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DELAY DISTRIBUTION BASED ROBUST H_∞ CONTROL OF NETWORKED CONTROL SYSTEMS WITH UNCERTAINTIES

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ABSTRACT

Network induced delay in networked control systems (NCS) is inherently non-uniformly distributed and behaves with multifractal nature. However, such network characteristics have not been well considered in NCS analysis and synthesis. Making use of the information of the statistical distribution of NCS network induced delay, a delay distribution based stochastic model is proposed to link Quality-of-Control and network Quality-of-Service for NCS with uncertainties. From this model together with a tighter bounding technology for cross terms, H_∞ NCS analysis is carried out with significantly improved stability results. Furthermore, a memoryless H_∞ controller is designed to stabilize the NCS and to achieve the prescribed disturbance attenuation level. Numerical examples are given to demonstrate the effectiveness of the proposed method.

Key Words: Networked control systems, H_∞ control, stochastic distribution, linear matrix inequalities, uncertainties.

I. INTRODUCTION

Networked control systems (NCS) use data networks to close both information and control loops. Significant effort is being made in the networked control community to develop general NCS networking technologies, e.g. those based on Ethernet or wireless Ethernet, for control applications requiring remote and distributed operations. Typical examples include remote medical treatment, urban heating systems, and wide area plant automation over Internet Protocol (IP) networks. Due to its well-developed infrastructure, wide acceptance, simplicity and affordability [1], IP networking has been promoted for networked control. Although IP networking is convenient for communications in large-scale and remotely distributed systems, there are still major difficulties in design and implementation of NCS over IP networks [2]. Fundamentals of NCS over data communication networks can be found in [1, 3] and references therein for related works.

An NCS integrates information, communications, and control with control loops being closed through the network. Among many problems in NCS, how to deal with time-varying network induced delay is challenging, and has been one of our recent research focuses, e.g. [2, 4–6]. Network induced delay degrades control performance or even causes system instability. There have been some reports in the open literature on disturbance attenuation and robust H_∞ control of systems without transmission delay of control signals [7].

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Much effort has also been made on this topic for NCS [3, 8, 9]. By introducing some slack matrix parameters and considering the lower bound of network-induced delay, Yue *et al.* have studied the robust H_∞ control of uncertain NCS and obtained less conservative results than previous work [8]; but the computational demand is high for the controller feedback gain due to the introduced slack matrices. Under the assumption that the network-induced delay is less than the sampling period and satisfies Bernoulli random binary distribution, Yang *et al.* have considered the observer-based H_∞ controller problem for NCS with random communication delays [3]; however, they have neglected the synchronization of the control signals from the controller to actuator. Gao, Chen and Lam have studied a special type of delay system, which contains multiple successive delay components in the state [10, 11]. Less conservative results can be expected from [10, 11] because more information on the inner delay distribution properties has been utilized.

Although much research has been conducted in NCS analysis and synthesis, the statistical features of NCS communication networks have not been fully captured. For example, many published papers of theoretical investigations have inherently assumed an uniform distribution of the NCS network-induced delay between its lower and upper bounds. Recent advances, have revealed that IP however, based NCS network-induced delay is non-uniformly distributed [1] and behaves with a multifractal nature [6]. It is our belief that the more the statistical features of the NCS networks are utilized in NCS analysis and synthesis, the better the control performance obtained. The problem of H_∞ robust analysis and synthesis for uncertain NCS with consideration of the stochastic distribution of the network delay remains open. Furthermore, simultaneous consideration of networks and control is crucial for system co-design, which links NCS quality-of-control (QoC) and NCS network quality-of-service (QoS).

The overall aim of this paper is to develop a delay distribution-based robust H_∞ control method for a class of uncertain NCS over IP networks. To make use of the delay characteristics of the NCS networks, we propose a new stochastic NCS model to capture the non-uniform distribution of the network delays. From this model together with a tighter bounding technology, delay dependent criteria for NCS stability and control synthesis are derived using a linear matrix inequality (LMI) approach. Because of the use of the delay distribution information, much less conservative results can be obtained than those from existing references. Since the obtained conditions for the existence of admissible controllers are not expressed as strict LMIs, the cone complementary linearisation procedure is employed to solve the non-convex feasibility problem.

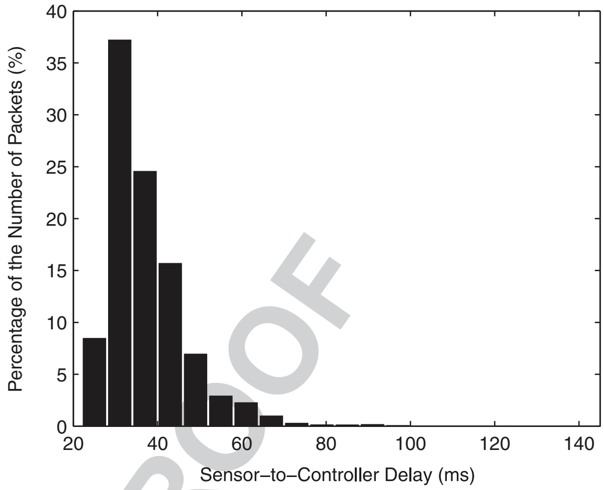

Fig. 1. Histograms of the network delays reported in [2].

Notation. $\mathbb{N}$ stands for positive integers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, I is the identity matrix of appropriate dimensions. Matrix $X>0$ (respectively, $X \geq 0$) for $X \in \mathbb{R}^{n\times n}$ means that X is a real symmetric positive definite (respectively, positive semi-definite). $\mathbb{E}\{x\}$ is the expectation of x. $\Pr\{\cdot\}$ means probability. For an arbitrary matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry. Ones$([n, m])$ is an m-by-n matrix of ones.

II. NON-UNIFORM DISTRIBUTION OF NETWORK DELAY IN IP BASED NCS

IP-based network delays display irregular behaviour. To illustrate the characteristics of such delays, Tipsuwan *et al.* measured Round-Trip Time delays from different Ethernet network nodes for 24 hours [1]. Recently, Tian *et al.* have used the open source package ns2 to model and simulate typical scenarios of NCS over IP networks. For the scenarios described in [2] for an NCS over 10 Mbps IP networks, the corresponding histograms of the delays are shown in Fig. 1 [4].

Figure 1 shows that the histograms skew to the left, indicating the higher probability to have delays shorter than the median and mean delays. Actual delays can be much longer than the median and mean, but with much lower probability. Generally speaking, although communication delay $\tau(t)$ is time-varying, it is non-uniform. In most cases, small delays are dominant while large delays are exiguous, implying that the probability of small delays is bigger than that of large delays. This phenomenon can be mathematically described as

$$\begin{cases} 1) \quad \tau_0 \le \tau(t) \le \tau_2 < \infty, \forall t \ge 0 \\ 2) \quad \Pr[\tau(t) \in [\tau_0, \tau_1)] = \delta(t), \\ \quad\quad \Pr[\tau(t) \in [\tau_1, \tau_2)] = 1 - \delta(t) \end{cases} \tag{1}$$

where $\delta(t) \in [0, 1]$ is a stochastic variable, τ_1 is the bound which means $\Pr\{\tau(t) \in [\tau_0, \tau_1)\} = \delta(t)$.

It is worth mentioning that the general statistical distribution of the network induced delay described in Eq. (1) can be regarded as one of the key characteristics of NCS network QoS. The fundamental difficulty from this new description of network QoS is how to improve the QoC under this new framework. In the following section, we devote to linking the QoS and QoC.

III. DELAY DISTRIBUTION BASED NCS CONTROL

Consider a class of control systems governed by

$$\dot{x}(t) = (A + \Delta A)x(t) + Bu(t) + B_1 w(t), \tag{2}$$

$$z(t) = Cx(t), \tag{3}$$

$$x(t) = \varphi(t), t \in [-\eta_2, -\eta_1] \tag{4}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $z(t) \in \mathbb{R}^p$ are the state vector, control input vector and controlled output vector, respectively. A, B, B_1 and C are constant matrices with appropriate dimensions. $\varphi(t)$ is the initial condition of the system. η_2 and η_1 are the lower and upper delay bounds, respectively. ΔA denotes the parameter uncertainties satisfying $\Delta A = HF(t)E_1$, where H and E_1 are constant matrices with appropriate dimensions, and $F(t)$ is an unknown time-varying matrix, which is Lebesque measurable in t and satisfies $F^T(t)F(t) \le I$. $w(t) \in L_2[t_0, \infty)$ denotes the external perturbation. Throughout this paper, it is assumed that system (2)–(4) is controlled over a network.

The following assumptions commonly used in NCS research in the literature are made in this work.

1. The sensors are clock-driven; and the controllers and actuators are event-driven;
2. All state variables are available for measurement; and the data is transmitted with a single-packet; and
3. Signal quantization and communication errors over the network are not considered.

Using a similar modeling technique in [5, 8, 9], we model the closed-loop control system for (2)–(4) as

$$\dot{x}(t) = (A + \Delta A)x(t) + Bu(t) + B_1 w(t), \quad t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}}) \tag{5}$$

$$u(t^+) = Kx(t - \tau_{i_k}), \quad t \in \{i_k h + \tau_{i_k}, k = 1, 2, \ldots\} \tag{6}$$

where $u(t^+) = \lim_{\hat{t} \to t+0} u(\hat{t})$, h is the sampling period, $i_k (k = 1, 2, 3, \ldots)$ are some integers such that $\{i_1, i_2, i_3, \ldots\} \subset \{0, 1, 2, 3, \ldots\}$, K is the state feedback gain, the network-induced delay τ_{i_k} is the time from the instant $i_k h$ when sensors sample from the plant to the instant when actuators send control actions to the plant. Here, it is assumed that the control computation and other overhead delays are included in τ_{i_k} [9].

In Equation (5), it is not required to have $i_{k+1} > i_k$. If $i_{k+1} - i_k = 1$, it means that there is no data packet dropout in the transmission. If $i_{k+1} > i_k + 1$, there are dropped packets but the received packets are in ordered sequence. If $i_{k+1} < i_k$, it means out-of-order packet arrival sequences occur; a typical scenario is that $u(i_{k+1}) = Kx(i_{k+1} h)$ is implemented after $u(i_k h) = Kx(i_k h)$. Discarding the old data packet will help reduce network-induced delay τ_{i_k}, which in turn makes the system tolerate a larger amount of data packet loss. Therefore, the time stamping technique is employed in this paper to implement the message rejection, implying that the latest message is kept and old massages are discarded.

Define $\eta(t) = t - i_k h$, $t \in [i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$, $k = 1, 2, 3, \ldots$, in every interval $[i_k h + \tau_{i_k}, i_{k+1} h + \tau_{i_{k+1}})$. It follows that

$$\tau_{i_k} \le \eta(t) \le (i_{k+1} - i_k)h + \tau_{i_{k+1}}. \tag{7}$$

Since $x(i_k h) = x(t - (t - i_k h))$, from eqs. (6) and (7), we have

$$u(t^+) = Kx(t - \eta(t)), \quad \eta_1 \le \eta(t) \le \eta_2 \tag{8}$$

where $\eta_1 = \inf_k \{\tau_{i_k}\}$, $\eta_2 = \sup_k [(i_{k+1} - i_k)h + \tau_{i_{k+1}}]$.

Similar to [12], define two sets $\Omega_1 = \{t : \eta(t) \in [\eta_0, \eta_1)\}$ and $\Omega_2 = \{t : \eta(t) \in [\eta_1, \eta_2)\}$. It is clear that $\Omega_1 \cup \Omega_2 = \mathbb{R}^+$ and $\Omega_1 \cap \Omega_2 = \mathbf{0}$. In terms of the definitions of Ω_1 and Ω_2, it is seen that $t \in \Omega_1$ means the event $\eta(t) \in [\eta_0, \eta_1)$ occurs and $t \in \Omega_2$ means the event $\eta(t) \in [\eta_1, \eta_2)$ occurs. Furthermore, we can define a stochastic variable $\delta(t)$ as

$$\delta(t) = \begin{cases} 1, & t \in \Omega_1 \\ 0, & t \in \Omega_2. \end{cases} \tag{9}$$

To make use of the distribution information of the NCS network described in (1), a new feedback control law is proposed below to replace control law (8):

$$\begin{cases} u(t) = \delta(t)Kx(t - \eta_1(t)) \\ \qquad +(1 - \delta(t))Kx(t - \eta_2(t)), \\ \eta_1(t) \in [\eta_0, \eta_1), \eta_2(t) \in [\eta_1, \eta_2) \end{cases} \tag{10}$$

where

$$\begin{cases} \eta_1(t) = \delta(t)\tau(t), \\ \qquad \text{and} \quad \delta(t) = 1 \text{ if } \eta(t) \in [\eta_0, \eta_1) \\ \eta_2(t) = (1 - \delta(t))\eta(t), \\ \qquad \text{and} \quad \delta(t) = 0 \text{ if } \eta(t) \in [\eta_1, \eta_2). \end{cases} \tag{11}$$

Substituting (10) into (5), we obtain the following model for the closed-loop control systems with the controlled output vector and initial conditions being copied to eqs. (3) and (4), respectively.

$$\dot{x}(t) = (A + \Delta A)x(t) + \delta(t)BKx(t - \eta_1(t)) + (1 - \delta(t))BKx(t - \eta_2(t)) + B_1 w(t) \tag{12}$$

Remark 1. Inspired by the work [3, 12–14], assume that $\delta(t)$ in (12) is a Bernoulli Distributed Sequence (BDS) with

$$\begin{cases} \Pr\{\delta(t) = 1\} = \mathbb{E}\{\delta(t)\} := \bar{\delta} \\ \Pr\{\delta(t) = 0\} = 1 - \mathbb{E}\{\delta(t)\} := 1 - \bar{\delta} \end{cases} \tag{13}$$

where $0 \le \bar{\delta} \le 1$ is a constant. Different from the work [3, 13, 14], where BDS is employed to deal with the systems with missing measurements, this paper utilises BDS to describe the probability distribution of the NCS network delay.

Remark 2. It is seen that system (12) makes use of more information about the distribution characteristics of the NCS network induced delay. Less conservative results can be expected from this formulation than those from the models without considering the non-uniform delay distribution. This will be verified through numerical examples in Section VI.

Remark 3. When $\delta(t) \triangleq 0$ or 1, model (12) is reduced to

$$\dot{x}(t) = (A + \Delta A)x(t) + BKx(t - \eta(t)) \tag{14}$$

$$x(t) = \varphi(t), \quad t \in [-\eta_2, 0] \tag{15}$$

This special case implies that the statistical distribution characteristics of input $u(t)$ is not considered in system analysis and synthesis. This case has been investigated in [5, 15, 16]. We will show in Section VI that their results are the special cases of our results.

To facilitate further development, Eq. (12) can be rewritten as

$$\dot{x}(t) = \varphi(t) + (\delta(t) - \bar{\delta})\psi(t) \tag{16}$$

$$\begin{cases} \varphi(t) \triangleq Ax(t) + \bar{\delta}BKx(t - \eta_1(t)) \\ \qquad +(1 - \bar{\delta})BKx(t - \eta_2(t)) \\ \qquad +B_1 w(t) + HF(t)E_a \\ \psi(t) \triangleq BKx(t - \eta_1(t)) \\ \qquad -BKx(t - \eta_2(t)) \end{cases} \tag{17}$$

Now, let us formulate some practically computable criteria for the asymptotic stability of the NCS described by (12). The following definition is useful in deriving the criteria.

Definition 1. For all admissible uncertainties, system (12) is said to be robustly asymptotically stable in the mean-square with an H_∞ norm bound γ if the following conditions hold:

1. For system (12) with $w(t) \triangleq 0$, and $\delta(t)$ described in (1) and (13), the trivial solution (equilibrium point) is globally and asymptotically stable in the mean square if the following holds:

$$\lim_{t \longrightarrow \infty} \mathbb{E}|x(t, \delta(t))|^2 = 0 \tag{18}$$

2. Under the assumption of zero initial condition, the controlled output $z(t)$ satisfies

$$\mathbb{E}\{\|z(t)\|_2\} \le \gamma \mathbb{E}\{\|w(t)\|_2\} \tag{19}$$

for any nonzero $w(t) \in L_2[0, \infty)$, where γ is a prescribed scalar.

IV. H_∞ PERFORMANCE ANALYSIS

Existing results of asymptotical stability of time-varying delay systems are delay distribution independent. This section will develop stability criteria that are delay distribution dependent. An innovative approach will be proposed to construct a Lyapunov functional that utilizes the information of the delay distribution and lower bound of the delay. The main results of our NCS stability analysis are summarized below in Theorem 1.

Theorem 1. For given constants $\eta_i (i=0,1,2)$, $\gamma>0$ and matrix K, if there exist matrices P, Q, R_j $(j=1,\ldots,4)>0$ with appropriate dimensions, scalar $\varepsilon>0$, such that the following matrix inequality holds, then system (12) is asymptotically stable in the mean-square with an H_∞ norm bound γ.

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ * & \Pi_{22} & 0 & 0 \\ * & * & \Pi_{33} & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} <0 \tag{20}$$

where

$$\Pi_{11} = \begin{bmatrix} \Psi_{11} & R_1+\bar{\delta}PBK & R_4 & \Psi_{14} & PB_1 & PH \\ * & -R_1-R_3 & R_3 & 0 & 0 & 0 \\ * & * & \Psi_{33} & 0 & 0 & 0 \\ * & * & * & -R_2 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix},$$

$$\Psi_{11} = -R_1-R_2-R_4+PA+A^TP+Q+C^TC,$$
$$\Psi_{33} = -Q-R_3-R_4, \Psi_{14}=R_2+(1-\bar{\delta})PBK,$$
$$\Pi_{12} = diag\{A, \bar{\delta}BK, 0, (1-\bar{\delta})BK, B_1, H\}^T \times Ones(6,4),$$
$$\Pi_{13} = diag\{0, (BK)^T, 0, -(BK)^T, 0, 0\} Ones(6,4),$$
$$\Pi_{22} = diag\left\{-\eta_1^{-2}R_1^{-1}, -\left(\frac{\eta_1-\eta_0}{2}\right)^{-2} R_3^{-1}, -\left(\frac{\eta_0+\eta_1}{2}\right)^{-2} R_4^{-1}, -\eta_2^{-2}R_2^{-1}\right\},$$
$$\Pi_{14} = [\varepsilon E_a \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$
$$\Pi_{33} = [\bar{\delta}(1-\bar{\delta})]^{-1}\Pi_{22}.$$

Proof. Define $\mu=\frac{\eta_0+\eta_1}{2}$. Construct a Lyapunov-Krasovskill functional candidate as

$$V(x_t) = \sum_{i=1}^{5} V_i(x_t) \tag{21}$$

where

$$V_1(x_t) = x^T(t)Px(t),$$
$$V_2(x_t) = \int_{t-\mu}^{t} x^T(s)Qx(s)ds,$$
$$V_3(x_t) = \sum_{i=1}^{2}\int_{-\eta_i}^{0}\int_{t+s}^{t} \dot{x}^T(v)\eta_i R_i \dot{x}(v)\,dv\,ds$$
$$V_4(x_t) = \int_{-\mu}^{-\eta_1}\int_{t+s}^{t} \dot{x}^T(v)(\mu-\eta_1)R_3\dot{x}(v)\,dv\,ds$$
$$V_5(x_t) = \int_{-\mu}^{0}\int_{t+s}^{t} \dot{x}^T(v)\mu R_4 \dot{x}(v)dvds,$$

$R_j>0$, $j=1,\cdots,4$. It follows from Eq. (13) that $\mathbb{E}\{\delta-\bar{\delta}\}=0$, $\mathbb{E}\{(\delta-\bar{\delta})^2\}=\bar{\delta}(1-\bar{\delta})$, where δ means $\delta(t)$. Then, the mathematical expectation of the generator $\mathcal{L}V(x_t)$ for the evolution of $V(x_t)$ along the solutions of system (12) is given by

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V_1(x_t)\} &= 2\mathbb{E}\{x^T(t)P\dot{x}(t)\} \\ &= 2\mathbb{E}\{x^T(t)P\varphi(t) \\ &\quad +(\delta-\bar{\delta})x^T(t)P\psi(t)\} \\ &= 2x^T(t)P\varphi(t) \end{aligned} \tag{22}$$

$$\begin{aligned} &\mathbb{E}\{\mathcal{L}V_2(x_t)\} \\ &= x^T(t)Qx(t) - x^T(t-\mu)Qx(t-\mu) \end{aligned} \tag{23}$$

According to (16), we have

$$\begin{aligned} &\mathbb{E}\left\{\sum_{i=1}^{2}\dot{x}^T(t)\eta_i^2 R_i\dot{x}(t)\right\} \\ &= \mathbb{E}\left\{\sum_{i=1}^{2}[\varphi(t)+(\delta-\bar{\delta})\psi(t)]^T\eta_i^2R_i \times[\varphi(t)+(\delta-\bar{\delta})\psi(t)]\right\} \\ &= \sum_{i=1}^{2}\varphi^T(t)\eta_i^2R_i\varphi(t) \\ &\quad +2\mathbb{E}(\delta-\bar{\delta})\sum_{i=1}^{2}\psi^T(t)\eta_i^2R_i\varphi(t) \\ &\quad +\mathbb{E}[(\delta-\bar{\delta})^2]\sum_{i=1}^{2}\psi^T(t)\eta_i^2R_i\psi(t) \\ &= \sum_{i=1}^{2}\varphi^T(t)\eta_i^2R_i\varphi(t) \\ &\quad +\bar{\delta}(1-\bar{\delta})\sum_{i=1}^{2}\psi^T(t)\eta_i^2R_i\psi(t) \end{aligned} \tag{24}$$

Applying Jessen's inequality [17], when $R_i>0$, we have

$$-\sum_{i=1}^{2}\int_{t-\eta_i(t)}^{t}\dot{x}^T(v)\eta_i R_i\dot{x}(v)dv \le \sum_{i=1}^{2}\begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \end{bmatrix}^T \times \begin{bmatrix} -R_i & R_i \\ * & -R_i \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\eta_i(t)) \end{bmatrix} \tag{25}$$

From (24) and (25), the following $\mathbb{E}\{\mathcal{L}V_3(x_t)\}$ can be more evolved.

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_3(x_t)\} &= \sum_{i=1}^{2}\mathbb{E}\left\{\dot{x}^T(t)\eta_i^2 R_i\dot{x}(t) - \int_{t-\eta_i}^{t}\dot{x}^T(v)\eta_i R_i\dot{x}(v)dv\right\} \\ &\le \sum_{i=1}^{2}\mathbb{E}\left\{\dot{x}^T(t)\eta_i^2 R_i\dot{x}(t) - \int_{t-\eta_i(t)}^{t}\dot{x}^T(v)\eta_i R_i\dot{x}(v)dv\right\}.\end{aligned} \tag{26}$$

□

The same operations can be applied to $\mathbb{E}\{\mathcal{L}V_4(x_t)\}$ and $\mathbb{E}\{\mathcal{L}V_5(x_t)\}$. Then, considering (22)-(26) together, and similar to the method in [18] to deal with uncertainties, we have

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(x_t)\} \le \mathbb{E}\{\xi^T(t)[\Pi_{11} - \Pi_{12}\Pi_{22}^{-1}\Pi_{12}^T \\ -\Pi_{13}\Pi_{33}^{-1}\Pi_{13}^T]\xi(t) - z(t)^T z(t) \\ +\gamma^2 w(t)^T w(t) \\ +\varepsilon[F(t)E_a]^T[F(t)E_a]\}\end{aligned} \tag{27}$$

where $\xi^T(t) = [x^T(t), x^T(t-\eta_1(t)), x^T(t-\mu), x^T(t-\eta_2(t)), \varpi^T(t), [F(t)E_a]^T]$, $\Pi_{ij}(i, j = 1, 2, 3, 4)$ is defined in Theorem 1.

By Schur complement, (20) implies that

$$\Pi_{11} - \Pi_{12}\Pi_{22}^{-1}\Pi_{12}^T - \Pi_{13}\Pi_{33}^{-1}\Pi_{13}^T + \Pi_{14}\varepsilon^{-1}\Pi_{14}^T<0. \tag{28}$$

It is clear that there exists a scalar $\varepsilon>0$ such that

$$\xi^T(t)\Pi_{14}\varepsilon^{-1}\Pi_{14}^T\xi(t) - \varepsilon[F(t)E_a]^T[F(t)E_a] \ge 0 \tag{29}$$

Then, combining (28) and (29), we have

$$\xi^T(t)[\Pi_{11} - \Pi_{12}\Pi_{22}^{-1}\Pi_{12}^T - \Pi_{13}\Pi_{33}^{-1}\Pi_{13}^T]\xi(t) + \varepsilon[F(t)E_a]^T[F(t)E_a] \le 0. \tag{30}$$

With (30), (27) implies that

$$\mathbb{E}\{\mathcal{L}V(x_t)\} \le \mathbb{E}\{\gamma^2 w(t)^T w(t) - z(t)^T z(t)\} \tag{31}$$

Under zero initial condition, integrating both sides of (31) from t_0 to t and letting $t\to\infty$, we have

$$\mathbb{E}\left\{\int_{t_0}^{\infty} z(s)^T z(s)ds\right\} \le \mathbb{E}\left\{\int_{t_0}^{\infty}\gamma^2 w(s)^T w(s)ds\right\}$$

thus $\mathbb{E}\{\|z(t)\|_2\} \le \gamma\mathbb{E}\{\|w(t)\|_2\}$.

Next, we can prove the asymptotic stability of system (12) in mean-square. From (20) and (27), we have $\mathbb{E}\{\mathcal{L}V(x_t)\}<0$. It can now be concluded from Lyapunov stability theory that the dynamics of system (12) is robustly asymptotically stable in the mean square. Then, by Definition 1, the result is established. This completes the proof.

Remark 4. Matrix inequality (20) is not an LMI due to nonlinear terms in Π_{22}. But it is easy to transformed into an LMI. Analysis of the transformation is omitted.

Remark 5. Model transformation is one of the sources of conservativeness [17]. Free weighting matrices also introduce conservativeness [19]. Peng *et al.* derived less conservative results based on an augmented matrix method, in which neither free weighting matrices nor any model transformation have been employed [19]. Using the convexity of the matrix function and employing fewer unknown variables, Park *et al.* obtained improved results [20]. Compared with Moon's inequality [21] and the slack variable matrix method [22] to deal with cross terms in [8, 15, 23], our proof of Theorem 1 uses neither model transformation nor free variable matrices. It is also worth mentioning that all variable matrices P and R_i in Theorem 1 appear in the Lyapunov-Krasovskii functional (21); and their physical interpretations are clear. Nevertheless, how to get less conservative results with reduced computational power demand is still challenging.

Remark 6. $\bar{\delta}$ reflects network QoS. The larger $\bar{\delta}$ is, the more network induced delays fall into the interval $[\eta_0, \eta_1)$. It will be seen later that a larger $\bar{\delta}$ and smaller interval $[\eta_0, \eta_1)$ correspond to a larger allowable η_2, which is the upper delay bound. This links the NCS QoC with the network QoS, and will help achieve an optimized co-design of NCS networks and control to maximize the overall performance of the NCS.

V. ROBUST H_∞ CONTROLLER DESIGN

With Theorem 1, we are now ready to design the feedback gain K to make system (12) robustly asymptotically stable in the mean square with an H_∞ norm bound γ. The main results of the control design are summarized below in Theorem 2.

Theorem 2. Given scalars $\eta_i (i=0,1,2)$, $\gamma>0$. System (12) is asymptotically stable in mean-square with feedback gain $K=VX^{-T}$ and the H_∞-norm constraint (19) is achieved for all nonzero $\omega(t)$, if there exist matrices $\tilde{Q}$, $\tilde{R}_j>0 (j=1,\ldots,4)$, a non-singular matrix X, matrix V with appropriate dimension, scalar $\varepsilon>0$, such that the following matrix inequality holds

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & XC^T \\ * & \Sigma_{22} & 0 & 0 & 0 \\ * & * & \Sigma_{33} & 0 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (32)$$

where

$$\Sigma_{11} = \begin{bmatrix} \tilde{\Psi}_{11} & \tilde{R}_1 + \bar{\delta} BV & \tilde{R}_4 & \tilde{\Psi}_{14} & B_1 & H \\ * & -\tilde{R}_1 - \tilde{R}_3 & \tilde{R}_3 & 0 & 0 & 0 \\ * & * & \tilde{\Psi}_{33} & 0 & 0 & 0 \\ * & * & * & -\tilde{R}_2 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix},$$

$$\tilde{\Psi}_{11} = -\tilde{R}_1 - \tilde{R}_2 - \tilde{R}_4 + AX^T + XA^T + \tilde{Q},$$

$$\tilde{\Psi}_{14} = \tilde{R}_2 + (1-\bar{\delta})BV, \quad \tilde{\Psi}_{33} = -\tilde{Q} - \tilde{R}_3 - \tilde{R}_4,$$

$$\Sigma_{22} = \mathrm{diag}\left\{ -\eta_1^{-2} X \tilde{R}_1^{-1} X, -\left(\frac{\eta_1 - \eta_0}{2}\right)^{-2} X \tilde{R}_3^{-1} X, -(\mu)^{-2} X \tilde{R}_4^{-1} X, -\eta_2^{-2} X \tilde{R}_2^{-1} X \right\}$$

$$\Sigma_{12} = diag\{XA^T, \bar{\delta} V^T B^T, 0, (1-\bar{\delta}) V^T B^T, B_1^T, H^T\} Ones(6,4),$$

$$\Sigma_{13} = diag\{0, V^T B^T, 0, -V^T B^T, 0, 0\} Ones(6,4),$$

$$\Sigma_{14} = [\varepsilon E_a X \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T,$$

$$\Sigma_{33} = [\bar{\delta}(1-\bar{\delta})]^{-1} \Sigma_{22}.$$

Proof. Pre- and post-multiply both sides of (20) with $\mathrm{diag}(X, X, X, X, I, I, I, I, I, I, I, I, I, I)$ and its transpose. Defining $V=KX^T$, $X=P^{-1}$, $\tilde{Q}=XQX$, $\tilde{R}_i = XR_iX (i=1,\cdots,4)$, and applying Schur complement, we have (32). This completes the proof. □

At this stage, if the above problem of (32) has a solution, we say that there exists a feedback gain $K=VX^{-T}$ that guarantees the asymptotical stability of system (12) in the mean square. However, the nonlinear equality matrix conditions $X\tilde{R}_i^{-1}X$ in (32) result in difficulties for obtaining such a solution. The following method is developed to overcome the difficulties.

Define M_i, $i=1,\ldots,4$, such that $X\tilde{R}_i^{-1}X \geq M_i$, and define $S_i = M_i^{-1}$ and $U_i = \tilde{R}_i^{-1}$. Replace condition (32) with the following (33) and (34)

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} \\ * & \Gamma_{22} & 0 & 0 \\ * & * & \Gamma_{33} & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0, \quad (33)$$

$$\begin{bmatrix} S_i & X \\ X & U_i \end{bmatrix} \geq 0, i=1,\ldots,4 \quad (34)$$

where $\Gamma_{22} = \mathrm{diag}\{-\eta_1^{-2} M_1, -(\frac{\eta_1-\eta_0}{2})^{-2} M_3, -\mu^{-2} M_4, -\eta_2^{-2} M_2\}$, $\Gamma_{33} = [\bar{\delta}(1-\bar{\delta})]^{-1}\Gamma_{22}$.

Using the cone complementarity approach [24], we formulate the following nonlinear minimization problem with considerations of LMI conditions instead of the original non-convex feasibility problem

$$\text{Min: Trace} \quad \left\{ XP + \sum_{i=1}^{4} (S_i M_i + U_i \tilde{R}_i) \right\} \quad (35)$$

$$\text{Subject to : (33), (34) and } \begin{bmatrix} P & I \\ * & X \end{bmatrix} > 0,$$

$$\begin{bmatrix} M_i & I \\ * & S_i \end{bmatrix} > 0, \begin{bmatrix} \tilde{R}_i & I \\ * & U_i \end{bmatrix} > 0$$

For this optimization problem, the following algorithm is employed to obtain a suboptimal maximum of η_2 for given η_0, η_1, γ and $\bar{\delta}$. The convergence of the algorithm is guaranteed in terms of the similar result of [24].

Algorithm 1. Finding a suboptimal maximum η_2 with respect to γ

Q1

1.
2. Choose a sufficiently little initial $\eta_2>0$, such that there exists a feasible solution to (33) and (34). Set $\eta_{02}=\eta_2$.
3. Search a feasible set $\{X_0, P_0, M_{0i}, U_{0i}, \tilde{R}_{0i}, S_{0i}$ $(i=1,\dots,4)\}$ satisfying LMI in (33) and (35). Set $k=0$.
4. Solve the following LMI problem for the variables $\{X, P, M_i, U_i, \tilde{R}_i, S_i (i=1,\dots,4)\}$

$$\text{Min}: \quad \text{tr}\left(X_k P + P_k X + \sum_{i=1}^{4}[S_{ki}M_i + M_{ki}S_i + U_{ki}\tilde{R}_i + \tilde{R}_{ki}U_i]\right) \tag{36}$$

Subject to: (33) and (34).

Set $X_{k+1}=X$, $P_{k+1}=P$, $\tilde{R}_{(k+1)i}=\tilde{R}_{ki}$, $S_{(k+1)i}=S_{ki}$, $M_{(k+1)i}=M_{ik}$, $U_{(k+1)i}=U_{ki}$.

5. If (32) is satisfied, return to *Step 2* after increasing η_2 to some extent; if (32) is not satisfied within a specified number of iterations, then exit. Otherwise, set $k=k+1$ and go to *Step 3*.

Remark 7. It is worth mentioning that there are alternative algorithms to solve out the non-LMIs in Theorem 3. One of such algorithms is to linearise the nonlinear item $X\tilde{R}_i^{-1}X$ by setting $X=\varepsilon_i\tilde{R}_i$ in (32), where ε_i is a prescribed constant. Another algorithm is to replace the nonlinear item $X\tilde{R}_i^{-1}X$ with $\tilde{R}_i-2X$ [25]. Although the cone complementarity-based Algorithm 1 will increase computational complexity in comparison with the aforementioned methods, it gives less conservative results for the examples to be discussed later in this paper.

So far, the controller has been designed to satisfy Definition 1. The results in Theorem 2 also suggest some other types of optimization problems for control design, e.g. for the optimal H_∞ control problem

$$\begin{cases}\text{Minimize:} & \gamma \\ \text{Subject to:} & (33) \quad \text{and} \quad (34)\end{cases} \tag{37}$$

VI. NUMERICAL EXAMPLES

This section aims to demonstrate the effectiveness and applications of the methodology developed in this paper.

6.1 Performance analysis

Consider the following system investigated in [16]

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix}x(t)+\begin{bmatrix}0\\ 0.1\end{bmatrix}u(t) \tag{38}$$

When the effect of external perturbations on the system is considered, (38) can be expressed as

$$\dot{x}(t)=\begin{bmatrix}0 & 1\\ 0 & -0.1\end{bmatrix}x(t)+\begin{bmatrix}0\\ 0.1\end{bmatrix}u(t) + \begin{bmatrix}0.1\\ 0.1\end{bmatrix}w(t) \tag{39}$$

$$z(t)=[0 \quad 1]x(t) \tag{40}$$

Two cases are considered below for stability analysis of system (39)-(40).

Case 1.1. For (38), the state feedback controller designed in [16] is $u(t)=[-3.75 - 11.5]x(t)$. It is designed without considering the presence of NCS networks.

1. Without consideration of the distribution of the network induced delay, the control law of [16] is employed, i.e. $u(t)=-[3.75, 11.5]x(t)$. Using the criteria from [5, 15, 16, 23, 26] and two special cases of Theorem 1 of this paper ($\eta_0=0, \bar{\delta}=0$ or 1), we obtain the respective maximum allowable transfer intervals (MATI), also called MADB [23], which guarantees the asymptotic stability of system (38) controlled over a network. The results are shown in Table I, which indicates that our Theorem gives the same result as the existing best.
2. When the delay distribution is considered, the control law of [16] is modified to $u(t)=-\delta(t)[3.75, -11.5]x(t-\eta_1(t))-(1-\delta(t))[3.75, 11.5]x(t-\eta_2(t))$. Our MATI results are listed in Table II. Existing methods in the open literature cannot handle this case.

Case 1.2. Consider the H_∞ performance of system (39)-(40) under the given controller and the H_∞ disturbance attenuation level γ.

Table I. MATI results without consideration of network characteristics.

Criterion	[16]	[26]	[23]	[5]
MATI	0.00045	0.0538	0.7805	0.8695
Criterion	Th.1†	[15]	Th.1‡	
MATI	0.8695	0.9410	0.9410	

†$\eta_0 = 0$, $\bar{\delta} = 0$, finding the maximum η_2.
‡$\eta_0 = 0$ and $\bar{\delta} = 1$, finding the maximum η_1.

Table II. MATI η_2 with consideration of network characteristics.

$\bar{\delta}$	$\eta_0 = 0.00$, $\eta_1 = 0.02$	$\eta_0 = 0.02$, $\eta_1 = 0.04$	$\eta_0 = 0.04$, $\eta_1 = 0.06$
0	0.8695	0.8695	0.8695
0.2	0.9721	0.9720	0.9719
0.4	1.1224	1.1222	1.1217
0.6	1.3746	1.3741	1.3732
0.8	1.9440	1.9429	1.9411
1	∞	∞	∞

Table III. MATI η_2 under $\gamma = 1$ and with consideration of network characteristics.

$\bar{\delta}$	$\eta_0 = 0.00$, $\eta_1 = 0.02$	$\eta_0 = 0.02$, $\eta_1 = 0.04$	$\eta_0 = 0.04$, $\eta_1 = 0.06$
0	0.8695	0.8695	0.8695
0.2	0.9710	0.9677	0.9622
0.4	1.1197	1.1114	1.0976
0.6	1.3696	1.3539	1.3273
0.8	1.9340	1.9042	1.8539
1	∞	∞	∞

1. When the distribution of the network induced delay is not considered, i.e. $\bar{\delta} = 0$, we obtain the minimum allowable $\gamma = 1$ for $\eta_1 = 0.8695$.
2. When the distribution of the network induced delay is taken into account, we obtain the upper bound η_2 under different lower bounds η_0 and η_1, as shown in Table III. Again, existing methods in the open literature cannot handle this case.

From Tables I, II and III, we can draw the following general conclusions.

- When the distribution of the network induced delay is not considered, the result in this paper is as good as the existing best in the open literature.
- It is seen from Tables II and III that when $\bar{\delta} \neq 0$, i.e. when the distribution of the network induced delay is considered, less conservative results can be obtained.
- For fixed $\bar{\delta}(\bar{\delta} \nrightarrow 0$ or 1), η_2 decreases as η_0 or η_1 increases. When η_0 and η_1 are fixed, η_2 increases with the increase of $\bar{\delta}$. These results reflect the relationship between QoS and QoC. For example, the larger the probability of the network induced delay in the lower interval $[\eta_0, \eta_1)$ (QoS), the larger the allowable network induced delay upper bound η_2 (QoC).
- In Tables II and III, for any $\eta_0 \le \eta_1 \le 0.8695$, when $\bar{\delta} \to 0$ we have $\eta_2 = 0.8695$. When $\bar{\delta} \to 1$, we have $\eta_2 \to \infty$, implying that most of the network induced delays $\eta(t)$ fall into $[\eta_0, \eta_1)$ and the probability of $\eta(t) \in [\eta_1, \eta_2)$ trends to zero. Therefore, when $\bar{\delta} \to 1$, the effect of η_2 on system stability is negligible.

6.2 Controller design

For controller design, we study the networked control problem for an uninterruptible power system (UPS). The objective is to control the PWM inverter over an IP based NCS network such that the output AC voltage is kept at the desired value while maintaining robustness against load disturbances. A UPS with 1 KVA is considered in this example. The continuous-time model can be obtained from the discrete model described in [3] at half-load operating point.

$$\dot{x} = \begin{bmatrix} 0.0449 & -0.093 & 0 \\ 0.1469 & -0.0906 & 0 \\ -6.6297 & 6.2635 & -4.2872 \end{bmatrix} x + \Delta A(t)x + \begin{bmatrix} 0.0238 \\ -0.0678 \\ -3.8693 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} w(t) \quad (41)$$

$$z(t) = [0.1 \quad 0 \quad 0]x(t) \quad (42)$$

where $\|\Delta A\| \le 0.01$.

Two cases are considered below for controller design of the system.

Case 2.1. Let us design an H_∞ controller with or without consideration of the IP network characteristics. Assume that the disturbance attenuation level $\gamma = 0.5$.

Without consideration of the delay distribution of the NCS network ($\bar{\delta} = 0$), using Theorem 2 and

Algorithm 1, we have the maximum allowable network delay $\eta_2 = 1.0$.

When the network characteristics are considered ($\bar{\delta} = 0.5, \eta_0 = 0, \eta_1 = 0.2$), we have the maximum allowable delay $\eta_2 = 1.7$. The corresponding

$$X = \begin{bmatrix} 12.9 & 18.2 & 8.45 \\ 18.2 & 20.5 & 42.27 \\ 8.45 & 42.27 & 930.03 \end{bmatrix}$$

and $V = [-137.6118, \quad 273.698, \quad -463.8711]$, respectively.

To practically simulate the time-varying network delay, a stochastic delay table is established, in which the number of delay values $\eta(t) \in [0, \eta_1)$ is N_1, and the number of delay values $\eta(t) \in [\eta_1, \eta_2)$ is N_2, respectively. For given $\bar{\delta}$, it is required to have $N_1 : N_2 = \bar{\delta} : (1 - \bar{\delta})$ in order to model the distribution of the network induced delay.

With the delay bounds obtained above, the feedback controller gain K can be computed as $K = [-14.1371, \quad 2.6782, \quad -0.0432]$. Furthermore, different distributions of the network induced delay will give different control results.

Case 2.2. Let us design the H_∞ controller over an IP network described in [1] with $\bar{\delta} = 0.5$, $\eta_1 = 0.0629$, and $\eta_2 = 0.7562$. The purpose is to minimize the H_∞ index, i.e. to solve the problem (37).

Solving the optimization problem (37) using the LMI Toolbox yields the the minimum allowable value of $\gamma = 0.23$ and $K = [-35.2031, \quad 1.9184, \quad 0.0472]$.

However, if the network characteristics are not considered, i.e. $\bar{\delta} = 0$, we have $\gamma = 0.36$ and $K = [-22.1321, \quad 2.0866, \quad 0.0223]$ for $\eta_2 = 0.7562$. The results are more conservative.

In summary, it is seen from the examples that the more information of the network characteristics is considered in NCS analysis and design, the less conservative results can be obtained. Therefore, the delay distribution based approach proposed in this paper is effective to improve the NCS control performance.

VII. CONCLUSION

The disturbance attenuation problem for NCS over IP based networks has been investigated based on a Lyapunov–Krasovskii functional method. A delay distribution based stochastic control approach that links QoC and QoC is developed. Improved criteria for H_∞ performance analysis and H_∞ control synthesis have been derived using this approach together with a tighter bounding technology to deal with cross terms. Simulation results have been given to demonstrate the effectiveness of the proposed approach.

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