AN ANALYSIS OF MIDDLE-YEARS SCHOOL MATHEMATICS TEXTBOOKS

Mal Shield
Queensland University of Technology
<m.shield@qut.edu.au

Shelley Dole
University of Queensland
<s.dole@uq.edu.au>

Abstract

Studies have shown that a set mathematics textbook chosen by teachers is the dominant resource in mathematics classrooms in Australia and many countries throughout the world. The textbook exerts a strong influence on both content and pedagogy. In this study, a method of analysing textbooks was developed and applied to three middle-years mathematics textbook series. The method was based on two pedagogical principles related to deep understanding, namely connectedness and structure and context, and focused on mathematical ideas based on proportional reasoning. The study found that the textbooks provided little support for the pedagogical principles. In most cases, the textbooks presented a specific procedure for each problem type, with little or no recognition of similar structures in different problem contexts.

Introduction

One of the dominant resources in many mathematics classrooms is the mathematics textbook. Different teachers use textbooks differently, and the specifics of textbook interactions between teachers, students and the textbook are complex (Remillard, 2000; Reys, 2004). The majority of teachers follow the textbook when planning and implementing the curriculum, particularly in the middle years of schooling (MYS) (Thompson & Fleming, 2004). As a strong influence upon planning and sequencing mathematics lessons, the presentation of topics and ideas within mathematics textbooks requires detailed analysis. Prior research has highlighted the disconnected nature in which topics are presented (Howson, 2005), and that too often textbooks are compilations of low-level procedural exercises that do little to promote conceptual understanding (Vincent & Stacey, 2008). In this paper we report on a model of textbook analysis that looks specifically at the big ideas of mathematics topics and the connectedness with which big ideas are developed. The model draws upon the work of Bell (1993), and others, who focus on teaching mathematics for understanding.

Background

Teaching for Mathematics for Understanding

The nature of mathematical understanding has been described in various ways. In 1976, Skemp raised the idea of students' developing relational and instrumental understanding of mathematics in light of the mathematics instruction to which they were exposed. As opposed to relational mathematics understanding, instrumental understanding is demonstrated through flawless execution of mathematical procedures, accompanied with little understanding of those procedures highlighted in evidence of lack of approaches for solving non-routine tasks. However, it is elements of Bell's (1993) model for the design of teaching that we draw upon in our model for textbook analysis. Bell (1993) described and exemplified five principles for the design of teaching, and our model draws on two of those principles: (1) connectedness, and (2) structure and context. The principle of connectedness highlights the idea that when students develop rich, connected mathematics knowledge, they can more readily retrieve and apply this knowledge in novel situations. Connectedness for mathematics teaching means connecting mathematical ideas within the discipline and to the world of students outside the classroom. The structure and context principal refers to the idea that understanding mathematics involves knowing the key mathematical structures that underlie mathematical procedures. Traditional mathematics instruction has concentrated on the learning of a large number of procedures with few connections amongst them and with no recognition of structural similarities. Bell noted that many students do not readily transfer structural knowledge from one context to another, even though the new procedure may be mathematically similar to one they have studied earlier.

In this study, Bell's (1993) principles of connectedness, and structure and context, provided the theoretical framework for our model of textbook analysis. Bell's principles prompted us to consider what connectedness, and structure and context, would mean for developing relational mathematics understanding in topics in the MYS mathematics curricula. Clearly, first steps would be to select a core mathematics topic and then consider the relevant literature and research base associated with teaching and learning that topic.

To operationalise the model we selected the topic of ratio and proportion. This topic was chosen because of the rich literature on its learning and teaching accumulated over several decades, and also because proportional reasoning is regarded as a key aspect of mathematical understanding in the middle years of schooling (Behr, Harel, Post & Lesh, 1992; Lamon, 2006). Proportional reasoning derives from multiplicative thinking (e.g, Vergnaud, 1983) and problems located in the multiplicative field involving proportional reasoning feature heavily in a wide range of mathematical topics in middle-years curricula.

Proportion in Middle-Years Mathematics

The multiplicative field includes the operations of multiplication and division as well as the rational number topics of fractions, decimals, ratios, rates, proportion and percent. Vergnaud (1983) argued the importance of building students' understanding of the multiplicative structures in these topics "so that all these different meanings are synthesised into the concept of rational number" (p. 158). When considering the extent of the conceptual field linked by multiplicative structures, the importance of teaching for connectedness is underscored. Each topic linked by multiplicative structures is a considerable research field in its own right. This was highlighted by Vergnaud, who commented on the long-term nature of the study of multiplicative structures.

The structure of proportion situations, according to Vergnaud, is the relationship between the two components of comparison, which he termed the 'measure-spaces'. For example, in considering the ratio of concentrate to water in a fruit juice mix, two components (concentrate and water) are being compared. When considering the relationship between these two components, two types of analyses can occur: a 'between' and a 'within' analysis. A between analysis is consideration of the multiplicative relationship that links the two measure spaces. A within analysis occurs when considering a new quantity in the same proportion.

The following example illustrates the different methods of analyses: *A juice mix is made with one part of concentrate for every five parts of water. How much water needs to be mixed with 40 mL of concentrate to make a drink of the required strength?* A between analysis shows that the amount of water is 5 times the amount of concentrate. A within analysis shows that the new concentrate amount has been multiplied by 40, so the water amount must be multiplied by 40 to have the same taste as the original mix. Representing this situation in a table organises the two measure spaces, and allows easier between and within analyses, as seen in Figure 1.

× 40
→

concentrate	× 5 ↓	1	40	↓× 5
water		5	?	

→
× 40

Figure 1. Measure spaces representation of *between* (x5) and *within* (x40) analysis of the juice problem

Typically, proportion problems in mathematics textbooks provide three quantities and the task is to find the fourth, proportional quantity, as in the example above. These are referred to as 'missing value' proportion problems. Fisher (1988) stated that "The most common textbook strategy for solving a proportion problem is to write an equation in the $^{a}/_{b} = ^{c}/_{d}$ form with an unknown as one of the four terms, cross-multiply and solve for the unknown" (p. 157). However, recent analyses of some Year 8 mathematics textbooks for Year 8 students (Dole & Shield, 2008), found that this was not the most common method suggested, as a plethora of methods were presented for various proportion situations in these textbooks, suggesting specific procedures for specific problems, with the cross-multiply rule used sparingly.

The proportion equation has been the focus of much research in the field of ratio and proportion. The proportion equation of $^{a}/_{b} = ^{c}/_{d}$ is regarded as encapsulating proportion situations (Touriniaire & Pulos, 1985). The cross-multiply approach, however, has been heavily criticised. For example, Hart (1981) stated, "Teaching an algorithm such as $^{a}/_{b} = ^{c}/_{d}$ is of little value unless the child understands the need for it and is capable of using it. Children who are not at a suitable level to the understanding of $^{a}/_{b} = ^{c}/_{d}$ will just forget the formula" (p. 101). Further, Cramer, Post and Currier (1992) stated,

The cross-product algorithm is efficient, [yet] it has little meaning. In fact, it is impossible to explain why one would want to find the product of contrasting elements from two different rate pairs. The cross-product rule has no physical referent and therefore lacks meaning for students and for the rest of us as well. (p. 170)

The main criticism of 'equationising' proportion situations is that the focus moves to equation solving rather than thinking in terms of the proportional nature of the problem situation. The representation of the problem also uses fraction notation, although these are not fractions in the sense of a part to whole multiplicative comparison.

Extensive work by Lamon (2006) has provided comprehensive guidelines for teaching about proportion. Despite the fact that some textbook series advocate a diverse range of approaches for solving proportion problems, as highlighted by Dole and Shield (2008), proportional situations are structurally similar (Vergnaud, 1983), and therefore it seems that textbooks would do well to support students' understanding of this structure in different contexts. For example, rates are structurally similar to ratios as they involve two quantities that are related multiplicatively, yet textbooks treat rates as separate from ratios as if they were unrelated. Speed, is also a special rate that involves two measure spaces of distance and time that are in proportion. But often speed is presented simply as an equation s=d/t. Similarly, problems relating to percentage, similarity of shapes, trigonometry, scale and density all involve two measures related multiplicatively. The connectedness of topics associated with ratio and proportion is clearly a major aspect of proportional reasoning and multiplicative thinking.

Model for textbook analysis

The literature on proportional reasoning, and Bell's (1993) principles of connectedness, and structure and context, provided the basis for identification of six Specific Curriculum Content Goals (SCCGs) and three indicators that enabled identification of features that would contribute to achieving that goal, summarised as follows.

SCCG1. Identification of multiplicative structure and proportional thinking

Indicators: (a) multiplicative comparative relationship of proportional situations clearly defined; (b) use of the operations of multiplication and division highlighted (inverse); (c) use of both 'within' and 'between' thinking.

SCCG2. Use of authentic life-related situations that involve proportional thinking

Indicators: (a) contexts relevant to students; (b) students encouraged to interpret solutions in terms of the context; (c) given examples and exercises use authentic comparisons.

SCCG3. Additive and multiplicative comparison contrasted

Indicators: (a) real-world examples of both additive and multiplicative comparisons are given; (b) opportunities to differentiate additive and multiplicative comparisons are provided; (c) the multiplicative relationship in proportional situations is made explicit (cf the additive nature of non-proportional comparisons)

SCCG4. Effective use of a range of representations

Indicators: (a) tables used to highlight multiplicative relationships; (b) graphs of proportional situations are straight lines that go through the origin; (c) graphs are used to extrapolate and interpolate solutions and/or make predictions.

SCCG5. Related fraction ideas are explicitly connected

Indicators: (a) clear links are made with ideas of fraction and equivalence; (b) part-whole fraction and part-part-whole ratio relationships explicitly distinguished; (c) clearly signals fraction notation meaning in use (e.g., part-whole, ratio, quotient).

SCCG6. Meaningful symbolic representation

Indicators:(a) representation supports identification of within and between relationships in the proportion situation; (b) links between symbolic representation across problem types are made explicit (ie, solution procedures are based on consistent symbolic representation of the problem rather than linked to problem situation); (c) the introduction of the formal 'proportion equation' is delayed until extensive experience has been gained with other representations.

The Study

The SCCGs and indicators were applied to three sets of textbooks currently used in many mathematics classrooms in the home state of the authors (Queensland, Australia). Each textbook series analysed consisted of three books, one each for years 8, 9 and 10. In our approach, the first step is to overview each of the books in the series and identify the chapters containing proportion-related mathematical topics. The chapters are then searched for evidence relating to each SCCGs. Based on the evidence, a rating on a four-point scale (N – non-existent, L – low, M – medium, H – high) is applied to each indicator. We describe our findings of analysis of one textbookseries in the next section.

Analysis of a Textbook Series

The textbook series by Burns and Lynagh (2004) consists of around 500 pages in each book, divided into 15, 14 and 15 chapters respectively. Each chapter focuses on a specific mathematical topic, for example *Decimals and Percentages* (Year 8, Chapter 4), *Linear Equations and Formulas* (Year 9, Chapter 8) and *Parabolas* (Year 10, Chapter 10). The books have a consistent presentation style. Each chapter is sub-divided into a number of sections with specific topic headings. For example, in the Year 8 book Chapter 8 (*Rates and Ratios*) consists of fourteen sections with topic names (for example, 8A Ratio and proportion, 8B Simplifying ratios, 8I Using rates: Speed, distance and time) as well as four extra sections at the end of the chapter (Puzzles, Applications and Activities, Enrichment and Extension, Revision Questions). Each of the topic sections consists of brief definitions and explanations, several worked examples and extensive practice exercises. In some of the sections there is a "learning task". Some learning tasks involve an activity (constructing a triangle) while others are somewhat similar to a set of exercises.

The key chapters identified for analysis were Chapter 8 (Rates and Ratios) in the Year 8 book and Chapter 2 (Ratio and Rates) in the Year 9 book. Other chapters with the potential to involve proportional thinking, including ideas such as equivalent fractions, percentages, unit conversions, scale, similar shapes, gradient, trigonometry and probability were also surveyed. Each of the SCCGs is now discussed in relation to the textbook series.

SCCG1. Identification of multiplicative structure and proportional thinking

Indicator (a) The Year 8 book defines ratio as follows. "A ratio is a comparison of two quantities where we look at how the two quantities are related to each other" (p. 198). The first example involves a ratio of students to teachers at an adventure camp of 5 to 1. At no point does the text mention that this means that the number of students is five times the number of teachers. Shortly after, there is the statement: "Ratios, like fractions, can be simplified using multiplication and division" (p. 200). A 'within' strategy is then shown to write equivalent ratios but there is no use of the 'between' strategy that would show the multiplicative nature of the comparison. Later in the chapter, rate is defined as follows. "A rate is a measure of how one quantity changes with respect to another" (p. 206). Again the only mention of multiplication or division comes when the specific context of speed is introduced. "Speed is the rate at which distance changes with respect to time. It is measured in units of distance divided by time, such as metres per second, kilometres per hour" (p. 208). Manipulation of the formula $d = s \times t$ is then demonstrated. Rating – L

Indicator (b) The books demonstrate the use of an identical 'within' proportion strategy in the ratio and rate chapters for years 8 and 9. Problems are represented consistently as two measure fields with the operations of multiplication and division highlighted, as in the following examples.

In a canoeing camp, the ratio of students to teachers needs to be 3 : 1. Find the number of teachers required if there are 27 students.

Ratio of students : teachers is 3 : 1

	3	1	
x 9 ↓			↓ x 9
	27	9	

If rope cost $2.20 per metre, find the cost of 12 m of rope.

	Cost	Metre	
	$2.20	1	
x 12 ↓			↓ x 12
	□	12	

While these examples involve the use of multiplication and others the use of division, there are no cases where the use of multiplication and division as inverse operations in the same example is shown. It is interesting to note that in the year 8 book, the idea of speed is introduced in the rate subsection, immediately following practice exercises modelled on the second example shown above. However, there is no connection with the prior method of solving a rate problem. The formula and a triangle diagram to help students remember the placement of the symbols in the formula is used. The same approach is used in the year 9 book when working with density. Rating – M

Indicator (c) The year 8 and 9 books illustrate the use of 'within' proportional thinking extensively. As well as its use with ratio and rate problems as illustrated above, the same thinking is illustrated with unit conversions, scale and some percentage situations. Use of 'between' thinking is not illustrated in any context, meaning that the multiplicative relationship between the two variables is not highlighted. Rating – L

SCCG2. Use of authentic life-related situations that involve proportional thinking

Indicator (a) The books display the usual range of problems contexts for ratio and rate. For example, ratio problems involve mixing quantities such as cordial, cement and mower fuel as well as sharing quantities in given ratios. In the year 9 book, (Chapter 2), there is a section headed "Changing quantities in a given ratio", an example being: "Decrease $50 in the ratio 3 : 4" (p. 38). We could not relate this operation to an authentic context. The solution of this worked example involved finding ¾ of $50 and most of the problems that followed involved fractions and were more recognisable as an appropriate context, such as this example: "If a 700 mL can of drink is decreased by one-fifth, find the new volume of the drink" (p. 38). Rate problems also cover a range of familiar authentic contexts. Rating – M

Indicator (b) In the majority of problems presented in the three books, students are asked to provide a specific unknown values for a given variable, with no further interpretation required. Some opportunities for students to interpret findings are provided in sections labelled "Explore", with students being asked to make a statement about their results. This was not the case in all of the relevant explorations. Rating – L

Indicator (c) Most of the situations addressed in the examples and exercise involve authentic comparisons. There were occasional problem contexts that did not represent an authentic use of ratio or rate. For example, income tax is normally calculated as cents in the dollar which is effectively a percentage, meaning that the following problem was not authentic. "For every $8 earned, $2.50 is paid in taxes. If I earn $13216 in a part-time job, find: the amount of tax I need to pay b the amount I take home" (Year 8, p. 37). However, the instances of diversions from the authentic use of the concepts were considerably fewer than in some of the other books examined, with many of the contexts used being quite informative to students. Rating – H

SCCG3. Additive and multiplicative comparison contrasted

Indicator (a) No examples of additive comparisons are provided. Rating – N

Indicator (b) Students are not provided with opportunities to differentiate between additive and multiplicative comparisons. Rating – N

Indicator (c) As discussed in criterion 1, the multiplicative relationship in proportional situations is not made explicit and compared with non-proportional situations. Rating – N

SCCG4. Effective use of a range of representations

Indicator (a) The books make consistent use of a 2 X 2 table for the two measure fields as illustrated earlier. In all cases, only the 'within' way of working is demonstrated. The use of these tables is extended to other topics including unit conversions and percentage. There are no instances of the use of extended tables showing more than two values for each variable that could be used to plot a graph of the relationship. Rating – L

Indicator (b) Proportional situations are not represented with linear graphs in any of the books in the series. Rating – N

Indicator (c) In a Learning Task in Book 1 (p. 277), number lines are used to provide a graphical representation of the sharing of a quantity in a given ratio. These provide a simple physical representation of ratio (length:length) but do not convey any notion of a relationship between two variables and equivalent ratios. Rating – N

SCCG5. Related fraction ideas are explicitly connected

Indicator (a) The use of fraction notation is delayed until Book 2 in the series. In Book 1, clear links are made with the ideas of fraction and equivalence. "Ratios, like fractions, can be simplified using multiplication and division" (p. 200). In Book 2, the use of fraction notation is introduced. "Ratios are usually written in the form *a* : *b* but can also be written as *a/b*" (p. 34). In Book 3, this is referred to "as a fraction, *a/b*" (p. 17). Rating – M

Indicator (b) In Book 2, without explicitly stating the difference between part : whole and part : part relationships, the part : part nature of a ratio is expressed with a rectangular area diagram showing the whole partitioned into three equal parts, two coloured red and one coloured blue. "A ratio expressed as 2 : 1 indicates that one part is twice the size of the other. Here the ratio of the red to blue areas is 2:1. In Book 3, fraction notation is used, including the proportion equation, without further qualification. Rating – M

Indicator (c) Explicit identification of the meaning of *a/b* in use is not provided, apart from the use of formulas for speed and density where division is highlighted. Rating – L

SCCG6. Meaningful symbolic representation

Indicator (a) As mentioned previously, the representations used strongly support 'within' thinking, without any use of inverse operations. However, the representations do not support the idea of relationship. Rating – L

Indicator (b) The use of the two measure fields representation is consistent across problem types in Books 1 and 2. However, in Book 1, even though the first rate examples (costs, exchange rates) continue the representation and solution method established for ratio problems, when speed is introduced, no connection to previous ideas are made. Instead, the formula is stated with a triangle pattern to assist students to remember the various forms of the formula. The same formula/triangle method is used for both speed and density in Book 2. Rating – M

Indicator (c) As mentioned earlier, fraction notation is not used for ratio until Book 2 and the proportion equation is not introduced until Book 3. Rating - H

Table 1 summarises the analysis above and reports on results of similar analyses of the other two textbook series undertaken in this study.

Table 1
Summary of textbook analyses

SCCG	Burns & Lynagh (2004)	Brodie & Swift (2004)	Elms (2003)
1. *Identification of multiplicative structure and proportional thinking*	*(a) L* *(b) M* *(c) L*	*(a) L* *(b) L* *(c) L*	*(a) M* *(b) L* *(c) L*
2. *Use of authentic life situations that involve proportional thinking*	*(a) M* *(b) L* *(c) H*	*(a) M* *(b) M* *(c) M*	*(a) M* *(b) L* *(c) L*
3. *Additive and multiplicative*	*(a) N*	*(a) N*	*(a) N*

SCCG	Burns & Lynagh (2004)	Brodie & Swift (2004)	Elms (2003)
comparison contrasted	*(b) N* *(c) N*	*(b) N* *(c) N*	*(b) N* *(c) N*
4. *Effective use of a range of representations*	*(a) L* *(b) N* *(c) N*	*(a) L* *(b) M* *(c) L*	*(a) M* *(b) H* *(c) M*
5. *Related fraction ideas are explicitly connected*	*(a) M* *(b) M* *(c) L*	*(a) H* *(b) N* *(c) N*	*(a) H* *(b) N* *(c) N*
6. *Meaningful symbolic representation*	*(a) L* *(b) M* *(c) H*	*(a) L* *(b) L* *(c) H*	*(a) N* *(b) M* *(c) N*

Discussion

The analysis of mathematics textbooks reported above commenced with the pedagogical principles of "connectedness" and "structure and context" (Bell, 1993). The SCCGs and associated indicators were derived from the extensive literature on the teaching and learning of the mathematics of the multiplicative field. The analysis revealed considerable similarities amongst the three series. Generally, there is an absence of recognition of the multiplicative relationship between the two variables in proportional situations. Proportional thinking is identified in problems labelled as "direct proportion", but otherwise there is little recognition of this mathematical structure in the solution methods demonstrated. No attention is paid to the difference between a multiplicative comparison and an additive comparison. A specific approach is given for each problem type, in some cases differing greatly from the solution method for earlier problems with essentially the same underlying mathematical structure. In terms of the principle of structure and context, students are not helped to see that different problems have a similar structure and that similar solution strategies can be used.

All three series make use of a range of everyday contexts, with many of them being common to all of the series. Many of the contexts provide reasonably authentic applications of ratio and rate. However, the problem type involving increasing or decreasing a quantity in a given ratio is difficult to associate with any authentic life-related situation. The use of mathematical representations was generally limited in these books. The series used in the illustration of the analysis above (Burns & Lynagh, 2004) makes use of the two measure fields representation but this is not evident in the other series. The representation of proportional situations with a linear graph is used rarely in the books. A positive feature was that the use of the standard proportion equation, solved using cross-multiplication, is delayed until later in each series.

All of the series link the idea of equivalent ratios with equivalent fractions, highlighting that the operations to find equivalent ratios were the same as those for equivalent fractions. Two of the three series introduce the use of a/b to represent a ratio in year 8, with the other series waiting until Year 9. Typically this is introduced with words stating that a ratio can be written "in fraction notation" or "as a fraction". None of the series makes any mention of the fact that this fraction is not the same as the fraction concept familiar to students, that is, in the ratio case it is a part : part comparison and not the previous part : whole comparison. When formulae are used for rates such as speed and density, the division interpretation of a/b is assumed. Overall, there was little evidence of assistance to students to recognise and interpret the three meanings of a/b.

Conclusion

The method of analysis demonstrated in this paper was aimed at uncovering the extent to which middle-years mathematics textbooks promote pedagogy that supports the learning of mathematics for deep understanding. While the examined textbooks series contain many features that are useful to teachers in implementing the curriculum, the analysis highlighted the need for teachers to take responsibility for developing the structural aspects of the mathematics and for helping students build robust connections in their knowledge and understanding. The textbooks did not promote such an approach to learning, although one (Burns & Lynagh, 2004) did at least have some consistency in solution strategies and did highlight some of the structural features of proportion.

We believe that it is critical that teaching and learning materials be scrutinised more closely and specifically mathematics textbooks when used as the foundation for the development of a school mathematics program. The accompanying resources for mathematics textbooks also need to be considered. Many mathematics textbooks are marketed with a CD version, and typically this is a digital copy of the printed book with some additional features such as the answers hyperlinked to the exercises. As learning materials publication moves further into digital media, it would be beneficial to explore ways in which the new environments can support presentations that provide for more effective pedagogy.

References

Bell, A. (1993). Principles for the design of teaching. *Educational Studies in Mathematics*, 24(1), 5-34.

Behr, M., Harel, G., Post, T and Lesh, R. (1992). Rational number, ratio and proportion. In D. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 296-333). New York: MacMillan.

Brodie, R. & Swift, S. (2004). *New Maths 8, 9 and 10*. Melbourne: Thomson Nelson.

Burns, P. & Lynagh, C. (2004). *Maths for Queensland 1, 2 and 3*. Sydney: Pearson Longman.

Cramer, K., Post, T. & Currier, S. (1992). Learning and teaching ratio and proportion: Research implications. In D. T. Owens (Ed.), *Research ideas for the classroom: Middle grade mathematics* (pp. 159-178). New York: MacMillan.

Dole, S. & Shield, M. (2008). The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning. *Research in Mathematics Education, 10*(1), 19-35.

Elms, L. (2003). *Maths Quest for Queensland 1, 2 and 3*. Milton, QLD: Wiley.

Fisher, L. C. (1988). Strategies used by secondary mathematics teachers to solve proportion problems. *Journal for Research in Mathematics Education, 19*(2), 157-168.

Hart, K. (1981). Ratio and proportion. In K. Hart, M. L. Brown, D. Kuchemann, D. Kerslake, G. Ruddock & M. McCartney (Eds.), *Children's understanding of mathematics: 11-16* (pp. 88-101). London: John Murray.

Howson, G. (2005). "Meaning" and school mathematics. In Kilpatrick, J., Hoyles, C., Skovsmose, O. & Valero, P. (Eds.), *Meaning in mathematics education*. New York, NY: Springer.

Lamon, S. J. (2006). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers (2nd ed.)*. Mahwah, NJ: Erlbaum.

Remillard, J. T. (2000). Can curriculum materials support teachers' learning? Two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal, 100*, 331-350.

Reys, B. (2004). Why mathematics textbooks matter. *Educational Leadership*, 61, 61-66.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching 77*, 20-26.

Thomson, S. & Fleming, N. (2004). *Summing it up: mathematics achievement in Australian schools in TIMSS 2002*. Melbourne: Australian Council for Educational Research.

Tourniaire, F., & Pulos, S. (1985). Proportional reasoning: A review of the literature. *Educational Studies in Mathematics*, *16*, 181-204.

Vergnaud, G. (1983). Multiplicative structures. In R. Lesh, & M. Landau, *Acquisition of mathematical concepts and processes* (pp. 127-174). Orlando, FL: Academic Press.

Vincent, J. & Stacey, K. (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal, 20*(1), 81-106.