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# A heuristic algorithm for fixed train runtime 

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#### Abstract

This paper proposes a train movement model with fixed runtime that can be employed to find feasible control strategies for a single train along an inter-city railway line. The objective of the model is to minimize arrival delays at each station along railway lines. However, train movement is a typical nonlinear problem for complex running environments and different requirements. A heuristic algorithm is developed to solve the problem in this paper and the simulation results show that the train could overcome the disturbance from train delay and coordinates the operation strategies to sure punctual arrival of trains at the destination. The developed algorithm can also be used to evaluate the running reliability of trains in scheduled timetables.


## Keywords

Fixed runtime, traction calculation, computer simulation

## I. Introduction

The train movement calculation is to compute a train's running trajectory under various conditions according to train traction calculation theory (Ho 1998, Mao 2000), and multi-train simulator is developed to analyze the disturbances between two consecutive trains (Liu 2005). Computer simulation of train performance has been widely used for the design and operation of railway systems (Goodman, 1998). Genetic algorithm was used to search coasting points for train movement (Chang, 1997; Wong, 2004). Wong (2007) presented a dynamic programming approach to control dwell times and runtimes for train operation adjustment and the results showed that run-time control was more effective than dwell-time control from the
viewpoint of energy saving. A fuzzy control model was presented to determine an economical running pattern for the compromise between trip time and energy consumption (Hwang, 1998) and genetic algorithm was used to solve the problem (Bocharnikov, 2007). A predictive fuzzy control approach was presented to adjust the train dwell time and the results showed that it was an effective method to maintain the service quality (Chang, 1996). The fuzzy controllers in automatic train operation were presented using the differential evolution algorithm (Chang, 2000).

A railway timetable is a deterministic plan and the trains should arrive at or depart from a station at specified time. However, real-time train operations are suffered from stochastic disturbances. Thus, on-line calculation is needed to achieve fixed runtime under actual running environment. It can be used to optimize the driving strategies and acquire energy consuming within different fixed runtime, and it can also reflect the ability to absorb stochastic disturbances of a scheduled train in a timetable. Meanwhile, it also can be used to evaluate level of service for a passenger train in dedicated passenger lines.

This paper discussed the method to achieve running trajectory under fixed runtime where traction characteristic, weight and railway infrastructure was considered. Computational results showed that the algorithm is feasible and the algorithm proposed here can also serve the simulation in timetable.

## II. The fixed runtime problem

The train movement is subject to the scheduled timetable, line geometries, rolling
stocks and traction weight, and these are static restrictions. Meanwhile, the train's running environment is dynamic, and the train may be influenced by temporary speed restrictions caused by device conflicts or the leading train. In the other hand, if there are many boarding or alighting passengers at a station, the dwell time may be extended and the train will have to adjust its target speed to a higher level in order to arrive next stations on time. The operation strategies should also be changed according to the specified runtime when the running environment changes.

The notations used in the context are defined as follows:
$C$, the set of the handles of the rolling stock.

$$
C=\{-b,-b+1, . ., 0,1, . ., r-1, r\}, b \in N, r \in N,
$$

$b$ and $r$ denote the highest braking and traction handle, respectively.
$T$, the total travel time specified in timetable, and the unit is seconds.
$n$, the total running steps of the train.
$t_{i}$, the travel time in step $i$, and the unit is second.
$c_{i}$, the handle in step $i, c_{i} \in C$.
$a_{i}$, the acceleration in step $i$, and the unit is $\mathrm{km} / \mathrm{h} \cdot$ seconds.
$s_{i}$, the travel distance in step $i$, and the unit is meter.
$v_{i}$, the speed in step $i$, and the unit is $\mathrm{km} / \mathrm{h}$.
$v_{i}^{M}$, the restricted speed in step $i$, and the unit is $\mathrm{km} / \mathrm{h}$.
$T_{c}^{+}$, the minimum duration time before the handle is increased, and the unit is seconds.
$T_{c}^{-}$, the minimum duration time before the handle is decreased, and the unit is seconds.
$k$, the switch times of the handles during the total travel process, $k \in N$.
$c_{l}^{\prime}$, the chosen handle, $1 \leq l \leq k, c_{l}^{\prime} \in C$.
$t\left(c_{l}^{\prime}\right)$, the actual duration time of handle $c_{l}^{\prime}, \quad 1 \leq l \leq k, c_{l}^{\prime} \in C$.
$S=\{1, \ldots, s\}$, the set of stations, and 1 and
$s$ denote the first and last station, respectively.
$T_{j}^{a}, T_{j}^{d}$ denote the scheduled arrive and depart time specified in timetable, respectively. $j \in \mathrm{~S}$
$t_{j}^{a}, t_{j}^{d}$ denote the actual arrive and depart time, respectively. $j \in \mathrm{~S}$

The fixed runtime model

$$
\begin{align*}
& \min \sum_{j=1}^{s}\left|t_{j}^{a}-T_{j}^{a}\right|  \tag{1}\\
& v_{i}<v_{i}^{M} \tag{2}
\end{align*}
$$

$$
\left\{\begin{array}{l}
\left|a_{i}\right| \leq \varphi  \tag{3}\\
\frac{a_{i+1}-a_{i}}{t_{i}} \leq \varepsilon
\end{array}\right.
$$

$$
\begin{align*}
& \text { If } c_{k+1}^{\prime}>c_{k}^{\prime} \text {, then } t\left(c_{k+1}^{\prime}\right) \geq T_{c}^{+} \\
& \text {If } c_{k+1}^{\prime}<c_{k}^{\prime} \text {, then } t\left(c_{k+1}^{\prime}\right) \geq T_{c}^{-} \tag{4}
\end{align*}
$$

The following formula (2) shows that it is not permitted for running speed $v_{i}$ to exceed the current speed restriction $v_{i}^{M}$. There are two types of speed restriction, one is caused by line geometries and railway infrastructure, such as curves, grades and switches, and the other is caused by the leading train because a certain distance should be kept between two consecutive trains for safety, and $v_{i}^{M}$ is the minimum of these two speed restrictions. The formula (3) shows the influence on passengers comfort caused by the acceleration. The above and below formula show the maximum of the acceleration and its changing rate, respectively. The preferred value $\varphi$ is 0.8 when accelerating and 1.0 when decelerating, and the preferred value of $\varepsilon=0.75 \mathrm{~m} / \mathrm{s}^{3}$ for passenger trains. Meanwhile, the handles are not allowed to switch frequently considering the maintenance of rolling stocks, and (4) expresses the restricted condition. If the handle $c_{k}^{\prime}$ is increased to $c_{k+1}^{\prime}$ at a time, then the duration time should not be less than $T_{c}^{+}$ before it is decreased to $c_{k}^{\prime}$ at the following switching time; and if the handle $c_{k}^{\prime}$ was decreased from $c_{k+1}^{\prime}$ at a time, then the duration time should not be less than $T_{c}^{-}$before it is increased to $c_{k}^{\prime}$ at the following switching time, and the preferred value of $T_{c}^{+}$and $T_{c}^{-}$is 60 seconds and 30 seconds, respectively.

If the train is scheduled to stop at station $j \in \mathrm{~S}$, and $d_{j}$ denotes the stop error at station
$j$. Stop error is the distance between train head position and the target stop position when the train is stopped, and the head of the train is not allowed to overpass the stop sign, so the train should stop precisely and $\sigma$ is here used to denote the stop error. The preferred value of $\sigma$ is 10 cm for subway trains equipped with screen doors.

$$
\begin{equation*}
\left|d_{j}\right| \leq \sigma \tag{5}
\end{equation*}
$$

The dwell times specified in timetable is generally composed by two parts: the minimum dwell times and its supplements, and the actual dwell times may be reduced to depart on time when a train arrives lately. If a train is scheduled to stop at station $j \in \mathrm{~S}, w_{j}$ denotes the minimum dwell times, $T_{j}^{d}-T_{j}^{a}>w_{j} \quad$ generally and $T_{j}^{d}-T_{j}^{a}-w_{j}$ denotes the dwell supplement times. Trains are not allowed to depart before the specified time, and then $t_{j}^{d}$ can be expressed with formula (6):

$$
t_{j}^{d}=\left\{\begin{array}{l}
T_{j}^{d}, T_{j}^{a} \leq t_{j}^{a} \leq T_{j}^{d}-w_{j}  \tag{6}\\
t_{j}^{a}+w_{j}, t_{j}^{a}>T_{j}^{d}-w_{j}
\end{array}\right.
$$

The train movement equations can be expressed by formula (7):

$$
\begin{align*}
& v_{i+1}=v_{i}+a_{i} \cdot t_{i}  \tag{7}\\
& s_{i+1}=v_{i} \cdot \Delta t+\frac{1}{2} a_{i} \cdot\left(t_{i}\right)^{2} \tag{8}
\end{align*}
$$

There are two decision variables: $c_{l}^{\prime}$ and $t\left(c_{l}^{\prime}\right)$, the adopted handles and its duration time. The actual travel time of the train is expressed by $t_{s}^{a}-t_{1}^{d}$, and we can figure out the following expression $t_{s}^{a}-t_{1}^{d}=\sum_{l=1}^{k} t\left(c_{l}^{\prime}\right)$. If the set of $c_{l}^{\prime}$ is denoted by $\Phi$, and then $c_{i} \in \Phi$. The simulation time step decides on the calculation precision and efficiency, and the recommended value for low speed trains is 1 to 5 seconds and high speed trains 0.1 to 1 seconds. In addition, in order to ensure the stop precision, short step time should be adopted in the process of stopping. The fixed runtime problem is a combination optimization problem and the feasible solutions are infinite. A heuristic algorithm is developed to solve the problem here and the satisfactory results of running trajectory are obtained.

## III. Algorithm

There are two factors which have effect on
the driving strategies of trains, the restricted speeds and the target speed. There are static and dynamic restricted speeds and the latter is forced to the train randomly, and then the complexity of the problem is increased. The target speed denotes the expected speed, and it means that the train would get good running performance if it could run with the speed at any time; however, it is difficult to control trains to run with the target speed. The target speed is decided by the scheduled travel time, actual travel time and whether train stops at the anterior station or not. Let $l_{j}, j \in S$ denotes the distance between station $j$ and $j+1$, and Let $l_{j}^{\prime}$ and $\dot{t}_{j}^{\prime}$ denote the current running distance and time. If the train was schedule to pass the anterior station, the target speed $v^{T}$ can be expressed by formula (9).

$$
\begin{equation*}
v^{T}=\frac{l_{j}-l_{j}^{\prime}}{T_{j+1}^{a}-t_{j}^{d}-t_{j}^{\prime}}, j \in S \tag{9}
\end{equation*}
$$

In above formula, the numerator denotes the remaining distance and denominator the available travel time. It is noticeable that the target speed $v^{T}$ just denotes the expected speed, and the actual speed it can reach is not allowed to exceed the speed restriction. If the train is scheduled to stop at the anterior station, the train would delay owing to stop supplement times if it still adopted the target speed expressed in formula (9). Thus, the target speed should be increased a little under this condition, and the target speed can be expressed by formula (10):

$$
\begin{equation*}
v^{T}=\frac{l_{j}-l_{j}^{\prime}}{T_{j+1}^{a}-t_{j}^{d}-t_{j}^{\prime}}(1+\beta), j \in S, 0<\beta<1 \tag{10}
\end{equation*}
$$

In the above formula, the parameter $\beta$ denotes an increased ratio, and the value of $\beta$ is determined by the current status of trains. The value of $\beta$ is less than 1.0 in general; it can be determined by one dimension search method, and we can choose the iteration step from 0.01 to 0.05 .

Let $p_{j}$ represent the scheduled trip between station $j$ and $j+1$. Let $p_{j . b}$ denotes the activity at start-station, and there are two kinds of the activity, start and pass, expressed by $b_{1}$ and $b_{2}$ respectively. Let $p_{\text {j.e }}$ denotes the activity at end-station, and there are also two kinds of the activity, pass and stop, expressed by $e_{1}$ and $e_{2}$ respectively. A scheduled train in a timetable is composed of several plan units. The optimization process is based on these plan units, and the running time of the finished plan units have effect on the following plan units. The flow
of the algorithm is expressed as follows:
Step 0: initialize the network and the static speed restrictions of trains, and set $j=1$;
Step 1: compute the target speed $v^{T}$ in the plan unit $p_{j}$, and then go to Step 2;
Step 2: if $p_{j . b}$ is equal to $b_{1}$, startup the train, otherwise go to Step 3;
Step 3: compute the proper handle according the $v^{T}$, and then update the train's position, speed and travel time; go to Step 4;
Step 4: if $p_{j . e}$ is equal to $e_{2}$, go to Step 5, otherwise go to Step 3;
Step 5: whether the train entered the stop verification area, and perform the stop verification procedure if true, otherwise go to Step 6;
Step 6: whether $p_{j}$ is over, and go to Step 7 if true, otherwise go to Step 3;
Step 7: whether all the plan units is over, go to Step 8 if true; Otherwise, update $j=j+1$, and then go to Step 2;
Step 8: Calculation is finished.
The travel route and stop target position of the train should be defined in step 0.The stop verification area in step 5 is used to compute the braking position to ensure the stop precision. The area could be located between the position in which the train can stop close to the homing signal if it executed braking control at the present time to the homing signal for the sake of safety and fast convergence.

## IV. Cases study

A 117 kilometer inter-city railway line with five stations and four sections is selected in the case study. The four sections are denoted by A, $\mathrm{B}, \mathrm{C}$ and D , and the distance are 21.23 km , $24.35 \mathrm{~km}, 37.67 \mathrm{~km}$ and 33.65 km , respectively. The static speed restriction in sections is $300 \mathrm{~km} / \mathrm{h}$ and $80 \mathrm{~km} / \mathrm{h}$ in sidings. Figure 1 shows the layout of the railway line and stations. The train is composed of high speed rolling stocks and the total weight is 578 tones, and the maximum traction acceleration can be calculated according to the traction curves and total weight of the train. Figure 2 shows the maximum traction acceleration and resistance deceleration curves of the train, and the maximum traction acceleration is $0.7 \mathrm{~m} / \mathrm{s}^{2}$.


Fig. 1 Layout of the inter-city railway line.

The minimum running times in each section are determinate after the line geometries and train parameters are specified. So, the time-optimal running results should be calculated in advance, and the time-optimal mode means the fastest running trajectory from origination to destination. The scheduled running times we arranged here for each section should be more than the minimum running times.


Fig. 2 Maximum traction acceleration and resistance deceleration of the train.

We defined two modes to calculate time-optimal runtime results:

Mode 1: nonstop
$\forall j=1, \ldots, s-2, p_{j, e}=e_{1} ; p_{s-1 . e}=e_{2} ;$
Mode 2: With stops
$\forall j=1, \ldots, s-1, p_{j . e}=e_{2}$.
Table 1 Time-optimal running results of two
modes

| modes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Mode 1 |  | Mode 2 |  |
| Section | Runtime | $\begin{aligned} & \text { Energy } \\ & \text { (KWH) } \end{aligned}$ | Runtime | $\begin{aligned} & \text { Energy } \\ & \text { (KWH) } \\ & \hline \end{aligned}$ |
| A | 6'58" | 617.87 | $7{ }^{\prime} 43$ " | 739.41 |
| B | 5 '28" | 461.39 | 8'17" | 782.92 |
| C | 8'9" | 666.18 | 11'15" | 1036.85 |
| D | 8'6" | 520.1 | 10'22" | 970.97 |
| Total | 28'41" | 2265.54 | 37'37" | 3530.15 |

The running times and the energy consuming in Mode 2 are increased by $31.14 \%$ and $43.18 \%$ than those in Mode 1, respectively. Fig 3 and Fig 4 show the speed and time curves in two modes, respectively.


Fig. 3 Speed and time curves in Mode 1.


Fig. 4 Speed and time curves in Mode 2.
Fig 5 and Fig 6 show the traction and braking forces in Model 1 and Model 2 respectively. The traction when the train started from still is great and the train applied the brake for four times in Mode 2, so the energy consuming in Mode 2 was more than that in Model.


Fig. 5 Traction and braking forces in Mode 1.


Fig. 6 Traction and braking forces in Mode 2.
According to the results in two modes, we increased the running time in Mode 2 by $10 \%$, and then got case 1 . Case 1 can be used to analysze the running results when the scheduled running time was enough. We designed case 2
on a case when the dynamic speed restrictions was add to case 1 , and the speed restrictions area was located at ten kilometers in section B and the restricted speed was $120 \mathrm{~km} / \mathrm{h}$. Case 2 can be used to analysis the strategies of train operation adjustment in order to arrive at the destination on time when the train service was disrupted by the occasional restricted speeds.

Table 2 Simulation results of case 1

| Sec- <br> tion | Sche- <br> duled | Actual | Delay <br> (secs) | Energy <br> $(\mathrm{KWH})$ |
| :--- | :--- | :--- | :--- | :--- |
| A | $8^{\prime}$ | $8^{\prime}$ | +3 | 774.03 |
|  | $30^{\prime \prime}$ | $33^{\prime \prime}$ |  |  |
| B | $9^{\prime}$ | $9^{\prime}$ | 0 | 839.37 |
|  | $10^{\prime \prime}$ | $10^{\prime \prime}$ |  |  |
| C | $12^{\prime}$ | $12^{\prime}$ | +2 | 1047.69 |
|  | $30^{\prime \prime}$ | $32^{\prime \prime}$ |  |  |
| D | $11^{\prime}$ | $11^{\prime}$ | +2 | 1029.65 |
|  | $30^{\prime \prime}$ | $32^{\prime \prime}$ |  |  |
| Total | $41^{\prime}$ | $41^{\prime} 46^{\prime \prime}$ | +7 | 3690.74 |
|  | $40^{\prime \prime}$ |  |  |  |

Table 2 shows the simulation results of case 1. The delay and stop error in each station didn't exceed 5 seconds and 100 mm , respectively. The running trajectory was smooth, and it can be seen from Fig 7. In addition, the trip time and energy were increased by 7 minutes and $4.55 \%$ compared to Mode 1, respectively.


Fig. 7 Speed and time curves in Case 1.

Table 3 Simulation results of case 2

| Sec- <br> tion | Sche- <br> duled | Actual | Delay <br> (secs) | Energy <br> $(\mathrm{KWH})$ |
| :--- | :--- | :--- | :--- | :--- |
| A | $8^{\prime}$ | $8^{\prime}$ | +3 | 774.03 |
|  | $30^{\prime \prime}$ | $33^{\prime \prime}$ |  |  |
| B | $9^{\prime}$ | $10^{\prime}$ | +61 | 483.69 |
|  | $10^{\prime \prime}$ | $11^{\prime \prime}$ |  |  |
| C | $12^{\prime}$ | $11^{\prime}$ | -48 | 1034.42 |
|  | $30^{\prime \prime}$ | $42^{\prime \prime}$ |  |  |
| D | $11^{\prime}$ | $11^{\prime}$ | -9 | 996.26 |
|  | $30^{\prime \prime}$ | $21^{\prime \prime}$ |  |  |
| Total | $41^{\prime}$ | $41^{\prime \prime}$ | +7 | 3288.4 |
|  | $40^{\prime \prime}$ | $46^{\prime \prime}$ |  |  |

Table 3 shows the simulation results of case
2. The actual running time in section $B$ was 61 seconds more than the scheduled running time, and it resulted in the arrival delay. The delay was decreased by 48 seconds and 9 seconds in section C and D , respectively. Thus, the arrival delay was only 7 seconds in destination. In addition, the stop error in each station didn't exceed 100 mm and the running trajectory can be seen from Fig 8.


Fig. 8 Speed and time curves in Case 2.
Compared to Case 1, the energy in total trip was decreased by $10.9 \%$ in Case 2. For section B, the energy in Case 2 was only $57.63 \%$ of that in Case 1. The main reason was that there was the temporary restricted speed in Section B, and the train applied coasting and decreased the traction in order not to exceed the restricted speed, thus the energy was saved. Fig 9 and 10 show the traction and braking forces in Case 1 and Case 2, respectively.


Fig. 9 Traction and braking forces in Case 1.


Fig. 10 Traction and braking forces in Case 2.

## V. Conclusion

The train movement problem with fixed runtime was modeled. The aim of this model is to minimize the arrival delay of the train and a heuristic algorithm was presented to solve the problem. We designed two cases to illustrate the problem. In Case 1, the scheduled running time was increased by $10 \%$ of time-optimal Model,
and the simulation results showed that the energy was increased by $4.55 \%$ than that in time-optimal Mode, and the arrival delay and stop error in each station didn't exceed 5 seconds and 100 mm , respectively. In Case 2, the condition of the temporary restricted speed in section B was added, and the simulation results showed that an arrival delay of 61 seconds occurred in station 3, then the train made up the delay in the following two sections and the arrival delay was only 7 seconds in destination. The total trip energy was decreased by $10.9 \%$ on account of the coasting strategy in section B. In addition, a railway timetable is a deterministic plan, however, the actual train operations is subject to stochastic disturbance. So, the algorithm presented here can be used to analysis the running reliability in punctuality of a scheduled train in a timetable.

## VI. Acknowledgement

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## VII. References

[1]. Ho, T.K., Mao, B.H., Yang, Z.X., et al., A general-purpose simulator for train operations. Proceedings of ICTTS'98, ASCE, 1998, pp. 830-839.
[2]. Mao, B.H., Ho, T.K., et al., A general-purposed simulation system on train movement. Journal of the China railway society. 2000, 22 (1), pp. 1-6.
[3]. Liu, J.F., Ding, Y., et al., Multi-train Movement Simulation System for Urban Rail Transit. Journal of Transportation Systems Engineering and Information Technology. 2005, 1 (1), pp. 79-82.
[4]. Goodman, C.J., Siu, L.K., and Ho, T.K. A review of simulation models for railway system. IEE - Int. Conf. Developments in Mass Transit Systems, 1998, vol. 543, pp. 80-85
[5]. Ho, T.K., Mao, B.H, et al., Computer simulation and modeling in railway applications. Computer Physics Communications, 2002, 143, pp. 1-10
[6]. Khmelnitsky, E., On an optimal control problem of train operation, IEEE Trans. on Automatic Control, 2000, VOL.45, No.7, pp.1257-1266
[7]. Wong, K.K., and Ho, T.K. Coast control for mass rapid transit railways with searching methods, IEE Proc., Electr. Power Appli., 2004,

151, (3), pp. 365-376
[8]. Chang, C.S., and SIM, S.S. Optimising train movement through coast control using genetic algorithms, IEE Proc. -Electr. Power Appl., 1997, 144, (I), pp. 65-73
[9]. Wong K.K., Ho T.K. Dynamic coast control of train movement with genetic algorithm, Int. J. Syst. Sci., 2004, 35, (13-14), pp. 835-846
[10]. Wong, K.K. and Ho, T.K. Dwell-time and run-time control for DC mass rapid transit railways. IET Electr. Power Appl., 2007, 1, (6), pp. 956-966
[11]. Hwang, H.S. Control Strategy for Optimal Compromise between Trip Time and Energy Consumption in a High-Speed Railway. IEEE Transactions on Systems, Man and Cybernetics, Part A, 1998, 28(6), pp. 791-802.
[12]. Bocharnikov, Y.V., Tobias, A.M., Roberts, C., Hillmansen, S. and Goodman, C.J., Optimal driving strategy for traction energy saving on DC suburban railways, 2007, IET Electr. Power Appl. 1(5), pp. 675-682
[13]. Chang, C.S., and Thia, B.S., Online rescheduling of mass rapid transit systems: fuzzy expert system approach. IEE Proc.- Electr. Power Appl., 1996, 143(4), pp. 307-316
[14]. Chang, C.S., and Xu, D.Y. Differential evolution based tuning of fuzzy automatic train operation for mass rapid transit system. IEE Proc.- Electr. Power Appl., 2000, 147, (3), pp. 206-212

