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# PRIMARY STUDENTS' SPATIAL VISUALIZATION AND SPATIAL ORIENTATION: AN EVIDENCE BASE FOR INSTRUCTION

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*This paper reports on the performance of 58 11 to 12-year-olds on a spatial visualization task and a spatial orientation task. The students completed these tasks and explained their thinking during individual interviews. The qualitative data were analysed to inform pedagogical content knowledge for spatial activities. The study revealed that “matching” or “matching and eliminating” were the typical strategies that students employed on these spatial tasks. However, errors in making associations between parts of the same or different shapes were noted. Students also experienced general difficulties with visual memory and language use to explain their thinking. The students’ specific difficulties in spatial visualization related to obscured items, the perspective used, and the placement and orientation of shapes.*

## INTRODUCTION

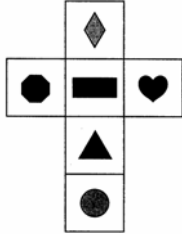
In 2006, the (US) National Academies (National Academy of Sciences, National Academy of Engineering, Institute of Medicine, National Research Council) published a landmark report titled “Learning to Think Spatially” in which they proposed the importance of embedding spatial thinking in the contemporary curriculum. They described thinking spatially as knowing about *space*, *representation* and *reasoning*. Although the National Academies’ report was directed towards achieving spatial literacy across the curriculum, it is particularly applicable in mathematics. For example, there is a relationship between adolescents’ performance on spatial ability tasks and their performance in and preference for mathematics (Stavridou & Kakana, 2008). Thus, the purpose of this paper is to explore students’ thinking on mathematics tasks that incorporate visual representations, in particular, those that place a heavy demand on spatial ability. Henceforth, the term “graphics” will be used to refer to visual representations because the term “representations” has multiple meanings in mathematics education.

## INTERPRETING GRAPHICS

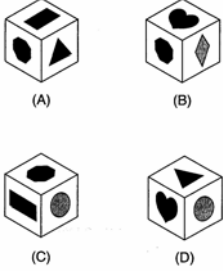
Routinely, in mathematics, students are required to interpret graphics (e.g., maps, number lines, graphs) as well as text and mathematical symbols. However, our previous research has revealed that for many students the interpretation of graphics is problematic rather than routine (e.g., Lowrie & Diezmann, 2007). The interpretation of graphics is problematic for students for at least three reasons.

First, the interpretation of graphics is complex and requires particular knowledge and skills. Specifically, it involves the interaction between a visual symbol system and perceptual and cognitive (i.e., conceptualisation) processes (Winn, 1994). The symbol system is composed of *visual elements* (e.g., shapes) that represent objects or ideas and the *spatial relationships* among the elements within the graphic (e.g., one shape inside another). Mackinlay (1999) argues that there are six visual symbol systems comprised of particular combinations of perceptual elements and spatial relationships which he terms “graphical languages”. These are *Map Languages* (e.g., topographic map), *Axis Languages* (e.g., number line), *Opposed Position Languages* (e.g., bar chart), *Connection Languages* (e.g., network), *Miscellaneous Languages* (e.g., calendar), and *Retinal List Languages* (e.g., mental rotation task). This latter language presents mathematical information through a combination of perceptual elements (e.g., colour, shape, size, saturation, texture, orientation) and capitalizes on these retinal properties to encode this information (E.g., Figures 1 and 2).

This is the net of a cube.



Which one of these cubes could be made by folding the net?

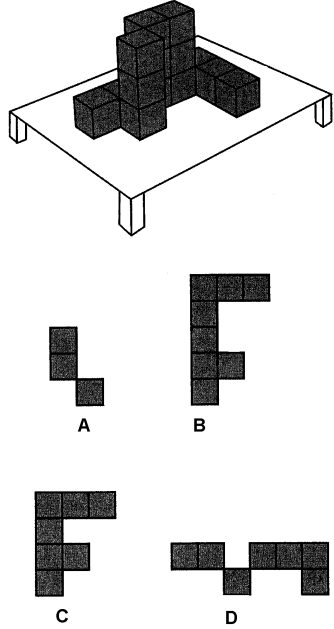


(A) (B)  
(C) (D)

Educational Testing Centre, 2002, p. 9.

Figure 1: Net task: Spatial visualization.

What does this model look like from above?



A B  
C D

Queensland Studies Authority, 2002, p. 10.

Figure 2: Model task: Spatial orientation.

The interpretation of graphics has dual perceptual and cognitive foci. Students need to make sense of various perceptual elements (e.g., shape, size, saturation, orientation, texture) and the spatial relationships among these elements. They also need to employ various spatial perception skills, such as eye-motor co-ordination;

figure-ground perception; perceptual constancy; position-in-space perception; perception of spatial relationships; visual discrimination or visual memory (Del Grande, 1990). In tandem, students need to employ the appropriate cognitive processes for the particular graphical language. For example, in *Retinal List Languages*, they might be required to employ particular spatial abilities such as spatial visualization (Figure 1) or spatial orientation (Figure 2). McGee (1979) argues that visualization and orientation are two distinct factors in spatial ability. Visualization and orientation are of particular importance in enabling the interpreter of a graphic to translate between different representations of the same object. In Figure 1, the translation is between the (2D) net of a cube and the drawn (3D) graphic of the cube. In Figure 2, the translation is between the (3D) model of a set of cubes and the (2D) bird's eye view of the model. Henceforth, these two tasks will be referred to as "cube tasks" because both tasks involve the interpretation of cubes.

Second, students experience particular difficulties in each of the graphical languages. For example, on structured *number lines* items (Axis languages), students' difficulties included overlooking the relative position of an unnumbered mark to identify its numerical value (Diezmann & Lowrie, 2006). Whereas on a *map* (Map languages), students experienced difficulty identifying which landmarks they should use in the solution process (Diezmann & Lowrie, 2008). Thus, we anticipate that students will experience unique difficulties interpreting Retinal List languages because it is a distinct graphical language.

Third, there is scant guidance for teachers to support students' interpretation of graphics in mathematics. Thus, it should be worthwhile to explore students' interpretation of graphics in relation to five aspects of pedagogical content knowledge (PCK) proposed by Carpenter, Fenema and Franke (1996): (1) what tasks students can typically solve and how they solve them; (2) an understanding of individual students' thinking; (3) how students connect new ideas to existing ideas; (4) common errors made by students; and (5) what is difficult and what is easy for students.

## **METHOD**

This investigation is part of a 3-year longitudinal study which sought to describe and monitor primary students' capacity to interpret information graphics in mathematical test items. The aims of this study were:

1. To describe students' knowledge and thinking about cube tasks;
2. To document the errors students made on cube tasks; and
3. To identify the difficulties that students experienced on cube tasks.

### **The Participants**

The participants were 58 primary students aged 11 to 12 years drawn from two schools in moderate socio-economic areas. Fewer than 5% of students had English as a second language.

## The Interviews

The interview tasks were a pair of Retinal List items (Figures 1 and 2) drawn from the 36-item Graphical Languages in Mathematics test (Diezmann & Lowrie, in press). This test comprises six sets of graphic items corresponding to each of the six graphical languages. The two selected Retinal List items are similar in that they each included the interpretation of 3D cubes. The items are dissimilar in that the Net task required students to identify the correct net for a cube whereas the Model task requires students to identify the bird's eye view of a set of cubes. The students completed the two items during an individual interview and then explained their thinking. They also explained which of these tasks was more difficult for them. The analyses of data were guided by Carpenter et al.'s (1996) five aspects of PCK. It involved the thematic coding of students' responses and frequency counts.

## RESULTS AND DISCUSSION

The results focus on three research questions. The first question addresses three facets of PCK (Carpenter et al., 1996): (1) which tasks students can typically solve and how they solve them, (2) students' thinking, and (3) connections students made between new ideas to existing ideas. The subsequent two questions focus on the other two aspects of PCK, namely (4) students' errors and (5) difficulties respectively.

### 1. What did students know about these cube tasks?

These tasks were not particularly difficult for Grade 6 students (N=58) with 75.9% and 65.5% of students successful on the Net and Model tasks respectively. Thus, the Net task was relatively easy for students and the Model task was of moderate difficulty. Higher results had been anticipated because these tasks are designed for students one to two years older than this cohort.

Across the two tasks, successful students used a variety of strategies. However there was one predominant strategy for each task. On the Net task (Figure 1), 68.1% of successful students (n=30) used a *matching strategy*. Paul's response, in which he identified the same parts of the shape on two different graphics, was typical.

I chose A (answer) because to make a cube, the one (shape) that's in the middle of the cross (net) is the one that's going to be on the top and A is the one on the top (*matching*).

Matching was also part of the typical strategy employed for the Model task (Figure 2) with 60.5% of successful students (n=23) using a *matching and eliminating strategy*. Heather's response of *matching* aspects of one representation to another and *eliminating* multiple choice answers was typical.

First I had a look at the model and I had a quick look at the A B C and D and then I counted how many blocks were along that side and I saw that it was 3 and on this side it was 4 and one down so I had a look on here and I thought cause that one (Answer A) was too small, so 1 2 3 (counting cubes) and then I saw 1 2 3 4 (*matching*) and I got that so I thought it was probably B and then I just checked C and D and I didn't think it was D

cause you would be seeing all the blocks and there's not a space there (*eliminating*), I can see it there and C was too short going this way so I thought it must be B.

The use of the matching and eliminating strategy appears to increase the likelihood of success on the Model task.

The exploration of students' thinking revealed two unanticipated results. First, notable in the successful (and unsuccessful) students' responses was a difficulty using language to describe their thinking. There were considerable pauses and reference to vague language such as "it", "that" and "there" as in Megan's response:

**It** folds down ... **that** would fold down to **there** and **that** would be on top like **that** and then **it** would be like **that** (emphases added).

Second, only one student made a link between one of the tasks and prior knowledge. Colin's comment provides evidence of a connection between the Net task and a previous task in an earlier year albeit using concrete materials.

Well we did this in Grade 5 folding the net of a cube and so and we did colour it in before so I learnt a bit about shapes and possible configurations ...

The paucity of explanations linking the tasks to prior knowledge is surprising given that a constructivist philosophy underpins the mathematics syllabus in this state and building on prior knowledge is a central tenet of constructivism. However, there are three plausible reasons why relevant prior experience might not have been described by the students. The students might have had no previous experience with similar tasks; they might have had previous experience with similar tasks but did not think to refer to these experiences in their explanations; or they may have failed to make a connection between prior knowledge and these tasks.

## 2. What errors do students make on the cube tasks?

Students made four types of errors across these two tasks. On the Net task (Figure 1), the dominant error was *incorrect association* with 92.9% (n=13) of the 14 unsuccessful students using this strategy. This code was assigned when students made an incorrect association between two parts of the same shape or between one part of a shape and the corresponding part on its alternative representation. For example, Sue made the correct assumption that the heart could be on the top but then made an incorrect spatial association between the hexagon and the heart.

I picked B and the love heart could be on the top (correct) and then the hexagon would be on the side (incorrect it would be on the bottom) so that means that the diamond would be on the other side.

Similar to the Net task, *incorrect association* was also the dominant error on the Model task (Figure 2). Seventy percent of the 20 unsuccessful students made this error. The second most frequent error on this task was *incorrect elimination* which was made by 15% of students. Paul's response was typical of an incorrect elimination error:

I chose C – it couldn't be that one (A) because there's more (cubes). I can see 3 blocks there and I can see another block there (pointing to the model). It couldn't be that one (B) because three down and two (incorrect elimination) there so I chose that one.

Across the two tasks, one to two students also made errors because they assumed the diagram *looked correct* (without checking) or because they *misread the graphic*.

### 3. What difficulties do students report experiencing on the cube tasks?

After the students had completed the two tasks, they were asked to identify which task was harder for them and why. The results indicate that students perceived the Net task to be more difficult than the Model task (Table 1). Students' perceptions of task difficulty mirrored their performance with approximately 10% more students perceiving the Net task to be more difficult than the Model task (53.4% : 41.4%) and being successful on the Net than Model tasks respectively (75.9% : 65.5%).

| Net task was harder | Model task was harder | Both tasks were similar | Students not questioned |
|---------------------|-----------------------|-------------------------|-------------------------|
| 53.4% (n=31)        | 41.4% (n=24)          | 1.7% (n=1)              | 3.4% (n=2)              |

Table 1: Relative difficulty of the tasks.

A thematic analysis of students' explanations of why particular tasks were difficult highlights the complexity of graphic interpretation and the specificity of difficulties with particular tasks. On the Net task, in which students were working from a 2D representation to a drawn 3D representation (Figure 1), students reported four difficulties: a lack of prior experiences; limited visual memory; difficulty imagining an obscured view; and difficulty imagining the placement and orientation of shapes when a net was folded (Table 2). On the Model task, in which students were working from a drawn 3D representation to a 2D representation (Figure 2), students reported two difficulties: imagining an obscured view and a bird's eye view (Table 3).

*Lack of prior concrete experiences:* "You have to have an (concrete) example or something because sometimes you're not sure" (Bridget)

*Visual memory:* "Hard to imagine them being folded and forgot which way each one went" (Rachel)

*Imagining an obscured view:* "Didn't see the whole cube, you could only see three sides" (Molly)

*Imagining the placement and orientation of shapes on a folded net:* "Hard to work out which shapes were next to each other" (Alan) (placement); "You had to work out which way to fold them and whether you could turn them around" (Ned) (orientation)

Table 2: Type of difficulties and examples for the Net task.

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*Imagining an obscured view:* “Because you can’t get the exact photo because you’ve got like blocks there and then you can’t see the blocks behind and you’ve got to sort of guess like those blocks or how many blocks there are.” (Isobel)

*Imagining a bird’s eye view:* “It was just hard imagining what it would look like from above.” (Colin)

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Table 3: Type of difficulties and examples for the Model task.

The students’ difficulties across both tasks highlight the importance of concrete experiences and strong visual perception skills particularly visual memory. Students’ difficulties with various aspects of imagining (obscured view, placement, orientation, bird’s eye view) suggest the importance of both spatial visualization and spatial orientation in these types of tasks. On the Model task, students’ difficulty imagining what blocks are hidden might have been exacerbated by the perspectives shown (See Parzysz, 1991 for a discussion of perspectives). On this task not only did students need to translate from a drawn 3D representation to a 2D representation but they also had to coordinate an orthogonal projection (model) with oblique projections (answers) (Figure 2).

## CONCLUSION

The importance of spatial skills in our technological world is increasing with new devices becoming commonplace (e.g., Global Positioning Systems [GPS], new virtual worlds to traverse (e.g., Google earth), and new careers that rely heavily on spatial abilities (e.g., deep sea imaging). Hence, spatial literacy is indisputably a fundamental literacy in the 21<sup>st</sup> century. Our investigation of students’ performance on spatial visualization and spatial orientation tasks indicates six ways that educators can foster students’ spatial abilities and work towards spatial literacy for all students. First, ensure spatial skill development and a variety of spatial activities are embedded in the mathematics curriculum. Second, support students to develop their spatial vocabulary and provide opportunities for them to use this language. Third, foster the development of students’ visual memory and spatial abilities with particular attention to the visualization of obscured views, the placement and orientation of shapes, and different viewpoints. Fourth, provide concrete examples of tasks prior to expecting students to visualize tasks and encouraging them to make links to these previous experiences. Fifth, follow up on students’ difficulties and errors and provide practice tasks on each of the sub components of problem tasks. Finally, capitalize on 21<sup>st</sup> century technologies to provide opportunities to develop spatial literacy. For example, 3D games that include virtual avatars provide multiple opportunities for students to learn about orientation in an informal environment (Amorim, 2003).

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