

Guidance and Control for Underactuated Autonomous Underwater Vehicles

Ryan N. Smith, University of Southern California, Los Angeles/USA, ryannsmi@usc.edu

Monique Chyba, University of Hawaii, Manoa, Hawaii/USA, mchyba@math.hawaii.edu

Song K. Choi, Giacomo Marani, University of Hawaii, Manoa, Hawaii/USA

{[schoi](mailto:schoi@wiliki.eng.hawaii.edu),[marani](mailto:marani@wiliki.eng.hawaii.edu)}@wiliki.eng.hawaii.edu

Abstract

The main focus of this paper is on the motion planning problem for an under-actuated, submerged, Omni-directional autonomous vehicle. Underactuation is extremely important to consider in ocean research and exploration. Battery failure, actuator malfunction and electronic shorts are a few reasons that may cause the vehicle to lose direct control of one or more degrees-of-freedom. Underactuation is also critical to understand when designing vehicles for specific tasks, such as torpedo-shaped vehicles. An under-actuated vehicle is less controllable, and hence, the motion planning problem is more difficult. Here, we present techniques based on geometric control to provide solutions to the under-actuated motion planning problem for a submerged underwater vehicle. Our results are validated with experiments.

1. Introduction

The motion planning problem for AUVs is a widely studied research area with interesting problems that are approachable from a variety of different fields. For example, general modeling and control for underwater vehicles can be found in *Fossen (1994)* or *Sagatun (1992)*. Motion planning, steering algorithms and tracking control can be found in *Frazzoli et al. (2002)*, *McIsaac and Otrowski (2001)* and *Yoerger and Slotine (1985)*, respectively. Research on under-actuated underwater vehicles can be found in *Leonard (1994,1995)*.

The research presented in this paper addresses the scenario of an Autonomous Underwater Vehicle (AUV) that malfunctions for one reason or another; battery failure, an actuator quits or electronics short out. Any of these three problems can lead to one or more actuators unable to function normally, if at all. Depending on the number and arrangement of the actuators, in the event that one or more actuators stop working, the vehicle can lose direct control in one or more degrees-of-freedom (DOF). Once we do not have direct control on all six DOF, we consider the vehicle to be under-actuated. In this scenario, the motion planning problem is much more difficult. Only partial answers can be found in the literature and usually only for oversimplified models. Our purpose is to fill this gap. By the use of geometric control theory, we bring some answers to this particular motion planning problem for a realistic model including external forces.

The motivation for our work is twofold. First, the equations of motion for a submerged rigid body present a very rich platform with which to study the motion planning problem for under-actuated, simple mechanical systems by use of a kinematic reduction in the presence of external forces (dissipative and restoring). Secondly, the existence of a test-bed AUV, owned and maintained by the Autonomous System Laboratory (ASL) (the ASL is part of the College of Engineering at the University of Hawai'i at Manoa) offers the possibility to implement our theory and to perform experimental validation. The designed trajectories provide new perspectives for ocean exploration utilizing under-actuated AUVs that move in six DOF.

An under-actuated system can be classified as a system with n degrees of freedom and m actuators, where $m < n$. It is critical to understand that, in this situation, constraints are imposed on the set of admissible accelerations for the vehicle. These dynamic constraints cannot be translated to constraints on the state space (configuration + velocities). This a priori prevents the development of a kinematic path planning algorithm (only on the configuration space) for a vehicle in an under-actuated

condition. As explained below, it is possible to overcome this difficulty.

In this paper, we focus on the experimental work that has been conducted with the test bed vehicle owned by ASL, the Omni-Directional Intelligent Navigator (ODIN). We omit the lengthy discussions involving the complex mathematics necessary to our approach. The basic idea behind the mathematics is to use kinematic controllability to design feasible trajectories for our under-actuated vehicle. Kinematic controllability was initially introduced in *Bullo and Lynch (2001)* for simple, controlled mechanical systems with no external forces. A system that is kinematically controllable has the property that motion planning can be reduced to the concatenation of trajectories obtained on a kinematic system. These trajectories are then appropriately reparameterized with respect to time, and an inverse-kinematic procedure is applied to calculate the control strategy for the dynamic system. The main idea of kinematic controllability that is used in our calculations is to decouple the trajectory into arcs that begin and end in zero velocity states. These arcs are actually obtained from the integral curves of vector fields that are defined on the configuration space only (not the entire state space). The motion planning problem is significantly reduced, as the dimension of the system is cut in half. To apply these techniques to the guidance and control of AUVs, we had to extend the existing theory to account for external forces such as dissipation forces and moments due to viscous drag and restoring or potential forces and moments due to gravity and buoyancy. For the later, we use an ad-hoc method related to the features our test-bed vehicle, as there is still a need for a systematic generalization of the theory in this area. Regarding the dissipative forces and moments, due to their assumed quadratic dependence with respect to the velocities, we were able to introduce a modification to the existing theory, which incorporates viscous drag into the general theory of our trajectory planning algorithms. The reader interested in the theoretical aspects of our research is invited to consult *Bullo and Lewis (2004)*, *Chyba et al. (2009)* and *Smith et al. (2008a, 2008b, 2009)*. In the next section, we describe the challenges faced when implementing our theoretical control strategies onto ODIN.

2. Experimental Work

2.1. Test-bed Vehicle and Facilities

The test-bed vehicle used for our experiment is the Omni-Directional Intelligent Navigator (ODIN), developed in 1991 by the ASL. This vehicle has been designed to test many essential features related to AUV research and operation, such as vehicle design optimization, thruster's layout, sensor arrays, sensor fusion and motion algorithms. ODIN can be seen operating in the pool in Fig.1 and opened up in the laboratory in Fig.2.

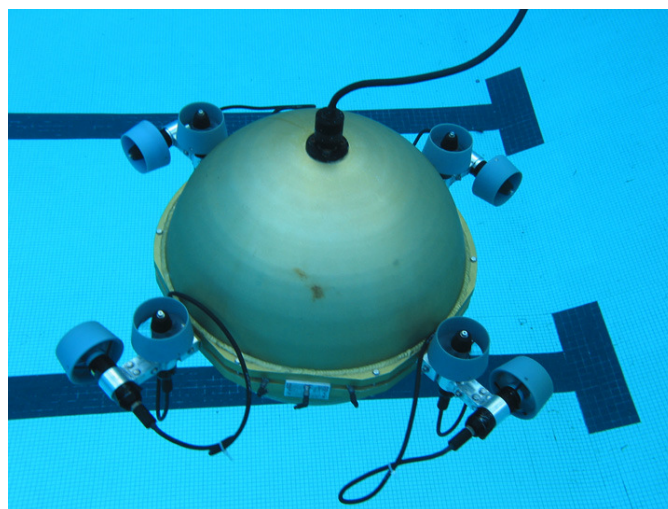


Fig.1: ODIN operating at the pool

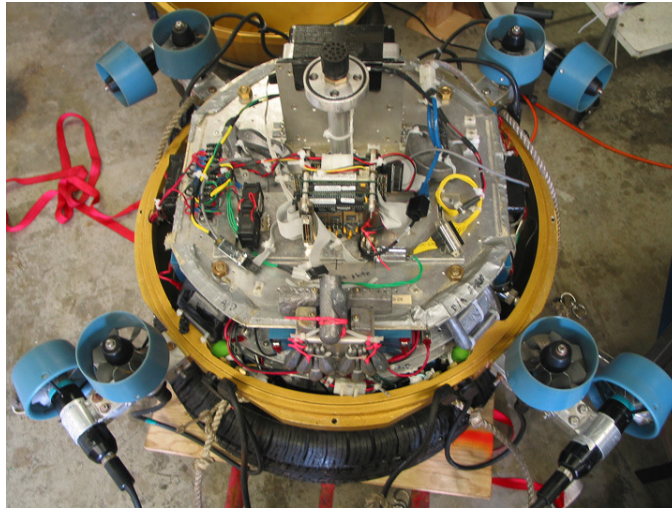


Fig.2: ODIN in the lab

As seen in these figures, ODIN's main body is a 0.64 m diameter sphere made of anodized aluminium (AL 6061-T6). Eight Tecnadyne brushless thrusters are attached to the sphere via four fabricated mounts, each holding two thrusters. These thrusters are evenly distributed around the sphere with four oriented vertically and four oriented horizontally. This design provides instantaneous and unbiased motion in all six DOF, contrary to the more usually seen torpedo-shaped vehicles. Unique to ODIN's construction is the control from an eight dimensional thrust to move in six degrees-of-freedom. To calculate the six-dimensional thrust resulting from the eight-dimensional thrust (from the thrusters), or vice-versa, we need to apply a linear transformation. We omit the details of this transformation here, but refer the interested reader to *Smith (2008a)* or *Chyba (2008c)*.

Fully assembled, ODIN's mass is 123.8 kg and she is positively buoyant by 1.3 N. ODIN is depth rated for 100 meters. The numerical values of additional various parameters used for modelling ODIN can be found in *Smith (2008a)* or *Chyba (2008c)*. These values were derived from estimations and experiments performed on ODIN. The added mass and drag terms were estimated from formulas found in *Allmendinger (1990)* and *Imlay (1961)*. Moments of inertia were calculated using experiments outlined in *Bhattacharyya (1978)*. We used inclining experiments to locate C_G , which we take as the center of our body-fixed reference frame (i.e., $C_G=O_B$). Due to the symmetry of the vehicle, the center of buoyancy C_B , is assumed to be the center of the spherical body of ODIN. The location of C_B is measured from C_G , and is 7mm above.

ODIN's internal CPU is a 800 MHz Intel based processor running on a PC104+ form factor with two external I/O boards providing A/D and D/A operations. The software is divided into two components. The first component is based on a real time extension to the Windows 2000 operating system, which provides ODIN real time autonomous control. The second component runs on the remote laptop and allows the operator to upload autonomous mission profiles to ODIN on the fly during testing, as well as monitor ODIN in real time during such missions. The communication from ODIN to the remote laptop is via a RS232c serial protocol. Other major internal components include a pressure sensor, inertial measurement unit, leakage sensor, heat sensor and 24 batteries (20 for the thrusters and four for the CPU). ODIN is able to compute and communicate real time, yaw, pitch, roll, and depth and can run autonomously for up to five hours from either a tethered or fully-autonomous mode.

ODIN does not have real time sensors to detect horizontal (x-y) position. Instead, experiments are videotaped from a platform 10m above the water's surface, giving us a near nadir view of ODIN's movements. Videos are saved and horizontal position data are post-processed for later analysis. A real-time system utilizing sonar was available on ODIN, but was abandoned for two main reasons. First, the sonar created too much noise in the diving well and led to inaccuracies. More significantly, in the implementation of our control strategies, ODIN is often required to achieve large ($>15^\circ$) list angles which render the sonars useless for horizontal position. Many alternative solutions were

attempted and video provided a cost-effective solution which produced accurate results. We are able to determine ODIN's relative position in the testing pool to $\pm 10\text{cm}$.

Along with the tests to determine hydrodynamic parameters, we also tested the thrusters. Each thruster has a unique voltage input to power output relationship. This relationship is highly nonlinear and is approximated using a piecewise linear function which we refer to as our thruster model. More information regarding the thruster modelling can be found in *Smith (2008a)*.

For the experiments presented in this paper, ODIN was operated from a tethered configuration, but the tether was only used to send commands to be run in autonomous mode. This setup allows for multiple tests to be conducted without removing ODIN from the water to upload mission sorties, and allows for ODIN to be immediately shut down in the event of an emergency.

The experiments presented in this study were all conducted in the diving well at the Duke Kahanamoku Aquatic Complex at the University of Hawai`i. This facility is 25×25 m by five meters deep, and provides a constant and controlled environment for our experiments. A picture of the entire Duke Kahanamoku Aquatic Complex is shown in Fig.3. The diving well is the far pool seen in the photo.



Fig.3: Duke Kahanamoku Aquatic Complex at the University of Hawaii

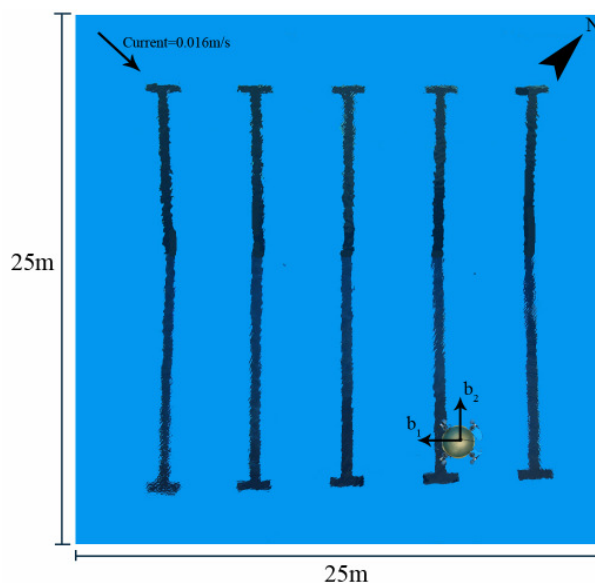


Fig.4: Duke Kahanamoku Aquatic Complex diving well

A close up of the diving well with appropriate dimensions, geographical orientation, ambient current and ODIN, as seen from the 10m diving platform, is presented in Fig.4. Note that this is only a representative image.

The water temperature in the pool is a consistent 28° C, and the density of the water is taken to be 997 kg/m³. The only other environmental factor to consider is a small current created by the circulation pumps. Many drifter tests were performed at multiple depths in and around the testing area. The pools current travels in a direction from west to east at an average rate of 0.016 m/s. The effect of this environmental disturbance is considered to be negligible.

3. Designing Implementable Trajectories

To understand our point of view in the design of implementable trajectories, it is important to note that we do not use any sensor feedback to correct the applied controls during the trajectory. Contrary to automatic or closed-loop control, such as adaptive or PID techniques which are model free, we create a model based control strategy and implement it in open-loop. The reader might be surprised by such a choice, as it is well known that uncertain disturbances play a major role in the guidance and control of underwater vehicles in an ocean environment. Our reasons and motivations for implementing open loop strategies stem from the following. First, our experiments are test trials to validate the mathematical theory as well as our model for the equations of motion of ODIN. Secondly, the ability to successfully implement our trajectories onto a test-bed vehicle in a stable environment, such as the pool, will drastically reduce the complexity of implementation when we migrate the experiments to the open ocean. From a practical point of view, the trajectories calculated using the techniques presented here will eventually play the role of the desired trajectories to track using well known feedback or adaptive techniques. We elaborate on this point in the forthcoming section on future work.

One consequence of working with an open-loop scheme is that continuous control, as a function of time, such as the ones generated by our geometric technique, cannot be readily implemented. Indeed, it would require a massive amount of data storage on the vehicle's on-board computer. Moreover, we also must consider the refresh rate of the actuator controller, the voltage to thrust relation used for the thrusters, and making an effort to keep the thrusters operating in a steady state to reduce their transient output response. Based on these considerations and our experimentation, it is clear that a piece-wise constant control strategy is best suited for implementation. Note that we must also link each piece-wise constant thrust via a linear junction since it is impossible for a physical actuator to change outputs instantaneously.

In summary, to test our strategies on ODIN, we must adapt the continuous control into piece-wise constant (PWC) controls. To do this, we consider the work which is required to perform a desired motion, and ensure that equivalent work is being done by both the continuous and PWC controls. We can compute the work done over a given time interval by integrating the control strategy. Thus, by appropriately choosing when the actuator switches outputs, we can design a PWC control from a given continuous control, and the work done on the system is equivalent. This method is explained in detail in *Smith (2008a)*.

Another aspect of the implementation that we need to address is the initialization procedure at the beginning of each trajectory. In order to begin with a stable, submerged vehicle, we implement a closed-loop initialization dive. This positions the vehicle at the origin of the trajectory and stabilizes depth, roll, pitch and yaw using ODIN's on-board PID controller. The depth is chosen to fully submerge the vehicle, reduce free surface effects and allow for substantial distance from the bottom of the pool.

4. Results

In this section, we examine two mission scenarios for an under-actuated ODIN. Based on the layout of the thrusters, the two under-actuated scenarios we examine are the loss of all four horizontally oriented thrusters and the loss of all four vertically oriented thrusters. Both of these scenarios may seem quite restrictive and a bit extreme; however based on ODIN's circuitry, safety factors and possibly conservation of energy along a trajectory, both of these under-actuated scenarios are quite probable. In an effort to heighten interest, we provide a practical application for each of the two scenarios presented. This application is just one example of path planning and implementation for an under-actuated AUV.

As previously mentioned, we will omit the details of the construction and calculation of the trajectory and dynamic control strategy to be implemented. In the following sections, we present only the PWC control scheme along with experimental results. For the interested reader, a detailed description of the calculations involved to produce the control strategies presented is contained in *Smith (2008a)*.

4.1. Vertical Thrusters Only

Suppose ODIN starts at rest at the origin and that we would like ODIN to realize a pure surge displacement finishing the motion at rest, except we assume that ODIN does not have the use of the horizontally-mounted thrusters. Thus, ODIN only has the use of the vertically-mounted thrusters, and has direct control only upon roll, pitch and heave. Is it possible to reach the final configuration $(a,0,0,0,0,0)$, for an any real number, in the proposed under-actuated condition? By Proposition 4.1 of *Smith (2008)*, we can conclude that ODIN is kinematically controllable, and hence, the answer to the question is yes; any configuration is reachable from any other via kinematic motions. A formal proof of this fact is contained in *Chyba et al. (2009)*. The motivation for this mission is to demonstrate the ability of ODIN to realize a displacement in an under-actuated scenario in a DOF upon which ODIN does not have direct control. Next, we discuss one way to accomplish this displacement and present the associated control strategy that was calculated by use of the geometric theory.

For this example, let us choose $a=1.25$ m. Since the pure heave motion is directly controllable, and ODIN is positively buoyant, it is clear that reaching the final configuration $(1.25, 0, b, 0, 0, 0)$, for b a real number, will prove that ODIN can realize the prescribed surge displacement. One way to reach the desired configuration is to pitch the vehicle an angle α and hold this pitch angle while applying a body-pure heave (i.e., apply a control along the z axis of the body-fixed reference frame) until the vehicle realizes the 1.25 m displacement with regard to the inertial reference frame. The value of b depends upon the pitch angle α . For this experiment, we choose $\alpha=30^\circ$, which corresponds to $b=2.165$ m, and thus the final configuration $(1.25,0,2.165,0,0,0)$.

If we want ODIN to realize a surge displacement greater than 1.25 m, we may concatenate the trajectory described above with one using the negative of the prescribed pitch angle and body-pure heave control to create a V-shaped path. This would have ODIN realize a 2.5 m displacement. Concatenating more V-shaped motions will allow ODIN to realize a greater surge displacement. On the other hand, we could successively implement the trajectory given here followed by a pure heave motion of 2.165 m. This would create a sawtooth-type trajectory. The distance of 1.25 m is arbitrarily chosen and depends upon the pitch angle prescribed and the length of the body-pure heave motion. Altering each of these parameters, we can create different surge displacements.

We give the PWC controls in Table I for the under-actuated case that ODIN only uses vertically-oriented thrusters. The control strategy is presented as a six-dimensional thrust, i.e., $\sigma^1, \sigma^2, \sigma^3$ are forces along the axes of the body-fixed reference frame and $\sigma^4, \sigma^5, \sigma^6$ are moments about these axes. This is different than the eight-dimensional control strategy which gives a thrust strategy for each of ODIN's eight thrusters.

Table I: PWC control structure

Time (s)	Applied Thrust (6-dim.) (N)
0	(0,0,0,0,0,0)
0.9	(0,0,1.006,0,5.392,0)
4.513	(0,0,1.006,0,5.392,0)
5.413	(0,0,1.006,0,5.392,0)
6.8	(0,0,1.006,0,5.392,0)
7.7	(0,0,31.166,0,4.2553,0)
13.273	(0,0,31.166,0,4.2553,0)
14.173	(0,0,-23.431,0,4.2553,0)
16.6	(0,0,-23.431,0,4.2553,0)
17.5	(0,0,0,0,0,0)

The implementation of this control strategy is presented in Fig.5. Here, the first column of three graphs presents the actual control forces and moments (in N and N m, resp.) applied by ODIN during the experiment. Note that no control was applied in the body-pure surge direction, as there was no use of the horizontally mounted thrusters. The following two columns of graphs display the evolution of ODIN in each of the six DOF with respect to the inertial reference frame. The solid (blue) line denotes the actual evolution of the vehicle and the dot dash (red) line denotes the theoretical prediction of the evolution. In this study, the theoretical evolution is computed by simulating ODINs motion with our numerical model using the actual control forces and moments that were applied during the experiment.

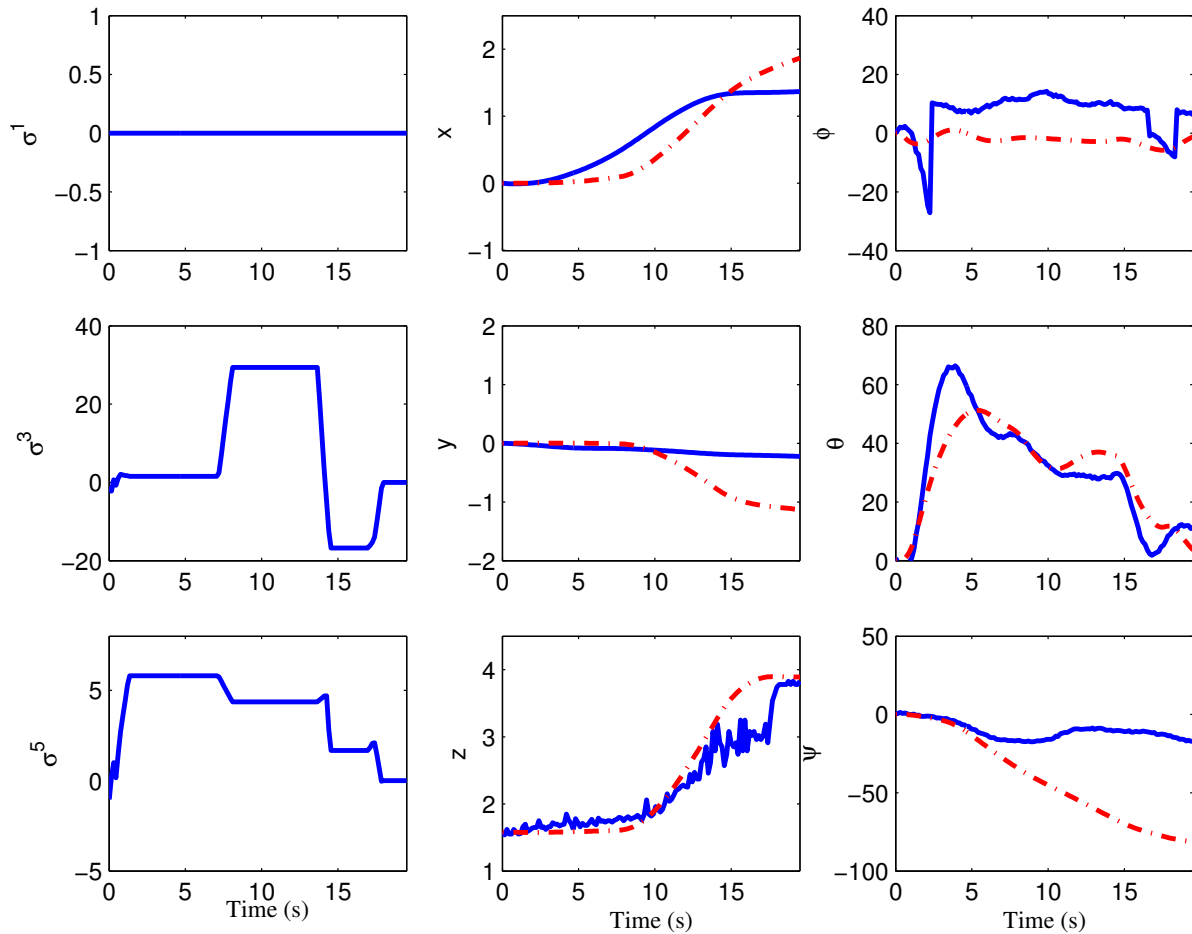


Fig.5: Evolution of the control strategy

In Fig.5, the pitch evolution has an overshoot at the beginning. This is probably a result of the initial start-up friction within the thrusters, which is not necessarily the same for each experiment. Consider the x evolution. Here, ODIN actually exceeds the prescribed surge displacement. Applying a pitch moment induces a force in the surge component of the inertial reference frame. Since ODIN is under-actuated, she cannot apply a counteracting force. Thus, the evolution overshoots the prescribed distance. However, we did show that achieving the motion is possible in the under-actuated situation. One concern with the experimental implementation of these strategies is that the pitch angle did not stabilize until midway through the body-pure heave segment of the trajectory. Other control methods to ensure accuracy in realizing the prescribed list angles are studied in *Smith (2008a)*.

Additionally, in Fig.5, we notice a slight roll angle throughout the trajectory. Since the center of buoyancy C_B and center of gravity C_G of ODIN are located close to each other, such slight instabilities are expected and generally are a result from an imbalance between the physical actuators. The theoretical evolution of yaw diverges from the actual evolution after the initial 5 seconds. For the transformation from a six-dimensional control strategy to an eight-dimensional control strategy, the null space is nonzero. Thus, a small, constant thrust may have been recorded for the yaw component, even though the thrusters were not on. This results in a deviation of the theoretical yaw, as well as sway, evolutions.

4.1. Vertical Thrusters Only

Now, let us assume that we are in the opposite scenario as presented in the previous section; we only have the use of the horizontal thrusters. Such an under-actuated situation only allows direct control on surge, sway and yaw. In other words, ODIN's motion is restricted to a horizontal plane. One reason to consider such a situation is energy conservation during long transects. If ODIN were neutrally buoyant, she would only need the horizontally-oriented thrusters to reach a prescribed location. Or, in the event of an emergency, could traverse along the ocean surface to reach a rendezvous point.

Rather than presenting a random planar path for ODIN to traverse, we consider a practical planar problem instead. A practical application for an AUV in this is to survey the cables or legs of an offshore platform. The vehicle would be equipped again with a forward facing camera to collect video of the supports and rigging. In this case, we will imagine a submerged vertical cylinder with a one-meter radius to be examined. We would like to design a trajectory which keeps the camera facing along a line normal to the surface of the cylinder to give the best view of the entire beam. One way to achieve this goal is to design a spiral trajectory around the cylinder. Depending on the vehicle, this may be a difficult motion to construct. To this end, we present an alternate solution.

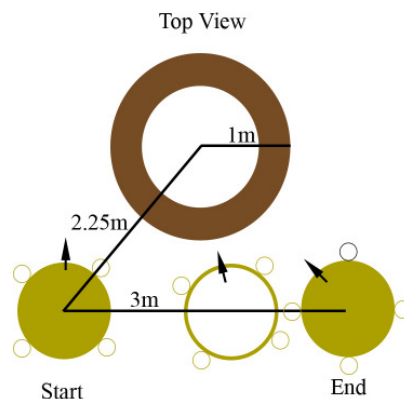


Fig.6: Trajectory around a cylinder for mission six

Maintaining a view normal to a curved surface can be accomplished by following a straight line while simultaneously yawing. The distance from the object may vary, however this motion is much easier to design than the spiral.

For our example, we suppose that the center of the vehicle is positioned 1.25 m from the surface of the pre-described cylinder. The initial position and orientation is depicted in Fig.6 along with the proposed path and ending configuration.

The basic idea is to first rotate 45° , by applying a control in yaw, to face the cylinder, then follow the integral curves of a vector field combining sway and yaw motions to effectively move in a straight line while rotating to maintain a head-on view of the cylinder. We remark here that at the end of this trajectory, the vehicle is in the appropriate position to begin another leg of the survey. After four applications of this control, the vehicle will have traversed around the cylinder via a square trajectory while keeping the camera view normal to the cylinder's surface. The PWC control structure for the cylinder examination mission is given in Table II.

During the implementation of this control strategy, ODIN was adjusted to be neutrally buoyant by adding external weights. The closed-loop initialization was run to start ODIN at a depth of 1.5 m before the mission started. The results from the implementation are presented in Fig.7.

Examining Fig.7, the desired sway motion was achieved; however the yaw orientation never reached the -45° as prescribed. However, the model predicted that it should have realized the prescribed yaw with the control moments that were applied. We see some instability in the pitch and roll components which are again, a result of the close proximity of C_G to C_B . The depth evolution remained constant, as we just see the noise of the pressure sensor. Overall, for this experiment, theoretical predictions matched very well with experimental results.

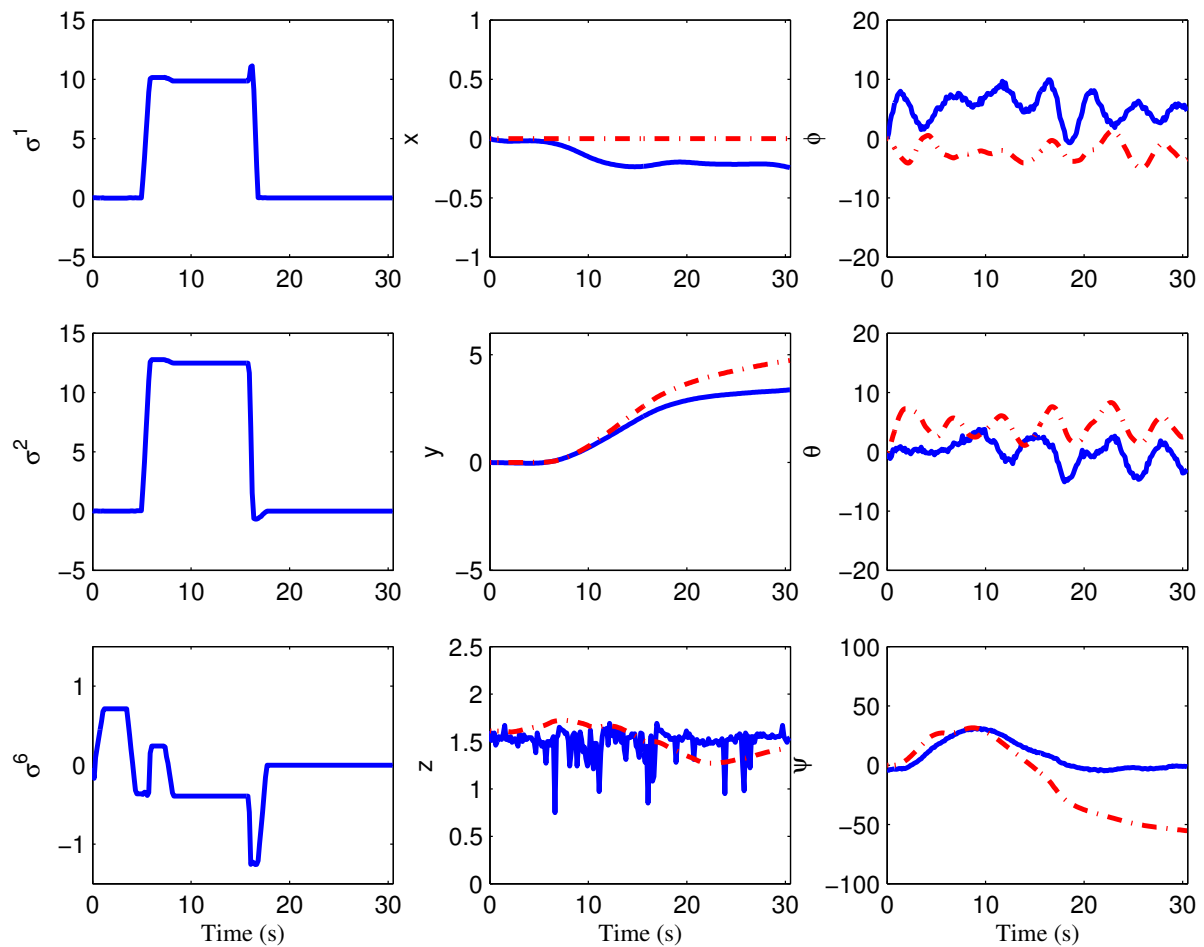


Fig7.: Submerged cylinder examination trajectory.

Table II: PWC control structure.

Time (s)	Applied Thrust (6-dim.) (N)
0	(0,0,0,0,0,0)
0.9	(0,0,0,0,0,0.7)
3.301	(0,0,0,0,0,0.7)
4.201	(0,0,0,0,0,-0.36)
4.8	(0,0,0,0,0,-0.36)
5.7	(11.084,13.698,0,0,0,-0.34)
7.137	(11.084,13.698,0,0,0,-0.34)
8.037	(11.084,13.698,0,0,0,-1.242)
15.186	(11.084,13.698,0,0,0,-1.242)
15.7	(11.084,13.698,0,0,0,-1.242)
16.086	(11.084,-0.66,0,0,0,-1.242)
16.6	(11.084,-0.66,0,0,0,-1.242)
17.5	(0,0,0,0,0,0)

5. Comments and Future Work

The excellent correlation between theoretical predictions and experimental results displayed above come from the fact that we work in a controlled environment with a negligible current. Many experimental trials have helped us model the pool environment and limit the uncertainties during our experiments. This is certainly not true in the open ocean. To migrate from the pool to the ocean, significant adjustments are necessary. An AUV cannot operate in the ocean in an open-loop mode, the magnitude of the poorly known disturbance forces and moments is too large to be neglected, and we would expect to observe large errors between theoretical predictions and experimental results. One reasonable approach is to use our trajectories as the desired theoretical prediction and to implement a robust, feedback trajectory-tracking controller that can compensate for the external disturbances. Initial steps in this direction can be found in *Singh et al. (2009)* and *Sanyal and Chyba (2009)*. Once the theory contained in these references becomes well developed and proven, we plan to implement the hybrid control scheme in the pool. Here, we will begin with simple disturbances such as a deviation in the initial state of the vehicle. Through our experiments, we have observed that a source of error, seen in the graphs displayed in the previous section, come from an initial offset in configuration, usually in yaw. Implementing a hybrid controller as described above will require upgrades to ODIN, as real-time feedback in all six DOF is necessary and not currently available. Eventually, ocean trials will follow.

A natural question arises regarding the applicability of our techniques to multiple types of underwater vehicles. First, the theoretical aspect, namely the geometric control portion, is independent of the choice of the vehicle. The geometric theory is solely based on the fact the underwater vehicle is an example of a simple mechanical system, and this is the case for any underwater vehicle. To generalize our work to an alternate vehicle design, we need only make slight modifications. Assuming that the vehicle has three planes of symmetry, which is common for AUVs, the basic foundations and formulations do not change. The physical attributes need to be altered, such as mass, inertia and added mass terms; this corresponds to the generation of a new kinetic energy metric for the kinematic reduction. Viscous drag coefficients need to be estimated, and the locations of the center of buoyancy and center of gravity need to be calculated to appropriately account for the restorative forces. Aside from the obvious physical properties, the major difference is the change in the input control vector fields, since these are the basis to determine the kinematic motions. This change is simply expressing the location and output of the actuators of the vehicle in question in the geometric formulation. In *Smith (2008a)*, the reader can find two examples that generalize the techniques presented here to other vehicles.

Acknowledgment

Research supported by NSF grant DMS-030641 and partially supported by ONR grant #N00014-03-1-0969, #N00014-04-1-0751 and #N00014-04-1-0751.

References

ALLMENDINGER, E. (1990), *Submersible Vehicle Design*, SNAME.

BHATTACHARYYA, R. (1978), *Dynamics of Marine Vehicles*, Wiley.

BULLO, F.; LEWIS, A.D. (2004), *Geometric Control of Mechanical Systems Modeling, Analysis, and Design for Simple Mechanical Control Systems*, Springer, 49 in Texts in Applied Mathematics

BULLO, f.; LYNCH, K.M. (2001), *Kinematic Controllability and Decoupled Trajectory Plannig for Underactuated Mechanical Systems*, IEEE Trans. Robotics and Autom., 17/4, pp. 402-412.

CHYBA, M.; HABERKORN, T.; SMITH, R.N.; WILKENS, G.R. (2009), *A Geometric Analysis of Trajectory Design for Underwater Vehicles*, Discrete and Continuous Dynamical Systems-B, Volume: 11, Number: 2, pp.233-262.

FOSSEN, T.I. (1994), *Guidance and Control of Ocean Vehicles*, John Wiley & Sons.

FRAZZOLI, E.; DALEH, M.A.; FERON, E. (2002), *Real-time Motion Planning for Agile Autonomous Vehicles*, AIAA Journal of Guidance, Control and Dynamics, 25/1, pp.116-129.

IMLAY, F.H. (1961), *The Complete Expressions for Added Mass of a Rigid Body Moving in an Ideal Fluid*, Technical Report DTMB 1528.

LEONARD, N.E. (1994), *Control Synthesis and Adaptation for an Underactuated Autonomous Underwater Vehicle*, Research Report T.R. 94-58, Department of Electrical Engineering, Institute for Systems Research, University of Maryland, College Park, MD.

LEONARD, N.E. (1995), *Periodic Forcing, Dynamics and Control of Underactuated Spacecraft and Underwater Vehicles*, 34th IEEE Conference on Decision and Control, pp.3980-3985, New Orleans.

MACIVER, M.A.; FONTAINE, E.; BURDICK, J.W. (2004), *Designing Future Underwater Vehicles: Principles and Mechanisms of the Weakly Electric Fish*, IEEE J. Oceanic Eng. 29/3.

McISAAC, K.; OSTROWSKI, J.P. (2001), *Steering Algorithms for Dynamic Robotic Locomotion Systems*, in B.R. Donald and K.M. Lynch and D. Rus, editors, "Algorithmic and Computational Robotics: New Directions", pp.221-231, A.K. Peters, Natick.

SAGATUN, S.I. (1992), *Modeling and Control of Underwater Vehicles: A Lagrangian Approach*, Dr.ing. thesis, Dept. of Engineering Cybernetics, The Norwegian Institute of Technology, Trondheim.

SANYAL A.; CHYBA M., (2009), *Robust Feedback Tracking of Autonomous Underwater Vehicles with Disturbance Rejection*, American Control Conference (ACC), St Louis

SINGH, S.; SANYAL A.; SMITH. R.; NORDKVIST N.; CHYBA M. (2009), *Robust Tracking Control of Autonomous Underwater Vehicles in the Presence of Disturbance Inputs*, 28th Int. Conf. Offshore Mechanics and Artic Engineering (OMAE), Honolulu

SMITH, R.N. (2008a), *Geometric control theory and its application to underwater vehicles*, Ph.D. dissertation, Ocean & Resources Engineering Department, University of Hawaii.

SMITH, R.N.; CHYBA, M. (2008b), *A First Extension of Geometric Control Theory to Underwater Vehicles*, Navigation Guidance & Control of Underwater Vehicles NGCUV08, Killaloe

SMITH, R.N.; CHYBA, M.; WILKENS, G.R.; CATONE, C. (2009), *A Geometrical Approach to the Motion Planning Problem for a Submerged Rigid Body*. International Journal of Control.

YOERGER, D.N.; SLOTINE, J.E. (1985), *Robust Trajectory Control of Underwater Vehicles*, IEEE Journal of Oceanic Engineering, 10/4, pp.462-470.