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Radiation effect on free convection laminar flow along a vertical flat plate with streamwise sinusoidal surface temperature

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Abstract: The effect of thermal radiation on a steady two-dimensional natural convection laminar flow of viscous incompressible optically thick fluid along a vertical flat plate with streamwise sinusoidal surface temperature has been investigated in this study. Using the appropriate variables; the basic governing equations are transformed to convenient form and then solved numerically employing two efficient methods, namely, Implicit finite difference method (IFD) together with Keller box scheme and Straight forward finite difference (SFFD) method. Effects of the variation of the physical parameters, for example, conduction-radiation parameter (Planck number), surface temperature parameter, and the amplitude of the surface temperature, are shown on the skin friction and heat transfer rate quantitatively are shown numerically. Velocity and temperature profiles as well as streamlines and isotherms are also presented and discussed for the variation of conduction-radiation parameter. It is found that both skin-friction and rate of heat transfer are enhanced considerably by increasing the values of conduction radiation parameter, R_d.

Keywords: Natural convection, radiation, sinusoidal surface temperature, vertical flat plate.

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Nomenclature

C_p	Specific heat at constant pressure
C_f	Skin-friction coefficient
f	Dimensionless stream function
g	Acceleration due to gravity
Gr	Grashof number
Gr_x	Local Grashof number
k	Thermal conductivity
l	Half wavelength of the variations
Nu	Local Nusselt number
Pr	Prandtl number
q_r	Radiation heat flux
q_c	Conduction heat flux
R_d	Radiation-conduction parameter or Plank number
T	Temperature of the fluid in the boundary layer
T_∞	Temperature of the ambient fluid
T_w	Mean temperature at the surface
U_∞	Free stream velocity
u,v	Dimensionless fluid velocities in the x, y directions
$\hat{u},\hat{v}$	Dimensional fluid velocities in the $\hat{x}$, $\hat{y}$ directions
U, V	Non-dimensional velocity components
x,y	Dimensionless axes in the direction along and normal to the surface
$\hat{x}$, $\hat{y}$	Dimensional axes in the direction along and normal to the surface
X, Y	Non-dimensional axes in the direction along and normal to the surface

Greek Symbols

α	Relative amplitude of the surface temperature variations
α_r	Rosseland mean absorption coefficient
β	Volumetric coefficient of thermal expansion
ΔT	Temperature difference
σ	Stefan-Boltzmann constant
σ_s	Scattering coefficient

ψ	Stream function
τ_w	Wall shear stress
ρ	Fluid density
μ	Dynamic viscosity of the fluid
ν	Kinematic viscosity of the fluid
θ	Dimensionless temperature function
θ_w	Surface heating parameter
η	Similarity variable

1. INTRODUCTION

It is known that power law surface temperature distributions give rise to self similar boundary layer flows [1-2]. However, Rees [3] and Roy and Hossain [4] proposed another form of surface temperature variation, namely, sinusoidal variations about a mean temperature which is held above the ambient temperature of the fluid. This type of surface temperature may be considered to define the periodic array of heaters behind or within the wall. A large number of literature exists which deals with the effects of surface variations. For example, the streamwise surface waviness of free and mixed convection were investigated by Yao [5] and Moulic and Yao [6-7] respectively. Afterwards Cheng and wang [8], Hossain et al. [9] and Kim [10] extended these analysis to micropolar fluids, magnetohydrodynamics and non-Newtonian convection, respectively. However, the radiation effect on free convection laminar flow along a vertical flat plate with streamwise sinusoidal surface temperature is still unrevealed which motivates the present study.

It is noteworthy that the thermal radiation effect on the free convection flow are important in many engineering applications, such as in advanced types of power plants for nuclear rockets, high-speed flights, re-entry vehicles and processes involving high temperatures. However, a little knowledge is unfolded from a vast effect of radiation on the boundary layer flow of radiating fluid past a body (see Ozisik [11], Ch-13). It is also noted that the radiative heat fluxes can be approximated by the Rosseland diffusion approximation [12] for an optically dense medium, which has been greatly used in many radiation related studies [13-22].

The interaction of thermal radiation with free convection heat transfer along a vertical flat plate was considered by Cess [13] and Cheng and Ozisic [15]. The authors considered the absorbing, emitting and non-scattering gas for their investigation. The singular perturbation technique was used to solve the set of non-linear partial differential equations. On the other hand, an analytical attempt was made by Arpaci [14] to understand the non-equilibrium interaction between the thermal radiation and the laminar natural convection from a heated vertical plate immersed in a radiating gas. Shwartz [16] studied the behavior of an emitting and absorbing gas including the entire range of optical thickness, from thin to thick.

The effect of radiation using the Rosseland diffusion approximation was analyzed by Hossain et al. [17-18]. The authors obtained the non-similarity solution for the forced and free convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. They incorporated a group of transformations which led the boundary layer equations to the local non-similarity equations validating both in the forced and free convection regimes.

The study of the thermal radiation effect in natural convection in the cylinder also draws many researchers attention. For example, Novotny and Kelleher [19] investigated the laminar free convection of an absorbing-emitting gas in the region of the stagnation point of a horizontal cylinder. Hossain et al. [20-22] investigated the radiation effect on free convection along an isothermal vertical cylinder, elliptic cylinder and mixed convection from a horizontal circular cylinder. The effect of radiation on natural convection on the wavy surface [23-24] is also available in the literature. Very recently sinusoidal temperature variation with time has been studied on the inclined walls of the attic space by Saha et al. [25] and on the vertical plate by Roy and Hossain [26].

In the present study, it is proposed to investigate the natural convection flow of an optically thick viscous incompressible fluid along a non-isothermal vertical flat plate considering the Rosseland diffusion approximation. The basic equations of motion are transformed to convenient form, and then solved numerically employing two efficient methods, namely (i) Implicit finite difference method or Keller box scheme [27] and (ii)

Straight forward finite difference method. Consideration has been given to the situation where the buoyancy forces assist the natural convection flow for various combinations of the radiation-conduction parameter R_d, surface heating parameter, θ_w and the Prandtl number, Pr. The numerical results allow us to predict the different behaviour that can be observed when the relevant parameters are varied.

2. FORMULATION OF PROBLEM

A steady two-dimensional laminar free convective flow from non-isothermal semi-infinite vertical flat plate, which is immersed in a viscous and incompressible optically thick fluid, is considered. It is assumed that the heated surface temperature of the plate is maintained at the steady temperature,

$$T = T_\infty + (T_w - T_\infty)(1 + \alpha \sin(\pi \hat{x} / l)) \quad (1)$$

where T_∞ is the ambient temperature, T_w is the mean surface temperature with $T_w > T_\infty$, α is the relative amplitude of the surface temperature variations and $2l$ is the wavelength of the variations. The physical configuration and the coordinate system considered here are shown in Fig. 1.

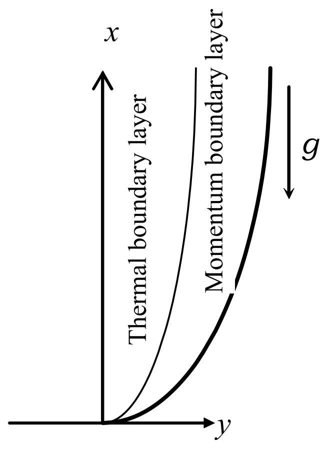


Fig. 1: Physical model and coordinate system.

Under the usual Bousinesq approximation, the equations governing the flow are

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \quad (2)$$

$$\rho\left(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}\right)=\mu\frac{\partial^2 \hat{u}}{\partial \hat{y}^2}+\rho g\beta(T-T_\infty), \tag{3}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}}+\hat{v}\frac{\partial T}{\partial \hat{y}}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \hat{y}^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial \hat{y}}, \tag{4}$$

where $(\hat{u},\hat{v})$ are the velocity components along the $(\hat{x},\hat{y})$ axes, g is the acceleration due to gravity, ρ is the fluid density, k is the thermal conductivity of the fluid, β is the coefficient of thermal expansion, μ is the viscosity of the fluid, C_p is the specific heat at constant pressure, and q_r on the right hand side of equation (4) represents the radiative heat flux in the $\hat{y}$ direction.

The appropriate boundary conditions to solve equations (2)-(4) are

$$\hat{u}=\hat{v}=0,\quad T=T_\infty+\Delta T\left[1+\alpha\sin(\pi\hat{x}/l)\right] \text{ at } \hat{y}=0, \tag{5a}$$

$$\hat{u}\to 0,\quad T\to T_\infty \quad \text{as} \quad \hat{y}\to\infty. \tag{5b}$$

This radiation heat flux, q_r, is simplified by the Rosseland diffusion approximation as

$$q_r=-\frac{4\sigma}{3k(\alpha_r+\sigma_s)}\frac{\partial T^4}{\partial \hat{y}} \tag{6}$$

where σ is the Stefan-Boltzmann constant, α_r is the Rosseland mean absorption coefficient and σ_s is the scattering coefficient. The limitation to the use of the Rosseland diffusion approximation should be recognized. It is valid in the interior of a medium but not near the boundaries. This method is appropriate only for intensive absorption, that is, for an optically thick boundary layer. The approximation cannot provide a complete description of the physical situation near the boundaries since it does not include any terms for radiation from the boundary surface. However, the boundary surface effects are negligible in the interior of an optically thick region since the radiation from the boundaries is attenuated before reaching the interior.

To obtain the non-dimensional governing equation let us introduce the following non-dimensional variables

$$x=\frac{\hat{x}}{l},\ y=Gr^{1/4}\left(\frac{\hat{y}}{l}\right),\quad u=\frac{l}{\nu}Gr^{-1/2}\hat{u},\quad v=\frac{l}{\nu}Gr^{-1/4}\hat{v},$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\qquad Gr=\frac{g\beta(T_w-T_\infty)l^3}{\nu^2}, \tag{7}$$

where ν (=μ/ρ) is the reference kinematic viscosity and Gr is the Grashof number, θ is the non-dimensional temperature function .

Substituting the variables (7) into equations (2)-(5) lead to the following non-dimensional equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,, \tag{8}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta\,, \tag{9}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\frac{\partial}{\partial y}\left[\left\{1+\frac{4}{3}R_d\left(1+(\theta_w-1)\theta\right)^3\right\}\frac{\partial \theta}{\partial y}\right], \tag{10}$$

and the corresponding boundary conditions are

$$u=v=0,\quad \theta=1+\alpha\sin(\pi x)\quad \text{at}\quad y=0\,, \tag{11a}$$

$$u\to 0,\quad \theta\to 0\quad \text{as}\quad y\to\infty\,, \tag{11b}$$

where R_d is the radiation-conduction parameter or Plank number, θ_w is the ratio of the mean surface temperature and the ambient, called the surface heating parameter and Pr is the Prandtl number, which are defined respectively as

$$R_d=\frac{4\sigma T_\infty^3}{k(\alpha_r+\sigma_s)},\ \theta_w=\frac{T_w}{T_\infty}\quad \text{and}\qquad Pr=\frac{\mu C_p}{k} \tag{12}$$

3. Numerical Methods

The governing equations (8)-(10) together with the boundary conditions (11a,b) have been solved numerically in this present investigation. Two numerical schemes namely, implicit finite difference method and straight forward finite difference method have been employed to validate the numerical code by comparing the numerical results. Note that in regard to solution procedure the stream-function formulation method used in [4] and straight forward finite difference method used here are different. In the stream function formulation the stream function, ψ is solved from the governing equations and the velocity components are calculated from the ψ. However, by using the straight forward method the velocity components can be solved directly. The brief description of the above two methods are given below.

3.1 Implicit Finite Difference Method

Before we employ the implicit finite difference method, we need to reduce the aforementioned equations to a convenient set of equations. To do that, we first, introduce the following transformations over the governing equation:

$$\psi = x^{3/4} f(x,\eta), \quad \eta = x^{-1/4} y, \quad \theta = \theta(x,\eta) \tag{13}$$

where ψ is the non-dimensional stream function defined in the usual way as

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \psi}{\partial x}. \tag{14}$$

Substituting (13-14) into equations (8)-(11) and after some algebraic manipulations, the transformed equations take the following form

$$f''' + \frac{3}{4} ff'' - \frac{1}{2} f'^2 + \theta = x\left(f' \frac{\partial f'}{\partial x} - f'' \frac{\partial f}{\partial x} \right), \tag{15}$$

$$\frac{1}{Pr}\left[\left\{ 1 + \frac{4}{3} R_d \left(1 + (\theta_w - 1)\theta\right)^3 \right\} \theta' \right]' + \frac{3}{4} f\theta' = x\left(f' \frac{\partial \theta}{\partial x} - \theta' \frac{\partial f}{\partial x} \right), \tag{16}$$

along with the boundary conditions

$$f(x,0) = f'(x,0) = 0,\ \theta(x,0) = 1 + \alpha \sin(\pi x), \tag{17a}$$

$$f'(x,\infty) \to 0,\ \ \theta(x,\infty) \to 0. \tag{17b}$$

It can be seen that near the leading edge i.e. when $x \approx 0$, equations (15) and (16) reduce to the following ordinary differential equations

$$f''' + \frac{3}{4} ff'' - \frac{1}{2} f'^2 + \theta = 0, \tag{18}$$

$$\frac{1}{Pr}\left[\left\{ 1 + \frac{4}{3} R_d \left(1 + (\theta_w - 1)\theta\right)^3 \right\} \theta' \right]' + \frac{3}{4} f\theta' = 0, \tag{19}$$

subject to the boundary conditions,

$$f(0) = f'(0) = 0, \quad \theta(0) = 1, \tag{20a}$$

$$f' \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty. \tag{20b}$$

The physical quantities of principle interest are the shearing stress and the rate of heat transfer in terms of the skin-friction coefficient C_f and the Nusselt number Nu respectively, which can be written as

$$C_f = \frac{2}{\rho U_\infty^2}(\tau_w)_{\hat{y}=0} \quad \text{and} \quad Nu = \frac{\hat{x}}{k(T_w - T_\infty)}(q_c + q_r)_{\hat{y}=0} \tag{21}$$

$$\text{where} \quad U_\infty = \nu Gr^{1/2} u, \qquad \tau_w = \mu\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right) \text{and} \quad q_c = -k\left(\frac{\partial T}{\partial \hat{y}}\right). \tag{22}$$

Using the variables (6), (7), (14) and the boundary condition (17a) into equations (21)-(22), we get

$$C_f Gr_x^{1/4} / 2 = f''(x,0), \tag{23}$$

$$NuGr_x^{-1/4} = -\left(1 + \frac{4}{3} R_d \theta_w^3\right)\theta'(x,0). \tag{24}$$

Now we employ one of the implicit finite difference method (the Keller box method) in solving the nonlinear system of partial differential equations (15)-(16) that govern the flow. To employ this method, the set of equations (15)-(16) is written in terms of a system of first order equations in y, which are then expressed in finite difference form by approximating the functions and the diffusion terms by the central difference and the convective terms by the backward difference. Denoting the mesh points in the (x, η) plane by x_i and η_j, where i = 1, 2, 3, …, M and j = 1, 2, 3, …, N, central difference approximations are made such that the equations involving x explicitly are centred at ($x_{i-1/2}$, $\eta_{j-1/2}$) and the remainder at (x_i, $\eta_{j-1/2}$), where $\eta_{j-1/2} = (\eta_j + \eta_{j-1})/2$, etc. This results in a set of nonlinear difference equations for the unknowns at x_i in terms of their values at x_{i-1}. These equations are then linearised by the Newton's quasi-linearization technique and are solved using a block-tridiagonal algorithm, taking as the initial iteration of the converged solution at $x = x_{i-1}$. Now to initiate the process at $x = 0$, we first provide guess profiles for all five variables (arising the reduction to the first order form) and use the Keller box method to solve the governing ordinary differential equations. Having obtained the lower stagnation point solution it is possible to march step by step along the boundary layer. For a given value of x, the iterative procedure is stopped when the difference in computing the velocity and the temperature in the next iteration is less than

10^{-5}, i.e. when $|\delta f^{i}| \le 10^{-5}$, where the superscript denotes the iteration number. The computations are performed using a non-uniform grid in the y direction which is by η_j = sinh ((j-1)/p), with j = 1, 2, …, 301 and p = 100.

3.2 Straight Forward Finite Difference Method

In order to incorporate the SFFD we introduce the following new transformations as

$$X = x, \quad Y = \frac{y}{x^{1/4}}, U = \frac{u}{x^{1/2}}, \quad V = x^{1/4}v, \quad \theta(X,Y) = \theta(x,y) \tag{25}$$

Using (25) into (8-10), we get

$$X\frac{\partial U}{\partial X} + \frac{1}{2}U - \frac{1}{4}Y\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} = 0, \tag{26}$$

$$XU\frac{\partial U}{\partial X} + \left(V - \frac{1}{4}UY\right)\frac{\partial U}{\partial Y} + \frac{1}{2}U^2 = \frac{\partial^2 U}{\partial Y^2} + \theta, \tag{27}$$

$$XU\frac{\partial \theta}{\partial X} + \left(V - \frac{1}{4}UY\right)\frac{\partial \theta}{\partial Y} = \frac{1}{Pr}\frac{\partial}{\partial Y}\left[\left\{1 + \frac{4}{3}R_d\left(1 + (\theta_w - 1)\theta\right)^3\right\}\frac{\partial \theta}{\partial Y}\right] \tag{28}$$

The corresponding boundary conditions are

$$U = V = 0, \quad \theta = 1 + \alpha\sin(\pi X) \quad \text{at} \quad Y = 0 \tag{29a}$$

$$U \to 0, \quad \theta \to 0 \quad \text{as Y} \to \infty \tag{29b}$$

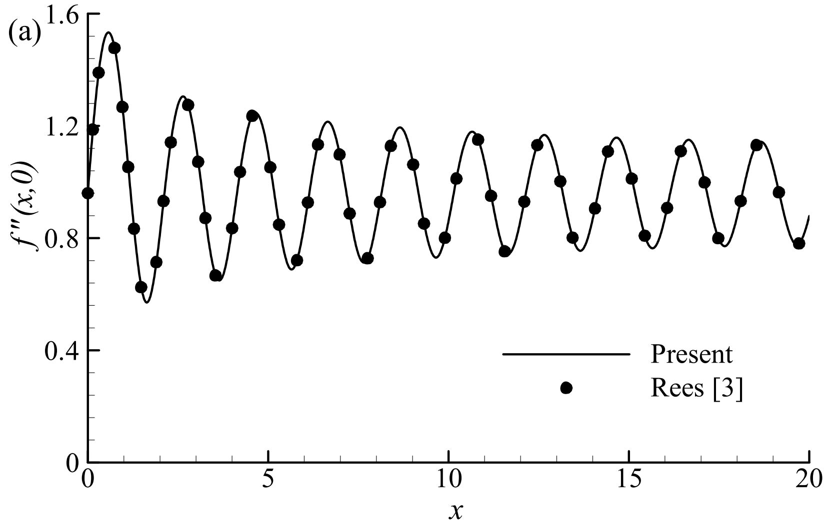
Now equations (26)-(28) subject to the boundary conditions (29) are discretized for straight forward finite difference (SFFD) method using central-difference for diffusion terms and the forward-difference for the convection terms ([24]). Finally we get a system of tri-diagonal algebraic equations which are solved by Gaussian elimination technique. The computation is started at X = 0.0 and then marches downstream implicitly. Here we have taken ΔX = 0.005 and ΔY = 0.01 for the X-and Y- grids respectively. The physical quantities, namely the rate of heat-transfer and the average rate of heat transfer which have great importance from the application point of view are defined as dimensionless form as follows,

$$C_f Gr_x^{1/4}/2 = \left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{30}$$

$$NuGr_x^{-1/4} = -\left(1+\frac{4}{3}R_d\theta_w^3\right)\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \tag{31}$$

4. RESULTS AND DISCUSSION

The numerical results of the natural convection adjacent to a semi-infinite vertical flat plate with sinusoidal streamwise surface heating have been presented in this study. The main objective of this study is to show the effect of radiative heat flux represented by Rosseland approximation on the flow field and the heat transfer of the boundary layer adjacent to the vertical flat plate. It is also shown the effect of the amplitude of the non uniform surface temperature on the flow field and the heat transfer. The results are discussed in terms of the skin-friction coefficient and the rate of heat transfer for different values of the physical parameters controlling the flow, for example, conduction-radiation parameter, ($0 \le R_d \le 1$), surface temperature parameter, ($1.1 \le \theta_w \le 1.9$) and the amplitude of the surface temperature, ($0 \le \alpha. \le 1$). Velocity and temperature profiles as well as streamlines and isotherms are also presented for the variation of conduction-radiation parameter.



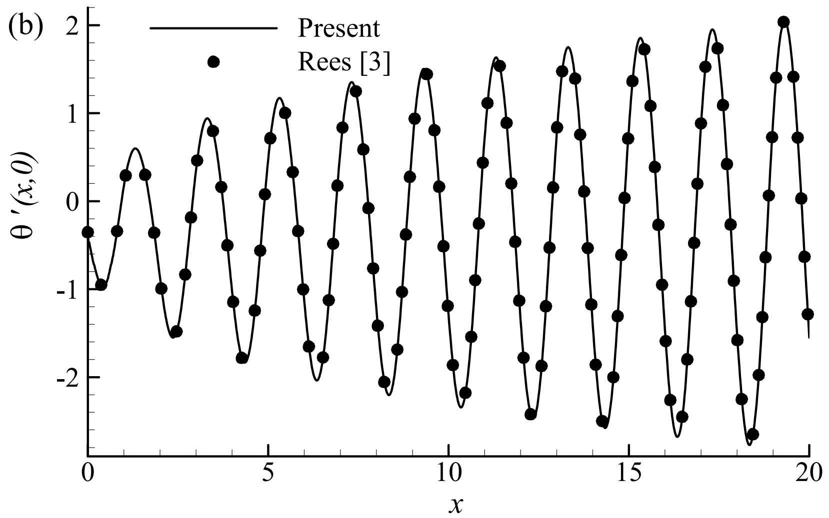


Fig. 2: Comparison with Rees [3], (a) $f''(x,0)$ and (b) $\theta'(x,0)$ for $R_d = 0.0$ while $Pr = 0.7$, $\alpha = 1.0$.

Since there is no experimental data available in the literature, the obtained results have been compared with two methods. Moreover, without radiation effect ($R_d = 0.0$) the solution can be compared with Rees [3] which is shown in Fig. 2(a-b). It is found that there is a good agreement between two results.

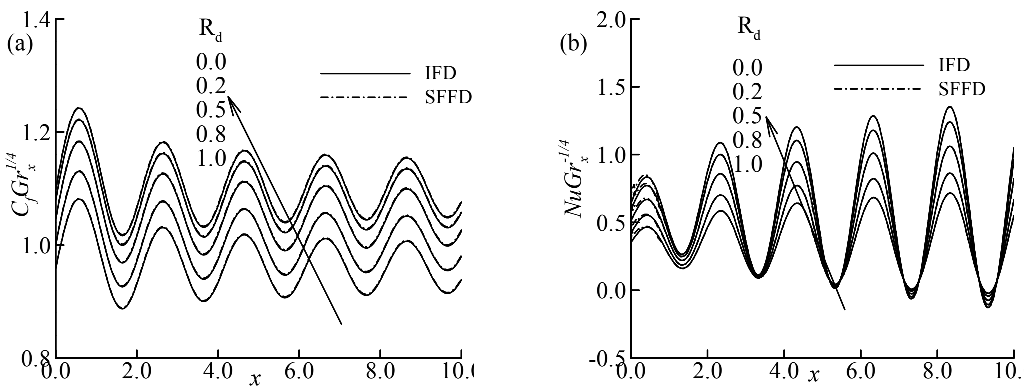


Fig. 3: (a) Skin-friction coefficient and (b) Nusselt number for different values of R_d while $Pr = 0.7$, $\alpha = 0.2$ and $\theta_w = 1.1$.

Numerical values of the skin friction, $C_fGr_x^{1/4}$, and the rate of heat transfer, $NuGr_x^{-3/4}$, are shown graphically, respectively, in Figs 3(a-b) against x for the radiation-conduction parameter, $R_d = 0.0$, 0.2, 0.5, 0.8 and 1.0 while keeping $Pr = 0.7$, $\alpha = 0.2$, and

θ_w = 1.1 fixed. Solutions are obtained using two different methods, implicit finite difference method (IFD) and the straight forward finite difference method (SFFD). Solid lines represent the results obtained by IFD and the dashed dot lines represent the results obtained by SFFD. It is anticipated that both lines merged together which conforms the efficiency of the numerical schemes. It is revealed from Fig. 3(a) and (b) that both skin-friction and rate of heat transfer are enhanced considerably, owing to increase in the values of conduction radiation parameter, R_d. This is happened because when the conduction radiation parameter increases, there is an increase in the rate of energy transport of the fluid within the boundary layer. Finally it upraises the temperature of the fluid layers in the vicinity of the plate surface. The waviness of the curves shows the effect of the streamwise temperature condition on the surface.

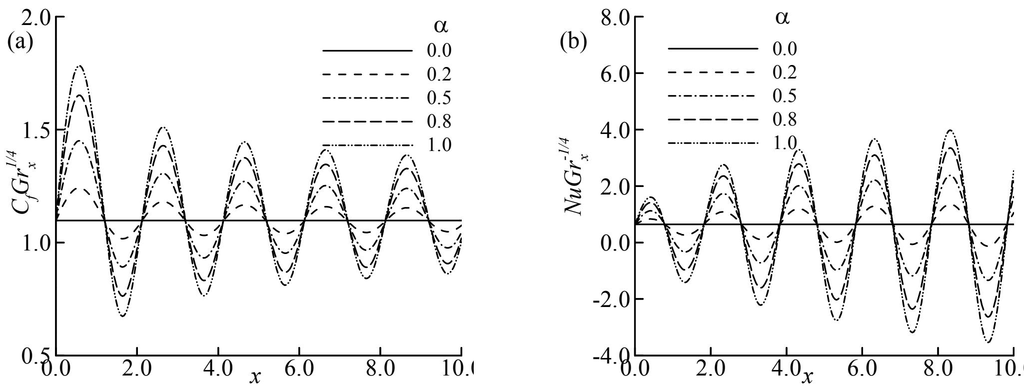


Fig. 4: (a) Skin-friction coefficient and (b) Nusselt number for different values of α while Pr = 0.7, R_d = 1.0 and θ_w = 1.1

Fig. 4 shows the variation of skin friction coefficient and rate of heat transfer against different values of the temperature wave amplitude on the plate for R_d = 1.0 and θ_w = 1.1. It is clear from the figures that both skin friction coefficient and the Nusselt number are uniform all over the plate if the amplitude is zero. However, with the increase of the amplitude of the heating effect both parameters also increase. The overall behaviour of these two figures may be discussed by observing the thermal and viscous boundary layers. As we know that when the temperature on the surface is relatively high, the fluids inside the boundary layer accelerate. On the other hand, if the surface temperature is low the fluids in the boundary layer have the minimum velocity. Therefore, it is expected that the shear stress and the rate of heat transfer will be higher at the position where the temperature is maximum. At the same time the ambient fluids receive the heat from the radiation and become heated. If the ambient temperature exceeds the mean temperature of the surface then the temperature difference will be maximum at the position where the surface temperature is minimum. As a result the overall heat transfer will be from the fluid into the surface which can be clearly seen in Fig. 4(b) where the Nusselt number is shown as negative.

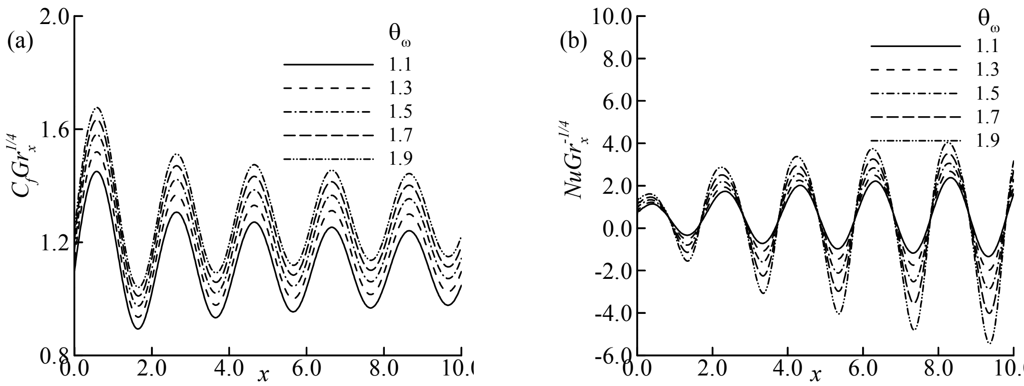


Fig. 5: (a) Skin-friction coefficient and (b) Nusselt number for different values of θ_w while $Pr = 0.7$, $\alpha = 0.2$ and $R_d = 1.0$

The coefficient of skin-friction and Nusselt number for different values of the ratio of the mean surface temperature and the ambient, θ_w while $Pr = 0.7$, $\alpha = 0.2$ and $R_d = 1.0$ are fixed have been shown in figure 4. It is observed that both skin friction and the rate of heat transfer increases with the increasing values of θ_w. It is also observed that the heat transfer rate is higher when the surface temperature is lower which can be clearly seen in figure 5(b).

Velocity and temperature profiles have been presented in figure 6 for different values of conduction radiation parameter, R_d for the fixed values of Pr - 0.7, $\alpha = 0.2$ and $\theta_w = 1.1$ at $x = 1.0$. Both viscous and thermal boundary layers increase with the increase of conduction radiation parameter, R_d. It is also observed that the maximum velocity also increases with increase of R_d. The maximum velocity for all values of R_d is at about $\eta = 1.8$. Therefore, it can be concluded that the inner viscous layer does not increase for variation of radiation parameter. Only the outer layer thickness has a great influence of R_d.

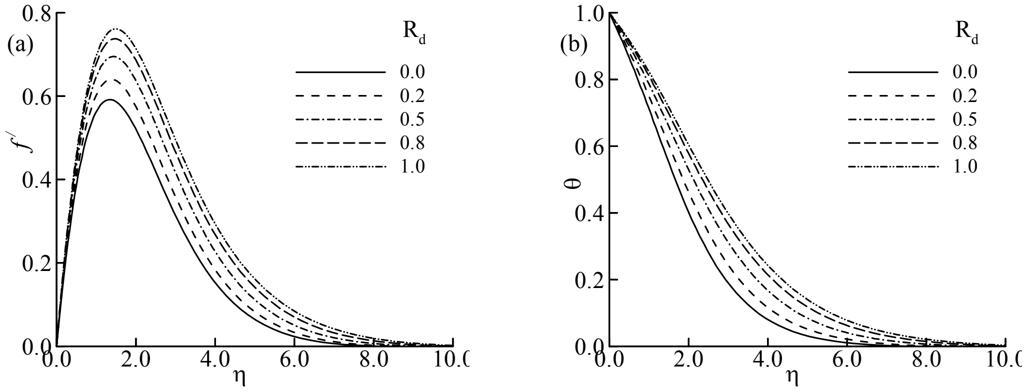


Fig. 6: (a) Velocity and (b) Temperature profiles for different values of R_d while Pr = 0.7, α = 0.2 and θ_w = 1.1 at x = 1.0

Figure 7 and 8 shows the streamlines and temperature contours for different conduction radiation parameter, R_d while α = 0.2, Pr = 0.7 and θ_w = 1.1. It is seen in figure 7 that as the radiation parameter increases the velocity inside the boundary layer increases as it is seen in the velocity profiles in figure 6. The streamlines concentrate near the wall for the higher values of radiation parameter. On the other hand the thermal boundary layer becomes thicker as R_d increases. The flow of the fluid appears to be oscillating near the surface of the plate due to streamwise variations of the surface temperature.

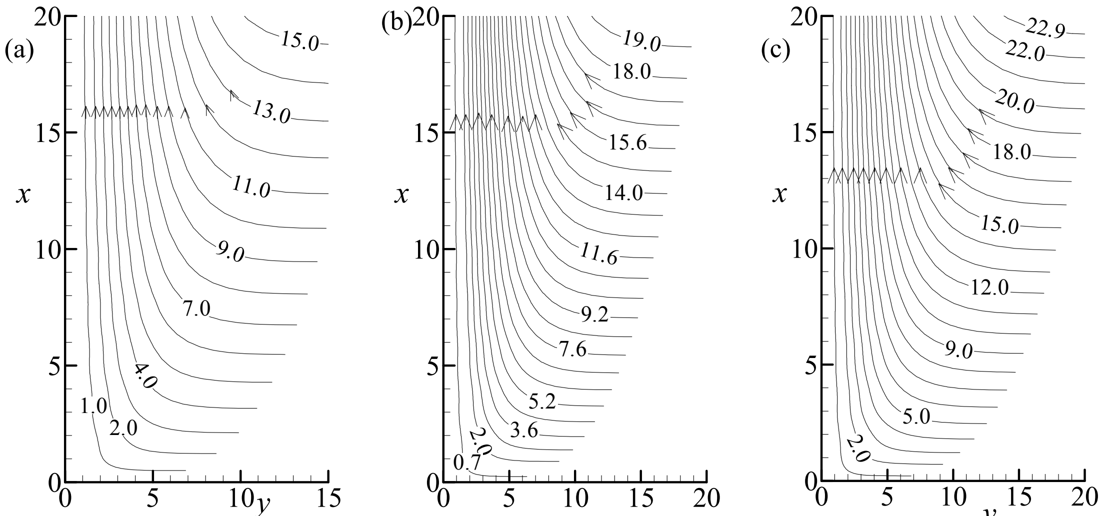


Fig. 7: Streamlines ψ for (a) R_d = 0.0 (b) R_d = 0.5 (c) R_d = 1.0 while Pr = 0.7, α = 0.2 and θ_w = 1.1

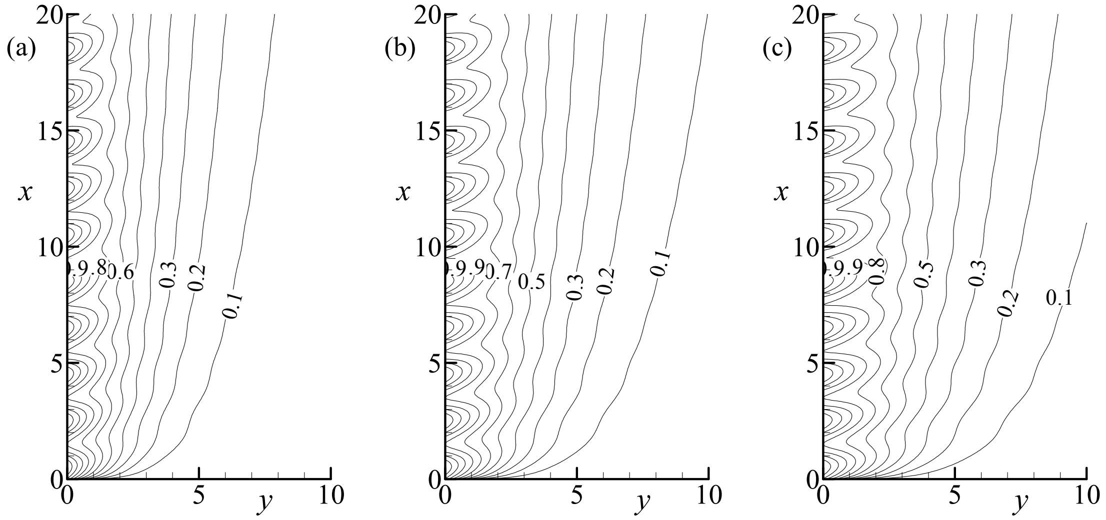


Fig. 8: Isotherms for (a) $R_d = 0.0$ (b) $R_d = 0.5$ (c) $R_d = 1.0$ while $Pr = 0.7$, $\alpha = 0.2$ and $\theta_w = 1.1$

Figs. 9 and 10 show the streamlines and isotherms for the effects of the surface heating parameter θ_w, respectively while $Pr = 0.7$, $\alpha = 0.2$ and $R_d = 1.0$. It is seen that within the boundary layer flow flux increases owing to the increase of surface heating parameter which is expected, since more surface heating intensify the flow velocity. From the isotherm it is very much clear for higher surface heating the thickness of the thermal boundary layer increases.

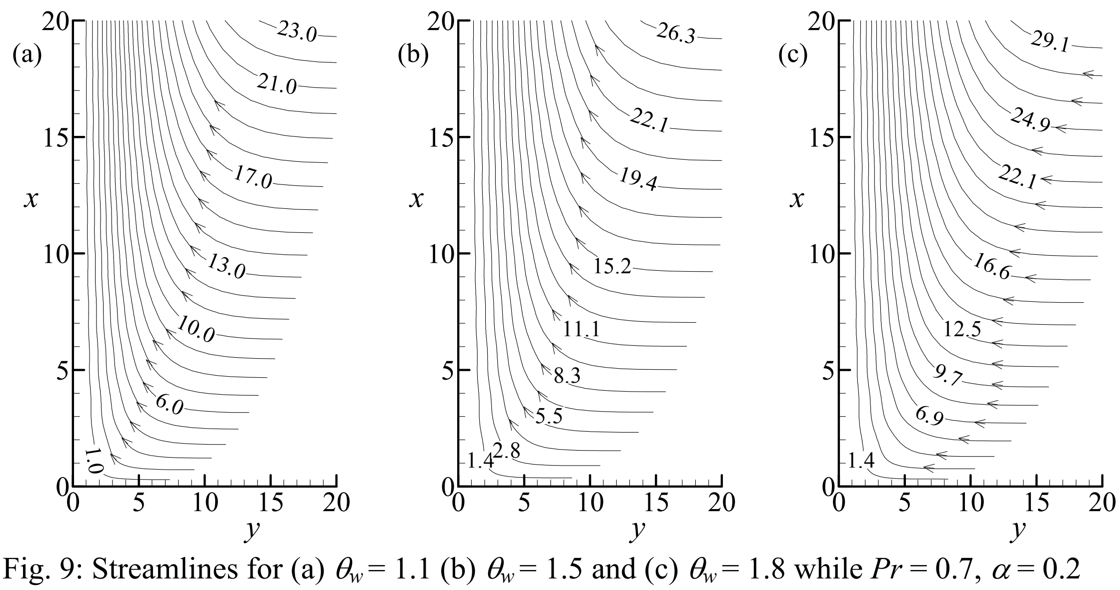


Fig. 9: Streamlines for (a) $\theta_w = 1.1$ (b) $\theta_w = 1.5$ and (c) $\theta_w = 1.8$ while $Pr = 0.7$, $\alpha = 0.2$ and $R_d = 1.0$

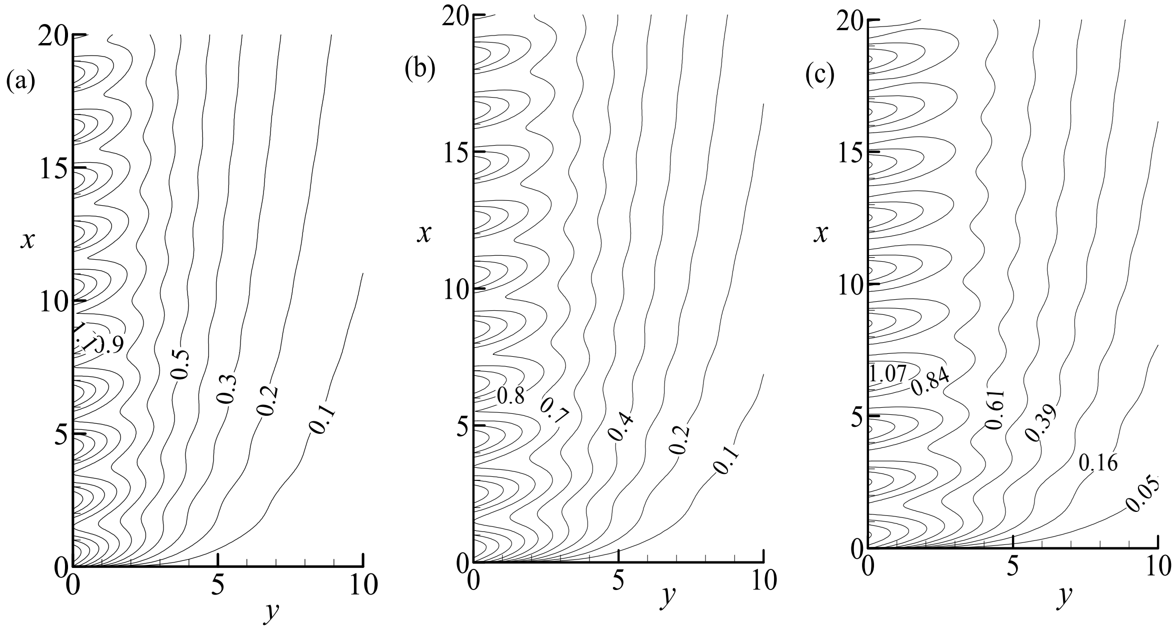


Fig. 10: Isotherms for (a) $\theta_w = 1.1$ (b) $\theta_w = 1.5$ and (c) $\theta_w = 1.8$ while $Pr = 0.7$, $\alpha = 0.2$ and $R_d = 1.0$

Conclusions

Here we have investigated the effect of thermal radiation on a steady two-dimensional natural convection laminar flow of viscous incompressible optically thick fluid along a vertical flat plate with streamwise surface temperature. There are two efficient methodologies, namely, Implicit finite difference method and Straight forward finite difference method have been used. Effects of various physical parameters arise from the governing equations and the boundary conditions on fluid flow and heat transfer has been shown. Velocity and temperature profiles as well as streamlines and isotherms are also presented. The outcomes of the results can be summarised as bellow:

- Both skin-friction and rate of heat transfer are enhanced considerably, owing to increase in the values of conduction radiation parameter, R_d.
- If the amplitude of the heating parameter is zero then both skin friction coefficient and the Nusselt number are uniform all over the plate. However, with the increase of the amplitude of the heating effect both parameters also increase.

- With the increasing values of θ_w, both skin friction and the rate of heat transfer increases.
- Viscous and thermal boundary layers increase with the increase of conduction radiation parameter, R_d. It is also seen that the maximum velocity also increases with increase of R_d.
- The streamlines concentrate near the wall for the higher values of radiation parameter. However, the thermal boundary layer becomes thicker as R_d increases. The flow of the fluid appears to be oscillating near the surface of the plate due to streamwise variations of the surface temperature.

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