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PROBABILISTIC LOAD FLOW CALCULATION FOR AC TRACTION SUPPLIES WITH AT FEEDING

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ABSTRACT

Power load flow analysis is essential for system planning, operation, development and maintenance. Its application on railway supply system is no exception. Railway power supplies system distinguishes itself in terms of load pattern and mobility, as well as feeding system structure. An attempt has been made to apply probability load flow (PLF) techniques on electrified railways in order to examine the loading on the feeding substations and the voltage profiles of the trains. This study is to formulate a simple and reliable model to support the necessary calculations for probability load flow analysis in railway systems with autotransformer (AT) feeding system, and describe the development of a software suite to realise the computation.

1. INTRODUCTION

An electrified railway system is indeed a huge electrical circuit. A train draws power as a load when it is moving. Its position helps complete the power system configuration whilst its speed, together with traction equipment characteristics and track geometry, determines the level of power it consumes. When regenerative braking is allowed, the train becomes a moving source during braking. In order to accomplish load flow analysis, it is necessary to derive the model to attain the power (both real and reactive) required for a train from its position and speed. The power system configuration is inevitably changing while the trains move.

To further complicate matter, a number of feeding systems have been widely used in AC railways and the main three are direct, booster transformer (BT) and autotransformer (AT) feed [1]. The introduction of additional feeding and return conductors with BT and AT feeding only aggravates the complexity of the calculation required.

Even if both P and Q of a train are readily available, the amount of calculation is still tedious because of the consideration of current distribution and the self and mutual impedances of the conductors. When there are more than one trains running on the line, their interaction through the power system and the relationship between the train separations and timetables have to be taken in account.

This paper will describe a simplified probabilistic load flow calculation approach which trade-offs between computation time and accuracy. Based on certain initial assumptions, the currents flowing through all feeding and return conductors are approximated as linear functions of the train current in the simplified model. The computational demand can thus be reduced at the expense of certain degree of accuracy. With the impedance of feeding conductors, the train voltage is then estimated. The probability density function (pdf) of the train voltage is computed with convolution theorem, followed by the pdfs for P and Q at the feeding points. The specifications and implementation of the software suite to attain the pdfs of train voltage, real power and reactive power in an AT feed system are also discussed here.

2. AT FEED POWER SUPPLY

The development of AC railways always focuses on two concerns: i) transmission of power over long distances and ii) electromagnetic interference to the equipment and environment in the vicinity. Autotransformer (AT) feeding system has been proven to provide the best performance on these two aspects [2]. The only disadvantage of AT feeding is the introduction of more overhead conductors and corresponding equipment.

With AT feed, a voltage twice of the train supply voltage is applied at the substation end. As shown in Fig. 1, the ATs are connected between the catenary (or contact wire) and an auxiliary feeder whilst they are centre-tapped to the rails. A train draws current from the two adjacent ATs so that current only flows through the rails between the two ATs. The current distribution among the conductors is therefore quite complicated and it is illustrated in Fig.2. To further reduce electromagnetic interference, a protection wire (not shown in Fig.1), clamped high along other overhead conductors and earthed, is connected to the rails at regular intervals including at AT locations and certain points between two adjacent ATs.

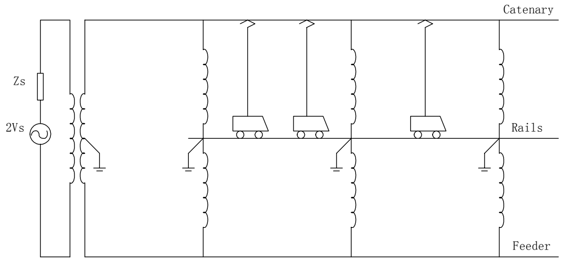

Fig. 1 AT feed system

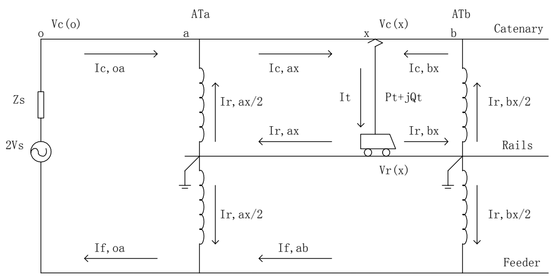

Fig. 2 Current distribution with AT feed

3. SIMPLIFIED MODEL FOR PROBABILITY LOAD FLOW

Probability load flow (PLF) calculation requires, first of all, the probability distribution functions (pdfs) of the real and reactive power (P and Q) of the train (as a load). As P and Q are related to the position, speed and traction equipment characteristics of the train, their pdfs should be derived from the above parameter, which is not within the scope of this study. The following discussions examine the model for PLF calculation once the pdfs of train load are known.

3.1 Feeding system

In addition to the pdfs of P & Q, an accurate model to represent the AT feed system, as well as a fast and reliable computation algorithm, is essential to PLF calculation.

As shown in Fig. 3, the AT feed system can be divided into 3 equivalent radial distribution system [3]. There are three main conductors carrying current, catenary, rail and feeder, whereas the two rails have been regarded as a combined conductor. Each conductor presents its own impedance and mutual impedances exist among the conductors. The admittances to ground is neglected to simplify the electrical network (even the rail-to-ground admittance is of the order of 0.5-1 Ω^{-1}km only).

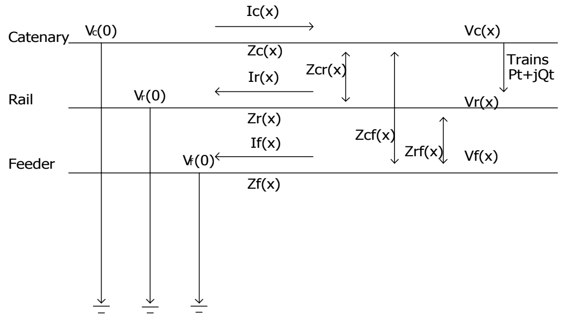

Fig. 3 Three equivalent radial distribution systems

Within a supply section where there is one supply feeding transformer of secondary voltage at 50kV, the catenary system is indeed a 25kV radical distribution with trains (loads) located at different positions. Fig. 4 shows an equivalent circuit for a total of *n* trains along the line.

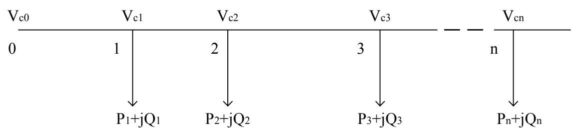

Fig. 4 Radial distribution with train loads

Mutual impedances play an important role in the calculation of volt-drop along the conductors. For example, in Fig. 3, the volt-drop in the catenary at train position *x* is

$$\begin{aligned}\Delta V(x) &= V_c(0) - V_c(x) \\ &= I_c(x)Z_c(x) - I_r(x)Z_{cr}(x) - I_f(x)Z_{cf}(x)\end{aligned}$$

which is different from the conventional radial distribution system.

3.2 Catenary voltage

Referring to Fig. 2, where there is a simple case of one train running and the two ATs are located at points *a* and *b*, the catenary voltage can be approximated by linearised equations [4] as follows:

The impedances of the catenary over *oa* and *ax* are $Z_{c,oa} = R_{c,oa} + jX_{c,oa}$ and $Z_{c,ax} = R_{c,ax} + jX_{c,ax}$ respectively.

The mutual impedances among the catenary amf the other two conductors along *oa* and *ax* are also denoted similarly,

$Z_{cf,oa} = R_{cf,oa} + jX_{cf,oa}$,
$Z_{cr,ax} = R_{cr,ax} + jX_{cf,ax}$ and
$Z_{cf,ax} = R_{cf,ax} + jX_{cf,ax}$.

$V_c(x)$ and $V_r(x)$ represent the catenary and rail voltage at point *x*.

When the train load is $S_t = P_t + jQ_t$, the current drawn by the train from the catenary is

$$I_t = \frac{S_t^*}{V_c^*(x) - V_r^*(x)} = \frac{P_t - jQ_t}{V_c^*(x) - V_r^*(x)} \qquad (1)$$

As V_r is usually much smaller than V_c (V_r is not larger than 50V while V_c should be very close to the nominal voltage, say 25kV, V_r can be neglected and (1) becomes

$$I_t = \frac{P_t - jQ_t}{V_c^*(x)} \qquad (2)$$

The two ATs share the current drawn by the train according to the location of train, *x*. The return currents toward the ATs through the rails are

$$I_{r,ax} = \frac{bx}{ab} I_t \text{ and } I_{r,bx} = \frac{ax}{ab} I_t \qquad (3)$$

To maintain the AT core ampere-turn balance, the inlet current should be shared equally between the two coils. Denote $\alpha = \frac{bx}{ab}$ and $\beta = \frac{ax}{ab}$, the currents within the supply system are thus given below.

$$\begin{aligned} I_{f,ab} &= 0.5 I_{r,ab} = 0.5\beta I_t \\ I_{f,oa} &= 0.5 I_{r,ax} + 0.5 I_{r,bx} = 0.5 I_t \\ I_{c,oa} &= I_{f,oa} = 0.5 I_t \\ I_{c,ax} &= I_{c,oa} + 0.5 I_{c,ax} = 0.5 I_t + 0.5\alpha I_t = 0.5(1+\alpha) I_t \\ I_{c,bx} &= 0.5 I_{r,bx} = 0.5\beta I_t \end{aligned} \qquad (4)$$

The catenary voltage drop, $\Delta V(x)$, at the train position *x*, is caused by the currents stated in (3) and (4) except $I_{c,bx}$, which does not flow within *ox*. Hence,

$$\begin{aligned} \Delta V(x) &= I_{c,oa} Z_{c,oa} + I_{c,ax} Z_{c,ax} - I_{r,ax} Z_{cr,ax} - I_{f,ab} Z_{cf,ax} \\ &\quad - I_{f,oa} Z_{cf,oa} \\ &= \Delta V_{c,oa} + \Delta V_{c,ax} - \Delta V_{cr,ax} - \Delta V_{cf,ax} - \Delta V_{cf,oa} \end{aligned} \qquad (5)$$

In order to simplify the volt-drop expressions for the PLF calculation, assumptions are made that i) the line voltage does not fluctuate much (which is a necessary requirement in railways); ii) the rail-to-earth voltage is negligible when compared with the nominal supply voltage; iii) the ATs are in ideal conditions of infinite transformer magnetising inductance, hence no iron loss and zero leakage and winding impedance; and iv) the imaginary part of the voltages are small enough to be ignored.

Hence, $V_c(x)$ can be initially replaced by a nominal supply voltage V_{nom} in (2) and the components of the voltage drop become:

$$\begin{aligned} \Delta V_{c,oa} &= 0.5 I_t Z_{c,oa} = 0.5(P_t R_{c,oq} + Q_t X_{c,oa}) / V_{nom} \\ \Delta V_{c,ax} &= 0.5(1+\alpha) I_t Z_{cr,ax} \\ &= 0.5(1+\alpha)(P_t R_{c,ax} + Q_t X_{c,ax}) / V_{nom} \\ \Delta V_{cr,ax} &= \alpha I_t Z_{cr,ax} = \alpha(P_t R_{c,ax} + Q_t X_{c,ax}) / V_{nom} \\ \Delta V_{cf,ax} &= 0.5\beta I_t Z_{cf,ax} = 0.5\beta(P_t R_{cf,ax} + Q_t X_{cf,ax}) / V_{nom} \\ \Delta V_{cf.oa} &= 0.5 I_t Z_{cf,oa} = 0.5(P_t R_{cf.oa} + Q_t X_{cf,oa}) / V_{nom} \end{aligned} \qquad (6)$$

As a result, the catenary voltage at point *x* is

$$V_c(x) = V(o) - \Delta V(x) = V_s - Z_s I_t - \Delta V(x) \qquad (7)$$

From the above linearised equations, it is possible to obtain the sensitivity coefficients as stated in [4] for the evaluation of the pdf of the voltage of any point along the catenary.

3.3 Power at feeding point

The current from the feeding transformer is $I_{c,oa}$ and the feeding point power is

$$S_s = V_s I_{c,oa}^* = 0.5 V_s (P_t + jQ_t) / V_c(x) \qquad (8)$$

From (8), the real and reactive power are

$$\begin{aligned} P_s &= 0.5 V_s P_t / V_c(t) \\ Q_s &= 0.5 V_s Q_t / V_c(t) \end{aligned} \qquad (9)$$

These two linearised equations are then used to derive the corresponding pdfs.

When there are more than one trains on the line, their current distributions are calculated individually. Currents are then summed together at each conductor to determine train voltage, P_s and Q_s by the superposition theorem.

4. SOFTWARE IMPLEMENTATION

The PLF calculation discussed in the previous section is part of a calculation tool for railway system analysis. A software suite has thus been developed to realise the railway feeding system model and the subsequent PLF calculation.

The flow diagram of the PLF calculation software is illustrated in Fig. 5. From the input data of feeding system, trains, power pdfs, the trains are located in turn, followed by the identification and calculation of the current distribution. Voltage drop at each node is then derived. The pdfs of the voltage and those of P and Q are attained by convolution. The program is written in C and run on a PC platform. The convolution routines are from the library of Matlab6.0 and they are wrapped in a DLL function for ready application.

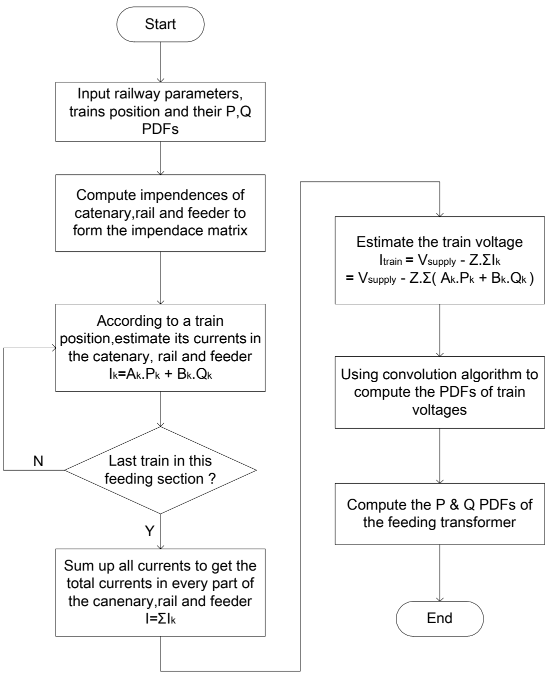

Fig. 5 Flow chart of the PLF calculation program

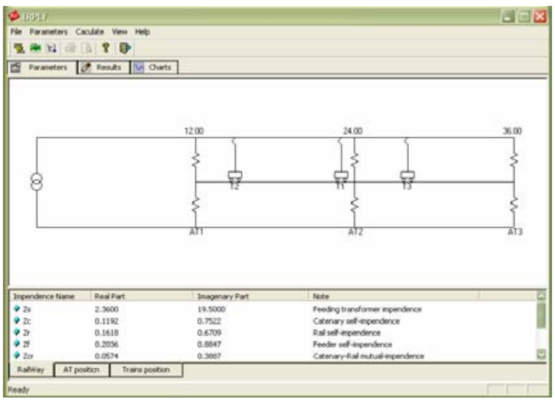

Fig. 6 An input interface

As the input process involves a large amount of data, the program also provides user-friendly interfaces for simple data input. Output, in term of pdf, is available in both numerical and graphical formats. Fig. 6 and Fig. 7 show example of an input and output interface respectively.

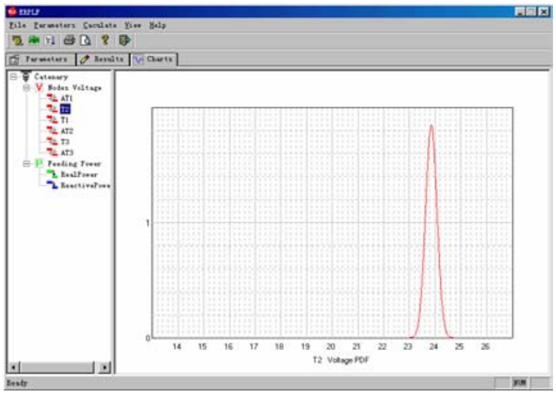

Fig. 7 An output interface

5. RESULTS AND DISCUSSIONS

A number of tests have been carried out to verify the calculation results from the program. A simple case is shown here to demonstrate the process. Referring to Fig. 2, the basic input data for a single-train PLF calculation is given below.

Feeding point voltage: V_s =27.5kV

Train load: P_t =5.66MW and Q_t =3.51MVar (power factor=0.85)

Train position: 23 km from the feeding transformer (i.e. *ox*=23km)

Pdf of P_t : $\mathrm{N}\left(5.66, 0.25^2\right)$

Pdf of Q_t : $\mathrm{N}\left(3.51, 0.25^2\right)$

P_t and Q_t are assumed to be normally distributed, which is of course not the case in real-life railway operation. However, they are set just for the purposes of result validation and indeed the calculation process works equally well for any distributions of P_t and Q_t .

Data from real systems are adopted for the self-impedances and mutual impedances of the conductors [3].

The pdf of the catenary voltage at the train position is shown in Fig. 8. $V_c(x)$ is also normally distributed with μ=25.06MW and σ^2=0.145. It is consistent with results obtained from probability theory, which reads: μ=25.09MW and σ^2=0.142.

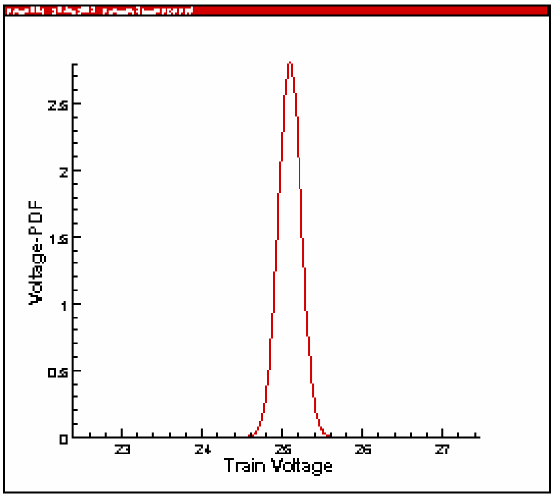

Fig. 8 Pdf of catenary voltage at train position

The computation model in this study is simplified with the radial distribution configuration, linearisation and approximations on the imaginary part of the voltages. While it provides a simple model and substantial reduction on computation, accuracy has inevitably been compromised in certain extent. Besides, this method is not applicable for train in regenerative braking operation as the train becomes a source and the system is no longer a radial one. Further studies are therefore required to quantify the reliability of the results.

Further works also include an attempt on a complete model which is capable of handling multiple-feeding sections, branches and junctions on track etc. One of the possibilities is to represent the three equivalent systems in Fig. 3 in matrix form

$$\begin{bmatrix} V_c(o)-V_c(x) \\ V_r(o)-V_r(x) \\ V_f(o)-V_f(x) \end{bmatrix} = \begin{bmatrix} Z_c(x) & Z_{cr}(x) & Z_{cf}(x) \\ Z_{cr}(x) & Z_r(x) & Z_{rf}(x) \\ Z_{cf}(x) & Z_{rf}(x) & Z_f(x) \end{bmatrix} \begin{bmatrix} I_c(x) \\ I_r(x) \\ I_f(x) \end{bmatrix} \quad (10)$$

By substituting $I = S^* / V^*$ in (10), the load flow equations become similar to those in conventional power system load flow analysis. Hence, they can be solved by the well-proven algorithms [5].

6. CONCLUSIONS

This paper presents a simplified PLF calculation model for electrified railway systems with AT feeding. It is a preliminary investigation to a comprehensive study of power load flow in railways and their calculation. It also describes the software suite developed to realise the model. Although accuracy is expected to be compromised, results from simple case studies show acceptable performance. This study certainly offers a solid foundation for further works on different PLF methods, feeding system modelling and software development.

ACKNOWLEDGEMENT

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