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SZACOWANIE WSPÓŁCZYNNIKÓW INTERAKCYJNYCH DO ANALIZY USZKODZEŃ INTERAKCYJNYCH

ESTIMATING INTERACTIVE COEFFICIENTS FOR ANALYSING INTERACTIVE FAILURES

Środki techniczne to często systemy złożone. W złożonym systemie, między komponentami często zachodzą interakcje uszkodzeniowe prowadzące do uszkodzeń interakcyjnych. Występowanie w systemie uszkodzeń interakcyjnych może prowadzić do wzmożonego prawdopodobieństwa uszkodzenia. Stąd też, przy projektowaniu i eksploatacji złożonych systemów inżynieryjnych, może zaistnieć potrzeba wzięcia pod uwagę uszkodzeń interakcyjnych. Odnosząc się do tego zagadnienia, Sun i in. stworzyli analityczny model uszkodzeń interakcyjnych. W modelu tym stopień interakcji między dwoma komponentami jest wyrażany przez współczynniki interakcyjne. Aby można było użyć tego modelu do analizy uszkodzeń należy więc oszacować współczynniki interakcyjne. Jednakże nie opisano jeszcze metod szacowania współczynników interakcyjnych. Aby wypełnić tę lukę, w niniejszej pracy zaprezentowano pięć metod szacowania współczynników interakcyjnych, wliczając w to metodę probabilistyczną, metodę analizy opartej na danych o uszkodzeniach, eksperymentalną metodę laboratoryjną, metodę opartą na mechanizmie interakcji między uszkodzeniami oraz metodę oceny eksperckiej. Podano przykłady pokazujące zastosowania proponowanych metod, a także dokonano porównania między nimi.

Słowa kluczowe: *Uszkodzenia zależne, uszkodzenia interakcyjne, współczynniki interakcyjne, niezawodność, system złożony.*

Engineering assets are often complex systems. In a complex system, components often have failure interactions which lead to interactive failures. A system with interactive failures may lead to an increased failure probability. Hence, one may have to take the interactive failures into account when designing and maintaining complex engineering systems. To address this issue, Sun et al have developed an analytical model for the interactive failures. In this model, the degree of interaction between two components is represented by interactive coefficients. To use this model for failure analysis, the related interactive coefficients must be estimated. However, methods for estimating the interactive coefficients have not been reported. To fill this gap, this paper presents five methods to estimate the interactive coefficients including probabilistic method; failure data based analysis method; laboratory experimental method; failure interaction mechanism based method; and expert estimation method. Examples are given to demonstrate the applications of the proposed methods. Comparisons among these methods are also presented.

Keywords: *Dependent failures, interactive failures, interactive coefficients, reliability, complex system.*

1. Introduction

The failures of engineering assets can be dependent or independent of other failure modes [3, 8]. Independent failures mean that these failures do not have relationship with each other while dependent failures indicate these failures have some influences between each other. The dependent failures can be divided into two categories: one-direction dependent failures and interactive failures.

One-direction dependent failures indicate that in a system, the failures of some components (affecting components) can affect the failure rates of other components (affected components) but the failures of the affected components do not have influence on the failure rates of the affecting components. Conventional dependent failures include three categories: cascading failures, negative dependency failures and common cause failures [7, 9]. Cascading failures are defined as multiple sequential failures. These failures are initiated by the failure of one component, then leads to sequential failures of other components. Negative dependency failures are defined as such failures that can avoid other components in a system from further failing. These two dependent failures are often analyzed using approaches for independent failures such as Fault Tree Analysis (FTA), Reliability Block Diagram (RBD) and Markov model [7]. Papers concerned with cascading failures and negative dependency failures have also been published [5]. Common cause failures are defined as multiple related events caused by a single common cause. Most existing research on dependent failures focuses on common cause failures [1, 4, 9, 10, 12, 13]. The Failure Mode and Effect Analysis (FMEA) and the FTA were extended for the analysis of common cause failures [9]. Methods for analysis of common cause failures quantitatively such as the square root models [6], β-factor models [7] and multivariate exponential distribution models [11] have also been reported.

On the other hand, interactive failures indicate the increased failure rates (hazard) of components due to the interactions among them, or more formally, the interactive failure is defined as mutually dependent failures, that is, the failures of some components will affect the failures of other components and vice versa [14]. Therefore, one-direction dependent failures are special cases of the interactive failures. Interactive failures can be further classified into two categories in terms of the relationship between two components:

1) Immediate Interactive Failures. The conditions of the two components before failure are independent. However, once one of them (influencing component) fails, the failure rate of the other component (affected component) will increase immediately. As a result, the affected component either has a faster deterioration process or fails immediately.
2) Gradual Degradation Interactive Failures. The conditions of two components before failure are dependent. A component deteriorates with time, that is, the failure rate of a component increases with time. The increase of failure rate of this component can result in an increase in deterioration of its affected components. The increase of failure rate of the "victims" can also increase the failure rate of this component - the original cause. This interaction can lead to a chain interaction process. As a result of this chain interaction, the two involved components may either achieve a new level of working status or eventually fail.

The above classification can be extended to multiple interactive components straightforwardly.

Interactive failures are relatively commonplace in engineering assets although research on this type of failures is still in infancy. Several examples given by Elsayed [3] belong to the interactive failures. One example is about an airplane with identical twin engines. When either of the engines fails, the other's working load will increase immediately. As a result, the failure rate of this engine increases. Therefore, the airplane engines suffer from the Immediate Interactive Failures. As for the Gradual Degradation Interactive Failures, one can consider a washing machine with a vertical shaft which is supported by two bearings. If the balls inside a bearing wear out severely, the clearance between the inner race and outer race will become excessive so that the shaft experienced eccentricity. As a consequence, the shaft will be subjected to rotary unbalance and vibrate significantly during its spin cycle. This vibration can accelerate the damage of the other bearing which will in turn increase the rotary unbalance of the shaft and cause further damage of the first bearing.

In order to maintain a system effectively and efficiently, one needs to take interactive failures in the system into account, or otherwise, the maintenance may be inadequate. For example, in the washing machine mentioned above, if both bearings have deteriorated, but only one of them are replaced by a new one, this new bearing will degrade very fast due to the influence of the unrepaired bearing. Therefore, understanding the characteristics of interactive failures in a system is important for optimal maintenance of the system.

However, the models/methods used for analyzing one-direction dependent failures are generally unsuitable for analyzing interactive failures. To address this issue, Sun, Ma and Mathew [14] have proposed a mathematical model to describe the interactive failures. In this model, the interactive coefficient is used to represent the degree of interaction between two dependent components. However, methodologies for estimating the interactive coefficients have not been investigated systematically. This paper aims to filling this gap. The rest of the paper is organized as follows. The mathematical model for the interactive failures and the concept of the interactive coefficients are briefly reviewed in Section 2. The estimation methods of the interactive coefficients are described in Section 3. The conclusions are presented in Section 4.

2. Concept of Interactive Coefficients

For an engineering system with M components, according to [14], its hazard can be described by the following equation:

$$\{h(t)\} = [I]\{h_I(t)\} + [\theta(t)]\{h(t)\}_B \tag{1}$$

where $\{h(t)\}$ is a $M\times1$ vector representing the interactive hazard (the increased hazard due to failure interactions) and $\{h(t)\}_B$ is the $M\times1$ hazard vector before an interaction. Vector $\{h_I(t)\}$ is the $M\times1$ independent hazard vector (the independent hazard of a component indicates the hazard of the component without influence of other components), $[I]$ is a $M\times M$ identity matrix and $[\theta(t)]$ is an interactive coefficient matrix which contains $M\times M$ interactive coefficients $\theta_{ij}(t)$ ($i,j = 1,2,...,M$) [$\theta_{ij}(t)$ represents the degree of the effect of failure of component j ($j = 1,2,...,M$) on component i ($i = 1,2,...,M$)]. The interactive coefficients have the following properties:

1) $\theta_{ij}(t)\geq0$ ($i,j = 1,2,...,M$)) (this paper does not consider negative dependency failures). If $\theta_{ij}(t)=0$, then the failure of component j has no effect on the failure of component i. If the failure of component j will cause component i to fail immediately, then $\theta_{ij}(t)=1$.
2) $\theta_{ii}(t)\equiv0$ ($i,j = 1,2,...,M$). This indicates that any component has no failure interaction with itself.

To analyze interactive failures using equation (1), all parameters in the interactive coefficient matrix have to be identified. Considering the properties of the interactive coefficients, one only needs to estimate the value of each $\theta_{ij}(t)$ ($i,j = 1,2,...,M$, $i\neq j$).

3. Estimation methods

Engineering assets and their operational conditions are various. As a result, different estimation methods are needed. In this paper, five methods are proposed as follows.

3.1. Probabilistic method

In this method, interactive coefficients are determined using probability theory. The procedure of the probabilistic method is shown in Fig. 1. The application of this method is demonstrated using the following example.

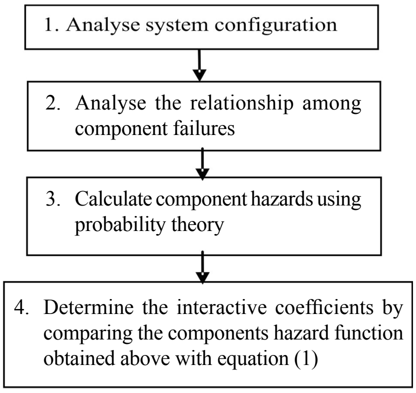


Fig. 1. Probabilistic method for estimating

Consider an engineering system with M Components 1, 2, …, M, each of which has an independent hazard $h_{Ij}(t)$ (i = 1,2,...,M). The conditions of these components before failure are independent of each other. Failure of any one of these will cause the rest of the components to fail immediately. This case demonstrates an interactive failure with the first category of interactive failures.

Let A_i represent the situation where Component i is fully operational at time t unaffected by any other component or common cause for $i = 1,2,...,M$. Then the independent reliability of Component i at time t, $R_{Ii}(t)$ is the probability that Component i remains fully operational at time t unaffected by other components or common cause, i.e., $R_{Ii}(t)=P(A_i)$ ($i = 1,2,...,M$). Based on the relationship between reliability function and hazard function, $R(t) = \exp[-\int_0^t h(t)dt]$, it can be stated that:

$$R_{Ii}(t) = P(A_i) = \exp[-\int_0^t h_{Ii}(t)dt] \qquad i = 1,2,...,M \qquad (2)$$

The probability that Component i remains operational at time t, $R_{Ii}(t)$ ($i = 1,2,...,M$), in this case is

$$R_i(t) = P(A_1 \cap A_2 \cap \ldots \cap A_M) \qquad i = 1,2,...,M \qquad (3)$$

Since events $A_1, A_2, \ldots, A_3$ are independent of each other,

$$P(A_1 \cap A_2 \cap \ldots \cap A_M) = \prod_{i=1}^{M} P(A_i) \qquad (4)$$

Using (2) and (4) for (3), gives

$$R_i(t) = \exp[-\int_0^t \sum_{i=1}^{M} h_{Ii}(t)dt] \qquad i = 1,2,...,M \qquad (5)$$

Eq. (5) indicates that the interactive hazard of Component i, $h_i(t)$, is

$$h_i(t) = \sum_{i=1}^{M} h_I(t) \qquad (\iota = 1,2,...,M) \qquad (6)$$

Since the system has the Immediate Interactive Failures, according to [14],

$$\{h(t)\}_B = \{h_I(t)\} \qquad (7)$$

Substituting eq.(7) into eq. (1) and then comparing the result with eq.(6), one can obtain the interactive coefficients of this system as follows:

$$\theta_{ij}(t) = 1 \quad i,j = 1,2,...,M \text{ and } i \neq j \qquad (8)$$

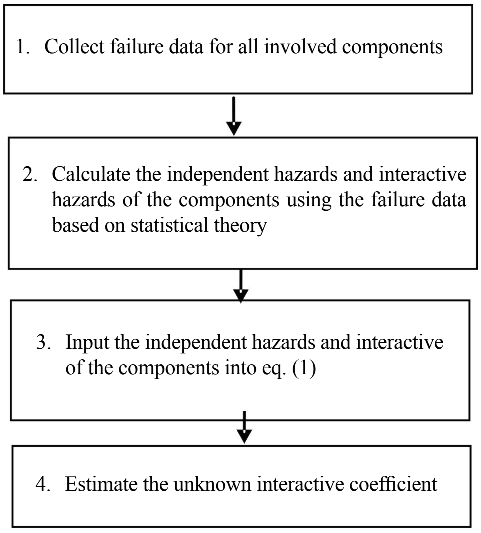
Eq. (8) shows that interactive coefficients in this particular example are all equal to one, but this is not a general case to all interactive failures. Probability theory enables interactive coefficients to be calculated accurately. However, calculating the interactive coefficients using probability theory is often inapplicable due to its mathematical complexity. Therefore, alternative approaches need to be considered.

3.2. Failure data based analysis method

Failure data based analysis method is suitable for those situations where sufficient failure data of an engineering system have been or can be collected. In this method, the independent hazards and interactive hazards of components are calculated based on the failure data. The procedure of this method is shown in Fig. 2. An example of using this method is interactive coefficient was assumed to be time-independent. In this case, according to [14],

$$\begin{Bmatrix} h_L(t) \\ h_R(t) \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} h_{LI}(t) \\ h_{RI}(t) \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ \theta_{RL} & 0 \end{bmatrix} \begin{Bmatrix} h_L(t) \\ h_R(t) \end{Bmatrix} \qquad (9)$$

where, $h_L(t)$ and $h_R(t)$ are the hazard functions of the lubricating system and the rotating system, respectively. Functions $h_{LI}(t)$ and $h_{RI}(t)$ are the independent hazard functions of the lubricating system and the rotating system, respectively. Coefficient $\theta_{RL}(t)$ represents the degree of the effect of failure of the lubricating system on the rotating system.



1. Collect failure data for all involved components
2. Calculate the independent hazards and interactive hazards of the components using the failure data based on statistical theory
3. Input the independent hazards and interactive of the components into eq. (1)
4. Estimate the unknown interactive coefficient

Fig. 2. Failure data based analysis method for estimating interactive coefficients

To demonstrate the estimation of the interactive coefficient using failure history, a number of failure times of the lubricating system, the rotating system with perfect lubrication conditions and the rotating system under the effects of failures of the lubricating system were simulated using MatLab software. The simulated failure data are shown in Tab. 1. Note that in Table 1, the observation time was assumed to be 2400 hours. The survival times that exceeded 2400 hours have been right-censored. In addition, the numbers of observations were different: 16 for the lubricating system, 12 for the rotating system with perfect lubrication conditions and 19 for the rotating system with the effect of failures of the lubricating system.

The failure times of the lubricating system were used to estimate the independent and the interactive hazard functions of the lubricating system. The result is:

$$h_L(t) = h_{LI}(t) = \exp(-0.00095t) \qquad (10)$$

The failure times of the rotating system with perfect lubrication conditions were used to estimate the independent hazard functions of the rotating system. The result is:

$$h_{RI}(t) = \exp(-0.000195t) \qquad (11)$$

The failure times of the rotating system were used to estimate the interactive hazard functions of the rotating system. The result is

$$h_R(t) = \exp(-0.000704t) \qquad (12)$$

Substituting equations (10), (11) and (12) into (9) gives $\theta_{RL} = 0.0536$. Fig. 3 shows the relationship between the hazards of the rotating system and the lubricating system.

Tab. 1 Assumed failure times and observation times

Failure times of the lubricating system (hours)	Failure times of the rotating system with perfect lubrication conditions (hours)	Failure times of the rotating system (hours)
1794.1	980.0	619.1
931.6	2400^a	493.9
652.5	1414.2	510.3
331.1	2080.3	172.2
563.5	2400^a	2400^a
774.8	2400^a	1532.6
809.0	2400^a	327.0
2400^a	2081.4	1545.7
813.1	2400^a	99.8
1004.3	2258.5	507.7
1195.6	2400^a	2400^a
160.4	2400^a	2400^a
275.1		2400^a
51.5		1941.6
583.5		866.1
2400^a		1022.4
		461.3
		708.0
		884.1

a. Observations are right-censored

Understanding interactive failures is often important for selecting correct asset maintenance strategies. To demonstrate this point, another simulation study has been conducted. In this simulation, the lubricating system was assumed to enter the wear-out stage after 2500 hours, i.e., the lubricating system had an increasing hazard (failure rate) after 2500 hours (see Fig. 4). Due to the failure interactions, the hazard of the rotating system after 2500 hours increased as well. Without considering interactive failures, one may choose preventive maintenance strategy for both lubricating system and rotating system. However, the hazard of the rotating system itself was still constant. In this case, preventive maintenance is ineffective to it at all.

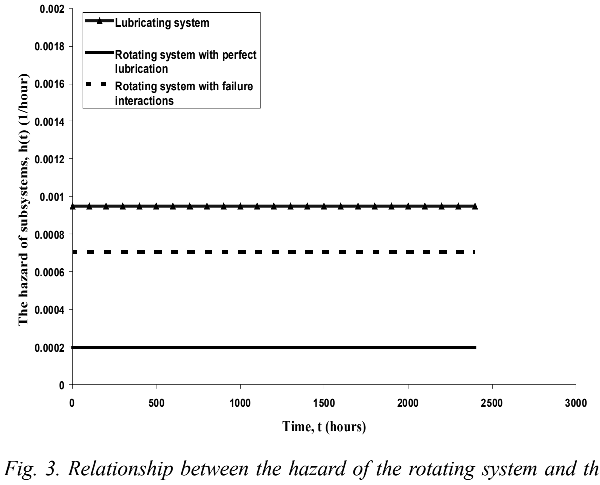


Fig. 3. Relationship between the hazard of the rotating system and the hazard of the lubricating system based on the observations

The interactive coefficients estimated using historical failure data can represent asset reliability more accurately. However, failure data are often difficult to collect. In this case, other methods should be adopted.

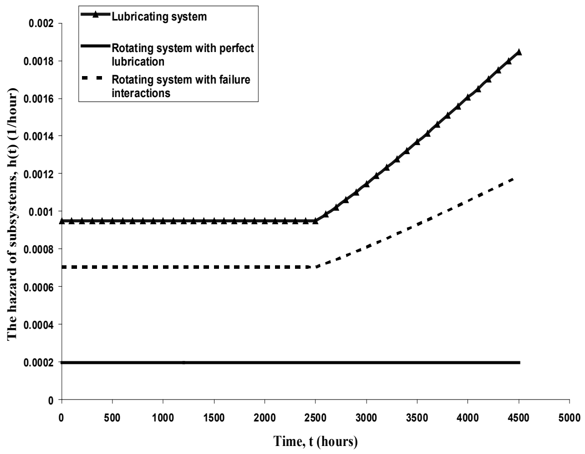


Fig. 4. The hazards of the rotating system and the lubricating system

3.3. Laboratory experimental method

Instead of collecting failure data from fields, one can collect failure data through accelerated life tests in laboratories and then estimate the interactive coefficients using the methods mentioned above.

In addition, in laboratory experiments, one can simulate and control degradation processes of affecting components, and measure the symptoms that indicate the deteriorations of affected components. In this case, the requirement on the number of failure data can be reduced. The degradation process data and condition monitoring data (i.e., the measurements of symptoms) can be used to estimate the hazards of the components using Proportional Hazard Model (PHM) [2] and Proportional Covariate Model (PCM) [15]. To the affected components, the influences of their affecting components can be treated to be external covariates. The procedure of this method is shown in Fig. 5. An example of using this method has been presented in [14]. In this example, a number of experiments were conducted on a mechanical test rig to investigate the effect of a misaligned shaft on the failures of its supporting bearings. The misalignment of the shaft was simulated and measured directly, but the degradations of the bearings were monitored through measuring their vibration signals.

The laboratory experimental method can be used to overcome the difficulty of collecting failure data from fields. However, accelerated life tests are often costly as one normally needs to design and build special test rigs. Moreover, this method often involves using failure mechanisms in developing hazard models as shown in the above example.

3.4. Failure interaction mechanism based method

Interactive coefficients can be calculated based on failure interaction mechanism and/or dynamics. For example, when a bearing has some defects, the related shaft will vibrate. This vibration will increase the failure probability of the shaft. The relationship between the defects of bearing and the failure of the shaft can be determined using dynamics and fatigue failure theo-

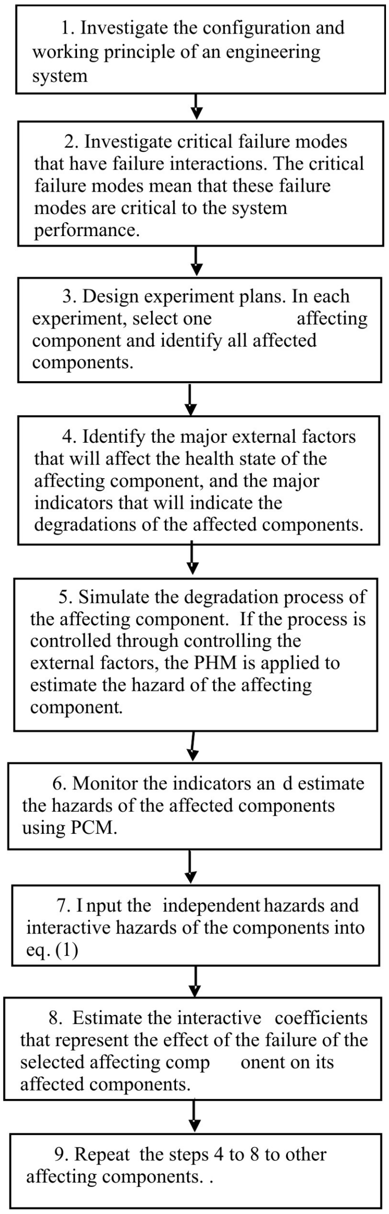


Fig. 5. Laboratory experiemental method for estimating interactive coefficients

ry. The interactive coefficient between the bearing and the shaft can then be calculated. The general procedure of the failure interaction mechanism based method is summarized in the Fig. 6.

Failure interaction mechanism based method can often provide accurate estimates of interactive coefficients. However, the applications of this method are often limited to some special cases because of the difficulty to understand the physical or chemical mechanisms of failure interactions between components.

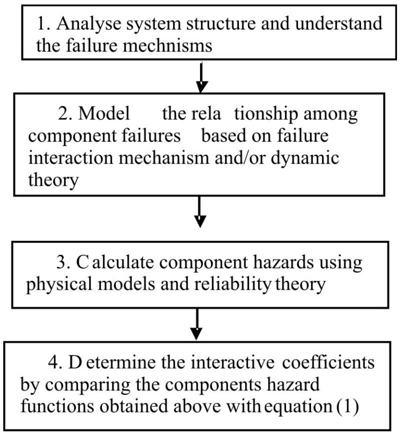


Fig. 6. Failure interaction mechanism based method for estimating interactive coefficients

3.5. Expert estimation method

Interactive coefficients can be estimated according to the experiences of designers, manufacturers and maintenance staff. A key issue for using this method is to eliminate subjective bias.

4. Conclusions

Accurately analysing interactive failures which are caused by the failure interactions between components in engineering assets is essential to optimally design and maintain the assets. The degree of failure interaction between two components can be represented using an interactive coefficient. To calculate the interactive failures, interactive coefficients need to be estimated accurately.

This paper proposed five methods for interactive coefficient estimation. Each method has its own advantages and limitations. The probability theory enables interactive coefficients to be calculated accurately. However, calculating the interactive coefficients using probability theory is often inapplicable due to its mathematical complexity. The interactive coefficients estimated using historical failure data can represent asset real conditions more accurately. However, this method requires sufficient failure data which are often difficult to collect with the improvement of asset reliability. To address the failure data insufficiency, the laboratory experimental method can be adopted. However, this method needs to build test rigs and is often costly. Failure interaction mechanism based methods can also provide accurate estimations. However, it relies on a thorough understanding of failure interactive mechanisms and therefore is often limited to special cases. The expert estimation method can be adaptive to broad scenarios. However, the accuracy of the estimated coefficients is greatly affected by the subjective bias of the estimators. One should select these methods based on the conditions of individual cases. Alternatively, one may use a combination of these methods. The authors will further investigate this issue in due course.

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