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# Analysis of Primary User Duty Cycle Impact on Spectrum Sensing Performance

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**Abstract**—Spectrum sensing is considered to be one of the most important tasks in cognitive radio. Many sensing detectors have been proposed in the literature, with the common assumption that the primary user is either fully present or completely absent within the window of observation. In reality, there are scenarios where the primary user signal only occupies a fraction of the observed window. This paper aims to analyse the effect of the primary user duty cycle on spectrum sensing performance through the analysis of a few common detectors. Simulations show that the probability of detection degrades severely with reduced duty cycle regardless of the detection method. Furthermore we show that reducing the duty cycle has a greater degradation on performance than lowering the signal strength.

## I. INTRODUCTION

Cognitive radio (CR) is a relatively new technology that has received major attention in the recent years [1]. A well coordinated and deployed CR network is a potential solution to the problem of spectral scarcity. Under the concept of CR, secondary users (SU) are allowed to occupy spectra not exclusively licensed to them provided that the primary, licensed users (PU) are not disrupted. The potential of CR has become more apparent as studies [2], [3] have shown that licensed spectra are commonly under utilised.

Spectrum sensing is one of the most important task in CR operation. CR users must reliably detect the presence and absence of PU signals for PU protection and increase spectra utilisation efficiency. To accurately detect a range of different signal types, spectrum detectors will need to extract different features that are unique to PU signals and distinguish them from noise and other unwanted interference [4], [5].

Numerous spectral sensing techniques have been proposed in the literature. Three commonly considered detectors are; the energy detector, waveform detector and cyclostationary-based detectors [6]–[8]. The energy detector computes the energy of the received signal and compares it to a threshold based on the noise floor. The waveform detector correlates a known sequence expected within the PU signal to the received signal for coherent detection, while cyclostationary-based detection extracts the cyclostationary features common in communication signals. Each of these detectors have their advantages and disadvantages and many authors have proposed solutions to improve their performances [6], [7], [9]–[12].

Assumptions are typically imposed to ensure the functionality of the detector. One condition often assumed in the literature is the full presence or absence of the PU, but scenarios exists where this is not the case. If sensing is performed at the end or the beginning of a PU transmission then the signal will only be present during the first or last portion of the observed sample. For burst transmission schemes characteristic of many communications systems, the probability that the PU signal may only occupy a fraction of the observation window increases. In this paper we analyse the impact on detection performance when the PU only occupies a fraction of the observation window. We shall define the duty cycle of the PU signal as the portion of the observed signal of which the signal is present. For example, with an observation length of 1000 samples and during which the PU signal only occupies 500 samples, the equivalent duty cycle is 50%. The common assumption is that the duty cycle of the signal is constant at 100%.

This paper analyses the deterioration of spectrum sensing performance due to the duty cycle of the PU using the afore mentioned detectors. The structure of the paper is organised as follows: Section II introduces the model of PU signal duty cycle and reviews and reformulates the models for each detector to include the signal's duty cycle. Section III outlines the simulation setup used to validate the new expressions obtained. Section IV presents the results of the investigation as well as providing an analysis and discussion. Finally, Section V concludes this investigation.

## II. DETECTOR MODELS AND DUTY CYCLE

We begin the analysis by reviewing the model of the detection problem and introduce the model to consider the effect of PU signal duty cycle. The models of the energy detector, waveform detector and cyclostationary-based detectors are then reformulated to include the duty cycle effect. For the purpose of this investigation, we shall assume that all the required parameters are completely known and readily available for the detectors.

### A. Detection Model and PU Duty Cycle

The received signal  $y(n)$  is typically modelled as having the following form

$$y(n) = s(n) + w(n). \quad (1)$$

Here,  $s(n)$  is the signal to be detected and  $w(n)$  is the background noise and other interference sources. When the user signal is absent,  $s(n) = 0$  and  $y(n) = w(n)$ . Therefore the detector is required to distinguish between two hypotheses,

$$\mathcal{H}_0 : y(n) = w(n) \quad (2)$$

$$\mathcal{H}_1 : y(n) = s(n) + w(n). \quad (3)$$

The duty cycle of the PU signal is introduced through a new variable  $D$  into the above model.  $D$  defines the proportion of the observed signal in which PU signal is present. That is,  $D$  takes a value between  $0 \leq D \leq 1$ , where  $D = 0$  is equivalent to no PU signal and  $D = 1$  is where PU is present over the entire observed duration.

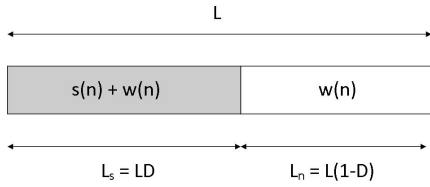


Fig. 1. Structure of the received signal with non 100% duty cycle

An example of an observed signal with sample length  $L$  and  $D < 1$  is illustrated in Fig. 1. The portion that contains the PU signal corrupted by noise has new length  $L_s = LD$  and the section that contains noise only has length  $L_n = L(1 - D)$ . The received signal with non 100% duty cycle is equivalent to the two sub portions concatenated and is defined as

$$y_D(n) = \begin{cases} s(n) + w(n) & \text{for } 0 \leq n \leq L_s \\ w(n) & \text{for } L_s + 1 \leq n \leq L. \end{cases}$$

Under the special case of  $D = 0$ , PU signal is completely absent and the received signal contains noise only as per  $\mathcal{H}_0$  of (2). When  $D = 1$ , PU signal is completely present similar to  $\mathcal{H}_1$  of (3).

The performance of detection is determined by the probability of false alarm  $P_F$  and probability of detection  $P_D$ . Probability of false alarm is the case where the detector decides a signal is present when there is no signal. Probability of detection is the case where the detector decides a signal is present when the signal is truly there. The test statistic  $T$  is a metric given at the output of a detector and compared to a threshold  $\lambda$ . The two probabilities are given by

$$P_F = P(T > \lambda | \mathcal{H}_0) \quad (4)$$

$$P_D = P(T > \lambda | \mathcal{H}_1). \quad (5)$$

For the purpose of this investigation we assume that the exact SNR of the PU signal is known. Thus an optimum threshold can be determined to optimise  $P_F$  and  $P_D$ . In

practice where SNR may be unknown, a constant false alarm rate (CFAR) threshold is often considered to fix a maximum false alarm rate.

The detectors do not know the duty cycle of the PU signal and must make a decision based on the threshold optimised for  $\mathcal{H}_0$  and  $\mathcal{H}_1$ . The deterioration in detection performance is due to the threshold not being optimised for a PU signal with non 100% duty cycle.

We now use the afore mentioned detectors as examples to demonstrate the effect of PU duty cycle on detection performances. The detectors are based on different models and hence the relation between probability of detection and PU duty cycle will be different. Firstly we review the models of these detectors and then introduce the effect of duty cycle  $D$  into their expressions. A similar analysis can be applied to any spectrum sensing detector. The location of the PU signal is also assumed to be at the beginning of the observed signal. In practice, the location of the PU signal will be unknown, but the same analysis holds true regardless of the location of the PU signal.

### B. Example 1: Energy Detector

The energy detector computes the total energy of an observed signal and compares it with a threshold dependent on the noise floor. The test statistic  $T$  is calculated by the sum of all samples squared,

$$T = \sum_{n=0}^{L-1} |y(n)|^2. \quad (6)$$

The detector uses this metric to decide the presence of a signal. When  $T$  is greater than a specific threshold  $\lambda$ ,  $s(n)$  is present. On the other hand, when  $T < \lambda$  the signal is decided to be absent.

For simplicity, noise and interference is modelled as zero-mean Gaussian random variable with a variance  $\sigma_n^2$ , i.e.  $w(n) \sim N(0, \sigma_n^2)$ . Furthermore, we also assume the signal to be a zero-mean Gaussian variable with variance  $\sigma_s^2$ , i.e.  $s(n) \sim N(0, \sigma_s^2)$ . We denote  $T_0$  as  $T$  under  $\mathcal{H}_0$  and  $T_1$  is  $T$  under  $\mathcal{H}_1$ . If  $L$  is sufficiently large, then both  $T_0$  and  $T_1$  will converge to a Gaussian distribution.

If we define the variance of  $y(n)$  under  $\mathcal{H}_1$  as  $\sigma_t^2 = \sigma_n^2 + \sigma_s^2 = \sigma_n^2(\text{SNR} + 1)$ , then the distributions of  $T_0$  and  $T_1$  are given as

$$T_0 \sim N(L\sigma_n^2, 2L\sigma_n^4) \quad (7)$$

$$T_1 \sim N(L\sigma_n^2(\text{SNR} + 1), 2L\sigma_n^4(\text{SNR} + 1)^2). \quad (8)$$

$P_F$  and  $P_D$  is obtained by finding the cumulative distribution function of  $T_0$  and  $T_1$  respectively,

$$P_F = Q\left(\frac{\lambda - L\sigma_n^2}{\sqrt{2L\sigma_n^4}}\right) \quad (9)$$

$$P_D = Q\left(\frac{\lambda - L\sigma_n^2(\text{SNR} + 1)}{\sqrt{2L\sigma_n^4(\text{SNR} + 1)^2}}\right). \quad (10)$$

where  $Q(\cdot)$  is one minus the cumulative distribution function of the standardised normal distribution.

As SNR of the received signal is assumed to be known, the optimum threshold  $\lambda$  that gives  $P_F = 1 - P_D$  is determined by rearranging (9) and (10),

$$\lambda = \frac{2L(\text{SNR} + 1)\sigma_n^2}{\text{SNR} + 2}. \quad (11)$$

We now analyse the energy detector when the receive signal is of the form illustrated in Fig. 1. The portion that contains the PU signal is normally distributed with zero mean and variance  $\sigma_t^2 = \sigma_n^2(\text{SNR} + 1)$ . The section that contains noise only is zero mean with variance  $\sigma_n^2$ . The complete observed signal,  $y_D(n)$ , remains normally distributed with zero mean, and a variance  $\sigma_D^2$  determined by the mean of the variance of the two portions, such that

$$\begin{aligned} \sigma_D^2 &= \sigma_t^2 D + \sigma_n^2(1 - D) \\ &= \sigma_n^2(\text{SNR} \times D + 1). \end{aligned} \quad (12)$$

Following the same approach to define the test statistic as in (6), we can define  $T_D$  as the test statistic of the observed signal with the effect of PU signal duty cycle. The distribution of  $T_D$  is

$$\begin{aligned} T_D &\sim N(L\sigma_D^2, 2L\sigma_D^4) \\ &= T_D \sim N(L\sigma_n^2(\text{SNR} \times D + 1), 2L\sigma_n^4(\text{SNR} \times D + 1)^2). \end{aligned} \quad (13)$$

The test statistic of the received signal is compared to the threshold calculated by (11) assuming that  $D = 1$ . We define the probability of detection of a PU signal under duty cycle  $D$  as  $P_{DD}$  and apply an expression similar to (10) using  $\lambda$  and the distribution of  $T_D$ ,

$$\begin{aligned} P_{DD} &= Q\left(\frac{\lambda - L\sigma_D^2}{\sqrt{2L}\sigma_D^2}\right) \\ &= Q\left(-\text{SNR}\sqrt{\frac{L}{2}}\left(\frac{\text{SNR} \times D + 2D - 1}{(\text{SNR} \times D + 1)(\text{SNR} + 2)}\right)\right). \end{aligned} \quad (14)$$

Under the special case where  $D = 0$ ,  $P_{DD}$  is the equivalent to the probability of false alarm as per (9). Also when  $D = 1$ ,  $P_{DD}$  gives the probability of detection as per (10). By analysing the effect of PU duty cycle we assume that the PU signal is always present at a specific duty cycle  $D > 0$ . Probability of false alarm is unaffected by PU duty cycle and therefore only the probability of detection is considered.

From (8) and (13) we can see that the distribution of  $T_D$  at a given SNR and  $D$  is equivalent to  $T_1$  at  $\text{SNR} \times D$ . However,  $P_D$  at  $\text{SNR} \times D$  and  $D = 1$  is not the same as  $P_{DD}$  at  $\text{SNR}$  and  $D < 1$ . This is because when  $D = 1$ , the threshold is calculated to optimise  $P_D$  while the threshold used to calculate  $P_{DD}$  at  $D < 1$  is not optimised.

### C. Example 2: Waveform Detector

Most modern communication signals introduce some pre-known patterns such as pilot tones, preambles and cyclic prefixes, etc. to assist synchronisation and other purposes. Waveform-based detectors utilise this a priori knowledge of

the PU signal to perform correlation detection. Thus, the waveform detectors require the assumption that some signature of the PU signal is known and is perfectly synchronised with the PU signal.

The same signal model as given in Section II-B. is used. Here  $s(n)$  is the known pilot data. The test statistic is calculated by

$$T = \sum_{n=1}^L y(n)s^*(n) \quad (15)$$

$$T_0 = \sum_{n=1}^L w(n)s^*(n) \quad (16)$$

$$T_1 = \sum_{n=1}^L |s(n)|^2 + \sum_{n=1}^L w(n)s^*(n). \quad (17)$$

where  $*$  denotes the complex conjugate. Since  $T$  is a linear combination of jointly Gaussian random variables  $y(n)$ ,  $T$  is also Gaussian under either hypothesis. If we define  $\varepsilon$  similar to the approach taken in [6],

$$\begin{aligned} \varepsilon &= \sum_{n=1}^L |s(n)|^2 \\ &= L \times \text{E}[s(n)^2] \\ &= L\sigma_n^2 \text{SNR}, \end{aligned} \quad (18)$$

where  $\text{E}[\cdot]$  is the expectation operator, then the distribution of  $T_0$  and  $T_1$  becomes

$$\begin{aligned} T_0 &\sim N(0, \sigma_n^2 \varepsilon) \\ &= T_0 \sim N(0, L\sigma_n^4 \text{SNR}) \end{aligned} \quad (19)$$

$$\begin{aligned} T_1 &\sim N(\varepsilon, \sigma_n^2 \varepsilon) \\ &= T_1 \sim N(L\sigma_n^2 \text{SNR}, L\sigma_n^4 \text{SNR}). \end{aligned} \quad (20)$$

$P_D$  and  $P_F$  are then evaluated as

$$P_F = Q\left(\frac{\lambda}{\sqrt{L} \times \text{SNR} \sigma_n^2}\right) \quad (21)$$

$$P_D = Q\left(\frac{\lambda - L\sigma_n^2 \text{SNR}}{\sqrt{L} \times \text{SNR} \sigma_n^2}\right). \quad (22)$$

Since  $T_0$  and  $T_1$  have the same variance, the optimum threshold is

$$\lambda = \frac{\varepsilon}{2} = \frac{L\sigma_n^2 \text{SNR}}{2}. \quad (23)$$

By introducing  $D$ , the length of  $s(n)$  is shortened to  $L_s = LD$ .  $T_D$  is then calculated as

$$T_D = \sum_{n=1}^{L_s} |s(n)|^2 + \sum_{n=1}^L w(n)s^*(n). \quad (24)$$

Similar to (18), the first term of  $T_D$  equates to  $\varepsilon_D = LD\sigma_n^2 \text{SNR}$ . The second term of  $T_D$  is a random variable having the same distribution as  $T_0$ . The distribution of  $T_D$  is thus given by

$$\begin{aligned} T_D &\sim N(\varepsilon_D, \sigma_n^2 \varepsilon) \\ &= T_D \sim N(LD\sigma_n^2 \text{SNR}, L\sigma_n^4 \text{SNR}). \end{aligned} \quad (25)$$

Once again we assume that the waveform detector does not know the duty cycle of PU signal and calculates a threshold

based on the assumption that  $D = 1$ . Therefore the probability of detection under the effect of duty cycle is given by

$$P_{DD} = Q\left(\frac{\lambda - LD\sigma_n^2 \text{SNR}}{\sqrt{L \times \text{SNR} \sigma_n^2}}\right) = Q\left(\frac{\sqrt{L \times \text{SNR}(1-2D)}}{2}\right). \quad (26)$$

For the waveform detector, reducing PU signal's duty cycle by  $D$  results in the test statistics to be distributed as  $T_D \sim N(LD\sigma_n^2 \text{SNR}, LD\sigma_n^4 \text{SNR})$ . On the other hand reducing the SNR of a fully present PU signal by  $D$  will result in a test statistic of  $T_1 \sim N(LD\sigma_n^2 \text{SNR}, LD\sigma_n^4 \text{SNR})$ . Earlier, with the energy detector, we showed that the distribution of  $T_1$  by introducing  $D$  to the PU duty cycle is the same as reducing the SNR by a factor of  $D$ . For waveform detectors, this is not the case because only the mean of  $T_D$  is affected by  $D$ . Nevertheless,  $P_{DD}$  degrades significantly with respect to  $D$  because the threshold is no longer optimised.

#### D. Example 3: Cyclostationary Detector

A signal is said to be cyclostationary if the signal's statistics such as the mean and autocorrelation are periodic with time. The cyclic frequency  $\alpha$  is the frequency at which these statistic vary. Communication systems signals typically have induced cyclostationary features because information data is often modulated onto periodic carriers which are cyclostationary in nature. The cyclic spectral density (CSD), is a function of frequency and cyclic frequency, and can be used to extract features that are unique to PU signal due to the fact that white noise has no correlation and its CSD is weak.

The CSD of the received signal is calculated as [7]

$$S(f, \alpha) = \sum_{\tau=-\infty}^{\infty} R_y^\alpha(\tau) e^{-j2\pi f \tau}, \quad (27)$$

where

$$R_y^\alpha(\tau) = \text{E} [y(n + \tau) y^*(n - \tau) e^{j2\pi \alpha n}] \quad (28)$$

is the cyclic autocorrelation function (CAF). The CSD function peaks when the cyclic frequency is equal to the fundamental frequencies of the transmitted signal  $s(n)$ . For a narrow band signal with a centre frequency  $f = f_0$ , the CSD has four peaks located at  $(f = \pm f_0, \alpha = 0)$  and  $(f = 0, \alpha = \pm 2f_0)$ . As the cyclostationary features will vary depending on signal type, the method to extract these features will also be dependent on the signal.

An explicit relation between the CSD and signal parameters such as noise variance, SNR and sample length is difficult to calculate and dependent upon PU signal type. Therefore a numerical approach is used to find the test statistic of the cyclostationary detector. For the purpose of this investigation, we assume that  $s(n)$  is a pure sinusoid at frequency  $f_0$ . Thus the CSD consists of four impulses located at the previously mentioned frequency pairs corrupted by the CSD of noise. We

then define the test statistic as the sum of the magnitude at these four locations:

$$T = |S(f_0, 0)| + |S(-f_0, 0)| + |S(0, 2f_0)| + |S(0, -2f_0)|. \quad (29)$$

The distribution of  $T$  were generated numerically through fifty thousand independent trials. The distribution closely resembles the Gamma distribution, and thus it is most reasonable to use the Gamma distribution to model them. The two parameters  $k$  and  $\theta$  were calculated to be the maximum likelihood estimate of the shape and scale parameters of the Gamma distribution respectively. The distribution of  $T_0$  and  $T_1$  are modelled as

$$T_0 \sim \Gamma(k_0, \theta_0) \quad (30)$$

$$T_1 \sim \Gamma(k_1, \theta_1), \quad (31)$$

where  $k_0, \theta_0, k_1$  and  $\theta_1$  are the shape and scale parameters for  $T_0$  and  $T_1$  respectively. The threshold is then chosen by evaluating the cumulative distribution function to optimise  $P_F$  and  $P_D$ .

To analyse the effect of PU duty cycle we consider the case of  $\mathcal{H}_1$  where  $y(n) = s(n)$ . From (27) and (28) we can see that the CSD involves the multiplication of the Fourier Transform of the signal,  $S(f)$ , by a frequency shifted version of itself. Therefore the peaks of the CSD have magnitude equal to the magnitude of  $S(f)^2$ . With the sample length of  $s(n)$  reduced to  $LD$ , the magnitude of  $S(f)$  is also reduced by  $D$ . As before we model  $T_D$  as

$$T_D \sim \Gamma(k_D, \theta_D). \quad (32)$$

The same threshold calculated from the distribution  $T_0$  and  $T_1$  is used to determine  $P_{DD}$  based on the distribution of  $T_D$ .

### III. SIMULATION SETUP

For the simulation it will be necessary to introduce further assumptions for each detector to focus the investigation on the duty cycle effect of PU signal. For the energy detector, noise and signal are normally distributed with no noise variance uncertainty. The pilot data is known and perfectly synchronised for the waveform detector. The cyclostationary detector requires perfect cyclostationary features of the PU signal and the knowledge of cyclic frequencies. Furthermore the SNR is needed for optimal threshold decision.

Four SNR values are chosen and a range of duty cycle,  $0 \leq D \leq 1$  is simulated to compute the corresponding  $P_{DD}$ . The special cases at  $D = 0$  gives  $P_F$  (noise only) and  $D = 1$  gives  $P_D$  (signal fully present). The first SNR is chosen to be 0dB and the other three SNR are calculated for each detector such that  $P_D = [0.99, 0.9, 0.8]$ . SNR = 0dB simulates the scenario of high SNR and extremely small chance of miss detection.  $P_D = 0.99$  is the case where there is a slight chance for miss detection.  $P_D = 0.9$  is commonly considered to be the minimum detection rate (and maximum false alarm rate) for a viable CR system [13].  $P_D = 0.8$  is a scenario where detection rate is less than ideal. The SNR used for each detector are outlined in Table I.

TABLE I  
SNR REQUIRED TO ACHIEVE DESIRED  $P_D$  FOR EACH DETECTOR.

	$\rightarrow 1$	$\approx 0.99$	$P_D$ $\approx 0.9$	$\approx 0.8$
Energy Detector	0	-6.4	-9.2	-11.1
Waveform Detector	0	-16.8	-22	-25.6
Cyclostationary	0	-13.6	-17	-19.3

For the energy detector, random signal and noise samples are generated and their variances are used to calculate threshold  $\lambda$  and  $P_{DD}$  at varying  $D$ . As for the waveform detector, a sinusoid is used as the pilot data and it is constant for a given SNR and only noise samples are generated. Since no explicit relationship is available for the cyclostationary detector, the distribution of  $T_D$  is obtained from multiple simulations. The threshold is then determined by  $T_D$  at  $D = 0$  and  $D = 1$  and  $P_{DD}$  is calculated for varying  $D$ .

We also compare the effect of PU duty cycle and the effect of lowering PU SNR. We define  $\text{SNR}_D$  as the higher SNR with  $D < 1$  and associated  $P_{DD}$ , and  $\text{SNR}_L$  as the lower SNR with  $D = 1$  and associated  $P_D$ . For a fixed  $\text{SNR}_D$ , we find the  $\text{SNR}_L$  such that  $P_{DD} = P_D$ . The ratio of  $\text{SNR}_L/\text{SNR}_D$  is then compared with  $D$ . As the threshold used to detect a signal at  $D < 1$  is not optimised, it is expected that the required  $\text{SNR}_L$  to achieve the same detection rate will be less than SNR reduced by  $D$ .

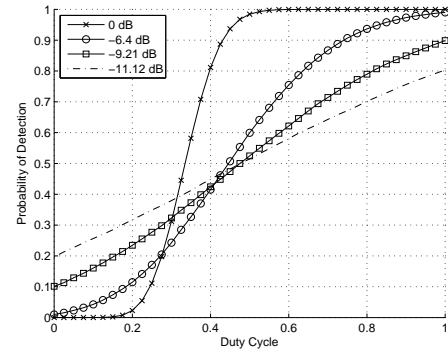
#### IV. RESULTS AND ANALYSIS

The relationship between the duty cycle of PU signal and the associated probability of detection is presented for each of the considered detector. Each line in Fig. 2, 3, 4 represent a SNR of interest. The  $P_{DD}$  at  $D = 0$  is the probability of false alarm and  $P_{DD}$  at  $D = 1$  is the probability of detection for the given SNR. Secondly, the ratio  $\text{SNR}_L/\text{SNR}_D$  is presented with respect to duty cycle to demonstrate the relation between duty cycle and changing SNR.

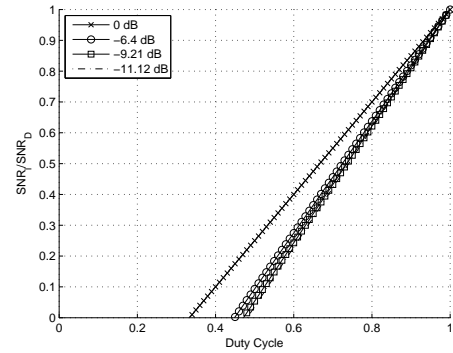
Fig. 2(a). shows the relation between  $P_{DD}$  and  $D$  for the energy detector. At high SNR ( $\text{SNR} = 0\text{dB}$ ),  $D$  does not have much effect on  $P_{DD}$  until  $D < 0.5$ . However for lower SNR the effect is more severe. Since this investigation implements optimum threshold detection,  $P_{DD} = 0.5$  is the minimum bound for detector performance. This implies that the detector cannot accurately detect any signals where the duty cycle result in  $P_{DD} \leq 0.5$ .

Fig. 2(b). compares the relative SNR required to achieve the same  $P_D$  as a signal with  $D < 1$ . It can be seen that the ratio  $\text{SNR}_L/\text{SNR}_D$  is less than  $D$  and the rate of decay is dependent on SNR. For example, the  $P_{DD}$  of a signal with  $\text{SNR} = 0\text{dB}$  and  $D = 0.8$  is the same  $P_D$  as a full duty signal with  $\text{SNR} \times 0.6$ . Such relationship suggests that when the duty cycle of a PU signal is reduced the detection rate degrades more severely compared to a PU signal with lower SNR.

For the waveform detector,  $T_1$  and  $T_D$  have the same variance but different means and the threshold is optimised to be  $\lambda = E[T_1]/2$ . When  $D = 0.5$ ,  $E[T_D] = \lambda$  and  $P_{DD} = 0.5$ .



(a)  $P_{DD}$  vs.  $D$



(b)  $\text{SNR}_L/\text{SNR}_D$  vs.  $D$

Fig. 2. Detection performance of energy detector with duty cycle.

This relation holds true regardless of the SNR as shown in Fig. 3(a). Thus, the Waveform Detector cannot reliably detect PU signals at any SNR with less than half the duty cycle.

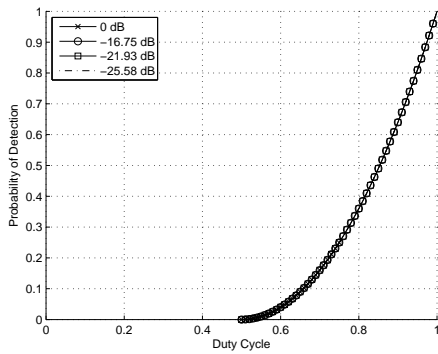
Fig. 3(b) shows that the relationship between the relative SNR and duty cycle is the same regardless of signal SNR due to the distribution of  $T_D$ . The figure also shows that reducing  $D$  reduces the detection rate more significantly than lowering SNR.

The cyclostationary detector is just as intolerant to  $D$  as the other two detectors as demonstrated in Fig. 4(a). At lower SNR,  $P_{DD}$  drops quickly to an unacceptable level as the duty cycle decreases.

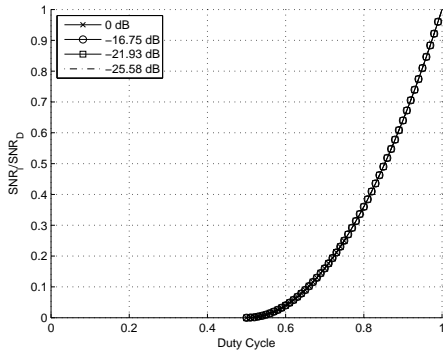
Fig. 4(b). shows that rate of decay of the relative SNR is more similar to the energy detector. In this figure the discontinuity for  $\text{SNR} = 0\text{dB}$  from  $0.35 \leq D \leq 1$  is due to the resolution of the numerical approach. The line should follow the general trend as the other SNRs and extend to  $\text{SNR}_L/\text{SNR}_D = D = 1$ .

#### V. CONCLUSION

In this paper, we presented an analysis on how the duty cycle of the primary user signal impacts the probability of detection in a cognitive radio spectrum sensing scenario. The three spectrum sensing detectors considered were; the energy detector, waveform detector and cyclostationary-based detectors. All of them were shown to perform poorly when

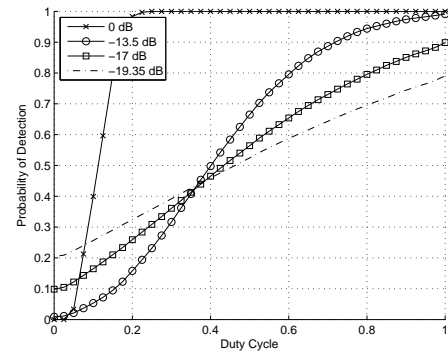


(a)  $P_{DD}$  vs.  $D$

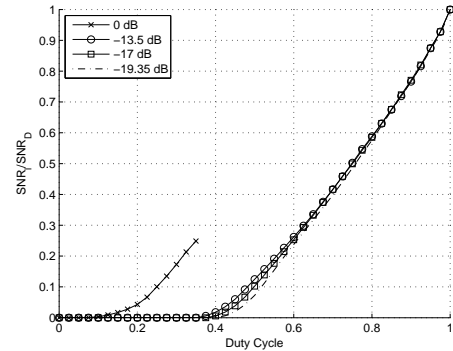


(b)  $\text{SNR}_L/\text{SNR}_D$  vs.  $D$

Fig. 3. Detection performance of waveform detector with duty cycle.



(a)  $P_{DD}$  vs.  $D$



(b)  $\text{SNR}_L/\text{SNR}_D$  vs.  $D$

Fig. 4. Detection performance of Cyclostationary-based Detector with duty cycle.

the primary user signal does not occupy the entire window of observation. Of the three detectors, the waveform detector is the least tolerant to reduced duty cycle and the detector cannot accurately detect a signal when the duty cycle is less than 50%. Finally we have shown that reducing the signal's duty cycle by a factor  $D$  will result in a smaller detection rate compared to lowering the signal SNR by  $D$ .

## REFERENCES

- [1] J. Mitola Iii, "Cognitive radio for flexible mobile multimedia communications," *Mobile Networks and Applications*, vol. 6, no. 5, pp. 435–441, 2001.
- [2] R. Chiang, G. Rowe, and K. Sowerby, "A quantitative analysis of spectral occupancy measurements for cognitive radio," in *Vehicular Technology Conference, 2007. VTC2007-Spring. IEEE 65th*, April 2007, pp. 3016–3020.
- [3] F. B. Frederiksen, "Improving spectral capacity and wireless network coverage by cognitive radio technology and relay nodes in cellular systems," *Wireless Personal Communications*, vol. 45, no. 3, pp. 355–368, 2008.
- [4] A. Hisham and H. Arslan, "Multidimensional signal analysis and measurements for cognitive radio systems," in *Radio and Wireless Symposium, 2008 IEEE*, Jan. 2008, pp. 639–642.
- [5] M. Hoyhtya, T. Chen, and A. Mammela, "Interference management in frequency, time, and space domains for cognitive radios," in *Wireless Telecommunications Symposium, 2009. WTS 2009*, April 2009, pp. 1–7.
- [6] D. Cabric, A. Tkachenko, and R. W. Brodersen, "Spectrum sensing measurements of pilot, energy, and collaborative detection," in *Military Communications Conference, 2006. MILCOM 2006. IEEE*, 2006, pp. 1–7.
- [7] S. Xu, Z. Zhao, and J. Shang, "Spectrum sensing based on cyclostationarity," aug. 2008, pp. 171–174.
- [8] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *Communications Surveys & Tutorials, IEEE*, vol. 11, no. 1, pp. 116–130, Quarter 2009.
- [9] P. De and Y.-C. Liang, "Blind spectrum sensing algorithms for cognitive radio networks," *Vehicular Technology, IEEE Transactions on*, vol. 57, no. 5, pp. 2834–2842, Sept. 2008.
- [10] Y. M. Kim, G. Zheng, S. H. Sohn, and J. M. Kim, "An alternative energy detection using sliding window for cognitive radio system," in *Advanced Communication Technology, 2008. ICACT 2008. 10th International Conference on*, vol. 1, Feb. 2008, pp. 481–485.
- [11] D. Kun and N. Morgan, "A new low-cost cfar detector for spectrum sensing with cognitive radio systems," in *Aerospace conference, 2009 IEEE*, March 2009, pp. 1–8.
- [12] M. Oner and F. Jondral, "Cyclostationarity based air interface recognition for software radio systems," sept. 2004, pp. 263–266.
- [13] D. Cavalcanti and M. Ghosh, "Cognitive radio networks: Enabling new wireless broadband opportunities," in *Cognitive Radio Oriented Wireless Networks and Communications, 2008. CrownCom 2008. 3rd International Conference on*, May 2008, pp. 1–6.