Structural reliability prediction of a steel bridge element using Dynamic Object Oriented Bayesian Network (DOOBN)

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Abstract—**Different from conventional methods for structural reliability evaluation, such as, first/second-order reliability methods (FORM/SORM) or Monte Carlo simulation based on corresponding limit state functions, a novel approach based on dynamic objective oriented Bayesian network (DOOBN) for prediction of structural reliability of a steel bridge element has been proposed in this paper. The DOOBN approach can effectively model the deterioration processes of a steel bridge element and predict their structural reliability over time. This approach is also able to achieve Bayesian updating with observed information from measurements, monitoring and visual inspection. Moreover, the computational capacity embedded in the approach can be used to facilitate integrated management and maintenance optimization in a bridge system. A steel bridge girder is used to validate the proposed approach. The predicted results are compared with those evaluated by FORM method.**

Keywords-structural reliability; limit state functions;Dynamic Object Oriented Bayesian Network (DOOBN)

I. INTRODUCTION

As bridges are regarded as critical components of a transport network, the safety of bridges is crucial to people's daily life and to the national economy. Structural reliability is an accurate and commonly used measure of the safety of a bridge as well as its elements, which indicates the failure probability can be defined through the limit state functions. Many structural design codes are based on structural reliability, such as, load and resistance factor design (LRFD). To date, a large amount of research work has been done to evaluate safety of bridge structures based on time-variant structural reliability [1-3]. Furthermore, it is of importance to predict structural reliability of a bridge structure in the future for the purpose of bridge life-time management optimization [4-6].Although most existing bridge life-time management optimizations are based on condition states , it is expected time-dependent structural reliability will play an important role in optimizing bridge life-time management in the near future[7].

Conventional bridge structural reliability is evaluated through approximated methods, such as, first/second-order reliability methods (FORM/SORM), Monte Carlo simulation (MCS) and Response Surface Modes (RSM).This paper proposes a novel approach for structural reliability prediction in a steel bridge element based on dynamic objective oriented Bayesian network (DOOBN). DOOBN is the extension of Bayesian Network (BN), which has been widely used in many areas such as risk assessment and reliability [8-10]. In contrast to the previous research, the proposed DOOBN approach can effectively model deterioration processes of a steel bridge element caused by corrosion, and predict its structural reliability over time. Another advantage of the proposed approach is the ability to achieve the Bayesian updating with observed information from measurements, monitoring and visual inspection. In addition, the implementation of the DOOBN approach in software facilitates integrated bridge management system for the purpose of maintenance optimization.

The rest of this paper is organised as follows. Brief introduction of BN theory and its dynamic object oriented representation are given in Section 2. Fundamental knowledge about deterioration mechanisms of a steel bridge element is presented in Section 3. The DOOBN based approach for bridge structural reliability prediction is illustrated in Section 4, which consists of modelling of structural reliability and corrosion deterioration processes and parameters estimations. Furthermore, the proposed DOOBN approach is demonstrated and validated through a steel bridge girder (Section 5). The results are compared with the ones obtained from an approximate method (FORM). Finally, Section 6 gives the conclusions and future work.

II. BN THEORY

According to Jensen and Nielsen[11], a BN is a probabilistic model in the form of directed acyclic graphs (DAG) with the directed edges and a table of conditional probabilities of each variable on all its parents. Fig.1 gives a simple example of BN. Each node represents a probability distribution of a variable, which may in principle be continuous states or discrete states. Nodes X_2 and X_3 with arrows directed from other nodes are called child nodes, and they have a common parent node X_1. Nodes without any arrows directed into them are called root nodes. An arrow between two nodes X_1 and X_2 indicates conditional dependence between the two variables that are represented by the two nodes. The dependence relationships are represented by a set of conditional probability distributions (CPDs). For instance, the probability of a dependent variable X_2 being in a particular state given for each state of variable X_1 is expressed as $P(X_2| X_1)$. Prior probability tables or functions are held by root nodes.

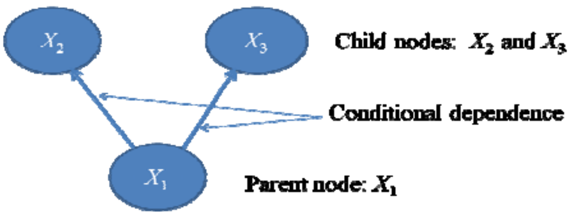

Figure1. A simple BN consisting of three variables

As the probability of each variable is defined conditional on its parents, the joint probability of this network $p\{X_1, X_2, X_3\}$ is specified as a product of these conditional probabilities

$$P\{X_1, X_2, X_3\} = P\{X_1\}P\{X_2|X_1\}P\{X_3|X_1\} \tag{1}$$

where $P\{X_2|X_1\}$ and $P\{X_3|X_1\}$ are conditional probabilities given X_1, respectively, and $P\{X_1\}$ is prior probability. Moreover, with the assumptions of Markov property and conditional independence (d-separation[12]), the joint probability for any BN is given as

$$p(X) = p\{X_1, \cdots, X_n\} = \prod_{i=1}^{n} p(X_i|Pa(X_i)) \tag{2}$$

where $Pa(Xi)$ is the set of parents of node X_i. One distinctive advantage of BN is the inference ability for calculation of beliefs of events based on new observed evidence. The beliefs (probabilities) are updated in accordance with observation using Bayesian updating. Assume an evidence e is observed, and then we have

$$p(X|e) = \frac{p(X,e)}{p(e)} = \frac{p(X,e)}{\sum_X p(X,e)} \tag{3}$$

As an extension of conventional BN, an Object Oriented Bayesian Network (OOBN) contains, in addition to the usual nodes, instance nodes[13]. In an OOBN, a physical or an abstract entity, or a relationship between two entities can be all modelled as an object. The object represents either a node or an instantiation of a network class (instance nodes). An example BN class is shown in Fig.2, where input nodes are ellipses with shadow dashed and output nodes are ellipses with shadow bold line borders. An instantiation of this network class is also given in the Fig.2, which has one input C, and two outputs A and B.

To address temporal behaviour of OOBN, time slices are added to represent each period of interest so that OOBN is changed into DOOBN. Fig.3 shows a three-slice DOOBN. The input comes from output in previous time slice, and temporal behaviour can be described.

III. DETERIORATION OF STEEL BRIDGE ELEMENTS

In light of bridge elements made of steel, the most common cause of deterioration is corrosion since all structural metals suffer from corrosion. The corrosion can lead to cracking (fracture), yielding or bucking, bending or distortion, and slipping, which can result in stress concentration, change in geometric parameters, and a build-up of the corrosion products. Consequently, the bridge reliability decreased over time. As much as a steel bridge girder is concerned, corrosion can cause a reduction in the cross-section area. Furthermore, the reduction of the web area will result in the shear capacity loss of the structure and the reduction of the plastic section modulus will result in the moment capacity loss[6].

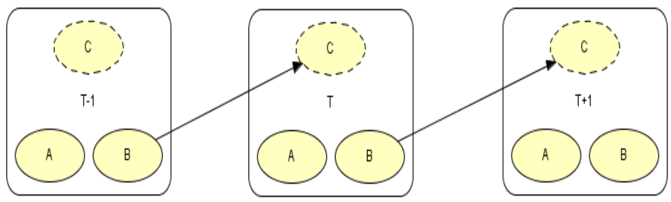

Figure3. A simple three-slice DOOBN

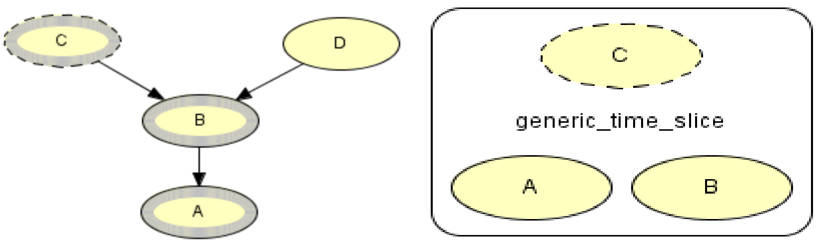

Figure2. A simplified BN class and its instantiation

For steel bridge elements, there are many factors that can influence the propagation of corrosion and also many different forms of corrosion, such as, pitting, crevice, galvanic and stress corrosion. However, only uniform corrosion is considered in this study. Current available data are not sufficient to formulate analytical models. Therefore, it is only possible to use approximate empirical formulas. Normally, if effects of paint and coating are not considered, a common agreement is to use a power function to describe corrosion propagation. An exponential function is given as [14].

$$C = A \cdot t^{B} \tag{4}$$

where C is average corrosion penetration from corrosion loss after t years in micrometers (10^{-6} m), A is the corrosion loss after one year, and B is a regression coefficient numerically equal to the slope of Eq 4 in log-log plot. Both A and B are based on the environment and the type of steel. For instance, in term of carbon steel and rural environment A=34 with coefficient of variation equal 0.09, and B= 0.65 with coefficient of variation equal 0.10[14].Based on Eq 4, new geometric parameters, such as, plastic section area and web area could be recalculated for the purpose of structural reliability estimation

IV. DOOBN BASED APPROACH FOR STRUCTURAL RELIABILITY PREDICTION OF STEEL BRIDGE ELEMENTS

A. DOOBN formulation

- Structural reliability aspect

This section aims to model structural reliability of a steel bridge element based on BN. Generally, each bridge element may have more than one failure mode, for instance, failure modes of shear and moment are often considered together. Overall structural reliability of a steel bridge element is calculated based on its structural reliabilities in each failure mode, and normally, series relationship is assumed among different failure modes. Fig.4 gives an example of a bridge element with failure modes of shear and moment.

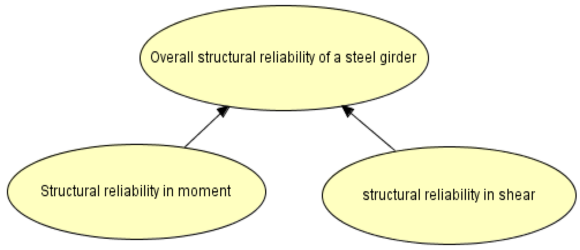

Figure 4.BN model of a steel girder considering both shear and moment

Consider a generic form of limit state function g that describes all types of failure modes as a function of steel yield strength F_y, a set of parameters $\boldsymbol{S}$ relating to section modulus or web area, a set of parameters $\boldsymbol{L}_d$ relating to dead load and a set of parameters $\boldsymbol{L}_l$ relating to live load. This limit state function g is expressed by the difference between resistance R and load L, and is written in generic form as Eq. (5). The limit state function g can represent all types of failure modes. For example, for moment of a steel girder, F_y denotes steel yield strength, $\boldsymbol{S}$ denotes a set of parameters relating to plastic section modulus, $\boldsymbol{L}_d$ denotes a set of parameters relating to moment due to dead load, and $\boldsymbol{L}_l$ denotes a set of parameters relating to moment owing to live load.

$$g = R - L = F_y \tag{5}$$

The generic limit state function is formulated as BN in Fig.5.The links between different nodes represent conditional relationship between different nodes. Since each node represents vectors of variables, the BN here can be further extended to model any limit state function in details. Therefore, the BN here is a generic model and can be used for the purpose of accurate structural reliability calculation of steel bridge elements. In addition, the BN model automates Bayesian updating efficiently based on the observations of each node. Moreover, the BN model enables bi-directional Bayesian updating, which means once an observation of one node is available, the whole BN will be updated automatically. The inclusion of observations will be discussed in details in next corrosion deterioration aspect.

- Corrosion deterioration aspect

This section aims to model deterioration processes of steel bridge elements based on DOOBN. If the live load is assumed to be time-invariant, only the deterioration of resistance contributes to time-dependent structural reliability of a steel bridge element. According to the discussion in Section 3, the most common cause of deterioration of resistance is corrosion. Furthermore, corrosion deterioration process can be described by a power function (Eq. (4)). In this research, the corrosion deterioration process is modelled as a discrete time process. The DOOBN modelling is given in Fig.6, where C, as an output, is corrosion loss (corrosion penetration depth) after t years, A is the corrosion loss after one year, and B is a regression coefficient numerically. The nodes T-1 and T represent time variables in two consecutive time slices and are assigned as input and output, respectively. The time variable T is conditional on previous time variable T-1.By introducing the time variable T, the commonly held Markovian assumption in most of BN applications is released in this research. The time-variant corrosion deterioration is implemented by connecting the object of corrosion deterioration in each time slice.

The DOOBN model is capable of computational and robust Bayesian updating when observation information (new evidence) is available. Observation information can be obtained through visual inspection, NDT (Non-Destructive Technology) and SHM (Structural health monitoring).Visual inspection provides straightforward information for bridge

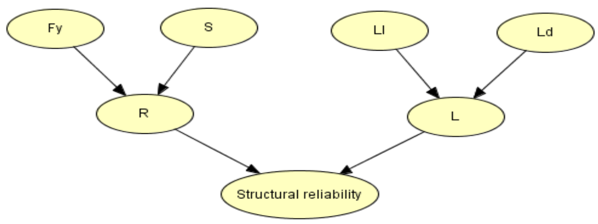

Figure 5.BN model for generic limit state function

engineers, while both NDT and SHM provide indirect information which need to be further processed. In terms of corrosion deterioration process in this research, observation relating to corrosion loss can be used to update the prediction of structural reliability. The inclusion of an observation node in the DOOBN model is shown in Fig.7. This observation node *Ob* could be a discrete random variable with two states "corrosion" and "no corrosion" or a continuous random variable, for instance, a measured corrosion penetration depth. In the previous case, a probability of detection (PoD) model is adopted to characterize the observation information; in the latter case, measurement error is utilised to characterize the observation information.

B. Parameters estimation

After the DOOBN formulation, this section is implemented with the purpose of estimation of conditional probabilities tables (CPTs) and priori probabilities of root nodes for the DOOBN, which could be the most difficult. However, before the estimation, it is necessary to discretize continuous nodes into discrete nodes. Owing to the limitation of current inference algorithms and slow convergence rate, continuous nodes cannot be dealt with efficiently. Furthermore, current inference algorithms cannot handle the situation adequately that continuous parent nodes have discrete children nodes, which actually does happen in this research. Therefore, continuous variables should be replaced by a finite number of discrete states. Univariate discretization is chosen in this paper and is carried out sequentially from parent nodes to children nodes. Equal length intervals are chosen. Next, the discretization interval length is determined within the probable values range to make sure that the discretized distribution represents the original continuous distribution in a reasonable and accurate way. The probability of each discrete state is assigned with cumulative distribution probability over the corresponding discretization interval.

- Estimation of CPTs and prior probabilities

In this research, deterministic equations are used to estimate the CPTs for the proposed DOOBN model. Since modelling of structural reliability is based on deterministic limit state functions, conditional probabilities could be derived from the equations directly, such as, Eq. (5). In this case, the relationship described by the deterministic equation, is directly encoded into CPTs, which means the conditional relationships are deterministic. Moreover, the knowledge of steel corrosion in bridge deterioration (Eq. (4)) is also utilised to estimate the conditional probabilities relating to variables for modelling of corrosion deterioration processes. In light of observations, CPTs can be estimated through a probability of detection (PoD) model or measurement error. In addition to deterministic equations, knowledge from existing literatures can be used for estimation of CPTs and prior probabilities. For instance, the prior probabilities of A and B in Eq. (4) are obtained from the literature[14]. Finally, overall structural reliability of a steel bridge element is conditional on structural reliabilities of different failure modes with an assumed series relationship among different failure modes.

V. APPLICATION

A. DOOBN modelling

The proposed DOOBN model is applied to a classical example of a steel bridge girder for structural reliability prediction. The results obtained from the proposed approach are compared with results obtained from some approximate methods, for instance, first order reliability method (FORM).

$$\begin{aligned} g &= d_w F_y (t_w - 2d_{corr}) - 18.04\lambda_{conc} - 5.26\lambda_{asph} - 2.89\lambda_{steel} - 28.33 V_{trk-l} DF_l I_{beam} \\ &= 18.183 F_y \left(0.58 - \frac{At^B}{12700}\right) - 18.04\lambda_{conc} - 5.26\lambda_{asph} - 2.89\lambda_{steel} - 28.33 V_{trk-l} DF_l I_{beam} \end{aligned} \tag{6}$$

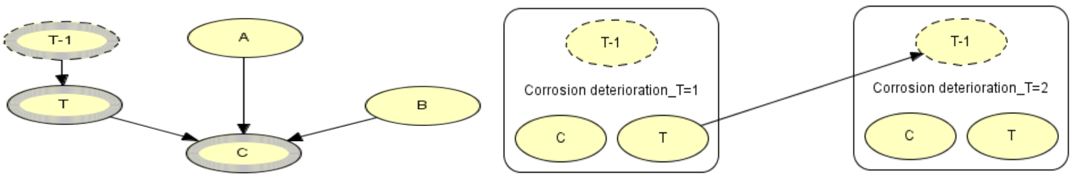

Figure 6.DOOBN modelling for corrosion deterioration process

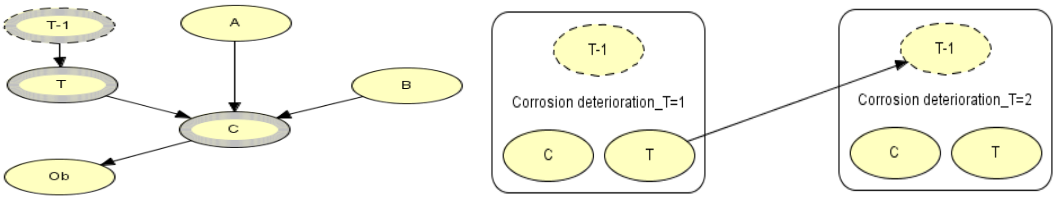

Figure 7.DOOBN modelling for corrosion deterioration process including an observation

The automatic Bayesian updating ability is also examined by integrating observation into structural reliability prediction. On behalf of this example, a limit state equation for shear failure of a steel bridge girder was chosen from a PhD thesis written by Estes[6]. To take into account temporal bridge deterioration, the limit state function is rewritten into Eq. (6), where d_w is the depth of the web; F_y is yield strength of steel in girders; t_w is the thickness of the web; d_{corr} is the depth of corrosion at the considered time; t is time variable; A is the corrosion loss after one year; B is a regression coefficient numerically; λ_{conc} is uncertainty factor for weight of concrete on deck; λ_{asph} is uncertainty factor for weight of asphalt on deck; λ_{steel} is uncertainty factor for weight of steel girders; $V_{trk\text{-}i}$ uncertainty factor for live load shear in girder; DF_i is uncertainty for live load girder distribution; I_{beam} uncertainty factor for impact on girder. To facilitate the CPTs estimation in this example, several new variables, R, L, V_{dl} and V_{ll}, are introduced, and the limit state function (Equation 7) is rewritten into Eqs. (7-12).

$$R = 18.183 F_y \left(0.58 - \frac{d_{corr}}{12700}\right) \quad (7)$$

$$L = V_{dl} + V_{ll} \quad (8)$$

$$d_{corr} = At^B \quad (9)$$

$$V_{dl} = 18.04\lambda_{conc} + 5.26\lambda_{asph} + 2.89\lambda_{steel} \quad (10)$$

$$V_{ll} = 28.33 V_{trk-i} DF_i I_{beam} \quad (11)$$

$$g = R - L \quad (12)$$

The DOOBN model for this application, including corrosion inspection results *Ob*, is given in Fig. 8. In this example, the observations can be either visual inspection for corrosion or measurements of depth of corrosion. In the former case, the PoD model (D is the event of corrosion indication of the steel girder) is considered as follows:

$$P(Ob_t = D \mid d_{corr}) = \mathrm{PoD}(d_{corr}) = 1 - \exp\left(-\frac{d_{corr}}{1000}\right) \quad (13)$$

The probability of an indication of corrosion is conditional on true corrosion depth. Therefore, the probability of no indication of corrosion at time t is expresses as 1-PoD (d_{corr}). In the later case, the measurements of corrosion depth at time t are assumed to be the true corrosion depth plus Normal distribution with μ=0 and σ=1.

As most of the variables are defined in continuous states, the implementation of discretization is necessary. Equal length discretization interval is chosen in this case study, and the probabilities of each discrete state are assigned with cumulative distribution function (CDF) over the corresponding interval. With discretized variables, the estimation of CPTs can be estimated based on deterministic equations above. By sampling the intervals of the parent nodes and inserting the sampled values into the equations, a large number of function values are available for each configuration of the parents' sampled values. By taking the relative frequency occurrence of the function values in each interval of the specified child node, the CPTs for each child node are obtained.

B. Numerical Results

By implementing inference algorithms, the reliability indexes obtained from DOOBN are compared with those obtained from FORM (Fig.9). The comparison demonstrates the accuracy of the proposed DOOBN model. In addition, the proposed DOOBN model facilitates Bayesian updating of all the variables within the model based on inspection results. To validate updating ability of this model, reliability indexes are updated based on the visual inspection results in Table I and the measurements results of corrosion depth in Table II, respectively. Fig.10 represents the resulting posterior reliability index based on visual inspection. The updated reliability indexes based on the measurements of corrosion depth are shown on Fig.11. Fig.10 and 11 illustrate automatic updating ability of the proposed DOOBN model, which brings in more accurate prediction results for the purpose of maintenance optimization.

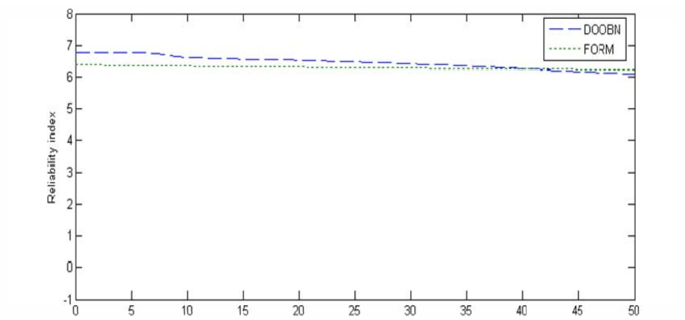

Figure 9.Comparison of reliability indexes obtained from DOOBN and FORM

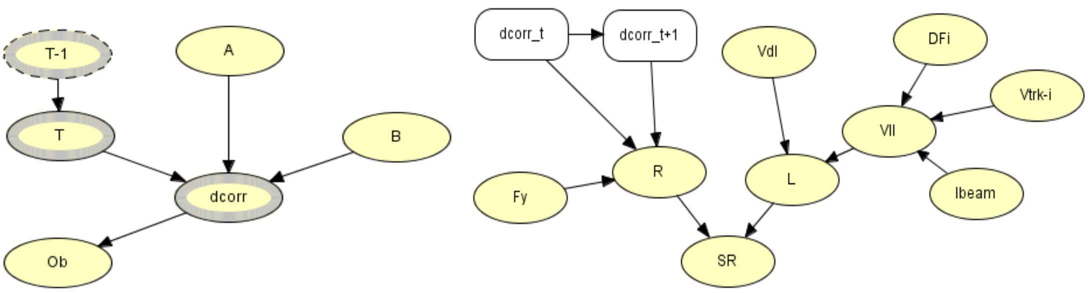

Figure 8.Shear structural reliability of a steel girder as a DOOBN

TABLE I. VISUAL INSPECTION

InspectionTimes (year)	5	10	15	20	25	30	35	40	45
Indication of corrosion	N	N	N	Y	Y	Y	Y	Y	Y

TABLE II. MEASUREMENT RESULTS OF CORROSION DEPTH

MeasurementTimes (years)	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**
Measurements(10^{-6}m)	95	125	145	197	287	361	503	616	799

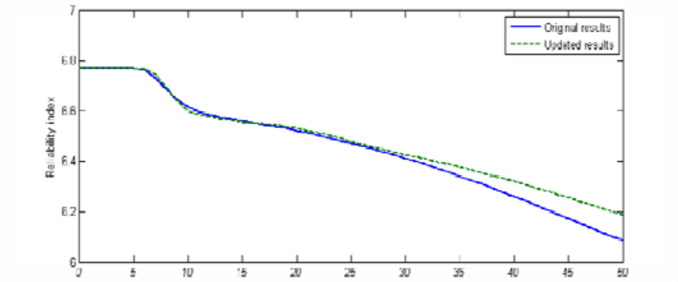

Figure 10.Updated reliability indexes with visual inspection results

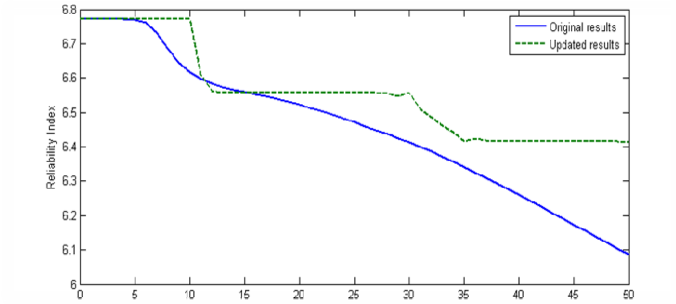

Figure 11.Updated reliability indexes with measurements of corrosion depth

VI. CONCLUSION

A DOOBN-based approach has been proposed in this paper to predict structural reliability of steel bridge elements. The approach includes DOOBN formulation and parameters estimation. A steel bridge girder has been selected to validate the applicability of the proposed approach. It has been confirmed that the DOOBN-based approach can accurately predict structural reliability of this bridge element. The approach is also able to model the temporal behaviour of the deterioration processes of steel bridge elements caused by corrosion. In addition, the proposed approach automates computational and robust Bayesian updating with observation information. The potential application includes bridge health prediction and integrated management for bridge maintenance optimization. The applicability of the proposed approach will further investigated with a focus on other failure modes, for instance, moment, and multiple steel bridge elements. We believe the proposed approach can be also extended to predict the structural reliability of a whole steel bridge system.

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