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KINDERGARTEN STUDENTS' UNDERSTANDING OF PROBABILITY CONCEPTS

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This study explored kindergarten students' intuitive strategies and understandings in probabilities. The paper aims to provide an in depth insight into the levels of probability understanding across four constructs, as proposed by Jones (1997), for kindergarten students. Qualitative evidence from two students revealed that even before instruction pupils have a good capacity of predicting most and least likely events, of distinguishing fair probability situations from unfair ones, of comparing the probability of an event in two sample spaces, and of recognizing conditional probability events. These results contribute to the growing evidence on kindergarten students' intuitive probabilistic reasoning. The potential of this study for improving the learning of probability, as well as suggestions for further research, are discussed.

INTRODUCTION AND THEORETICAL FRAMEWORK

The importance of having all students develop a sound awareness of probability concepts and appropriately use these concepts in solving problems has been recognized in recent curriculum documents (e.g., National Council of Teachers of Mathematics, 2000). These recommendations adopt the position that young students, even at the kindergarten level, need to explore the processes of probability (NCTM, 2000). The teaching of probability is, however, not an easy task (Fischbein & Schnarch, 1997; Langrall & Mooney, 2005). As argued by Shaughnessy (1992), modeling probabilistic situations is complex and the teaching of probability concepts is often hindered by students' primitive intuitions and alternative conceptions. Following recommendations for early introduction of probability concepts in school curricula and for students to exhibit probabilistic thinking, there is a need for students to understand probability concepts that are multifaceted and develop over time (Jones, Langrall, Thornton, & Mogill, 1997). Although there has been substantial research on young children's probabilistic thinking (e.g., Fischbein, 1975; Fischbein, & Schnarch, 1997; Piaget & Inhelder, 1975; Shaughnessy, 1992), little recent research has been done in the field of teaching and learning probabilities to young learners and further on how young learners' intuitive models and strategies on probability concepts are incorporated into solving problems related to probability.

Fischbein (1975) reported that 'probability matching', "the expression of … the intuition of relative frequency" (p.58), had been observed and generally well established in pre school children. Although the concept of ratio appears to be crucial to the development of probabilistic reasoning (Piaget & Inhelder, 1951) and therefore the concept of chance cannot be obtained before proportional reasoning is mastered

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(Greer, 2001), the intuitive foundations of pre-school students can serve for the development of probabilistic knowledge. As primary intuitions of chance and the concept of change certainly exist in pre-school students (Greer, 2001; Langrall & Mooney, 2005), it is important to take these intuitions into consideration in designing and implementing problem-solving activities in probability. Moreover, it is generally agreed that even before formal instruction in probability, children already acquire an elementary understanding of probability and are able to compare the probability of two situations in a qualitative way (e.g., English, 1993; Fischbein, 1975; Fischbein & Gazit, 1984; Sharma, 2005).

For the purposes of the present study we used the cognitive framework proposed by Jones and colleagues (1997, 1999), which can be used to describe and predict students' probabilistic thinking. In line with previous research, the proposed framework assumes that probabilistic thinking is multifaceted and develops slowly over time. Four key constructs are incorporated in the framework, to satisfactorily capture the manifold nature of probabilistic thinking and its interconnections. These constructs are sample space, probability of an event, probability comparisons, and conditional probability. Furthermore, young children's probabilistic thinking is described across four levels for each of the four constructs: the subjective level, the transitional level, the informal quantitative level, and the numerical level (Jones et al., 1997, 1999).

Since the present study focuses on exploring and identifying young learners' probabilistic thinking, students' actions at the subjective and transitional level are presented next. At the subjective level, children can list an incomplete set of outcomes for a one-stage experiment, predict most/least likely events partially based on subjective judgments, and recognize certain and impossible events. Children can also compare the probability of the same event in two different sample spaces, cannot distinguish "fair" probability situations from "unfair" ones, and recognize when certain and impossible events arise in a non-replacement situation (Jones et al., 1997, p.111). At the transitional level, the children list a complete set of outcomes for a one-stage experiment and sometimes list a complete set of outcomes for a two-stage experiment using limited and unsystematic strategies. Children can predict most/least likely events based on quantitative judgments (but sometimes may revert to subjective judgments), and make probability comparisons based on quantitative judgments (may not quantify correctly and may have limitations when non-contiguous events are involved). At the transitional level children begin to distinguish "fair" probability situations from "unfair" ones, recognize that the probability of some events changes in a non-replacement situation. Recognition is, however, incomplete and is usually restricted only to events that have previously occurred (Jones et al., 1997, p.111).

The aim of the present study was to investigate kindergarten students' intuitive probabilistic strategies and understandings in solving problems related to probabilities. For this purpose, the framework developed by Jones and colleagues

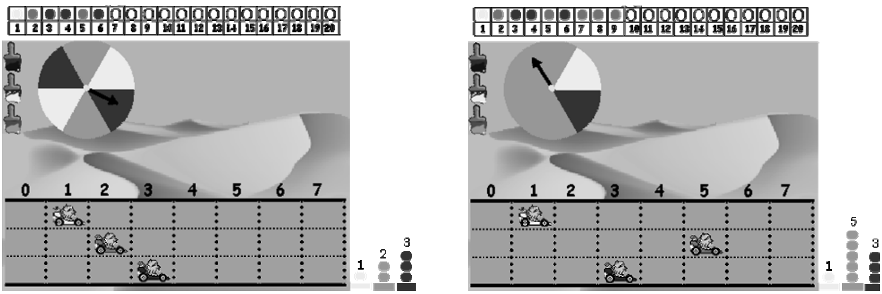
(1997) was used as a basis for identifying, exploring, and providing an in depth analysis of kindergarten students' thinking strategies.

DESCRIPTION OF THE STUDY

Participants and Procedures

Students in a large rural kindergarten school formed the population for this study. Four classes of the school are currently participating in a 2-year longitudinal study of students' probabilistic thinking and mathematical modeling. The school population is representative of a broad spectrum of multicultural and socioeconomic backgrounds. Twelve students, six from each of the two grade levels (one grade for 3-4 year olds and one for 5-6 year olds) were randomly selected and served as case studies. Prior to the start of this study, none of the students had been exposed to probability instruction. Due to space limitations, the interview of one pair of students (one from each grade level) is presented in this paper, namely Alex, 4 years and 3 months and Chris, 6 years and 1 month. It should be noted that both students are ranked (by their teachers) among the best in their classes.

The data reported here are from the first year of the respective longitudinal study and are drawn from one of the problem activities the children completed during the first year. The *Car Racing* problem (see Figure 1a and 1b) is a math applet, developed in Scratch (http://scratch.mit.edu), a freeware visual programming software, that can directly run from the Web. The problem presented a spinner (see Figure 1a for initial colours), three cars and a number of different representations related to the car racing.



(a) (b)

Figure 1: The Car Racing Activity.

These included the position of each car, a bar chart for the three colours and a "pattern style" representation for the different trials. Additionally, the applet gave students and teacher the opportunity to recolour the spinner (see Figure 1b for an example).

Data Collection and Instrumentation

A semi-structured interview protocol based on the framework proposed by Jones and colleagues (1997) was administered by the authors. The interview assessment comprised tasks related to the Car Racing problem. The tasks were associated with *sample space*, with *probability of an event,* with *probability comparisons*, and with *conditional probability* (see selected tasks, Table 1). The tasks enabled the researchers to explore students' probabilistic thinking across the two levels of the framework. The data sources included video-tapes of students' responses to the interview questions and our own field notes. The two students worked together. Some questions, however, were directed to one of them, while in other questions students were asked to first discuss the question between them and then answer.

Sample Space	Probability of an Event	Probability Comparisons	Conditional Probability
What colour will you get if you spin the spinner again and again? Is that all? How do you know?	Which colour has the least chance to appear? (1/2 was yellow, 1/3 was blue and 1/6 was green)	Colour the spinner in a way that you will have the best chance to win, using at least two colours.	What colour has the best chance of getting? Why? (no yellow in last four trials and all colours were 1/3)

Table 1: Selected Tasks from the Interview.

The transcripts were reviewed by the authors and data were analysed using interpretative techniques (Miles & Huberman, 1994) to explore and identify developments in students' probabilistic thinking with respect to: (a) the four key constructs of the proposed framework (sample space, probability of an event, probability comparisons and conditional probability), and (b) the two levels of probabilistic thinking (subjective and transitional).

RESULTS AND DISCUSSION

We report here on the students' understanding of probability concepts in terms of the two levels of probabilistic thinking as reported by Jones and colleagues (1997) and discuss possible further enhancements of the proposed framework, based on the results of the study. The individual responses and discussions between the two students were analyzed, and summaries and exemplars were produced to illuminate a number of the probabilistic thinking strategies outlined in the proposed framework and to suggest new thinking strategies. None of the students tended to generate the same level of probabilistic thinking for all four constructs. We therefore decided to present their results are follows: First we focus on students' probabilistic thinking strategies that are related to Level 1 (Subjective), and then we focus on their strategies that appear to be linked to Level 2 (Transitional).

Level 1 Probabilistic Thinking Strategies

Alex, the younger child exhibited both level 1 and level 2 probability thinking strategies. It should be noted, however, that he did not provide correct answers for all questions and problem situations related to the four constructs at level 1. Consequently, he provided fewer correct responses to problems corresponding to level 2. Chris, the older child successfully answer all questions related to all four level 1 constructs.

An explicit difference in the two students' responses was the absence of any subjective beliefs in Chris' judgements. He totally based his answers and comments on his probabilistic related intuitions and on his understandings on other mathematical constructs. On the contrary, Alex quite frequently based his comments on subjective beliefs. However, he did not consistently use subjective knowledge, but he rather used it when he felt that he could not use any of his prior mathematical or other understandings. On sample space related questions, he easily listed all possible outcomes when, for example, colours had equal probabilities. Sometimes, in questions that colour probabilities were not equal, he only listed his favourite colour or the colour that was more likely to happen. On a task, for example, where 5/6 of the spinner was shaded yellow and 1/6 blue, he reported that it was not fair because green was missing. He responded that only yellow would appear, since blue was too small compared to yellow. Somehow contradictory to what Jones (1997) reported, sometimes Alex spontaneously listed all expected outcomes. He could even recolor the spinner in a number of ways as to match a predefined list of outcomes. So, for example, when he was prompted to recolor the spinner in a way that only green and blue were the possible outcomes, he coloured 4/6 green and 2/6 blue. When asked if that was the only solution, he coloured one green slice into blue. His two solutions are presented in Figure 2.

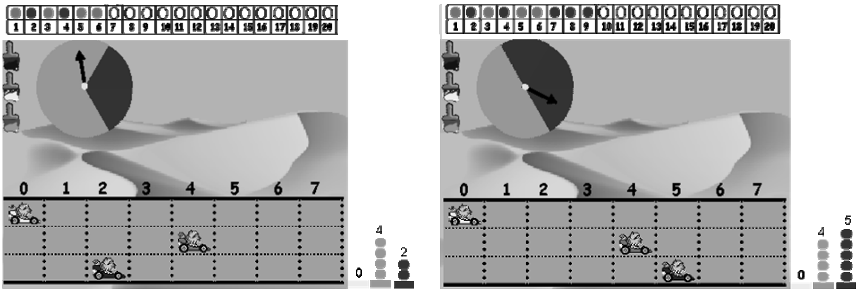


Figure 2: Alex's solutions.

A typical thinking strategy of Alex in probability comparisons, which was consistent in almost all his actions and responses, was his tendency to believe that the number of slices was more important than the size of them. When he was presented with a task where 1/3 was yellow, 2/6 was green and 2/6 was blue, he reported that it was not fair for yellow. He said, “Green is best because it has two slices and it is my favourite colour. Blue is the same …has more than yellow”.

On conditional probability tasks, both children experienced difficulties. Our problem setting did not include any tasks related to item replacement (or not). Alternatively, we used the pattern related representation that appear on the top of the applet screen and that presented a history of the game results. In a task where 1/3 was yellow, 1/3 was green and 1/3 was blue, in the first five attempts the spinner returned blue, blue, green, blue and green. When asked what colour had the best chance of getting, both students identified yellow as the best for the next spinning, since according to them “it has not appeared yet” and “it is time now for yellow”.

Level 2 Probabilistic Thinking Strategies

Quite impressive, Chris, the 6-year old pupil, reported typical level 2 probabilistic thinking strategies, in almost all four constructs. This was impressive not only because of his age, but also because of the absence of any formal instruction. Chris consistently identified a complete set of outcomes. We do not claim here that he used a generative strategy, since there are not enough data to support this claim. Consequently, in Chris’ answers, similar to Alex’s, there was quite frequently a tendency to overlook outcomes, rather than consider sample space and probability in combination. Chris exemplified quantitative reasoning in comparing probabilities. Similar to what Jones (1997) reported, Chris always correctly used the “more of” the target colour strategy. In stark contrast to Jones’ proposed framework, Chris tended to recognize the effect of conditional probability of related events. When asked, for example, how he could increase the probability of green without using the green painter in a setting where 1/2 was green and 1/2 was blue, he reported that he could use the yellow painter to paint one or more blue slices.

Another difference from Jones’ second level of probabilistic cognitive framework was the absence of any subjective reasoning in Chris’ answers. No doubt, Chris is not a level 3 pupil in any of the four constructs and he is probably not a level 2 pupil in all constructs. He tried his best to employ quantitative reasoning on all items relating to the probability of an event. Since his knowledge of fractions was very limited, he used the number of slices for each colour as the basis for his quantitative reasoning. When presented, for example, with a task where 4/6 was green, 1/6 was yellow and 1/6 was blue, he reported that “the probability of green was four times bigger than the probability of blue”. Quite interesting, in a consecutive task where 3/6 was green, 1/6 was yellow and 2/6 was blue, when asked to compare probabilities of the different colours, he replied that “probability of green was 3 times bigger than the probability of yellow…I can not compare green and blue…it is two times…no…I do not know”.

CONCLUDING POINTS

Although there has been substantial research on the probability constructs investigated in this study (e.g., English, 1993; Fischbein, & Schnarch, 1997; Piaget &

Inhelder, 1975), we claim that the present study provides some interesting insights into kindergarten students' probabilistic thinking, insights that are needed to guide classroom instruction and assessment. Although the purpose of the study was not to validate the framework proposed by Jones and colleagues (1997) at the kindergarten level, the results of the study revealed that students at the kindergarten school and before any formal instruction on probabilities hold and successfully employ in problem solving a number of probabilistic concepts. Even at the age of four, the case study student's probabilistic thinking across all four constructs appeared to be consistent. Further, the six-year-old student not only did not use any subjective knowledge in his work, but he also further realised the appropriateness of the quantitative reasoning in comparing probabilities and in calculating the probability of events, without any formal instruction on fractions. This kind of knowledge on students' probabilistic thinking should enhance information available to curriculum designers and teachers.

In accord with the framework of Jones and colleagues (1997, 1999), we claim that even at the age of four, without any formal instruction and based on their intuitive strategies, students start developing strategies for some of the four constructs at level 1 of the proposed framework. Further, the results showed that the six-year-old who participated in the study started developing successful quantitative and qualitative strategies for all four constructs at both levels. Further, even problem posing was not part of the tasks, the older student managed to pose correct probability problems for the younger student in order to exemplify his thinking during their discussion on several interview tasks. We do not claim that this is the case for all or for the majority of students and we are aware that very often students, especially at this age level are often distracted and misled by subjective knowledge, contradictory intuitions and other irrelevant aspects of the problems presented to them (English, 1993; Langrall & Mooney, 2005). However, the results provide some evidence that probability concepts should be introduced to students at the kindergarten level and teaching needs to consider all aspects related to students' prior intuitive strategies and cognitive models related to probability and number sense.

The results of the study illuminate the framework constructs by identifying more in-depth insights into students' probabilistic thinking. We need to address here the contribution of the software applet in framing the context of the problem situation presented to students and in providing fundamentally new representational resources (Greer, 2001). Clearly substantial more research is needed to identify the extent to which the car race scenario, the different representations (spinner, bar-chart like graph, pattern style), and the active manipulation of the spinner (changing colours at the beginning and during an experiment) contributed in enhancing student's probabilistic thinking.

The small sample size and given that both students were high achievers may limit the extent to which conclusions about the probabilistic thinking strategies students hold at the kindergarten level can be drawn. Further studies are needed to investigate in depth the probabilistic thinking of young students, covering a broad spectrum of multicultural and socioeconomic backgrounds. Clearly, more research is needed to examine the extent to which instructional programs influence the development of probabilistic thinking and to identify the critical steps in students' development of probability concepts. Such research would result in a more pervasive description of students' probabilistic thinking and could be even more useful in informing instruction in kindergarten and elementary school.

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