Trust and Reputation Management in Web-based Social Network

Touhid Bhuiyan[1], Audun Jøsang[2] and Yue Xu[1]
[1]*Queensland University of Technology*
[2]*University of Oslo*
[1]*Australia*
[2]*Norway*

1. Introduction

In a web-based social network, people may communicate with their friends whom they know personally. They also communicate with other members of the network who are the friends of their friends and may be friends of their friend's network. They share their experiences and opinions within the social network about an item which may be a product or service. The user faces the problem of evaluating trust in a service or service provider before making a choice. Opinions, reputations and recommendations will influence users' choice and usage of online resources. Recommendations may be received through a chain of friends of friends, so the problem for the user is to be able to evaluate various types of trust recommendations and reputations. This opinion or recommendation has a great influence to choose to use or enjoy the item by the other user of the community. Users share information on the level of trust they explicitly assign to other users. This trust can be used to determine while taking decision based on any recommendation. In case of the absence of direct connection of the recommender user, propagated trust could be useful.

The first problem for the user is how much he/she can trust on a particular opinion to select an item. The opinion or recommendation may come from a friend's of a friend's friend. So, the problem for the member is, how much to trust on the opinion giver. The quality of an opinion in terms of reliability may increase if we can consider the overall public reputation of that particular item. For example, if a member is interested to choose a hotel to stay in Sydney, he may browse the experiences of his/her friends who have stayed in that hotel in past. While receiving a recommendation about a particular hotel from a trusted friend, it is also possible to include the general opinion of the users, or the reputation of the same hotel, in order to be better informed about the quality of service, and thereby to enable a better decision.

As the social network is growing very fast by doubling the number of people joining every year (Golbeck, 2006), the possibility of getting a huge number of opinion regarding a particular item is very common. It is another problem for a member to read all these opinions from other members of the social network. This requires a recommender system to summarize or filter the top opinions or recommendation in terms of quality of the opinion and the trust between the user and the opinion giver. Social networking has been around for some time. Facebook and MySpace have become iconic, and other sites such as LinkedIn, hi5, Bebo, CyWorld and Orkut are becoming important as well. At the end of 2007, Microsoft paid $240 million for a 1.6% stake in Facebook, sparking a fierce debate about the theoretical valuation of Facebook. While few would go along with the $15 billion price tag, nobody would deny the huge potential of Facebook. The relevance of social networking for advertisers is very high considering they want to invest their money where the potential customers are located on social networking sites. The success of social networking should not come as a surprise. Social interaction is deeply rooted in human nature and is one of the most fundamental needs. Wireless and Internet technology act as enablers and facilitators for enhanced social interaction with a global reach. While social networking has been and still is dominated by teenagers and young adults, it is quickly spreading to all age groups and beyond the confines of consumer entertainment. Corporations are discovering the power of networking sites to enhance their brands, communities, and overall interaction with their customers by seamlessly linking corporate Web sites to public sites such as Facebook. And something even bigger is about to take place.

There has been dramatic growth in the number and size of Web-based social network. The number of sites almost doubled over the two year period from December 2004 to December 2006, growing from 125 to 223. Over the same period, the total number of members among all sites grew four-fold from 115 million to 490 million (Golbeck, 2006). The growth is continuing for last two years at the same rate, even more. The recent emergence of location-based mobile social networking services offered by providers such as Rummble, GyPSii, Whrrl and Loopt is revolutionizing social networking allowing users to share real-life experiences via geo-tagged user-generated multimedia content, see where their friends are and meet up with them. This new technology-enabled social geo-lifestyle will drive the uptake of Location-based services and provide opportunities for location-based advertising in the future.

In this research, we have tried to consider trust among the members while they select an item based on the opinion of friends. We calculate the public reputation of that item based on the general opinion given by previous users or customers. Then we combine this reputation with the trust among the opinion giver and the member who is going to select the item. As the recommendation comes from a trusted friend and it also includes the general public opinions, the quality of the opinion may improve. Currently, none of the web-based social network is considering combining the public reputation of an item with the trust among the members of the network to suggest or recommend an item. In general, people like to express their opinion and are interested about others opinion regarding the items they have concern. One popular way of obtaining customer feedback is collecting ratings about the product or services by the end users. In addition to the customer ratings about the product or services, there is also a good number of online customer feedback information available over the Internet as free text customer reviews, comments, newsgroups post, discussion forums or blogs. This information also can be used to generate the public reputation of the service providers'. To do this, data mining techniques, specially recently emerged opinion mining (Hu & Liu, 2004a), (Popescu & Etzioni, 2005), (Ku, Liang, & Chen, 2006) could be a useful tool. Mining and organizing opinions from the feedback of the customer or user of an item could be useful for the person or organization that is going to use the item in future.

2. Fundamentals of Trust and Reputation

2.1 Defining Trust

Trust has become important topic of research in many fields including sociology, psychology, philosophy, economics, business, law and IT. It is not a new topic to discuss. In fact, it has been the topic of hundreds books and scholarly articles over a long period of time. Trust is a complex word with multiple dimensions. A vast literature on trust has grown in several area of research but it is relatively confusing and sometimes contradictory, because the term is being used with a variety of meaning (McKnight & Chervany, 2002). Also a lack of coherence exists among researchers in the definition of trust. Though dozens of proposed definitions are available in the literature, a complete formal unambiguous definition of trust is rare. In many occasions, trust is used as a word or concept with no real definition. Hussain et al. present an overview of the definitions of the terms of trust and reputation from the existing literature (Hussain & Chang, 2007). They have shown that none of these definitions is fully capable to satisfy all of the context dependence, time dependence and the dynamic nature of trust.The most cited definition of trust is given by Dasgupta where he define trust as "the expectation of one person about the actions of others that affects the first person's choice, when an action must be taken before the actions of others are known" (Dasgupta, 1990). This definition captures both the purpose of trust and its nature in a form that can be reasoned about. Deutsch (Deutsch, 2004) states that "trusting behaviour occurs when a person encounters a situation where she perceives an ambiguous path. The result of following the path can be good or bad and the occurrence of the good or bad result is contingent on the action of another person" (Hussain & Chang, 2007). Another definition for trust by Gambetta is also often quoted in the literature: "trust is a particular level of the subjective probability with which an agent assesses that another agent or group of agents will perform a particular action, both before he can monitor such action and in a context in which it affects his own action" (Gambetta, 2000). But trust can be more complex than these definitions.

Trust is the root of almost any personal or economic interaction. Keser states "trust as the expectation of other persons goodwill and benign intent, implying that in certain situations those persons will place the interests of others before their own" (Keser, 2003). Golbeck (Golbeck , 2006) defines trust as "trust in a person is a commitment to an action based on belief that the future actions of that person will lead to a good outcome". This definition has a great limitation that it considers trust as always leading to positive outcome. But in reality, it may not be always true. Trust is such a concept that crosses disciplines and also domains. The focus of definition differs on the basis of the goal and the scope of the projects. Two generalized definitions of trust defined by Jøsang (Jøsang et al. 2007) which they called reliability trust (the term "evaluation trust" is more widely used by the other researchers, therefore we use this term) and decision trust respectively will be used for this work. Evaluation trust can be interpreted as the reliability of something or somebody and the decision trust captures broader concept of trust.

Evaluation Trust: Trust is the subjective probability by which an individual, A, expects that another individual, B, performs a given action on which its welfare depends.

Decision Trust: Trust is the extent to which one party is willing to depend on something or somebody in a given situation with a feeling of relative security, even though negative consequences are possible.

2.2 Defining Reputation

Reputation systems represent a significant trend in decision support for Internet mediated service provision (Resnick et al, 2000). The Feedback Forum on eBay is the most prominent example of online reputation systems (Keser, 2003). The basic idea is to let parties rate each other, for example after the completion of a transaction, and use the aggregated ratings about a given party to derive a reputation score, which can assist other parties in deciding whether or not to transact with that party in the future. A natural effect is that it also provides an incentive for good behavior, and therefore tends to have a positive impact on market quality.

Reputation is generally defines as the opinion or view of one about the character of somebody or an entity. Here, an entity could be an agent, a product or a service. Reputation is frequently used as the basis of a judgment to trust an individual or organization particularly in the absence of previous direct experience or contact with them. Mui et al. (Lik Mui, 2002) define reputation "as a perception that an agent creates through past actions about its intentions and norms". A similar definition given by Abdul-Rahman et al. (Abdul-Rahman & Hailes, 2000) who defines "a reputation is an expectation about an agents behaviour based on information about or observations of its past behavior".

We will use the Concise Oxford dictionary definition of reputation for the purpose of this work. This definition supports the view of social network researchers (Jøsang et al., 2007).

Reputation: Reputation is what is generally said or believed about a persons or things character or standing.

2.3 Characteristics of Trust and Reputation

The characteristics of trust and reputation may differ from business to business or their applications. But there are some common delimiters that indicate the existence of general principles governing trust in online environments. Dimitrakos (Dimitrakos, 2003) surveyed and analyzed the general properties of trust in e-services and listed the general properties of trust (and distrust) as follows:

- Trust is relativised to some business transaction. A may trust B to drive her car but not to baby-sit.
- Trust is a measurable belief. A may trust B more than A trusts C for the same business.
- Trust is directed. A may trust B to be a profitable customer but B may distrust A to be a retailer worth buying from.
- Trust exists in time. The fact that A trusted B in the past does not in itself guarantee that A will trust B in the future. Bs performance and other relevant information may lead A to re-evaluate her trust in B.
- Trust evolves in time, even within the same transaction. During a business transaction, the more A realizes she can depend on B for a service X the more A trusts B. On the other hand, A's trust in B may decrease if B proves to be less dependable than A anticipated.
- Trust between collectives does not necessarily distribute to trust between their members. On the assumption that A trusts a group of contractors to deliver (as a group) in a collaborative project, one cannot conclude that A trusts each member of the team to deliver independently.

- Trust is reflexive, yet trust in oneself is measurable. A may trust her lawyer to win a case in court more than she trusts herself to do it. Self-assessment underlies the ability of an agent to delegate or offer a task to another agent in order to improve efficiency or reduce risk.
- Trust is a subjective belief. A may trust B more than C trusts B with the same trust scope.

Wang et al. (Wang & Vassileva, 2007) identifies that trust and reputation share some common characteristics such as context specific, multi-faceted and dynamic. They argue that trust and reputation both depend on some context. Even in the same context there is a need to develop differentiated trust in different aspects of a service. As the dynamic character, they refer that trust and reputation can increase or decrease with further experiences of interactions or observations. Both of them also decay with time. Jennifer (Golbeck, 2006) proposes there are three main properties of trust in the web-based social environment. They are (i) transitivity, (ii) asymmetry and (iii) personalization. She explains transitivity as the propagation capability, asymmetry as the direction of trust which may be different depends on the direction and personalization as the personal opinion on a particular object by different agents.

2.4 Difference between Trust and Reputation

Reputation systems are closely related to the concept of trust. Mui et al. (Lik Mui, 2002) differentiate the concepts of trust and reputation by defining reputation is the perception that an agent creates through past actions about its intentions and norms and trust as a subjective expectation an agent has about another's future behavior based on the history of their encounters. The difference between trust and reputation can be illustrated by the following perfectly normal and plausible statements:

1. I trust you because of your good reputation.
2. I trust you despite your bad reputation.

Statement (1) reflects that the relying party is aware of the trustee's reputation, and bases his or her trust on that. Statement (2) reflects that the relying party has some private knowledge about the trustee, e.g. through direct experience or intimate relationship, and that these factors overrule any reputation that the trustee might have. This observation reflects that trust ultimately is a personal and subjective phenomenon that is based on various factors or evidence, and that some of those carry more weight than others. Personal experience typically carries more weight than second hand recommendations or reputation, but in the absence of personal experience, trust often has to be based on reputation. Reputation can be considered as a collective measure of trustworthiness (in the sense of reliability) based on ratings from members in a community. Any individual's subjective trust in a given party can be derived from a combination of reputation and personal experience.

That an entity is trusted for a specific task does not necessarily mean that it can be trusted for everything. The scope defines the specific purpose and semantics of a given assessment of trust or reputation. A particular scope can be narrow or general. Although a particular reputation has a given scope, it can often be used as an estimate of the reputation of other scopes (Jøsang et al., 2007). In general, we may say that trust is the subjective view of an agent to another but reputation is overall impression of members of the community on an agent based on its previous activities.

3. A Survey of Online Trust and Reputation Systems Research

The issue of trust has been gaining an increasing amount of attention in a number of research communities including online service provision. There are many different views of how to measure and use trust. Some researchers use trust and reputation as same meaning while others are not. Though the meaning of trust is different to different people, a brief review on these models is a good starting point to research in the area of Trust and Reputation. As trust is a social phenomenon, the model of trust for the artificial world like Internet should be based on how trust works between people in society (Abdul-Rahman & Hailes, 2000). The rich literature growing around trust and reputation systems for Internet transactions, as well as the implementations of reputation systems in successful commercial application such as eBay and Amazon, give a strong indication that this is an important technology (Jøsang et al., 2007). Feedback on an online marketplace like eBay is an expression of reputation which provides a simple accumulative model for reputation (Sundaresan, 2007). In Amazons reputations scheme, reviews consist of a rating in the range between 1 and 5 stars. The average of all ratings gives a books reputation (Zou, Gu, Li, Xie, & Mei, 2007). Commercial implementations seem to have settled around relatively simple principles, whereas a multitude of different systems with advanced features are being proposed by the academic community. A general observation is that the proposals from the academic community so far lack coherence and are rarely evaluated in a commercial/industrial application environment. The systems being proposed are usually designed from scratch, and only in very few cases are authors building on proposals by other authors. The period we are in can therefore be seen as a period of pioneers. Consolidation around a set of sound and well recognized principles is needed in order to get the most benefit out of reputation systems.

Stephen Marsh (Marsh, 1994) is one of the pioneers to introduce a computational model for trust in the computing literature. For his PhD thesis, Marsh investigates the notions of trust in various contexts and develops a formal description of its use with distributed, intelligent agents. His model is based on social and psychological factors. He defines trust in three categories; namely the basic trust, general trust and situational trust. These trust values are used to help an agent to decide if it is worth it or not to cooperate with another agent. To calculate the risk and the perceived competence, different types of trust (basic, general and situational) are used. But the model is complex, mostly theoretical and difficult to implement. He did not considered reputation in his work. Zacharia et al. (Zacharia & Maes, 1999) have suggested that reputation in an on-line community can be related to the ratings that an agent receives from others. Their *Sporas* and *Histos* systems use the notions of global versus personalized reputation. Reputation in Sporas is similar to that used in eBay or Amazon, based on average of all ratings given to an agent. Sporas incorporates a measure of the reliability of the users' reputation based on the standard deviation of reputation values. Histos retrieves reputation based on who makes a query and the local environment surrounding the inquirer. It was designed as a response to the lack of personalization that Sporas reputation values have. The model can deal with direct information and witness information. Contrary to Sporas, the reputation value is a subjective property assigned particularly by each individual. Abdul-Rahman et al. (Abdul-Rahman & Hailes, 2000) proposed a model for supporting trust in virtual communities, based on direct experiences and reputation. They have proposed that the trust concept can be divided into direct and recommender trust. Recommended trust can be derived from word-of-mouth recommendations, which they consider as reputation. However, there are certain aspects of their model that are ad-hoc which limits the applicability of the model in broader scope. Schillo et al (Schillo, Funk, & Rovatsos, 2000) proposed a trust model for scenarios where interaction result is Boolean, either good or bad, between two agents trust relationship. Though, they did not consider the degrees of satisfaction. Resnick (Resnick et al., 2000) described reputation management as a system that collects, distributes and aggregates feedback about past behaviour.

Classification of Trust and Reputation Systems Research

Model Type	**Implementation Environment**	
	Centralized (Less complex system)	**Decentralized** (e.g. a peer-to-peer system))
Trust Management	Representative research examples: • Marsh 1994 • Schillo et al. 2000 • Esfandiari & Chandrasekharan 2001 • McKnight & Chervany, 2002 • Dimitrakos 2003 • Levien 2004 • Guha et al. 2004 • O'Donovan & Smyth 2005 • Ziegler 2005 • Pitsilis & Marshall, 2008	Representative research examples: • Golbeck 2006 • Ziegler & Golbeck 2007 • Coetzee & Eloff 2007 • Peng et al, 2008 • Tian et al, 2008
Reputation Management	Representative research examples: • Zacharia & Maes, 1999 • Resnick et al. 2000 • Malaga 2001 • Pujol et al. 2002 • Sen & Sajja 2002 • Carbo et al. 2002 • Carter et al. 2002 • Grishchenko 2004 • Folkerts 2005 • Whitby et al 2005	Representative research examples: • Aberer et al. 2001 • Damiani et al. 2002 • Yu & Singh 2002 • Kamvar et al. 2003 • Xiong 2005 • Jin et al, 2008
Trust & Reputation Management (Trust based reputation/ Reputation based trust))	Representative research examples: • Abdul-Rahman & Halies 2000 • Yu & Singh 2001 • Sabater & Sierra, 2005 • Mui et al. 2002 • Lin et al. 2005 • Jøsang et al. 2006, 2007 • Hussain & Chang 2007 • Silaghi et al. 2007 • Zou et al. 2007 • Xue & Fan, 2008 • Bi et al, 2008 • Bachrach, 2009	Representative research examples: • Venkatraman et al 2000 • Selcuk et al. 2004 • Nada et al. 2007 • Fuller et al 2007 • Sundaresan 2007 • Wang 2008, 2009 • Bharadwaj, 2009

Table 1. Research on trust and reputation systems

Venkatraman et al. (Venkatraman, Yu, & Singh, 2000) express their views of e-commerce community as a social network which supports reputations both for providing good services and for providing good referrals. Their model preserves the autonomy and privacy of the user by allowing the choice of ignoring such requests, if a user wishes not to give referrals.
Two one-on-one trust acquisition mechanisms are proposed by (Esfandiari & Chandrasekharan, 2001) in their trust model. The first is based on observation. They proposed the use of Bayesian networks and to perform the trust acquisition by Bayesian learning. The second trust acquisition mechanism is based on interaction. A simple way to calculate the interaction-based trust during the exploratory stage is using the formula

$$T_{inter}(A,B) = \frac{number\_of\_correct\_replies}{total\_number\_of\_replies} \quad (1)$$

(Sen & Sajja, 2002) present a method for ensuring robustness of a reputation model that is used to select processor resources. The model uses service selection as its measure of success. In this model, an agent selects the service provider that has the highest reputation from a pool. Aberer et al. (Aberer, 2001) describe a reputation system for Peer-to-peer (P2P) systems which is intended to meet needs that are left unfulfilled by other reputation systems: scalability to large numbers of nodes, and reduced amounts of required data storage and network communications. In order to reduce the amount of data stored and communicated, the model works on a binary rating system – an agent is either considered trustworthy or not. In the model proposed by Yu and Singh (Yu & Singh, 2002), the information stored by an agent about direct interactions is a set of values that reflect the quality of these interactions. Only the most recent experiences with each concrete partner are considered for the calculations. This model failed to combine direct information with witness information. When direct information is available, it is considered the only source to determine the trust of the target agent. Only when the direct information is not available, the model appeals to witness information.
Sabater et al. (Sabater & Sierra, 2005) have proposed a modular trust and reputation system oriented to complex small/mid-size e-commerce environments which they called ReGreT, where social relations among individuals play an important role. Mui et al. (Lik Mui, 2002) proposed a computational model based on sociological and biological understanding. The model can be used to calculate agent's trust and reputation scores. They also identified some weaknesses of the trust and reputation study which is the lack of differentiation of trust and reputation and the mechanism for inference between them is not explicit. Trust and reputation are taken to be the same across multiple contexts or are treated as uniform across time and the existing computational models for trust and reputation are often not grounded on understood social characteristics of these quantities. They did not examine effects of deception in this model. Pujol (Pujol et al, 2002) proposed a method for calculating the reputation of a given member in a society or in a social network by making use of PageRank™ algorithm. Dimitrakos (Dimitrakos, 2003) presented and analysed a service-oriented trust management framework based on the integration of role-based modelling and risk assessment in order to support trust management solutions. They provided evidence of emerging methods, formalisms and conceptual frameworks which, if appropriately integrated, can bridge the gap between systems modelling, trust and risk management in e-commerce.

Kamvar et al. (Kamvar, Schlosser, & Garcia-Molina, 2003) proposes a reputation system which makes use of matrices of reputation information which are maintained and stored by agents in their system. The authors explicitly target their system at providing reputation for peer-to-peer systems where malicious peers can generate illegitimate files for sharing and the general population of peers have no way of distinguishing illegitimate files from the legitimate ones. O'Donovan et al (O'Donovan & Smyth, 2005) distinguished between two types of profiles in the context of a given recommendation session or rating prediction. The consumer profile and the producer profile. They described "trust" as the reliability of a partner profile to deliver accurate recommendations in the past. They described two models of trust which they called profile-level trust and item-level trust. Selcuk et al. (Selcuk, Uzun, & Pariente, 2004) proposed a reputation-based trust management protocol for P2P networks where users rate the reliability of the parties they deal with and share this information with their peers.

Guha et al (Guha, Kumar, Raghavan, & Tomkins, 2004) proposed a method based on PageRank™ algorithm for propagating both trust and distrust. They identified four different methods for propagating the net beliefs values, namely direct propagation, co-citation, transpose and coupling. The Advogato maximum flow trust metric has been proposed by Levien (Levien, 2004) in order to discover which users are trusted by members of an online community and which are not. Trust is computed through one centralized community server and considered relative to a seed of users enjoying supreme trust. Local group trust metrics compute sets of agents trusted by those being part of the trust seed. Advogato, only assigns Boolean values indicating presence or absence of trust. It is a global trust algorithm which uses the same trusted nodes to make trust calculation for all users. It makes the algorithm suitable for P2P networks. As the trust inference algorithm has released under a free software license, it became the basis of many research paper. Appleseed trust metric was proposed by Ziegler (Zieglera, 2005). AppelSeed is closely based on PageRank™ algorithm. It allows rankings of agents with respect to trust accorded. One of the major weakness is that a person who has made many high trust ratings will have lower value than if only one or two people had been rated. Another weakness of this model is; it requires exponentially higher computation with increasing number of user which makes it non-scalable.

Shmatikov et al. (Shmatikov & Talcott, 2005) proposed a reputation-based trust management model which allows mutually distrusting agents to develop a basis for interaction in the absence of central authority. The model is proposed in the context of peer-to-peer applications, online games or military situations. Folkerts (Folkerts, 2005) proposed a simulation framework to perform comparison analysis between reputation models. They have implemented two reputation models and compared with regard to accuracy, performance and resistance to deception. Teacy (Teacy, 2005) proposed a probabilistic framework for assessing trust based on direct observations of a trustees behavior and indirect observations made by a third party. They claimed that their proposed mechanism can cope with the possibility of unreliable third party information in some context. Xiong (Xiong, 2005) also proposed a decentralized reputation based trust supporting framework called PeerTrust for P2P environment. The have focused on models and techniques for resilient reputation management against feed back aggregation, feedback oscillation and loss of feedback privacy. Jøsang (Jøsang et al, 2006) proposed a model for trust derivation with Subjective Logic. They argued that Subjective logic represents a practical belief calculus which can be used for calculative analysis trust networks. TNASL requires trust relationships to be expressed as beliefs, and trust networks to be expressed as DSPGs in the form of canonical expressions. They have described how trust can be derived with the belief calculus of subjective logic. Xue and Fan (Xue & Fan, 2008) proposed a new trust model for the Semantic Web which allows agents to decide which among different sources of information to trust and thus act rationally on the semantic web. Tian et al (Tian, Zou, Wang, & Cheng, 2008) proposed trust model for P2P networks in which the trust value of a given peer was computed using its local trust information and recommendation from other nodes.

4. Trust Network Analysis

Trust networks consist of transitive trust relationships between people, organisations and software agents connected through a medium for communication and interaction. By formalising trust relationships, e.g. as reputation scores or as subjective trust measures, trust between parties within a domain can be derived by analysing the trust paths linking the parties together. A method for trust network analysis using subjective logic (TNA-SL) has been described by Jøsang et al (2006, 2007). TNA-SL takes directed trust edges between pairs as input, and can be used to derive a level of trust between arbitrary parties that are interconnected through the network. Even in case of no explicit trust paths between two parties exist; subjective logic allows a level of trust to be derived through the default vacuous opinions. TNA-SL therefore has a general applicability and is suitable for many types of trust networks. A potential limitation with the previously described TNA-SL is that complex trust networks must be simplified to *series-parallel* networks in order for TNA-SL to produce consistent results. The simplification consisted of gradually removing the least certain trust paths until the whole network can be represented in a series-parallel form. As this process removes information it is intuitively sub-optimal.

In the following sections, we describe how TNA-SL can preserve consistency without removing information. Inconsistency can result from dependence between separate trust paths, which when combined will take the same information into account several times. Including the same trust edges multiple times will by definition produce an inconsistent result. Optimal TNA-SL avoids this problem by allowing the trust measure of a given trust edge to be split into several independent parts, so that each part is taken into account by separate trust paths. The result of this approach is compared with the analysis based on networks simplification.

4.1 Serial Trust Paths

Trust transitivity means, for example, that if A trusts B who trusts D, then A will also trust D. This assumes that A is actually aware that B trusts D. This could be achieved through a *recommendation* from B to A as illustrated in Fig.1, where the indexes on each arrow indicate the sequence in which the trust relationships/recommendation is formed.

It can be shown that trust is not always transitive in real life (Christianson, 2003). For example the fact that A trusts B to look after her child, and B trusts D to fix his car, does not imply that A trusts D for looking after her child, or for fixing her car. However, under certain semantic constraints (Jøsang and Pope, 2005), trust can be transitive, and a trust system can be used to derive trust. In the last example, trust transitivity collapses because the scopes of A's and B's trust are different. Trust scope is defined as the specific type(s) of trust assumed in a given trust relationship.

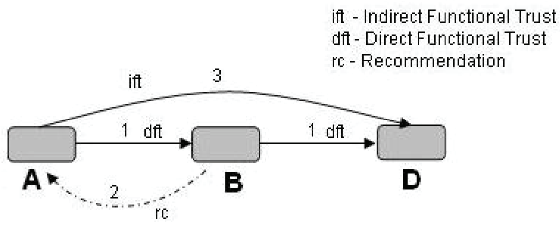

Fig. 1. Trust transitivity

It is important to separate between trust in the ability to recommend a good car mechanic which represents *referral trust*, and trust in actually being a good car mechanic which represents *functional trust*. The scope of the trust is nevertheless the same, namely to be a good car mechanic. Assuming that, on several occasions, B has proved to A that he is knowledgeable in matters relating to car maintenance, A's referral trust in B for the purpose of recommending a good car mechanic can be considered to be *direct*. Assuming that D on several occasions has proved to B that he is a good mechanic, B's functional trust in D can also be considered to be direct. Thanks to B's advice, A also trusts D to actually be a good mechanic. However, this functional trust must be considered to be *indirect*, because A has not directly observed or experienced D's skills in car mechanics. Let us slightly extend the example, wherein B does not actually know any car mechanics himself, but he knows C, whom he believes knows a good car mechanic. As it happens, C is happy to recommend the car mechanic named D. As a result of transitivity, A is able to derive trust in D, as illustrated in Fig.2, where the indexes indicate the order in which the trust relationships and recommendations are formed. The "drt" denotes direct referral trust, "dft" denotes direct functional trust, and "ift" denotes indirect functional trust.

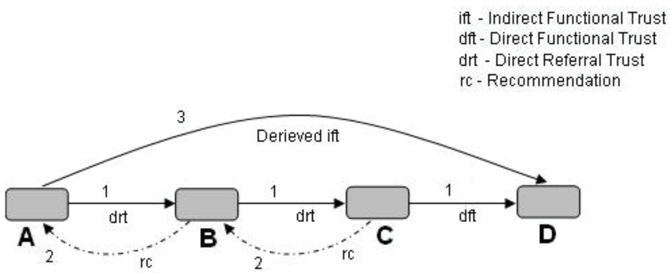

Fig. 2. Serial trust path

The "referral" variant of a trust scope can be considered to be recursive, so that any transitive trust chain, with arbitrary length, can be expressed using only one trust scope with two variants. This principle can be expressed as the derivation of functional trust through referral trust, requires that the last trust edge represents functional trust and all previous trust edges represent referral trust. It could be argued that negative trust in a transitive chain can have the paradoxical effect of strengthening the derived trust. Take for example the case of Fig.1, but in this case A distrusts B, and B distrusts D. In this situation, A might actually derive positive trust in D, since she does not believe B when he says: "D is bad mechanic, do not use him". So the fact that B recommends distrusts in D might count as a pro-D argument from A's perspective. The question boils down to "is the enemy of my enemy my friend?''. However this question relates to how multiple types of untrustworthiness, such as dishonesty and unreliability, should be interpreted in a trust network.

4.2 Parallel Trust Paths

It is common to collect advice from several sources in order to be better informed when making decisions. This can be modelled as *parallel trust combination* illustrated in Fig.3, where again the indexes indicate the order in which the trust relationships and recommendations are formed.

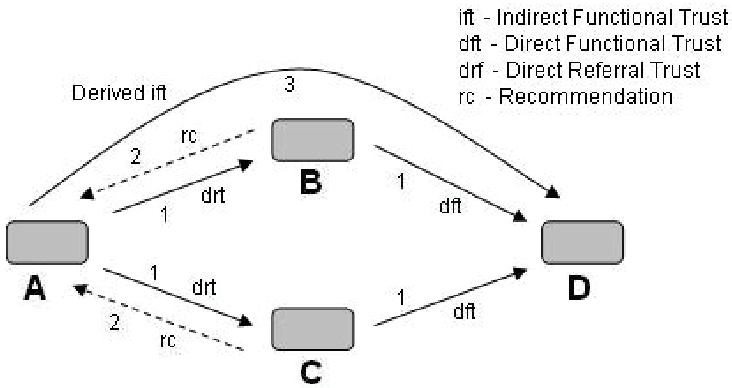

Fig. 3. Parallel trust paths

Let us assume again that A needs to get her car serviced, and that she asks B to recommend a good car mechanic. When B recommends D, A would like to get a second opinion, so she asks C whether she has heard about D. Intuitively, if both B and C recommend D as a good car mechanics, A's trust in D will be stronger than if she had only asked B. Parallel combination of positive trust thus has the effect of strengthening the derived trust. In the case where A receives conflicting recommended trust, e.g. trust and distrust at the same time, she needs some method for combining these conflicting recommendations in order to derive her trust in D. Our method, which is described in Sec.7, is based on subjective logic which easily can handle such cases. Subjective logic is suitable for analysing trust networks because trust relationships can be expressed as subjective opinions with degrees of uncertainty.

4.3 Operators for Deriving Trust

Subjective logic is a belief calculus specifically developed for modeling trust relationships. In subjective logic, beliefs are represented on binary state spaces, where each of the two possible states can consist of sub-states. Belief functions on binary state spaces are called *subjective opinions* and are formally expressed in the form of an ordered tuple ω_x^A = (b, d, u, a), where b, d, and u represent belief, disbelief and uncertainty respectively where $b, d, u \in [0, 1]$ and $b+d+u = 1$. The base rate parameter $a \in [0, 1]$ represents the base rate probability in the absence of evidence, and is used for computing an opinion's probability expectation value $E(\omega_x^A) = b + au$, meaning that a determines how uncertainty shall contribute to $E(\omega_x^A)$. A subjective opinion is interpreted as an agent A's belief in the truth of statement x. Ownership of an opinion is represented as a superscript so that for example A's opinion about x is denoted as ω_x^A.

The fact that subjective logic is compatible with binary logic and probability calculus means that whenever corresponding operators exist in probability calculus, the probability expectation value $E(\omega)$ of an opinion ω that has been derived with subjective logic, is always equal to the probability value that would have been derived had simple probability calculus been applied. Similarly, whenever corresponding binary logic operators exist, an absolute opinion (i.e. equivalent to binary logic TRUE or FALSE) derived with subjective logic, is always equal to the truth value that can be derived with binary logic. Subjective logic has a sound mathematical basis and is compatible with binary logic and traditional Bayesian analysis. Subjective logic defines a rich set of operators for combining subjective opinions in various ways (Jøsang, 2009). Some operators represent generalizations of binary logic and probability calculus, whereas others are unique to belief calculus because they depend on belief ownership. With belief ownership it is possible to explicitly express that different agents have different opinions about the same issue.

The advantage of subjective logic over probability calculus and binary logic is its ability to explicitly express and take advantage of ignorance and belief ownership. Subjective logic can be applied to all situations where probability calculus can be applied, and to many situations where probability calculus fails precisely because it can not capture degrees of ignorance. Subjective opinions can be interpreted as probability density functions, making subjective logic a simple and efficient calculus for probability density functions. Subjective logic defines a number of operators. Some operators represent generalizations of binary logic and probability calculus operators, whereas others are unique to belief theory because they depend on belief ownership. Here we will only focus on the *transitivity* and the *fusion* operators. The transitivity operator can be used to derive trust from a trust path consisting of a chain of trust edges, and the fusion operator can be used to combine trust from parallel trust paths. These operators are described below.

Transitivity is used to compute trust along a chain of trust edges. Assume two agents A and B where A has referral trust in B, denoted by ω_B^A, for the purpose of judging the functional or referral trustworthiness of C. In addition B has functional or referral trust in C, denoted by ω_C^B. Agent A can then derive her trust in C by discounting B's trust in C with A's trust in B, denoted by $\omega_C^{A:B}$. By using the symbol '$\otimes$' to designate this operator, we define

$$\omega_C^{A:B} = \omega_B^A \otimes \omega_C^B \qquad \begin{cases} b_C^{A:B} = b_B^A b_C^B \\ d_C^{A:B} = b_B^A d_C^B \\ u_C^{A:B} = d_B^A + u_B^A + b_B^A u_C^B \\ a_C^{A:B} = a_C^B. \end{cases} \tag{2}$$

The effect of discounting in a transitive chain is that uncertainty increases, not disbelief. Cumulative *Fusion* is equivalent to Bayesian updating in statistics. The cumulative fusion of two possibly conflicting opinions is an opinion that reflects both opinions in a fair and equal way. Let ω_C^A and ω_C^B be *A*'s and *B*'s trust in *C* respectively. The opinion $\omega_C^{A\Diamond B}$ is then called the fused trust between ω_C^A and ω_C^B, denoting an imaginary agent [*A*,*B*]'s trust in *C*, as if she represented both *A* and *B*. By using the symbol '$\oplus$' to designate this operator, we define $\omega_C^{A\Diamond B} = \omega_B^A \oplus \omega_C^B$

$$\omega_C^{A\Diamond B} = \omega_B^A \oplus \omega_C^B \quad \begin{cases} b_C^{A\Diamond B} = (b_C^A u_C^B + b_C^B u_C^A)/(u_C^A + u_C^B - u_C^A u_C^B) \\ d_C^{A\Diamond B} = (d_C^A u_C^B + d_C^B u_C^A)/(u_C^A + u_C^B - u_C^A u_C^B) \\ u_C^{A\Diamond B} = (u_C^A u_C^B)/(u_C^A + u_C^B - u_C^A u_C^B) \\ a_C^{A\Diamond B} = a_C^A. \end{cases} \tag{3}$$

where it is assumed that $a_C^A = a_C^B$. Limits can be computed (Jøsang, 2007) for $u_C^A = u_C^B = 0$. The effect of the cumulative fusion operator is to amplify belief and disbelief and reduce uncertainty.

4.4 Example Derivation of Trust Measures

The transitivity and fusion operators will be used for the purpose of deriving trust measures applied to the trust graph of Fig.2 and Fig.3.

In case of Fig.2, the edge trust values will all be set equal as:

$$\omega_B^A = \omega_C^B = \omega_D^C = (0.9, 0.0, 0.1, 0.5) \tag{4}$$

By applying the transitivity operator to the expression of Eq.(2), the derived trust value evaluates to:

$$\omega_D^{A:B:C} = \omega_B^A \otimes \omega_C^B \otimes \omega_D^C = (0.729, 0.000, 0.271, 0.5) \tag{5}$$

In case of Fig.3, the edge trust values will all be set equal as:

$$\omega_B^A = \omega_D^B = \omega_C^A = \omega_D^C = (0.9, 0.0, 0.1, 0.5) \tag{6}$$

By applying the transitivity and cumulative fusion operators to the expression of Eq(3), the derived indirect trust measure can be computed. The expression for the derived trust measure and the numerical result is given below.

$$\omega_D^A = (\omega_B^A \otimes \omega_D^B) \oplus (\omega_C^A \otimes \omega_D^C) = (0.895, 0.000, 0.105, 0.5) \tag{7}$$

5. Trust Fusions of Opinion

Computational trust allows new trust relationships to be derived from pre-existing trust relationship through mathematical computations. Trust fusion is an important element in computational trust, meaning that A can combine B's recommendation with her own personal experience in dealing with C, or with other recommendations about C, in order to derive a more reliable measure of trust in C. These simple principles, which are essential for human interaction in business and everyday life, manifest it in many different forms. This section identifies the parameter dependence problem in trust fusions and investigates possible formal computational models that can be implemented using belief reasoning based on subjective logic. We have proposed three definitions of trust fusion for independent, dependent and partially dependent opinions. We explain the definitions by respective examples. With adequate computational trust models, the principles of trust propagation can be ported to online communities of people, organizations and software agents, with the purpose of enhancing the quality of those communities.

5.1 Fusion of Independent Trust

This operator is most naturally expressed in the evidence space, so we define it first and subsequently map it over to the opinion space.

Definition 1 (Consensus Operator for Independent Opinions).
Let $\omega_x^A=(b_x^A, d_x^A, u_x^A, a_x^A)$ and $\omega_x^B=(b_x^B, d_x^B, u_x^B, a_x^B)$ be trust in x from A and B respectively. The opinion $\omega_x^{A\Diamond B}=(b_x^{A\Diamond B}, d_x^{A\Diamond B}, u_x^{A\Diamond B}, a_x^{A\Diamond B})$ is then called the consensus between ω_x^A and ω_x^B, denoting the trust that an imaginary agent [A,B] would have in x, as if that agent represented both A and B. In case of Bayesian (totally certain) opinions, their relative weight can be defined as $\gamma^{A/B} = \lim(u_x^B / u_x^A)$.

Case I:

$$u_x^A + u_x^B - u_x^A u_x^B \neq 0$$

$$\begin{cases} b_x^{A\Diamond B} = \dfrac{b_x^A u_x^B + b_x^B u_x^A}{u_x^A + u_x^B - u_x^A u_x^B} \\ d_x^{A\Diamond B} = \dfrac{d_x^A u_x^B + d_x^B u_x^A}{u_x^A + u_x^B - u_x^A u_x^B} \\ u_x^{A\Diamond B} = \dfrac{u_x^A u_x^B}{u_x^A + u_x^B - u_x^A u_x^B} \\ a_x^{A\Diamond B} = \dfrac{a_x^A u_x^B + a_x^B u_x^A - (a_x^A + a_x^B) u_x^A u_x^B}{u_x^A + u_x^B - 2u_x^A u_x^B} \end{cases}.$$

Case2:

$$u_x^A + u_x^B - u_x^A u_x^B) = 0$$

$$\begin{cases} b_x^{A\Diamond B} = \dfrac{(\gamma^{A/B} b_x^A + b_x^B)}{(\gamma^{A/B} + 1)} \\ d_x^{A\Diamond B} = \dfrac{(\gamma^{A/B} d_x^A + d_x^B)}{(\gamma^{A/B} + 1)} \\ u_x^{A\Diamond B} = 0 \\ a_x^{A\Diamond B} = \dfrac{\gamma^{A/B} a_x^A + a_x^B)}{\gamma^{A/B} + 1} \end{cases}.$$

By using the symbol '$\oplus$' to designate this operator, we can $\omega_x^{A\diamond B} = \omega_x^A \oplus \omega_x^B$.

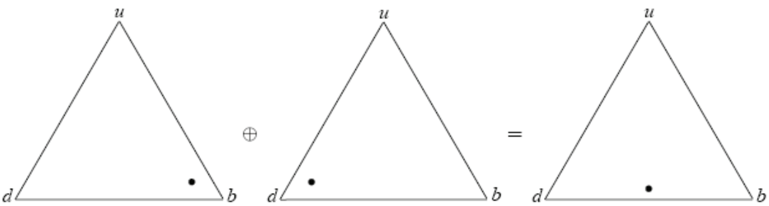

Fig. 4. Example of applying the consensus operator for fusing independent trust

It can be shown that $\oplus$ is both commutative and associative which means that the order in which opinions are combined has no importance. Opinion independence must be assured, which obviously translates into not allowing an entity's opinion to be counted more than once. The effect of independent consensus is to reduce uncertainty. For example the case where several witnesses give consistent testimony should amplify the judge's opinion, and that is exactly what the operator does. Consensus between an infinite number of not totally uncertain (i.e. u < 1) opinions would necessarily produce a consensus opinion with u = 0. Fig.1 illustrates an example of applying the consensus operator for independent opinions where ω_x^A = {0.8, 0.1, 0.1, a} and ω_x^B = {0.1, 0.8, 0.1, a}, so that $\omega_x^{A\diamond B} = \omega_x^A \oplus \omega_x^B$ = {0.47, 0.47, 0.06, a} .

5.2 Fusion of Dependent Trust

Assume two agents *A* and *B* having simultaneously observed the same process. Because their observations are identical, their respective opinions will necessarily be dependent, and a consensus according to Def.1 would be meaningless. If the two observers have made exactly the same observations, and their estimates are equal, it is sufficient to take only one of the estimates into account. However, although two observers witness the same phenomenon, it is possible (indeed, likely) that they record and interpret it differently. The observers may have started and ended the observations at slightly different times; one of them may have missed or misinterpreted some of the events, resulting in varying, but still dependent opinions.

Definition 2 (Consensus Operator for Dependent Opinions).

Let $\omega_x^{A_i} = (b_x^{A_i}, d_x^{A_i}, u_x^{A_i}, a_x^{A_i})$ where i $\in$ [1, n], be n dependent opinions respectively held by agents A_1, ...,A_n about the same proposition x. The depended consensus is then $\omega_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n} = b_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n}, d_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n}, u_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n}, a_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n}$ where:

$$
\begin{cases}
b_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n} = \dfrac{\sum_1^n (b_x^{A_i}/\mathrm{u}_x^{A_i})}{\sum_1^n (b_x^{A_i}/\mathrm{u}_x^{A_i}) + \sum_1^n (d_x^{A_i}/\mathrm{u}_x^{A_i}) + n} \\
d_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n} = \dfrac{\sum_1^n (d_x^{A_i}/\mathrm{u}_x^{A_i})}{\sum_1^n (b_x^{A_i}/\mathrm{u}_x^{A_i}) + \sum_1^n (d_x^{A_i}/\mathrm{u}_x^{A_i}) + n} \\
u_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n} = \dfrac{n}{\sum_1^n (b_x^{A_i}/\mathrm{u}_x^{A_i}) + \sum_1^n (d_x^{A_i}/\mathrm{u}_x^{A_i}) + n} \\
a_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n} = \dfrac{\sum_1^n a_x^{A_i}}{n}
\end{cases}
$$

where all the $u_x^{A_i}$ are different from zero. By using the symbol $\underline{\oplus}$ to designate this operation, we get $\omega_x^{A_1\underline{\diamond}\ldots\underline{\diamond}A_n} = \omega_x^{A_1} \,\underline{\oplus} \ldots \underline{\oplus}\, \omega_x^{A_n}$.

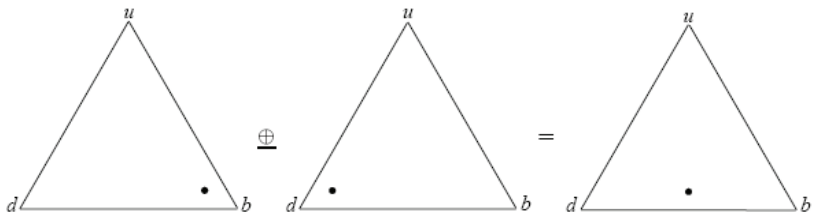

Fig. 5. Example of applying the consensus operator for dependent opinions

The $\underline{\oplus}$ operator is both commutative and associative. The effect of the dependent consensus operator is to produce an opinion which is based on an average of positive and an average of negative evidence. Fig.2 illustrates an example of applying the consensus operator for dependent opinions where ω_x^A = {0.8, 0.1, 0.1, a} and ω_x^B = {0.1, 0.8, 0.1, a}, so that $\omega_x^{A\underline{\diamond}B}$ = $\omega_x^A \,\underline{\oplus}\, \omega_x^B$ = {0.45, 0.45, 0.10, a} .

5.3 Fusion of Trust Under Partial Dependence

Let two agents *A* and *B* observed the same process during two partially overlapping periods. If it is known exactly which events were observed by both, one of the agents can simply dismiss these observations, and their opinions will be independent. However, it may not always be possible to determine which observations are identical.

Fig.6 illustrates a situation of partly dependent observations. Assuming that the fraction of overlapping observations is known, the dependent and the independent parts of their observations can be estimated, so that a consensus operator can be defined.

In the figure, $\omega_x^{Ai(B)}$ and $\omega_x^{Bi(A)}$ represent the independent parts of *A* and *B*'s opinions, whereas $\omega_x^{Ad(B)}$ and $\omega_x^{Bd(A)}$ represent their dependent parts.

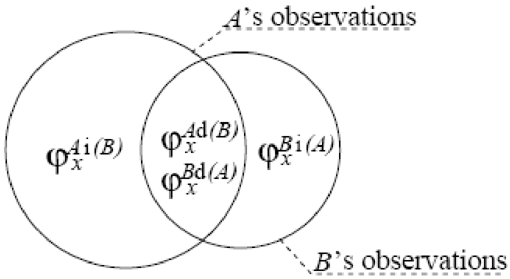

Fig. 6. Beta PDFs based on partly dependent observations

The representation of dependent and independent opinions can be defined by using reciprocal dependence factors denoted by $\lambda^{Ad(B)}$ and $\lambda^{Bd(A)}$.

$$\omega_x^{Ai(B)}:\begin{cases} b_x^{Ai(B)} = b_x^A \mu_x^{Ai(B)} \\ d_x^{Ai(B)} = d_x^A \mu_x^{Ai(B)} \\ u_x^{Ai(B)} = u_x^A \mu_x^{Ai(B)} /(1-\lambda_x^{Ad(B)}, \end{cases} \qquad \mu_x^{Ai(B)} = \frac{1-\lambda_x^{Ad(B)}}{(1-\lambda_x^{Ad(B)})(b_x^A + d_x^A) + u_x^A}$$

$$\omega_x^{Ad(B)}:\begin{cases} b_x^{Ad(B)} = b_x^A \mu_x^{Ad(B)} \\ d_x^{Ad(B)} = d_x^A \mu_x^{Ad(B)} \\ u_x^{Ad(B)} = u_x^A \mu_x^{Ad(B)} / \lambda_x^{Ad(B)}, \end{cases} \qquad \mu_x^{Ad(B)} = \frac{\lambda_x^{Ad(B)}}{\lambda_x^{Ad(B)}(b_x^A + d_x^A) + u_x^A}$$

$$\omega_x^{Bi(A)}:\begin{cases} b_x^{Bi(A)} = b_x^B \mu_x^{Bi(A)} \\ d_x^{Bi(A)} = d_x^B \mu_x^{Bi(A)} \\ u_x^{Bi(A)} = u_x^B \mu_x^{Bi(A)} /(1-\lambda_x^{Bd(A)}, \end{cases} \qquad \mu_x^{Bi(A)} = \frac{1-\lambda_x^{Bd(A)}}{(1-\lambda_x^{Bd(A)})(b_x^B + d_x^B) + u_x^B}$$

$$\omega_x^{Bd(A)}:\begin{cases} b_x^{Bd(A)} = b_x^B \mu_x^{Bd(A)} \\ d_x^{Bd(A)} = d_x^B \mu_x^{Bd(A)} \\ u_x^{Bd(A)} = u_x^B \mu_x^{Bd(A)} / \lambda_x^{Ad(A)}, \end{cases} \qquad \mu_x^{Bd(A)} = \frac{\lambda_x^{Bd(A)}}{\lambda_x^{Bd(A)}(b_x^B + d_x^B) + u_x^B} \qquad (8)$$

Having specified the separate dependent and independent parts of two partially dependent opinions, we can now define the consensus operator for partially dependent opinions.

Definition 3 (Consensus Operator for Partially Dependent Opinions).
Let A and B have the partially dependent opinions ω_x^A and ω_x^B respectively, about the same proposition x, and let their dependent and independent parts be expressed according to Eq.(8).We will use the symbol $\tilde{\oplus}$ to designate consensus between partially dependent opinions. As before $\underline{\oplus}$ is the operator for entirely dependent opinions. The consensus of A and B's opinions can then be written as:

$$
\begin{aligned}
&\omega_x^A \,\tilde{\oplus}\, \omega_x^B \\
&= \omega_x^{A \tilde{\lozenge} B} \\
&= \omega_x^{(Ad(B)\underline{\lozenge}Bd(A))\lozenge Ai(B)\lozenge Bi(A)} \\
&= (\omega_x^{Ad(B)} \underline{\oplus} \omega_x^{Bd(A)}) \oplus \omega_x^{Ai(B)} \oplus \omega_x^{Bd(A)}
\end{aligned}
\tag{9}
$$

It could be proved that for any opinion ω_x^A with a dependence factor $\lambda_x^{Ad(B)}$ to any other opinion ω_x^B the following equality holds:

$$\omega_x^A = \omega_x^{Ai(B)} \oplus \omega_x^{Ad(B)} \tag{10}$$

6. Trust Paths Dependency and Network Simplification

Transitive trust networks can involve many principals, and in the examples below, capital letters *A,B,C* and *D* will be used to denote principals We will use basic constructs of directed graphs to represent transitive trust networks, and add some notation elements which allow us to express trust networks in a structured way. A single trust relationship can be expressed as a directed edge between two nodes that represent the trust source and the trust target of that edge. For example the edge [*A,B*] means that *A* trusts *B*. The symbol ":" will be used to denote the transitive connection of two consecutive trust edges to form a transitive trust path. The trust relationships of Fig.1 can be expressed as:

$$([A,D]) = ([A,B]:[B,C]:[C,D]) \tag{11}$$

where the trust scope is implicit. Let the trust scope e.g. be defined as σ: "*trust to be a good car mechanic*". Let the functional variant be denoted by "fσ" and the referral variant by "rσ". A distinction can be made between initial *direct trust* and derived *indirect trust*. Whenever relevant, the trust scope can be prefixed with "d" to indicate direct trust (dσ), and with "i" to indicate indirect trust (iσ). This can be combined with referral and functional trust, so that for example indirect functional trust can be denoted as "ifσ". A reference to the trust scope can then be explicitly included in the trust edge notation as e.g. denoted by $[A,B,\mathrm{dr}\sigma]$. The trust network of Fig.2 can then be explicitly expressed as:

$$([A,B,\mathrm{if}\sigma]) = ([A,B,\mathrm{dr}\sigma]:[B,C,\mathrm{df}\sigma]:[C,D,\mathrm{df}\sigma] \tag{12}$$

Let us now turn to the combination of parallel trust paths, as illustrated in Fig.3. We will use the symbol "◊" to denote the graph connector for this purpose. The "◊" symbol visually resembles a simple graph of two parallel paths between a pair of agents, so that it is natural to use it for this purpose. In short notation, A's combination of the two parallel trust paths from her to D in Fig.3 is then expressed as:

$$([A,D]) = (([A,B]:[B,D]) \lozenge ([A,C]:[C,D])) \tag{13}$$

It can be noted that Fig.3 contains two parallel paths.

Trust networks can have dependent paths. This is illustrated on the left-hand side of Fig.7. The expression for the graph on the left-hand side of Fig7 would be:

$$([A,D]) = (([A,B]:[B,D]) \lozenge ([A,C]:[C,D]) \lozenge ([A,B]:[B,C]:[C,D])) \tag{14}$$

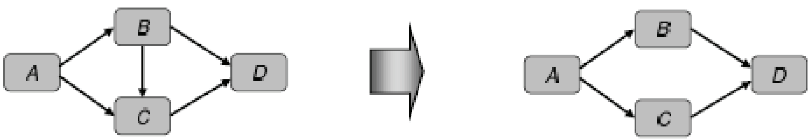

Fig. 7. Network simplification by removing weakest path

A problem with Eq.(14) is that the arcs $[A,B]$ and $[C,D]$ appear twice, and the expression is therefore not canonical. Trust network analysis with subjective logic may produce inconsistent results when applied directly to non-canonical expressions. It is therefore desirable to express graphs in a form where an arc only appears once. A canonical expression can be defined as an expression of a trust graph in structured notation where every edge only appears once.

A method for canonicalization based on network simplification was described in (Jøsang, 2006). Simplification consists of removing the weakest, i.e. the least certain paths, until the network becomes a directed series-parallel network which can be expressed on a canonical form. Assuming that the path $([A,B]:[B,C]:[C,D])$ is the weakest path in the graph on the left-hand side of Fig.7, network simplification of the dependent graph would be to remove the edge $[B,C]$ from the graph, as illustrated on the right-hand side of Fig.7. Since the simplified graph is equal to that of Fig.3, the formal expression is the same as Eq.(13).

7. Trust Network Canonicalization by Node Splitting

The existence of a dependent edge in a graph is recognized by multiple instances of the same edge in the trust network expression. Node splitting is a new approach to achieving independent trust edges. This is achieved by splitting the target edge of a given dependent edge into as many different nodes as there are different instances of the same edge in the exploded notation. A general directed trust graph is based on directed trust edges between pairs of nodes. It is desirable not to put any restrictions on the possible trust arcs except that they should not be cyclic. This means that the set of possible trust paths from a given source X to a given target Y can contain dependent paths. The left-hand side of Fig.8 shows an example of a trust network with dependent paths.

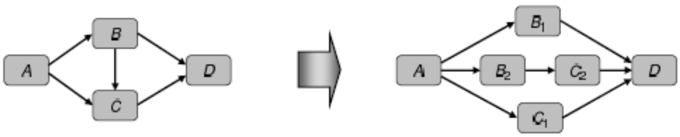

Fig. 8. Node splitting of trust network to produce independent paths

In the non-canonical expression for the left-hand side trust network of Fig.8:

$$([A,D]) = ([A,B]:[B,D]) \lozenge ([A,C]:[C,D]) \lozenge ([A,B]:[B,C]:[C,D])) \tag{15}$$

the edges $[A,B]$ and $[C,D]$ appear twice. Node splitting in this example consists of splitting the node B into B_1 and B_2, and the node C into C_1 and C_2. This produces the right-hand side trust network in Fig.5 with canonical expression:

$$([A,D]) = ([A,B_1]:[B_1,D]) \lozenge ([A,C_1]:[C_1,D]) \lozenge ([A,B_2]:[B_2,C_2]:[C_2,D])) \tag{16}$$

Node splitting must be translated into opinion splitting in order to apply subjective logic. The principle for opinions splitting will be to separate the opinion on the dependent edge into two independent opinions that when cumulatively fused produce the original opinion. This can be called fission of opinions, and will depend on a fission factor ø that determines the proportion of evidence assigned to each independent opinion part. The mapping of an opinion ω = (b,d,u,a) to Beta evidence parameters Beta(r,s,a) and linear splitting into two parts Beta(r_1,s_1,a_1) and Beta(r_2,s_2,a_2) as a function of the fission factor ø is:

$$\text{Beta}(r_1,s_1,a_1): \begin{cases} r_1 = \dfrac{\phi 2b}{u} \\ s_1 = \dfrac{\phi 2d}{u} \\ a_1 = a \end{cases} \qquad \text{Beta}(r_2,s_2,a_2): \begin{cases} r_2 = \dfrac{(1-\phi)2b}{u} \\ s_2 = \dfrac{(1-\phi)2d}{u} \\ a_2 = a \end{cases} \tag{17}$$

The reverse mapping of these evidence parameters into two separate opinions according to Eq.(2) produces:

$$\omega_1 : \begin{cases} b_1 = \dfrac{\phi b}{\phi(b+d)+u} \\ d_1 = \dfrac{\phi d}{\phi(b+d)+u} \\ u_1 = \dfrac{u}{\phi(b+d)+u} \\ a_1 = a \end{cases} \qquad \omega_2 : \begin{cases} b_2 = \dfrac{(1-\phi)b}{(1-\phi)(b+d)+u} \\ d_2 = \dfrac{(1-\phi)d}{(1-\phi)(b+d)+u} \\ u_2 = \dfrac{u}{(1-\phi)(b+d)+u} \\ a_2 = a \end{cases} \tag{18}$$

It can be verified that $\omega_1 \oplus \omega_2 = \omega$, as expected.

When deriving trust values from the cannibalized trust network of Eq.(14) we are interested in knowing its certainty level as compared with a simplified network. We are interested in the expression for the uncertainty of ω_D^A corresponding to trust expression of Eq.(16). Since the node splitting introduces parameters for splitting opinions, the uncertainty will be a function of these parameters. By using Eq.(2) the expressions for the uncertainty in the trust paths of Eq.(16) can be derived as:

$$\begin{aligned} u_D^{A:B_1} &= d_{B_1}^A + u_{B_1}^A + b_{B_1}^A u_D^{B_1} \\ u_D^{A:C_1} &= d_{C_1}^A + u_{C_1}^A + b_{C_1}^A u_D^{C_1} \\ u_D^{A:B_2:C_2} &= b_{B_2}^A d_{C_2}^{B_2} + d_{B_2}^A + u_{B_2}^A + b_{B_2}^A u_D^{B_2} + b_{B_2}^A b_{C_2}^{B_2} u_D^{C_2} \end{aligned} \tag{19}$$

By using Eq.(3) and Eq.(19), the expression for the uncertainty in the trust network of Eq.(16) can be derived as:

$$u_D^A = \frac{u_D^{A:B_1} u_D^{A:C_1} u_D^{A:B_2:C_2}}{u_D^{A:B_1} u_D^{A:C_1} + u_D^{A:B_1} u_D^{A:B_2:C_2} + u_D^{A:C_1} u_D^{A:B_2:C_2} - 2u_D^{A:B_1} u_D^{A:C_1} u_D^{A:B_2:C_2}} \tag{20}$$

By using Eq.(17), Eq.(19) and Eq.(20), the uncertainty value of the derived trust ω_D^A according to the node splitting principle can be computed. This value depends on the trust edge opinions and on the two splitting parameters ϕ_B^A and ϕ_D^C. By fixing the opinion values as in the example of Eq.(4) according to

$$\omega_B^A = \omega_D^B = \omega_C^A = \omega_D^C = \omega_C^B = (0.9, 0.0, 0.1, 0.5) \tag{21}$$

a plot of the uncertainty u_D^A as a function of ϕ_B^A and ϕ_D^C is shown in Fig.9

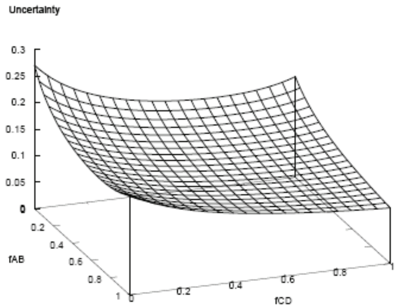

Fig. 9. Uncertainty u_D^A as a function of ϕ_B^A and ϕ_D^C

The conclusion which can be drawn from this is that the optimal value for the splitting parameters are $\phi_B^A = \phi_D^C = 1$ because that is when the uncertainty is at its lowest. In fact the uncertainty can be evaluated to $u_D^A = 0.105$ in that case, which is equal to the uncertainty of Eq.(10). This is equivalent to the case of trust network simplification where the edge [*B*,*C*] is removed from the left-hand side graph of Fig.5.

The least optimal values for the splitting parameters is when $\phi_B^A = \phi_D^C = 0$, resulting in $u_D^A = 0.271$ which is equal to the uncertainty of Eq.(12). This is thus equivalent to the absurd trust network simplification where the edges [*A*,*C*] and [*B*,*D*], and thereby the most certain trust paths are removed from the left-hand side graph of Fig.8. Given the edge opinion values used in this example, ([*A*,*B*]:[*B*,*C*]:[*C*,*D*]) is the least certain path of the left-hand side graph of Fig.8. It turns out that the optimal splitting parameters for analysing the right-hand side graph of Fig.8 produces the same result as network simplification where this particular least certain path is removed.

8. Calculating Public Reputation

Opinion Mining is the area of research that attempts to make automatic systems to determine human opinion from free text written in natural language as a feedback. It is a recent discipline at the crossroads of information retrieval and computational linguistics. The discipline is also known as Sentiment Mining, Sentiment Analysis, Sentiment Classification, Opinion Extraction etc. Unlike the text mining, opinion Mining is concerned with the opinion it expresses instead of the topic of a document. Inspiring by the algorithm proposed by Ding (Ding et al, 2008), we can calculate the public reputation from a given opinion text. Usually an item has several features, for example, a hotel can have features such as room, food, etc. One review expresses one customer's comments toward one item. From each review, we first generate the customer's sentimental orientation to each feature of the item, such as positive or like, negative or dislike, and neutral etc (Popescu et al, 2005), then generate a score to this item according to the user's feature sentimental orientation, finally generate an overall score to this item based on all users' scores.

9. Integrating Trust and Reputation

While we calculate the public reputation of an item, we may combine that with the trust between the opinion giver and the potential user of that item. How it can be done is shown in the framework given in Fig.10. As thousands of web offers to provide opinions from their users, from the Internet, we can download a large amount of opinion data and calculate the general public opinion about an item based on those opinions. We can also calculate the existence of the degree of trust between two members in a trust network and that can be considered while suggesting an item to each other. If any suggestion or recommendation comes from a trusted member, it is more likely to be the right choice of item for a member.

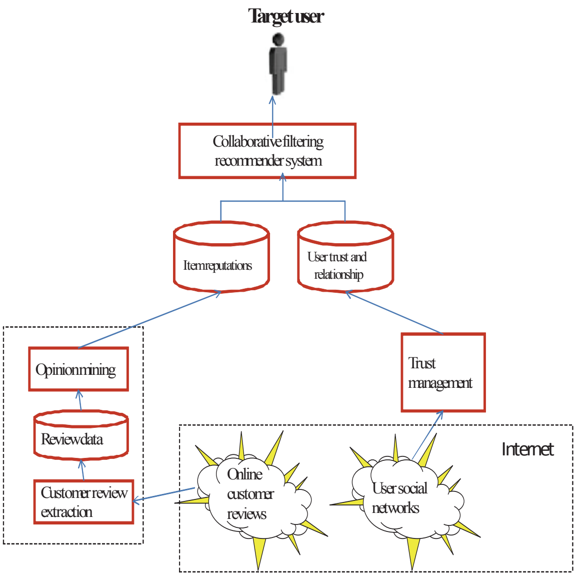

Fig. 10. Framework for integrating trust and public reputation

10. Conclusion

The current online community is suffering the lack of trust or confidence on the opinion expressed in the web-based social network where the degree of trust among the members is absent. The members are facing the quality problem in terms of poor quality and even deceptive opinions or recommendations. In this research work, we have surveyed the current scholars work in the area of trust and reputation management in online social network. We also discuss the method of trust propagation in a trust network. We have described node splitting which is a new principle for trust network analysis with subjective logic. This method which consists of splitting dependent trust edge opinions in order to avoid inconsistencies seems to produce the same result as the previously described method of network simplification. Our analysis was based on a fixed set of edge opinion values. Because of the large number of parameters involved, it is a relatively complex task to verify if our conclusion is valid for all possible trust edge opinion values, so a complete study must be the subject of future work. The present study has given a strong indication that trust network simplification produces the optimal result even though edges are removed from the trust graph. Trust and reputation management represents an important approach for stabilizing and moderating online communities including the members of a social network. Integration of different systems would be problematic with incompatible trust and reputation systems. We have also described how it is possible to gracefully integrate public reputation and trust management with recommender system. This provides a flexible and powerful framework for online trust and reputation management.

11. References

A. A. Selcuk, E. Uzun, and M. R. Pariente. (2004). A Reputation-based Trust Management System for P2P Networks. IEEE Int. Symposium on Cluster Computing and the Grid, 2004.

A. Abdul-Rahman and S. Hailes.(2000). Supporting Trust in Virtual Communities. Proceedings of the Hawaii International Conference on System Sciences, USA.

A. Jøsang (2009). Subjective Logic, Draft Book.

A. Jøsang and S. Pope. Semantic Constraints for Trust Tansitivity.(2005). Proceedings of the Asia-Pacific Conference of Conceptual Modelling. Vol. 43, Australia.

A. Jøsang, R. Hayward, and S. Pope. (2006). Trust Network Analysis with Subjective Logic. In Proceedings of the Australasian Computer Science Conference, Vol. 48, Australia.

A. Jøsang, R. Ismail, and C. Boyd. (2007). A Survey of Trust and Reputation Systems for Online Service Provision. Decision Support Systems, 43(2):618-644.

Aberer, K and Despotovic, Z. (2001). Managing Trust in a Peer-2-Peer Information System. Proceedings of the 10th International Conference on Information and Knowledge Management, USA.

B. Christianson and W. S. Harbison. (2003). Why Isn't Trust Transitive? In Proceedings of the 6th International Conference on Information Fusion, 2003.

Bachrach, Y., A. Parnes et al. (2009). Gossip-based aggregation of trust in decentralized reputation systems. Autonomous Agents and Multi-Agent Systems. 19(2): 153-172.

Bharadwaj, K.K. and M.Y.H. Al-Shamri. (2009). Fuzzy computational model for trust and reputation systems. Electronic Commerce Research and Applications. 8(1): 37-47.

C. Ziegler. (2005). Towards Decentralized Recommender Systems. PhD Thesis, University of Freiburg, Germany.

Ding, X., Liu, B., & Yu, P. S. A, (2008). Holistic Lexicon-Based Approach to Opinion Mining. In the Proceedings of the Int. Conference on Web Search and Web Data Mining.

G. Zacharia, P. Maes (1999). Collaborative Reputation Mechanisms in Electronic Marketplaces. Proceedings of 32nd Hawaii International Conf on System Sciences.

Golbeck, J. (2006). Combining Provenance with Trust in Social Networks for Semantic Web Content Filtering. IPAW 2006: 101-108

Guha, R. V., Kumar, R., Raghavan, P., Tomkins, A., (2004). Propagation of trust and distrust. Proceedings of the 13th International World Wide Web Conference, USA, pp. 403-412.

J. B. Folkerts. (2005). A comparison of reputation-based trust management, MSc thesis, Rochester Institute of Technology, USA.

J. O'Donovan and B. Smyth. (2005). Trust in Recommender Systems. IUI'05, USA.

Kamvar, S., Schlosser, M. and Garcia-Molina, H. (2003). EigenRep: Reputation Management in P2P Networks. 12th International World Wide Web Conference.

L. Mui, M. Mohtashemi, and A. Halberstadt. (2002). A Computational Model of Trust and Reputation. In Proceedings of the 35th Hawaii International Conference on System Science.

L. Xiong. (2005). Resilient Reputation and Trust Management: Models and Techniques. PhD thesis, Georgia Institute of Technology, USA.

Levien, R. Attack-resistant trust metrics. (2004). Ph.D. thesis, University of California at Berkeley, USA.

M. Venkatraman, B. Yu, and M.P. Singh. (2000). Trust and Reputation Management in a Small-World Network. In 4th International Conference on Multi Agent Systems.

P. Resnick, R. Zeckhauser, R. Friedman, and K. Kuwabara. (2000). Reputation Systems. Communications of the ACM, 43(12):45–48.

Popescu, A.-M., & Etzioni, O. (1996). Extracting Product Features and Opinions from Reviews. In the Proceedings of Human Language Technology Conference, Canada.

Pujol, J. M., Sanguesa, R., Delgado, J. (2002). Extracting reputation in multi-agent system by means of social network topology. Proceedings of the first international joint conference on autonomous agents and multi-agent systems, Italy, pp. 467-474.

S. Marsh. (1994). Formalising Trust as a Computational Concept. PhD thesis, University of Stirling, 1994.

Sabater, J. and Sierra, C. (2005). Review on Computational Trust and Reputation Models. Artificial Intelligence Review, 2005 Vol.24, pp. 33-60.

Schillo, M., Funk, P. & Rovatsos, M. (2000). Using Trust for Detecting Deceitful Agents in Aritficial Societies. Applied Artificial Intelligence.

Sen, S. and Sajja, N. (2002). Robustness of Reputation-based Trust: Boolean Case. Proceedings of the First International Joint Conference on Autonomous Agents and Multi-Agent Systems, Italy.

Sundaresan, N. (2007). Online Trust and Reputation Systems. In 8th ACM Conference on Electronic Commerce.

Theo Dimitrakos. (2003). A Service-Oriented Trust Management Framework. International Workshop on Deception, Fraud, and Trust in Agent Societies, pp. 53-72.

Tian, C.-Q., Zou, S.-H., Wang, W.-D., & Cheng, S.-D. (2008). Trust model based on reputation for peer-to-peer networks. Journal on Communication, 29(4), 63-70.

Wang, L., Y.J.Guo, et al. (2009). A reputation-based trust evaluation model for P2P E-Commerce. International Journal of Distributed Sensor Networks. 5(1): 39-49.

Wang, Y., & Vassileva, J. (2007). A Review on Trust and Reputation for Web Service Selection. In the 27th International Conference on Distributed Computing Systems.

Xue, W., & Fan, Z. (2008). A New Trust Model based on Social Characteristic and Reputation Mechanism for the Semantic Web. In the Workshop on Knowledge Discovery and Data Mining.

Ziegler, C. and Golbeck, J. (2007). Investigating interactions of trust and interest similarity. Decision Support Systems 43(2): 460-475

Zou, Y., Gu, L., Li, G., Xie, B., & Mei, H. (2007). Rectifying Prejudicial Feedback Ratings in Reputation based Trust Management. In the IEEE International Conference on Services Computing.