Closed-form design equations for decoupling networks of small arrays

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Small element spacing in compact arrays results in strong mutual coupling between the array elements. A decoupling network consisting of reactive cross-coupling elements can alleviate problems associated with the coupling. Closed-form design equations for the decoupling networks of symmetrical arrays with two or three elements are presented.

Introduction: Space-time communication techniques like antenna diversity and MIMO have been shown to provide improvements in quality, capacity and coverage. Multiport antennas usually have the design goal of isolated ports and uncorrelated radiation patterns. This goal is normally achieved by using an inter-element spacing of at least half a wavelength ($\lambda/2$) to inhibit the effects of mutual coupling. For antenna diversity in applications where space is limited, an element spacing significantly smaller than $\lambda/2$ may be required, which would in turn result in strong mutual coupling between array ports. Ludwig states that tight radiator coupling in conjunction with superdirective modes of operation results in significant gain reduction and distorted beam patterns [1]. A decoupling network may compensate for the mutual coupling effects to yield decoupled ports [2-4].

In its simplest form, the decoupling network consists of reactive elements connected between neighbouring array elements, but this technique is only applicable special cases where off-diagonal elements of the admittance matrix of the original array are all purely imaginary [2]. Decoupling networks for arrays with arbitrary complex mutual admittances were described in [3] and [4]. In [3], the numerical values for the reactive elements in the decoupling network were obtained by solving sets of nonlinear simultaneous equations. In this letter, closed-form design equations for the decoupling network elements of symmetrical 2-element and 3-element arrays are presented. They are also applicable to the design of networks used to provide improved isolation between closely-spaced, co-polarized antennas in transmit/receive applications [5], or to minimize signal correlation of a two-port antenna diversity system [6].

Design: A symmetrical array of n=3 identical elements is characterized by a scattering matrix $\mathbf{S^a}$ given by

$$\mathbf{S^a} = \begin{bmatrix} \begin{bmatrix} S_{11}^a & S_{12}^a \\ S_{12}^a & S_{11}^a \end{bmatrix} & \begin{matrix} S_{12}^a \\ S_{12}^a \end{matrix} \\ \begin{matrix} S_{12}^a & S_{12}^a \end{matrix} & S_{11}^a \end{bmatrix}. \qquad (1)$$

For an array with n=2 elements, the scattering matrix will be the 2×2 sub-matrix indicated in (1). The corresponding impedance matrix can in both cases be calculated from [7]

$$\mathbf{Z^a} = Z_0(\mathbf{I} + \mathbf{S^a})(\mathbf{I} - \mathbf{S^a})^{-1}, \qquad (2)$$

where **I** is an $n\times n$ identity matrix and Z_0 is the characteristic impedance of the system. If n=2, the eigenvalues of the impedance matrix are given by $Z_a = R_a + jX_a = Z_{11}^a + Z_{12}^a$ and $Z_b = R_b + jX_b = Z_{11}^a - Z_{12}^a$, while the corresponding orthogonal eigenvectors are $\mathbf{e}_a = [1,1]^T$ and $\mathbf{e}_b = [1,-1]^T$. For n=3, the eigenvalues are $Z_a = R_a + jX_a = Z_{11}^a + 2Z_{12}^a$ and $Z_b = R_b + jX_b = Z_{11}^a - Z_{12}^a = Z_c$, with orthogonal eigenvectors $\mathbf{e}_a = [1,1,1]^T$, $\mathbf{e}_b = [2,-1,-1]^T$ and $\mathbf{e}_c = [0,1,-1]^T$.

These arrays can be decoupled using the circuits shown in Fig. 1(a) and (b), respectively. Using the characteristic circuits for the eigenmodes [3], the modal admittances as seen from the new input ports (1', 2', etc.) are obtained as

$$
\begin{aligned}
Y_a' &= \left(Z_a + jX_1\right)^{-1}, \\
Y_b' &= \left(Z_b + jX_1\right)^{-1} + jnB_2 .
\end{aligned}
\tag{3}
$$

For n=3, note that $Y_c' = Y_b'$. The array can be decoupled by ensuring that the modal impedances are matched. Setting $Y_a' = Y_b'$ and evaluating the real and imaginary parts give the following closed-form results for the elements of the decoupling network:

$$
\begin{aligned}
X_1 &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}, \\
B_2 &= \frac{1}{n}\left(\frac{X_b + X_1}{{R_b}^2 + \left(X_b + X_1\right)^2} - \frac{X_a + X_1}{{R_a}^2 + \left(X_a + X_1\right)^2} \right),
\end{aligned}
\tag{4}
$$

where $A = R_a - R_b$, $B = 2(R_a X_b - R_b X_a)$, $C = R_a\left(R_b^2 + X_b^2\right) - R_b\left(R_a^2 + X_a^2\right)$. The input impedance at each port will then be equal to the modal impedances defined in (3). The ports can be matched to the system impedance Z_0 using L-section impedance matching networks [7].

Example: To verify the theory, decoupling networks for 2-element and 3-element monopole arrays were designed. The array elements were wires measuring λ/4 in length and λ/40 in diameter. An element spacing of λ/10 was used for both arrays. Assuming a system impedance of $Z_0 = 50\,\Omega$, the scattering parameters of the arrays were calculated using IE3D [8] and converted into impedance parameters using (2). The impedance parameters at the centre frequency, f_0, and decoupling network elements calculated from (4) are specified in Table 1. The elements of the L-section impedance matching networks are also shown, with B_3 being the susceptance of a parallel element to ground and X_4 the reactance of a series element. The scattering parameters of the arrays were calculated over a frequency range of $0.95\,f_0$ to $1.05\,f_0$ and the results are shown in Fig. 2 and Fig. 3. The results clearly illustrate the validity of the theory. The bandwidth of each decoupled system is determined by the level of mutual coupling in the original array. Simulations for the normalized azimuth radiation pattern of the 2-element and 3-element arrays are shown in Fig. 4. The arrays have superdirective radiation patterns suitable for applications in frequency multiplexing, direction finding and adaptive nulling.

Conclusion: The closed-form equations greatly simplify the design of decoupling networks for small symmetrical arrays or for applications where improved isolation between transmitting and receiving antennas is required. At lower frequencies, the networks can be directly implemented with lumped elements. Alternatively, the procedures described in [3] may be used to realize the reactive elements as transmission line stubs.

References

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Figure captions

Fig. 1 Decoupling networks for (a) a 2-element array and (b) a symmetrical 3-element array.

Fig. 2 Scattering parameters of the decoupled and matched 2-element array versus normalized frequency.

——— $|S_{11}^{a}|$

– – – – $|S_{12}^{a}|$

………… $|S_{11}|$ (decoupled & matched)

—·—·— $|S_{12}|$ (decoupled & matched)

Fig. 3 Scattering parameters of the decoupled and matched 3-element array versus normalized frequency.

——— $|S_{11}^{a}|$

– – – – $|S_{12}^{a}|$

………… $|S_{11}|$ (decoupled & matched)

—·—·— $|S_{12}|$ (decoupled & matched)

Fig. 4 Simulated radiation patterns of the 2-element and 3-element arrays obtained by feeding port 1 and terminating the other ports in matched loads. For excitation at another port, the radiation pattern will be rotated by 180° (2-element array) or ±120° (3-element array).

——— 2-element array

– – – – 3-element array

Table 1 Impedance parameters and decoupling network elements for the 2-element and 3-element arrays.

Figure 1

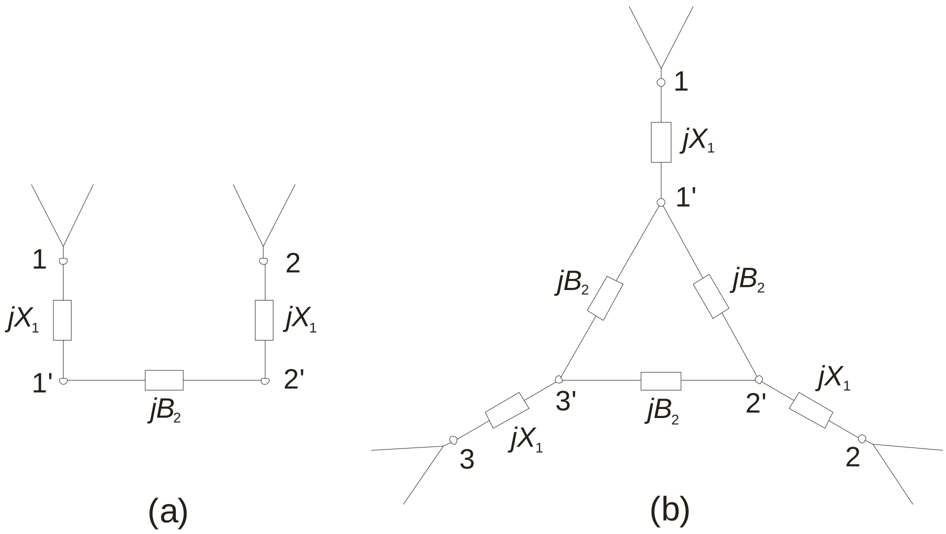

Figure 2

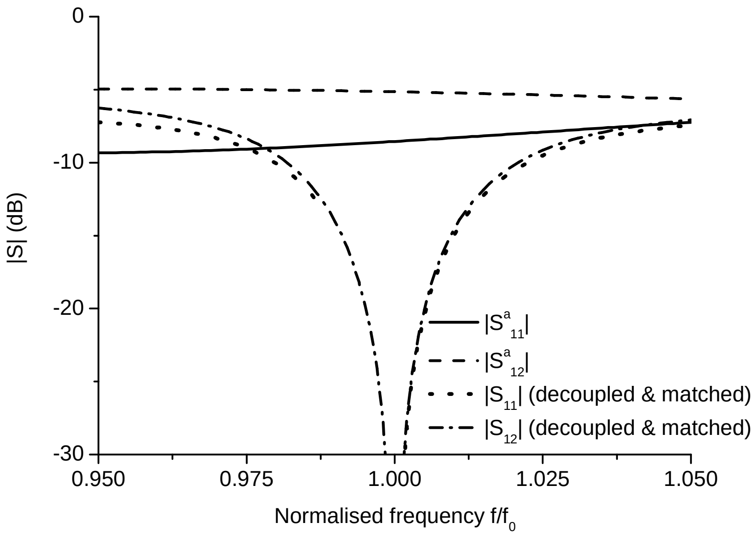

Figure 3

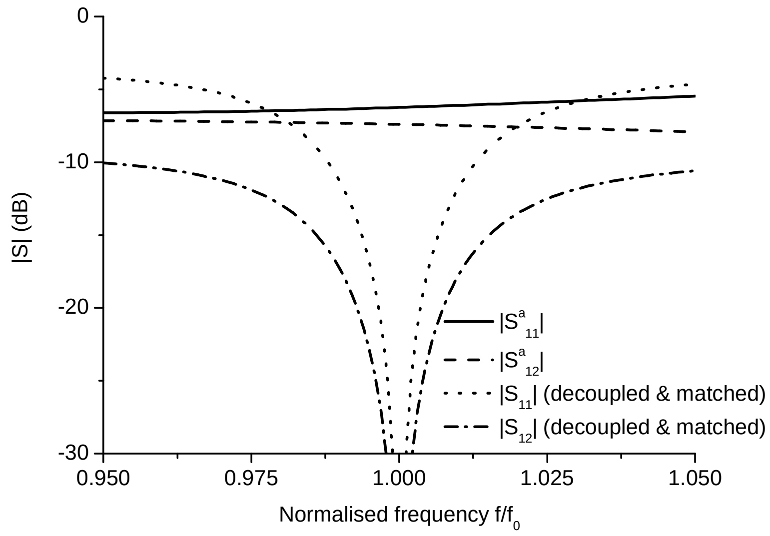

Figure 4

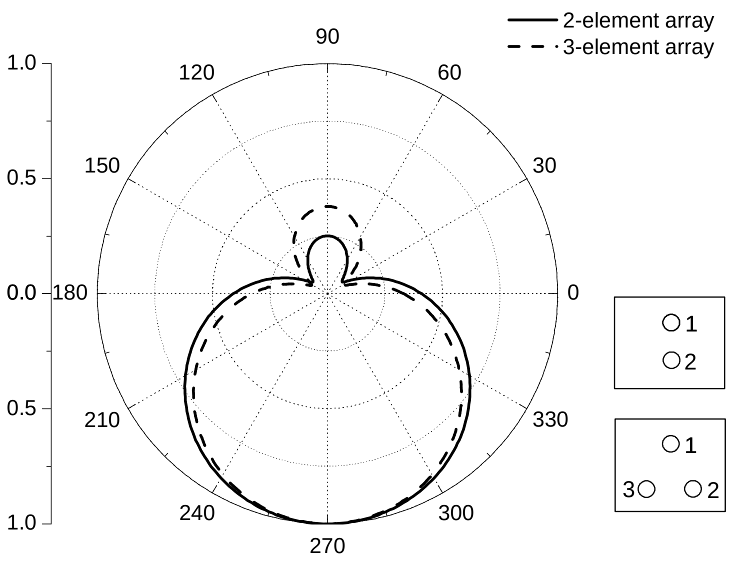

Table 1

	2-element array	3-element array
Array impedance parameters (Ω)	$Z_{11}^{a} = 50.70 + j15.34$ $Z_{12}^{a} = 47.05 - j7.02$	$Z_{11}^{a} = 48.48 + j9.45$ $Z_{12}^{a} = 44.96 - j12.61$
Decoupling network elements (Ω)	$X_1 = -41.98$ $B_2 = -0.023$	$X_1 = -45.95$ $B_2 = -0.0128$
Decoupled port impedance (Ω)	$Z_a' = 97.75 - j33.66$	$Z_a' = 138.4 - j61.71$
Matching network elements (Ω)	$B_3 = 0.00681$ $X_4 = 54.47$	$B_3 = 0.00649$ $X_4 = 76.13$