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Widanagamage, Anushka, Jayalath, Dhammika, & O'Shea, Peter J. (2011) On the empirical eigenvalue distribution of slotted amplify-and-forward relaying protocol model. In *eddBE2011 : Sustainable Wellbeing : The First International Conference on Engineering, Designing and Developing the Built Environment for Sustainable Wellbeing*, 27-29 April 2011, Gardens Point Campus, Queensland University of Technology, Brisbane.

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ON THE EMPIRICAL EIGENVALUE DISTRIBUTION OF SLOTTED AMPLIFY-AND-FORWARD RELAYING PROTOCOL MODEL

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pp. 338-342

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Abstract: In this paper we consider the case of large cooperative communication systems where terminals use the protocol known as slotted amplify-and-forward protocol to aid the source in its transmission. Using the perturbation expansion methods of resolvents and large deviation techniques we obtain an expression for the Stieltjes transform of the asymptotic eigenvalue distribution of a sample covariance random matrix of the type $\mathbf{HH}^{\dagger}$ where $\mathbf{H}$ is the channel matrix of the transmission model for the transmission protocol we consider. We prove that the resulting expression is similar to the Stieltjes transform in its quadratic equation form for the Marcenko-Pastur distribution.

Key words: cooperative communication, amplify-and-forward, resolvent, Stieltjes transform

1 INTRODUCTION

It is well known that MIMO (multiple-input, multiple-output) systems are capable of producing high spectral efficiencies with the added advantage of multipath diversity. In recent times cooperative systems where source node is assisted by other nodes in transmitting a message to the destination have been investigated in order to understand whether they provide the same results as that of the MIMO systems. A key element in analysing such systems is to investigate the eigenvalue distribution of the sample covariance random matrix $\mathbf{HH}^{\dagger}$ where $\mathbf{H}$ is the channel matrix for the transmission model $\mathbf{Y} = \mathbf{HX} + \mathbf{Z}$; $\mathbf{X}$, $\mathbf{Y}$ and $\mathbf{Z}$ being the input, output and the complex white Gaussian noise vectors respectively while $\mathbf{H}^{\dagger}$ denotes the Hermitian adjoint of $\mathbf{H}$. Apart from the case of MIMO systems for which the elements of the $\mathbf{H}$ matrix are independent and identically distributed (i.i.d) random variables eigenvalue distributions for other types of random matrices that occur in communication theory still remain unknown. Examples of such matrices are: the non-orthogonal amplify-and-forward system and orthogonal amplify-and-forward system.

Recently some new developments has surfaced in the field of Random Matrix Theory (RMT) in application to problems that arise in relation to communication theory, notable works being (Fawaz, Zarifi, Debbah, & Gesbert, 2008; Hachem, Khorunzhiy, Loubaton, Najim, & Pastur, 2008; Levy, Somekh, Shamai, & Zeitouni, 2009). Authors of (Hachem et al., 2008) analyse the $n \times m$ MIMO channels with correlated antenna paths, using perturbation formulas and Poincare-Nash inequality and have proved that the rescaled mutual information variable converges to a Gaussian random variable with unit variance as $m,n \to \infty$. In (Fawaz et al., 2008) once again a closed form expression for the end-to-end mutual information for a multi-hopped relay network is derived using the tools of free-probability theory. The application considered in (Levy et al., 2009) involves the soft-handover procedure encountered by a mobile in a cellular network and the system model is analysed using product random matrices and the theory of Harris Markov Chains.

Cooperative communication systems, in particular relay assisted communication has become a heavily researched area in recent times with regard to communication theory, the most popular relaying methods being amplify-and-forward and decode-and-forward although recently these simple protocols has been vastly modified, one such being the slotted amplify-and-forward protocol that we consider here. The major factor of benchmarking the performance of such systems is the Shannon capacity, and as already noted above a major ingredient necessary for obtaining bounds for the expected value of this random variable is the empirical eigenvalue distribution of the sample covariance matrix $\mathbf{HH}^{\dagger}$ where $\mathbf{H}$ is the channel matrix.

It has been noticed that the channel model of most of the transmission models of cooperative protocols can be proven to be band random matrices where the band width is finite. If one can obtain the limiting eigenvalue distribution of such matrices as matrix dimensions grow infinitely large, a direct consequence would be ―asymptotic" results for the expected value of the channel capacity random variable as shown in (Tulino & Verdu, 2004). By the term ―asymptotic" we mean that the system dimension grows infinitely large. For a MIMO system the growth factors would be the number of transmit and receive antennas, while for cooperative communication this may be the number of cooperative time-slots or the number of participating relays.

Although the applications of band-random matrices have found their application in communication theory quite recently they are not new topics among the theoretical physics research community. These types of matrices have been studied in relation to applications found in areas like statistical mechanics of disordered systems, quantum chaos theory, solid state physics and quantum field theory (Crisanti, Paladin, & Vulpiani, 1993; Guhr, Muller-Groeling, & Weidenmuller, 1998; Khorunzhy & Kirsch, 2002). But most cases investigated involve the form of $n \times n$ square matrices which are of the form $b/n \to c$, $c>0$, $b,n \to \infty$ where b represents the number of diagonals which are non-zero, i.e. the band width.

The organization of the paper is as follows. First we discuss the system model of the slotted amplify-and-forward protocol with multiple relays transmitting in one slot and derive the state equations for the model. Thereafter we state our main result and outline the proof of this before finally we discuss in brief consequences of this result.

2 SYSTEM MODEL AND ANALYSIS

2.1 Operation of the Slotted Amplify-and-Forward Protocol

In this section we explain the operational details of the slotted-and-forward method investigated in (Sheng & Belfiore, 2007) which was shown to reach the asymptotic multiple-input single-output (MISO) diversity multiplexing trade-off bound when the number of relays aiding the source grows asymptotically large. A MISO scheme is defined as a transmission system where the source is equipped with multiple antennas but the destination possesses only one antenna. Furthermore in a MISO system the link between the source and the destination is direct. We lay out the system equations of the scheme we intend analyse below and before that it is needed to briefly describe the system model.

➢ A cooperation frame is composed of N slots each of l symbols, denoted by $x_i \in C^{l\times 1}, i = 1,\ldots,N;$

➢ During the i^{th} slot, the source s transmits x_i and the p-1 relays $r_{ij}, j = 1,\ldots, p-1$ transmits $x_{r_j,i} \in C^{l\times 1}$ and another set of $(p-1)$ relays listen in this slot.

➢ Assuming that the relays are isolated up to the extent that inter-relay gain is zero, meaning that the transmission of one relay does not interfere with the source-transmitted signal reception of another relay, the received symbols at the j^{th} relay and at the destination are given by, $y_{r_j,i}, y_i \in C^{l\times 1}$ respectively, with ;

$$y_i = g_0^{(i)} x_i + \sum_{j=1}^{p-1} g_j^{(i)} b_{j,i} x_{r_j,i} + z_{d,i} \qquad \cdots(1.a)$$
$$y_{r_j,i} = h_j^{(i)} x_i + z_{r_j,i} \qquad \cdots(1.b)$$

where $g_0^{(i)}$ denotes the source to destination link channel gain assumed to be a complex circularly symmetric Gaussian random variable with real and imaginary parts distributed i.i.d with zero mean and variance 1. For two timeslots i and j of a cooperative frame, source to destination channel gains are assumed to be independent that is we assume the cooperative frame time-slots to be considerably long. Furthermore $g_j^{(i)}$ and $h_j^{(i)}$ represent the source to j^{th} relay, and j^{th} relay to destination gains in the i^{th} slot respectively which are also assumed to be independent complex circularly symmetric Gaussian random variables with real and imaginary parts identically and independently distributed with mean zero and variance 1. $z_{r_j,i} \in C^{l\times 1}$ denotes additive white Gaussian noise components which are i.i.d with zero mean and variance 1. Furthermore $b_{j,i}$ denotes the amplifying factors of the j^{th} relay at the i^{th} timeslot.

➢ The transmitted signals x_i and x_{i-1} are subjected to short term power constraint;

$$E\left[\|x_i\|^2 + (p-1)^2 \|x_{r_{i-1},i}\|^2\right] \le l.SNR, \ \forall i \qquad \cdots (2)$$

where SNR is the signal to noise ratio which is kept constant.

➢ Solving the equations (1.a) and (1.b) we obtain the relations (3.a) and (3.b);

$$y_{d,i} = g_0^{(i)} x_i + \sum_{j=1}^{p} b_{j,i-1} g_j^{(i)} y_{r_j,i-1} + z_{d,i} \qquad \cdots(3.a)$$
$$y_{r_j,i} = h_j^{(i)} x_i + z_{r_j,d} \qquad \cdots(3.b)$$

➢ Now since each of the signal models (3.a) and (3.b) denote the signal model for one slot and the cooperative frame consists of N slots we may convert the equations (3.a) and (3.b) into the vector form given by (4.a) and (4.b) following the same lines of reasoning as in (Sheng & Belfiore, 2007).

$$\mathbf{y_{d,i}} = diag(\mathbf{a}).x + \mathbf{u}.\mathbf{y_{r,i}} + \mathbf{z_{d,i}} \qquad ...(4.a)$$
$$\mathbf{y_{r,i}} = diag(\mathbf{h}).\mathbf{x} + \mathbf{z_{r_i,d}} \qquad ...(4.b)$$

where , $a \in R^{M\times 1}$, $a_i = g_0^{(i)}$ and $\mathbf{u}$ is defined as;

$$\mathbf{u} = \begin{bmatrix} 0^T & 0 \\ diag(\mathbf{c}) & 0 \end{bmatrix} \qquad ...(5)$$

and $\mathbf{c}$ is defined by $\quad c_i = \sum_{j=1}^{p} b_{j,i-1} g_j^{(i)} h_j^{(i)}$

➢ Hence finally from 4.a and 4.b we finally obtain the vector channel

$$\mathbf{y_d} = \mathbf{Hx} + \mathbf{z} \qquad ...(6)$$

where, $\mathbf{H}$ is defined by

$$\mathbf{H} = \mathbf{a} + \mathbf{u}.diag(\mathbf{h}) \qquad ...(7)$$

$$\mathbf{z} = \mathbf{z_{d,i}} + \mathbf{u}.\mathbf{z_{r,i}} \qquad ...(8)$$

➢ It is evident that the equivalent channel matrix $\mathbf{H}$ of the transmission model forms a lower triangular bi-diagonal matrix. Furthermore we may consider that the amplification factors $b_{j,i-1}$ to be constant for analysis purposes. Furthermore we may write down this matrix in the following form (9) as well.

$$H(x,y) = \frac{1}{\sqrt{p}} \begin{cases} g_0^{(x)} & ,x = y \\ g_0^{(x)} \sum_{j=1}^{p-1} g_j^{(x)} h_j^{(x)} & ,x = y+1 \\ 0 & ,otherwise \end{cases} \qquad ...(9)$$

➢ Now we are in a position to carry out our main analysis, which is the empirical eigenvalue distribution analysis of this model as the number of relays transmitting in a cooperative timeslot and the number of slots in the cooperative phase are very large, i.e. $p, N\to\infty$.

2.2 Analysis of the Empirical Eigenvalue Distribution for the SAF protocol

2.2.1 Main Result

Our main result concerns the empirical eigenvalue distribution of the random matrix (9). We show that the Stieltjes transform $\rho(z)$ given by (10) of this distribution is similar to the Stieltjes transform of the Marcenko-Pastur distribution.

$$\rho(z) = \int_{-\infty}^{\infty} \frac{1}{\lambda - z} df_x(\lambda) \qquad ...(10)$$

Theorem-1:

Let $\mathbf{H}$ be defined as in (9) and consider the normalized trace $m(z) = (1/N)Tr(G(z))$ of its resolvent $G(z)=(H-z)^{-1}$. Then for small δ, we have

$$|m(z) - m_\rho(z)| < \delta \qquad ...(11)$$

where,

$$m_\rho(z) = \frac{1}{1 - r - z - zrm_\rho(z)} \qquad ...(12)$$

with high probability where $r=N/p$ as $p, N\to\infty$, $m_\rho(z)$ given by (12) being the Stieltjes transform of the Marcenko-Pastur distribution.

That is, as $p,N\to\infty$, with $r=N/p<1$ the eigenvalue distribution of $\mathbf{HH}^\dagger$ with $\mathbf{H}$ given by (9) tends to the Marcenko-Pastur distribution.

2.2.2 Outline of the Proof

Consider the resolvent defined by $\mathbf{G}(z) = 1/(\mathbf{H} - z)$. First part of the proof is to expand the resolvent of the matrix using its minors to obtain a self-consistent formula for $m_n(z) = (1/n)Tr(\mathbf{G}(z))$. The resulting formula is close to the final expression except for an additional term $Y^{(j)}$. Our next step is to apply large deviation techniques to obtain a bound for this term. In other words by studying the stability of the self-consistent equation obtained by applying perturbation methods we prove that for large N the empirical eigenvalue distribution of matrix (9) tends to (12) with high probability.

Denote the first column of **H** as $\mathbf{h_1}$ ($\mathbf{h_1}^*$ stand for the conjugate transpose vector) and **B** as the $N \times (N-1)$ obtained by ordering the last $N-1$ columns of **H**. Thus we may write $\mathbf{A=HH^\dagger}$ as,

$$\mathbf{A} = \begin{pmatrix} \mathbf{h}_1\mathbf{h}_1^* & \mathbf{Bh}_1^* \\ \mathbf{B}^\dagger\mathbf{h}_1 & \mathbf{BB}^\dagger \end{pmatrix} \qquad ...(13)$$

Denote the first column of the resolvent **G** as $(G_{11}, G_{12} \ldots, G_{1N})^T = (x, w)^T$ where $x = G_{11}$, then we have,

$$(\mathbf{h_{11}} - z)x + \mathbf{Bh}_1^*.w = 1 \qquad ...(14.a)$$

$$x\mathbf{B}^\dagger\mathbf{h_1} + (\mathbf{BB}^\dagger - z)w = 0 \qquad ...(14.b)$$

From (14.b) we have $w = -x\mathbf{G}^{(1)}\mathbf{B}^\dagger\mathbf{h_1}$ where $\mathbf{G^{(1)}}$ is the resolvent of the matrix $\mathbf{BB}^\dagger$, after which this being substituted to (14.a) we have,

$$G(1,1) = \frac{1}{\mathbf{h}_1\mathbf{h}_1^* - z - \mathbf{h_1}.\mathbf{BB}^\dagger(\mathbf{BB}^\dagger - z)^{-1}\mathbf{h_1^*}} \qquad ...(15)$$

Now let us define the normalized eigenvectors and the non-zero eigenvalues of $\mathbf{BB}^\dagger$ as v_α and μ_α respectively with $\alpha = (1,..,N\text{-}1)$. Clearly we have the matrix elements of $\mathbf{BB}^\dagger$ given by,

$$(\mathbf{BB}^\dagger)_{ij} = \sum_{\alpha=1}^{N-1} \mu_\alpha v_\alpha^*(i) v_\alpha(j) \qquad ...(16)$$

Thus using (16) we can rewrite (15) as,

$$G(1,1) = \frac{1}{\mathbf{h}_1\mathbf{h}_1^* - z - \frac{1}{p}\sum_{\alpha=1}^{N-1}\frac{\mu_\alpha\xi_\alpha}{\mu_\alpha - z}} \qquad ...(17)$$

where $\xi_\alpha = p|\mathbf{h_1}v_\alpha|^2$, and note that $\mathbf{E}[\xi_\alpha] = 1$, i.e.

$$E[p|\mathbf{h_1}v_\alpha|^2] = pE[\mathbf{h_1}\mathbf{h_1^*}]E[|v_\alpha^2|]$$

$$E[\frac{1}{p}|g_0^{(i)}|^2 + \frac{1}{p}\sum_{k=1}^{p-1}|g_i^{(k)}|^2|h_i^{(k)}|^2] = 1 \qquad ...(18)$$

Now we may generalize the result for $G(1,1)$ as given in (14) to obtain an expression for the normalized trace as,

$$m_N(z) = \frac{1}{N}\sum_{j=1}^{N}\frac{1}{\mathbf{h}_1\mathbf{h}_1^* - z - \frac{N-1}{p} - \frac{z}{p}\sum_{\alpha=1}^{N-1}\frac{1}{\mu_\alpha^{(j)} - z} - X^{(j)}} \qquad ...(19)$$

where,

$$X^{(j)} = \frac{1}{p}\sum_{\alpha=1}^{N-1}\frac{\mu_\alpha^{(j)}}{\mu_\alpha^{(j)} - z}(\xi_\alpha^{(j)} - 1) \qquad ...(20)$$

Note that since $\mathbf{E}[\xi_\alpha] = 1$, $\mathbf{E}[X^{(j)}] = 0$. Let,

$$m_{N-1}^{(j)}(z) = \frac{1}{N-1}Tr(\mathbf{B}^{(j)\dagger}\mathbf{B}^{(j)} - z)^{-1} \qquad ...(21)$$

Defining $r = N/p<1$ we may rewrite (19) as,

$$m_N(z) = \frac{1}{N}\sum_{j=1}^{N}\frac{1}{1 - z - r - zrm_N(z) + Y^{(j)}} \qquad ...(22)$$

where,

$$Y^{(j)} = (\mathbf{h}_j.\mathbf{h}_j^* - 1) - \frac{1}{N} - \frac{z}{N}\{(N-1)m_{N-1}^{(j)}(z) - Nm_N(z)\} - X^{(j)}(z) \qquad ...(23)$$

Now given that, $Y = \max_j(Y^j)$ is small we can expand (22) as,

$$m + \frac{1}{z + zrm + r - 1} = O(Y) \qquad ...(24)$$

Provided that, $|z + zrm + r - 1| \geq C > 0$.

We can clearly see that (24) is very much close to the quadratic equation for the Stieltjes transform of the Marcenko-Pastur density function. Now we shall obtain a rough bound for $Y^{(j)}$. It is evident that,

$$\mathbf{h_1}\mathbf{h_1^*} = \frac{1}{p}|g_0^{(i)}|^2 + \frac{1}{p}\sum_{u,v=1}^{p-1} g_i^{(u)}h_i^{(u)}g_i^{(v)}h_i^{(v)} \qquad ...(25)$$

Now recalling the fact that $\mathbf{E}[\mathbf{h_1}\mathbf{h}^\dagger]=1$ we may use proposition-1 of **Appendix-A.1** (Erdos, Schlein, & Yau, 2010a) which yields,

$$\mathrm{P}(|\mathbf{h_1}\mathbf{h_1^*} - 1| \geq Kr^{-1/2}) \leq Ce^{-cK} \qquad ...(26)$$

for some large fixed K.

Furthermore we notice that the eigenvalues of $\mathbf{HH}^\dagger$ and $\mathbf{BB}^\dagger$ are interlaced hence,

$$\frac{z}{N}\left[(N-1)m_{N-1}^{(j)}(z) - Nm_N(z)\right] \leq \left(\frac{\eta}{N}\right)\left(\frac{C}{\eta}\right) = CN^{-1} \qquad ...(27)$$

We have used here the trivial bounds,

$$\|G(z)\| \leq \eta, \quad |G_{xy}(z)| \leq \eta \qquad ...(28)$$

where, $\varepsilon^{-1} = \mathbf{Im}(G(z))$. Now we obtain a bound for $X^{(j)}$.

$$X^{(j)} = \frac{1}{N}\sum_{\alpha=1}^{N-1}\frac{\mu_\alpha^{(j)}}{\mu_\alpha^{(j)} - z}(\xi_\alpha^{(j)} - 1)$$

$$= \frac{1}{N}\sum_{\alpha=1}^{N-1}\frac{\mu_\alpha^{(j)}}{\mu_\alpha^{(j)} - z}(N\sum_{l,k=1}^{N}|\mathbf{h}_l\mathbf{h}_k^\dagger v_\alpha(l)v_\alpha(k)| - E[N\sum_{l,k=1}^{N}|\mathbf{h}_l\mathbf{h}_k^\dagger v_\alpha(l)v_\alpha(k)|]) \qquad ...(29)$$

Recall that the eigenvectors are normalized and $\mathbf{E}[|\mathbf{h_j}|^2]=1$. Now noting that the eigenvectors $v_\alpha^{(j)}$ and $\mathbf{h_j}$ are independent we may write (29) as,

$$X^{(j)} = \sum_{l,k=1}^{N} \sigma_{lk}[u_l u_k^* - E[u_l u_k^*]] \quad ...(30)$$

$$\sigma_{lk} = \frac{1}{N}\sum_{\alpha=1}^{N} \frac{\mu_\alpha v_\alpha(l) v_\alpha^*(k)}{\mu_\alpha - z} \quad ...(31)$$

Now taking into account the trivial bound (28) we can see that,

$$\sum_{l,k} |\sigma_{lk}|^2 = \frac{1}{N^2}\sum_\alpha \frac{\mu_\alpha^2}{|\mu_\alpha - z|^2} \le \frac{K}{N\eta} \quad ...(32)$$

Now again using proposition-1 of **Appendix-A** for small δ we have,

$$\mathrm{P}(\max_j |X^{(j)}| \ge \delta) \le Ce^{-c\delta\sqrt{N\eta}} \quad ...(33)$$

Hence together with (26), (27) and (33) for some large fixed *K* and small enough δ,

$$Y^{(j)} \le Ce^{-cK} + Ce^{-c\delta\sqrt{N\eta}} + CN^{-1} + N^{-1} \quad ...(34)$$

Consider the equation,

$$m_\rho + \frac{1}{z + zrm_\rho + r - 1} = 0 \quad ...(35)$$

Now we can apply Lemma-2 (**Appendix-A.2**) to obtain a bound on the stability of the equation (24) as follows. From (24) for a certain

$Y = \Delta$ small enough such that,

$$m + \frac{1}{z + zrm + r - 1} \le \Delta \quad ...(36)$$

We can compute the stability of (24) as,

$$|m(z) - m_\rho(z)| \le \frac{C\Delta}{\sqrt{\kappa + \Delta}} \quad ...(37)$$

for κ > 0, which proves **Theorem-1**.

2.3 Discussion

We note that the Stieltjes transform of the eigenvalue distribution of the random matrix model we have considered which tends to a non-random limit with high probability is closer to the Marcenko-Pastur distribution with one exception. Although the Marcenko-Pastur distribution takes the form of (11) with $r = M/N$ and $M<N$ for a sample covariance matrix $\mathbf{HH}^\dagger$, **H** being an MxN matrix, for the matrix we considered here $r=N/p$ with $p<N$. This difference is due to the channel being in the form of multiple-input, single-output (MISO) as opposed to the transmission model of a multiple-input, multiple-output (MIMO) system of which the empirical eigenvalue distribution becomes the Marcenko-Pastur distribution. Another interesting scenario is the case when out of *p* relays *q* number of relays may be available for cooperation for a certain timeslot. We call this partial availability. Assume each relay is available for relaying for a particular slot with probability distribution,

$$f_q(q) = \frac{1}{\sqrt{q}}\begin{cases} 1 \text{ with probability } q \\ 0 \text{ with probability } 1 - q \end{cases} \quad ...(37)$$

Then equation (9) may be rewritten as,

$$H(x,y) = \frac{1}{\sqrt{p}}\begin{cases} g_0^{(x)} & ,x = y \\ g_0^{(x)} \sum_{j=1}^{p-1} q_j^{(x)} g_j^{(x)} h_j^{(x)} & ,x = y+1 \\ 0 & ,otherwise \end{cases} \quad ...(38)$$

It is easy to verify that the single entry distribution still obeys an exponential decay due to Rayleigh fading and given that $f_q(q)$, yet $\mathbf{E}[\xi_\alpha]$ is not equal to unity thus the analysis becomes more complicated. We intend to address this problem in future work.

3 CONCLUSION

In this paper we have analysed the eigenvalue distribution of a random matrix model one encounters when studying cooperative communication, particularly with regards to amplify-and-forward protocols and obtained the Stieltjes transform of the eigenvalue distribution of the resulting random matrix. This result would facilitate the investigation of the Shannon capacity of this protocol and obtain an upper bound for it.

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APPENDIX

Below we note down a large deviation result and a perturbation theory result that we have used in our analysis. The reader is referred to the mentioned references for the proofs of these results.

A.1 Proposition-1, ((Erdos, Schlein, & Yau, 2010a) Lemma-4.2)

Let $z = \gamma + i\varepsilon$, with $\gamma>0$. Suppose that v_α and λ_α are the eigenvectors and eigenvalues of an NxN random Hermitian matrix **B** with the single entry distribution satisfying sub-exponential decay. Let,

$$X = \frac{1}{N}\sum_{\alpha}\frac{\xi_\alpha - 1}{\lambda_\alpha - z}\ ,\ \xi_\alpha = \left|\mathbf{b}.v_\alpha\right|^2 \qquad ...(A.1)$$

where components of **b** are i.i.d random variables, independent of **B** and also satisfying exponential decay. Then there exists a positive constant c depending on γ so that for every $\delta > 0$,

$$\mathrm{P}[\left|X\right| \geq \delta] \leq Ce^{-c\delta\sqrt{N}} \qquad ...(A.2)$$

A2. Lemma-2 ((Erdos, Schlein, & Yau, 2010b), Lemma-8.4)

Let X_+ and X_- be the solutions of the equation,

$$X(z) + \frac{1}{z + r + zrX(z) - 1} = \Delta \qquad ...(A.3)$$

For small enough Δ depending on r and large $\kappa(\gamma)$ where $z = \gamma + i\varepsilon$,

$$\max\left\{\left|X_+ - S_+\right|, \left|X_- - S_-\right|\right\} \leq C\frac{\Delta}{\sqrt{\kappa(\gamma) + \Delta}} \qquad ...(A.4)$$

where S_+ and S_- are the solutions of the quadratic equation for the Stieltjes transform of the Marcenko-Pastur distribution.