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Free convection in a triangular enclosure with fluid-saturated porous medium and internal heat generation

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Abstract

Unsteady natural convection inside a triangular cavity has been studied in this study. The cavity is filled with a saturated porous medium with non-isothermal left inclined wall while the bottom surface is isothermally heated and the right inclined surface is isothermally cooled. An internal heat generation is also considered which is dependent on the fluid temperature. The governing equations are solved numerically by finite volume method. The Prandtl number, Pr of the fluid is considered as 0.7 (air) while the aspect ratio and the Rayleigh number, Ra are considered as 0.5 and 10^5 respectively. The effect of heat generation on the fluid flow and heat transfer have been presented as a form of streamlines and isotherms. The rate of heat transfer through three surfaces of the enclosure is also presented.

Contents

1 Introduction

The characteristics of heat and fluid flow for a configuration of isothermal vertical walls, maintained at different temperatures and with adiabatic horizontal walls, are well understood [1, 2]. Less work has been carried out for more complex thermal boundary conditions, such as an imposed thermal gradient that is neither purely horizontal nor purely vertical. Shiralkar and Tien [3] numerically investigated the natural convection in an enclosure with temperature gradients imposed in both the horizontal and vertical directions simultaneously. There are number of studies related to simultaneously heating and cooling of the adjacent walls of a square/rectangular enclosure are available in the literature [4–6]. Velusamy et al. [7] investigated the steady two-dimensional natural convection flow in a rectangular enclosure with a linearly varying surface temperature on the left vertical wall, cooled right vertical and top walls and a uniformly heated bottom wall. The similar work is also conducted by Hossain and Wilson [8].

In recent decades, flow in a confined porous medium has received considerable attention from researchers working with various methodologies such as experimental, theoretical as well as numerical. The main application of studying in this area is to develop technology and industry, such as prevention of sub-oil water pollution, storage of nuclear waste and geothermal energy systems (see extensive review in [10]).

The fluid flow and heat transfer inside a triangular enclosure is also considered by many researchers recently because of its engineering application in attic-shaped buildings, electronic devices, solar collectors etc. An extensive review of natural convection in the triangular enclosure can be found in [11]. An investigation of natural convection inside a triangular enclosure with linear heating/cooling on the inclined surfaces is investigated by Basak et al. [9]. Natural convection in attic space filled with porous media has also been conducted by a number of researchers [12–15]. In this study, we consider triangular enclosure filled with porous media and heat generating fluid with non-uniform temperature conditions of one of the slopping walls. A detailed development of the present investigation is given in the subsequent sections.

2 Mathematical formulation

Under consideration is a triangular cavity of height H, half length of the base l, in a saturated porous medium containing a Newtonian fluid with $\mathrm{Pr} = 0.7$ which is initially at rest with a temperature T_c. At the time $t = 0$, a non-isothermal temperature condition is applied on the left inclined wall where

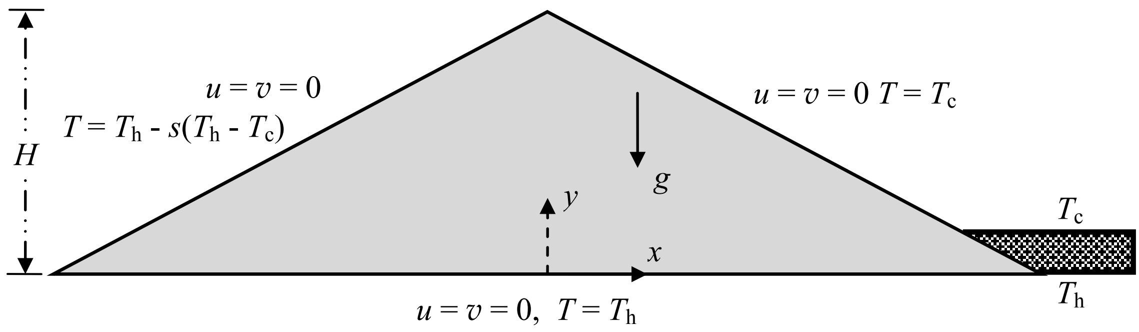

Figure 1: Schematic of the geometry and the coordinate system

the temperature reduces linearly from bottom to top. The bottom surface is uniforly heated at T_h and the top mejor portion of the right inclined wall is cold at T_c. The boundary condition at the lower portion of the right inclined wall is explained as: a small gap of length $s_1 = s/10$ (s is the total length of the inclined surface) where the temperature varies linearly from T_h to T_c. The effect of temperature-dependent heat generation in the flow region is also taken into consideration. The volumetric rate of heat generation, $Q(W/m^3)$, is assumed to be:

$$Q = \begin{cases} Q_0(T - T_c), & T \geq T_c \\ 0, & T \leq T_c \end{cases} \tag{1}$$

where Q_0 is the heat generation constant. The above relation, as explained in Saha et al. [16], is valid as an approximation of the state of some exothermic process, which means that heat flows from the surface to the enclosure. We further assume unsteady laminar flow of a viscous incompressible fluid having constant properties. The effect of buoyancy is included through the well-known Boussinesq approximation. Under the above assumptions, the conservation equations for mass, momentum and energy in a two-dimensional Cartesian co-ordinate system are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{\nu u}{K} \tag{3}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\nu v}{K} + g\beta_T(T - T_c) \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{Q_0}{\rho C_p}(T - T_c) \tag{5}$$

where u and v are the fluid velocity components in the x and y-direction, respectively. t is the time, T is the fluid temperature, p is the fluid pressure, β is the volumetric thermal expansion coefficient, K is the permeability of the porous medium, and ρ, α and C_p are, respectively the density of the fluid, the thermal diffusivity and the specific heat at constant pressure. In the present investigation, porous medium inertia effects have been neglected in the momentum equations, and the effects of viscous dissipation are neglected from the energy equation. In Eqs. (3) and (4), K is the measure of the permeability of the porous medium (a packed bed of spheres) (see [8]), defined by

$$\frac{\epsilon^{+3} d^2}{180\,(1-\epsilon^{+})^2} \tag{6}$$

where, d is the diameter of the solid sphere and ϵ^{+} is known as the porosity of the media and is defined by

$$\epsilon^{+} = \frac{V_f}{V_c} \tag{7}$$

Here V_f is the volume of the fluid and V_c is the control volume. The following dimensionless variables are constructed:

$$\begin{gathered} X = \frac{x}{H}, \quad y = \frac{y}{H}, \quad \tau = \frac{t}{H/U_0}, \quad P = \frac{p}{\rho U_0} \\ U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad \theta = \frac{T-T_c}{T_h-T_c} \end{gathered} \tag{8}$$

where $U_0 = \frac{\nu}{H}$ is the reference velocity. Introducing the above dimensionless dependent and independent variables in the governing Eqs. (3)(4) the following equations are obtained:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{9}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} - \gamma U\right) \tag{10}$$

$$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} - \gamma V\right) + RaPr\theta \tag{11}$$

$$\frac{\partial T}{\partial t} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \lambda\theta \tag{12}$$

where

$$Ra = \frac{g\beta_T(T_H - T_C)H^3}{\kappa\nu}, \quad Pr = \frac{\nu}{\kappa}, \quad \text{and} \quad \lambda = \frac{Q_0 H^2}{\rho\nu C_p} \tag{13}$$

The dimensionless initial and boundary conditions:
All boundaries are rigid and non-slip;
at $\tau \leq 0, \quad U = V = \theta = 0$;
On the bottom surface: $\theta = 1$;
On the left inclined surface: $\theta = 1 - s$;
Upper portion of the right inclined surface: $\theta = 0$;
Lower portion of the right inclined wall: $\theta = 1 - s/s_1$.

Using the numerically obtained values of the temperature function we may obtain the rate of heat flux from each of the walls. The non-dimensional heat flux from any surface is given by -$\frac{\partial T}{\partial \mathbf{n}}$, where $\mathbf{n}$ is the direction normal to the wall. For example, the non-dimensional heat transfer rate, Nu, per unit length in the depthwise direction for the left vertical surface is:

$$Nu = -\int_0^1 \left(\frac{\partial T}{\partial X}\right)_{X=0} dY \tag{14}$$

3 Numerical procedure

Two-dimensional numerical simulations have been carried out in this study. For this purpose, an isosceles triangular domain is considered, and a Cartesian coordinate system is adopted with the origin located at the centre of the base, the x-direction pointing horizontally, and the y-direction pointing vertically. The initial and boundary conditions for the numerical simulations are also specified. That is, the air in the enclosure is initially quiescent and isothermal with a uniform temperature of $\theta = 0$. At the time $\tau = 0$, the left inclined surface is linearly cooled. On the right inclined wall, the bottom gap is maintained as linearly cooling like left wall and top portion is maintained as uniform cooling and the base of the enclosure is uniformly heated. All three surfaces of the enclosure are assumed as rigid and no slip.

The governing equations (9)-(12), along with the specified initial and boundary conditions are solved numerically. The Finite Volume scheme has been chosen to discretize the governing equations, with the QUICK scheme approximating the advection term. The diffusion terms are discretized using central-differencing with second order accurate. A second order implicit time-marching scheme has also been used for the unsteady term. A detailed grid sensitivity test has been carried out to ensure the accuracy of the numerical results. The grid size of 300×200 with non-uniformly distributed has been

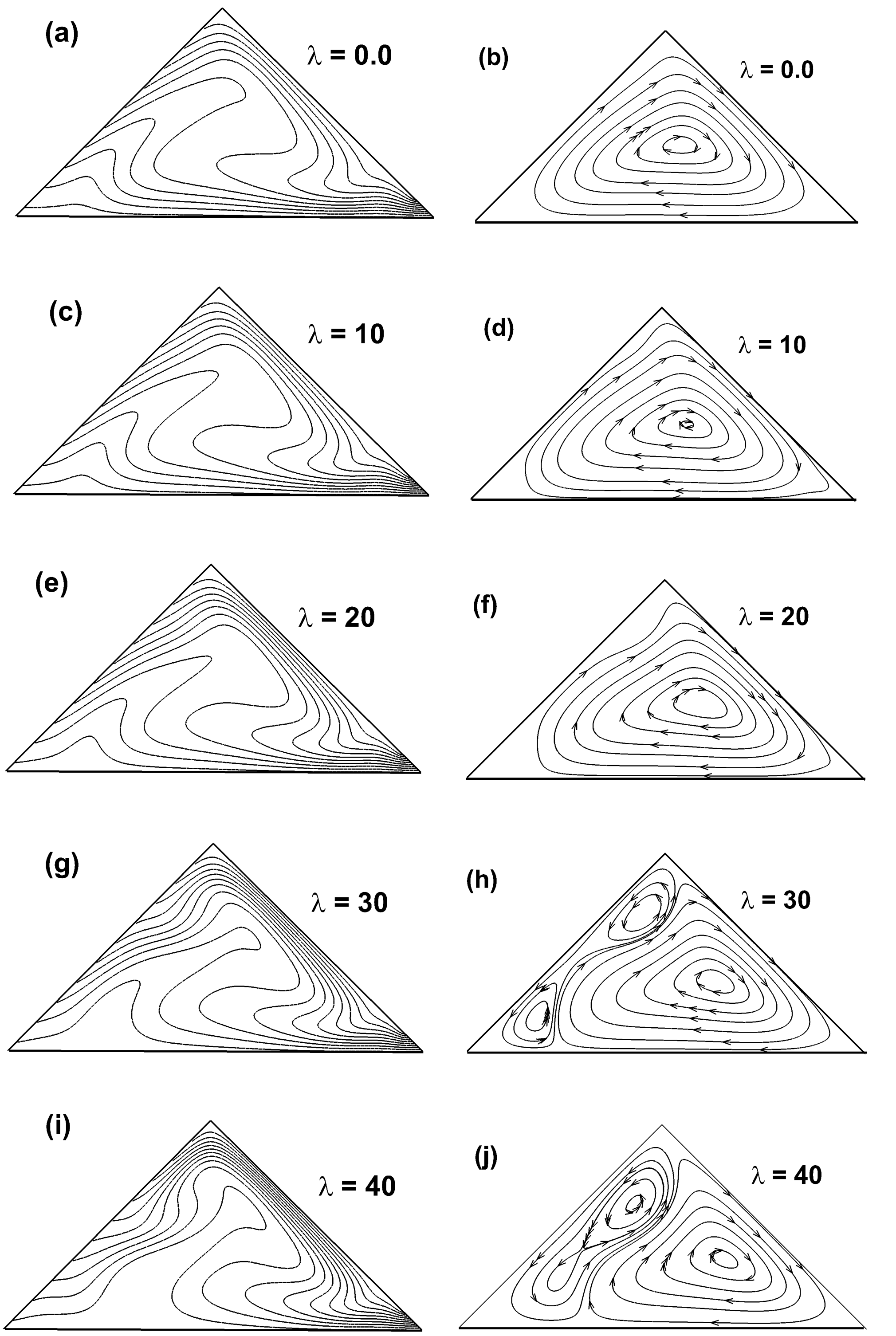

Figure 2: Isotherms (left) and velocity field with vector arrow (right): (a-b) $\lambda = 0$, (c-d) $\lambda = 10$, (e-f) $\lambda = 20$, (g-h) $\lambda = 30$, (i-j) $\lambda = 40$ while $Ra = 10^5$, $\Pr = 0.7$, $K = 10.0$.

adopted for the present simulations. For brevity, the test results of grid dependency are not presented here.

4 Results and discussions

Numerical results for natural convection heat transfer for the fluid with internal heat generation in a triangular enclosure filled with a saturated porous medium with uniform porosity are described. The left inclined wall is non-uniformly cooled and the right inclined and the bottom base are uniformly cooled and heated respectively.

We plot the isotherms and contours of stream function in Fig. 2 for different values of heat generation parameter, λ with $Ra = 10^5$, $Pr = 0.7$ and $K = 10.0$. Fig. 2(a) represents the isotherm and streamlines for pure fluid in the absence of internal heat generation ($\lambda = 0$). Since the bottom surface is uniformly heated, a heated boundary layer develops adjacent to the bottom surface. Due to the buoyancy effect, the hot fluid inside the boundary layer moves upward from the bottom left tip. The non-uniform heating effect of the left inclined surface triggers the hot fluid to move upward and to disparse into the core of the enclosure. On the otherhand, the cold fluid near the right slopping wall moves downwards through the cold inclined boundary layer. In the corresponding streamlines, a large convecting cell occupies the whole enclosure with clockwise circulation. We see in the isotherm that in the presence of heat generation and for increasing value of the heat generation parameter, λ, (See Fig. 2) the region of clustered isotherms moves to the right inclined cold surface of the enclosure. The gradient of temperature also appears to increase in the core section of the enclosure. As λ increases, the flow becomes stronger and a secondary vortex appears near the left inclined wall. This secondary vortex becomes larger as the heat generation parameter increases. It is expected that the secondary vortex would dominant the flow if the heat generation parameter cinreases further.

In Figs. 3(a)-(c) we plot the total heat transfer along three surfaces of the enclosure for values of the permeability, $10^{-5} \leq K \leq -10^{-2}$, for $Pr = 0.7$ and $Ra = 10^5$. It can be seen from these figures that, as the value of the permeability increases, the heat transfer from the bottom surface increases. For the linearly varying temperatureon left surface, the heat flux is lower near the bottom and higher near the mid portion of the surface from where hot fluids discharge into the core of the enclosure. However, the heat transfer near the top end is the lowest where two surfaces meets with the same temperature. Heat transfer through the right surface shows that the increase of the permeability accelerates the heat transfer. However, there

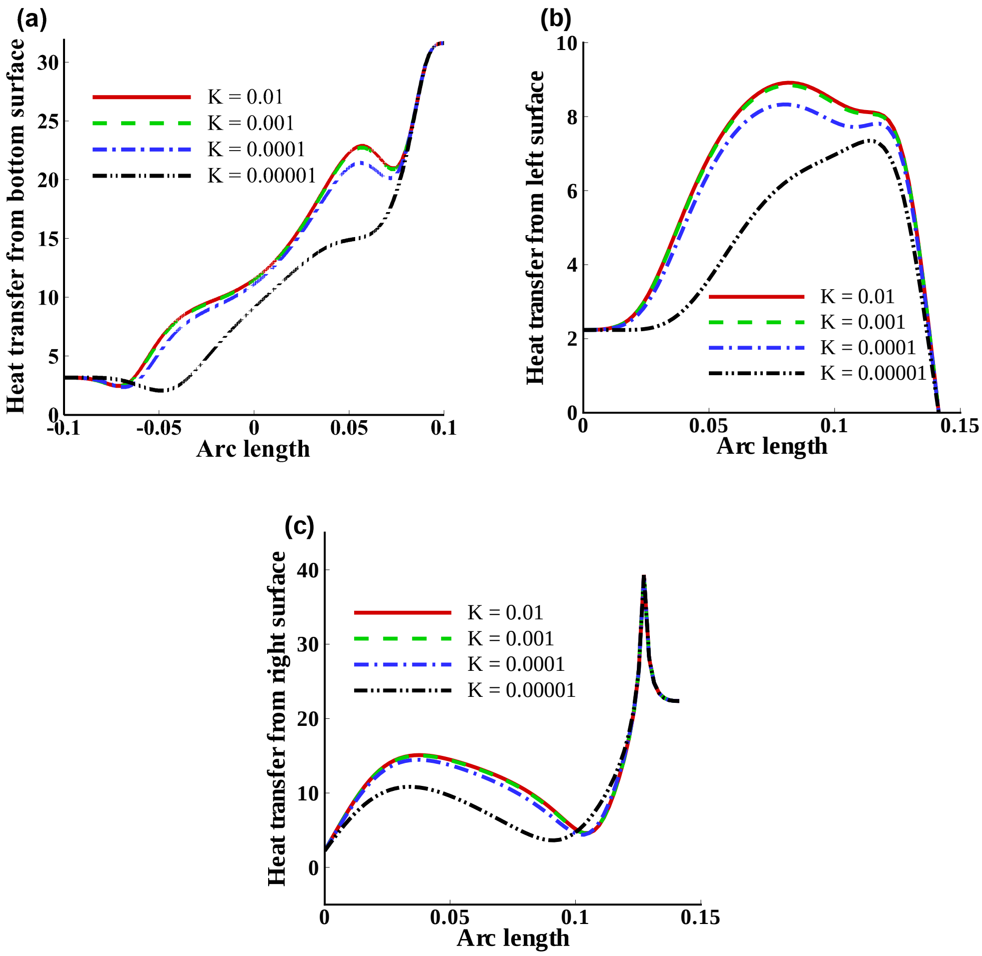

Figure 3: Rate of heat transfer from the surfaces for different values of K while $Ra = 10^5, Pr = 0.7, \lambda = 0.0$ (a) bottom surface, (b) left surface and (c) right surface.

is almost no effect of the permeability on heat transfer through the portion where the temperature is linearly distributed.

We have shown the time series of average heat transfer through three surfaces of the enclosure for different values of heat generation parameter, λ in Fig. 4. Fig. 4(a) shows the average heat transfer through the uniformly heated bottom surface. Initially, the heat transfer rate is very high for all values of λ due to conduction. As the flow grows with time, there exists undershoots when the convection starts to dominate the heat transfer. We also see that as heat generation parameter increases, the heat transfer through this surface decreases which is expected as discussed before. Time evolution of average heat transfer on the left inclined surface, which is linearly heated, is shown in Fig. 4(b). A large variation of heat transfer is seen for increase values of λ when the convection starts to dominant in the duration of $0.02 < \tau < 0.04$. After that the heat transfer reduced slightly and becomes steady state. Fig. 4(c) calculates the average heat transfer on cold right inclined surface. There is a large variation of heat transfer at the time of steady state for increasing values of heat generation parameter.

5 Conclusions

The effects of internal heat generation and porosity of the medium on the natural convection laminar flow and heat transfer in a triangular enclosure with one of the inclined surfaces is non-isothermal. We have used finite volume solution technique and apply the Boussinesq approximation to treat the buoyancy effects. The studies have been carried out for a fluid having Prandtl number 0.7 and for a Rayleigh number of 10^5. Increasing heat generation in the fluid reduces the thermal gradients near the heated bottom surface of the enclosure. The strength of the dominant vortex induced by bouyancy is reduced due to increasing internal heat generation, and a more, nearly equal double vortex structure develops. Increasing the permeability of the medium for a fixed heat generation parameter value also increases heat transfer at the surfaces.

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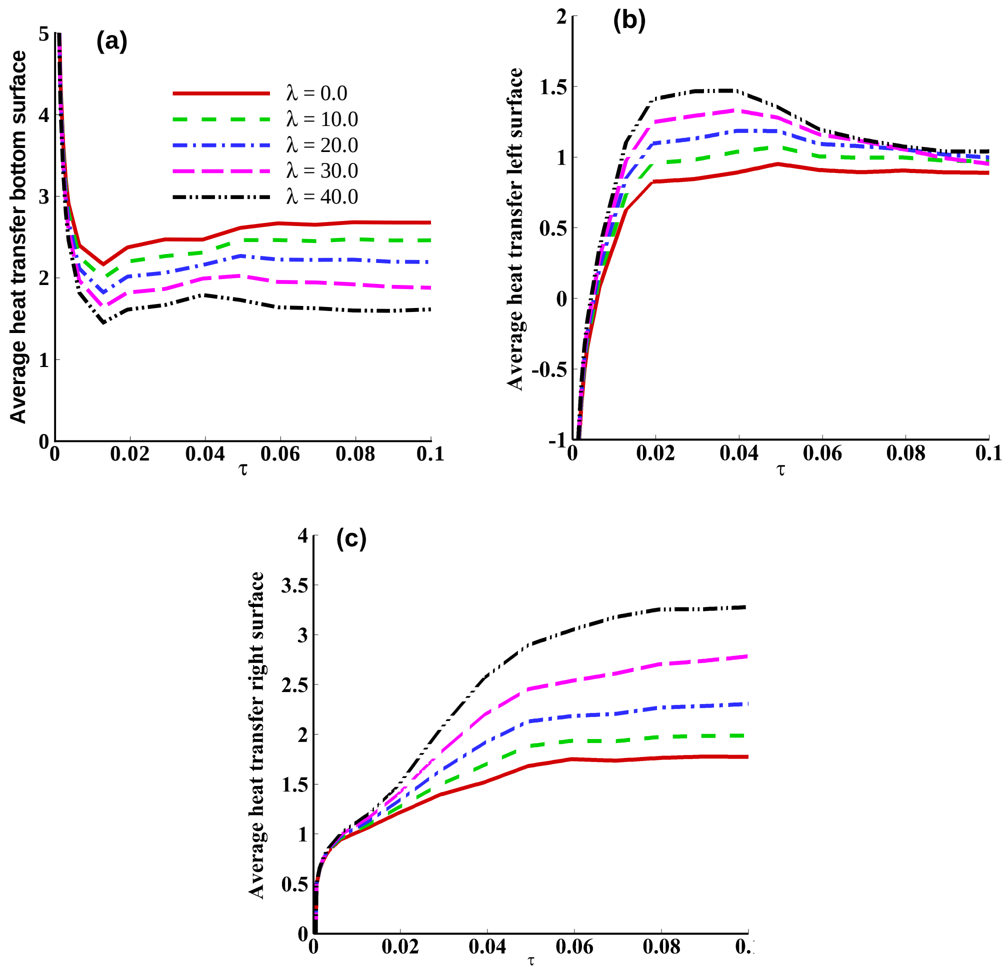

Figure 4: Time series of average heat transfer from the surfaces for different values of λ while $Ra = 10^5$, $\Pr = 0.7$, $K = 0.0$ (a) bottom surface, (b) left surface and (c) right surface.

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