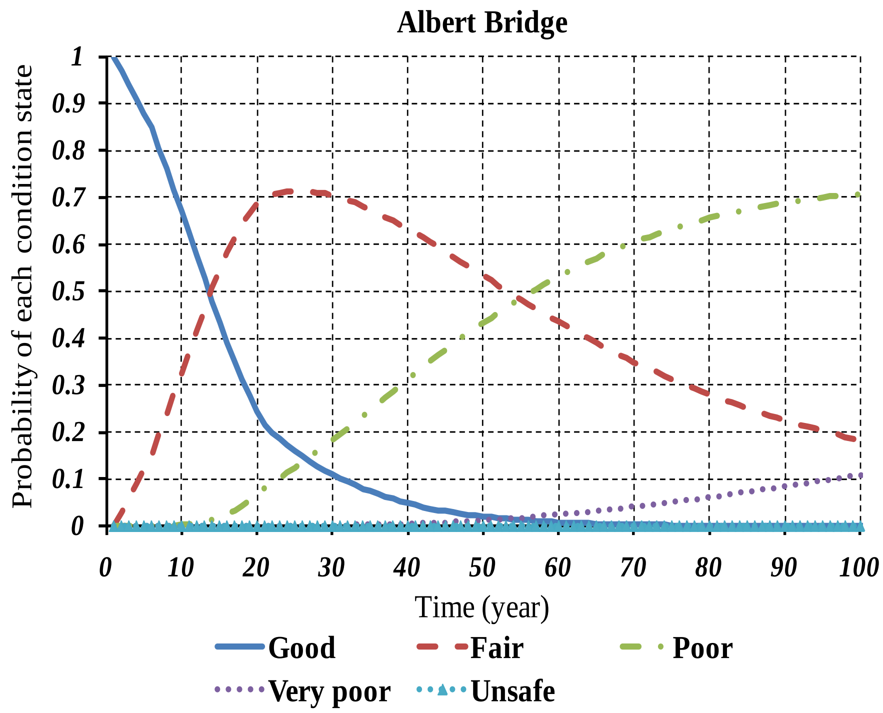

Figure 4-36. Updated condition states probabilities of Albert Bridge with observation

4.4 Summary

The objective of this chapter is to develop a Model I based on DOOBNs for generally applicable condition states prediction. The proposed model is characterised by probabilistic modelling of bridge deterioration in hierarchical way, and is outlined through three steps: modelling consideration, DOOBN development and parameters estimation. The first step carries out a systematic analysis aiming to provide the necessary information to establish DOOBN conceptual model. Then the DOOBNs are built up from two parts: bridge hierarchies and bridge elements. The last step focuses on the estimation of the CPTs for the DOOBN model. To demonstrate the practicability and benefits of our proposed DOOBN model, an application is given to a steel truss railway bridge. The tailored Model I enables object oriented representation of bridge systems in a hierarchical way. As long as bridge deterioration over 100 years was concerned, the condition states evolutions from bridge elements to the whole bridge under different environmental conditions were all predicted. Two simulated scenarios were conducted to demonstrate that the Model I can take into account the observed information and deterioration dependencies from maintenance actions and environmental effects.

Further investigation should be implemented to apply Model I for bridges in different BMSs. The ability to model deterioration dependencies among bridge elements has not been verified in practice. With regards to CPTs estimation, if historical bridge condition data are available, it is always better to rely on bridge condition data rather than expert knowledge. Prediction results of Model I should be compared with the ones from other methods. Further study can be dedicated to the extension of the proposed model. By expanding the DOOBN model with utility nodes and decision nodes, influence diagrams (IDs) can be formulated as a decision tool for bridge maintenance optimization [13].

Chapter 5: Bridge health prediction in safety aspect

5.1 Introduction

In this chapter, Model II for bridge structural reliability prediction is proposed. In Section 5.2 the development is addressed in details. Model II is outlined through three steps: modelling consideration, DOOBN development and parameters estimation. The proposed model not only evaluates time-variant structural reliability of bridge elements based on limit state functions, but also allows hierarchically representation of a complex bridge system with the consideration of complex probabilistic relationship among bridge systems. The Model II possesses the Bayesian updating ability and enhances the computational efficiency of reliability updating. Therefore, information from observation, maintenance and environment can be easily incorporated to deal with uncertainties in bridge deterioration. To validate the Model II, an application of the proposed model based on the existing literature is given in Section 5.3 to demonstrate its practicability.

5.2 Model II: using structural reliability

Considering the requirements of versatility for different types of bridges and of extensibility for maintenance optimization, the proposed model is designed through three steps: modelling consideration, DOOBNs development and parameters estimation. Bridge systems are presented in a hierarchical way by DOOBNs so that the system structural reliability can be evaluated based on structural reliability of bridge elements. The advantage of the proposed model lies in the consideration of complex probabilistic dependencies among bridge system rather than only parallel and/or series logical relationship. Temporal deterioration processes of bridge elements are modelled to achieve time-variant structural reliability. With the ability to handle uncertainty, the Model II provides an alternative computational method for structural reliability evaluation. The following sections discuss the three modelling steps in details.

5.2.1 Modelling consideration

The first step aims to analyse bridge systems hierarchically to facilitate the development of DOOBN. The identification of bridge sub-systems and bridge components as well as bridge elements, and the development of limit state functions of each bridge element are included in this step.

5.2.1.1 Bridge hierarchical decomposition

Structural reliability evaluation of bridge systems requires the identification of all the individual bridge elements that contribute to the safety of the entire structure. Similar to the bridge decomposition in Section 4.2.1.1, different bridge decomposition methods can be taken. The decomposition focuses on the identification of structural bridge hierarchies. As for the evaluation of structural reliability of bridge systems, the same bridge hierarchies located in different area are treated as different individual bridge hierarchies so that the whole estimation is based on individual bridge elements.

5.2.1.2 Limit state functions development

Regarding structural reliability analysis of bridge elements, limit state functions need to be developed at first. As mentioned in previous section, basic limit state functions are always in the form of Equation 2.8. Specialised limit state functions should be formulated in details for each bridge element. And there may be not only one failure mode for each bridge element. The development of limit state functions starts from the selection of essential failure modes. Table 5-1 lists essential failure modes normally considered for some typical bridge elements. Generally, shear and moment are most commonly considered failure modes. And the performance functions for moment and shear failures in ultimate limit states are shown by Equation 5-1 and Equation 5-2, respectively.

$$g_m = M_u - M_{dl} - M_{ll} \tag{5-1}$$

where M_u, M_{dl}, M_{ll} are moment capacity, moment due to dead load and moment due to live load, respectively.

$$g_{sh} = V_u - V_{dl} - V_{ll} \tag{5-2}$$

where V_u, V_{dl}, V_{ll} are shear capacity, shear due to dead load and shear due to live load, respectively.

Table 5-1. Critical failure modes for typical bridge elements

Structural elements	**Failure modes considered**
Deck/slab	Moment/flexure
Girder	Moment, shear
Bearing	Expansion
Piers cap	Shear, positive flexure, negative flexure
Columns	Top columns-crushing, bottom columns-crushing
Footing	One-way shear, two-way shear, flexure

Furthermore, because there are different kinds of uncertainties associated to structural reliability estimation, such as material strength, dimensions that cannot be easily measured, live loads and unit weight of materials, all the variables related to limit state functions should be treated as random variables and their distributions should be defined as well [47]. Such random variables can be found from some standard specifications and literature. Overall, limit state functions could be developed for each structural element with basic knowledge of structural mechanics. An instance for detailed development of limit state functions is given by Estes [47].

5.2.2 DOOBN development

In the second step, a conceptual model based on DOOBNs is constructed from top level (the whole bridge system) to bottom level (bridge elements). The conceptual model consists of two parts: bridge system and bridge elements. The first part focuses on hierarchical representation of bridge systems through several bridge factors. The complex relationship rather than parallel and/or series logical relationship can be considered. The second part focuses on the calculation of structural reliability of bridge elements over time. The limit state functions and deterioration processes of bridge elements are both modelled in this part.

5.2.2.1 OOBNs model of bridge hierarchies

In this part, bridge systems are modelled based on OOBNs in a similar way to the modelling of bridge system in Section 4.2.2.1. Consider that a bridge system is hierarchically decomposed into a number of bridge hierarchies in L (L>2) levels with the whole bridge in the highest Level *1* and bridge elements in the lowest Level L. Additionally, each bridge factor is defined with two states: failed and safe. Suppose a generic bridge hierarchy *S* in Level *M* (*M*<*L*) is further decomposed into *N* bridge sub-hierarchies *Bi* (*i* =1, …, N) in Level *M*+1, particularly B_i denotes primary bridge elements when *M*+1 equals to *L*. Similarly, the object oriented representation of BNs (OOBNs) is employed so that each time modelling process concentrates on only one bridge hierarchy and will not be overwhelming by plenty of other bridge hierarchies. All the bridge sub-hierachies related to this bridge hierarchy are encapsulated in one object. The modelling of this genetic bridge hierarchy *S* by means of OOBNs is given by Figure 5-1. Then the whole bridge system is constructed by connecting all the individual bridge hierarchies that are modelled by OOBNs from Level *1* to Level *M*. Since in each object the logic relationships have been modelled clearly, the whole OOBNs of bridge system can be modelled by linking all the individual OOBNs.

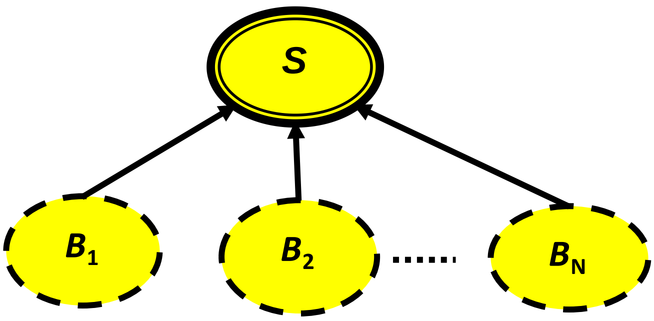

Figure 5-1. OOBNs model of a generic bridge hierarchy *C* for structural reliability prediction

By contrast with traditional series and/or parallel representation of bridge systems, the OOBNs model not only has the equal ability to model series and/or parallel relationship, but also possesses more flexibility to handle probabilistic relationship in a complex bridge systems rather than deterministic relationship only. For instance, one bridge superstructure consists of four girders, and the superstructure is assumed to be failed if three adjacent girders are failed. The series-parallel model for this superstructure is shown by Figure 5-2. Whereas the equivalent BN model of this superstructure as well as a CPT are displayed in Figure 5-3 and Table 5-2, respectively. The failure assumption of three adjacent girders is expressed by the CPT so that the conditional failure relationship between the superstructure and girders can be implemented in the BNs model. The CPT decodes the deterministic series-parallel relationship into probabilities that are equal to 1 or 0. Therefore, in this case the BNs model is equivalent to the traditional series-parallel model. Moreover, because of this CPT, the BNs model can easily model other types of failure assumptions by changing the values of the CPT. For example, if this bridge superstructure is assumed to be failed only if two adjacent girders are failed, the BN model can be adapted by means of a new CPT in Table 5-3.

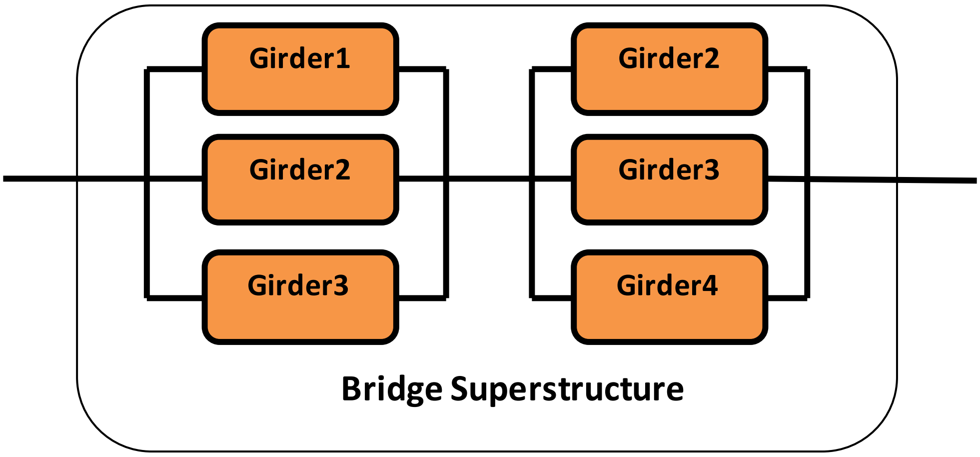

Figure 5-2. An example of series-parallel models for structural reliability of a bridge superstructure

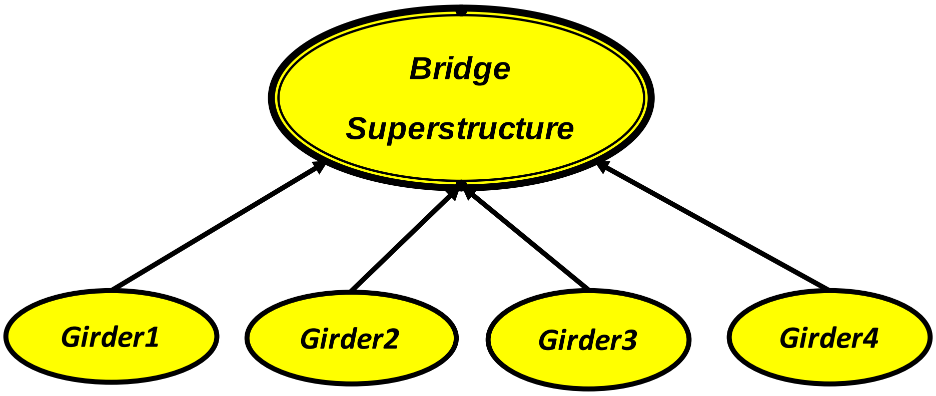

Figure 5-3. BNs model of a bridge superstructure for structural reliability prediction

Table 5-2. CPT model with failure assumption of three adjacent girders

	Girder1	**F**								**S**							
	Girder2	**F**				**S**				**F**				**S**			
	Girder3	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder4	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*1*
	Failed(F)	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

Table 5-3. CPT with failure assumption of two adjacent girders

	Girder1	**F**								**S**							
	Girder2	**F**				**S**				**F**				**S**			
	Girder3	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder4	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*0*	*0*	*0*	*1*	*1*	*1*	*0*	*0*	*1*	*1*	*0*	*1*	*1*	*1*
	Failed(F)	*1*	*1*	*1*	*1*	*1*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*0*

One feature of the OOBNs model outperforming the traditional series-parallel model is the ability to model complex probabilistic relationship. So far, deterministic relationship is commonly held. However, because a bridge system is a complex system being composed of many inter-related bridge elements, this representation is never verified favourably in practice. Taking the same superstructure as an example, as it is not sure if only one or two failed girders will certainly cause the failure of the superstructure or not, it may not be correct to hold the failure assumption that only three adjacent girders or more will make the superstructure failed. Additionally, the failure of the superstructure should be treated in a probabilistic way based on the failure probabilities of the four girders. Owing to the CPTs, the OOBNs model can easily deal with complex probabilistic relationships in bridge systems. By setting different probabilities between 0 and 1 in CPTs, different types of probabilistic failure relationships can be modelled appropriately. For instance, a new CPT defined in Table 5-4 encodes the probabilistic failure relationship between bridge superstructure and the four bridge girders. The CPT accounts for all the combinations of girders' failures and are filled with different values. For each combination, the sum of probabilities over "safe" and "failed" equals to 1. Based on this example, the OOBNs model has demonstrated the advantage over the traditional series-parallel model, and it is more suitable to model structural reliability of complex bridge systems. The specifications for further bridge elements modelling will be given below.

Table 5-4. CPT with the consideration of probabilistic failure relationship

	Girder 1	**F**								**S**							
	Girder 2	**F**				**S**				**F**				**S**			
	Girder 3	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder 4	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*0.1*	*0.2*	*0.1*	*0.4*	*0.4*	*0.8*	*0*	*0.2*	*0.4*	*0.8*	*0.2*	*0.8*	*0.8*	*1*
	Failed (F)	*1*	*1*	*0.9*	*0.8*	*0.9*	*0.6*	*0.6*	*0.2*	*1*	*0.8*	*0.6*	*0.2*	*0.8*	*0.2*	*0.2*	*0*

5.2.2.2 DOOBNs model of bridge elements

This part aims to model time-variant structural reliability of bridge elements by means of DOOBNs. The development of DOOBNs model consists of two sections: structural reliability and temporal deterioration processes. In the first section, as bridge elements may suffer from multiple failure modes, such as, fatigue, moment and shear, the structural reliability should be estimated based on multiple limit state functions. However, in this research, only ultimate limit state functions are considered. Other types of limit state functions can be modelled in a heuristic way by adapting the relevant variables in DOOBN model. In the second section, temporal deterioration processes of bridge elements made of reinforcement concrete and steel are modelled.

➢ Structural Reliability

Consider a general bridge element E of which structural reliability is dependent on a set of limit state functions $\mathbf{g}=\{ g_1,\ldots,g_n \}$. A network class of BNs shown in Figure 5-4 is adopted to express overall structural reliabilities of the bridge element E with different types of failure modes. Furthermore, consider a generic limit state function g shown in Equation 5-3 that generally describes all types of limit state functions. This limit state function g is expressed by the difference between bridge resistance R and bridge demand load L, where L is further composed of dead load L_{dl}, live load L_{ll}, wind load L_{wl} and earthquake load L_{el}. Additionally, the bridge resistance R is modelled as the function f_{R} of a set parameters $\mathbf{F}$ related to yield strength/stress of steel, steel reinforcement or concrete; a set of parameters $\mathbf{A}$ related to section area of steel reinforcement, web area, or section modulus; and a set of parameters $\boldsymbol{\lambda}_{\mathrm{R}}$ related to uncertainty factors of bridge resistance. While dead load L_{dl}, live load L_{ll}, wind load L_{wl} and earthquake load L_{el} are further modelled as function f_{dl} of a set of parameters $\boldsymbol{\lambda}_{\mathrm{dl}}$ related to uncertainty factors of dead load, function f_{ll} of a set of parameters $\boldsymbol{\lambda}_{\mathrm{ll}}$ related to uncertainty factors of live load, function f_{wl} of a set of parameters $\boldsymbol{\lambda}_{\mathrm{wl}}$ related to uncertainty factors of wind load, and function f_{el} of a set of parameters $\boldsymbol{\lambda}_{\mathrm{el}}$ related to uncertainty factors of earthquake load, respectively.

$$
\begin{aligned}
g &= R - L \\
&= R - (L_{dl} + L_{ll} + L_{wl} + L_{el}) \\
&= f_R(\mathbf{F}, \mathbf{A}, \boldsymbol{\lambda}_R) - (f_{dl}(\boldsymbol{\lambda}_{\mathrm{dl}}) + f_{ll}(\boldsymbol{\lambda}_{\mathrm{ll}}) + f_{wl}(\boldsymbol{\lambda}_{\mathrm{wl}}) + f_{el}(\boldsymbol{\lambda}_{\mathrm{el}}))
\end{aligned} \tag{5-3}
$$

This generic limit state function can represent different limit states defined according to different failure modes. For example, in terms of flexure in a steel girder, by ignoring wind load and earthquake load, the limit state function is adapted to Equation 5-4

$$
\begin{aligned}
g &= R - L \\
&= R - (L_{dl} + L_{ll}) \\
&= f_R\left(\mathbf{F_y}, \mathbf{A_z}, \boldsymbol{\lambda}_{\mathrm{flexure}}\right) - (f_{dl}(\boldsymbol{\lambda}_{\mathrm{flexure\_dl}}) + f_{ll}(\boldsymbol{\lambda}_{\mathrm{flexure\_ll}}))
\end{aligned} \tag{5-4}
$$

where $\mathbf{F}_{\mathbf{y}}$ denotes parameters associated to steel yield strength; $\mathbf{A}_{\mathbf{z}}$ denotes parameters associated to plastic section modulus; $\boldsymbol{\lambda}_{\mathrm{flexure}}$ denotes parameters associated to uncertainty factors of flexure in girders; $\lambda_{flexure\_dl}$ and $\lambda_{flexure\_ll}$ denote parameters associated to uncertainty factors of dead load moment and live load moment, respectively.

The generic limit state function is formulated as BNs in Figure 5-5. The adapted BNs for flexure limit state function in steel girders are shown in Figure 5-6.

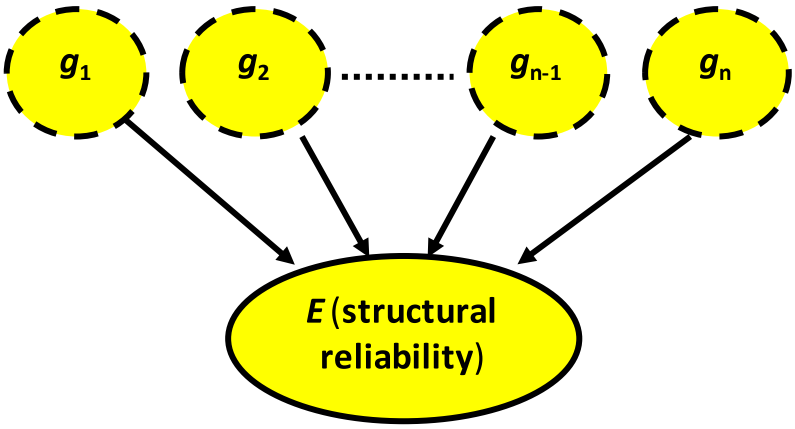

Figure 5-4. A network class of BNs for structural reliability of a general bridge element based on *n* different limit state functions

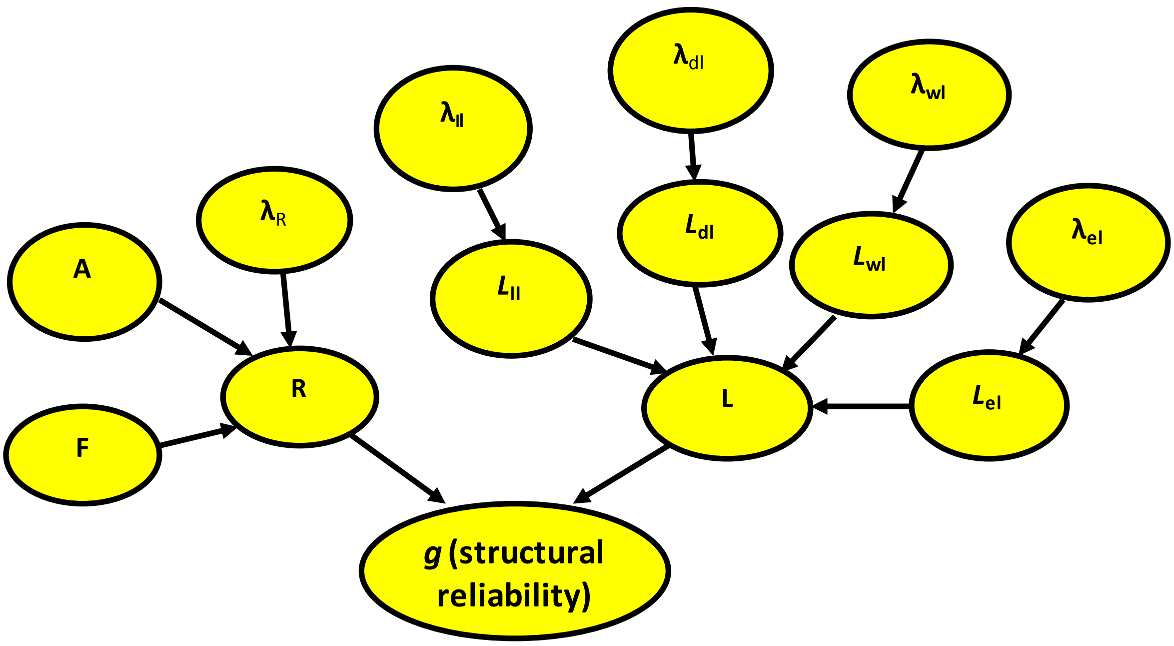

Figure 5-5. BNs model of a generic limit state function

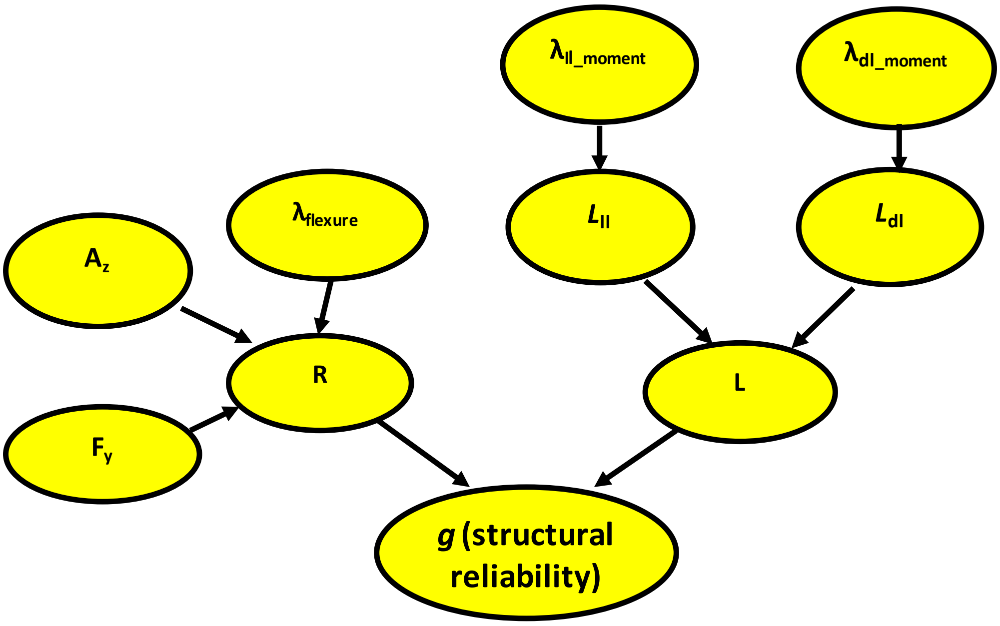

Figure 5-6. BNs model of a steel girder in flexure limit state function

When a limit state function is modelled by means of BNs for structural reliability estimation, one thing has to bear in mind is that if one child node has many parent nodes, the computational efficiency of the whole BNs inference can be significantly affected or sometimes intractable. Therefore, it is really necessary to reduce the number of its parent nodes for each node as many as possible. One viable way for this problem is to introduce new nodes between the child variable and the parent variables. Each new node indicates part of the original child node and becomes new parent node of the original child node so that the number of parent nodes for original child node is decreased. Meanwhile, each added node will also have fewer parent nodes, which improves the computational efficiency dramatically. One example referring to this solution is given in Figure 4-2.

➢ Temporal deterioration processes

Modelling of temporal deterioration processes of bridge elements is addressed in this part. The live load is assumed to be time-invariant distribution. As in practice the permitted weight of truck for certain bridge can be controlled, it is reasonable to hold this assumption. Another reason is that as this study focuses on modelling of structural reliability based on DOOBN rather than modelling of live load, it is possible to simplify the problem by hold this assumption. The live load is a deterministic distribution calculated from the 50 years load of Nowak live load model [118]. Therefore, only deterioration of bridge resistance contributes to the time-dependent structural reliability of bridge elements.

According to the discussion in Section 3.2, the main cause of bridge resistance deterioration is due to corrosion, which may result in the reduction of cross-section area of reinforcing steel, plastic section modulus, shear web area and so on. In this research, the corrosion deterioration process is modelled as a discrete time process. According to different materials, Equation 3-7 and Equation 3-11 are employed as the basis for DOOBNs modelling of corrosion deterioration process in steel and reinforced concrete, respectively. The two corresponding DOOBNs models are illustrated in Figure 5-7 and Figure 5-8, respectively. In Figure 5-7, C, as an output, is corrosion loss (corrosion penetration depth) after t years; A is the corrosion loss after one year, and B is a regression coefficient numerically. In Figure 5-8, T_{corr} denotes the corrosion initiation time; R_{corr} denotes the corrosion rate; D(t) denotes the diameter of reinforced steel bar at time t. Additionally, the node “corrosion indication” is discrete variable with two states “Yes” and “No”.

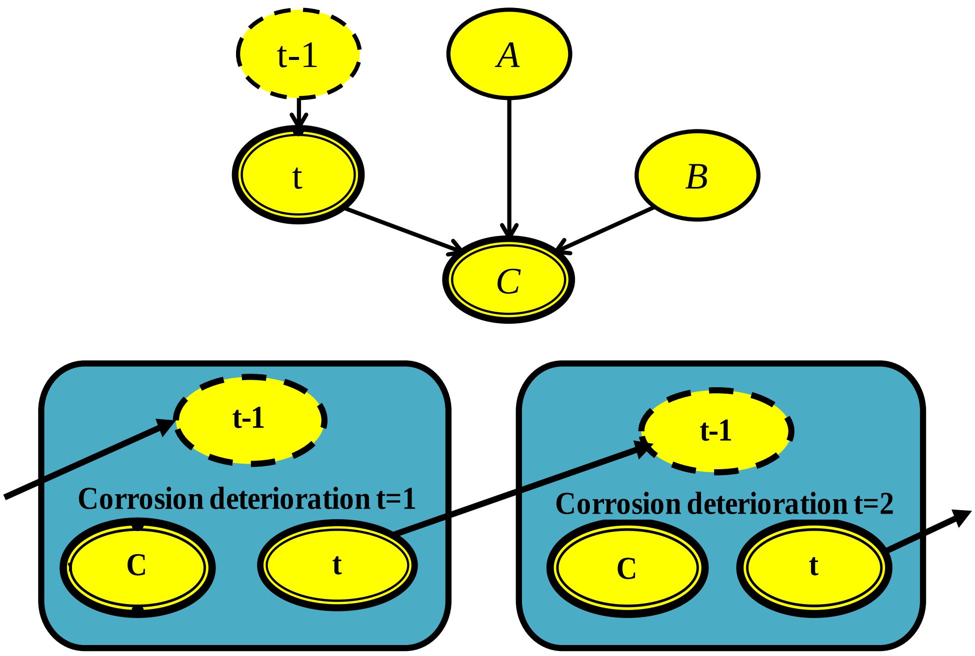

Figure 5-7. DOOBN modelling for corrosion deterioration process in bridge elements made of steel

In both cases, the nodes t-1 and t represent time variables in two consecutive time slices and are assigned as input and output, respectively. Here, t and t-1 are defined as discrete time variables. The time variable t is conditional on previous time variable t-1. By introducing the time variable t, the commonly held Markovian assumption in most of BNs applications is released for structural reliability modelling in this research. The time-variant corrosion deterioration is implemented by connecting the object of corrosion deterioration in each time slice.

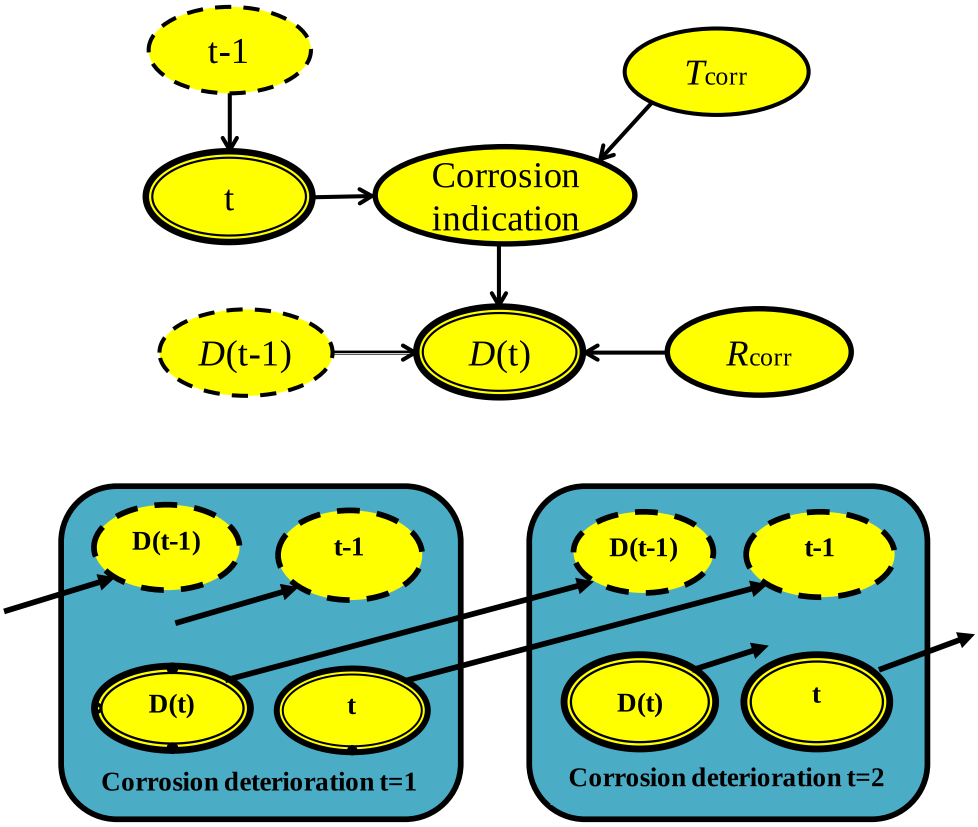

Figure 5-8. DOOBN modelling for corrosion deterioration process in bridge elements made of reinforced concrete

To ensure the correctness of DOOBNs modelling for bridge elements' real temporal deterioration, available event information about maintenance intervenes, environmental effects and observation information should be taken into consideration as well. As these factors are independent of each other and the past, a set of nodes standing for maintenance actions, environment levels and observations can be individually added in each time slice according to data availability. For instance, the extended DOOBNs models for corrosion deterioration process in steel and reinforced concrete, which include all types of event information, are displayed in Figure 5-9 and Figure 5-10, respectively. For bridge elements made of steel, different maintenance actions have effects on corrosion loss and time variable *t*. The maintenance intervene variable is defined with several states according to available maintenance actions. For instance, replacement and perfect repair remove corrosion loss and reset the time variable *t* to zero. Minimal repair and no maintenance leave corrosion loss the same as before. Imperfect maintenance relieves the corrosion loss and resets time variable *t* to early time value. The probabilities over all the possible time value can be used to represent imperfect maintenance actions. Furthermore, observation variable accounts for corrosion information obtained from visual inspection, NDT and monitoring techniques. This observation could be a discrete random variable with two states “corrosion” and “no corrosion” or a continuous random variable, for instance, a measured corrosion penetration depth. In the previous case, a probability of detection (PoD) model can be adopted to characterize the observation information, in the latter case, measurement error can be utilised to characterize the observation information. With regarding to environmental variable in Figure 5-9, the effects of different environmental levels on variables *A* and *B* should be defined. For example, two environmental states “urban environment” and “rural environment” can be simply used to address the influence of environments on variables *A* and *B* [8].

Similarly, in Figure 5-10, the maintenance intervene variable is also defined with available maintenance actions, which renew the diameter of reinforced steel bar and time variable *t* to some extents. Additionally, observation information reflecting real corrosion deterioration processes inside concrete is characterised as well to facilitate Bayesian updating of related corrosion variables. For corrosion deterioration in reinforced concrete, the corrosion initiation time and corrosion rate are supposed to be largely dependent on environmental factors. To address the effects of environmental factors, one example is that three environmental states “Low”, “Medium” and “High” could be adopted to express the environmental effects [152].

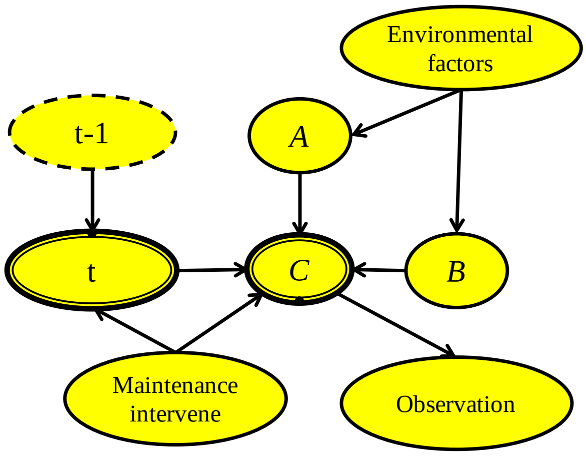

Figure 5-9. OOBN modelling for corrosion deterioration process in steel considering maintenance intervene, environmental effects and observation

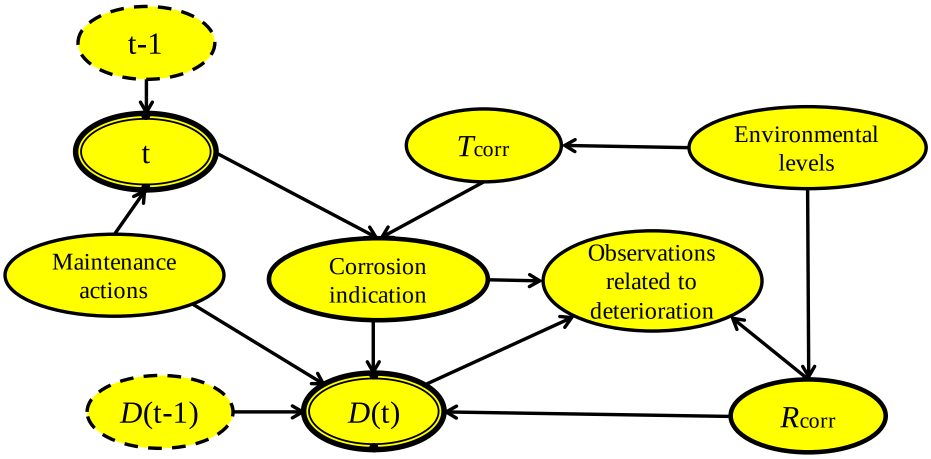

Figure 5-10. OOBN modelling for corrosion deterioration process in reinforced concrete considering maintenance actions, environmental effects and inspection results

By now, the modelling of structural reliability and temporal deterioration processes are completed. These two parts are connected so as to present time-variant structural reliability. The connection is implemented through a set of variables **A** that are conditional on variable *C* in Figure 5-9 or variable *D*(t) in Figure 5-10 depending on the material used. For time-variant structural reliability, DOOBNs models of a generic limit state function with regarding to steel and reinforced concrete are displayed in Figure 5-11 and Figure 5-12, respectively. Finally, the output of DOOBNs models of bridge elements are connected to the corresponding bridge elements modelled in OOBNs model of bridge system so as to implement time-variant structural reliability of bridge system.

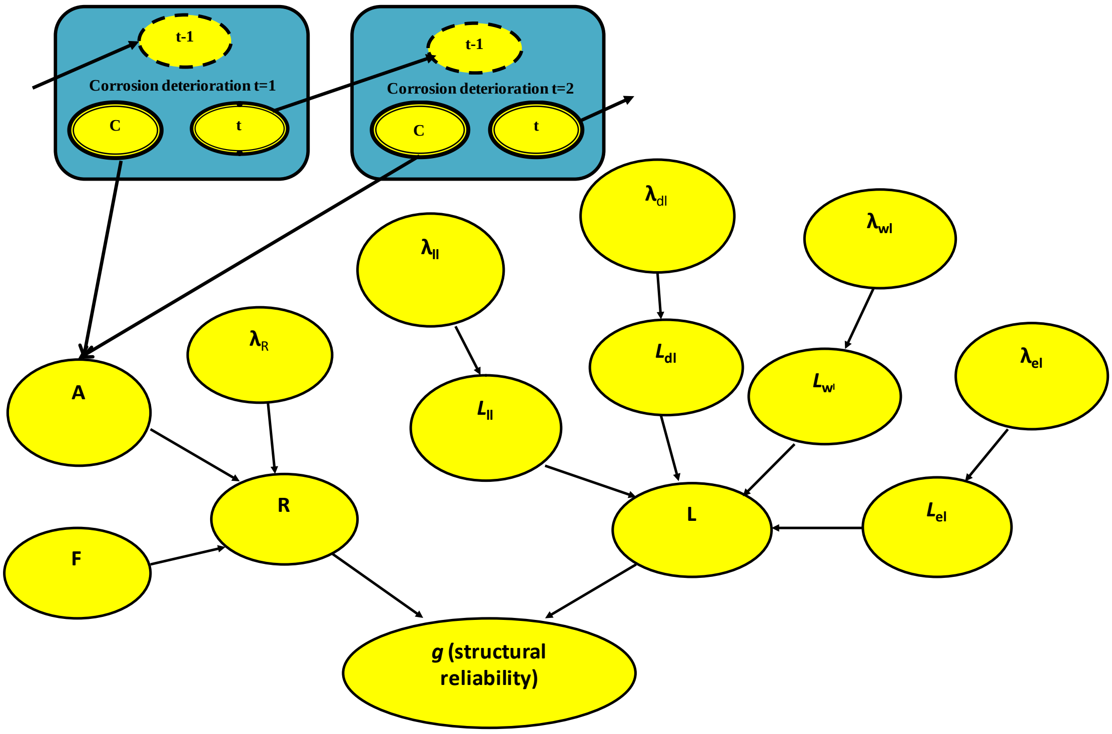

Figure 5-11. DOOBNs model for time-variant structural reliability based on a general limit state function in steel

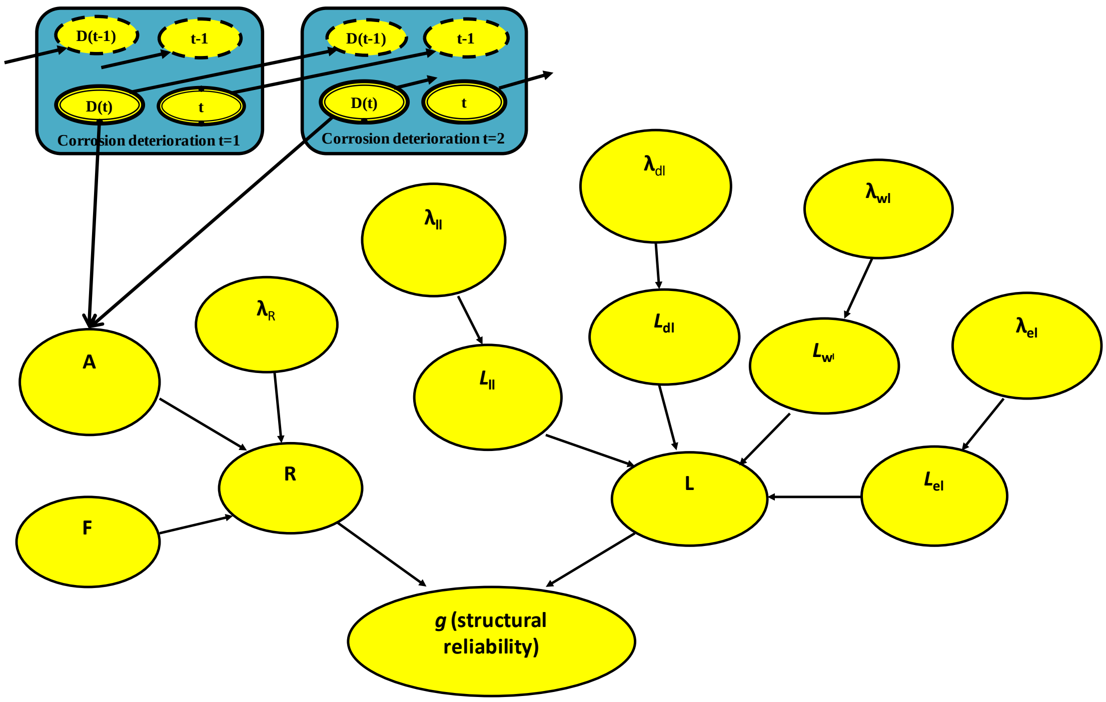

Figure 5-12. DOOBNs model for time-variant structural reliability based on a general limit state function in reinforced concrete

5.2.3 Parameters estimation

The final step of the proposed model is to parameterise the conceptual DOOBN model by means of estimation of conditional probabilities tables (CPTs) and priori probabilities of root nodes, which could be the most difficult in the whole modelling process. Overall, plenty of probabilities need to estimate so that the whole network functions well and renders a global distribution of bridge system health. Since no single method is adaptable and is able to fulfil all the estimation, it needs combine all different kinds of data sources. However, to complete the estimation, it is also necessary to discretize continuous variables into discrete variables. Due to the limitation of current inference algorithms and slow convergence rate, continuous variables cannot be dealt with efficiently. Furthermore, current inference algorithms cannot handle the situation adequately that continuous parent variables have discrete children variables, which actually happens in this research. As a result, continuous variables should be replaced by a finite number of discrete states so that CPTs and priori probabilities based on discrete states can be derived.

5.2.3.1 CPTs and priori probabilities estimation

➢ Bridge hierarchies

The CPTs in bridge systems part can be straightforwardly estimated based on traditional series and/or parallel representation of bridge systems. However, if probabilistic relationship is considered, CPTs should be evaluated through statistical data and expert knowledge. When a number of historical failure data about bridge systems are available, CPTs are filled in based on these data preferentially. Taking the general bridge hierarchy *S* in Figure 5-1 as an example, given a certain combination of *N* parent variables B_i (i=1,…,N), the conditional failure probability of *S* under this combination is estimated by

$$p_S^{(F)} = \frac{n_S^{(F)}}{n_S^{(F)}+n_S^{(S)}} \tag{5-5}$$

where $n_S^{(F)}$ is the observed number of failure events under this combination; $n_S^{(S)}$ is the observed number of safe events under this combination. Nonetheless, this kind of data always suffers from insufficiency in practice. As a result, expert knowledge is utilised as it is quite straightforward for CPTs estimation. CPTs are filled in based on daily obtained knowledge of bridge experts and bridge engineers. The elicitation processes introduced in section 4.2.3.2, can be also applied here.

➢ Bridge elements

In this part, parameters estimation mainly relies on the existing literature and deterministic equations. Most priori probabilities can be obtained from the existing literature directly. However, some priori probabilities, for example, corrosion initiation time, cannot be obtained directly. In this case, Monte Carlo simulation based on physical corrosion initiation equation like Equation 3-9 can be used to calculate the simulated priori probabilities. The simulation for corrosion initiation time is shown in the Appendix D. Moreover, priori probabilities of root nodes related to bridge demand load are calculated based on Novak's live load [118] or AASHTO specification [2].

In this research, deterministic equations are largely used to estimate the CPTs of DOOBNs model that is built for structural reliability estimation of bridge elements. Since modelling of structural reliability is built based on deterministic limit state functions, CPTs could be derived from the functions directly. In other words, the relationship described by the deterministic equations, is directly encoded into CPTs. Moreover, Equation 3-7 and Equation 3-10 describing corrosion in bridge deterioration are utilised to estimate the CPTs related to modelling of temporal deterioration processes. As deterministic equations are formulated based on objective information, subjective judgement involved in expert knowledge could be avoided.

In addition, miscellaneous knowledge is utilised as well. For instance, maintenance variables have a dominant influence on the bridge elements deterioration. By defining the impacts of different maintenance activities on the time variable t, corrosion loss C or diameter of reinforced steel, the CPTs related to maintenance variables can be filled in partially. Normally, replacement and perfect repair remove bridge resistance loss caused by corrosion and reset the time variable t to zero. Minimal repair and no maintenance leave bridge deterioration as the same as before. Imperfect maintenance relieves the bridge deterioration and resets time variable t to early time value. The probabilities over all the possible discrete time values and deterioration variables can be used to represent imperfect maintenance actions. Also, CPTs related to environmental variables are estimated based on the effects of different environmental states. For instance, under different environmental states both variables A and B in Equation 3-7 are assigned with different probability distributions according to the existing literature [8]. Similarly, corrosion initiation time in Equation 3-10 is supposed to result in different simulated initiation time distributions based on different deterioration parameters under different environmental states [152]. In light of observations, CPTs can be estimated based on the accuracy of inspection methods. For observations obtained from NDT and monitoring techniques, CPTs can be estimated from probability of detection (PoD) model and measurement accuracy, respectively. Finally, shown in Figure 5-4, overall structural reliability of a bridge element is conditional on structural reliabilities in multiple failure modes. As each failure mode plays the same role in bridge safety, series relationship among different failure modes is held for CPT estimation.

5.2.3.2 Discretization of continuous variables

In the conceptual DOOBNs model, most nodes in bridge elements part are continuous variables that follow continuous distributions. To facilitate the inference algorithms, discretization has to be carried out sequentially from parent nodes to children nodes. In this research univariate discretization is chosen simply because bivariate discretization is incompatible with BNs. In addition, the same discretization scheme is utilised for all the time slices. To represent a continuous distribution as several discretized intervals, the continuous distribution needs to truncate at both ends. The truncating points are rather important for the discretization accuracy. In practice, it has been learned from empirical knowledge that truncating a continuous distribution at five standard deviations from the mean generates a reasonable approximation. In Section 5.3, a large number of continuous distributions truncated based on this rule with more or fewer improvements are given. Before formal discretization is implemented to a coutinous distribution, another issue is to deal with the tails of the distribution. With regarding to structural reliability, the probability mass of the tail is lumped into the outermost state of the probable value range as recommended [55]. The probable values range of continuous nodes can be identified based on truncated probability distributions of variables concerned. Moreover, there are also some continuous nodes of which distributions are unknown. For those nodes, simulation techniques, such as, MCS can be used to evaluate the probable value range based on limit state functions. Other information, such as, literature, empirical and common knowledge can be are used for those nodes as well. For instance, the value of reinforced steel should be between zero and its maximum diameter.

Next, discretization interval length should be determined carefully within the probable values range to make sure that the discretized distribution meets the requirement of minimum accuracy for a continuous distribution. Interval length could be equal length or equal frequency as well as other alternatives. The choice of certain discretization interval length is determined by the objectives of study and types of continuous distributions, such as, symmetric distributions or asymmetric distributions. In this research, equal length interval is mostly chosen, since it has been approved as a simple and effective way to discretization [149]. The number of discretization intervals is crucial for accurate results and should be chosen under the optimal balance between accuracy and speed. Normally, the important variables need more discretization intervals so that important information content can be captured. To keep the discretization step as simple as possible, we will not consider achieving the optimal number of discretization intervals in this research. However, considering the importance, different numbers of discretization intervals for different nodes can be found in Section 5.3.

Afterwards, for root nodes, the probability of each discrete state can be easily assigned with cumulative distribution function (CDF) over the corresponding discretization interval. For example, if a root continuous variable *r* is discretized into *n* exclusive discrete states $r^{(i)}$ (*i*=1,…,n), the probability of each discrete state $p(r^{(i)})$ is expressed as follows:

$$p(r^{(i)}) = F_r(r_{ub}^{(i)}) - F_r(r_{lb}^{(i)}) \tag{5-6}$$

where F_r is the cumulative distribution function (CDF) of *r*; $r_{ub}^{(i)}$ and $r_{lb}^{(i)}$ are the upper bound and lower bound of the discrete state $r^{(i)}$, respectively.

However, for other nodes, it is rather difficult to estimate the probability of each discrete state. A detailed discussion about discretization for continuous random variables can be found in the paper [149]. Now, we consider a general variable *X* that is discretized into m exclusive discrete states $x^{(j)}$ (*j*=1, …, m). The variable *X* has a set of parent variables $\mathbf{Y} = \{\mathrm{y}_\mathbf{1}, \mathrm{y}_\mathbf{2}, \dots, \mathrm{y}_l\}$, (*l*=1,…, *L*). Owing to the sequential discretization from parent nodes to children nodes, all the parent variables **Y** have been discretized into $\mathbf{Y}^{(k)} = \{y_1^{(k_1)}, y_2^{(k_2)}, \dots, y_l^{(k_l)}\}$, (k_1=1,…,K_1; k_2=1,…,K_2;…; k_l=1,…,K_l). For the continuous variable *X*, the probability of each discrete state $x^{(j)}$ conditional on $\mathbf{Y}^{(k)}$ is expressed as follows:

$$p(x^{(j)}|\mathbf{Y}^{(k)}) = F_X'(x_{ub}^{(j)}|\mathbf{Y}^{(k)}) - F_X'(x_{lb}^{(j)}|\mathbf{Y}^{(k)}) \tag{5-7}$$

where F_X' is the cumulative distribution function (CDF) of *X* conditional on $\mathbf{Y}^{(k)}$; $x_{ub}^{(j)}$ and $x_{lb}^{(j)}$ are the upper bound and lower bound of the discrete state $x^{(j)}$, respectively. Additionally, F_X' can be estimated by

$$F_X\left(X\middle|\mathbf{Y}^{(k)}\right) = \int_{\mathbf{Y}^{(k-)}}^{\mathbf{Y}^{(k+)}} F_X(X|\mathbf{Y})\, f_Y\left(\mathbf{Y}\middle|\mathbf{Y}^{(k)}\right) d\mathbf{Y} \tag{5-8}$$

where F_X is the original cumulative distribution function (CDF) of X conditional on $\mathbf{Y}$; $\mathbf{Y}^{(k+)}$ and $\mathbf{Y}^{(k-)}$ denote the upper bounds and lower bounds of all the discrete state components in $\mathbf{Y}^{(k)}$, respectively; $f_Y\left(\mathbf{Y}\middle|\mathbf{Y}^{(k)}\right)$ is the original probability density function (PDF) of $\mathbf{Y}$, $f_Y(\mathbf{Y})$, truncated in the interval $\mathbf{Y}^{(k-)} < \mathbf{Y}^{(k)} < \mathbf{Y}^{(k+)}$.

To solve the above equation, we must know $f_Y(\mathbf{Y})$ at first. However, normally, as only conditional distribution is defined in BNs, some assumptions, such as, Gaussian distribution and exponential distribution, have to be made for $f_Y(\mathbf{Y})$ so as to facilitate the calculation. Specifically, when all the parent variables $\mathbf{Y}$ are root variables, $f_Y\left(\mathbf{Y}\middle|\mathbf{Y}^{(k)}\right)$ can be easily obtained since $f_Y(\mathbf{Y})$ is already known. Nonetheless, in practice, the introduced method is not often applied. Instead, the more efficient sampling algorithms, such as, likelihood sampling, logic sampling and backward sampling, are used by current commercial BNs softwares to derive discrete CPTs. The sampling techniques can be easily implemented and meanwhile provide reasonable accuracy at relative fast computational speed.

5.3 Case study of Bridge E-17-AH: structural reliability prediction

The proposed DOOBNs model for bridge structural reliability prediction is applied to a highway bridge “E-17-AH” (Figure 5-13) located in Denver, Colorado. The bridge has three equal length spans and is mainly made of reinforced concrete. More information about this bridge can be found in the PhD thesis [47]. Since the bridge has been modelled for system structural reliability prediction by using traditional methods in the previous study [47], some information presented in that study are directly utilised for this application. A customized DOOBNs model is developed to predict the structural reliability of this bridge over 50 years. The Comparisons between the DOOBNs model and traditional methods, for instance, FORM, are given to demonstrate the accuracy. Moreover, the advantages of proposed DOOBNs model outperforming the traditional method are also illustrated in this application.

Figure 5-13. Bridge E-17-AH, Denver, Colorado

5.3.1 DOOBNs development of Bridge E-17-AH for bridge structural reliability

5.3.1.1 System analysis of Bridge E-17-AH

On behave of DOOBNs model development, the structure of Bridge E-17-AH need to analyse at first. In the previous study, a series-parallel model has been presented by Estes, which identifies all the bridge structural elements systematically [47]. To reduce the computation burden owing to a large number of bridge elements and to keep this application as simple as possible, the same simplified series-parallel model (Figure 5-14) is chosen as the basis for DOOBNs model with the assumption that the failure of bridge system requires the failure of three adjacent girders [47]. Basically, the bridge structure is composed of major bridge elements, including slab, exterior Girder 1, interior-exterior Girder 2, interior Girders 3 to 5, pier and column footing.

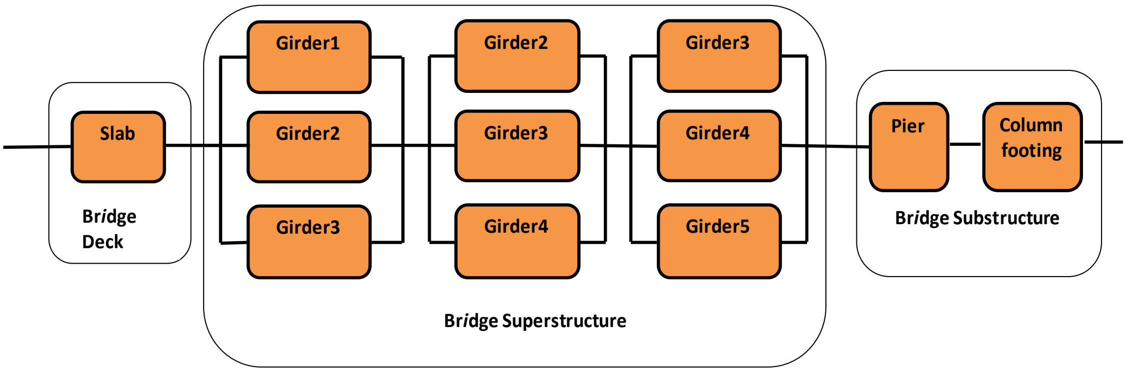

Figure 5-14. Simplified series-parallel representation of Bridge E-17-AH[47]

Furthermore, to estimate bridge structural reliability, limit state functions for each identified bridge element are necessary to formulate. The previous study had displayed limit state functions for all the major failure modes of each bridge element. Generally, slab and column footing suffer from flexure failure mode; pier suffers from shear failure mode; girders suffer from both failure modes. The two limit state functions for bridge slab and exterior Girder 1 in shear are shown as follows [47]:

Slab flexure

$$g_{slab\ flex} = \gamma_{mfc}\left(0.349\lambda_{rebar}f_y\lambda_{deff} - \frac{0.3844\lambda_{rebar}^2 f_y^2}{244.8f_c'}\right) - 0.137\lambda_{asph}$$

$$-0.471\lambda_{conc} - 4.27\lambda_{trk} \quad (5\text{-}9)$$

where γ_{mfc} is uncertainty factor for concrete flexure; λ_{rebar} is uncertainty factor for reinforcing steel area; f_y is yield stress of reinforcing steel; λ_{deff} is effective depth of reinforcing steel; f_c' is 28 day yield strength of concrete; λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{trk} is uncertainty factor for HS-20 truck in analysis of deck.

Exterior Girder 1 in shear

$$g_{girder1\ shear} = 10.38F_y\gamma_{msg} - (13.27\lambda_{conc} + 3.4\lambda_{steel} + 28.33V_{trk-e}DF_eI_{beam}) \quad (5\text{-}10)$$

where F_y is yield strength of steel in girders; γ_{msg} is model uncertainty factor regarding to shear in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{steel} is uncertainty factor for weight of steel girders; V_{trk-e} uncertainty factor for live load shear in exterior girder; DF_e is uncertainty for live load distribution of exterior girders; I_{beam} uncertainty factor for impact on girders. The other limit state functions for all the other bridge elements can be found in Appendix B.

Furthermore, to take into account temporal bridge deterioration owing to corrosion, physical deterioration equations, such as, Equation 3-7 and Equation 3-10, are integrated into the limit state functions. Additionally, to build up DOOBNs model if one child node has many parent nodes, the estimation of CPTs would be intractable and quite time consuming. Therefore, to facilitate the CPTs estimation in this example, the original limit state functions are rewritten into several new equations with newly introduced variables. Equation 5-9 is rewritten as follows:

Slab flexure

$$g_{slabflex} = R_{slab\ moment} - L_{slab\ \ moment} = R_{slab\ moment} - (M_{slab\ dl} + M_{slab\ \ ll}) \quad (5\text{-}11)$$

$$R_{slab\ moment} = \gamma_{mfc}(R_{slab\ moment\ 1} - R_{slab\ moment\ 2}) \quad (5\text{-}12)$$

$$R_{slab\ moment\ 1} = \frac{6.75A_{t\ slab}f_y\lambda_{deff}}{12} \quad (5\text{-}13)$$

$$R_{slab\ moment\ 2} = \frac{A_{t\ slab}^2 f_y^2}{244.8f_c'} \quad (5\text{-}14)$$

$$A_{t\ slab} = \frac{2\pi D_{slab}^2\lambda_{rebar}}{4} \quad (5\text{-}15)$$

$$D_{slab}(t) = D_{slab}(0) - R_{corr\ slab}(t - T_{corr\ slab}) \quad (5\text{-}16)$$

$$M_{slab\ dl} = 0.137\lambda_{asph} + 0.471\lambda_{conc} \quad (5\text{-}17)$$

$$M_{slab\ \ ll} = 4.26\lambda_{trk} \quad (5\text{-}18)$$

where $R_{slab\ moment}$ is the flexure capacity of the slab; $L_{slab\ \ moment}$ is the flexure demand for the slab; $M_{slab\ dl}$ is the dead load demand in flexure for the slab; $M_{slab\ \ ll}$ is the live load demand in flexure for the slab; $R_{slab\ moment\ 1}$ and $R_{slab\ moment\ 2}$ are two parts of total flexure capacity of slab; $A_{t\ slab}$ is the temporal changed cross section area of reinforced steel in slab; $D_{slab}(t)$ is the temporal changed diameter of single reinforced steel bar in slab; $R_{corr\ slab}$ is the corrosion rate in slab; $T_{corr\ slab}$ is the corrosion initiation time in slab.

For steel girders, if corrosion is assumed to penetrate the top and sides of the bottom flanges in addition to each side of the web, new equations based on Equation 5-10 for time-variant structural reliability are given by

Exterior Girder 1 in shear

$$g_{girder1\ shear} = R_{girder1\ shear} - L_{girder1\ shear} \quad (5\text{-}19)$$

$$R_{girder1\ shear} = 0.58F_y\gamma_{msg}d_wt_w = 18.2062F_y\gamma_{msg}\left(0.57 - \frac{d_{corr1}}{12700}\right) \quad (5\text{-}20)$$

$$L_{girder1\ shear} = V_{girder1\ dl} + V_{girder1\ \ ll} \quad (5\text{-}21)$$

$$d_{corr\ 1} = A_1 t^{B_1} \quad (5\text{-}22)$$

$$V_{girder1\ dl} = 13.27\lambda_{conc} + 3.4\lambda_{steel} \quad (5\text{-}23)$$

$$V_{girder1\,ll} = 28.33 V_{trk-e} DF_e I_{beam} \tag{5-24}$$

where $R_{girder1\,shear}$ is the shear capacity of exterior girder 1; $L_{girder1\,shear}$ is the shear demand for exterior girder 1; d_{corr1} is the corrosion loss of exterior girder 1 at the considered time; A_1 and B_1 are the corrosion loss after one year and a regression coefficient numerically; $V_{girder1\,dl}$ and $V_{girder1\,ll}$ are dead load and live load demand in shear for girder 1. In the same way, all the other limit states functions are rewritten for time-variant structural reliability shown in Appendix A.

5.3.1.2 DOOBNs model of Bridge E-17-AH

Based on the system analysis, conceptual DOOBNs model is set up in this part. First, in terms of the whole bridge system, because of the simplified series-parallel model (Figure 5-14), the bridge system OOBNs model for Bridge E-17-AH can be easily built up in Figure 5-15, where the nodes with different colours centred denote three hierarchical levels of Bridge E-17-AH. The next step is to further model time-variant structural reliability of each bridge element based on DOOBNs. Taking the slab and exterior Girder 1 as examples, the modelling consists of two components: structural reliability model and temporal deterioration model. According to the identified variables in limit state function of slab flexure, the BNs model for slab structural reliability in flexure is formulated in Figure 5-16. Moreover, without any information about maintenance intervene, environmental effects and observation, the DOOBN model of slab for temporal deterioration processes is shown in Figure 5-17. As indicated by Equation 5-15, the node “At (slab)” is conditional on the node “D (slab)_T”, so the DOOBNs model of slab for time-variant structural reliability is achieved by connecting these two nodes. In Figure 5-18, the two red dash lines indicate temporal relationship between two nodes, which enable dynamic evolution to facilitate time-variant estimation of structural reliability. Additionally, the nodes with white colour centred signify the variables related to bridge corrosion deterioration; the node with black colour centred signifies the goal of slab structural reliability.

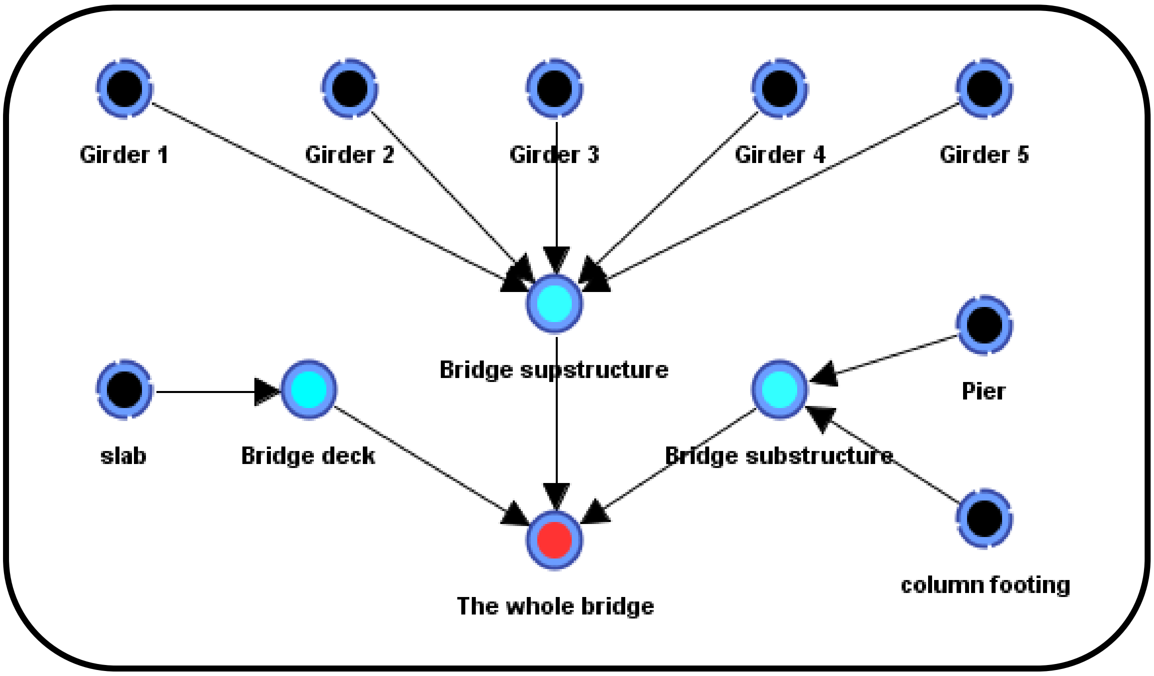

Figure 5-15. OOBNs model of the whole bridge system for structural reliability

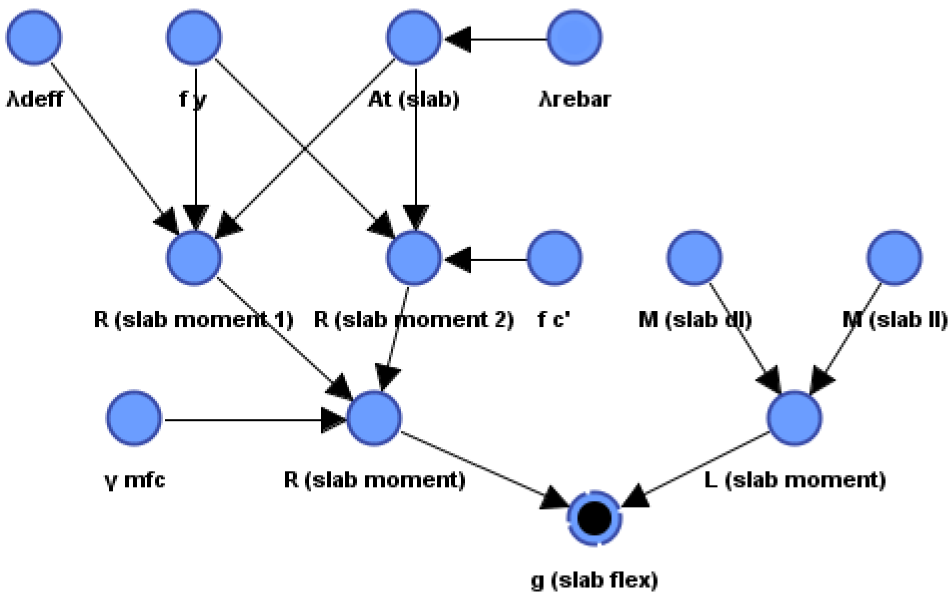

Figure 5-16. BNs model for slab structural reliability in flexure

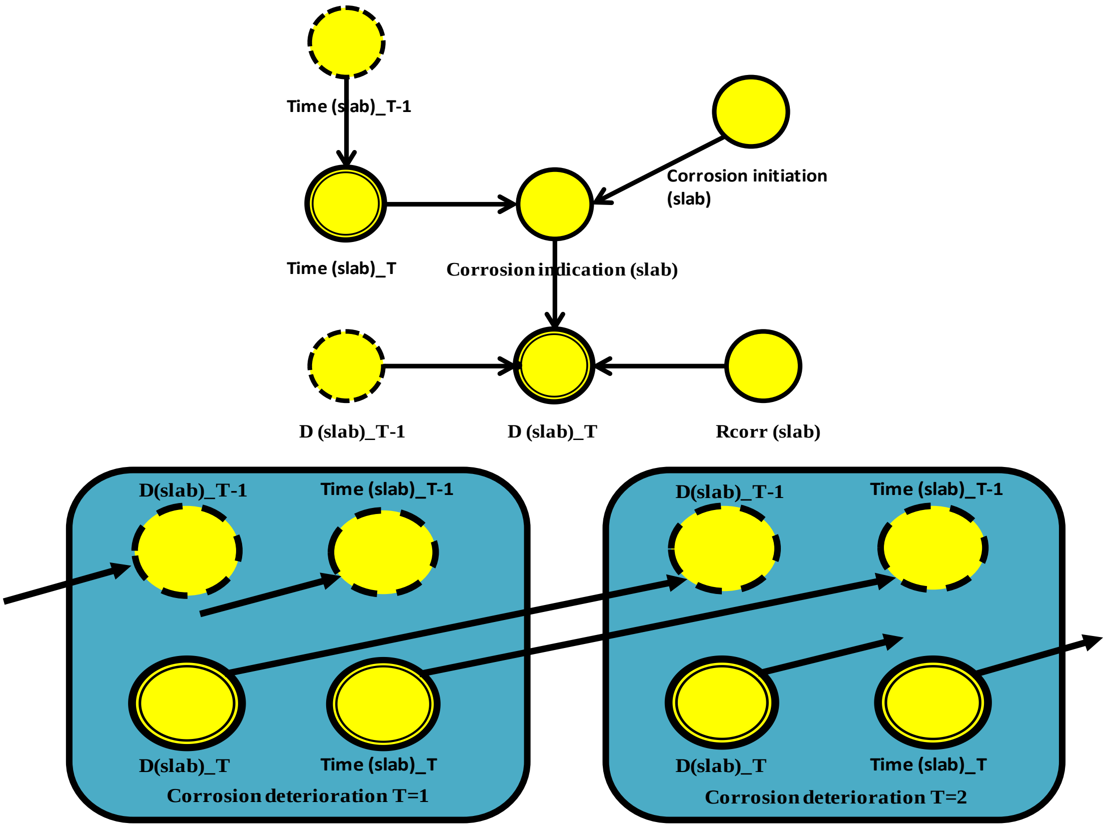

Figure 5-17. DOOBN model of slab for temporal deterioration processes

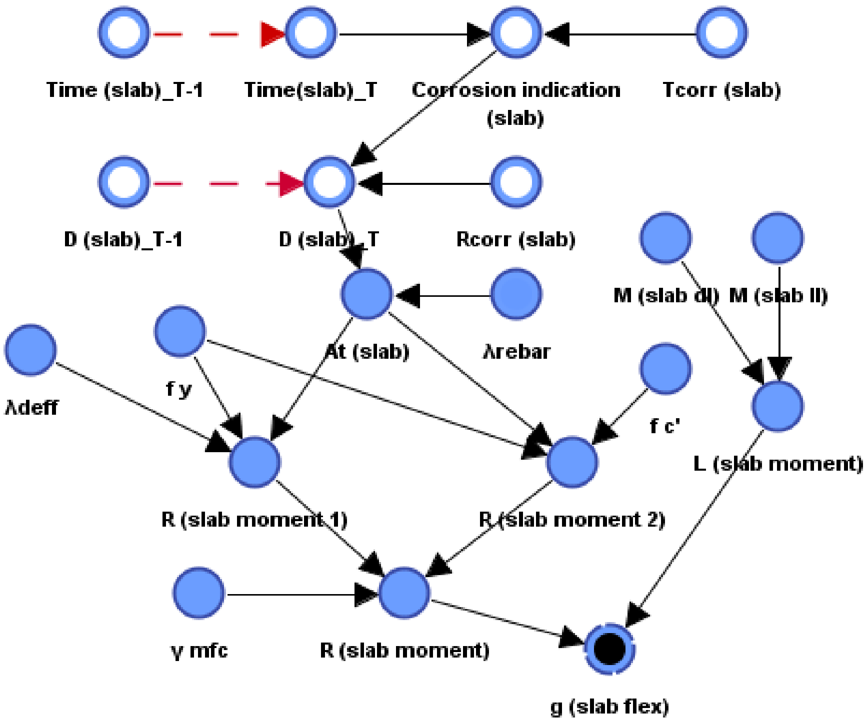

Figure 5-18. DOOBN model of slab for time-variant structural reliability

In light of exterior Girder 1, one thing has to be born in mind is that Girder 1 suffers from both flexure and shear. Based on the revised limit state functions, the BNs models for structural reliability without the consideration of temporal deterioration in both failure modes are formulated in Figure 5-19 and Figure 5-20, respectively. To address corrosion deterioration process, DOOBNs model is given as well (Figure 5-21). According to Equation 5-20 and B-12, both flexure and shear capacity is dependent on corrosion loss. As a result, by linking the temporal deterioration component and structural reliability component as well as some common variables, such as, *F*y, the DOOBNs model of exterior Girder 1 is described by Figure 5-22. Overall, the structural reliability of Girder 1 is dependent on its both flexure structural reliability and shear structural reliability. In a similar way, the DOOBNs model of other bridge elements are also displayed from Figure 5-23 to Figure 5-27. For column footing, as the deterioration are assumed to be ignored compared with other elements [47], the BNs model is presented in Figure 5-28. The outputs of all the these bridge elements models are used as inputs of OOBNs model in Figure 5-15 to estimate time-variant structural reliability of the whole bridge system.

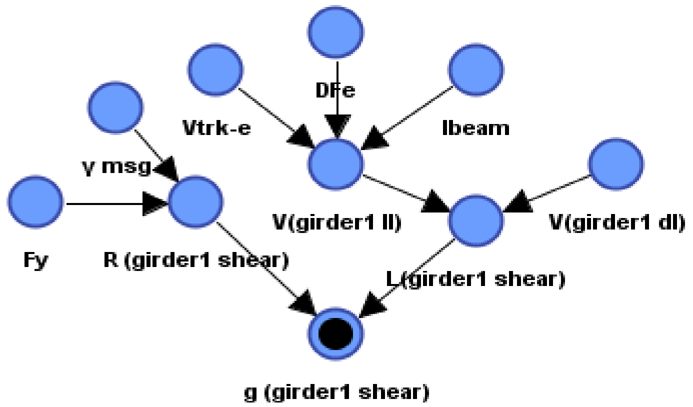

Figure 5-19. BNs model for Girder 1 structural reliability in shear

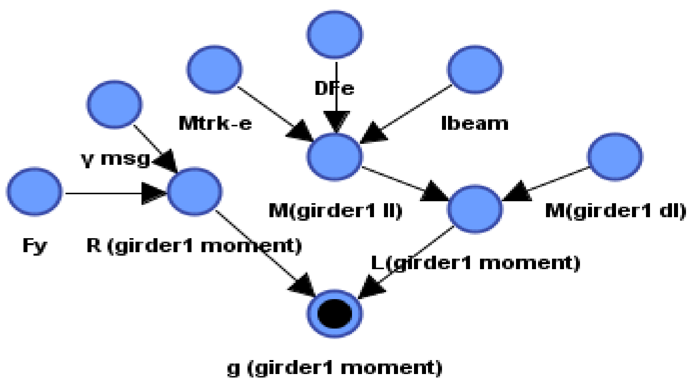

Figure 5-20. BNs model for Girder 1 structural reliability in flexure

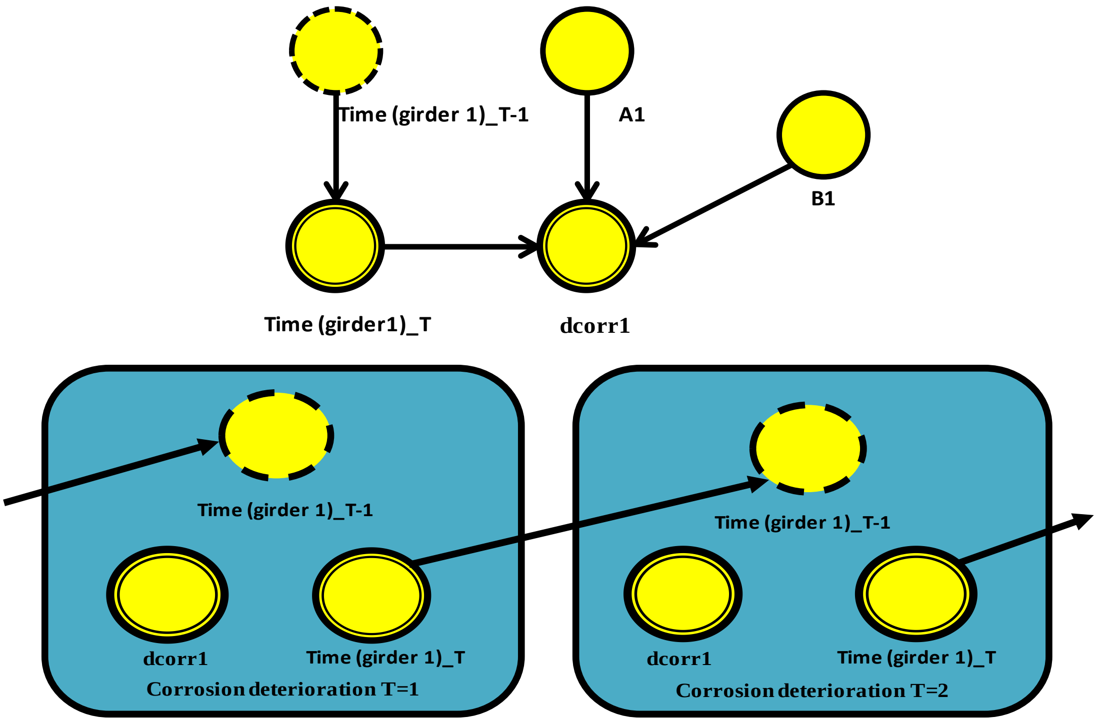

Figure 5-21. DOOBNs model of Girder 1 for temporal deterioration processes

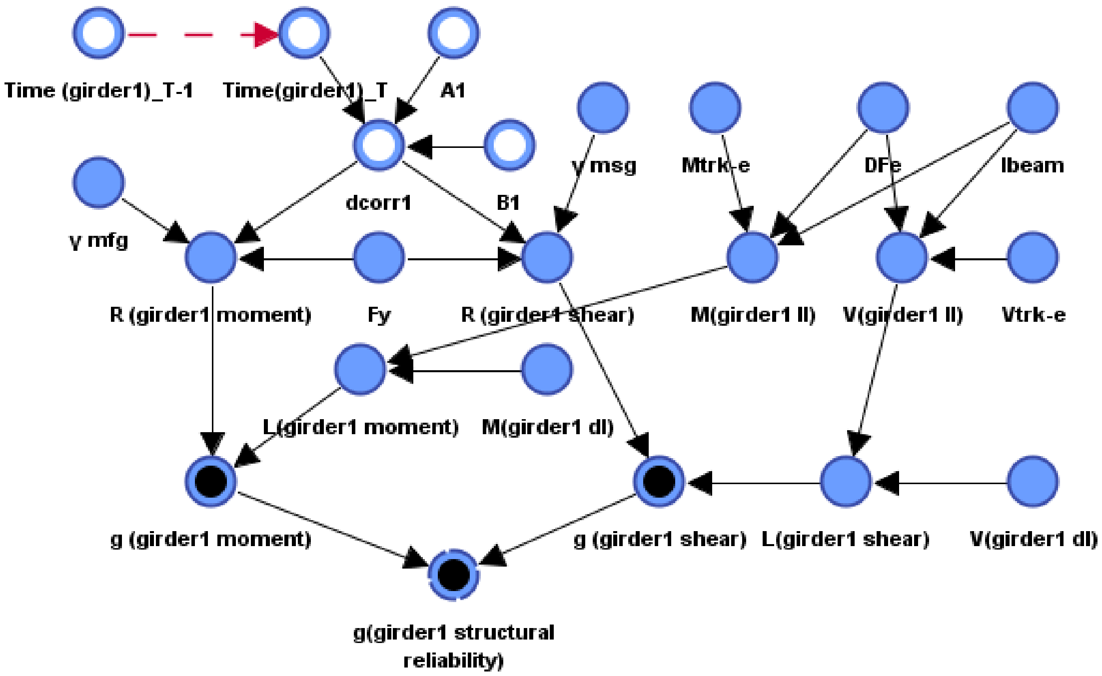

Figure 5-22. DOOBNs model of Girder 1 for time-variant structural reliability

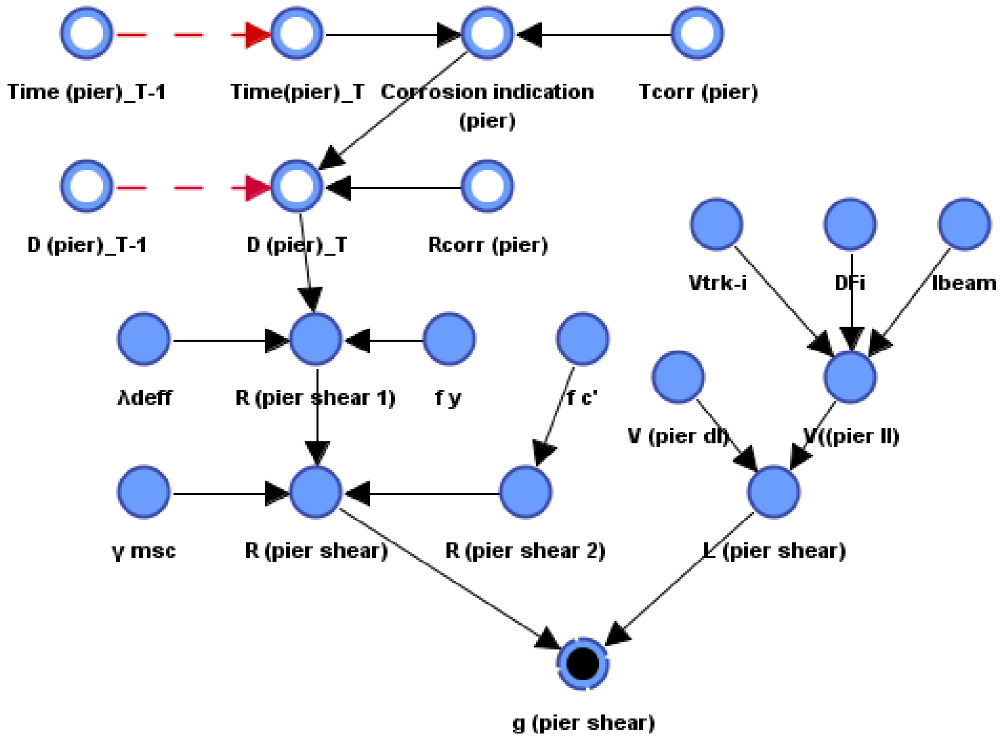

Figure 5-23. DOOBNs model of pier for time-variant structural reliability

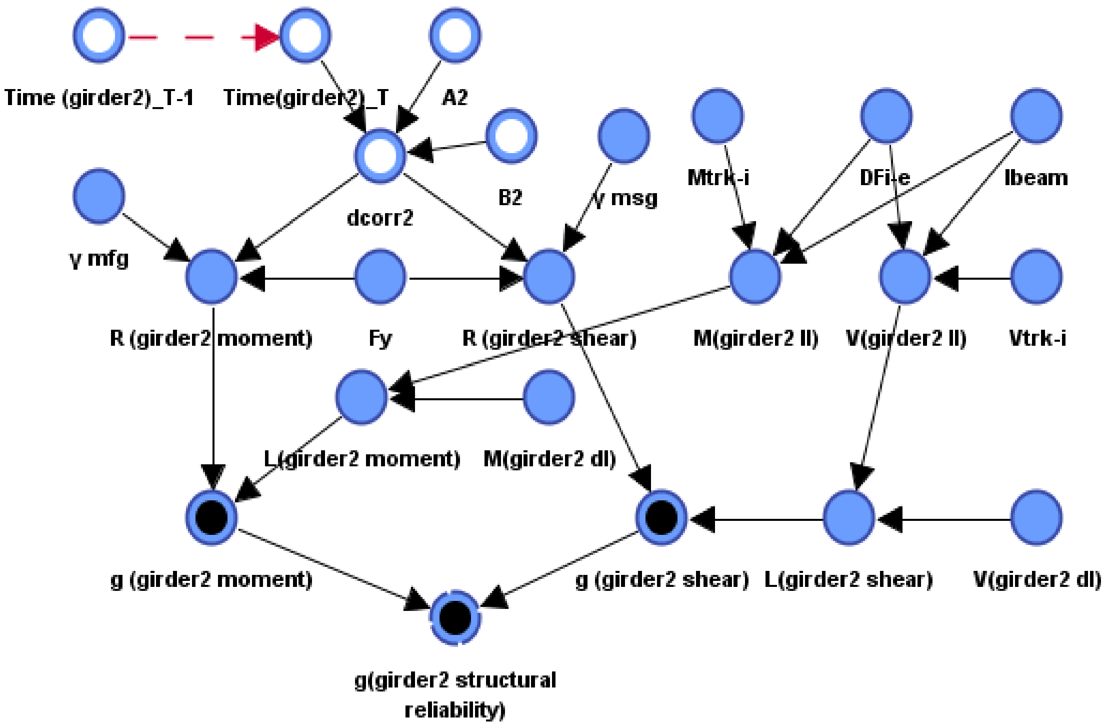

Figure 5-24. DOOBNs model of interior-exterior Girder 2 for time-variant structural reliability

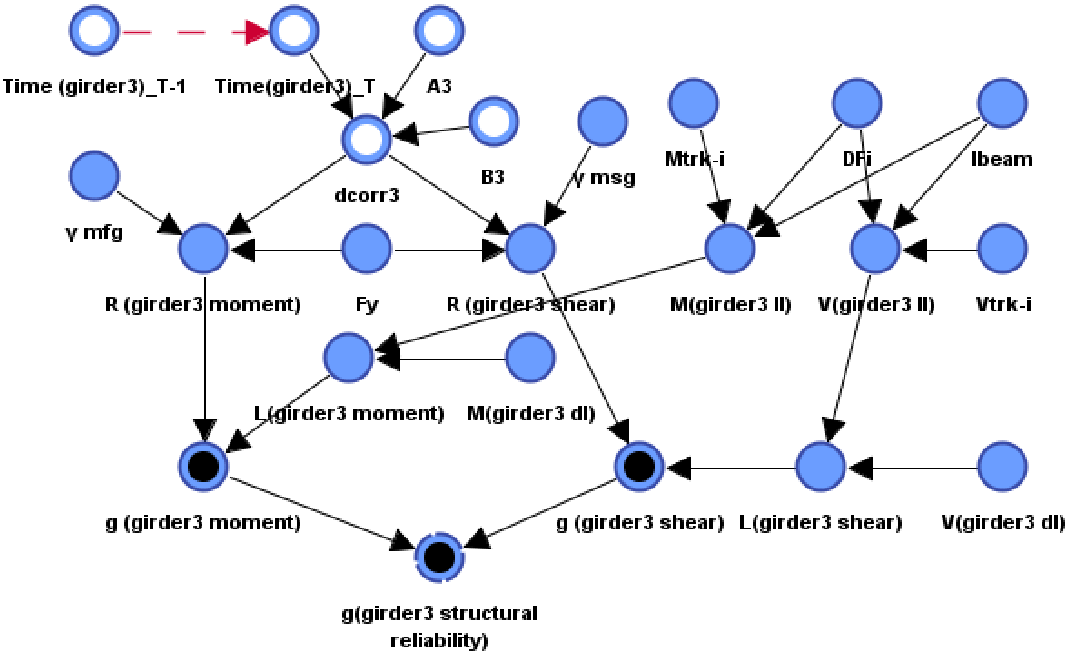

Figure 5-25. DOOBNs model of interior Girder 3 for time-variant structural reliability

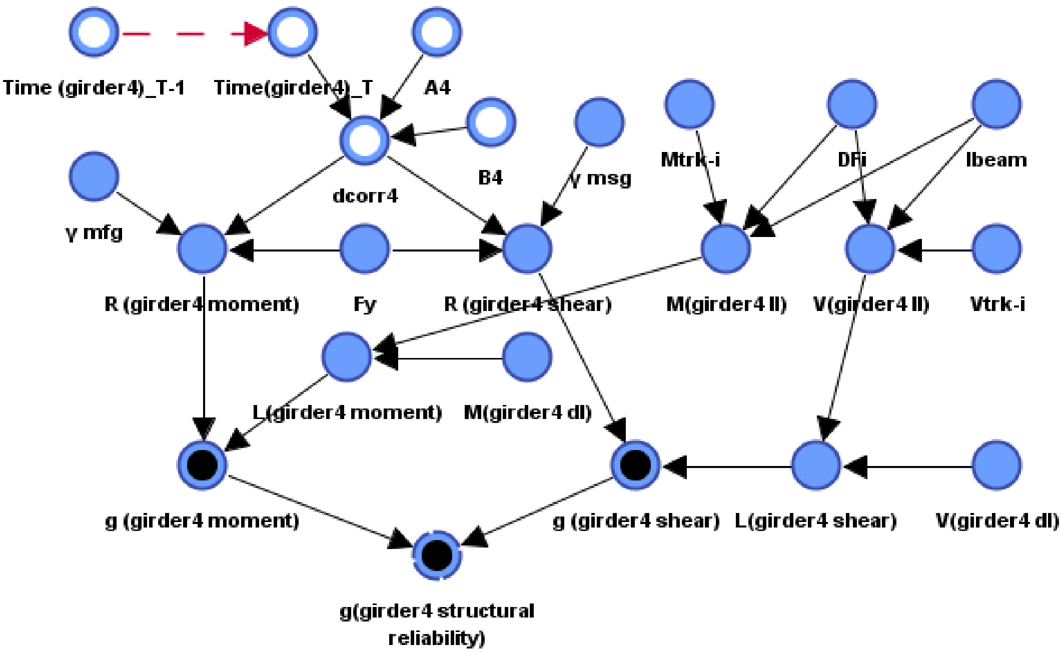

Figure 5-26. DOOBNs model of interior Girder 4 for time-variant structural reliability

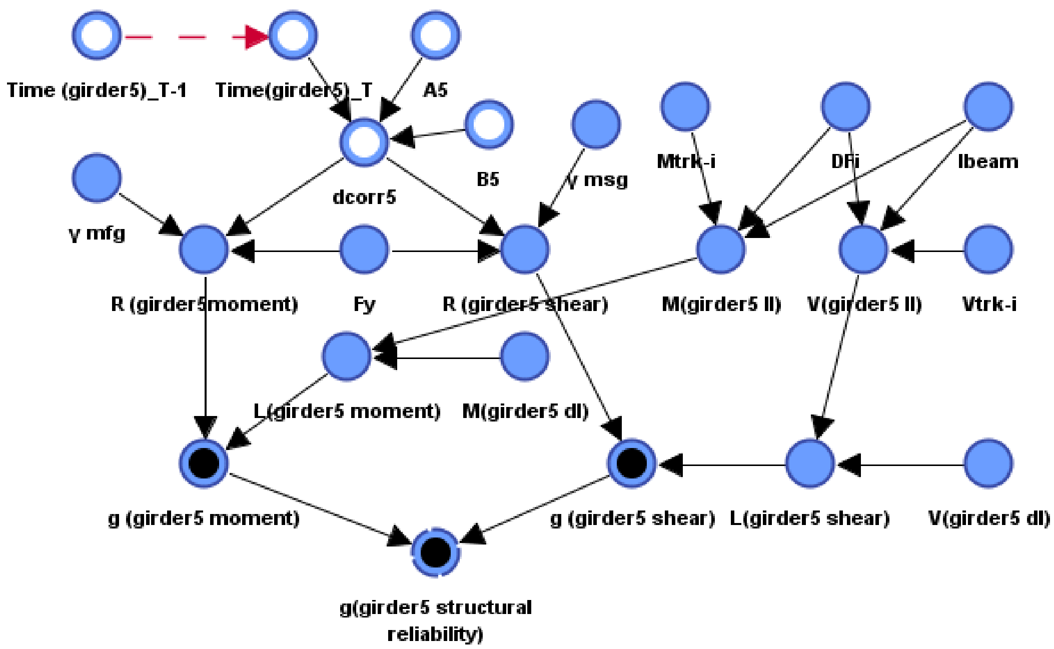

Figure 5-27. DOOBNs model of interior Girder 5 for time-variant structural reliability

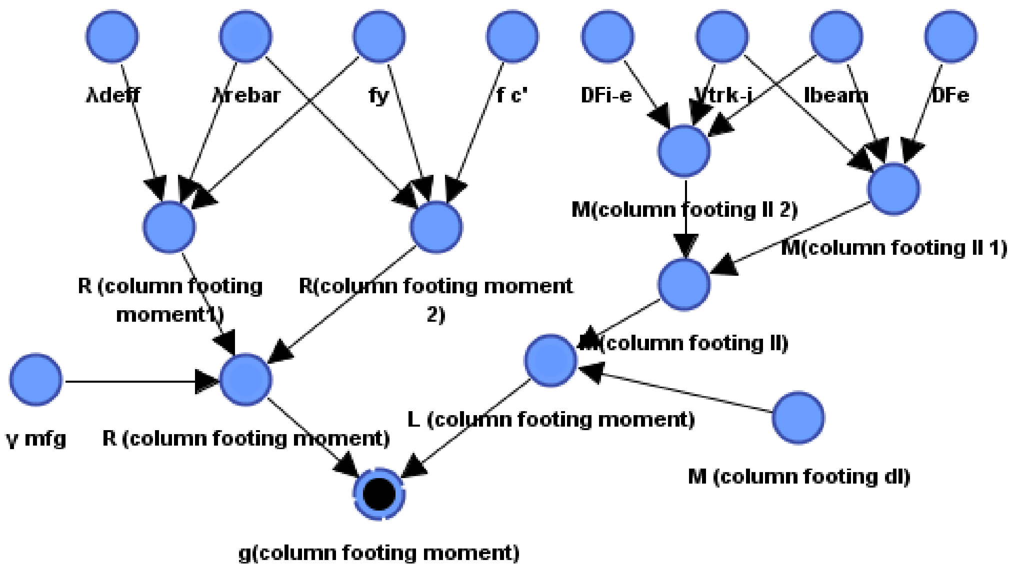

Figure 5-28. BNs model of column footing for time-variant structural reliability

5.3.1.3 Parameters estimation for developed DOOBNs model

After the conceptual DOOBNs model of Bridge E-17-AH is completed, the CPTs and priori probabilities have to be filled out. According to the data availability in this application, parameters estimation is carried out largely based on the existing literature and the limit state equations. Moreover, each continuous node of DOOBNs model is discretized into a finite number of discrete states to facilitate the implementation of inference algorithms.

➢ Assignment of CPTs and priori probabilities

For comparison purpose, CPTs associated to bridge hierarchies are easily estimated from traditional series-parallel relationship and the assumption that the failure of bridge system requires the failure of three adjacent girders. Tables 5-5, 5-6 and 5-7 illustrate the estimated CPTs, where we can see that the CPTs decode the deterministic series-parallel relationship into probability of either 1 or 0. Additionally, Table 5-8 shows the CPT based on the failure assumption of two adjacent girders. However, all these CPTs express deterministic relationship that is never verified in practice, and it is more reasonable to hold probabilistic relationship for bridge systems. For this application, since neither statistical data nor expert knowledge is available, the accurate estimation of probabilistic CPTs is impossible. To demonstrate the advantages and flexibility of proposed DOOBNs model, hypothetical CPTs are used to represent the probabilistic relationship in bridge systems. Tables 5-9 and 5-10 display the hypothetical CPTs considering probabilistic failure likelihood. In Table 5-9, the concern is that the deterioration of column footing can be ignored compared with the deterioration of pier, therefore, the failure of column footing plays less important role in the failure of bridge substructure. While, in Table 5-10, the concern is about how the failures of different girders contribute to the failure of the whole superstructure. To consider the real case, different probabilities in the interval of value [0, 1] are assigned.

Table 5-5. CPT of the whole bridge based on series-parallel relationship

	Deck	**F**				**S**			
	Superstructure	**F**		**S**		**F**		**S**	
	Substructure	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
The whole bridge	**Safe(S)**	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*1*
	Failed(F)	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*0*

Table 5-6. CPT of bridge substructure based on series relationship

	Column footing	F		S	
	Pier	F	S	F	S
Bridge substructure	Safe(S)	*0*	*0*	*0*	*1*
	Failed(F)	*1*	*1*	*1*	*0*

Table 5-7. CPT of bridge superstructure with the failure assumption of three adjacent girders

	Girder1	F															
	Girder2	F								S							
	Girder3	F				S				F				S			
	Girder4	F		S		F		S		F		S		F		S	
	Girder5	F	S	F	S	F	S	F	S	F	S	F	S	F	S	F	S
Bridge superstructure	Safe(S)	*0*	*0*	*0*	*0*	*1*	*1*	*1*	*1*	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*1*
	Failed(F)	*1*	*1*	*1*	*1*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*
	Girder1	S															
	Girder2	F								S							
	Girder3	F				S				F				S			
	Girder4	F		S		F		S		F		S		F		S	
	Girder5	F	S	F	S	F	S	F	S	F	S	F	S	F	S	F	S
Bridge superstructure	Safe(S)	*0*	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*1*	*1*	*1*	*1*	*1*	*1*	*1*
	Failed(F)	*1*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*1*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

Table 5-8. CPT of bridge superstructure with the failure assumption of two adjacent girders

	Girder1	**F**															
	Girder2	F								S							
	Girder3	**F**				**S**				**F**				**S**			
	Girder4	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder5	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*1*	*1*	*0*	*1*	*1*	*1*
	Failed(F)	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*0*
	Girder1	**S**															
	Girder2	**F**								**S**							
	Girder3	**F**				**S**				**F**				**S**			
	Girder4	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder5	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*0*	*0*	*0*	*1*	*1*	*1*	*0*	*0*	*1*	*1*	*0*	*1*	*1*	*1*
	Failed(F)	*1*	*1*	*1*	*1*	*1*	*0*	*0*	*0*	*1*	*1*	*0*	*0*	*1*	*0*	*0*	*0*

Table 5-9. CPT of bridge substructure with the consideration of failure uncertainty

	Column footing	**F**		**S**	
	Pier	**F**	**S**	**F**	**S**
Bridge substructure	**Safe(S)**	*0*	*0.8*	*0.2*	*1*
	Failed(F)	*1*	*0.2*	*0.8*	*0*

Table 5-10. CPT of bridge superstructure with the consideration of failure uncertainty

	Girder1	**F**															
	Girder2	F								S							
	Girder3	**F**				**S**				**F**				**S**			
	Girder4	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder5	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*0*	*0*	*0.1*	*0.2*	*0.2*	*0.2*	*0*	*0.2*	*0.3*	*0.4*	*0.2*	*0.4*	*0.4*	*0.5*
	Failed(F)	*1*	*1*	*1*	*1*	*0.9*	*0.8*	*0.8*	*0.8*	*1*	*0.8*	*0.7*	*0.6*	*0.8*	*0.6*	*0.6*	*0.5*
	Girder1	**S**															
	Girder2	**F**								**S**							
	Girder3	**F**				**S**				**F**				**S**			
	Girder4	**F**		**S**		**F**		**S**		**F**		**S**		**F**		**S**	
	Girder5	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**	**F**	**S**
Bridge superstructure	**Safe(S)**	*0*	*0*	*0.2*	*0.2*	*0.2*	*0.4*	*0.4*	*0.5*	*0*	*0.2*	*0.4*	*0.5*	*0.2*	*0.5*	*0.5*	*1*
	Failed(F)	*1*	*1*	*0.8*	*0.8*	*0.8*	*0.6*	*0.6*	*0.5*	*1*	*0.8*	*0.6*	*0.5*	*0.8*	*0.5*	*0.5*	*0*

With regarding to the CPTs associated to bridge elements, deterministic equations used for the conceptual model are directly utilised to elicit the parameters. Since most of the original variables are continuous variables, discretization is necessary for CPTs in discrete states so as to enhance inference computational efficiency. Moreover, the prior parameters of each bridge element can be found from the PhD thesis [47] and the existing literatures [8, 47, 154]. For instance, all the available probabilistic parameters of slab are summarised in Table 5-11. The available parameters of other bridge elements are listed in Appendix B. To estimate the priori probabilities of corrosion initiation time of slab and pier, the MCS based on physical equations is implemented, respectively. The detailed codes by means of MATLAB and the values of the parameters related to corrosion deterioration can be found in Appendix D.

Table 5-11. Probabilistic parameters of slab [47, 154]

variable	Distribution	Mean	Standard deviation
D_{slab} (inch)	Normal	0.625	0.0187
$R_{corr\ slab}$ (mils/year)	Normal	1.989	0.231
λ_{rebar}	Normal	1	0.015
γ_{mfc}	Normal	1.02	0.061
λ_{deff}	Normal	1	0.02
f_y (ksi)	Normal	56	6.16
f'_c (ksi)	Normal	2.76	0.497
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
$M_{slab\ dl}$ (ft-kip)	Normal	0.63	0.084
λ_{trk}	Normal	1.27	0.036
$M_{slab\ ll}$ (ft-kip)	Normal	5.41	0.153

➢ Discretization schemes

As most of the variables are defined in continuous states, and the identified CPTs and priori probabilities for DOOBNs model are continuous as well, the discretization is implemented to derive CPTs and priori probabilities in discrete states. The discretization scheme for slab flexure is summarized in Table 5-12. Equal length discretization interval is chosen in this application, different variables are assigned with different numbers of discrete intervals. The probabilities of each discrete state are assigned with cumulative distribution function (CDF) over the corresponding interval. With discretized nodes, the new CPTs can be estimated based on the deterministic equations. By sampling the intervals of the parent nodes and inserting the sampled values into the equations, a large number of function values are available for each configuration of the parents' sampled values. By taking the relative frequency occurrence of the function values in each interval of the specified child node, the CPTs with discrete states are obtained for each child node. In fact, the whole process is supported by the software BayesiaLab [16] and GeNie [56], which actually run the sampling algorithms for discretization. The discretization schemes for other bridge elements are presented in Appendix C. Since there are so many discretized

CPTs and the size of each CPT is quite large, the discretized CPTs in this application are only provided upon request.

Table 5-12. Discretization schemes for slab flexure

Variable	Probable range	Number of states	Final interval boundaries
D_{slab} (inch)	0.32-0.72	12	0,0.32:0.04:0.72,∞
$T_{corr\ slab}$ (year)	0-50	51	0:1:50
Time(slab)	0-50	51	0:1:50
$R_{corr\ slab}$ (mils/year)	0.8-3.2	12	0,0.8:(3.2-0.8)/10:3.2,∞
λ_{rebar}	0.9-1.1	12	0,0.9:(1.1-0.9)/10:1.1,∞
γ_{mfc}	0.7-1.3	12	0,0.7:0.6/10:1.3, ∞
λ_{deff}	0.9-1.1	12	0,0.9:0.02:1.1, ∞
f_y (ksi)	26-86	12	0,26:(86-26)/10:86,∞
f'_c (ksi)	0.3-5.3	12	0,0.2:5/10:5.3, ∞
$A_{t\ slab}$ (in^2)	0.14-0.82	12	0,0.14:0.68/10:0.82,∞
$M_{slab\ dl}$ (ft-kip)	0.2-2.2	12	0,0.2:2/10:2.2, ∞
$M_{slab\ ll}$ (ft-kip)	4.6-8.2	12	0,4.6:3.6/10:8.2, ∞
$R_{slab\ moment\ capacity}$	0-28	11	0:28/10:28,∞
$R_{slab\ moment\ capacity\ 2}$	0-4.8	11	0:4.8/10:4.8, ∞
$R_{slab\ moment\ capacity\ 1}$	0-30	11	0:30/10:30, ∞
$L_{slab\ load\ moment}$	4.8-11	12	0,4.8:6.2/10:11,∞

5.3.2 Prediction results of structural reliability

In this part, the completed DOOBN deterioration model is operated to predict the structural reliability of the whole bridge and its bridge elements over 50 years. With the support from the software GeNIe [56] and BayesiaLab [16], the inference algorithms for the structural reliability prediction can be easily executed. In this application, all the bridge elements are initialized with no deterioration at all. The corrosion loss and time are assumed to be zero in the beginning. Three scenarios are conducted for the verification of proposed DOOBNs model. First of all, based on series-parallel relationship and the assumption of three adjacent girders, the prediction results are compared with the ones calculated in the previous study [47] where the traditional method, FORM, had been adopted for structural reliability prediction. Second, Prediction results based on the assumption of two adjacent girders and hypothetical probabilistic relationship, respectively, are utilised to display the merit of proposed DOOBNs outdoing the traditional methods. At last, to further demonstrate the advantages, simulated event information, such as, maintenance actions, observations and environmental effects are included for updated structural reliability prediction.

➢ Scenario one

In order to compare with traditional methods, the CPTs shown in Tables 5-5, 5-6 and 5-7 are used for bridge system in this scenario. By running the DOOBNs model, time-variant structural reliability of the whole bridge as well as bridge elements is predicted during a period of 50 years. Figures 5-29, 5-30 and 5-31 display the comparisons between predicted results from DOOBNs and those calculated in previous study based on FORM [47]. For the purpose of convenience, the calculated failure probabilities are expressed in the form of reliability index. From the pictures, we can observe that both the results based on DOOBNs model and FORM indicate the same trends of reliability indexes. Although there are minor differences between the two groups of results, they are keeping close to each other all the time. Moreover, since the results calculated from FORM are only approximate estimation of structural reliability rather than accurate assessment, the minor differences never impede the prediction results from DOOBNs model to be reasonable evaluation. As a result, the comparisons have demonstrated the accuracy of the proposed DOOBNs model.

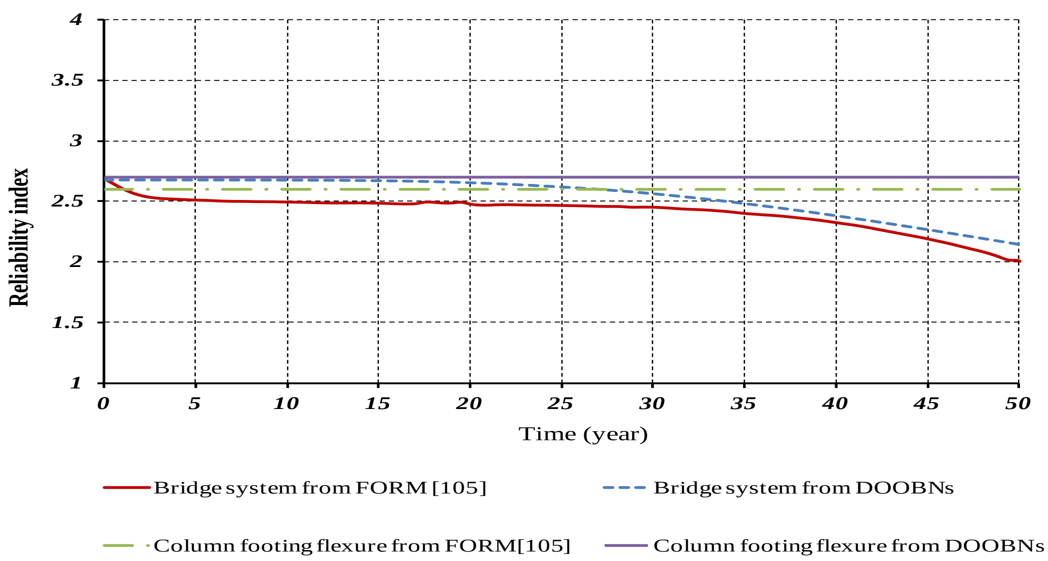

Figure 5-29. Comparison of reliability index of bridge system and column footing over time for Bridge E-AH-17

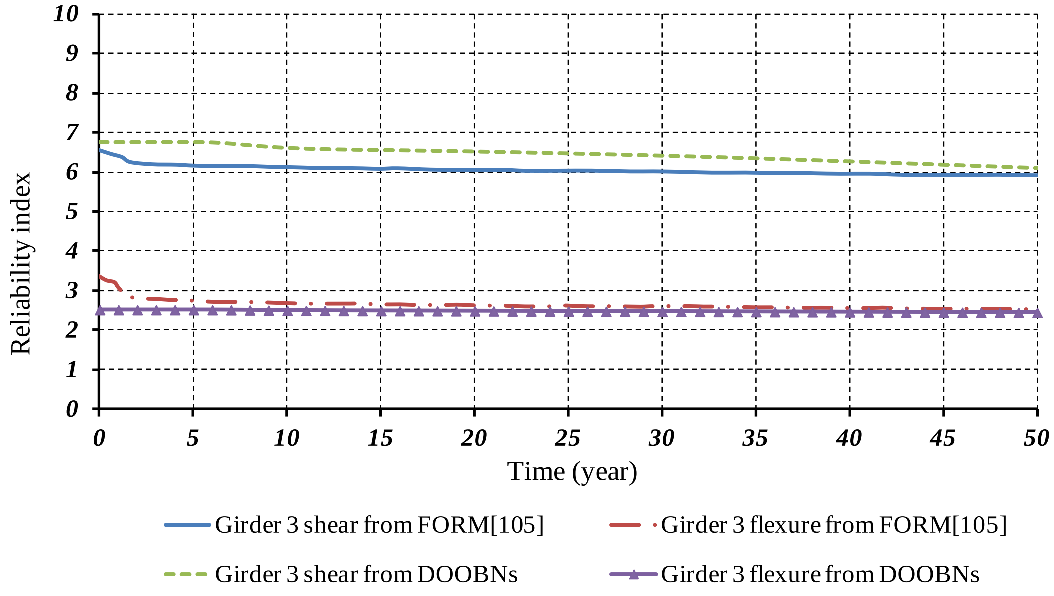

Figure 5-30. Comparison of reliability index of Girder 3 in both shear and flexure over time for Bridge E-AH-17

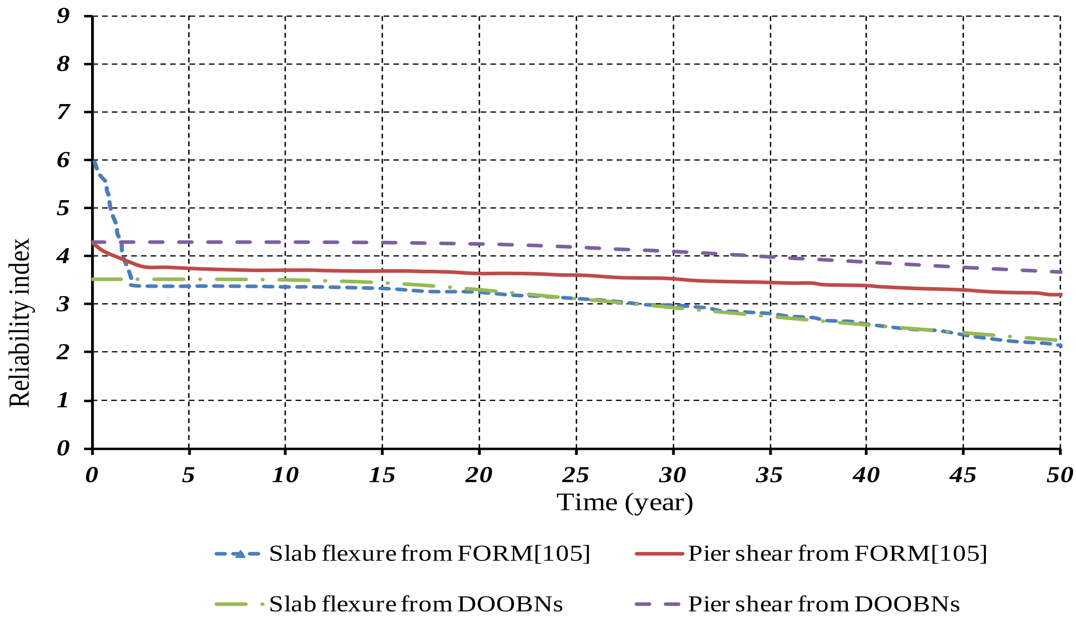

Figure 5-31. Comparison of reliability index of slab and pier over time for Bridge E-AH-17

➢ Scenario two

This scenario aims to explore the advantage of the proposed DOOBNs model. The DOOBNs model allows hierarchically representation of a complex bridge system with the consideration of not only deterministic parallel and/or series relationship but also probabilistic failure dependency between bridge system and bridge elements. Moreover, by means of CPTs, any types of failure assumptions can be easily modelled by changing the corresponding values in the CPTs. Therefore, if the failure assumption of two adjacent girders is held, we can simply adjust to the CPT shown in Table 5-8. The comparisons of prediction results resulting from the two different failure assumptions are presented in Figure 5-32. We can see that the reliability indexes of bridge superstructure with the failure assumption of two adjacent girders decrease. However, the reliability indexes of bridge system are identical all the time. This is due to the failure probability of bridge superstructure is relatively small compared with bridge substructure and deck. Additionally, reliability indexes of all the other bridge elements are also the same.

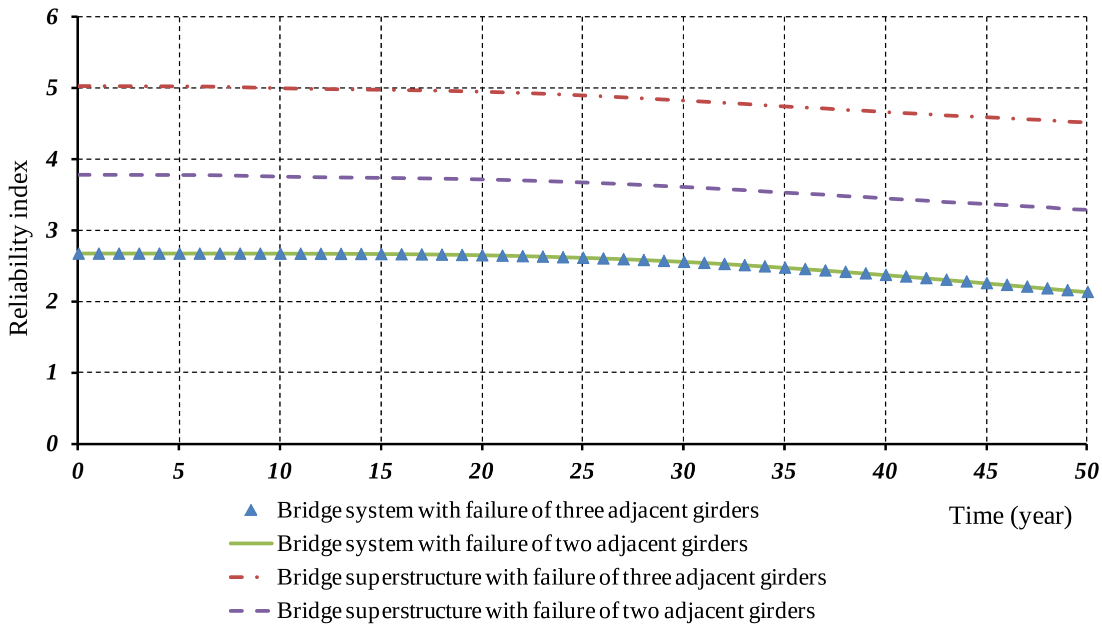

Figure 5-32. Comparisons of reliability index with two different failure assumptions

Furthermore, probabilistic relationship needs to consider as in practice there are lots of bridge failures happened occasionally. Different values between 0 and 1 are assigned in CPTs so as to model all kinds of failure relationships appropriately. For instance, with regarding to hypothetical probabilistic likelihoods of failure encoded by CPTs given by

Table 5-9 and Table 5-10, the updated prediction results are compared with the ones with failure assumption of three adjacent girders (Figure 5-33). By contrast, the failure probabilities of both bridge system and bridge superstructure escalate, while the failure probability of bridge substructure reduces, inversely. This is caused by the fact that the failure assumption turns to underestimate the failure probability of bridge superstructure, while series relationship overestimates the failure probability of bridge substructure. In addition, failure probabilities of other bridge elements are unchanged with the new CPTs. Based on the two examples the merit of proposed DOOBNs model to handle not only deterministic relationship but also probabilistic relationship in bridge systems has been well displayed.

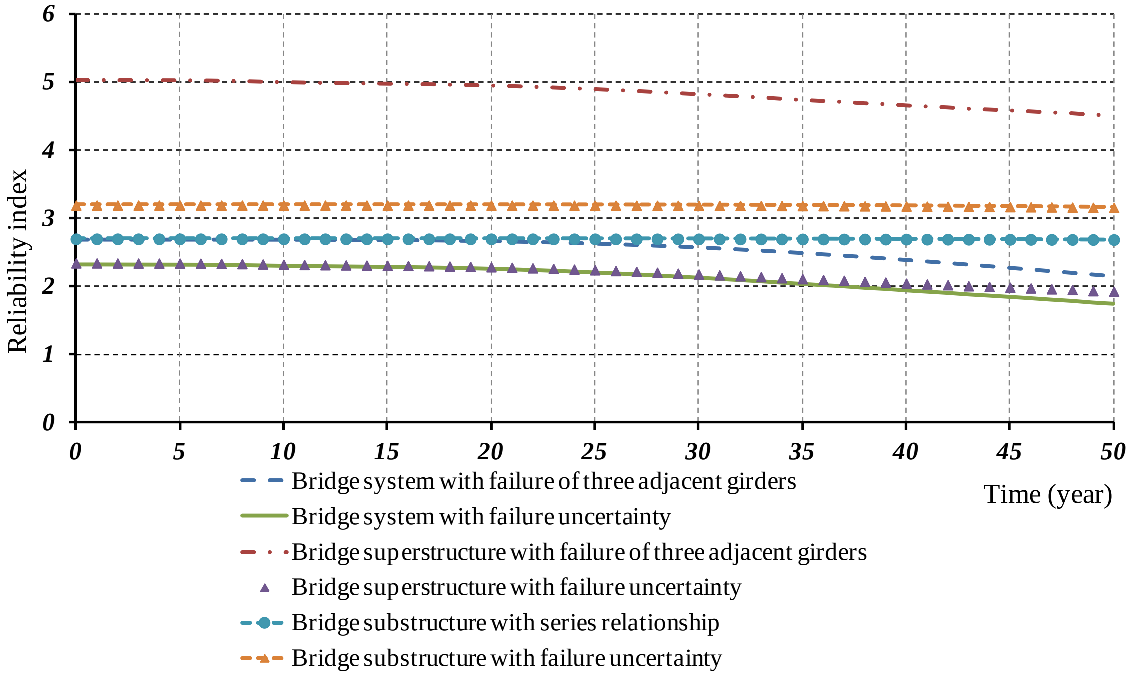

Figure 5-33. Comparisons of reliability index with failure uncertainty and failure assumption of three adjacent girders

- Scenario three

To demonstrate the automatic Bayesian updating ability of the proposed DOOBNs model, event information regarding to bridge deterioration processes are considered in this scenario. Additionally, since none of this information is available for this application in practice, simulated information is utilised to exhibit the ability. Here, available information from observation, maintenance actions and environment is simulated to bridge interior Girder 3, Girder 4 and Girder 5. Measurements of corrosion depth shown in Table 5-13 are simulated to these interior Girders. These measurements are assumed to be the true corrosion depth plus Normal distribution with μ=0 and σ=1 as follows:

$$\text{Measurements}=\text{dcorr}+N(0, 1) \tag{5-25}$$

Moreover, the same discretization scheme as dcorr is taken to the measurements. Maintenance actions scheduled in Table 5-14 are simulated to interior girders as well. There are three kinds of maintenance actions: no maintenance, imperfect maintenance and perfect maintenance. We assume that the perfect maintenance compensates corrosion loss and renew interior girders, while imperfect maintenance is assumed to have 50% chance to compensate corrosion loss and to renew interior girders. According to the literature[8], environmental levels is defined with two states “rural environment” and “urban environment”. The distributions of variable *A* and *B* under different environmental levels are described in Table 5-15[8]. Taking interior Girder 3 as an example, the DOOBN model for temporal deterioration processes is revised to consider the available information (Figure 5-34). Also, the revised DOOBNs model for time-variant structural reliability is given by Figure 5-35. The environmental level of Girder 3 is deemed to be “rural environment”.

Table 5-13. Measurement results of corrosion depth

Measurement Times (years)	5	10	15	20	25	30	35	40	45
Measurements (10^{-6}m)	95	275	423	601	818	934	1080	1422	1721

Table 5 14. Maintenance activities on interior girders during 50 years

Time (year)	15	45
Maintenance actions	**Imperfect maintenance** renew interior girders with 50% likelihood	**Perfect maintenance** renew interior girders with 100% likelihood

Table5-14. Distributions of *A* and *B* for different environmental levels[8]

Variables	Rural environment	Urban environment
A	N(34,3.06)	N(80.2,33.684)
B	N(0.65,0.065)	N(0.593,0.2372)

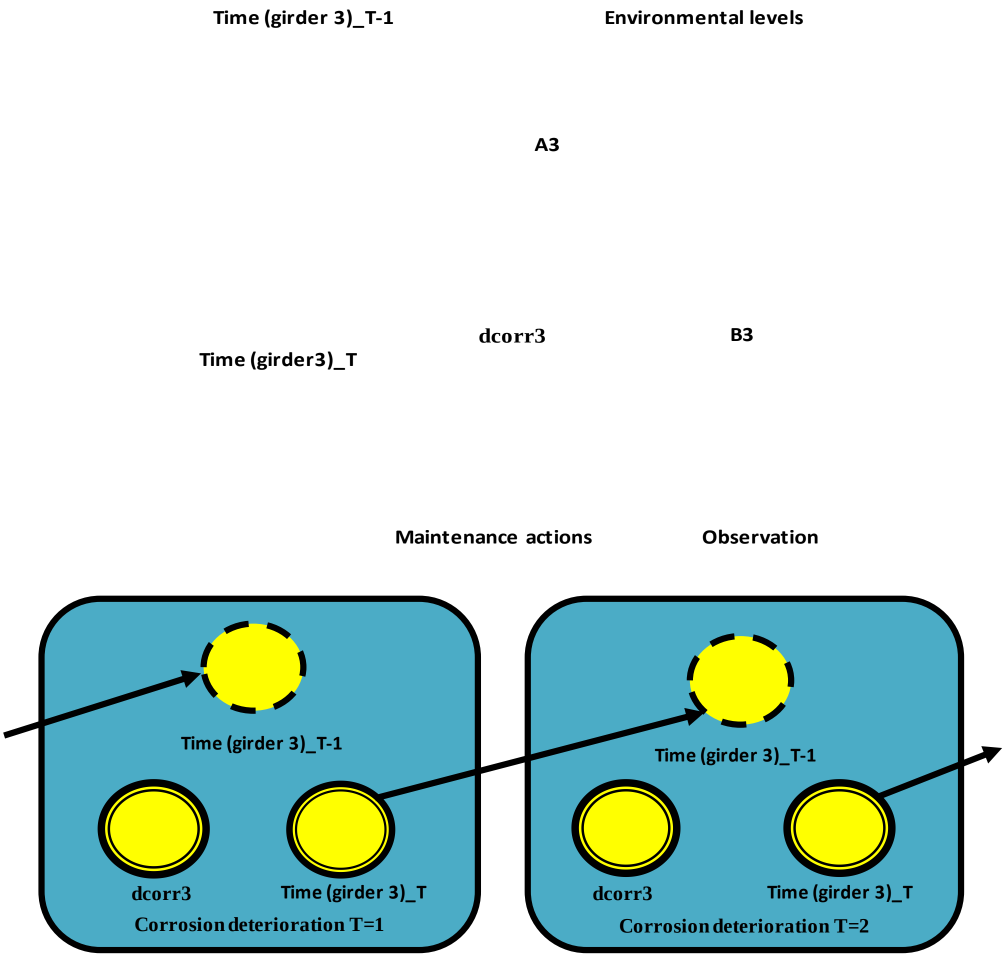

Figure 5-34. Revised DOOBNs model of interior Girder 3 for temporal deterioration processes including available event information

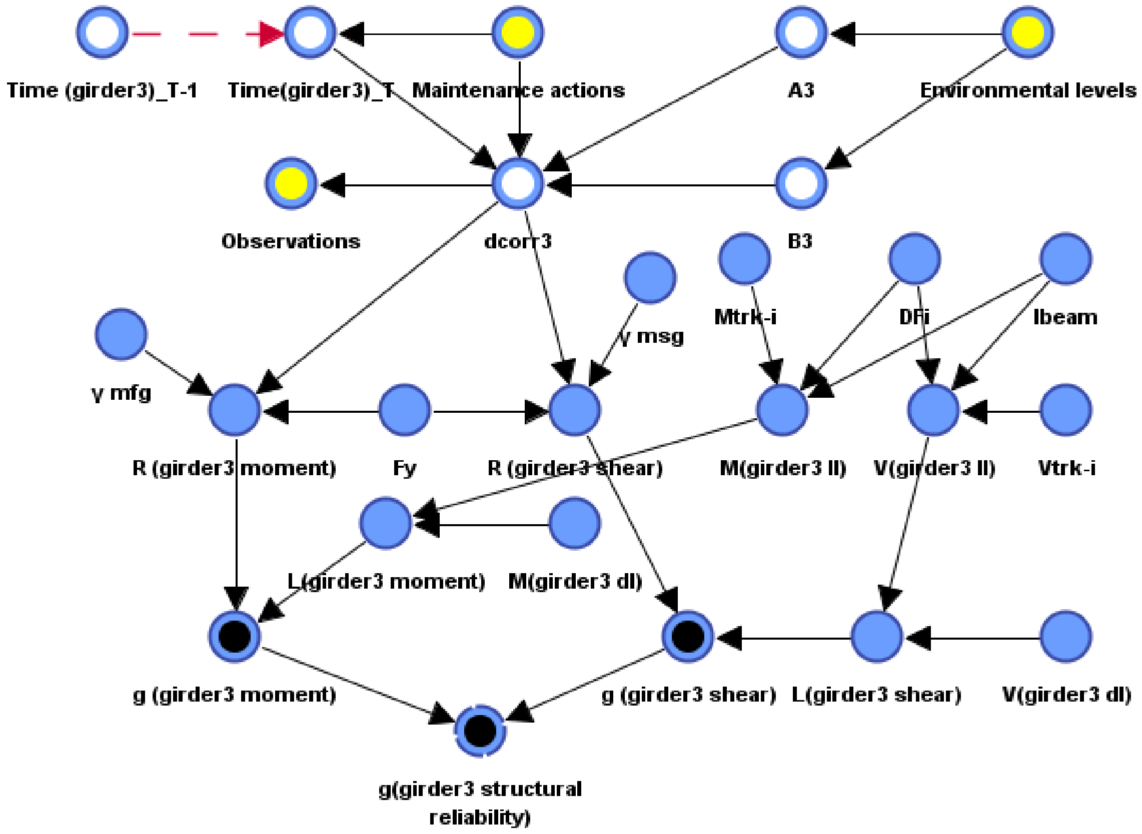

Figure 5-35. Revised DOOBNs model of interior Girder 3 for time-variant structural reliability including available event information

To validate the automatic updating ability, reliability indexes of interior girders as well as bridge superstructure are updated based on the simulated event information and the failure assumption of three adjacent girders. Based on the simulated measurements Figure 5-36 represents the resulted posterior reliability indexes of Girder 3 and bridge superstructure under the "rural environment". Under the same environmental level, Figure 5-37 shows the updated evolution of reliability index of Girder 3 in shear considering the simulated maintenance activities. The updated reliability indexes of Girder 3 in flexure and bridge superstructure are also given by Figures 5-38 and 5-39, respectively. By comparing these figures, we can find that the shear of Girder3 is so sensitive to the simulated information. In contrast, reliability indexes of Girder 3 in flexure and bridge superstructure are not changed so much due to the simulated information. The reasons are that the corrosion deterioration does not contribute to the reduction of flexure structural reliability very much, and the failure probability in flexure is relatively larger. In addition, the whole bridge system is nearly not influenced by the simulated information since the failure probability of bridge superstructure is too small to affect the final result slightly. Through

Figures 5-36 to 5-39, automatic updating ability of the proposed DOOBN model has been illustrated. This ability brings in more accurate prediction results of structural reliability, which benefits bridge maintenance optimization, eventually.

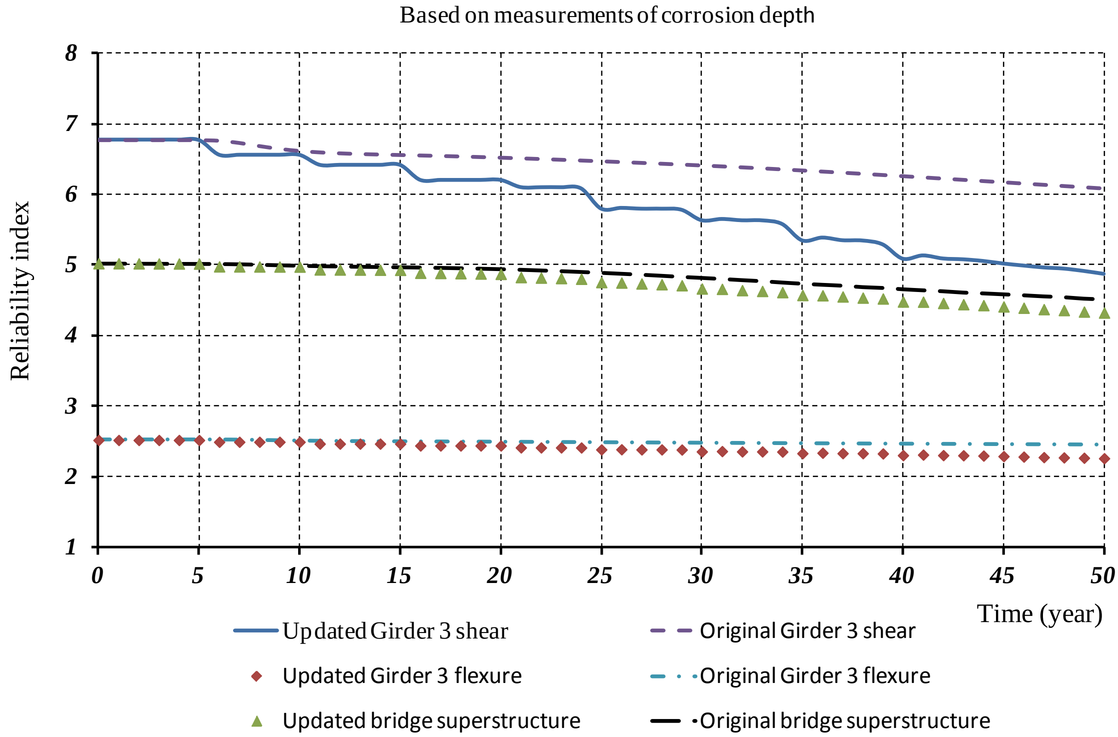

Figure 5-36. Updated reliability indexes of Girder 3 in both shear and flexure, and bridge superstructure based on simulated measurements and DOOBNs model II

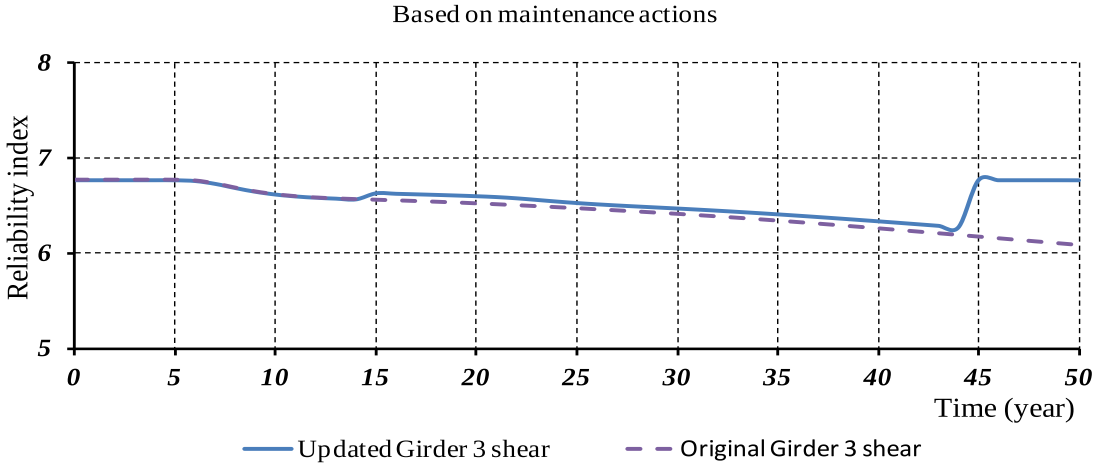

Figure 5-37. Updated reliability index of Girder 3 in shear based on simulated maintenance actions and DOOBNs model II

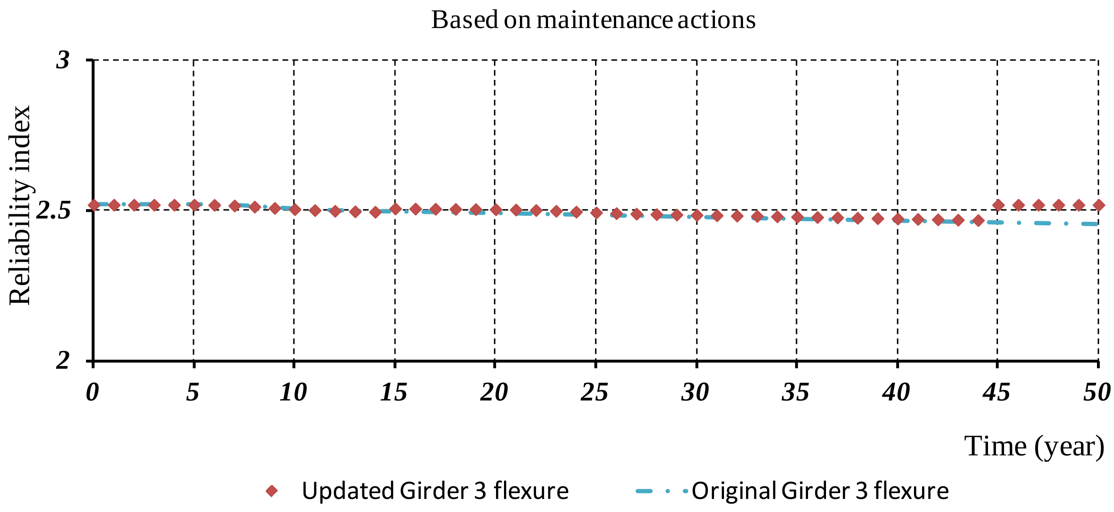

Figure 5-38. Updated reliability index of Girder 3 in flexure based on simulated maintenance actions and DOOBNs model II

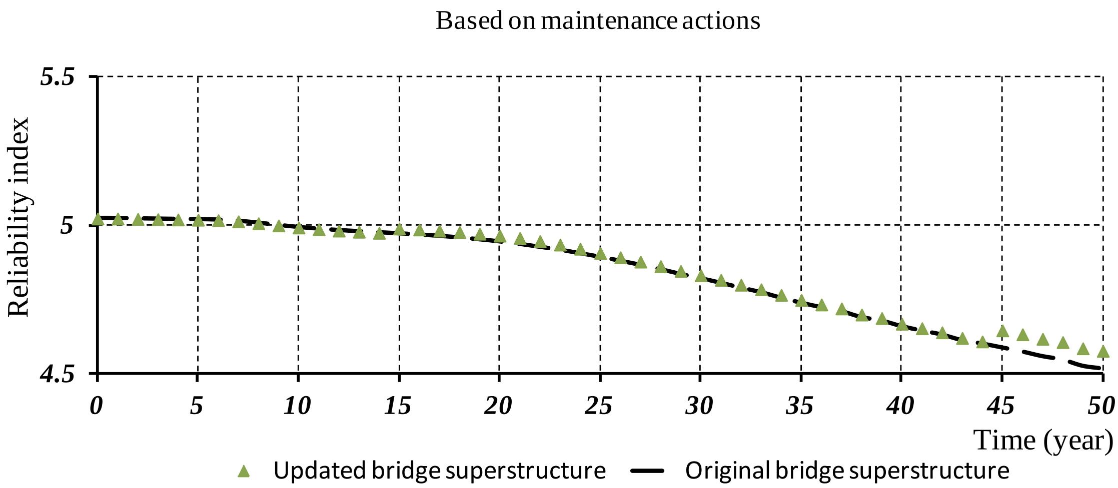

Figure 5-39. Updated reliability index of bridge superstructure based on simulated maintenance actions and DOOBNs model II

Overall, through the three scenarios above, the proposed DOOBNs model is validated to be superior to other traditional methods for structural reliability estimation of bridge system. The feasibility and merits of the proposed DOOBNs model have been demonstrated. The comparisons show that the DOOBNs model can perform reasonable results like traditional methods, such as FORM. In addition, the DOOBNs model is more suitable for the modelling of complex bridge system. Not only deterministic relationship but also probabilistic relationship can be handled by the DOOBNs model. The automatic Bayesian updating ability enhances computational efficiency of reliability updating. Therefore, information from observation, maintenance and environment can be easily incorporated to deal with uncertainties in bridge deterioration.

5.4 Summary

In this chapter, Model II for structural reliability prediction has been developed. The proposed DOOBNs model is generally applicable for different bridge structures, and is outlined through three steps: modelling consideration, DOOBN development and parameters estimation. In the first step, a bridge is hierarchically decomposed into a number of bridge elements. For each bridge element, limit state functions are developed. Then conceptual DOOBNs model is formulated through two parts: bridge system and bridge elements. In the part of bridge system, By means of the CPTs, both series-parallel logical relationship and complex probabilistic relationship can be effectively modelled. In the part of bridge elements, limit state functions and corrosion deterioration processes are modelled by DOOBNs. Moreover, event information about observations, maintenance actions and environment is included to reduce the prediction uncertainty. The last step focuses on the estimation of the CPTs, where deterministic equations are mainly used. To facilitate the inference computational efficiency, discretization is implemented for all the continuous variables. To verify the proposed DOOBN model, one application was given based on previous study [47]. As long as safety performance over 50 years is concerned, structural reliability from bridge elements to the whole bridge system is predicted. Three scenarios were conducted to demonstrate the advantages. The Model II is better choice for the modelling of complex bridge system. The Bayesian updating ability can improves the reliability updating efficiency so that event information can be incorporated efficiently. Based on all the merits, we can draw the conclusion that the proposed DOOBNs model is more appropriate for structural reliability prediction.

In the further, more research work should be focused on applying the proposed DOOBNs model to other bridge structures. Instead of simulated event information used in the application, real event information is expected for the validation. The discretization used in the DOOBNs model could bring the errors when a finite number of discrete states are utilised to approximate a continuous distribution. Therefore, optimal discretization schemes are demanded to eliminate these errors. Further study should be dedicated to the extension of the proposed model for bridge maintenance optimization. By expanding the proposed DOOBNs model with utility nodes and decision nodes, influence diagrams (IDs) as a decision tool can be derived [13].

Chapter 6: Integrated health prediction

6.1 Introduction

Based on the two previous chapters, this chapter integrates Models I and II for bridge health predictions in both serviceability and safety aspects. Cost-effective maintenance strategies are achieved based on health prediction in these two performance criteria. However, since the existing approaches are segregated and mutually exclusive, their prediction results in these two aspects cannot be used jointly for maintenance optimization. The integrated model, Model III, has the ability to model bridge deterioration in terms of both condition ratings and structural reliability. In Section 6.2, the modelling of bridge condition ratings is modified and the modelling integration is implemented by means of bridge essential failure modes. To validate Model III, an application based on data from open database and the existing literature is presented in Section 6.3.

6.2 Model III: using condition ratings and structural reliability jointly

In this section, an extended model for integrated bridge health prediction is developed based on Model I and Model II. To facilitate the integration, modified Model I for bridge condition ratings are presented at first. Then Models I and II are integrated through the modelling of bridge essential failure modes, such as, corrosion, cracking and spalling. The integration also includes the event information, such as, observed information, maintenance intervenes and environmental effects for each bridge element. At last, parameters estimation for the extended model needs to be accomplished. By modelling the underlying relationship between condition ratings and structural reliability, Model III generates enhanced prediction results to the same deterioration processes. As a result, prediction results in two performance criteria are calibrated and improved. Compared with Models I and II, Model III not only provides health prediction in two performance criteria but also calibrates prediction results with the consideration of latent correlations between condition ratings and structural reliability so that more accurate and consistent results can be achieved. Because Models I and II have been specified in previous chapters, only the extension part will be addressed in the following sections.

6.2.1 DOOBNs development

6.2.1.1 Modelling modification of bridge condition ratings

Regarding Model I, bridge condition ratings are predicted without distinguishing the locations of the same type of bridge elements. This is due to the routine inspection procedures used by the current BMSs. As a result, the same bridge elements with different locations are treated as one bridge entity. However, in Model II, bridge elements are treated individual entities. The difference has obstructed the modelling integration. Inspired by the segment-based inspection procedure [133], this compatible inspection method are adopted so that both previous and new inspection data can be used concurrently. Every time when an inspection is implemented, condition ratings of all the individual bridge elements will be assigned and recorded. Hence, Model I is modified to model not only one entity of the same bridge elements but also a number of individual bridge elements with different locations. The entity is conditional on these new individual bridge elements. For instance, if there are *N* different girders in a bridge, the entity of bridge girder is modelled as Figure 6-1.

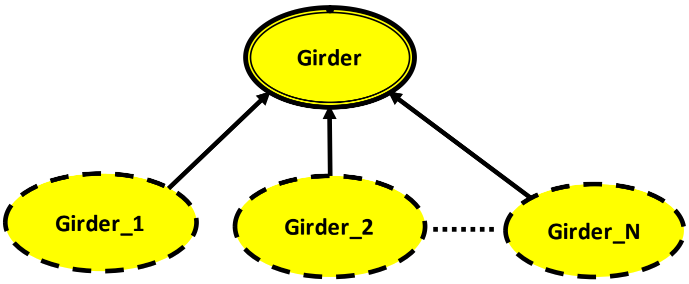

Figure 6-1. Modified OOBNs model of a bridge girder

In addition, without any specifications, the weights of all the new bridge elements are treated equally. The same condition ratings definition and DOOBNs modelling are applied to new individual bridge elements. However, since data and expert knowledge about condition ratings evolution of each individual bridge element are always insufficient, the transition probabilities of all these new bridge elements are assumed to be the same. Apparently, with this assumption the parameters estimation for these new bridge elements is simplified. The modelling modification enables basic modelling unit in Model I representing physical bridge elements rather than one entity of the same bridge elements, which paves the roads for the further modelling integration.

6.2.1.2 Modelling integration through critical failure modes

Although the condition ratings and structural reliability are two different bridge health performance criteria, they both concern the deterioration processes correlated to bridge critical failure modes, such as corrosion, crack and spalling. Therefore, these essential failure modes can be modelled in order to achieve the integrated modelling of bridge deterioration in both performance criteria. As only reinforced concrete and steel are considered in this research, their modelling integrations are illustrated, respectively.

- Steel

In light of bridge elements made of steel, only corrosion is identified as a critical failure mode. The corrosion deterioration process described by Equation 3-7 has been already modelled by DOOBN in Figure 5-7. However, this equation cannot reflect the real corrosion deterioration processes since painting for corrosion prevention on the surface of bridge elements is not considered. More information about painting effects is needed. On the other hand, valuable information about painting effects has been already expressed in details through the specification of condition ratings. As a result, for a steel bridge element, the modelling of corrosion deterioration processes regarding structural reliability prediction can be calibrated by its condition ratings from visual inspection. Based on this point, modellings of condition ratings and structural reliability can be integrated. The modified DOOBN modelling in steel is shown as Figure 6-2. In condition ratings definition, the extent of the corrosion on bridge elements is described from no corrosion to advanced corrosion. Considering painting effects for more accurate results, the node of corrosion starting time " t " is dependent on the node of condition ratings in Figure 6-2.

Moreover, if event information regarding to deterioration observations, maintenance actions and environmental effects is available for bridge elements, the event variables regarding condition ratings and structural reliability, respectively, can be integrated to reduce the number of nodes and size of the whole network. Generally, Figure 6-3 presents the integrated OOBNs modelling for steel bridge elements with the consideration of observations, maintenance intervene and environmental levels. The integration will bring observation variables with more parents, so new CPTs need to be estimated. Bearing in mind that integrated event variables may have different effects on the condition ratings and structural reliability. For instance, some condition loss oriented maintenance actions may have no influence on structural reliability. Hence, the corresponding CPTs may be different as well.

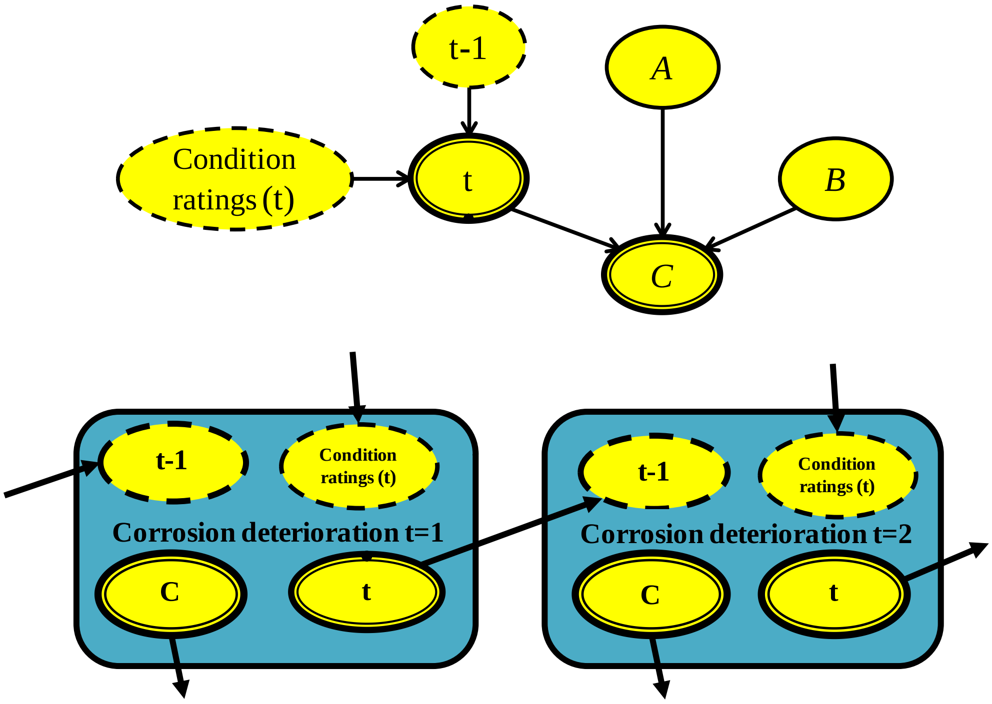

Figure 6-2. Modified DOOBN model for corrosion deterioration process in steel bridge elements

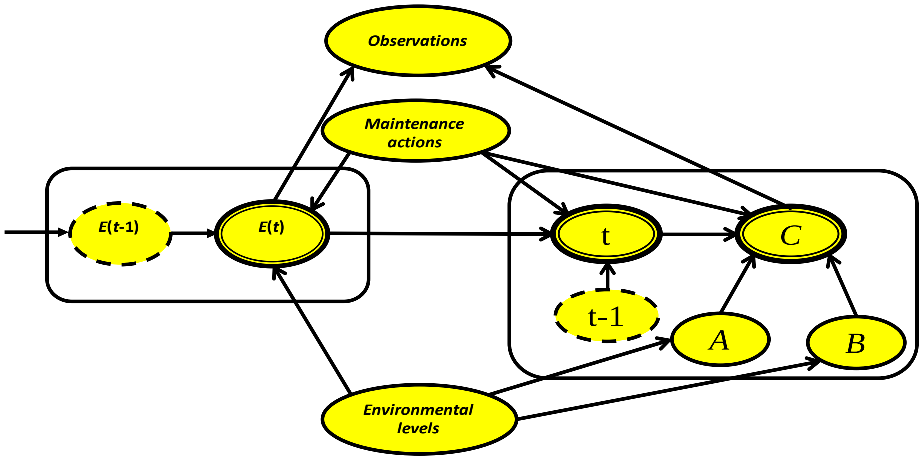

Figure 6-3. Modelling integration for steel bridge elements considering event information

➢ Reinforced concrete

For bridge elements made of reinforced concrete, there are three identified essential failure modes: corrosion, crack and spalling. Temporally, corrosion is initiated at first when critical chloride concentration is reached. After the initiation of corrosion, further deterioration will cause reinforced concrete cover to crack. Afterward larger and larger crack width will bring in spalling eventually. The deterioration processes in reinforced concrete have been explained in details by Section 3.2.2. Based on Equation 3-11, Figure 5-8 has specified the DOOBN modelling for corrosion deterioration process. However, failure modes of crack and spalling are excluded in this model. Based on Equations 3-17 and 3-19, the DOOBN model is extended to account for the other failure modes. In Figure 6-4, T_{corr} denotes the time to corrosion initiation since the beginning; T_{crack} denotes the time to crack initiation since the beginning; T_{spalling} denotes the time to spalling initiation time since the beginning. The node "crack indication" and node "spalling indication" are both discrete variables with two states "Yes" and "No".

To predict condition ratings accurately, the estimated deterioration rates are essential. Nonetheless, the calculated deterioration rates always tend to be underestimated because there are less observed data regarding severe bridge deterioration. As observed condition data are assessed directly from bridge inspectors' experience, intuition and judgement, or indirectly from advanced sensing techniques and NDT, some errors are expected among these data. Moreover, expert knowledge used to estimate deterioration rates may not be often available. For reinforced concrete bridge elements, bridge experts cannot easily acquire good knowledge about bridge deterioration processes since deterioration happening inside bridge elements is invisible. Therefore, the predicted results based on Model I are not so reliable. To enhance the accuracy, one alternative way is to calibrate deterioration rates with physical and chemical models of deterioration processes. As the essential failure modes of bridge deterioration have been modelled, prediction results of condition ratings can be easily calibrated. Figure 6-5 illustrates the modelling integration, where the condition ratings of a general bridge element "E(t)" is conditional on the indications of three essential failure modes. Event information can be integrated as Figure 6-3.

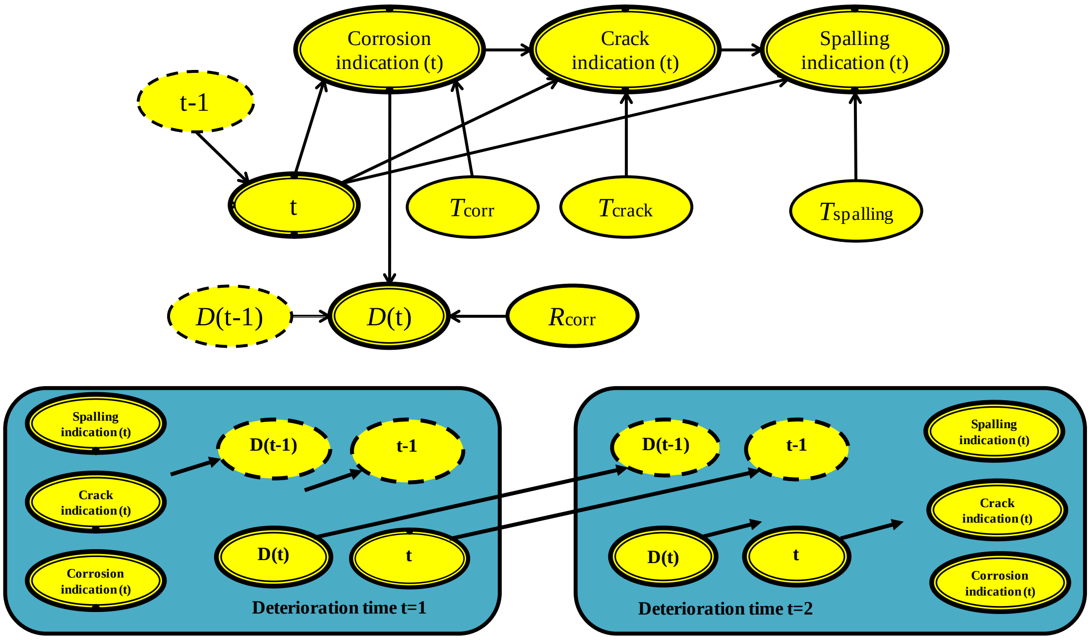

Figure 6-4. Modified DOOBN model for temporal deterioration process in bridge elements made of reinforced concrete

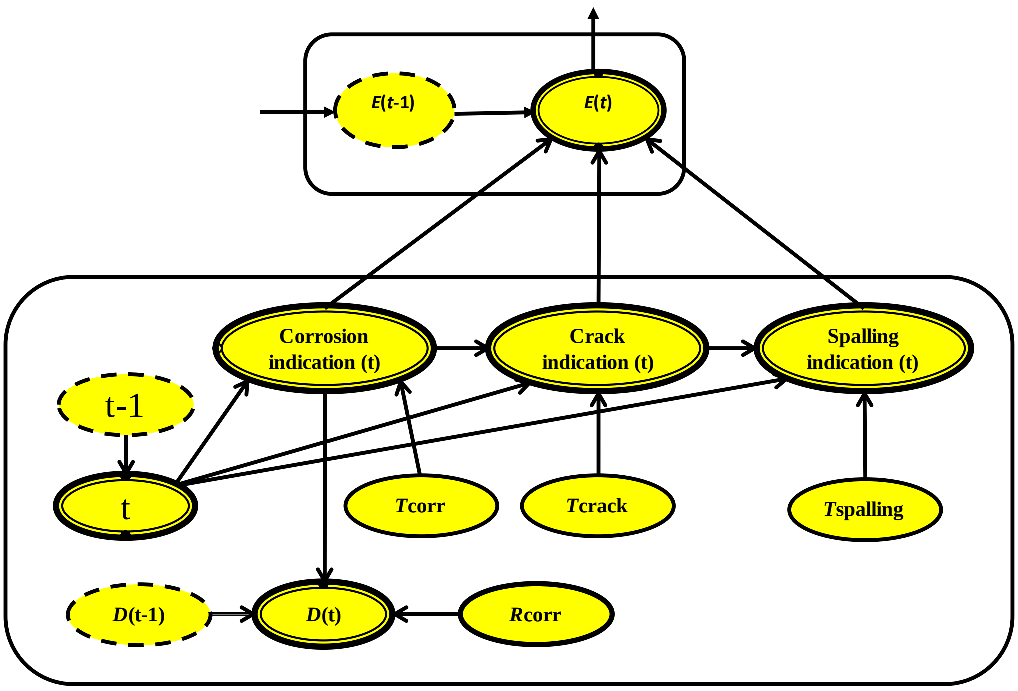

Figure 6-5. Modelling integration for reinforced concrete bridge elements

6.2.2 Parameters estimation

Model III is completed with the extended work above. As parameters estimation Models I and II have been specified in previous chapters, only CPTs and priori probabilities related to the extension part are addressed in this section. Overall, the parameters estimation is mainly based on physical equations of deterioration processes and condition ratings definition.

As the basis of essential failure modes, such as, corrosion, crack and spalling, physical equations of deterioration processes are directly used to estimate priori probabilities and CPTs. For instance, priori probabilities of crack and spalling initiation time are derived from Equations 3-9, 3-14 and 3-18 based on MCS. The distributions of crack and spalling initiation time are estimated by MCS with parameter values from the existing literature. In accordance with other variables, the estimated distributions need to be discretized to facilitate inference algorithms. The detailed simulation processes can be found in Appendix D. The CPTs related to node “crack indication” and node “spalling indication” are estimated based on Equations 3-17 and 3-19.

For the CPTs regarding modelling integration, condition ratings specification is used. The extents of corrosion deterioration are described, for instance, no corrosion, paint distress, rust deformation, active corrosion and section loss. Considering practical paint protection, in Figure 6-2, the node of corrosion time “ t ” is conditional on the node of condition ratings in order to determine whether structural reliability deterioration of steel bridge elements has actually started or not. The corresponding CPT is set to zero until corrosion is actually initiated. In terms of bridge elements made of reinforced concrete, the evolutions of condition ratings are adjusted by bridge essential failure modes. For example, if the state of node “crack indication” in Figure 6-5 is true, probabilities of all the condition ratings describing no crack are set to zero.

6.3 Case study of Bridge E-17-AH: integrated health prediction

For the purpose of validation, Model III is applied to Bridge E-17-AH for integrated health prediction. The application is based on open database “National Bridge Inventory” (NBI) and the previous study from the literature [47]. Moreover, as Bridge E-17-AH has been modelled in the last chapter, the repeated work will not be addressed in this application again. A customized Model III based on DOOBNs is developed to predict both condition ratings and structural reliability of this bridge over 50 years. To demonstrate the integrated health prediction, the predicted results are compared with the ones obtained from Models I and II. With simulated event information, the automatic Bayesian updating ability can be also illustrated in this application, where prediction results in both performance criteria are updated.

6.3.1 National Bridge Inventory (NBI) database

Towards integrated health prediction, historical condition information about Bridge E-17-AH is necessary. In this application, we resort to the National Bridge Inventory (NBI) which is a huge database covering just fewer than 600,000 of the United States' bridges located on public roads, including Interstate Highways, U.S. highways, State and county roads, as well as publicly-accessible bridges on Federal lands [50]. The summary analysis of the number, location, and general condition of highway bridges within each State is given state by state. The NBI information is collected annually to Federal Highway Administration (FHWA) by the state highway agencies all around the U.S. The data are all recorded in NBI data format, which has 116 items and totally 432 characters for each data record. The explanations for each item can be found from the report [51]. The NBI data are easily downloaded from the internet in ASCII files and used for different purposes.

For this application, a number of bridges located in Colorado, similar to Bridge E-17-AH, were chosen to secure enough amounts of data for parameters estimation. The selection criteria are to consider “Record Type”, “Route Signing Prefix”, “Kind of Material/Design” and “Type of Design/Construction” in NBI data record [51]. As a result, a number of condition records at NBI#58 (deck), #59(superstructure) and #60 (substructure) were selected out. The history of the selected data lasts from 1992 to 2010. For unknown reasons, the condition data of some bridges are not available all the time. Therefore, the selected data are regarded as incomplete data.

6.3.2 DOOBNs development of Bridge E-17-AH for integrated health prediction

6.4.2.1 Modelling consideration for integrated health prediction

In order to construct an integrated DOOBNs model, the system analysis of Bridge E-17-AH in the last chapter is extended. Based on the simplified series-parallel model (Figure 5-14) the bridge has been hierarchically decomposed into three levels with identified bridge elements including five girders, one slab, one pier and one column footing. Since the deterioration of column footing can be ignored compared with other elements [47], it is excluded from the condition ratings modelling of Bridge E-17-AH. Moreover, all the five girders are treated equally and assigned with the same relative weights. The condition ratings definition outlined by FHWA (Table 2-2) is employed [51], where condition rating 9 denotes excellent condition and condition rating 0 denotes failed condition. Because the worst record in the selected data is condition rating 3, the predicted probabilities of condition ratings less than 3 will be zero all the time. Therefore, only condition ratings more than 2 are considered in this application. Finally, no deterioration dependencies among these bridge elements are identified.

6.4.2.2 Integrated DOOBNs model

Customised Model III is conceptually formulated in this section. Based on the system analysis, Model I for Bridge E-17-AH can be developed as Figure 6-6, where the node with red colour centred signifies the whole bridge; the nodes with light blue colour centred signify bridge deck, superstructure and substructure; the nodes with black colour centred signify bridge elements; the red dash lines represent temporal deterioration for each bridge element: a bridge element node (T-1) and its evolution node (T) are defined as a discrete time Markov process modelled by the CPT. As the developed Models I and II are separate and irrelevant, the latent correlations between condition ratings and structural reliability are missed and prediction results may not be consistent reflecting true deterioration. Therefore, these two models are integrated into Model III.

The integration is actually performed in bridge elements level. For the bridge girders made of steel, the condition states embodying valuable information about painting effects are used to calibrate the corrosion deterioration processes for structural reliability prediction. The integrated DOOBNs models of the five girders are displayed from Figure 6-7 to Figure 6-11. From the figures, we can see that both condition states and structural reliability are modelled in one network, and the node of structural reliability is indirectly conditional on the node of condition states. For the bridge slab and pier made of reinforced concrete, essential failure modes of corrosion, crack and spalling are modelled to assist the integration. Taking the bridge slab as an example, the extended DOOBNs model of its essential failure modes is illustrated in Figure 6-12. By means of this temporal deterioration model, the evolutions of these two performance criteria are linked each other for integrated health prediction. Figure 6-13 and Figure 6-14 present the integrated DOOBNs models of the slab and pier. For Bridge E-17-AH, as long as the integrated DOOBNs model of each bridge element is completed, the nodes of condition states and structural reliability in these models are further connected to the previous OOBNs models of the whole bridge system. The updated health prediction results of bridge elements are further utilised to update the condition states and structural reliability of the whole bridge system.

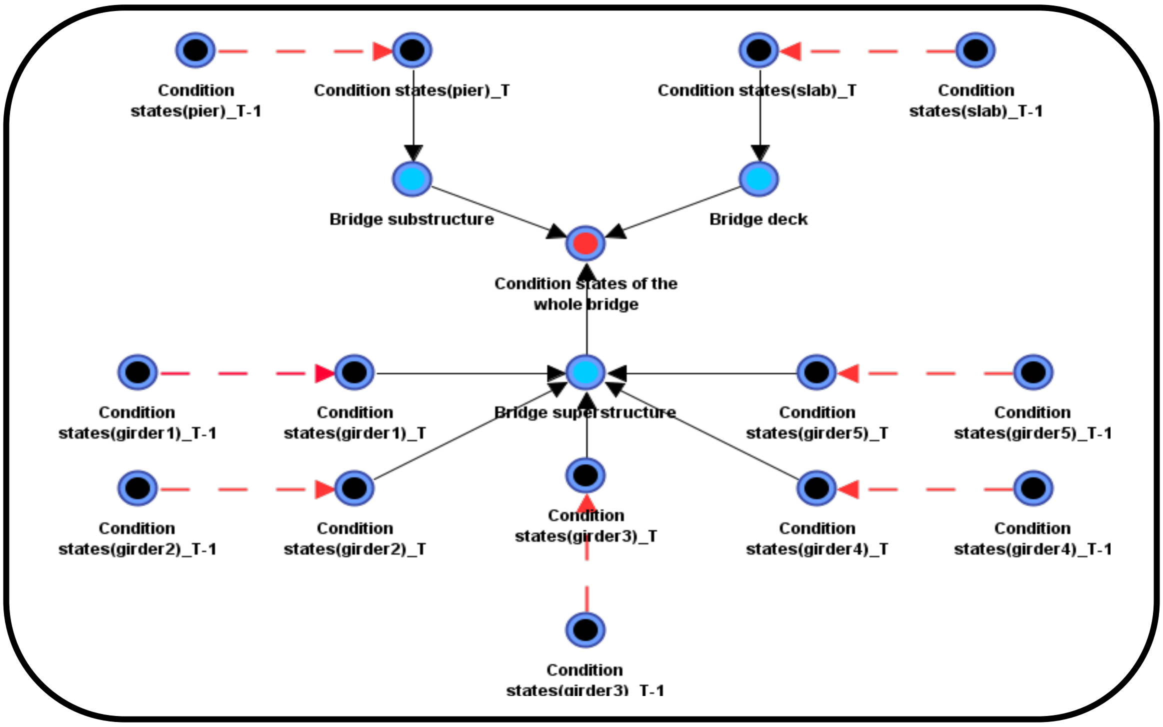

Figure 6-6. DOOBNs model of bridge system for condition states

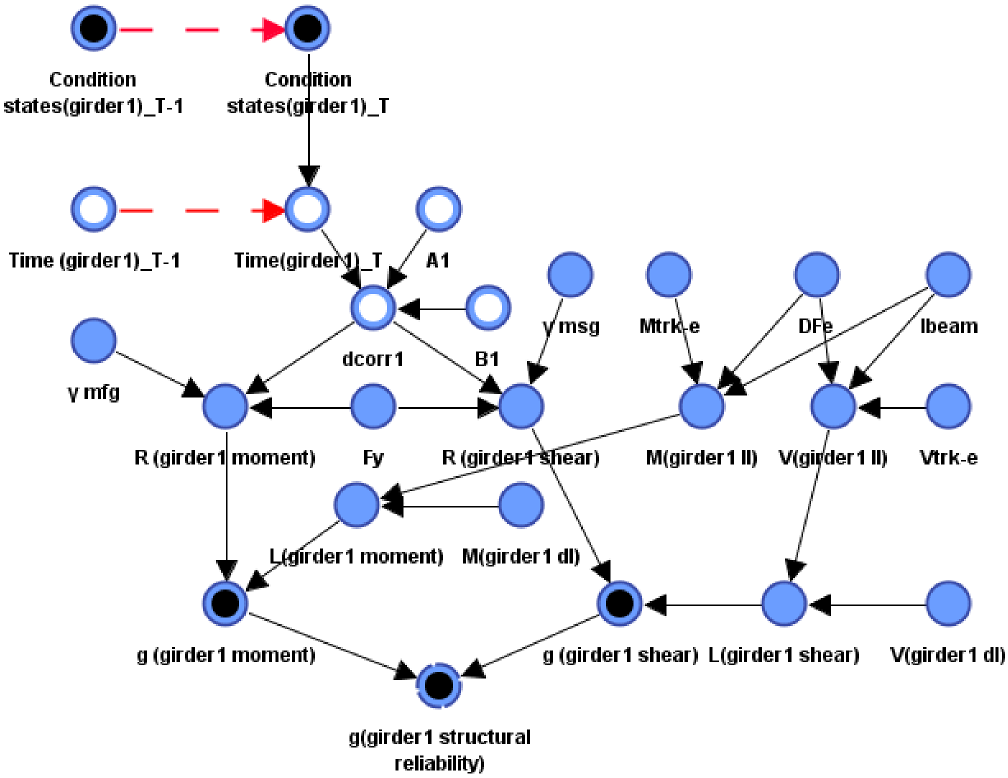

Figure 6-7. DOOBNs model of Girder 1 for integrated health prediction

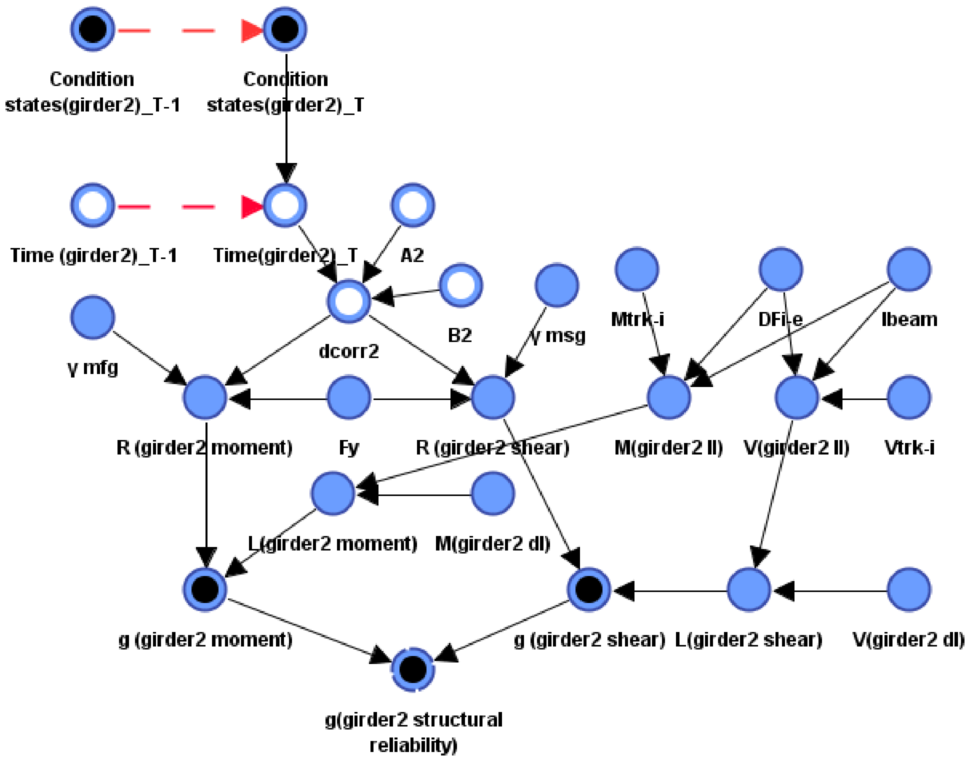

Figure 6-8. DOOBNs model of Girder 2 for integrated health prediction

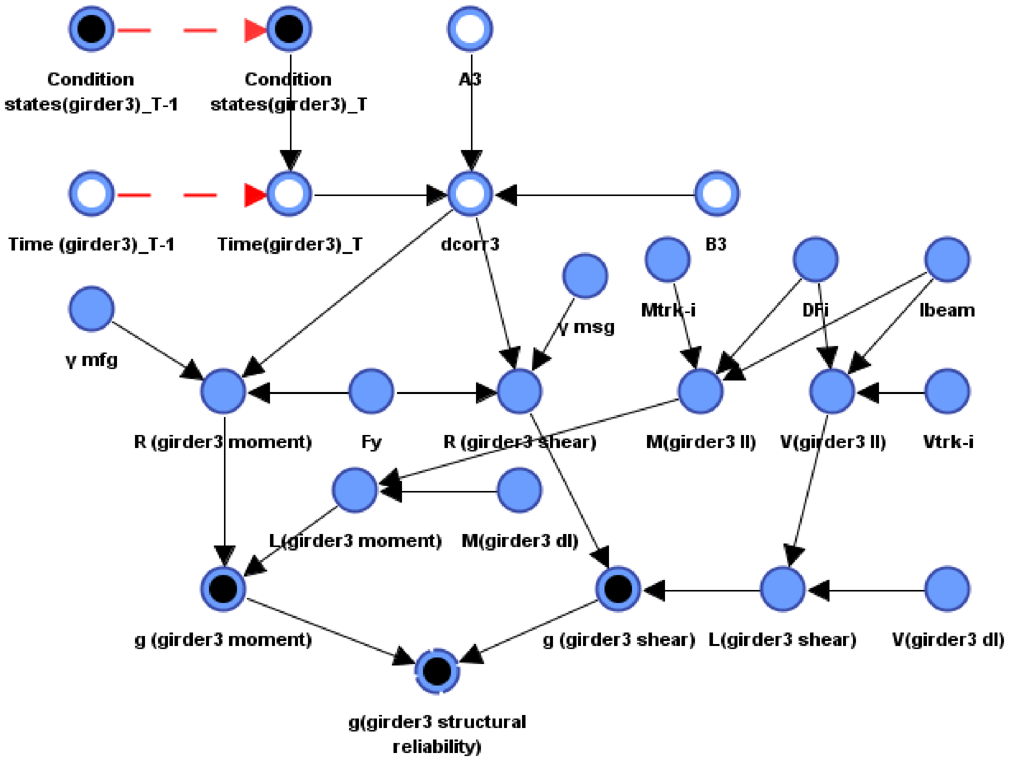

Figure 6-9. DOOBNs model of Girder 3 for integrated health prediction

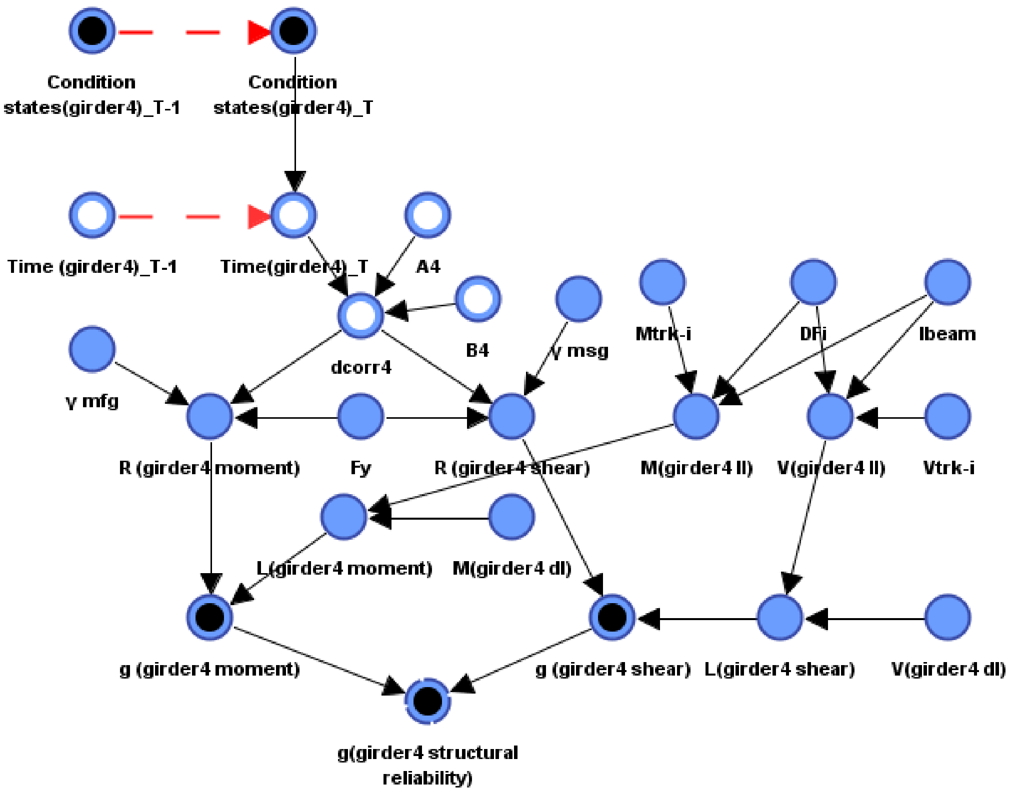

Figure 6-10. DOOBNs model of Girder 4 for integrated health prediction

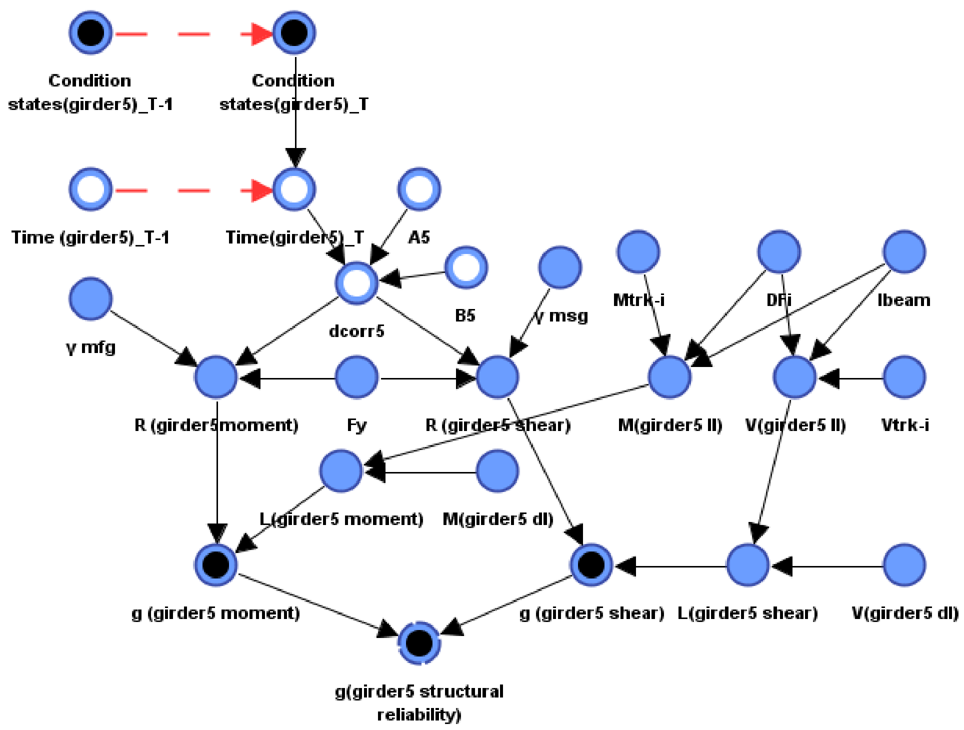

Figure 6-11. DOOBNs model of Girder 5 for integrated health prediction

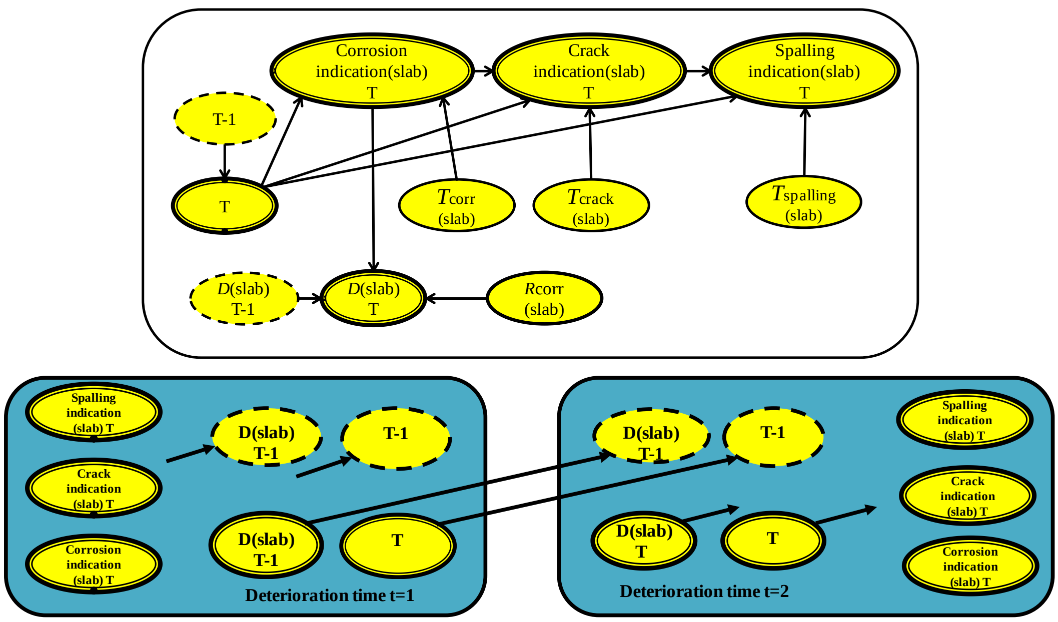

Figure 6-12. DOOBNs model of slab critical failure modes

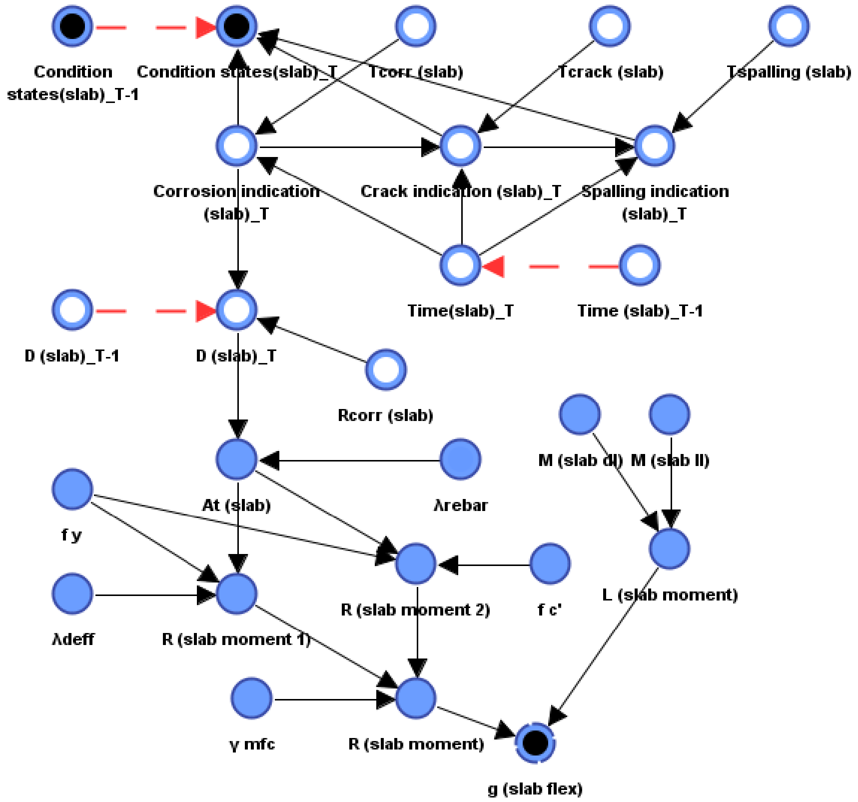

Figure 6-13. DOOBNs model of slab for integrated health prediction

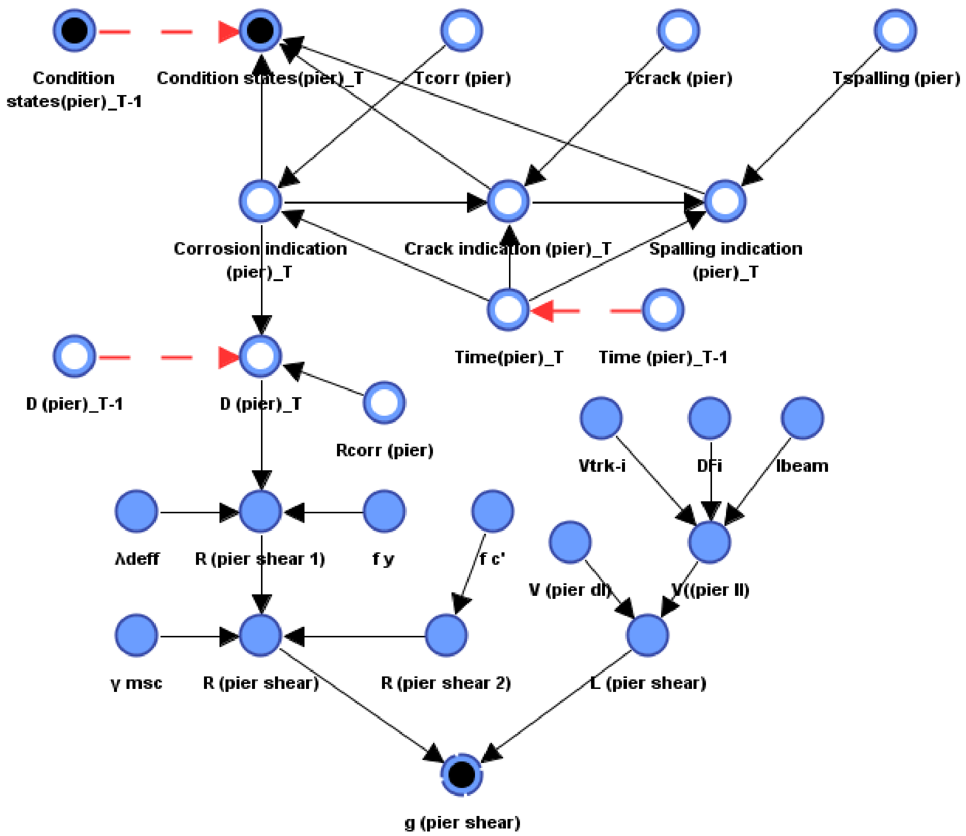

Figure 6-14. DOOBNs model of pier for integrated health prediction

6.4.2.3 Parameters estimation for the integrated DOOBNs model

The parameters estimation of the newly obtained DOOBNs model for integrated health prediction is addressed here. As CPTs and priori probabilities regarding structural reliability of Bridge E-17-AH have been tackled in the last chapter, this parameters estimation focuses on condition states part and integration part.

➢ Condition ratings part

With the relative weight of each bridge element, the CPTs of the whole bridge system are easily estimated based on Equation 4-2. Therefore, more effects are made to estimate the CPTs associated to bridge elements. According to the data availability, the selected data from NBI database are chosen in this application. However, the condition data are only related to bridge deck, superstructure and substructure rather than bridge elements. For the sake of CPTs estimation, the bridge deck, superstructure and substructure are assumed to be slab, girders and pier based on the simplified series-parallel model (Figure 5-14). The selected data denote the condition states evolutions of bridge elements. Since the data are deemed as incomplete data, an appropriate algorithm, the Expectation-maximization (EM), is employed. To implement the EM algorithm, the number of transitions from condition rating i to condition rating j (i,j=9,8,…,3) for each bridge element is calculated at first. Based on the MATLAB Software, all the CPTs associated to bridge elements can be estimated. Table 6-1 shows the CPT of bridge slab, which actually models the discrete time Markov process of the slab. Furthermore, with the assumption of the same deterioration rates, the CPT of the girders is given by Table 6-2. From the two CPTs, we can see that after experiencing the initial condition rating 9, bridge elements tend to stay at their current condition ratings.

Table 6-1. CPT of bridge slab based on EM algorithm

Condition states Slab (t-1)	Condition 9	Condition 8	Condition 7	Condition 6	Condition 5	Condition 4	Condition 3
Condition9	*0.3333*	*0.6667*	*0*	*0*	*0*	*0*	*0*
Condition8	*0*	*0.8732*	*0.1268*	*0*	*0*	*0*	*0*
Condition7	*0*	*0*	*0.9619*	*0.0381*	*0*	*0*	*0*
Condition6	*0*	*0*	*0*	*0.9591*	*0.0409*	*0*	*0*
Condition5	*0*	*0*	*0*	*0*	*0.9516*	*0.0484*	*0*
Condition4	*0*	*0*	*0*	*0*	*0*	*0.9789*	*0.0211*
Condition3	*0*	*0*	*0*	*0*	*0*	*0*	*1*

Table 6-2. CPT of girders based on EM algorithm

Condition states Slab (t-1)	Condition 9	Condition 8	Condition 7	Condition 6	Condition 5	Condition 4	Condition 3
Condition9	*0.3333*	*0.6667*	*0*	*0*	*0*	*0*	*0*
Condition8	*0*	*0.8925*	*0.1075*	*0*	*0*	*0*	*0*
Condition7	*0*	*0*	*0.9547*	*0.0453*	*0*	*0*	*0*
Condition6	*0*	*0*	*0*	*0.9693*	*0.0307*	*0*	*0*
Condition5	*0*	*0*	*0*	*0*	*0.9788*	*0.0212*	*0*
Condition4	*0*	*0*	*0*	*0*	*0*	*0.9655*	*0.0345*
Condition3	*0*	*0*	*0*	*0*	*0*	*0*	*1*

- The integration part

After the parameters estimation for condition states is finished, some new parameters raised by the modelling integration need to be estimated as well. As discussed in Section 6.2.2, physical equations of deterioration processes and condition ratings definition are adopted. For bridge elements made of reinforced concrete, the priori probabilities of corrosion, crack and spalling initiation time in Figure 6-12 are estimated based on Equations 3-9, 3-14 and 3-18. With specified probabilistic parameters for slab and pier, MCS is implemented to derive the corresponding distributions. In addition, these initiation time distributions are dsicretized with 1 year equal interval over 50 years. The detailed codes by means of MATLAB and all the parameters used for the estimation can be found in Appendix D. These equations help estimate the CPTs of the failure modes indications. To estimate the CPTs regarding the modelling integration, we rely on the condition states definition outlined by FHWA (Table 2-2) [51]. For steel bridge elements, when the condition rating is more than 6, the corresponding CPT of the node "Time (girders)" should be set to zero. For bridge elements made of reinforced concrete, when the node "corrosion indication" is true, probabilities of the condition ratings more than 7 are set to zero; when the node "crack indication" is true, probabilities of the condition ratings more than 6 are set to zero; when the node "spalling indication" is true, probabilities of the condition ratings more than 5 are set to zero.

6.3.3 Prediction results of integrated health performance

The complete DOOBNs model for integrated health prediction is implemented to predict both condition states and structural reliability over 50 years. The operation is supported by the same software GeNIe [163] and BayesiaLab [165]. To begin with, all the bridge elements are initialized with the condition states showing no deterioration at all (CS9). The corrosion loss and initiation time are all assumed to be zero at the beginning. Furthermore, to validate the proposed DOOBNs model III, we have conducted two scenarios. In order to demonstrate integrated health prediction, in the first scenario the prediction results of condition states and structural reliability are compared with the ones calculated from the separated DOOBNs models. The second scenario aims to validate the Bayesian updating ability of the proposed DOOBNs model III. With simulated maintenance actions, both condition states and structural reliability predictions are updated from the bridge elements to the whole bridge system.

➢ Scenario one

By running the DOOBNs model III, we can predict both condition states and structural reliability of Bridge E-17-AH during a period of 50 years. Owing to the modelling integration, all the original prediction results are expected to be updated from bridge elements to the whole bridge system. For the purpose of comparison, we also predict condition states and structural reliability based on Models I and II. Figures 6-15, 6-16, 6-17 and 6-18 illustrate the prediction results of condition states of bridge girders, bridge slab, bridge pier as well as the whole bridge system, respectively, based on Model I. By contrast, Figures 6-19, 6-20 and 6-20 present the updated prediction results after the integration. Except bridge girders, the condition states of all the other bridge elements and the whole bridge system are updated. This is because that condition states of bridge girders can be easily observed through visual inspection. The prediction results are believed to be accurate and employed to calibrate the prediction results of structural reliability. As the condition states of bridge slab and pier are invisible, special equipments are needed for inspection. Therefore, the prediction results are supposed to be inaccurate. They can be calibrated by essential failure modes.

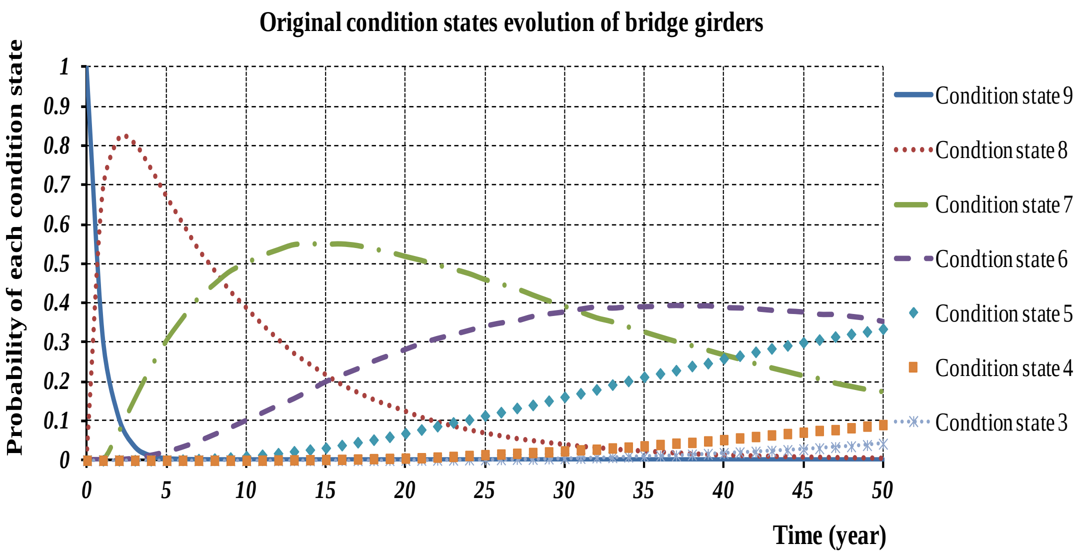

Figure 6-15. Condition states evolution of bridge girders based on model I

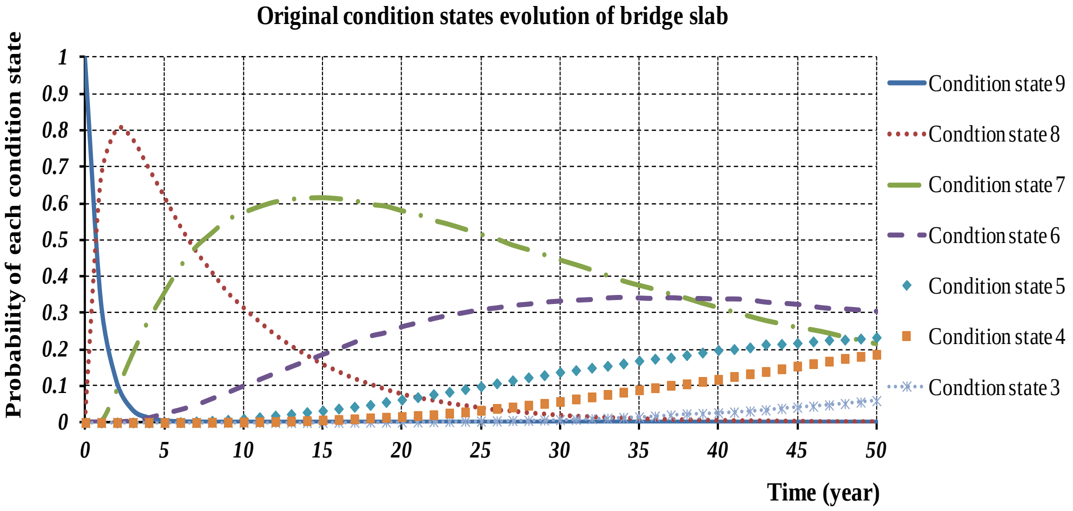

Figure 6-16. Condition states evolution of bridge slab based on model I

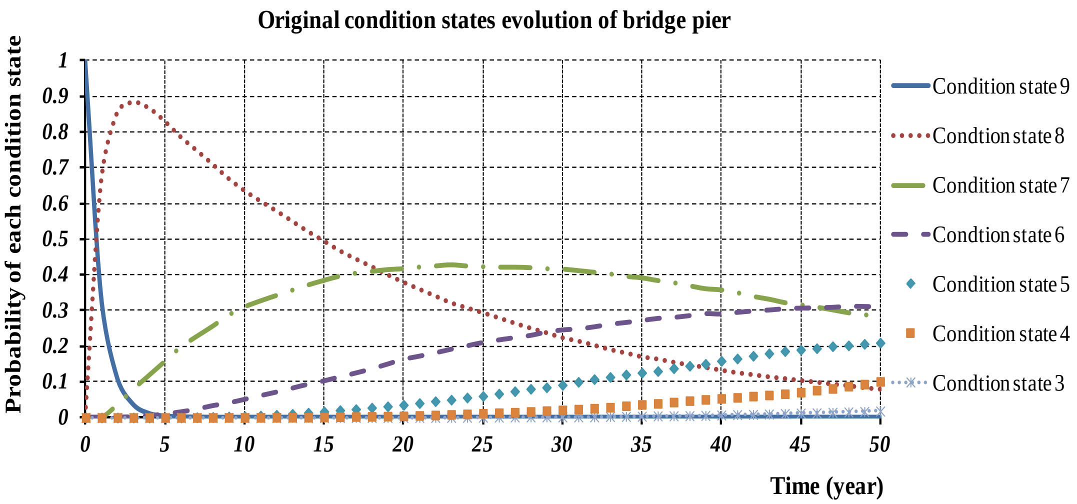

Figure 6-17. Condition states evolution of bridge pier based on model I

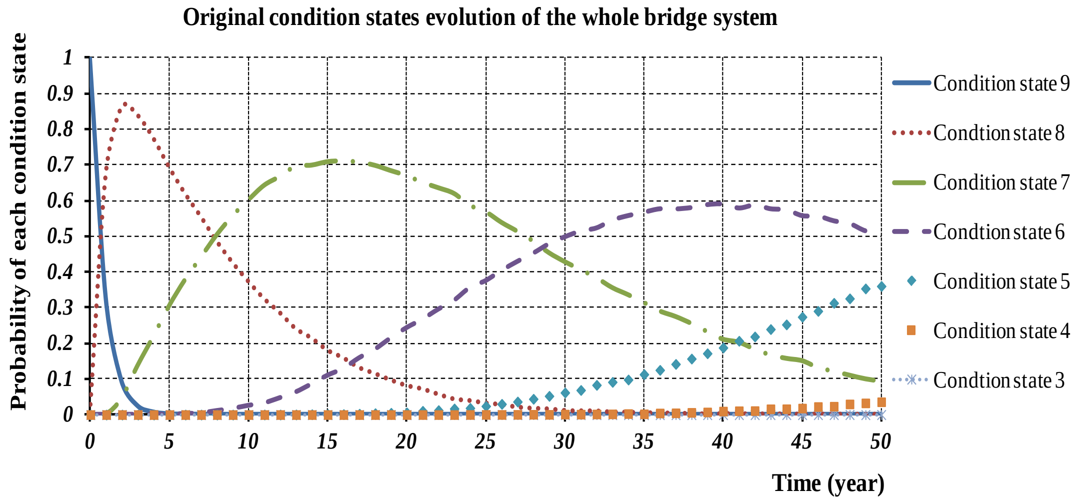

Figure 6-18. Condition states evolution of the whole system based on model I

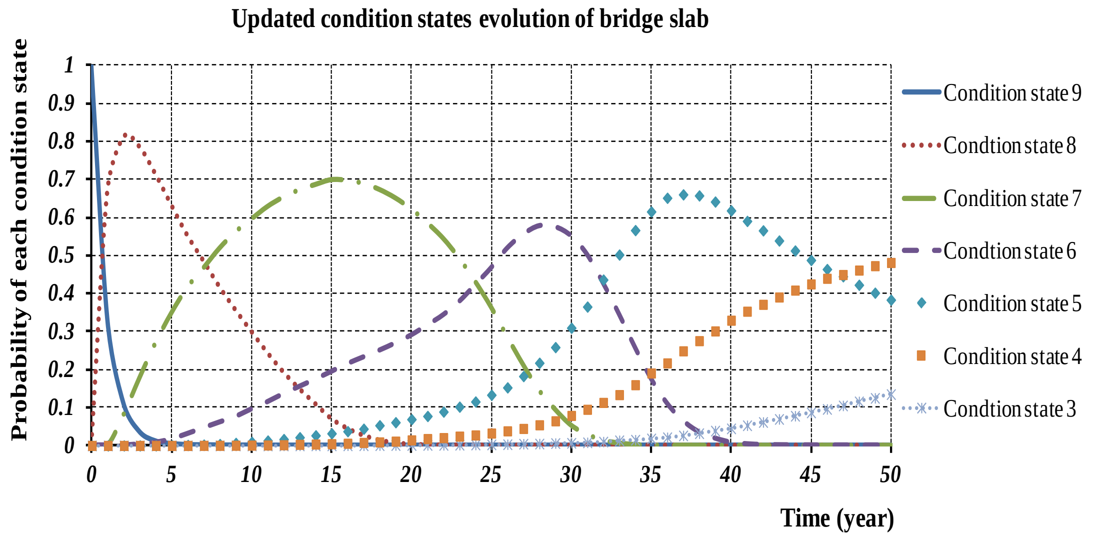

Figure 6-19. Updated condition states evolution of bridge slab based on model III

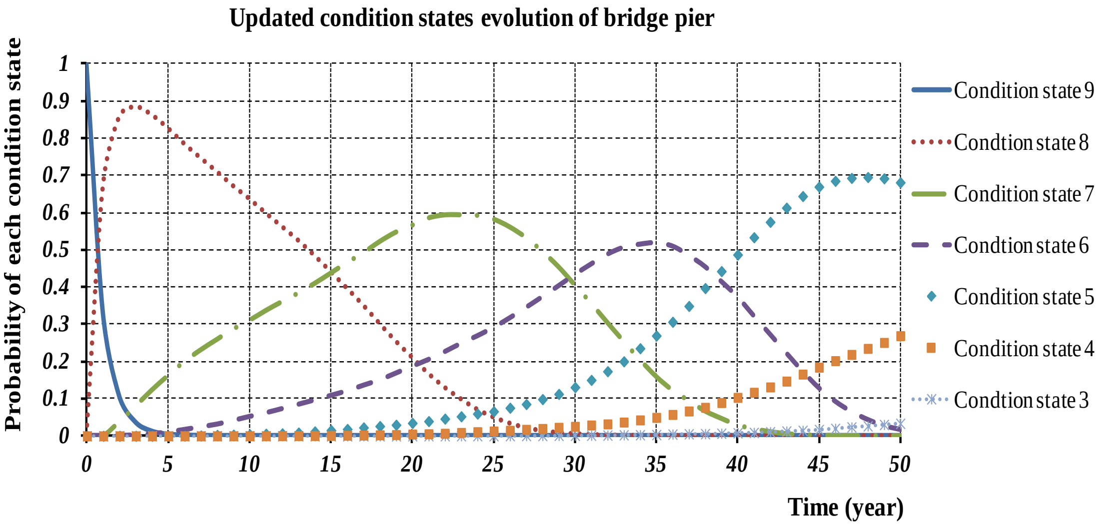

Figure 6-20. Updated condition states evolution of bridge pier based on model III

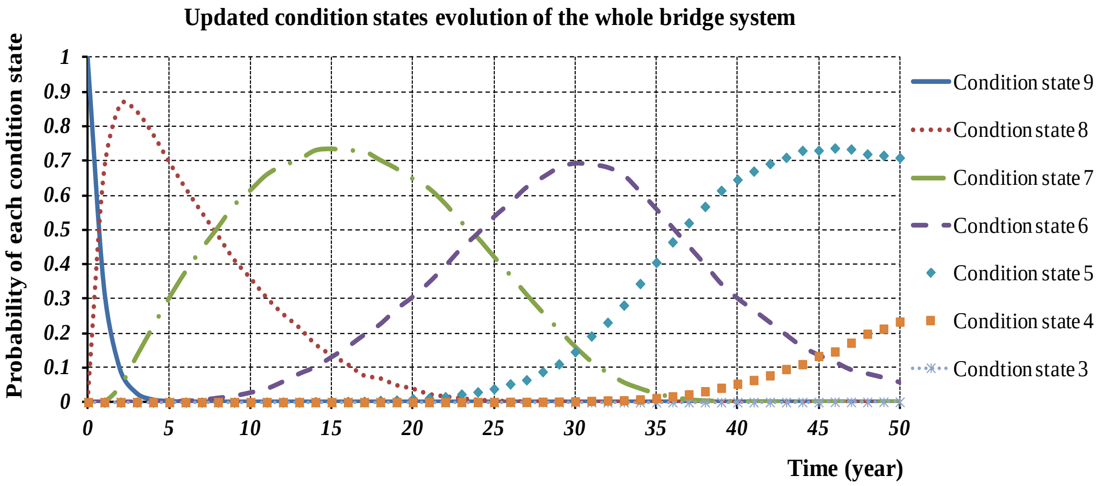

Figure 6-21. Updated condition states evolution of bridge pier based on model III

Based on the CPTs shown in Table 5-5, 5-6 and 5-7, the prediction results of structural reliability based on Models II and III are compared each other. The prediction results are expressed in the forms of reliability index. Figures 6-22, 6-23 and 6-24 display some of the comparison results. Except bridge slab and pier, reliability indexes of all the other bridge elements resulting from Model III are supposed to increase owing to the considered painting effects. However, the reliability index of the whole bridge system seems identical all the time. This is due to the failure probability of bridge superstructure is relatively small compared with those of bridge substructure and deck. As the reliability indexes of bridge deck and substructure remain the same, it is impossible that the reliability index of the whole bridge system augments dramatically.

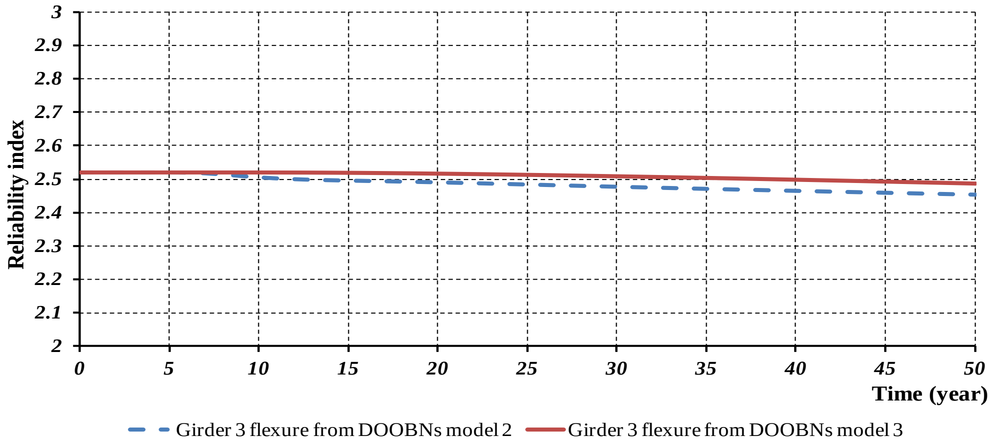

Figure 6-22. Comparison of reliability indexes of Girder 3 in flexure resulting from model II and model III

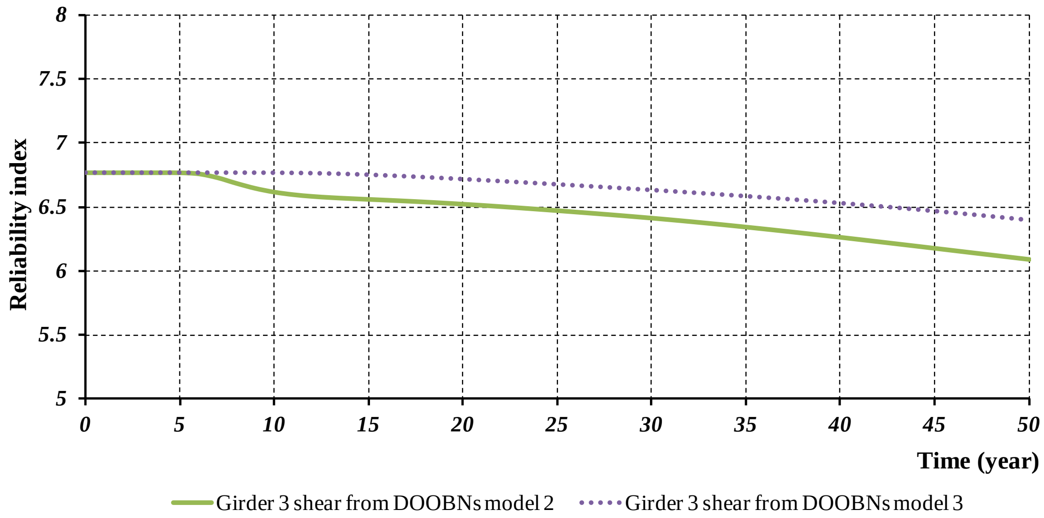

Figure 6-23. Comparison of reliability indexes of Girder 3 in shear resulting from model II and model III

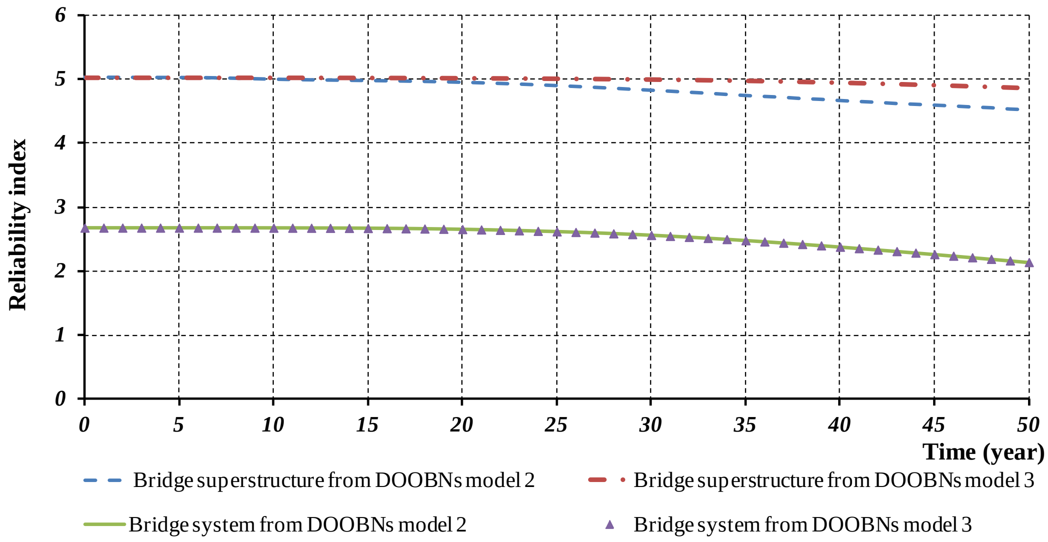

Figure 6-24. Comparisons of reliability indexes of bridge superstructure and the whole bridge system resulting from model II and model III

- Scenario Two

To ensure the correctness of prediction results, event information reflecting bridge real deterioration processes needs to be taken into account. The Model III possessing the Bayesian updating ability can effectively incorporate the event information for the latest prediction results. In this section, the Bayesian updating ability is verified through simulated maintenance activities upon bridge Girder 1. Table 6-3 shows the scheduled maintenance activities. Similar to the last chapter, there are also three kinds of maintenance actions: no maintenance, imperfect maintenance and perfect maintenance. We assume that the perfect maintenance compensates corrosion loss and renews Girder 1; imperfect maintenance has 50% chance to do so. Moreover, the DOOBNs model of Girder 1 is amended to include the maintenance actions as well (Figure 6-25).

Table 6-3. Maintenance activities for Girder 1 during 50 years

Time (year)	**20**	**45**
Maintenance actions	**Imperfect maintenance** renew exterior Girder 1 with 50% likelihood	**Perfect maintenance** renew exterior Girder 1 with 100% likelihood

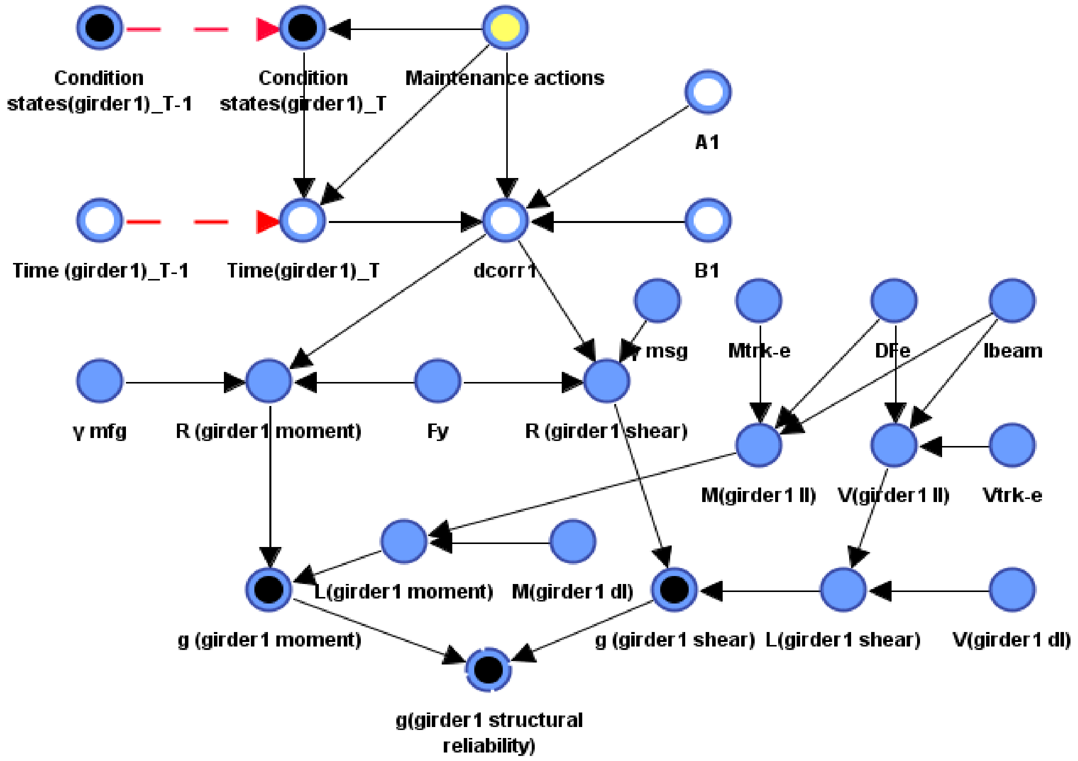

Figure 6-25. Modified DOOBNs model of Girder 1 for integrated health prediction including maintenance actions

Based on the simulated maintenance actives, the prediction results of Girder 1 are updated primarily. With these updated results, condition states and structural reliability of bridge superstructure and the whole bridge system are both recalculated. Figures 6-26, 6-27 and 6-28 display the updated evolution curves of bridge condition states. At 20th year and 45th year, the maintenance activities conducted has raised significant impacts on the condition states evolution of Girder 1. Because of the causal relationships, the maintenance activity has influence on the deterioration evolution of bridge superstructure and the whole bridge system. However, the influence becomes weaker when it comes to the whole bridge system owing to deterioration effects of other bridge elements.

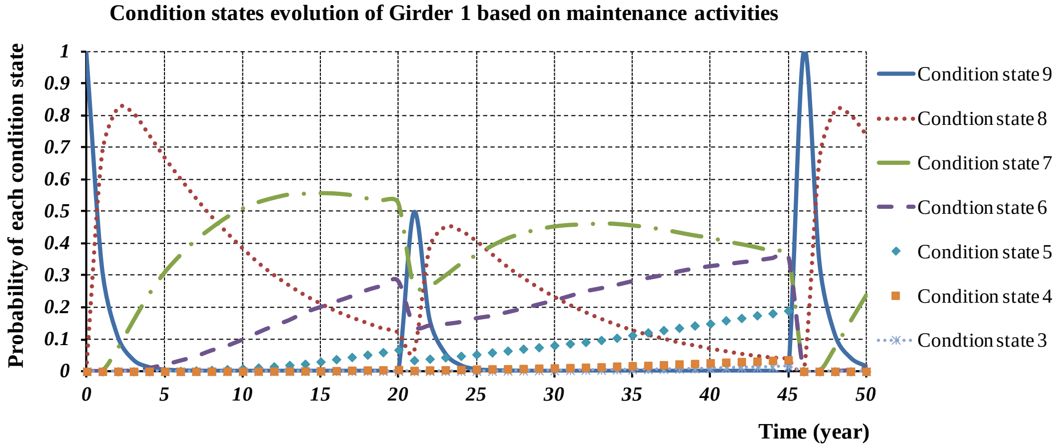

Figure 6-26. Updated condition states evolution of Girder 1 based on model III and maintenance activities

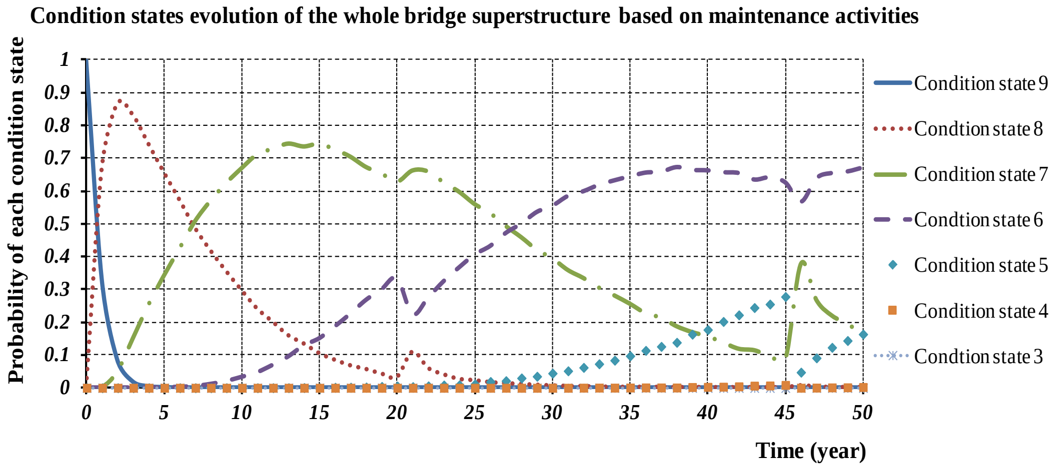

Figure 6-27. Updated condition states evolution of bridge superstructure based on model III and maintenance activities

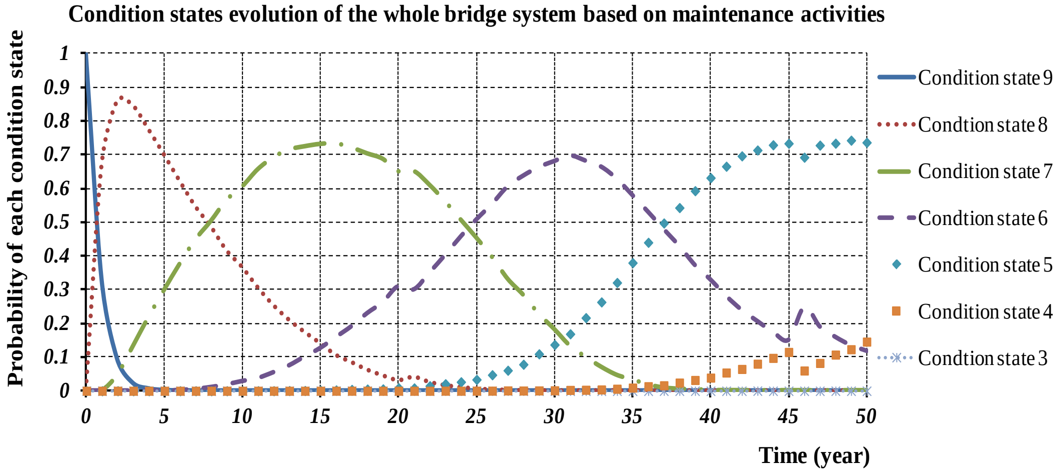

Figure 6-28. Updated condition states evolution of the whole bridge system based on model III and maintenance activities

Similarly, Figures 6-29, 6-30 and 6-31 present the updated reliability indexes of Girder 1 and bridge superstructure based on the simulated maintenance activities. Comparing Figures 6-29 and 6-30, we can observe that the shear of Girder 1 is more sensitive to the simulated information than the flexure of Girder 1. This is because that the corrosion deterioration only plays a minor role in the reduction of flexure structural reliability, and the failure probability in flexure is relatively large compared with shear. In addition, bridge superstructure only subjects to minor influences of maintenance activities. In this scenario, the whole bridge system is almost not influenced by the maintenance actions because the failure probability of bridge superstructure is even not enough to change the final result slightly. The Bayesian updating ability has been demonstrated based on the results above.

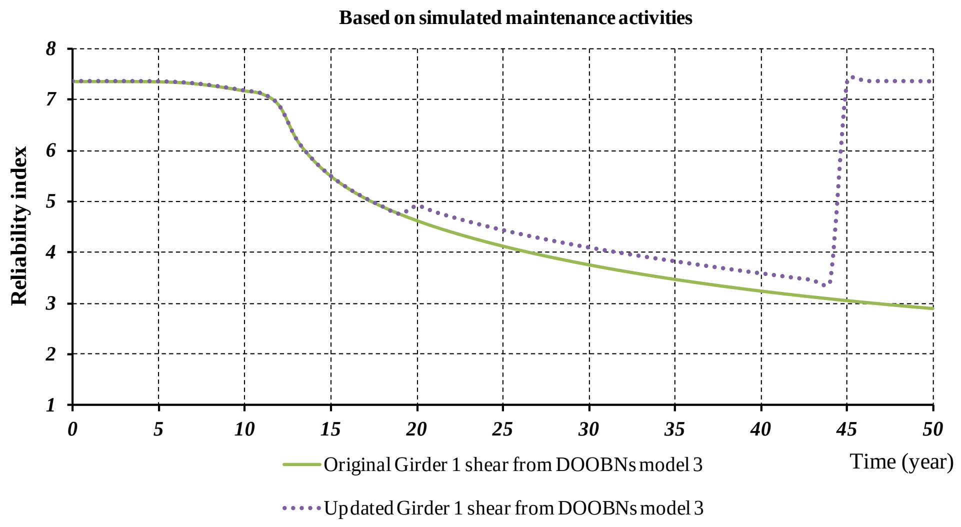

Figure 6-29. Updated reliability index of Girder 1 in shear based on model III and maintenance activities

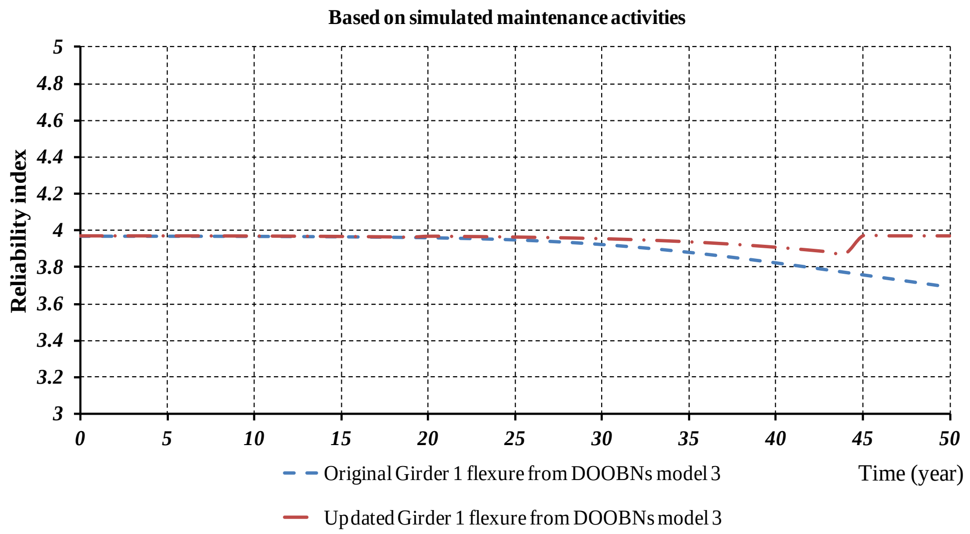

Figure 6-30. Updated reliability index of Girder 1 in shear based on model III and maintenance activities

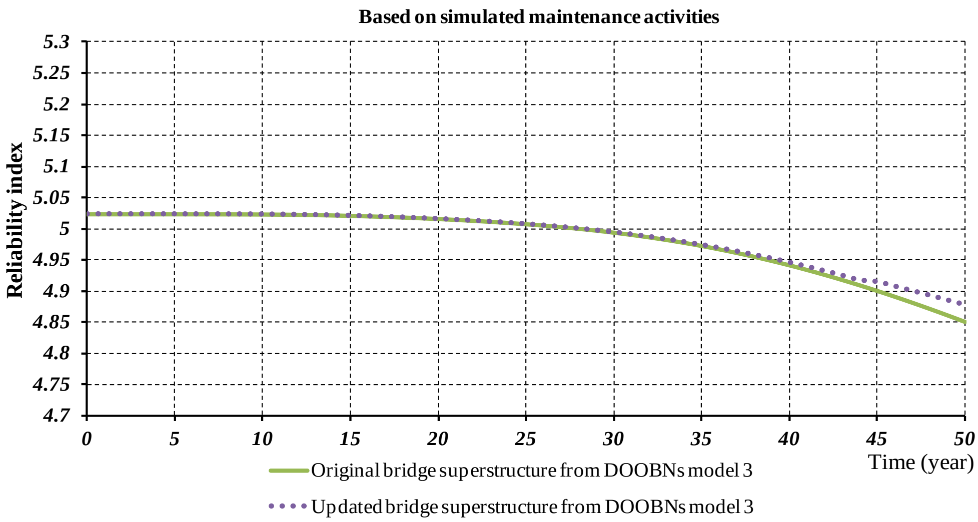

Figure 6-31. Updated reliability index of Girder 1 in shear based on model III and maintenance activities

Overall, the two conducted scenarios have proved the ability of Model III for integrated health prediction in both serviceability and safety aspects. By recognizing the latent correlation between condition ratings and structural reliability, the proposed model generates enhanced prediction results to the same deterioration processes so that the prediction in two performance criteria are calibrated and improved. By comparison with Models I and II, the updated long-term prediction outcomes are specifically demonstrated. Moreover, the proposed model is able to incorporate event information to incessantly improve prediction results. Although the proposed model is not particularly compared with other conventional methods due to the limited data, the Bayesian updating ability convinces that the proposed model will continue enhancing the accuracy of the prediction results with more available data.

6.4 Summary

To achieve integrated health prediction of bridge systems, a model III based on DOOBNs with the ability to address bridge deterioration in terms of both condition ratings and structural reliability is proposed in this chapter. Based on the DOOBNs model I and II, the proposed model inherits all their merits and is designed to be generally applicable for different types of structures. The proposed model is developed through two steps: DOOBN development and parameters estimation. In the first step, the modelling of bridge condition states is adapted to facilitate the modelling integration. According to the used materials, the integration of condition states and structural reliability is implemented through critical failure modes. Moreover, with available event information, the corresponding variables about observations, maintenance actions and environment can be also integrated to reduce the size of the whole network. In the second step, the new raised CPTs and priori probabilities are mainly estimated based on physical equations of deterioration processes and condition states definition. To demonstrate the feasibility of the proposed DOOBNs model III, we applied the proposed model to Bridge E-17-AH. Based on an open database "National Bridge Inventory" (NBI) and the previous study from the literature [47], both condition states and structural reliability of this bridge are predicted over 50 years. We implemented two scenarios to display the advantages. In the first scenario, the predicted results based on the DOOBNs model III are compared with the ones obtained from DOOBNs model I and II to demonstrate the integrated health prediction. By considering the correlations between condition states and structural reliability, more reasonable prediction results are expected. In the second scenario, based on the simulated maintenance actions, the automatic Bayesian updating ability of the DOOBNs model III is also illustrated in this application. The ability helps incorporate event information efficiently so that the prediction results reflecting bridge real deterioration processes can be delivered. Based on the two scenarios, the proposed DOOBNs model III is deemed more desirable.

In the future, more research work is demanded to apply the proposed DOOBNs model III to other bridge structures. More data are also required so that some unnecessary assumptions could be released. Instead of simulated event information, event information obtained in practice is expected for the model validation as well. The most importantly, more efforts should be devoted to the extension of the proposed model for bridge maintenance optimization. By adding utility nodes and decision nodes to the proposed model, influence diagrams (IDs) are obtained to function as a powerful maintenance decision tool [13].

Chapter 7: Conclusions and Future work

7.1 Summary of background and established models

To ensure the reliability and serviceability of a bridge, appropriate maintenance strategies need to be implemented. Recently, there is an increasing demand of reduction of maintenance cost without compromising the serviceability. To this end, identification of bridge deterioration models is crucial for health prediction and optimization of the maintenance strategy. In this work, comprehensive literature review indicated several research problems that have not been sufficiently investigated, such as

- Existing approaches are not capable of modelling bridge deterioration performance in both serviceability and safety aspects in an integrated manner so that both performance criteria can be evaluated coherently.
- Although it is a well accepted that a bridge is a complex system composed of many inter-related bridge elements, system approaches have not been successfully developed for bridge deterioration modelling.
- The existing models are not able to deal with multiple bridge deterioration factors concurrently, such as deterioration dependencies between different bridge elements, different inspection and maintenance methods and environmental effects. Consequently, accurate and robust prediction models are still lacking.
- Existing models are deficient in updating the prediction methodology. Bayesian method has been proved to be an effective tool for this purpose. However, its applicability for bridge health prediction needs to be investigated.
- An effective platform is required so that a variety of information, such as monitoring data, expert knowledge and physical laws can be integrated for uncertainties reduction.
- The assumption of series and/or parallel system relationship for bridge level reliability is always held in all structural reliability estimation of bridge systems

To adopt a complex system modelling approach to deal with the above deficiencies, three novel models based on DOOBNs have been proposed. The Model I focuses on bridge deterioration in serviceability using condition ratings as the health index. The probabilistic deterioration is represented in a hierarchical way so that the contribution of each bridge element to bridge deterioration can be included. Deterioration of bridge elements over time is modelled based on a discrete-time Markov process. The Model II concentrates on bridge deterioration in safety. The structural reliability of bridge systems is estimated from structural elements to the entire bridge. With CPTs, both series-parallel relationship and complex probabilistic relationship in bridge systems can be effectively modelled. The structural reliability of each bridge element is based on its limit state functions, considering the probability distributions of resistance and applied load. Both Models I and II are established in three steps: modelling consideration, DOOBN development and parameter estimation. Model III integrates Models I and II to address bridge health performance in both serviceability and safety. The modelling of bridge condition ratings is modified so that every basic modelling unit represents one physical bridge element. According to the specific materials used, the integration of condition ratings and structural reliability is implemented through essential failure modes. Overall, this work developed three novel DOOBNs based bridge deterioration models with the following features:

- Recognition of implicit correlation between condition ratings and structural reliability. Although condition ratings and structural reliability are two different performance indicators for bridge health they both reflect fundamental bridge deterioration processes. By combining these two parameters, bridge deterioration in serviceability and safety can be addressed in an integrated way.
- The object oriented representation of bridge dynamic deterioration behaviours from bridge elements to the entire bridge. This representation eases integrated bridge health management for the purpose of maintenance optimization.
- Adaptive representation of bridge systems for structural reliability estimation. Limit state functions regarding bridge elements are modelled as the basis of bridge systems estimation. Without relying on deterministic series and/or parallel relationship among bridge elements, inappropriate assumptions can be realised so that potential errors about bridge system estimation can be minimised. With this adaptive ability, structural reliability of bridge systems under different circumstances can be estimated more practically and accurately.

- Concurrent consideration of bridge deterioration factors, such as, deterioration dependency, observation and environmental conditions as well as maintenance intervene. This method provides more accurate health prediction.
- Multiple data sources for parameters estimation. Considering different data availabilities, detailed specifications for CPTs and priori probabilities estimation based on bridge inspection data, expert knowledge, theoretical deterioration equations and limit state functions as well as miscellaneous knowledge are all formulated. The specifications guarantee the proposed prediction model is ready to use. By including various types of data, prediction accuracies can be enhanced and data scarcity problems of current research can be mitigated.
- Automatic Bayesian ability for better updating efficiency and more accurate prediction results

To validate these models, three case studies have been conducted. Carefully selected data and knowledge from bridge experts, the National Bridge Inventory (NBI) and the existing literature [47] have been utilised for model validation. In addition, event information has been generated using simulation to demonstrate the automatic Bayesian updating ability of these models. The prediction results of condition ratings and structural reliability have been presented and interpreted for the basic bridge elements and the whole bridge system. The results obtained from Model II were compared with the ones obtained from traditional structural reliability methods. Overall, the results have confirmed the feasibility of these models for bridge health prediction. Note that three models can be used separately or jointly. The implementation of the three new models is expected to be more effective and vigorous than the existing modelling approaches.

7.2 Additional functionality of the model

Some other potential functionalities of the established models, which are not demonstrated in this thesis, include the following:

- The proposed models are capable of handling both time-variant and time-invariant live loading effects. Although a time-invariant distribution of live load is assumed in this research, the live load is generally expected to be dynamic and follow a time-variant distribution. This can be achieved by incorporating different live load models. For instance, Novak's live load [118] can be integrated for dynamic loading effects. Compared with static live loading effects, more computational time is expected.

- The proposed models are capable of handling the change of environment temperature. This is important for bridge health prediction as the climate change is expected to affect bridge health dramatically. For condition ratings prediction, deterioration rates of each bridge element under different temperature conditions can be estimated separately with actual condition data. For structural reliability prediction, temperature will influence the deterioration processes, such as corrosion, crack and spalling. Parameter values of physical deterioration equations are assigned conditional on their temperature conditions.
- The proposed models are capable of considering different material and load conditions. For bridge elements made of different materials, the corresponding prior probabilities related to material yield strength/stress are assigned with different distributions. For different load types, such as axial, bending and shear, pertinent limit state functions can be developed for each element. Its structural reliability will be estimated based on the formulated multiple limit state functions.
- As a causal modelling approach, the proposed models are capable of capturing the dependencies among the whole bridge system, and doing “what-if ” analysis. This analysis is a common characteristic of bridge health management and will help to identify significant bridge structural elements among a complex bridge system so that maintenance actions can be implemented effectively.

7.3 Future work

To make the proposed models more applicable in practice, several new research challenges have been identified as follows:

- Further modifications and validations of the proposed models

  a) Overall, Model I is not restricted to any special type of bridge materials. However, Models II and III are constrained to bridges made of reinforced concrete and carbon steel only, since these two types of materials are the most commonly used for bridges. However, there are bridges made of other material, for instance, timber, stone masonry and other composite materials.

In order to cater to the entire bridge population, the proposed models have to be extended for other materials.

b) For structural reliability prediction, limit state functions for each bridge element are established. In this research, only ultimate limit state functions are considered. However, other types of limit state functions, such as, serviceability limit state functions and fatigue limit state functions, need to be considered as well. By adapting the relevant variables, Model II is capable of modelling other types in a heuristic way. In the future research, all types of limit state functions should be taken into account concurrently.

c) Modelling of critical failure modes for bridge deterioration is based on a number of chosen physical deterioration equations for corrosion, crack and spalling. A better understanding of the bridge deterioration process will help improve and broaden the applicability of the proposed models.

d) With the development of inference algorithms, the errors caused by discretization can be eliminated. One alternative way is to choose hybrid BNs, which allow both discrete and continuous variables within one network. Langseth et al [88] summarised and discussed all the inference algorithms in hybrid BNs.

e) More bridge data are required for complete model validation. Real event information can be utilised to verify the Bayesian updating ability. Prediction results based on different data sources can be verified by comparing with each other.

f) To facilitate the application, proper software tools need to be developed. This will certainly help the bridge practitioners to practice the proposed models.

- Influence Diagrams (IDs) for bridge maintenance optimization

Based on health prediction results, further study should be dedicated to the extension of the proposed models. By adding decision nodes and utility nodes to DOOBNs, influence diagrams (IDs) based on the proposed integrated health prediction model can be constructed as an effective decision tool for bridge maintenance optimization. The decision nodes define the maintenance actions concerned by the user. The utility nodes conditional on probabilistic and/or decision nodes are the measures of decision nodes. The developed

IDs can compute the expected utility (EU) of each maintenance action. Based on the maximum expected utility (MEU) principle, the alternative with the highest EU is chosen. With the consideration of the impacts of each maintenance action on bridge deterioration in both serviceability and safety aspects, IDs provide the best maintenance decisions for decision makers in a cost-effective and a sustainable manner. IDs have been appropriately utilised as a decision tool for a number of applications, such as marine and offshore application, industrial process control, steam turbine maintenance decisions and pavement management decisions [11, 12, 43, 55, 77, 169]. Both inspection planning and maintenance actions can be optimised based on IDs. Attoh-Okine and Chajes [13] discussed advantages and disadvantages of IDs in bridge health management. IDs are more effective than decision trees, especially in compactness and information flow. Further investigation is needed to apply IDs to bridge maintenance optimization.

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Appendices

Appendix A: Limit state functions

Pier shear [47]

$$g_{pier\ shear} = 8.28\lambda_{deff} A_{v\ pier} f_y \gamma_{msc} + 2.682\sqrt{f_c'}\gamma_{msc} - 15.78\lambda_{asph} - 68.04\lambda_{conc} - 10.02\lambda_{steel} - 42.50 V_{trk-i} DF_i I_{beam} \tag{A-1}$$

where λ_{deff} is effective depth of reinforcing steel; $A_{v\ pier}$ is the area of shear steel in pier; f_y is yield stress of reinforcing steel; γ_{msc} is uncertainty factor for concrete shear; f_c' is 28 day yield strength of concrete; λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; V_{trk-i} uncertainty factor for live load shear in interior girder; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{pier\ shear} = R_{pier\ shear} - L_{pier\ shear} \tag{A-2}$$

$$R_{pier\ shear} = \gamma_{msc}(R_{pier\ shear\ 1} + R_{pier\ shear\ 2}) \tag{A-3}$$

$$R_{pier\ shear\ 1} = 8.28\lambda_{deff} f_y \left(\frac{4\pi D_{pier}^2}{4}\right) \tag{A-4}$$

$$R_{pier\ shear\ 2} = 2.682\sqrt{f_c'} \tag{A-5}$$

$$D_{pier}(t) = D_{pier}(0) - R_{corr\ pier}(t - T_{corr\ pier}) \tag{A-6}$$

$$L_{pier\ shear} = V_{pier\ dl} + V_{pier\ ll} \tag{A-7}$$

$$V_{pier\ dl} = 15.78\lambda_{asph} + 68.04\lambda_{conc} + 10.02\lambda_{steel} \tag{A-8}$$

$$V_{pier\ ll} = 42.50 V_{trk-i} DF_i I_{beam} \tag{A-9}$$

where $R_{pier\ shear}$ is the shear capacity of the pier; $L_{pier\ shear}$ is the shear demand for the pier; $V_{pier\ dl}$ is the dead load demand in shear for the pier; $V_{pier\ ll}$ is the live load demand in shear for the pier; $R_{pier\ shear\ 1}$ and $R_{pier\ shear\ 2}$ are two parts of total shear capacity of the pier; $A_{t\ slab}$ is the temporal changed cross section area of reinforced steel in slab;

$D_{pier}(t)$ is the temporal changed diameter of single reinforced steel bar in pier; $R_{corr\,pier}$ is the corrosion rate in the pier; $T_{corr\,pier}$ is the corrosion initiation time in the pier.

Exterior girder 1 flexure [47]

$$g_{girder1\,moment} = 36.54F_y\gamma_{mfg} - (145.32\lambda_{conc} + 37.3\lambda_{steel} + M_{trk-e}DF_eI_{beam}) \tag{A-10}$$

where F_y is yield strength of steel in girders; γ_{mfg} is model uncertainty factor regarding to flexure in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{steel} is uncertainty factor for weight of steel girders; V_{trk-e} uncertainty factor for live load shear in exterior girder; DF_e is uncertainty for live load distribution of exterior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder1\,moment} = R_{girder1\,moment} - L_{girder1\,moment} \tag{A-11}$$

$$R_{girder1\,moment} = \frac{F_yZ\gamma_{mfg}}{12} = \frac{F_y\gamma_{mfg}\left(439.6 - \frac{407.78d_{corr1}}{25400} - 341.64\left(\frac{d_{corr1}}{25400}\right)^2\right)}{12} \tag{A-12}$$

$$d_{corr1} = A_1t^{B_1} \tag{A-13}$$

$$L_{girder1\,moment} = M_{girder1\,dl} + M_{girder1\,ll} \tag{A-14}$$

$$M_{girder1\,dl} = 145.32\lambda_{conc} + 37.3\lambda_{steel} \tag{A-15}$$

$$M_{girder1\,ll} = M_{trk-e}DF_eI_{beam} \tag{A-16}$$

where$R_{girder1\,moment}$ is the flexure capacity of exterior girder 1; $L_{girder1\,moment}$ is the flexure demand for exterior girder 1; d_{corr1} is the corrosion loss of exterior girder 1 at the considered time; A_1 and B_1 are the corrosion loss after one year and a regression coefficient numerically; $M_{girder1\,dl}$ and $M_{girder1\,dl}$ are dead load and live load demand in flexure for girder 1.

Interior-exterior girder 2 shear [47]

$$g_{girder2\,shear} = 10.55F_y\gamma_{msg} - (22.29\lambda_{conc} + 2.63\lambda_{asph} + 2.89\lambda_{steel} + 28.33V_{trk-i}DF_{i-e}I_{beam}) \tag{A-17}$$

where F_y is yield strength of steel in girders; γ_{msg} is model uncertainty factor regarding to shear in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{steel} is uncertainty factor for weight of steel girders; $V_{trk\text{-}i}$ uncertainty factor for live load shear in interior girder; $DF_{i\text{-}e}$ is uncertainty for live load distribution of interior-exterior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder2\ shear} = R_{girder2\,shear} - L_{girder2\ shear} \tag{A-18}$$

$$R_{girder2\ shear} = 0.58F_y\gamma_{msg}d_w t_w = 18.183F_y\gamma_{msg}\left(0.58 - \frac{d_{corr2}}{12700}\right) \tag{A-19}$$

$$L_{girder2\ shear} = V_{girder2\,dl} + V_{girder2\ ll} \tag{A-20}$$

$$d_{corr2} = A_2 t^{B_2} \tag{A-21}$$

$$V_{girder2\,dl} = 22.29\lambda_{conc} + 2.63\lambda_{asph} + 2.89\lambda_{steel} \tag{A-22}$$

$$V_{girder2\,ll} = 28.33V_{trk-i}DF_{i-e}I_{beam} \tag{A-23}$$

where $R_{girder2\,shear}$ is the shear capacity of interior-exterior girder 2; $L_{girder2\ shear}$ is the shear demand for interior-exterior girder 2; d_{corr2} is the corrosion loss of interior-exterior girder 2 at the considered time; A_2 and B_2 are the corrosion loss after one year and a regression coefficient numerically; $V_{girder2\,dl}$ and $V_{girder2\ ll}$ are dead load and live load demand in shear for girder 2.

Interior-exterior girder 2 flexure [47]

$$g_{girder2\,moment} = 39.8F_y\gamma_{mfg} - (244.08\lambda_{conc} + 28.8\lambda_{asph} + 31.7\lambda_{steel}$$

$$+M_{trk-i}DF_{i-e}I_{beam}) \tag{A-24}$$

where F_y is yield strength of steel in girders; γ_{mfg} is model uncertainty factor regarding to flexure in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; M_{trk-i} uncertainty factor for live load flexure in interior girder; $DF_{i\text{-}e}$ is uncertainty for live load distribution of interior-exterior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder2\,moment} = R_{girder2\,moment} - L_{girder2\,moment} \tag{A-25}$$

$$R_{girder2\ moment} = \frac{F_y Z \gamma_{mfg}}{12} = \frac{F_y \gamma_{mfg}(477.79 - \frac{407.78 d_{corr2}}{25400} - 341.64(\frac{d_{corr2}}{25400})^2)}{12} \quad \text{(A-26)}$$

$$d_{corr2} = A_2 t^{B_2} \quad \text{(A-27)}$$

$$L_{girder2\ moment} = M_{girder2\ dl} + M_{girder2\ ll} \quad \text{(A-28)}$$

$$M_{girder2\ dl} = 244.08\lambda_{conc} + 28.8\lambda_{asph} + 31.7\lambda_{steel} \quad \text{(A-29)}$$

$$M_{girder2\ ll} = M_{trk-i} DF_{i-e} I_{beam} \quad \text{(A-30)}$$

where $R_{girder2\ moment}$ is the flexure capacity of interior girder 2; $L_{girder2\ moment}$ is the flexure demand for interior girder 2; d_{corr2} is the corrosion loss of interior girder 2 at the considered time; A_2 and B_2 are the corrosion loss after one year and a regression coefficient numerically; $M_{girder2\ dl}$ and $M_{girder2\ ll}$ are dead load and live load demand in flexure for girder 2.

Interior girder 3 shear [47]

$$g_{girder3\ shear} = 10.55 F_y \gamma_{msg} - (18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} + 28.33 V_{trk-i} DF_i I_{beam}) \quad \text{(A-31)}$$

where F_y is yield strength of steel in girders; γ_{msg} is model uncertainty factor regarding to shear in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; V_{trk-i} uncertainty factor for live load shear in interior girder; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder3\ shear} = R_{girder3\ shear} - L_{girder3\ shear} \quad \text{(A-32)}$$

$$R_{girder3\ shear} = 0.58 F_y d_w t_w = 18.183 F_y \left(0.58 - \frac{d_{corr3}}{12700}\right) \quad \text{(A-33)}$$

$$L_{girder3\ shear} = V_{girder3\ dl} + V_{girder3\ ll} \quad \text{(A-34)}$$

$$d_{corr3} = A_3 t^{B_3} \quad \text{(A-35)}$$

$$V_{girder3\ dl} = 18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} \quad \text{(A-36)}$$

$$V_{girder3\ ll} = 28.33 V_{trk-i} DF_i I_{beam} \tag{A-37}$$

where $R_{girder3\ shear}$ is the shear capacity of interior girder 3; $L_{girder3\ shear}$ is the shear demand for interior girder 3; d_{corr3} is the corrosion loss of interior girder 3 at the considered time; A_3 and B_3 are the corrosion loss after one year and a regression coefficient numerically; $V_{girder3\ dl}$ and $V_{girder3\ ll}$ are dead load and live load demand in shear for girder 3.

Interior girder 3 flexure [47]

$$g_{girder3\ moment} = 39.8 F_y \gamma_{mfg} - (197.65\lambda_{conc} + 57.64\lambda_{asph} + 31.7\lambda_{steel} + M_{trk-i} DF_i I_{beam}) \tag{A-38}$$

where F_y is yield strength of steel in girders; γ_{mfg} is model uncertainty factor regarding to flexure in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; M_{trk-i} uncertainty factor for live load flexure on interior girders; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder3\ moment} = R_{girder3\ moment} - L_{girder3\ moment} \tag{A-39}$$

$$R_{girder3\ moment} = \frac{F_y Z \gamma_{mfg}}{12} = \frac{F_y \gamma_{mfg}\left(477.79 - \frac{407.78 d_{corr\ 3}}{25400} - 341.64\left(\frac{d_{corr\ 3}}{25400}\right)^2\right)}{12} \tag{A-40}$$

$$d_{corr\ 3} = A_3 t^{B_3} \tag{A-41}$$

$$L_{girder3\ moment} = M_{girder3\ dl} + M_{girder3\ ll} \tag{A-42}$$

$$M_{girder3\ dl} = 197.65\lambda_{conc} + 57.64\lambda_{asph} + 31.7\lambda_{steel} \tag{A-43}$$

$$M_{girder3\ ll} = M_{trk-i} DF_i I_{beam} \tag{A-44}$$

where $R_{girder3\ moment}$ is the flexure capacity of interior girder 3; $L_{girder3\ moment}$ is the flexure demand for interior girder 3; d_{corr3} is the corrosion loss of interior girder 3 at the considered time; A_3 and B_3 are the corrosion loss after one year and a regression coefficient numerically; $M_{girder3\ dl}$and $M_{girder3\ ll}$ are dead load and live load demand in flexure for girder 3.

Interior girder 4 shear [47]

$$g_{girder4\ shear} = 10.55F_y\gamma_{msg} - (18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} + 28.33V_{trk-i}DF_iI_{beam})$$

(A-45)

where F_y is yield strength of steel in girders; γ_{msg} is model uncertainty factor regarding to shear in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; $V_{trk\text{-}i}$ uncertainty factor for live load shear in interior girder; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder4\ shear} = R_{girder4\ shear} - L_{girder4\ shear} \quad \text{(A-46)}$$

$$R_{\ girder4\ shear} = 0.58F_yd_wt_w = 18.183F_y\left(0.58 - \frac{d_{corr4}}{12700}\right) \quad \text{(A-47)}$$

$$L_{girder4\ shear} = V_{girder4\ dl} + V_{girder4\ ll} \quad \text{(A-48)}$$

$$d_{corr4} = A_4t^{B_4} \quad \text{(A-49)}$$

$$V_{girder4\ dl} = 18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} \quad \text{(A-50)}$$

$$V_{girder4\ ll} = 28.33V_{trk-i}DF_iI_{beam} \quad \text{(A-51)}$$

where $R_{girder4\ shear}$ is the shear capacity of interior girder 4; $L_{girder4\ shear}$ is the shear demand for interior girder 4; d_{corr4} is the corrosion loss of interior girder 4 at the considered time; A_4 and B_4 are the corrosion loss after one year and a regression coefficient numerically; $V_{girder4\ dl}$ and $V_{girder4\ ll}$ are dead load and live load demand in shear for girder 4.

Interior girder 4 flexure [47]

$$g_{girder4\ moment} = 39.8F_y\gamma_{mfg} - (197.65\lambda_{conc} + 57.64\lambda_{asph} + 31.7\lambda_{steel} + M_{trk-i}DF_iI_{beam})$$

(A-52)

where F_y is yield strength of steel in girders; γ_{mfg} is model uncertainty factor regarding to flexure in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; M_{trk-i} uncertainty factor for live load flexure on interior girders; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder4\ moment} = R_{girder4\ moment} - L_{girder4\ moment} \tag{A-53}$$

$$R_{girder4\ moment} = \frac{F_y Z \gamma_{mfg}}{12} = \frac{F_y \gamma_{mfg}(477.79 - \frac{407.78 d_{corr4}}{25400} - 341.64(\frac{d_{corr4}}{25400})^2)}{12} \tag{A-54}$$

$$d_{corr4} = A_4 t^{B_4} \tag{A-55}$$

$$L_{girder4\ moment} = M_{girder4\ dl} + M_{girder4\ ll} \tag{A-56}$$

$$M_{girder4\ dl} = 197.65\lambda_{conc} + 57.64\lambda_{asph} + 31.7\lambda_{steel} \tag{A-57}$$

$$M_{girder4\ ll} = M_{trk-i} DF_i I_{beam} \tag{A-58}$$

where $R_{girder4\ moment}$ is the flexure capacity of interior girder 4; $L_{girder4\ moment}$ is the flexure demand for interior girder 4; d_{corr4} is the corrosion loss of interior girder 4 at the considered time; A_4 and B_4 are the corrosion loss after one year and a regression coefficient numerically; $M_{girder4\ dl}$and $M_{girder4\ ll}$ are dead load and live load demand in flexure for girder 4.

Interior girder 5 shear [47]

$$\begin{aligned} g_{girder5\ shear} = 10.55 F_y \gamma_{msg} - (18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} \\ + 28.33 V_{trk-i} DF_i I_{beam}) \end{aligned} \tag{A-59}$$

where F_y is yield strength of steel in girders; γ_{msg} is model uncertainty factor regarding to shear in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; V_{trk-i} uncertainty factor for live load shear in interior girder; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder5\ shear} = R_{girder5\ shear} - L_{girder5\ shear} \tag{A-60}$$

$$R_{girder5\ shear} = 0.58 F_y d_w t_w = 18.183 F_y \left(0.58 - \frac{d_{corr5}}{12700}\right) \tag{A-61}$$

$$L_{girder5\ shear} = V_{girder5\ dl} + V_{girder5\ ll} \tag{A-62}$$

$$d_{corr5} = A_5 t^{B_5} \tag{A-63}$$

$$V_{girder5\ dl} = 18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} \tag{A-64}$$

$$V_{girder5\ ll} = 28.33 V_{trk-i} DF_i I_{beam} \tag{A-65}$$

where $R_{girder5\ shear}$ is the shear capacity of interior girder 5; $L_{girder5\ shear}$ is the shear demand for interior girder 5; d_{corr5} is the corrosion loss of interior girder 5 at the considered time; A_5 and B_5 are the corrosion loss after one year and a regression coefficient numerically; $V_{girder5\ dl}$ and $V_{girder5\ ll}$ are dead load and live load demand in shear for girder 5.

Interior girder 5 flexure [47]

$$g_{girder5\ moment} = 39.8 F_y \gamma_{mfg} - (197.65\lambda_{conc} + 57.64\lambda_{asph} + 31.7\lambda_{steel} + M_{trk-i} DF_i I_{beam}) \tag{A-66}$$

where F_y is yield strength of steel in girders; γ_{mfg} is model uncertainty factor regarding to flexure in girders. λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; M_{trk-i} uncertainty factor for live load flexure on interior girders; DF_i is uncertainty for live load distribution of interior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{girder5\ moment} = R_{girder5\ moment} - L_{girder5\ moment} \tag{A-67}$$

$$R_{girder5\ moment} = \frac{F_y Z \gamma_{mfg}}{12} = \frac{F_y \gamma_{mfg}(477.79 - \frac{407.78 d_{corr5}}{25400} - 341.64(\frac{d_{corr5}}{25400})^2)}{12} \tag{A-68}$$

$$d_{corr5} = A_5 t^{B_5} \tag{A-69}$$

$$L_{girder5\ moment} = M_{girder5\ dl} + M_{girder5\ ll} \tag{A-70}$$

$$M_{girder5\ dl} = 197.65\lambda_{conc} + 57.64\lambda_{asph} + 31.7\lambda_{steel} \tag{A-71}$$

$$M_{girder5\ ll} = M_{trk-i} DF_i I_{beam} \tag{A-72}$$

where $R_{girder5\ moment}$ is the flexure capacity of interior girder 5; $L_{girder5\ moment}$ is the flexure demand for interior girder 5; d_{corr5} is the corrosion loss of interior girder 5 at the considered time; A_5 and B_5 are the corrosion loss after one year and a regression coefficient numerically; $M_{girder5\ dl}$and $M_{girder5\ ll}$ are dead load and live load demand in flexure for girder 5.

Column footing flexure [47]

$$g_{column\ footing\ moment} = 7.75\lambda_{rebar}f_y\lambda_{deff}\gamma_{mfc} - 0.132\frac{\lambda_{rebar}^2 f_y^2\gamma_{mfc}}{f_c'} - 10.65\lambda_{asph} - 64.44\lambda_{conc} - 6.93\lambda_{steel} - 27V_{trk-i}DF_iI_{beam} - 3.4V_{trk-i}DF_{i-e}I_{beam} \quad \text{(A-73)}$$

Where λ_{rebar} is uncertainty factor of reinforcing steel are; f_y is yield stress of reinforcing steel; λ_{deff} is effective depth of reinforcing steel; γ_{mfc} is model uncertainty factor regarding to concrete flexure; f_c' is 28 day yield strength of concrete; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{steel} is uncertainty factor for weight of steel girders; V_{trk-i} uncertainty factor for live load shear on interior girder; DF_i is uncertainty for live load distribution of interior girders; DF_{i-e} is uncertainty for live load distribution of interior-exterior girders; I_{beam} uncertainty factor for impact on girders. The new equations for time-variant structural reliability are given by

$$g_{column\ footing\ moment} = R_{column\ footing\ moment} - L_{column\ footing\ moment} \quad \text{(A-74)}$$

$$R_{column\ footing\ moment} = \gamma_{mfc}(R_{column\ footing\ moment\ 1} - R_{column\ footing\ moment\ 2}) \quad \text{(A-75)}$$

$$R_{column\ footing\ moment\ 1} = 7.75\lambda_{rebar}f_y\lambda_{deff} \quad \text{(A-76)}$$

$$R_{column\ footing\ moment\ 2} = 0.132\frac{\lambda_{rebar}^2 f_y^2}{f_c'} \quad \text{(A-77)}$$

$$L_{column\ footing\ moment} = M_{column\ footing\ dl} + M_{column\ footing\ ll} \quad \text{(A-78)}$$

$$M_{column\ footing\ dl} = 10.65\lambda_{asph} + 64.44\lambda_{conc} + 6.93\lambda_{steel} \quad \text{(A-79)}$$

$$M_{column\ footing\ ll} = M_{column\ footing\ ll\ 1} + M_{column\ footing\ ll\ 2} \quad \text{(A-80)}$$

$$M_{column\ footing\ ll\ 1} = 27V_{trk-i}DF_iI_{beam} \quad \text{(A-81)}$$

$$M_{column\ footing\ ll\ 2} = 3.4 V_{trk-i} DF_{i-e} I_{beam} \quad \text{(A-82)}$$

where $R_{column\ footing\ moment}$ is the flexure capacity of column footing; $L_{column\ footing\ moment}$ is the flexure demand for column footing; $R_{column\ footing\ moment\ 1}$ and $R_{column\ footing\ moment\ 2}$ are two parts of total flexure capacity of column footing; $M_{column\ footing\ dl}$ and $M_{column\ footing\ ll}$ are dead load and live load demand in flexure for column footing, respectively; $M_{column\ footing\ ll\ 1}$ and $M_{column\ footing\ ll\ 2}$ are two parts of total live load demand in flexure for column footing.

Appendix B: Probabilistic parameters of each bridge element

Table B-1. Probabilistic parameters of Pier in shear [47, 154]

variable	Distribution	Mean	Standard deviation
D_{pier} (inch)	Normal	0.5	0.015
$R_{corr\ pier}$ (mils/year)	Normal	1.989	0.231
γ_{msc}	Normal	1.075	0.108
λ_{deff}	Normal	1	0.02
f_y (ksi)	Normal	56	6.16
f'_c (ksi)	Normal	2.76	0.497
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
V_{trk-i}	Normal	1.27	0.036
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114
$V_{pier\ dl\ shear}$ (kips)	Normal	97.54	11.9

Table B-2. Probabilistic parameters of Exterior Girder 1 in shear [47, 154]

variable	Distribution	Mean	Standard deviation
A_1	Normal	80.2	33.684
B_1	Normal	0.593	0.2372
F_y(ksi)	Normal	36.33	4.21
γ_{msg}	Normal	1.14	0.137
λ_{conc}	Normal	1.05	0.105
λ_{steel}	Normal	1.03	0.082
$V_{girder1\ dl}$	Normal	17.4355	1.67215
$V_{trk\text{-}e}$	Normal	0.905	0.064
DF_e	Normal	0.982	0.122
I_{beam}	Normal	1.14	0.114

Table B-3. Probabilistic parameters of Exterior Girder 1 in flexure [47, 154]

variable	Distribution	Mean	Standard deviation
A_1	Normal	80.2	33.684
B_1	Normal	0.593	0.2372
F_y(ksi)	Normal	36.33	4.21
γ_{mfg}	Normal	1.11	0.128
λ_{conc}	Normal	1.05	0.105
λ_{steel}	Normal	1.03	0.082
$M_{girder1\ dl}$ (fit-kip)	Normal	191	18.32
$M_{trk\text{-}e}$ (fit-kip)	Normal	306	22.76
DF_e	Normal	0.982	0.122
I_{beam}	Normal	1.14	0.114

Table B-4. Probabilistic parameters of Interior-exterior Girder 2 in shear [47, 154]

variable	Distribution	Mean	Standard deviation
A_2	Normal	80.2	33.684
B_2	Normal	0.593	0.2372
F_y(ksi)	Normal	36.33	4.21
γ_{msg}	Normal	1.14	0.137
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
$V_{girder2\ dl}$	Normal	29	3.235
V_{trk-i}	Normal	1.27	0.036
DF_{i-e}	Normal	1.14	0.142
I_{beam}	Normal	1.14	0.114

Table B-5. Probabilistic parameters of Interior-exterior Girder 2 in flexure [47, 154]

variable	Distribution	Mean	Standard deviation
A_2	Normal	80.2	33.684
B_2	Normal	0.593	0.2372
F_y(ksi)	Normal	36.33	4.21
γ_{mfg}	Normal	1.11	0.128
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
$M_{girder2\ \ dl}$ (fit-kip)	Normal	317.735	35.43
M_{trk-i} (fit-kip)	Normal	435.6	14.76
DF_{i-e}	Normal	1.14	0.142
I_{beam}	Normal	1.14	0.114

Table B-6. Probabilistic parameters of Interior Girder 3 in shear [47, 154]

variable	Distribution	Mean	Standard deviation
A_3	Normal	34	3.06
B_3	Normal	0.65	0.065
F_y(ksi)	Normal	36.33	4.21
γ_{msg}	Normal	1.14	0.137
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
$V_{girder3\ dl}$	Normal	27.1787	3.45
$V_{trk\text{-}i}$	Normal	1.27	0.036
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114

Table B-7. Probabilistic parameters of Interior Girder 3 in flexure [47, 154]

variable	Distribution	Mean	Standard deviation
A_3	Normal	34	3.06
B_3	Normal	0.65	0.065
F_y(ksi)	Normal	36.33	4.21
γ_{mfg}	Normal	1.11	0.128
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
$M_{girder3\ dl}$ (fit-kip)	Normal	297.8235	37.76
$M_{trk\text{-}i}$ (fit-kip)	Normal	435..6	14.76
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114

Table B-8. Probabilistic parameters of Interior Girder 4 shear [47, 154]

variable	Distribution	Mean	Standard deviation
A_4	Normal	34	3.06
B_4	Normal	0.65	0.065
F_y(ksi)	Normal	36.33	4.21
γ_{msg}	Normal	1.14	0.137
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
$V_{girder4\ dl}$	Normal	27.1787	3.45
$V_{trk\text{-}i}$	Normal	1.27	0.036
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114

Table B-9. Probabilistic parameters of Interior Girder 4 flexure [47, 154]

variable	Distribution	Mean	Standard deviation
A_4	Normal	34	3.06
B_4	Normal	0.65	0.065
F_y(ksi)	Normal	36.33	4.21
γ_{mfg}	Normal	1.11	0.128
λ_{conc}	Normal	1.05	0.105
λ_{asph}	Normal	1	0.25
λ_{steel}	Normal	1.03	0.082
$M_{girder4\ dl}$ (fit-kip)	Normal	297.8235	37.76
$M_{trk\text{-}i}$ (fit-kip)	Normal	435..6	14.76
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114

Table B-10. Probabilistic parameters of Exterior Interior Girder 5 shear [47, 154]

variable	Distribution	Mean	Standard deviation
$\mathbf{A_5}$	Normal	34	3.06
$\mathbf{B_5}$	Normal	0.65	0.065
$\mathbf{F_y}$**(ksi)**	Normal	36.33	4.21
$\gamma_{\mathbf{msg}}$	Normal	1.14	0.137
$\lambda_{\mathbf{conc}}$	Normal	1.05	0.105
$\lambda_{\mathbf{asph}}$	Normal	1	0.25
$\lambda_{\mathbf{steel}}$	Normal	1.03	0.082
$\mathbf{V_{girder5\ dl}}$	Normal	27.1787	3.45
$\mathbf{V_{trk\text{-}i}}$	Normal	1.27	0.036
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114

Table B-11. Probabilistic parameters of Interior Girder 5 flexure [47, 154]

variable	Distribution	Mean	Standard deviation
$\mathbf{A_5}$	Normal	34	3.06
$\mathbf{B_5}$	Normal	0.65	0.065
$\mathbf{F_y}$**(ksi)**	Normal	36.33	4.21
$\gamma_{\mathbf{mfg}}$	Normal	1.11	0.128
$\lambda_{\mathbf{conc}}$	Normal	1.05	0.105
$\lambda_{\mathbf{asph}}$	Normal	1	0.25
$\lambda_{\mathbf{steel}}$	Normal	1.03	0.082
$\mathbf{M_{girder5\ \ dl}}$ **(fit-kip)**	Normal	297.8235	37.76
$\mathbf{M_{trk\text{-}i}}$ **(fit-kip)**	Normal	435..6	14.76
DF_i	Normal	1.309	0.163
I_{beam}	Normal	1.14	0.114

Table B-12. Probabilistic parameters of Column footing in flexure [47, 154]

variable	Distribution	Mean	Standard deviation
λ_{rebar}	Normal	1	0.015
γ_{mfc}	Normal	1.02	0.061
λ_{deff}	Normal	1	0.02
f_y (ksi)	Normal	56	6.16
f'_c (ksi)	Normal	2.76	0.497
λ_{conc}	Normal	1.05	0.105
λ_{steel}	Normal	1.03	0.082
λ_{asph}	Normal	1	0.25
$M_{column\ footing\ dl}$	Normal	85.45	9.997
$V_{trk\text{-}i}$	Normal	1.27	0.36
DF_i	Normal	1.309	0.163
$DF_{i\text{-}e}$	Normal	1.14	0.142
I_{beam}	Normal	1.14	0.114

Appendix C: Discretization schema

Pier shear

Variable	Probable range	Discretized states	Final interval boundaries
D_{pier} (inch)	0.4-0.6	12	0,0.4: (0.6-0.4)/10:0.6,∞
$T_{corr\ pier}$ pier (year)	0-50	51	0:1:50
Time(slab)	0-50	51	0:1:50
$R_{corr\ pier}$ (mils/year)	0.8-3.2	12	0,0.8:(3.2-0.8)/10:3.2,∞
γ_{msc}	0.6-1.6	12	0,0.6:0.1:1.6, ∞
λ_{deff}	0.9-1.1	12	0,0.9:0.02:1.1, ∞
f_y (ksi)	26-86	12	0,26:(86-26)/10:86,∞
f'_c (ksi)	0.3-5.3	12	0,0.2:5/10:5.3, ∞
$V_{pier\ dl\ shear}$ (kips)	40-160	12	0,40:120/10:160, ∞
$V_{trk\text{-}i}$	1-1.5	12	0,1:0.05:1.5,∞
DF_i	0.5-2.1	12	0,0.5:(2.1-0.5)/10:2.1,∞
I_{beam}	0.6-1.8	12	0,0.6:(1.8-0.6)/10:1.8,∞
$V_{pier\ ll\ shear}$ (kips)	10-240	22	0,10:10.5:240, ∞
$R_{pier\ shear\ capacity}$ (kips)	70-1000	12	0,70:93:1000,∞
$R_{pier\ shear\ capacity\ 2}$ (kips)	75-205	12	0,75:130/10:205, ∞
$R_{pier\ shear\ capacity\ 1}$ (kips)	78-900	12	0,78:82.2:900, ∞
$L_{pier\ load\ shear}$ (kips)	60-300	32	0,60:8:300,∞

Exterior Girder 1 in shear

Variable	Probable range	Discretized states	Final interval boundaries
A_1	0-200	41	0:5:200,∞
B_1	0-1.5	31	0:1.5/30:1.5,∞
Time (girder 1)	0-50	51	0:1:50
d_{corr1} (10^{-6}m)	0-7239	50	0:7239/50:7239
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{msg}	0.45-1.8	22	0,0.45:1.35/20:1.8,∞
$V_{girder1\ dl}$ (kips)	9-25	42	0,9:16/40:25,∞
$V_{girder1\ ll}$ (kips)	12-150	52	0,12:138/50:150,∞
V_{trk-e}(kips)	0.5-1.3	22	0,0.5:.0.8/20:1.3,∞
DF_e	0.4-1.6	22	0,0.4:1.2/20:1.6,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$R_{girder1\ shear}$ (kips)	0-600	102	0:600/100:600,∞
$L_{girder1\ shear}$ (kips)	20-180	82	0,20:(180-20)/80:180,∞

Exterior Girder 1 in flexure

Variable	Probable range	Discretized states	Final interval boundaries
A_1	0-200	41	0:5:200,∞
B_1	0-1.5	31	0:1.5/30:1.5,∞
Time (girder 1)	0-50	51	0:1:50
d_{corr1} (10^{-6}m)	0-7239	50	0:7239/50:7239
F_y(ksi)	15-57	12	0,15:(57-15)/10:57,∞
γ_{mfg}	0.5-1.8	12	0,0.5:0.13:1.8, ∞
$M_{girder1\ dl}$ (ft-kip)	100-280	12	0,100:180/10:280,∞
M_{trk-e}(ft-kip)	190-420	12	0,190:23:420,∞
DF_e	0.4-1.6	12	0,0.4:0.12:1.6,∞
I_{beam}	0.6-1.8	12	0,0.6:0.12:1.8,∞
$M_{girder1\ ll}$ (ft-kip)	100-660	12	0,100:56:660, ∞
$R_{girder1\ moment}$ (ft-kip)	500-3100	12	0,500:260:3100,∞
$L_{girder1\ moment}$ (ft-kip)	250-850	12	0,250:60:850,∞

Interior-Exterior Girder 2 in shear

Variable	Probable range	Discretized states	Final interval boundaries
A_2	0-200	41	0:5:200,∞
B_2	0-1.5	31	0:1.5/30:1.5,∞
Time (girder 2)	0-50	51	0:1:50
d_{corr2} (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{msg}	0.45-1.8	22	0,0.45:1.35/20:1.8,∞
$V_{girder2\ dl}$ (kips)	13-45	42	0,13:32/40:45,∞
$V_{girder2\ ll}$ (kips)	10-110	52	0,10:20:110,∞
$V_{trk\text{-}i}$(kips)	1-1.5	22	0,1:0.5/20:1.5,∞
$DF_{i\text{-}e}$	0.4-1.9	22	0,0.4:1.5/20:1.6,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$R_{girder1\ shear}$ (kips)	0-600	81	0:600/100:600,∞
$L_{girder1\ shear}$ (kips)	20-160	82	0,20:140/80:140,∞

Interior-Exterior Girder 2 in flexure

Variable	Probable range	Discretized states	Final interval boundaries
A_2	0-200	41	0:5:200,∞
B_2	0-1.5	31	0:1.5/30:1.5,∞
Time (girder 2)	0-50	51	0:1:50
d_{corr2} (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	12	0,15:(57-15)/10:57,∞
γ_{mfg}	0.5-1.8	12	0,0.5:0.13:1.8, ∞
$M_{girder2\ dl}$ (ft-kip)	150-490	12	0,150:34:490,∞
$M_{trk\text{-}i}$(ft-kip)	360-510	12	0,360:15:510,∞
$DF_{i\text{-}e}$	0.4-1.9	12	0,0.4:0.15:1.6,∞
I_{beam}	0.6-1.8	12	0,0.6:0.12:1.8,∞
$M_{girder2\ ll}$ (ft-kip)	200-1100	12	0,200:90:1100, ∞
$R_{girder2\ moment}$ (ft-kip)	500-3100	102	0,500:2600/100:3100,∞
$L_{girder2\ moment}$ (ft-kip)	300-1600	62	0,300:1300/60:1600,∞

Interior Girder 3 in shear

Variable	Probable range	Discretized states	Final interval boundaries
A_3	19-49	32	0,19:1:49,∞
B_3	0.3-1	32	0,0.3: (1-0.3)/30:1,∞
Time (girder 3)	0-50	51	0:1:50
$d_{corr\ 3}$ (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{msg}			
$V_{girder3\ dl}$ (kips)	15-49	42	0,15:(49-15)/40:49,∞
$V_{girder3\ ll}$ (kips)	1-106	42	0,1:(106-1)/40:106,∞
$V_{trk\text{-}i}$(kips)	1-1.5	22	0,1:0.5/20:1.5,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$R_{girder3\ shear}$ (kips)	0-600	81	0:600/80:600,∞
$L_{girder3\ shear}$ (kips)	10-165	52	0,10:(165-10)/50:165,∞

Interior Girder 3 in flexure

Variable	Probable range	Discretized states	Final interval boundaries
A_3	19-49	32	0,19:1:49,∞
B_3	0.3-1	32	0,0.3: (1-0.3)/30:1,∞
Time (girder 3)	0-50	51	0:1:50
$d_{corr\ 3}$ (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{mfg}	0.5-1.8	22	0,0.5:1.3/20:1.8, ∞
$M_{girder3\ dl}$ (ft-kip)	130-450	42	0,130:320/40:450,∞
$M_{trk\text{-}i}$(ft-kip)	360-510	52	0,360:3:510,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$M_{girder3\ ll}$ (ft-kip)	120-1200	52	0,120:1080/50:1200, ∞
$R_{girder3\ moment}$ (ft-kip)	700-3000	82	0,700:2300/80:3000,∞
$L_{girder3\ moment}$ (ft-kip)	500-1500	52	0,500:20:1500,∞

Interior Girder 4 in shear

Variable	Probable range	Discretized states	Final interval boundaries
A_4	19-49	32	0,19:1:49,∞
B_4	0.3-1	32	0,0.3: (1-0.3)/30:1,∞
Time (girder 4)	0-50	51	0:1:50
$d_{corr\ 4}$ (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{msg}			
$V_{int\ girder\ dl}$ (kips)	15-49	42	0,15:(49-15)/40:49,∞
$V_{girder4\ ll}$ (kips)	1-106	42	0,1:(106-1)/40:106,∞
$V_{trk\text{-}i}$(kips)	1-1.5	22	0,1:0.5/20:1.5,∞
$V_{trk\text{-}i}$(kips)	0-3	21	0:3/20:3,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$R_{girder4\ shear}$ (kips)	0-600	81	0:600/80:600,∞
$L_{girder4\ shear}$ (kips)	10-165	52	0,10:(165-10)/50:165,∞

Interior Girder 4 in flexure

Variable	Probable range	Discretized states	Final interval boundaries
A_4	19-49	32	0,19:1:49,∞
B_4	0.3-1	32	0,0.3: (1-0.3)/30:1,∞
Time (girder 4)	0-50	51	0:1:50
$d_{corr\ 4}$ (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{mfg}	0.5-1.8	22	0,0.5:1.3/20:1.8, ∞
$M_{girder4\ dl}$ (ft-kip)	130-450	42	0,130:320/40:450,∞
$M_{trk\text{-}i}$(ft-kip)	360-510	52	0,360:3:510,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$M_{girder4\ ll}$ (ft-kip)	120-1200	52	0,120:1080/50:1200, ∞
$R_{girder4\ moment}$ (ft-kip)	700-3000	82	0,700:2300/80:3000,∞
$L_{girder4\ moment}$ (ft-kip)	500-1500	52	0,500:20:1500,∞

Interior Girder 5 in shear

Variable	Probable range	Discretized states	Final interval boundaries
A_5	19-49	32	0,19:1:49,∞
B_5	0.3-1	32	0,0.3: (1-0.3)/30:1,∞
Time (girder 5)	0-50	51	0:1:50
$d_{corr\ 5}$ (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{msg}			
$V_{girder5\ dl}$ (kips)	15-49	42	0,15:(49-15)/40:49,∞
$V_{girder5\ ll}$ (kips)	1-106	42	0,1:(106-1)/40:106,∞
$V_{trk\text{-}i}$(kips)	1-1.5	22	0,1:0.5/20:1.5,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$R_{girder5\ shear}$ (kips)	0-600	81	0:600/80:600,∞
$L_{girder5\ shear}$ (kips)	10-165	52	0,10:(165-10)/50:165,∞

Interior Girder 5 in flexure

Variable	Probable range	Discretized states	Final interval boundaries
A_3	19-49	32	0,19:1:49,∞
B_3	0.3-1	32	0,0.3: (1-0.3)/30:1,∞
Time (girder 5)	0-50	51	0:1:50
$d_{corr\ 3}$ (10^{-6}m)	0-7366	50	0:7366/50:7366
F_y(ksi)	15-57	42	0,15:(57-15)/40:57,∞
γ_{mfg}	0.5-1.8	22	0,0.5:1.3/20:1.8, ∞
$M_{girder5\ dl}$ (ft-kip)	130-450	42	0,130:320/40:450,∞
$M_{trk\text{-}i}$(ft-kip)	360-510	52	0,360:3:510,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$M_{girder5\ ll}$ (ft-kip)	120-1200	52	0,120:1080/50:1200, ∞
$R_{girder5\ moment}$ (ft-kip)	700-3000	82	0,700:2300/80:3000,∞
$L_{girder5\ moment}$ (ft-kip)	500-1500	52	0,500:20:1500,∞

Column footing in flexure

Variable	Probable range	Discretized states	Final interval boundaries
λ_{rebar}	0.9-1.1	22	0,0.9:(1.1-0.9)/20:1.1,∞
γ_{mfc}	0.7-1.3	22	0,0.7:0.6/20:1.3, ∞
λ_{deff}	0.9-1.1	22	0,0.9:0.01:1.1, ∞
f_y (ksi)	26-86	32	0,26:(86-26)/30:86,∞
f'_c (ksi)	0.3-5.3	27	0,0.2:5/25:5.3, ∞
V_{trk-i}(kips)	0-3	21	0:3/20:3,∞
DF_i	0.5-2.1	22	0,0.5:(2.1-0.5)/20:2.1,∞
DF_{i-e}	0.4-1.9	22	0,0.4:1.5/20:1.6,∞
I_{beam}	0.6-1.8	22	0,0.6:(1.8-0.6)/20:1.8,∞
$M_{column\ footing\ dl}$ (ft-kip)	45-135	42	0,45:90/40:135, ∞
$M_{column\ footing\ ll}$ (ft-kip)	10-110	82	0,10:100/80:110, ∞
$M_{column\ footing\ ll\ 1}$ (ft-kip)	10-100	42	0,10:90/40:100, ∞
$M_{column\ footing\ ll\ 2}$ (ft-kip)	1.6-9	42	0,1.6:7.4/40:9, ∞
$R_{column\ footing\ moment}$	0-500	102	0,100:620/100:720,∞
$R_{column\ footing\ moment\ 2}$	30-500	42	0,2:1.2:50, ∞
$R_{column\ footing\ moment\ 1}$	200-640	52	0,200:440/50:640, ∞
$L_{column\ footing\ moment}$	70-230	102	0,70:160/100:230,∞

Appendix D: Estimation of corrosion, crack and spalling initiation time

Based on the knowledge in Section 3.2.2, the initiation time of corrosion, crack and spalling for bridge elements made of reinforced concrete can be estimated. With identified parameters, the detailed simulation for the deterioration processes of bridge slab and pier are implemented based on MCS as follows:

Slab

Table D-1. Parameters of slab for corrosion initiation time [47, 154]

variable	Distribution	Mean	Standard deviation
C_0 (slab, %)	Normal	1.08	0.072
D_c (in^2/sec, 10^{-9})	Normal	5.42	0.387
X (slab, in)	Normal	2.25	0.337
C_{cr} (%)	Normal	0.4	0.05

Table D-2. Parameters of slab for the time from corrosion initiation to cracking [47, 97, 154]

variable	Distribution	Part1	Part2
D (slab, in)	Normal	0.625	0.0187
α	Uniform	0.523 ($Fe(OH)_3$)	0.622 ($Fe(OH)_2$)
i_{corr} (mA/ft^2)	Normal	2.35	0.27
d_0(4.9mils)	Deterministic	4.9	—
f_t (psi)	Deterministic	472	—
f'_c (ksi)	Normal	2.76	0.497
C(slab, in)	Normal	2.25	0.337
ρ_{rust} (lb/ft^3)	Deterministic	225	—
E_c (ksi)	Deterministic	3900	—
φ_{cr}	Deterministic	2	—
V_c	Deterministic	0.18	—
ρ_{steel} (lb/ft^3)	Uniform	7750	8050

Table D-3. Parameters of slab for the time from crack initiation to spalling [47, 154, 164]

variable	Distribution	Part1	Part2
W_{lim}(mm)	Uniform	0.3	1
i_{corr} ($\mu A/cm^2$)	Normal	2.51	0.29
f'_c (MPa)	Normal	19	3.24
C(slab,mm)	Normal	57.15	8.56

****

% **MATLAB codes for slab deterioration processes**

****

% Calculate the corrosion initiation time T_{corr}

```
NPar=100000; % the sampling size
C=zeros(NPar,1);
t1=zeros(NPar,1); %sampled corrosion initiation time
C0=normrnd(1.08,0.072,NPar,1); % the chloride concentration on the concrete surface, C0
% (slab,%)
x=normrnd(2.25,0.337,NPar,1); % Distance to reinforcement X (slab, in)
```

```
Dc=normrnd(5.42,0.387,NPar,1); % the diffusion coefficient for chloride in concrete, Dc
% (in^2/sec, 10^-9)
Cc=normrnd(0.4,0.05,NPar,1); %critical chloride concentration Ccr (%)
C=1-Cc./C0;
for i=1:100000
 t1(i,1)=x(i,1)^2/(4*Dc(i,1))*((erfinv(C(i,1)))^-2)*(10^9)/31536000;
end
Tcorr=t1; %sampled corrosion initiation time
T=0:1:50; % time horizon
Ncorr=histc(Tcorr,T)';%counts the number of values of sampled corrosion initiation time that fall
%between the elements in the edges vector
Mcorr=cumsum(Ncorr); % calculate cumulative sum of elements of Ncorr
Hist_corr= Ncorr / Mcorr (1,51);% calculate the histogram of corrosion initiation time
```

% Calculate the crack initiation time T_{crack}

```
D2=normrnd(15.875,0.475,NPar,1); % the diameter of reinforcement steel, D (slab, in)
C2=normrnd(2.25,0.337,NPar,1); % cover depth, C(slab, in)
d0=4.9; % the thickness of the pore band around the steel/concrete interface
Ec=3900000; % elastic modulus of the concrete, Ec (ksi)
qcr=2;    % the creep coefficient of the concrete, φ_cr
i_corr2=normrnd(2.35,0.27,NPar,1); % the annual mean corrosion rate, icorr (mA/ft^2)
afa=0.523+0.099*rand(NPar,1); % the molecular weight of steel weigh divided by the molecular
% weight of corrosion products, α
Den=7750+300*rand(NPar,1); % the density of steel, ρsteel (lb/ft^3)
kp=0.098*(1./afa)*3.14.*D2.*i_corr2; % the rate of rust production
a2=(D2/25.4+2*d0/1000)/2;  % inner radius of a thick-wall concrete cylinder, (in)
b2=C2+(D2/25.4+2*d0/1000)/2; % is outer radius of a thick-wall concrete cylinder, (in)
Eef=Ec/(1+qcr); % effective elastic modulus of the concrete
W=3.6.*D2.*3.14.*(472.*C2.*((a2.*a2+b2.*b2)./(b2.*b2-
a2.*a2)+0.18)/Eef+d0/1000)*25.4./(1+3600.*afa./Den); % the critical amount of
% corrosion products
t2=W.*W./(2.*kp); % the time from corrosion initiation to cracking
Tcrack=t1+t2; % crack initiation time from the beginning
Ncrack=histc(Tcrack,T)';% counts the number of values of sampled crack initiation time that fall
% between the elements in the edges vector
Mcrack=cumsum(Ncrack); % calculate cumulative sum of elements of Ncrack
Hist_crack= Ncrack / Mcrack (1,51); %calculate the histogram of crack initiation time
```

% Calculate the spalling initiation time $T_{spalling}$

```
C3=normrnd(57.15,8.56,NPar,1); % concrete cover of slab (mm)
i_corr3=normrnd(2.51,0.29,NPar,1); % corrosion rate icorr (μA/cm^2)
fc=normrnd(19,3.24,NPar,1); % concrete compressive strength (MPa)
Wlim=0.3+0.7*rand(NPar,1); % limit crack width, (mm)
wc=27./(fc+13.5); % water-cement ratio estimated from Bolomey's formula
t3=0.0167.*i_corr3.^(-1.1).*(42.9*(wc./C3).^(-0.54)+((Wlim-
0.3)/0.0062).^1.5); % the time from crack initiation to spalling
Tspalling=t1+t2+t3; %spalling initiation time from beginning
Nspalling=histc(Tspalling,T)'; %counts the number of values of sampled spalling initiation
% time that fall between the elements in the edges vector
Mspalling=cumsum(Nspalling); % calculate cumulative sum of elements of Nspalling
Hist_spalling = Nspalling/ Mspalling(1,51); %calculate the histogram of spalling initiation
% time
```

Pier

Table D-4. Parameters of pier for corrosion initiation time [47, 154]

variable	Distribution	Mean	Standard deviation
C_0 (pier, %)	Normal	0.72	0.048
D_c (in^2/sec, 10^{-9})	Normal	5.42	0.387
X (pier, in)	Normal	2.0	0.3
C_{cr} (%)	Normal	0.4	0.05

Table D-5. Parameters of pier for the time from corrosion initiation to cracking [47, 97, 154]

variable	Distribution	Part1	Part2
D (pier,in)	Normal	0.5	0.015
α	Uniform	0.523 ($Fe(OH)_3$)	0.622 ($Fe(OH)_2$)
i_{corr} (mA/ft^2)	Normal	2.35	0.27
d_0(4.9mils)	Deterministic	4.9	—
f_t (psi)	Deterministic	472	—
f'_c (ksi)	Normal	2.76	0.497
C(pier, in)	Normal	2	0.3
ρ_{rust} (lb/ft^3)	Deterministic	225	—
E_c (ksi)	Deterministic	3900	—
φ_{cr}	Deterministic	2	—
V_c	Deterministic	0.18	—
ρ_{steel} (lb/ft^3)	Uniform	7750	8050

Table D-6. Parameters of pier for the time from crack initiation to spalling [47, 154, 164]

variable	Distribution	Part1	Part2
W_{lim}(mm)	Uniform	0.3	1
i_{corr} (μA/cm^2)	Normal	2.51	0.29
f'_c (MPa)	Normal	19	3.24
C(pier,mm)	Normal	50.8	7.62

****

% **MATLAB codes for pier deterioration processes**

****

% Calculate the corrosion initiation time T_{corr}

```
NPar=100000; % the sampling size
C=zeros(NPar,1);
t1=zeros(NPar,1); %sampled corrosion initiation time
C0=normrnd(0.72,0.048,NPar,1); % the chloride concentration on the concrete surface, C0
% (pier,%)
x=normrnd(2,0.3,NPar,1); % Distance to reinforcement X (pier, in)
Dc=normrnd(5.42,0.387,NPar,1); % the diffusion coefficient for chloride in concrete, Dc
% (in^2/sec, 10^-9)
Cc=normrnd(0.4,0.05,NPar,1); %critical chloride concentration Ccr (%)
C=1-Cc./C0;
for i=1:100000
 t1(i,1)=x(i,1)^2/(4*Dc(i,1))*((erfinv(C(i,1)))^-2)*(10^9)/31536000;
end
Tcorr=t1; %sampled corrosion initiation time
T=0:1:50; % time horizon
Ncorr=histc(Tcorr,T)';%counts the number of values of sampled corrosion initiation time that fall
%between the elements in the edges vector
Mcorr=cumsum(Ncorr); % calculate cumulative sum of elements of Ncorr
Hist_corr= Ncorr / Mcorr (1,51);% calculate the histogram of corrosion initiation time
```

% Calculate the crack initiation time T_{crack}

```
D2=normrnd(12.7,0.381,NPar,1); % the diameter of reinforcement steel, D (pier, in)
C2=normrnd(2,0.3,NPar,1); % cover depth, C(pier, in)
d0=4.9; % the thickness of the pore band around the steel/concrete interface
Ec=3900000; % elastic modulus of the concrete, Ec (ksi)
qcr=2;    % the creep coefficient of the concrete, φ_cr
i_corr2=normrnd(2.35,0.27,NPar,1); % the annual mean corrosion rate, icorr (mA/ft^2)
afa=0.523+0.099*rand(NPar,1); % the molecular weight of steel weigh divided by the molecular
% weight of corrosion products, α
Den=7750+300*rand(NPar,1); % the density of steel, ρsteel (lb/ft^3)
kp=0.098*(1./afa)*3.14.*D2.*i_corr2; % the rate of rust production
a2=(D2/25.4+2*d0/1000)/2;  % inner radius of a thick-wall concrete cylinder, (in)
b2=C2+(D2/25.4+2*d0/1000)/2; % is outer radius of a thick-wall concrete cylinder, (in)
Eef=Ec/(1+qcr); % effective elastic modulus of the concrete
W=3.6.*D2.*3.14.*(472.*C2.*((a2.*a2+b2.*b2)./(b2.*b2-
a2.*a2)+0.18)/Eef+d0/1000)*25.4./(1+3600.*afa./Den); % the critical amount of
% corrosion products
t2=W.*W./(2.*kp); % the time from corrosion initiation to cracking
Tcrack=t1+t2; % crack initiation time from the beginning
Ncrack=histc(Tcrack,T)';% counts the number of values of  sampled crack initiation time that fall
% between the elements in the edges vector
Mcrack=cumsum(Ncrack); % calculate cumulative sum of elements of Ncrack
Hist_crack= Ncrack / Mcrack (1,51); %calculate the histogram of crack initiation time
```

% Calculate the spalling initiation time $T_{spalling}$

```
C3=normrnd(50.8,7.62,NPar,1); % concrete cover of pier (mm)
```

```
i_corr3=normrnd(2.51,0.29,NPar,1);  % corrosion rate icorr (μA/cm2)
fc=normrnd(19,3.24,NPar,1);  % concrete compressive strength (MPa)
Wlim=0.3+0.7*rand(NPar,1);  % limit crack width, (mm)
wc=27./(fc+13.5);  % water-cement ratio estimated from Bolomey's formula
t3=0.0167.*i_corr3.^(-1.1).*(42.9*(wc./C3).^(-0.54)+((Wlim-
0.3)/0.0062).^1.5);  % the time from crack initiation to spalling
Tspalling=t1+t2+t3;  %spalling initiation time from beginning
Nspalling=histc(Tspalling,T)';  %counts the number of values of sampled spalling initiation
%  time that fall between the elements in the edges vector
Mspalling=cumsum(Nspalling); % calculate cumulative sum of elements of Nspalling
Hist_spalling = Nspalling/ Mspalling(1,51); %calculate the histogram of spalling initiation
% time
```

Appendix E: Publication

Conference papers:

Wang. Ruizi, Ma. Lin, Yan. Cheng, & Mathew. Joseph (2010), Preliminary study on bridge health prediction using Dynamic Objective Oriented Bayesian Networks (DOOBNs). In *Proceedings of WCEAM 2010 : Fifth World Congress on Engineering Asset Management*, *World Congress on Engineering Asset Management*, Brisbane, Qld.

Wang. Ruizi, Ma. Lin, Yan. Cheng, and Mathew. Joseph, Structural reliability prediction of a steel bridge element using dynamic object oriented Bayesian networks (DOOBNs). In *Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2011 International Conference on*, pp. 7-12.

Wang. Ruizi, Ma. Lin, Yan. Cheng, and Mathew. Joseph, Condition deterioration prediction of bridge elements using Dynamic Bayesian Networks (DBNs). In Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), 2012 International Conference (in press).

Journal Papers:

Wang. Ruizi, Ma. Lin, Yan. Cheng, and Mathew. Joseph, *Stochastic modelling of bridge serviceability deterioration using Dynamic Object Oriented Bayesian Networks (DOOBNs)*. Journal of bridge engineering. (To be submitted)

Wang. Ruizi, Ma. Lin, Yan. Cheng, and Mathew. Joseph, *Structural reliability prediction of bridge systems using Dynamic Object Oriented Bayesian Networks (DOOBNs)*. Journal of bridge engineering. (To be submitted)

Wang. Ruizi, Ma. Lin, Yan. Cheng, and Mathew. Joseph, *Integrated bridge deterioration prediction using Dynamic Object Oriented Bayesian Networks (DOOBNs)*. Journal of infrastructure systems. (To be submitted)