Reducing Actuator Switchings for Motion Control of Autonomous Underwater Vehicles

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***Abstract*—A priority when designing control strategies for autonomous underwater vehicles is to emphasize their cost of implementation on a real vehicle. Indeed, due to the vehicles' design and the actuation modes usually under consideration for underwater plateforms the number of actuator switchings must be kept to a small value to insure feasibility and precision. This is the main objective of the algorithm presented in this paper. The theory is illustrated on two examples, one is a fully actuated underwater vehicle capable of motion in six-degrees-of freedom and one is minimally actuated with control motions in the vertical plane only.**

I. Introduction

Investigating the control of mechanical systems becomes increasingly interesting when one considers systems that have unique methods of actuation and locomotion with specific constraints, or are underactuated, subject to dynamic external forces. Underwater vehicles are subjected to dynamic and unpredictable external forces (i.e., ocean currents), and are propelled in many different and novel ways (e.g., biomimetic [1], energy harvesting [2] and variable buoyancy [3]). In many cases they are also underactuated (e.g., torpedo-shaped vehicles). For these reasons, control theory for Autonomous Underwater Vehicles (AUVs) is a heavily studied area with many open questions, practical applications and interesting problems.

With an ever increasing autonomy, AUVs are becoming a fundamental research tool for studying the ocean and conducting science underwater. These vehicles facilitate simultaneous and rapid measurements that capture the appropriate scale of spatiotemporal variability for many dynamic phenomena that we seek to understand in marine ecosystems. Intelligent implementations of AUVs can provide fine-scale data resolution far surpassing existing sampling methods, such as infrequent measurements from ships, or static measurements from buoys. Intelligent planning requires a path for the vehicle(s) to traverse, gathering samples in the right place at the right time for maximal information gain. Based on the limited endurance of most AUVs, computing optimal controls (with respect to time or energy conservation) along these paths increases their abilities to gather data of high scientific importance.

Theoretical algorithms very often produce control strategies that require a large number of instantaneous changes in some of the components of the control. These instantaneous changes are referred to as *switchings*. It is especially true, in optimal control for instance, that optimal trajectories often contain a singular arc, a large number of switchings or even chattering [4], [5]. In some systems, such as AUVs, this is undesirable or unimplementable due to physical constraints on the actuators, or other concerns. A common solution is to develop piecewise-constant (PWC) control to achieve the desired goal as well as possible. Based on practical constraints, these new strategies are designed to have a small number of switchings. In practice, these PWC controls are constructed through heuristics and extensive numerical simulations. The PWC controls can be designed in many different ways. We can, in optimal control, put the emphasize on the final configuration by fixing it, and construct a sub-optimal path (see [6], [7] for applications of this approach to time and energy minimization for AUVs).

A major difficulty is to provide a formal estimation that compares the newly computed PWC strategies to the non-feasible strategy that was initially calculated. We briefly (due to the lack of space) address this question here. More precisely, we develop the theory to explicitly calculate an error bound (in terms of difference in final state vectors) of a PWC control with a fixed (small) number of switchings with respect to a desired control, and we provide an algorithm to determine such a PWC control. We remark that an integral cost can be appended to the system as a state variable.

The paper is organized as follows. In Section II we introduce all necessary definitions and state the problem. Section III describes the algorithm and provides theoretical error estimates. Finally, Section IV addresses the motion planning problem of an AUV via PWC controls. Results are presented for two different types of AUV, followed by concluding remarks on the application of our proposed methods.

II. Problem statement

Consider a nonlinear, control-affine control system

$$\dot{x}(t) = f\left(x(t)\right) + g\left(x(t)\right) u(t) \doteq \bar{f}\left(x(t), u(t)\right), \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathcal{U} \subset \mathbb{R}^m$ is a measurable bounded function called the control and assume the functions $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ to be continuously differentiable. Let x_0 be an initial condition, and $u^*(\cdot)$ a reference control defined on $[0,T]$ such that its corresponding trajectory $x^*(\cdot)$ is the unique solution of (1) defined on $[0,T]$as well.

Our goal is to construct a PWC control $\tilde{u}(\cdot)$ defined on $[0,T]$ with a fixed number of switchings σ such that its associated trajectory $\tilde{x}(\cdot)$ minimizes the difference of the final states $\|x^*(T)-\tilde{x}(T)\|$. We also provide an upper bound for the state error at the final time, it depends on the functions $f(\cdot), g(\cdot)$, on the given control signal $u^*(\cdot)$ and on the switching times of the PWC control.

III. Algorithm for the PWC control with a fixed number of switchings

The analysis of PWC controls has received some attention in the literature but the state of art is still uncomplete. Some related work can be found in [8] in an optimal-control setting, in [9] for driftless control systems, and in [10], [11] for time-optimal syntheses of bang-bang controls. The major difficulty is to prove that the algorithms constructing PWC controls with a small number of switchings produce very efficient solutions (either in terms of the cost or of the final configuration). In this paper we take a novel approach by considering the one-sided Lipschitz constant. It should actually be no surprise since it is known that the classical Lipschitz constant estimates only exponential growth even if the errors decay [12]. The details regarding the theoretical estimation bounds will be provided in a forthcoming work, here we limit the discussion to a few results without proofs since the emphasize is on the applicability of our techniques to AUVs.

Let us first introduce some basic definitions.

Definition 1 (Logarithmic norm [13]): Given a square matrix A, its logarithmic norm is defined as

$$\mu(A) \doteq \lim_{h\to 0^+} \frac{\|I+hA\|-1}{h}. \tag{2}$$

where $\|.\|$ is a given norm.
In [12] the reader can find formula for the logarithmic norm. For instance, for the Euclidean norm we have that $\mu(A) = \lambda_{max} =$ largest eigenvalue of $\frac{1}{2}(A^T+A)$.

Theorem 1: Assume that $(x(.), u(.))$ and $(\tilde{x}(.), \tilde{u}(.))$ are both solutions to the affine control system (1), and that the following estimates hold

1) $\mu\left(\frac{\partial \bar{f}}{\partial x}(\xi, u(t))\right) < l(t) \quad \forall \xi \in [x(t), \tilde{x}(t)]$;
2) $\left\|\bar{f}(\tilde{x}(t), u(t)) - \bar{f}(\tilde{x}(t), \tilde{u})\right\| \leq \delta(t)$;
3) $\|x(0)-\tilde{x}(0)\| \leq \rho$.

Let $L(t) = \int_0^t l(\tau)d\tau$. Then the controlled trajectories satisfy

$$\|x(t)-\tilde{x}(t)\| \leq e^{L(t)}\left(\rho + \int_0^t e^{-L(\tau)}\delta(\tau)d\tau\right). \tag{3}$$

Proof: We here only sketch the prrof since it folllows very closely the proof of Theorem 10.6 in [12], where a similar result in the framework of ordinary differential equations is presented. First, we introduce $\epsilon(t) \doteq \|x(t)-\tilde{x}(t)\|$. It is easy to see using triangle inequality and assumption 2 in the theorem that for small $h \in \mathbb{R}^+$,

$$\epsilon(t+h) \leq \left\|x(t)-\tilde{x}(t) + h\left(\bar{f}(x(t),u(t)) - \bar{f}(\tilde{x}(t),u(t))\right)\right\| + h\delta(t) + \mathcal{O}(h^2). \tag{4}$$

Applying the Mean Value Theorem on the compact interval $[x(t), \tilde{x}(t)]$ to the function $F(x(t)) = x + h\bar{f}(x, u(t))$ we obtain

$$\epsilon(t+h) \leq \epsilon(t) \max_{\xi\in[x(t),\tilde{x}(t)]} \left\{\left\|I + h\frac{\partial \bar{f}}{\partial x}(\xi, u(t))\right\|\right\} + h\delta(t) + \mathcal{O}(h^2). \tag{5}$$

Then

$$\frac{\epsilon(t+h)-\epsilon(t)}{h} \leq \epsilon(t) \max_{\xi\in[x(t),\tilde{x}(t)]} \left\{\frac{\left\|I + h\frac{\partial \bar{f}}{\partial x}(\xi, u(t))\right\| - 1}{h}\right\} + \delta(t) + \mathcal{O}(h) \tag{6}$$

and by taking the limit for $h \to 0^+$,

$$D_+\epsilon(t) \leq \epsilon(t) \max_{\xi\in[x(t),\tilde{x}(t)]} \mu\left(\frac{\partial \bar{f}}{\partial x}(\xi, u(t))\right) + \delta(t), \tag{7}$$

where $\mu(\cdot)$ is defined as in (2).

Using the first inequality in the statement of the theorem, we obtain

$$D_+\epsilon(t) < \epsilon(t)l(t) + \delta(t). \tag{8}$$

According to the definition of $L(t)$, the solution of the differential equation $\gamma'(t) = \gamma(t)l(t) + \delta(t)$ is

$$\gamma(t) = e^{L(t)}\left(\gamma(0) + \int_0^t e^{-L(\tau)}\delta(\tau)d\tau\right). \tag{9}$$

From the third assumption of the theorem, there exists ρ such that $\epsilon(0) \leq \rho$. We define $\gamma(0) = \rho$. Since $\epsilon(t)$ and $\gamma(t)$ are continuous, to complete the proof we simply have to show that $\epsilon(t) \leq \gamma(t) \quad \forall t \in [0, \Delta]$.

By contradiction, let t_2 be a point with $\epsilon(t_2) > \gamma(t_2)$ and consider the first point t_1 to the left of t_2 with $\epsilon(t_1) = u(t_1)$. Then for small $h > 0$ we have

$$\frac{\epsilon(t_1+h)-\epsilon(t_1)}{h} > \frac{\gamma(t_1+h)-\gamma(t_1)}{h} \tag{10}$$

and, taking the limits, $D_+\epsilon(t_1) \geq D_+\gamma(t_1)$. This contradicts (8) as

$$D_+\epsilon(t_1) < l(t_1)\epsilon(t_1) + \delta(t_1) = l(t_1)\gamma(t_1) + \delta(t_1) = D_+\gamma(t_1). \tag{11}$$

■

A. An upper bound for the final error

Important is to provide an upper bound for the right-hand side of equality (3). This can be done as follows. Let σ be the number of fixed switchings for the new control and $t_0 < t_1 < ... < t_\sigma$ be the switching times (including the initial and final time). we define $\Delta_0 = t_1 - t_0$ the first time interval and we define u_0 as

$$u_0 = \frac{1}{2} \max_{t,s\in[0,\Delta_0]} (u^*(t) - u^*(s)) \tag{12}$$

where $u^*(.)$ is the reference trajectory. Using the fact that $\forall\tau \in [0, \Delta_0]$ there exists $\bar{l}_\tau$ such that

$$\int_\tau^{\Delta_0} l(t)dt \ = \ \bar{l}_\tau(\Delta_0 - \tau), \tag{13}$$

we have

$$e^{L(\Delta)} \int_0^\Delta e^{-L(\tau)} \|g(x(\tau))(u^*(\tau) - \tilde{u})\| \, d\tau \leq \|\delta u_0\| \, \bar{e}(\Delta_0, \bar{l}_0) \tag{14}$$

where $\bar{l}_0 = \max_{\tau\in 0,\Delta_0} \bar{l}_\tau$, $\|\delta u_0\| = \frac{k_g}{2} \max_{t,s\in[0,\Delta]} \|u^*(t) - u^*(s)\|$, k_g defined such that $\|g(x)\| \leq k_g \ \forall x$, and $\bar{e}(\Delta_0, \bar{l}_0) \doteq \frac{e_0^{\bar{l}_0\Delta}-1}{\bar{l}_0}$.

Theorem 1 applied with $x(0) = \tilde{x}(0)$ implies

$$\epsilon_1 \leq \|\delta u_0\| \, \bar{e}(\Delta_0, \bar{l}_0). \tag{15}$$

Iterating this construction, we obtain

$$\epsilon_\sigma \ \leq \ \sum_{k=0}^{\sigma-1} \left\{ \|\delta u_k\| \, \bar{e}(\Delta_k, \bar{l}_k) \prod_{i=k+1}^{\sigma-1} \{\bar{l}_i \Delta_i\} \right\}. \tag{16}$$

B. Practical Algorithm

In this section we provide a constructive algorithm to design a PWC control with a fixed number of switchings that minimizes the error on the final state of the system with respect to the desired one. The PWC controls with a fixed number of switchings are computed via general nonlinear programming with initial guess computed as follows[1]. Notice that, as also outlined in [14], an accurate initial guess might play a central role in computing approximating PWC controls via nonlinear programming. For instance for our two examples, the fully actuated is very robust in terms of dependence with respect to initial guess while the minimally actuated is unstable.

We describe a numerical way to determine the best choice of PWC control, that usually differs from the one used above for theoretical purposes. For a given time step Δ between two switchings, according to the result of Theorem 1, we have

$$\|x(\Delta) - \tilde{x}(\Delta)\| \ \leq \ e^{L(\Delta)}\rho + e^{L(\Delta)} \int_0^\Delta e^{-L(\tau)} \|g(\tilde{x}(\tau))(u(\tau) - \tilde{u})\| \, d\tau, \tag{17}$$

[1]The code of the algorithm is available at http://math.hawaii.edu/~marriott/pwc/

and therefore the best choice for the constant value $\tilde{u} \in \mathbb{R}^m$ would be

$$\tilde{u} = \arg\min_{\upsilon\in\mathcal{U}} \int_0^\Delta e^{-L(\tau)} \|g(\tilde{x}(\tau))(u(\tau) - \upsilon)\| \, d\tau. \tag{18}$$

The main difficulty is that $\tilde{x}(t)$ is not known a priori. To address this issue, a good initial guess can be obtained by assuming $g(\tilde{x}(t)) \simeq g(x(t))$ and solving the convex optimization problem:

$$\tilde{u} = \arg\min_{\upsilon\in\mathcal{U}} \sum_{k=0}^{N} e^{-L(t^k)} \left\|g(x^k)(u^k - \upsilon)\right\|^2 \tag{19}$$

where t^k are samples over the interval $[0, \Delta]$, divided in $N+1$ steps. Fixing the length of the time steps for the PWC control $\Delta_0, \Delta_1, ..., \Delta_{\sigma-1}$, problem (19) can then be solved for each time step Δ_i, thus computing the values $\tilde{u}_0, \tilde{u}_1, ..., \tilde{u}_{\sigma-1}$ of the initial-guess PWC control. Finally, after selection of an initial guess, general nonlinear minimization techniques are used to find the PWC control, i.e. a set of switching times and constant control values, which minimizes the final error $\|x(T) - \tilde{x}(T)\|$.

IV. Application to the Motion Planning for Underwater Vehicles

In this section, we examine the implementation of our methods on two types of underwater plateforms: a fully actuated underwater vehicle capable of motion in all six degrees-of-freedom (DOF) and a profiling float which is a minimally actuated vehicle.

A. Application on a Fully-Actuated AUV

We first apply our analysis to the non-linear, control-affine model for AUVs as seen in [15] or [16]. The reference control is taken as a time-optimal control to steer the fully actuated AUV from one configuration to another, with the motion starting and ending at rest (zero velocity). Time-optimal trajectories are formed by concatenations of bang arcs (corresponding to either maximal or minimal thrust) and singular arcs [6]. In [6], the authors develop a numerical algorithm to design control strategies to steer the AUVs to the exact configuration with a fixed number of switchings. It is shown that this method can produce control strategies with as few as 2-3 switchings and be within 10% of the optimal time. The approach taken here is different, as we fix the final time to the optimal time and minimize the error between the achievable final configuration and the desired configuration.

The simulations are based on the AUV analyzed in [6]. The test-bed vehicle is the Omni-Directional Intelligent Navigator (ODIN) [16], [17], developed by the Autonomous Systems Laboratory, College of Engineering at the University of Hawai‘i. The vehicle's main body is a 0.64 m diameter sphere, with eight thrusters evenly distributed around the sphere; four oriented vertically and four oriented horizontally. The numerical values of vehicle parameters, derived from estimations and experiments, see [6], are given in Table I. Based on these vehicle parameters, this fully actuated AUV

mass	123.8 kg	$B = \rho g \mathcal{V}$	1215.8 N
diameter	0.64 m	$W = mg$	1214.5 N
C_B	$(0, 0, -7)$ mm	C_G	$(0, 0, 0)$ mm
$M_f^{\nu_1}$	70 kg	$M_f^{\nu_2}$	70 kg
$M_f^{\nu_3}$	70 kg	I_{xx}	5.46 kg m^2
I_{yy}	5.29kg m^2	I_{zz}	5.72 kg m^2

TABLE I
MAIN DIMENSIONS AND HYDRODYNAMIC PARAMETERS FOR ODIN.

is capable of unbiased movement in six DOF, and has acted as an excellent platform for the development, implementation and comparison of a wide-rage of control strategies, e.g., [18]–[20].

For an underwater vehicle moving in six DOF, we consider the generalized position to be $\eta = (b^\top, \pi^\top)^\top = (b_1, b_2, b_3, \phi, \theta, \psi)^\top$, with respect to an inertial frame, and let $\nu = (v^\top, \omega^\top)^\top$ be the vehicle velocity in the body-fixed frame. Let $u = (u_1, ..., u_6)$ represent the controls. Then the equations of motions are

$$\begin{aligned} \dot{\eta} &= J(\eta)\nu \\ M\dot{\nu} &+ C(\nu)\nu + D(\nu)\nu + g(\eta) = u, \end{aligned} \tag{20}$$

where $J(\cdot)$ maps a vector expressed in the body-fixed frame to the same vector expressed in the inertial frame, M is the mass and inertia matrix, $C(\cdot)$ the Coriolis matrix, $D(\cdot)$ the damping matrix and $g(\cdot)$ is the gravity vector (see [15] for details).

The initial and final are taken respectively as the origin and $(\eta^\top, \nu^\top) = (5, 4, 1.3, 0_{1\times3}, 0_{1\times6})$. The time-optimal control signals have many discontinuities, see Fig. 1, and therefore are not implementable on physical actuators. Moreover there are singular arcs which require a continuous change in the actuation.

Applying our algorithm to this scenario, we can produce a PWC control with only two switchings, with an error with respect to the desired final state equals to (0.0414, 0.0474, 0.0171, 0.0112, 0.0104, 0.0273, -0.0137, -0.0132, -0.0010, -0.0289, -0.0226, -0.0085). This error corresponds to approximately 6 cm in position, 2 deg in orientation, 2 cm/s in translational velocity, and 2 deg/s in angular velocity.

Figures 2–4 show the simulation results. Note that the problem formulation is such that the control-vector components are allowed to jump all at the same time instants.

The precision of the algorithm is particularly relevant based on the fact that, unlike [6] and [7], the final time is fixed to be the optimal one, 17.39 s. The estimate (16) provides a reasonable theoretical bound even for two control switchings.

The gain here is significant. Indeed, the number of swicthings were reduced from a large number of switchings (together with about 10 singular arcs) for the time-optimal control, to only 2 switchings, and reaching with good precision the final state. The advantage in reducing the number of control switchings has many practical reasons, for instance: less stress on the control actuators, besides less onboard memory-storage requirements. From the computational-time point of view there is also a significant gain by considering a small number of switchings. Solving the PWC control problem for a nonlinear system with 12 states and 6 inputs (including 2 switching times) required 36.943 s on a dual-core, 2.66 GHz processor. A computational time of 349.754 s was required for 10 switching times.

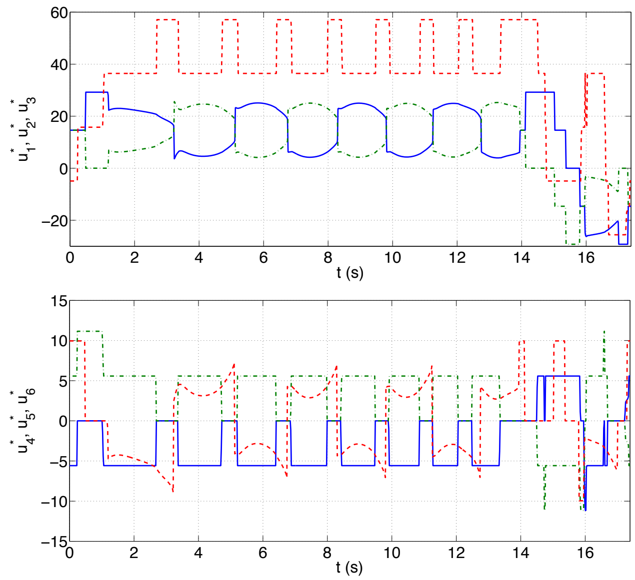

Fig. 1. Time-optimal control signals from [7]: u_1^* (solid blue), u_2^* (dashed red), u_3^* (dot-dashed green) in the top plot; u_4^* (solid blue), u_5^* (dashed red), u_6^* (dot-dashed green) in the bottom plot. In [7, Fig. 2], the corresponding signals for the eight thrusters are shown, where they are computed using a static control allocation.

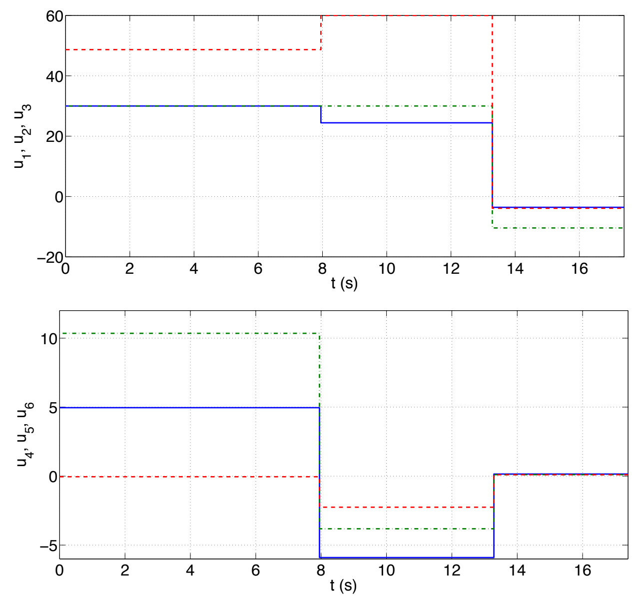

Fig. 2. PWC control signals: u_1 (solid blue), u_2 (dashed red), u_3 (dot-dashed green) in the top plot; u_4 (solid blue), u_5 (dashed red), u_6 (dot-dashed green) in the bottom plot.

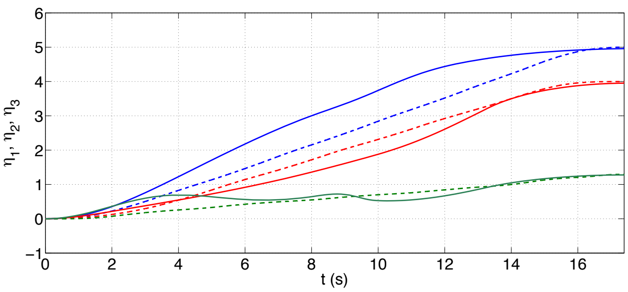

Fig. 3. State trajectories for the position vector: η_1 (blue), η_2 (red), η_3 (green). The solid line represents the trajectory obtained via the PWC control synthesis while the dashed line is the time-optimal trajectory.

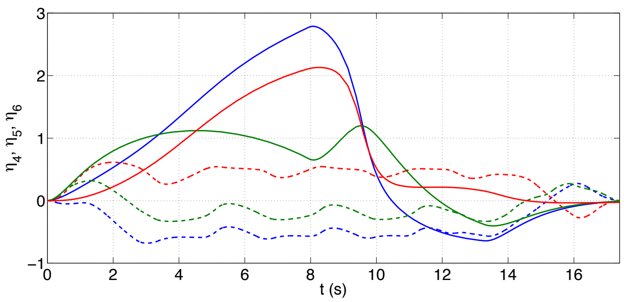

Fig. 4. State trajectories for the orientation vector: η_4 (blue), η_5 (red), η_6 (green). The solid line represents the trajectory obtained via the PWC control synthesis while the dashed line is the time-optimal trajectory.

B. Application on a Minimally-Actuated AUV

Here, we consider applying our method to the control of profiling float. This vehicle is free-drifting, and can only actively control its depth. By intelligently altering its depth, the vehicle uses ocean currents to carry it to a desired destination. Apart from the hotel load, changes in depth (buoyancy) are the only source of energy consumption. Extending the endurance of this vehicle directly corresponds to minimizing the number of depth changes (switchings).

1) System Model: For the analysis here, we assume a simple model of a profiling float. The vehicle moves in $\mathbb{R}^3$, and is invariant in roll (ϕ), pitch (θ), and yaw (ψ). The vertical position of the float ($b_3(t)$) can be controlled, and the horizontal velocities ($\dot{b_1}, \dot{b_2}$) are determined strictly by ocean currents. We assume that the float can be represented dynamically as a point mass. We can express the equations of motion in $\mathbb{R}^3$ by expressed by the equations

$$\dot{b_1} = f(b_3, t), \quad \dot{b_2} = g(b_3, t), \quad \dot{b_3} = v_3(t) + h(b_3, t), \quad (21)$$

where $f(b_3, t), g(b_3, t)$ and $h(b_3, t)$ represent the horizontal (x, y) and vertical components of the ocean velocity, respectively. Note that this system is not control affine as in the previous example, however the system does fit into the flexible framework of the proposed algorithm.

Here, we will ignore vertical currents, i.e., $h(b_3, t) = 0$, and thus $\dot{b_3} = v_3(t)$. As ocean currents are not constant functions with respect to depth, we can indirectly control the horizontal motion of the float by intelligently controlling the depth of the vehicle. Hence, rather than considering the ocean currents as drift to the system, we write them as a dependent input control, see [21] for a detailed treatment of this concept. Thus, for a given location, choosing a depth defines the direction of motion. Let $H(b_3, t) = (f(b_3, t), g(b_3, t), 0)^\top$, where $f, g : \mathbb{R}^+ \mapsto \mathbb{R}$ represent the north-south and east-west currents, respectively. Here, velocity is positive eastward and northward. Since we are interested in steering the float from one position on the sea surface to another position on the sea surface, the motion planning can actually be projected into $\mathbb{R}^2$, with the depth (b_3) acting as the input control that defines the velocities in the plane. Thus, we express the system as

$$(\dot{b}_1, \dot{b}_2, b_3)^\top = (0, 0, 1)^\top u_3(t) + H(b_3, t). \quad (22)$$

The only direct control input is the depth of the vehicle, i.e., $b_3(t) = u_3(t)$, and $u \in [0, 2000]$ meters. Since $H(b_3)$ is a function of depth and time, we obtain indirect control on the vehicle velocity as we change the depth over time. For our analysis, $H(b_3)$ is obtained from an ocean model. Based on the time scale of the horizontal motion, we can assume that depth can be changed instantaneously. We remark that although the vehicle is moving in $\mathbb{R}^3$ the motion of interest only occurs in $\mathbb{R}^2$. Since the depth is a control, the state variables are simply (b_1, b_2), corresponding to longitude and latitude, respectively.

2) Algorithm Results: We compare the results of our algorithm to an existing control strategy for an ocean-science application, see [22]. In [22], an A* algorithm is used to determine a set of waypoints, with a lower-level algorithm determining the control necessary to navigate between these waypoints. We compare our result to the output of this lower-level algorithm. For a reference control, we take the initial point $(-118.4800, 33.4600)$ with a final point of $(-118.4600, 33.4800)$. The reference algorithm from [22] produces a control with a duration of 62 hours and 45 switchings, shown in Fig. 5. The final point reached is $(-118.4632, 33.4800)$, which lies within the destination tolerance (400 m) for the low-level actuator decision algorithm. A PWC control is found with only eight switchings. This computed PWC control reaches the point $(-118.4633, 33.4801)$, which is just more than 300 m from the desired destination. Hence, we achieve the goal location within the desired accuracy with only 18% of the switchings. This is a significant reduction in energy consumption. Note that the path executed with only eight switchings deviates more significantly from the desired, straight-line path connected the waypoints than the algorithm presented in [22], however this accuracy is application specific. In general, such deviations are acceptable given the significant energy savings, especially at the large spatiotemporal scales examined in ocean science.

In this application, we notice a particular sensitivity to the initial guess on the number of switchings. In many systems, an acceptable PWC control can be found without putting great effort into the initial guess. In the first system considered, “opposite” initial guesses for the control, one with zero motor actuation and the other with constant full motor actuation, have

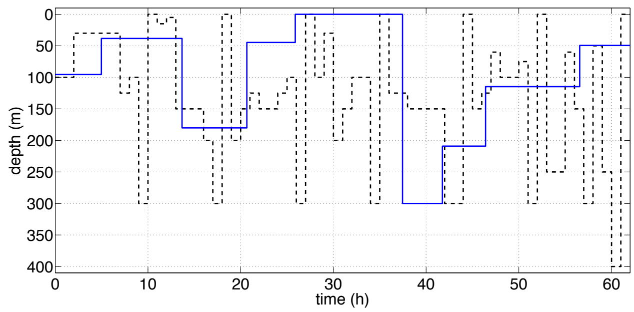

Fig. 5. Reference depth control (dashed black line) and the computed PWC depth control (solid blue line).

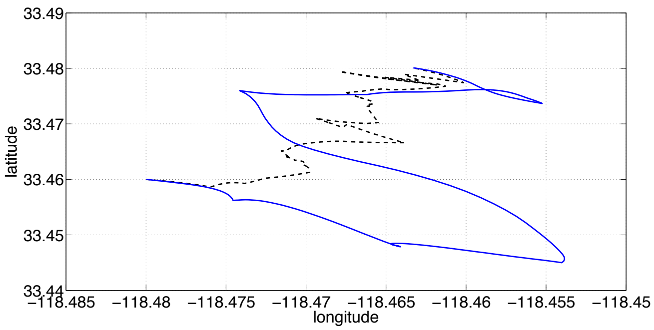

Fig. 6. Reference trajectory projected to the sea surface (dashed black line) and the computed PWC trajectory (dashed blue line) trajectory with eight switchings projected to the sea surface.

a difference of only 9.3% in final state error of the optimized PWC control.

In this example the importance of the initial guess is easily illustrated. For a PWC control with ten switchings, if the initial guess is constructed by sampling the given control at evenly spaced switching times, the algorithm arrives at a control with a final state error of 9.79 km. However we have shown that with just eight switchings we can produce an error of just 300 m.

V. Conclusion

Reducing the number of control switchings is considered important in many classes of nonlinear control systems, for many practical reasons, especially when there are physical limitations on the control actuators. This paper considers the use of PWC controls with a small number of switchings to obtain a state trajectory whose final state is as close as possible to the final point of a reference trajectory, and the trajectory time is the calculated optimal one.

For the case of underwater vehicles, it has been shown that very efficient trajectories can be designed by only using a few switchings in the control. Such heuristic observations motivate our study, and are validated here in a theoretical framework. We provide an effective algorithm which constructs such a control. Although we only considered applications of our algorithms to the motion control of underwater vehicles, this general approach has a wide range of applications. Further publications will give further treatment to the theoretical framework and a wider range of applications. Most important, future work will focus on discussion of the most suitable norm to be used in the error estimate arguments.

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