Gorjian, Nima and Ma, Lin and Mittinty, Murthy and Yarlagadda, Prasad K. and Sun, Yong (2010) *The explicit hazard model – part 1: theoretical development.* In: PHM 2010 Macau : IEEE - Prognostics & System Health Management Conference, 12-14, January 2010, University of Macau, Macau, China. (In Press)

The Explicit Hazard Model – Part 1: Theoretical Development

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Abstract – Modern Engineering Asset Management (EAM) requires the accurate assessment of current and the prediction of future asset health condition. Appropriate mathematical models that are capable of estimating times to failures and the probability of failures in the future are essential in EAM. In most real-life situations, the lifetime of an engineering asset is influenced and/or indicated by different factors that are termed as covariates. Hazard prediction with covariates is an elemental notion in the reliability theory to estimate the tendency of an engineering asset failing instantaneously beyond the current time assumed that it has already survived up to the current time. A number of statistical covariate-based hazard models have been developed. However, none of them has explicitly incorporated both external and internal covariates into one model. This paper introduces a novel covariate-based hazard model to address this concern. This model is named as Explicit Hazard Model (EHM). Both the semi-parametric and non-parametric forms of this model are presented in the paper. The major purpose of this paper is to illustrate the theoretical development of EHM. Due to page limitation, a case study with the reliability field data is presented in the *applications* part of this study.

I. INTRODUCTION

Prognostics and asset life prediction is one of the major research problems in Engineering Asset Management (EAM). The development of mathematical models that are capable of predicting the times to failures (or survival times) of engineering assets and the probability of failures in future time has become an essential scientific research problem in EAM. Hazard prediction is a significant approach to forecast the probability of failures and evaluate the reliability and safety of systems. Hazard is commonly used in reliability and survival analysis because it has an intuitive explanation, which appeals to engineers and researchers in the field of EAM [1]. The notion of the hazard is appealing and imperative for reliability engineers since an engineering asset is aging (or degrading) with time, and its conditional probability of failure is increasing with time.

The fundamental notion in hazard analysis is the failure times of an engineering asset and its covariates. These covariates change stochastically and may influence and/or indicate the failure time. Literature shows that a number of statistical models have been developed to estimate the hazards of engineering assets with covariates in both the reliability and biomedical fields. Most of these covariate-based hazard models (also termed as hazard models with covariates) have been developed based on the proportional hazard model which was developed by Cox in 1972 for the biomedical field [2]. The proportional hazard model was quickly and widely adopted in various fields including biomedical, reliability, and economics, due to its generality and flexibility. However, due to the prominence of this model, most other covariate-based hazard models have not attracted much attention in the field of reliability. In addition, attempts to develop an alternative covariate-based hazard model, to some extent, have been stifled.

In traditional reliability models, the lifetime of engineering assets is estimated in terms of the probability distribution of the times to failures of population, which reflects the average behavior of the population's reliability characteristics. However, a dynamic multivariate model such as a covariate-based hazard model is capable of estimating individual system reliability under dynamic operational and environmental conditions. In most real-life situations and industry applications, the lifetime of an engineering asset is influenced and/or indicated by different factors, which are termed as covariates. Operating environment factors (e.g. ambient temperature and pressure, humidity, dust, rate of working load, and skill of operator) can influence the hazard of an engineering asset. Furthermore, certain diagnostic factors (e.g. vibration of fitted rotating machinery and the level of metal particles in engine oil analysis) can be associated with the hazard of an engineering asset. *Operating environment factors* are usually termed as *external covariates* and *diagnostic factors* are often termed as *internal covariates*.

External covariates may accelerate or decelerate the failure time of an engineering asset. Some covariate-based hazard models (e.g. the proportional hazard model) are originally developed based on the influences (acceleration and deceleration effects) of external covariates on the hazard of an engineering asset/individual. The proportional hazard model with external covariates is broadly applied and verified to estimate the hazard and reliability of engineering assets in EAM [3-8]. However, this model with internal covariates is applied in the reliability field, too [9-14]. Nevertheless, care must be exercised in differentiating internal covariates from external covariates. It is noticeable that internal covariates must be handled differently from external covariates in any hazard models with covariates since such covariates may only carry information about the failure time.

Both external and internal covariates can be included in a covariate-based hazard model to predict the hazard of an engineering asset. In this paper, a new covariate-based hazard model, named as Explicit Hazard Model (EHM), is developed to address this issue. The major purpose of this paper is to present the theoretical development of this covariate-based hazard model. The verification and application of EHM is explained in [15]. The remainder of this paper is organized as follows. Section II provides the overview of existing covariate-based hazard models in both the reliability and biomedical fields. Section III aims to identify the different types of covariates in the reliability field. Section IV and its subsections describe the development of both the semi-parametric and non-parametric EHM. Additionally, likelihood functions of both the semi-parametric and non-parametric EHM are presented in the sections. The calculation of the residual life in EHM is briefly introduced in Section V. Section VI provides the conclusions.

II. OVERVIEW OF EXISTING COVARIATE-BASED HAZARD MODELS

A number of covariate-based hazard models have been developed and applied to calculate the hazards of engineering assets. Gorjian et al. [16] reviews covariate-based hazard models and provides comments on their merits and limitations. The basic theory of these models is to build the baseline hazard (or underlying hazard) using historical failure time data and the covariate function using covariate data. With this in mind, it is found that all of these models are derived from Cox's proportional hazard model. Only a few of them have been applied to estimate the hazard of engineering assets in the reliability field. The proportional hazard model [2] is the only model that has been widely applied in the reliability area to investigate the multiplicative effect of covariates (or explanatory variables) associated with an engineering asset on its life span.

Kay [17] develops the stratified proportional hazard model which is the simplest and most useful extension of the proportional hazard model. Anderson and Senthilselvan [18] extends Cox's proportional hazard model into the two-step regression model to allow for changing covariate effects in time. Additive hazard model is developed to investigate the additive effect of covariates on the baseline hazard [19]. To enhance modeling capability about covariates, the mixed model considers the hazard of an engineering asset/individual, which contains both a multiplicative and an additive component [20]. The accelerated failure time model is one of the most common approaches that estimate the hazard of an engineering asset/individual under stress conditions [21, 22].

The extended hazard regression model that includes both the proportional hazard model and accelerated failure time model is developed by Etezadi-Amoli [23]. The proportional intensity model was introduced by Cox in 1972 [2]. McCullough [24] generalizes the idea of constant odds ratio to more than two samples by means of a regression model which is termed as the proportional odds model [24]. Logistic and log-logistic regression models are two specific cases of the proportional odds model [25]. Sun et al. [26] proposes a novel covariate-based hazard model (i.e. proportional covariate model) to deal with internal covariates. Aalen [27] introduces a linear regression model to assess additive time-dependent covariate effects in possibly right-censored survival data.

Literature review depicts that some of covariate-based hazard models are appropriate to conduct with external covariates and some with internal covariates. However, all of these approaches neglect the existence of both external and internal covariates in the hazard of engineering assets. In order to have an effective asset life prediction, this concern is required to be thoroughly addressed.

III. COVARIATE CONCEPTS

Covariate is a significant element of covariate-based hazard models. Understanding the concept of covariates and distinguishing between different types of covariates are necessary in these models. In reliability and survival analysis, the factors which influence and/or indicate the hazard of an engineering asset/individual have been termed as covariates. Covariates can be classified into two major groups:

1. **External covariates:** This type of covariates as the stress factor may accelerate or decelerate the failure time of an engineering asset/individual. External covariates in reliability analysis are the so-called operating environment factors. Ambient temperature and pressure, humidity, dust, maintenance effects, age, rate of utilization, rate of working load, and skill of operator are some examples of external covariates in the reliability field. External covariates can be classified into *time-independent* and *time-dependent* covariates.

   Fixed covariates are the only type of time-independent external covariates [22]. Design modification of an engineering asset and base location for a vehicle such as smooth terrain, rough terrain are examples of fixed covariates in reliability analysis [9]. Time-dependent external covariates are divided into *defined* and *ancillary* covariates [22]. A stress factor under control of the experimenter in a laboratory experiment and the age of an engineering asset are typical examples of defined covariates in the reliability area. Dust and contaminations in the air as well as humidity can be common examples of ancillary covariates.

2. **Internal covariates:** Internal covariates are observed and measured only as long as an engineering asset is operational or an individual survives. These observed values as failure indicators may contain information about the failure time of an engineering asset/individual. This type of covariate measures the current status of an engineering asset rather than acting as a causal predictor. Internal temperature and pressure generated by an engine, vibration of fitted rotating machinery, the level of metal particles in engine oil analysis, the thickness of a brake pad, and the wear in a component are examples of internal covariates in the reliability field.

   Generally, internal covariates in the reliability field can be classified into *direct* and *indirect* covariates [28]. The thickness of a brake pad and the wear in a component are general examples of direct internal covariates [28]. Internal temperature and pressure generated by an engine, vibration of fitted rotating machinery, and the level of metal particles in engine oil analysis are examples of indirect internal covariates.

It is noticeable that if the values of internal covariates reach the pre-specified covariate threshold, the covariates can have effects of both external and internal covariates. However, what the threshold should be and how it should be specified has not been made clear. Expert knowledge information can be used to identify this covariate threshold.

IV. THE EXPLICIT HAZARD MODEL – MODEL DEVELOPMENT

Hazard models with covariates have been well studied in the biomedical field. Due to two separate groups of populations which are termed as *control* and *treatment* populations in clinical trials, selecting and formulating of influenced covariates for these types of models is not a difficult task in the field. On the other hand, this issue is an important concern in the reliability field since the control sample is not available. Furthermore, due to complexity of engineering assets and engineering systems, the study and analysis of clinical trial data is easier than industrial reliability data. As it mentioned in the earlier section, both external and internal covariates can be incorporated into a covariate-based hazard model to more effectively predict the hazard and reliability of an engineering asset. In fact, the presence and reality of both external and internal covariates in life span of engineering assets cannot be ignored.

In industrial applications, multiple failure mechanisms may be recognized by certain diagnostic factors (internal covariates). However, multiple failure mechanisms may not be identified by these certain diagnostic factors alone, as some failures occur due to random shocks (e.g. loads and stress factors) caused by the environment in which the engineering assets operate. Moreover, some of failures happened as a result of latent degradation processes. Hence, in addition to diagnostic factors (internal covariates), the operating environment factors (external covariates) should be considered in a covariate-based hazard model in order to have more effective prediction results for the hazard and reliability of an engineering asset.

In this study, an original covariate-based hazard model is developed to explicitly model both the external and internal covariates associated with the hazard of an engineering asset. This model accepts the existence of the two covariates to efficiently predict the hazard and reliability of an engineering asset so as to prevent costly failures and to reduce the frequency of unnecessary maintenance and repair. This model allows the external covariate to be considered as a stress factor and the internal covariate as a failure indicator for updating the current status of an engineering asset. This model, which is termed as Explicit Hazard Model (EHM) can be presented in two different forms: semi-parametric and non-parametric. Section A describes the semi-parametric EHM and its parameter estimation function is discussed in Section B. Section C explains the non-parametric EHM, and its parameter estimation function is explained in Section D.

If $h\left(t;\vec{z}_1(t),\vec{z}_2(t)\right)$ denotes the hazard of an engineering asset and $h_0\left(\exp(\vec{\gamma}_1\vec{z}_1(t)).t\right)$ is its baseline hazard (or underlying hazard), therefore the generic form of EHM can be expressed as:

$$h(t;\vec{z}_1(t),\vec{z}_2(t)) = h_0\left(\exp(\vec{\gamma}_1\vec{z}_1(t))\cdot t\right)\exp(\vec{\gamma}_2\vec{z}_2(t)) \qquad \textbf{(1)}$$

Where, $\vec{z}_1(t)$ and $\vec{z}_2(t)$ are vectors of internal and external covariates, respectively. $\vec{\gamma}_1$ and $\vec{\gamma}_2$ are vectors of regression coefficients.

The relationship between the hazard and the underlying hazard in the presence of both external and internal covariates in EHM are illustrated in Figure 1. The figure shows that the underlying hazard in EHM is changing by the time scale and the factor of internal covariates, hence the major difference between EHM and the proportional hazard model.

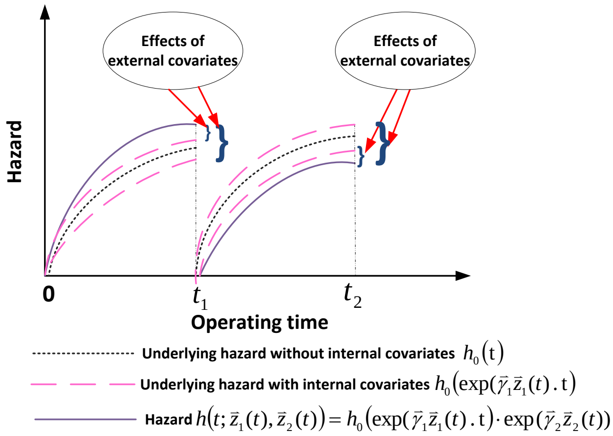

Figure 1: Influences of internal and external covariates in EHM

From a mathematical point of view, Equation (1) can be changed to the proportional hazard model for $\vec{z}_1(t)=0$. However, from an engineering point of view, internal covariates are observed values as long as an engineering asset is operational. Therefore, the existence of internal covariates should be considered for an engineering asset on its life span.

A. THE SEMI-PARAMETRIC EXPLICIT HAZARD MODEL

The semi-parametric EHM involves a specified function (i.e. Weibull distribution) in the form of the baseline hazard. In other words, the form of degradation paths or distribution of degradation measure is specified in the model. Alike other semi-parametric statistical models, this model incorporate a parametric modelling of the relationship between the hazard and specified covariates. The Weibull distribution is commonly applied in semi-parametric models for good reasons [5]. The hazard of the Weibull distribution is [29]:

$$h(k) = \begin{cases} \left(\dfrac{\beta}{\eta}\right)\left(\dfrac{k}{\eta}\right)^{\beta-1} & \text{for } k > 0 \\ 0 & \text{for } k \le 0 \end{cases} \qquad \textbf{(2)}$$

Where, $\beta > 0$ and $\eta > 0$ are shape and scale parameters of the Weibull distribution, respectively.

If $k = (\exp(\bar{\gamma}_1 \bar{z}_1(t)) \cdot t)$, the semi-parametric EHM can be expressed as:

$$h(t; \bar{z}_1(t), \bar{z}_2(t)) = \left[\frac{\beta}{\eta} \left(\frac{t}{\eta} \right)^{\beta-1} \left(\exp(\bar{\gamma}_1 \bar{z}_1(t)) \right)^{\beta-1} \right] \times \exp\left(\bar{\gamma}_2 \bar{z}_2(t) \right) \quad \textbf{(3)}$$

If t denotes the lifetime of an engineering asset with $0 \le \tau \le t$, the related reliability function of the semi-parametric EHM is given by:

$$R(t; \bar{z}_1(\tau), \bar{z}_2(\tau)) = \exp\left[-\int_0^t \frac{\beta}{\eta} \left(\frac{\tau}{\eta} \right)^{\beta-1} \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right] d\tau \right] \quad \textbf{(4)}$$

Suppose S has a unit negative exponential distribution, therefore the reliability function is:

$$S = \int_0^t \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right] d\left(\frac{\tau}{\eta} \right)^{\beta} \quad \textbf{(5)}$$

The value of S can be calculated by substituting the estimated values $\hat{\beta}, \hat{\eta}, \hat{\bar{\gamma}}_1$, and $\hat{\bar{\gamma}}_2$ of the parameters $\beta, \eta, \bar{\gamma}_1$, and $\bar{\gamma}_2$ into the preceding equation, provided that the values of $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ are known for all t. In other words, this assumption may only hold with *continuous time* samples of the covariates. A likelihood function is applied to estimate these parameters. Section B describes parameter estimation procedures for the above equation.

In real-life situations the values of $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ in numerous cases (e.g. *discrete time* samples such as an oil analysis sample) are not known for all τ. To address this concern where the values of internal and external covariates are unknown for all τ, an approach to spawn an approximate sample path for $\{ \bar{z}_1(\tau), \bar{z}_1(\tau) | t \ge 0 \}$ is necessitated. There are various approaches for this approximation in both the semi-parametric and non-parametric statistical models. Kalbfleisch and Prentice [22] suggests the step-function as a way to perform this approximation. Spline approximation is one way to carry out this in the non-parametric statistical models [30]. This paper provides an approximate sample path that should be substituted for $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ by the right continuous jump process [10, 31].

Assume that the sample path of the stochastic process $\left\{ \left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right] \middle| 0 \le \tau \le t \right\}$ is known only in $\tau_0, \ldots, \tau_n$. Thus, this sample path of the stochastic process can be approximated by the right continuous jump process $\left\{ \left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right]^* \middle| 0 \le \tau \le t \right\}$ that decreases and increases by jumps at times $\tau_0, \ldots, \tau_n$. Otherwise, it is constant and it takes the same values as $\left\{ \left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right] \middle| 0 \le \tau \le t \right\}$ at $\tau_0, \ldots, \tau_n$.

Using Equation (5) and the integrating by parts, S can be expressed as:

$$S = \left(\frac{t}{\eta} \right)^{\beta} \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(t) \right) - \bar{\gamma}_1 \bar{z}_1(t) + \bar{\gamma}_2 \bar{z}_2(t) \right] - \int_0^t \left(\frac{\tau}{\eta} \right)^{\beta} \mathrm{d} \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right] \quad \textbf{(6)}$$

If $0 \le \tau_0 \le \cdots \le \tau_n \le t$ and the sample mean is used as an estimate of the population mean, then the residual (or fitting error), S, which shows the deviation of the sample from the observable sample mean can be approximated by the right continuous jump process:

$$S^* = \left(\frac{t}{\eta} \right)^{\beta} \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(t) \right) - \bar{\gamma}_1 \bar{z}_1(t) + \bar{\gamma}_2 \bar{z}_2(t) \right] - \int_0^t \left(\frac{\tau}{\eta} \right)^{\beta} \mathrm{d} \exp\left\{ \left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right]^* \right\}$$

$$S^* = \left(\frac{t}{\eta} \right)^{\beta} \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(t) \right) - \bar{\gamma}_1 \bar{z}_1(t) + \bar{\gamma}_2 \bar{z}_2(t) \right] - \left(\int_0^{\tau_0} + \int_{\tau_0}^{\tau_n} + \int_{\tau_n}^{t} \right) \times \left(\frac{\tau}{\eta} \right)^{\beta} \mathrm{d} \exp\left\{ \left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right]^* \right\} \quad \textbf{(7)}$$

Suppose that $\varphi_i = \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau_i) \right) - \bar{\gamma}_1 \bar{z}_1(\tau_i) + \bar{\gamma}_2 \bar{z}_2(\tau_i) \right]$, thus the three integrals in the previous equation are solved by using the Reimann-Stieltjes integral as follows (Equations (8), (9), and (10)):

$$\int_0^{\tau_0} \left(\frac{\tau}{\eta} \right)^{\beta} \mathrm{d} \exp\left\{ \left[\beta\left(\bar{\gamma}_1 \bar{z}_1(\tau) \right) - \bar{\gamma}_1 \bar{z}_1(\tau) + \bar{\gamma}_2 \bar{z}_2(\tau) \right]^* \right\} = \left(\frac{\tau_0}{\eta} \right)^{\beta} \left\{ \varphi_0 - \exp\left[\beta\left(\bar{\gamma}_1 \bar{z}_1(0) \right) - \bar{\gamma}_1 \bar{z}_1(0) + \bar{\gamma}_2 \bar{z}_2(0) \right] \right\} \quad \textbf{(8)}$$

$$\int_{\tau_0}^{\tau_n}\left(\frac{\tau}{\eta}\right)^{\beta} \mathrm{d}\exp\left\{\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau)) - \bar{\gamma}_1\bar{z}_1(\tau) + \bar{\gamma}_2\bar{z}_2(\tau)\right]^*\right\}$$

$$= \left(\frac{\tau_n}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau_n)) - \bar{\gamma}_1\bar{z}_1(\tau_n) + \bar{\gamma}_2\bar{z}_2(\tau_n)\right]$$

$$- \left(\frac{\tau_0}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau_0)) - \bar{\gamma}_1\bar{z}_1(\tau_0) + \bar{\gamma}_2\bar{z}_2(\tau_0)\right]$$

$$= \sum_{i=1}^{n}\left(\frac{\tau_i}{\eta}\right)^{\beta}(\varphi_i - \varphi_{i-1}) \quad \textbf{(9)}$$

$$\int_{\tau_n}^{t}\left(\frac{\tau}{\eta}\right)^{\beta} \mathrm{d}\exp\left\{\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau)) - \bar{\gamma}_1\bar{z}_1(\tau) + \bar{\gamma}_2\bar{z}_2(\tau)\right]^*\right\}$$

$$= \left(\frac{t}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(t)) - \bar{\gamma}_1\bar{z}_1(t) + \bar{\gamma}_2\bar{z}_2(t)\right] - \left(\frac{t}{\eta}\right)^{\beta}\varphi_n \quad \textbf{(10)}$$

If $t = \tau_{n+1}$, by removing the lower limit of the three Reimann-Stieltjes integrals in Equations (8), (9), and (10) and then substituting the results into Equation (7), then with some arrangement S^* can be described as:

$$S^* = \left(\frac{t}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(t)) - \bar{\gamma}_1\bar{z}_1(t) + \bar{\gamma}_2\bar{z}_2(t)\right]$$
$$-\left\{\left(\frac{\tau_0}{\eta}\right)^{\beta}\left\{\varphi_0 - \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(0)) - \bar{\gamma}_1\bar{z}_1(0) + \bar{\gamma}_2\bar{z}_2(0)\right]\right\}\right.$$
$$+\left(\frac{\tau_1}{\eta}\right)^{\beta}\varphi_1 + \left(\frac{\tau_2}{\eta}\right)^{\beta}\varphi_2 + \cdots + \left(\frac{\tau_n}{\eta}\right)^{\beta}\varphi_n \quad \textbf{(11)}$$
$$+\left(\frac{t}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(t)) - \bar{\gamma}_1\bar{z}_1(t) + \bar{\gamma}_2\bar{z}_2(t)\right]$$
$$\left.-\left(\frac{t}{\eta}\right)^{\beta}\varphi_n\right\}$$

$$\mathrm{S}^* = \left(\frac{\tau_0}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(0)) - \bar{\gamma}_1\bar{z}_1(0) + \bar{\gamma}_2\bar{z}_2(0)\right]$$
$$+\varphi_n\left[\left(\frac{\tau_{n+1}}{\eta}\right)^{\beta} - \left(\frac{\tau_n}{\eta}\right)^{\beta}\right]$$
$$+\sum_{k=0}^{n-1}\varphi_k\left[\left(\frac{\tau_{k+1}}{\eta}\right)^{\beta} - \left(\frac{\tau_k}{\eta}\right)^{\beta}\right]$$

$$S^* = \left(\frac{\tau_0}{\eta}\right)^{\beta} \exp\left[\beta(\bar{\gamma}_1\bar{z}_1(0)) - \bar{\gamma}_1\bar{z}_1(0) + \bar{\gamma}_2\bar{z}_2(0)\right]$$
$$+\sum_{k=0}^{n}\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau_k)) - \bar{\gamma}_1\bar{z}_1(\tau_k) + \bar{\gamma}_2\bar{z}_2(\tau_k)\right] \quad \textbf{(12)}$$
$$\times\left[\left(\frac{\tau_{k+1}}{\eta}\right)^{\beta} - \left(\frac{\tau_k}{\eta}\right)^{\beta}\right]$$

The value of S^* can be estimated by replacing the values of internal and external covariates as well as the estimated values $\hat{\beta}, \hat{\eta}, \hat{\bar{\gamma}}_1$, and $\hat{\bar{\gamma}}_2$. A likelihood function is applied to estimate these parameters. The following section describes parameter estimation procedures for Equation (12).

B. PARAMETER ESTIMATION OF THE SEMI-PARAMETRIC EHM

Generally, the likelihood function is one way of statistical inference in covariate-based hazard models. A likelihood function is the joint density of the observed values considered as a function of the unknown parameters. The Expectation-Maximization (EM) algorithm is another practical tool for estimating the unknown parameters at a variety of incomplete data problems (e.g. missing covariate data) in covariate-based hazard models.

In order to estimate the parameters of the semi-parametric EHM, it is required to have the historical failure time data, external and internal covariates data. Suppose that a random sample of r items yields n distinct failure times and $r-n$ censoring (or suspended) times. Therefore, the likelihood function of the semi-parametric EHM is given by:

$$L(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) = \prod_{i\in\Theta_F} h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i))$$
$$\times \prod_{j\in\{\Theta_F\cup\Theta_C\}} \mathrm{R}(t_j;\bar{z}_1(\tau),\bar{z}_2(\tau)\,|\,0\le\tau\le t_j)$$

$$L(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) = \prod_{i\in\Theta_F}\left[\frac{\beta}{\eta}\left(\frac{t_i}{\eta}\right)^{\beta-1}\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(t_i)) - \bar{\gamma}_1\bar{z}_1(t_i) + \bar{\gamma}_2\bar{z}_2(t_i)\right]\right]$$
$$\times\prod_{j\in\{\Theta_F\cup\Theta_C\}}\exp\left[-\int_0^{t_j}\frac{\beta}{\eta}\left(\frac{\tau}{\eta}\right)^{\beta-1}\right. \quad \textbf{(13)}$$
$$\left.\times\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau)) - \bar{\gamma}_1\bar{z}_1(\tau) + \bar{\gamma}_2\bar{z}_2(\tau)\right]d\tau\right]$$

Where Θ_F indexes the set of failure times and Θ_C indexes the set of censoring (suspended) times, t_i is the failure time of the i^{th} item, and t_j is either the observed failure time or the suspended (censoring) time of the j^{th} item.

If $S_j = \int_0^{t_j}\exp\left[\beta(\bar{\gamma}_1\bar{z}_1(\tau)) - \bar{\gamma}_1\bar{z}_1(\tau) + \bar{\gamma}_2\bar{z}_2(\tau)\right]d\left(\frac{\tau}{\eta}\right)^{\beta}$ and it has a unit negative exponential distribution, $\Lambda h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i)) = \ln h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i))$, and n is the total number of failure times available, therefore the log-likelihood function of the semi-parametric EHM is defined as:

$$l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2) = \sum_{i\in\Theta_F}\Lambda h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i)) - \sum_{j\in\{\Theta_F\cup\Theta_C\}}\mathrm{S}_\mathrm{j}$$

$$l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2)=n\ln\left(\frac{\beta}{\eta}\right)+\sum_{i\in\Theta_F}\ln\left[\left(\frac{t_i}{\eta}\right)^{\beta-1}\right]$$
$$+\sum_{i\in\Theta_F}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(t_i)\right)-\bar{\gamma}_1\bar{z}_1(t_i)+\bar{\gamma}_2\bar{z}_2(t_i)\right]$$
$$-\sum_{j\in\{\Theta_F\cup\Theta_C\}}\int_0^{t_j}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau)\right)-\bar{\gamma}_1\bar{z}_1(\tau)+\bar{\gamma}_2\bar{z}_2(\tau)\right]d\left(\frac{\tau}{\eta}\right)^{\beta} \quad \textbf{(14)}$$

All parameters can be estimated by maximizing the log-likelihood function using a global optimization approach. Equation (14) is applied where the values of $\bar{z}_1(\tau)$ and $\bar{z}_2(\tau)$ are known for all τ. Otherwise the approximate sample path by the right continuous jump process for $\left\{\bar{z}_1(\tau),\bar{z}_1(\tau)\middle|t\geq 0\right\}$ is required. If $S_j^*=S^*$, as a result the log-likelihood function can be expressed as:

$$l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2)=\sum_{i\in\Theta_F}\Lambda h(t_i;\bar{z}_1(t_i),\bar{z}_2(t_i))-\sum_{j\in\{\Theta_F\cup\Theta_C\}}\mathrm{S}_{\mathrm{j}}^*$$
$$l(\beta,\eta,\bar{\gamma}_1,\bar{\gamma}_2)=n\ln\left(\frac{\beta}{\eta}\right)+\sum_{i\in\Theta_F}\ln\left[\left(\frac{t_i}{\eta}\right)^{\beta-1}\right]$$
$$+\sum_{i\in\Theta_F}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(t_i)\right)-\bar{\gamma}_1\bar{z}_1(t_i)+\bar{\gamma}_2\bar{z}_2(t_i)\right]$$
$$-\left\{\left(\frac{\tau_0}{\eta}\right)^{\beta}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(0)\right)-\bar{\gamma}_1\bar{z}_1(0)+\bar{\gamma}_2\bar{z}_2(0)\right]\right. \quad \textbf{(15)}$$
$$+\sum_{\substack{k=0\\k\in\{\Theta_F\cup\Theta_C\}}}^{n}\exp\left[\beta\left(\bar{\gamma}_1\bar{z}_1(\tau_k)\right)-\bar{\gamma}_1\bar{z}_1(\tau_k)+\bar{\gamma}_2\bar{z}_2(\tau_k)\right]$$
$$\left.\times\left[\left(\frac{\tau_{k+1}}{\eta}\right)^{\beta}-\left(\frac{\tau_k}{\eta}\right)^{\beta}\right]\right\}$$

C. THE NON-PARAMETRIC EXPLICIT HAZARD MODEL

In reality, ideal industrial historical failure data are generally not available; therefore, fitting a specific distribution to lifetime data is a violated assumption. To avoid making assumptions about the distribution of lifetime data in EHM which often is difficult to test, non-parametric EHM is devised. The non-parametric EHM involves an unspecified function in the form of an arbitrary baseline hazard. In other words, the form of degradation paths or distribution of degradation measure is unspecified in this model. Similar to other non-parametric statistical models, the key advantage of this model compared to the semi-parametric EHM is to provide a decent relative efficiency for the estimation of regression coefficients without having to make assumptions about baseline hazard.

Here, the generic form of EHM in Equation (1) is to be expressed as the non-parametric EHM. The baseline hazard of this model is a function of time and internal covariates; hence, an approximation of the baseline hazard is required. This approximation is performed by a transformation function of the baseline hazard which is suggested by Etezadi-Amoli and Ciampi [32]. Shyur et al. [33] modifies this transformation function for external time-dependent covariates. In this study, we follow their assumptions to build a transformation function to approximate the baseline hazard of the non-parametric EHM.

In general, splines are an evolution of classical parametric inference, and bridge the gap between parametric and non-parametric techniques [34]. In fact, a spline function is a natural choice for approximating the covariate transformation. A $n-$degree spline function is a piecewise polynomial of degree n with pieces joining at defined points, which are called knots [35]. The degrees of polynomial pieces (e.g. linear, quadratic, and cubic) as well as the number and position of the knots may vary in different situations. To represent and approximate the baseline hazard of the non-parametric EHM, a quadratic spline function, proposed by Etezadi-Amoli and Ciampi [32], is utilized. In this study a quadratic spline function is selected since it requires fewer parameters than a cubic spline and in several cases may provide a reasonably smooth and accurate fit to data [36]. In addition, a quadratic spline with one knot can fit to data almost as well as a quadratic spline with two knots and a cubic spline with one knot [37]. Luxhoj and Shyur [35] asserts that one knot spline is found to be sufficient to approximate the baseline hazard. A quadratic spline with m knots is given by:

$$q(t)=\sum_{j=0}^{2}\lambda_{0j}t^j+\sum_{i=1}^{m}\lambda_{i2}(t-\zeta_i)_+^2 \quad \textbf{(16)}$$

Suppose a monotone transformation from the baseline time scale u to the observed time scale t is a function of internal covariates history up to time t, then u can be defined as:

$$u=\left[\exp(\bar{\gamma}_1\bar{z}_1(t))\cdot t\right]\Rightarrow h_0\left(\exp(\bar{\gamma}_1\bar{z}_1(t))\cdot t\right)=h_0(u) \quad \textbf{(17)}$$

For convenience, the derivative of the transformation function $\left(du/dt\right)$ is considered as a function of the internal covariate $\bar{z}_1(t)$. Therefore, it is assumed that $du/dt=\exp(\bar{\gamma}_1\bar{z}_1(t))$ correspond to the relative rate where the baseline time at the baseline hazard is being compared to the actual time as a function of the history of the internal covariates, $\omega(t)$, up to that time. It is essential that $du/dt=\exp(\bar{\gamma}_1\bar{z}_1(t))$ must be a nonnegative function. This transformation function can be expressed as:

$$u=u(\omega(t),t)=\int_0^t\exp(\bar{\gamma}_1\bar{z}_1(\tau))d\tau \quad \textbf{(18)}$$

The transformation function in Equation (18) can be rewritten by a quadratic spline function in Equation (16). The non-parametric EHM can be described as:

$$h(t;\bar{z}_1(t),\bar{z}_2(t)) = h_0\left[u(\omega(\tau),t)\right]\exp(\bar{\gamma}_2\bar{z}_2(t)) \qquad \textbf{(19)}$$

By knowing the hazard equation, the corresponding cumulative or reliability function of the non-parametric EHM is given by:

$$R(t) = \exp\left[-\int_0^t h(\tau;\bar{z}_1(\tau),\bar{z}_2(\tau))d\tau\right] \qquad \textbf{(20)}$$

D. PARAMETER ESTIMATION OF THE NON-PARAMETRIC EHM

In order to estimate the parameters in Equation (19), the partial (or marginal) likelihood function is applied. Cox [38], Oakes [14], Kalbfleisch and Prentice [39] describe the concepts of the partial (or marginal) likelihood in a very clear manner. If the number of items in the risk set is equal to l, and $R(t_j)$ is the risk set of the items which have not failed and have not been censored just prior to the observed failure at time t_j. Thus, the partial likelihood of the non-parametric EHM is given by:

$$L(\bar{\gamma}_1,\bar{\gamma}_2,\zeta) = \prod_{j=1}^{k}\frac{h_0\left[u(\omega_j(t_j),t_j)\right]\cdot\exp\left(\bar{\gamma}_2\bar{z}_{2j}(t_j)\right)}{\sum_{l\in R(t_j)} h_0\left[u(\omega_l(t_j),t_j)\right]\cdot\exp\left(\bar{\gamma}_2\bar{z}_{2l}(t_j)\right)} \qquad \textbf{(21)}$$

Where, the failures of n items occur at time t_j $(j=1,2,\ldots,k)$, $k<n$, and $t_1<t_2<\ldots<t_k$ be uncensored times to failure of k items and let there be $n-k$ censored failure times. The log partial likelihood function is:

$$\begin{aligned} l(\bar{\gamma}_1,\bar{\gamma}_2,\zeta) &= \sum_{j=1}^{k}\Bigg[\ln\left\{h_0\left[u(\omega_j(t_j),t_j)\right]\cdot\exp\left(\bar{\gamma}_2\bar{z}_{2j}(t_j)\right)\right\} \\ &\quad -\ln\left\{\left(\sum_{l\in R(t_j)} h_0\left[u(\omega_l(t_j),t_j)\right]\cdot\exp\left(\bar{\gamma}_2\bar{z}_{2l}(t_j)\right)\right)\right\}\Bigg] \\ l(\bar{\gamma}_1,\bar{\gamma}_2,\zeta) &= \sum_{j=1}^{k}\ln h_0\left[u(\omega_j(t_j),t_j)\right]+\left(\bar{\gamma}_2\bar{z}_{2j}(t_j)\right) \\ &\quad -\sum_{j=1}^{k}\ln\left(\sum_{l\in R(t_j)} h_0\left[u(\omega_l(t_j),t_j)\right]\cdot\exp\left(\bar{\gamma}_2\bar{z}_{2l}(t_j)\right)\right) \end{aligned} \qquad \textbf{(22)}$$

It is clear that the hazard must always be greater or equal zero. In other words, the baseline hazard must be nonnegative. Therefore, care must be exercised in representing the baseline hazard. Kooperberg et al. [40] suggests one way of doing that. If $\alpha(t|\bar{z}_1(t)) = \ln h_0(t|\bar{z}_1(t))$, this assumption can be used in the approximation of the non-parametric EHM to ensure the baseline hazard is always positive. Therefore, the log partial likelihood function of the non-parametric EHM is expressed as:

$$\begin{aligned} l(\bar{\gamma}_1,\bar{\gamma}_2,\zeta) &= \sum_{j=1}^{k}\ln h_0\left[u(\omega_j(t_j),t_j)\right]+\left(\bar{\gamma}_2\bar{z}_{2j}(t_j)\right) \\ &\quad -\sum_{j=1}^{k}\ln\left(\sum_{l\in R(t_j)}\exp\left\{\alpha\left(\left[u(\omega_l(t_j),t_j)\right]\right)\right\}\right. \\ &\quad \left.\times\exp\left(\bar{\gamma}_2\bar{z}_{2l}(t_j)\right)\right) \end{aligned} \qquad \textbf{(23)}$$

All parameters can be estimated by maximizing this log partial likelihood function using a global optimization approach.

V. CALCULATION OF REMAINING USEFUL LIFE

The mean residual life function is the expected Remaining Useful Life (RUL), $T-t$, given that an engineering asset/individual has survived to time t [41, 42]. The expected residual life function $r(t)$ is defined as:

$$r(t) = E\left[T-t\,|\,T\geq t,\bar{z}_1(t),\bar{z}_2(t)\right]$$

$$r(t) = \frac{1}{R(t)}\cdot\int_t^{\infty}R(\tau)d\tau$$

$$r(t) = \frac{\int_t^{\infty}\exp\left[-\int_0^{\tau}h(u;\bar{z}_1(u),\bar{z}_2(u))du\right]d\tau}{\exp\left[-\int_0^{t}h(\tau;\bar{z}_1(\tau),\bar{z}_2(\tau))d\tau\right]} \qquad \textbf{(24)}$$

As it can be seen in Equation (24), both of internal and external covariates $\left\{\hat{\bar{z}}_1(u),\hat{\bar{z}}_2(u)\,|\,t<u<\infty\right\}$ need to be predicted first in order to estimate RUL. There are different statistical approaches to predict these covariates [13].

VI. CONCLUSIONS

Hazard assessment and prediction of engineering assets is one of essential scientific research problems in EAM. Due to the demands for reducing catastrophic failures, minimizing the frequency of unnecessary maintenance and logistic cost, as well as maximizing system availability and reliability, hazard notion becomes a crystal ball in asset life prediction. In order to predict the hazards of engineering assets, a number of covariate-based hazard models have been developed. These models are more effective than traditional reliability models in the real-life situations and industry applications, where an engineering asset operates under dynamic operational and environmental conditions.

Literature review shows that amongst a variety of covariate-based hazard models, only a few have been applied in the reliability field. These models consider either external covariates or internal covariates, but not both explicitly. In this study a new covariate-based hazard model, which termed as EHM, is developed to explicitly include both external and internal covariates associated with the hazard of an engineering asset. This model proposes a new approach to effectively predict the hazard and reliability of an engineering asset utilizing three different sources of data (i.e. historical failure data, internal and external covariates data). EHM is presented in two forms: semi-parametric and non-parametric. This paper focuses on the theoretical development of the semi-parametric and non-parametric EHM. The likelihood functions for each of these models are also derived. Due to page constraint, the related case study is presented in the second part of this work [15]. In the future work, both the semi-parametric and non-parametric EHM will be verified using more case studies.

ACKNOWLEDGMENTS

The authors gratefully acknowledge the financial support provided by both the Cooperative Research Centre for Integrated Engineering Asset Management (CIEAM), established and supported under the Australian Government’s Cooperative Research Centers Program, and the School of Engineering Systems of Queensland University of Technology (QUT).

REFERENCES

1. Singpurwalla, N.D., Foundational issues in reliability and risk Analysis. SIAM Review, 1988. 30(2): p. 264-282.
2. Cox, D.R., Regression models and life-tables. Royal Statistical Society, 1972. 34(2): p. 187-220.
3. Booker, J., et al., Applications of Cox's proportional hazards model to light water reactor component failure data. 1981. p. Size: Pages: 13.
4. Kumar, D. and B. Klefsjoe, Proportional hazards model-an application to power supply cables of electric mine loaders. International Journal of Reliability, Quality and Safety Engineering, 1994. 1(3): p. 337-352.
5. Landers, T.L. and W.J. Kolarik, Proportional hazards models and MIL-HDBK-217. Microelectronics and Reliability, 1986. 26(4): p. 763-772.
6. Leitao, A.L.F. and D.W. Newton, Proportional hazards modelling of aircraft cargo door complaints. Quality and Reliability Engineering International, 1989. 5(3): p. 229-238.
7. Mazzuchi, T.A. and R. Soyer, Assessment of machine tool reliability using a proportional hazards model. Naval Research Logistics, 1989. 36(6): p. 765-777.
8. Park, S., Identifying the hazard characteristics of pipes in water distribution systems by using the proportional hazards model: 2. applications. KSCE Journal of Civil Engineering, 2004. 8(6): p. 669-677.
9. Jardine, A.K.S. and M. Anders, Use of concomitant variables for reliability estimation. Maintenance Management International, 1985. 5: p. 135-140.
10. Jardine, A.K.S., P.M. Anderson, and D.S. Mann, Application of the Weibull proportional hazads model to aircraft and marine engine failure data. Quality and Reliability Engineering International, 1987. 3(2): p. 77-82.
11. Jardine, A.K.S., et al., Optimizing a mine haul truck wheel motors' condition monitoring program: Use of proportional hazards modeling. Quality in Maintenance Engineering, 2001. 7(4): p. 286.
12. Jardine, A.K.S., et al., Proportional hazards analysis of diesel engine failure data. Quality and Reliability Engineering International 1989. 5(3): p. 207-16.
13. Liao, H., W. Zhao, and H. Guo. Predicting remaining useful life of an individual unit using proportional hazards model and logistic regression model. in Annual Reliability and Maintainability Symposium 2006.
14. Oakes, D., Survival times: aspects of partial likelihood. International Statistical Review, 1981. 49: p. 235-252.
15. Gorjian, N., et al., The explicit hazard model - part 2: applications, in Prognostics & System Health Management Conference. 2010, IEEE: Macau - China.
16. Gorjian, N., et al. A review on reliability models with covariates. in The 4rd World Congress on Engineering Asset Management. 2009. Athens-Greece: Springer.
17. Kay, R., Proportional hazard regression models and the analysis of censored survival data. Applied Statistics, 1977. 26(3): p. 227-237.
18. Anderson, J.A. and A. Senthilselvan, A two-step regression model for hazard functions. Applied Statistics, 1982. 31(1): p. 44-51.
19. Hastie, T. and R. Tibshirani, Generalized additive models. 1st ed. Monographs on statistics and applied probability ;. 1990, London ; ; New York: Chapman and Hall. xv, 335 p.
20. Lin, D.Y. and Z. Ying, Semiparametric analysis of general additive-multiplicative hazard models for counting processes. The Annals of Statistics, 1995. 23(5): p. 1712-1734.
21. Cox, D.R. and D. Oakes, Analysis of survival data. 1984, London ; ; New York: Chapman and Hall. viii, 201 p.
22. Kalbfleisch, J.D. and R.L. Prentice, The statistical analysis of failure time data. Second ed. Wiley series in probability and statistics. 2002, New Jersey: Wiley. 439.
23. Ciampi, A. and J. Etezadi-Amoli, A general model for testing the proportional hazards and the accelerated failure time hypotheses in the analysis of censored survival data

with covariates. Communications in Statistics - Theory and Methods, 1985. 14(3): p. 651 - 667.

24. McCullagh, P., Regression models for ordinal data. Royal Statistical Society. Series B, 1980. 42(2): p. 109-142.
25. Bennett, S., Log-logistic regression models for survival data. Applied Statistics, 1983. 32(2): p. 165-171.
26. Sun, Y., et al., Mechanical systems hazard estimation using condition monitoring. Mechanical Systems and Signal Processing, 2006. 20(5): p. 1189-1201.
27. Aalen, O.O., A linear regression model for the analysis of life times. Statistics in Medicine 1989. 8(8): p. 907-925.
28. Wang, W., P.A. Scarf, and M.A.J. Smith, On the application of a model of condition-based maintenance. Operational Research Society, 2000. 51(11): p. 1218.
29. Jardine, A.K.S. and A.H.C. Tsang, Maintenance, replacement, and reliability : theory and applications. 2006, Boca Raton: CRC/Taylor & Francis. 322 p.
30. Eubank, R.L., Spline smoothing and nonparametric regression. Statistics, textbooks and monographs ;. 1988, New York: Dekker. xvii, 438 p.
31. Kay, R., Goodness of fit methods for the proportional hazards regression model: a review. Revue d'épidémiologie et de santé publique, 1984. 32(3-4): p. 185.
32. Etezadi-Amoli, J. and A. Ciampi, Extended Hazard Regression for Censored Survival Data with Covariates: A Spline Approximation for the Baseline Hazard Function. Biometrics, 1987. 43(1): p. 181-192.
33. Shyur, H.-J., E.A. Elsayed, and J.T. Luxhøj, A general model for accelerated life testing with time-dependent covariates. Naval Research Logistics, 1999. 46(3): p. 303-321.
34. Wegman, E.J. and I.W. Wright, Splines in statistics. American Statistical Association, 1983. 78(382): p. 351-365.
35. Luxhoj, J.T. and H.-J. Shyur, Comparison of proportional hazards models and neural networks for reliability estimation. Intelligent Manufacturing, 1997. 8(3): p. 227-234.
36. Sleeper, L.A. and D.P. Harrington, Regression splines in the Cox model with application to covariate effects in liver disease. American Statistical Association, 1990. 85(412): p. 941-949.
37. Wold, S., Spline functions in data analysis. Technometrics, 1974. 16(1): p. 1-11.
38. Cox, D.R., Partial likelihood. Biometrika, 1975. 62(2): p. 269-276.
39. Kalbfleisch, J.D. and R.L. Prentice, Marginal likelihoods based on Cox's regression and life model. Biometrika, 1973. 60(2): p. 267-278.
40. Kooperberg, C., C.J. Stone, and Y.K. Truong, Hazard regression. American Statistical Association, 1995. 90(429): p. 78-94.
41. Blischke, W.R. and D.N.P. Murthy, Reliability : modeling, prediction, and optimization. 2000, New York: Wiley. xxvii, 812 p.
42. Tang, L.C., Y. Lu, and E.P. Chew, Mean residual life of lifetime distributions. IEEE Transactions on Reliability, 1999. 48(1): p. 73-78.

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