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GLOBAL 3D RIGID REGISTRATION OF MEDICAL IMAGES

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ABSTRACT

We present in this paper an iterative algorithm for the simultaneous registration of multiple 3D medical images. The proposed algorithm is a point-based registration method and is based on global registration techniques rather than the traditional pair-wise registration methods. Corresponding feature points, known as extremal points, are first automatically extracted from the 3D images and are used as the matching features in the registration process. These extremal points are stable landmarks, as the relative positions of these points are known to be invariant according to 3D rigid transformations. The registration algorithm is based on a novel weighted least squares formulation and it also incorporates 3D noise models on the extracted feature points. Results will be presented for the 3D rigid registration of three successive images of the same patient taken at different periods of time.

1. INTRODUCTION

Rigid registration of 3D images is a fundamental task in the field of medical imaging. It is the process of aligning different images into a common coordinate system by application of certain rotations and translations. It plays an essential role in procedures such as surgical planning, image-guided surgery and other systems that are used for both diagnosis and therapy.

In the field of computer vision, 3D registration has applications in areas such as object reconstruction and reverse engineering. This involves taking multiple scans of an object, which are then registered and combined to form the complete surface model.

An important issue to consider when discussing registration algorithms is the concept of global registration verses pair-wise registration. As the name suggests, pair-wise registration techniques involve matching of pairs of images or data sets. However, global registration techniques perform the registration across multiple images or data sets simultaneously. A pair-wise technique can be used to solve a global problem. This can be accomplished by performing the registration of the multiple data sets two at a time. However, this is not necessarily the most appropriate solution as an uneven distribution of inaccuracy can result. Although individual pairs may have high registration accuracy, the entire system may not be in the most optimal registered state. Thus, a truly global system will evenly distribute registration errors throughout the entire data sets.

Another important issue to consider is the treatment of errors and uncertainty in the point features of 3-D images. There are a number of factors that lead to the generation of noise in the 3-D point sets. These include the inherent noise that is generated during the imaging process, and also the noise due to the actual feature extraction methods. If the statistical properties of this noise can be modelled, then it is beneficial to incorporate this information to improve the registration accuracy.

The registration algorithm proposed in this paper is based on points. Thus, in order to register 3-D or volume medical images, a set of feature points must first be extracted from each of the images. The method used is that proposed by Thirion [1] in which a set of feature points, called extremal points, are extracted from the images.

Many previous registration techniques have attempted to reduce the complexity of the features that are matched between images. The majority of these techniques have used surfaces as the matching components. This represents the information contained in the image in a 2-D variety. There are also methods that extract feature lines (such as crest lines proposed by Thirion et al. [2]). These feature lines represent a 1-D variety of the information contained in an image. These extremal points are a further extension of these previous methods as they represent a 0-D variety of the information contained in an image.

In this paper we present a solution to the global registration problem based on the method proposed by Williams et al. [3], using extremal points as the matching features. In section 2, we will give a brief explanation of extremal points and their extraction. Section 3 will outline the graph matching method that was used to find the correspondences between the successive sets of extremal points. Section 4 will outline the novel weighted least squares formulation of the point set registration problem. Section 5 will

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show results obtained from registration of MR images of the same patient taken at 3 different intervals of time (ie: mono-modality, mono-patient registration case). The paper is then concluded in section 6.

2. EXTREMAL POINTS

The method used by Thirion [1] to extract extremal points is based on a modified version of the Marching Lines algorithm, which is used to extract extremal lines. These extremal lines are defined as the intersection of two implicit surfaces and are geometric invariant 3D curves. Extremal points are defined as the intersection of extremal lines with a third implicit surface.

Before any further explanation however, the following must be stated. The total set of curvatures at any point on a surface can be described with two principle directions, $\vec{t_1}$ and $\vec{t_2}$, and two associated principle curvatures k_1 and k_2 (except for umbilic points).

Thus, an extremal point is defined as the intersection of three implicit surfaces: $f = I$, $e_1 = 0$ and $e_2 = 0$, where f represents the intensity value of the image, I is an iso-intensity threshold, and e_1 and e_2 are extremality functions defined as the directional derivative of k_1 and k_2 in the direction of $\vec{t_1}$ and $\vec{t_2}$ respectively. ie:

$$
\begin{aligned}
e_1 &= \vec{\nabla} k_1 \cdot \vec{t_1} \\
e_2 &= \vec{\nabla} k_2 \cdot \vec{t_2}
\end{aligned} \tag{1}
$$

The two extremality functions are geometric invariants of the implicit surface (ie: invariant to rigid transformations). Therefore, the relative positions of extremal points are also geometric invariants.

The extremal points are calculated using a modified version of Thirion's extensive computation method whereby the extremality functions e_1 and e_2, are calculated for all the voxels in the 3-D image, directly from the differentials of the image intensity function. Then, each cubic 8-cell (cell formed with 8 voxels) is considered individually to see if an extremal point can be extracted. For each vertice of the cube, 3 values can be defined: f, e_1 and e_2.

Thus, the extraction process requires first checking to see if the iso-surface $f = I$ crosses the 8-cell. Then, interpolating the extremality coefficients to these intersection points so that they can be used to check if a crest line exists. If so, then the extremality coefficients are further interpolated to the end points of the crest line to finally check if an extremal point exists. Tri-linear interpolation is used as a good first order approximation for the interpolation process.

The final result produced is a set of extremal points that are dispersed throughout the entire volume image. The above method however is only an extremely brief description of the extraction process and once again we refer the reader to Thirion [1] for a full explanation of the extremal points extraction method.

3. CORRESPONDENCE SELECTION

Once the set of extremal points have been extracted from the successive images, it is necessary to establish the correspondence between the point sets. This problem is solved using a recursive descent tree traversal algorithm originally proposed by Cheng et al. [4]. Given two sets of extremal points, the points in the first image are constructed as a graph and then a match graph is derived from the second set of extremal points so that a maximal matching point and minimum matching error is obtained. This algorithm is also able to handle the case when occluded points exist in either or both images.

This graph matching algorithm however, is extremely computationally expensive. (The computation time will be exponentially proportional to the size of the data set). In order to reduce the calculation time, a data splitting algorithm, also proposed by Cheng et al. [4], is used to significantly reduce the calculation time. However, application of this approach to the correspondence selection between the sets of extracted extremal points is still too computationally expensive. One method we used to reduce the amount of computation even further was by introduction of constraints into the graph matching process. These were based on labels that were associated to the extracted point sets. For every extremal point, one of four possible labels were assigned to it based on the sign of both its principal curvatures, k_1 and k_2. So, before considering the correspondence of one point to another, their associated labels were first checked to be compatible. This method also helped to reduce the search space of the graph matching process considerably.

4. REGISTRATION METHOD

We will now describe an iterative algorithm for the simultaneous registration of multiple 3-D point sets. The concepts utilised in this section come from a framework for probabilistic points and motions, as described in [5].

4.1. Representation of motions

A rigid transformation can be described by a rotation and a translation. Let the rotational component be specified by a rotation matrix $\mathbf{R}$ which represents a rotation of angle θ around an axis specified by a unit vector $\mathbf{n}$. Denoting r as $\theta\mathbf{n}$, the coordinate transformation may be represented by the 6-element vector $\mathbf{f}^T = [\mathbf{r}^T, \mathbf{t}^T]$, where $\mathbf{t}$ is the component responsible for the translation. The notation $\mathbf{f} = (\mathbf{r}, \mathbf{t})$ is also used to represent the rotational and translational components of a motion.

4.2. Probabilistic points

Extracted feature points (in our case extremal points) inherently suffer from some random noise due to imperfect feature extraction methods and other noise sources during the imaging process. Denoting $\tilde{\mathbf{p}}$ as the true feature point position, and $\mathbf{p}$ as the extracted or measured position, the extraction process is modelled by:

$$\mathbf{p} = \tilde{\mathbf{p}} + \mathbf{e}_p \tag{2}$$

where $\mathbf{e}_p$ is a random variable characterising the noise that corrupts the feature point. The expected value $E[\mathbf{e}_p]$ is assumed to be zero. The probability distribution of $\mathbf{e}_p$ depends largely on the feature extraction method, however for computational reasons it is common to only consider its covariance matrix as given by:

$$\Sigma_{pp} = E[\mathbf{e}_p \mathbf{e}_p^T] \tag{3}$$

A probabilistic point may be denoted by the pair $(\mathbf{p}, \Sigma_{pp})$, which indicates the expected value of the true feature point position $\tilde{\mathbf{p}}$, and the likely deviation around that position. Note that for an exact point, $\Sigma_{pp} = \mathbf{0}$.

4.3. Problem description

Assume the existence of M separate images each with their associated local coordinate systems represented by $\mathbf{f}_m, m \in 1 \ldots M$. There also exists P sets of pairwise feature point correspondences between the successive images, with $\mu \in 1 \ldots P$ referring to any particular correspondence set. The mappings $\alpha(\mu)$ and $\beta(\mu)$ define two sets of feature points which contain the μth correspondence. The quantity N_μ denotes the number of corresponding feature point pairs in each set.

The individual points comprising each correspondence set are denoted by $\mathbf{x}_i^\mu$ and $\mathbf{y}_i^\mu$, $i = 1 \ldots N_\mu$. The values $\mathbf{x}_i^\mu$ and $\mathbf{y}_i^\mu$ are the point locations as measured in the local coordinate system of the point sets $\alpha(\mu)$ and $\beta(\mu)$ respectively. They are probabilistic in the sense of section 4.2, with covariance matrices denotes by $\Sigma^\mu_{x_i x_i}$ and $\Sigma^\mu_{y_i y_i}$ respectively.

4.4. Problem formulation

The registration process lies in trying to compute a transformation for each set of feature points which, when applied to the points in that set, maximises the coincidence of each corresponding feature point pair. The feature points in the first image are chosen arbitrarily to be fixed to the canonical coordinate system and thus are not included in the estimation process.

The required transformations are contained in the vectors $\mathbf{f}^2, \mathbf{f}^3, \ldots, \mathbf{f}^M$, which are concatenated into a single vector

$$\theta^T = [\mathbf{f}^{2^T} \ldots \mathbf{f}^{M^T}] \tag{4}$$

We then define the residuals $\mathbf{v}_i^\mu$ as

$$\begin{aligned} \mathbf{v}_i^\mu &= \mathbf{f}^\alpha * \mathbf{x}_i^\mu - \mathbf{f}^\beta * \mathbf{y}_i^\mu \\ &= \mathbf{R}^\alpha \mathbf{x}_i^\mu + \mathbf{t}^\alpha - \mathbf{R}^\beta \mathbf{y}_i^\mu + \mathbf{t}^\beta \end{aligned} \tag{5}$$

where $\mathbf{R}^\alpha$ and $\mathbf{R}^\beta$ are the 3×3 rotation matrices formed from $\mathbf{f}^\alpha$ and $\mathbf{f}^\beta$ respectively, and $\mathbf{t}^\alpha$ and $\mathbf{t}^\beta$ are the translation components of the transformations.

We now form a covariance weighted least squares cost function $\Phi(\theta)$

$$\Phi(\theta) = \frac{1}{2} \sum_{\mu=1}^{P} \sum_{i=1}^{N_\mu} \mathbf{v}_i^{\mu^T} \mathbf{C}_i^{\mu(-1)} \mathbf{v}_i^\mu \tag{6}$$

where $\mathbf{C}_i^\mu$ is the covariance of $\mathbf{v}_i^\mu$ and is given by

$$\mathbf{C}_i^\mu = \mathbf{R}^\alpha \Sigma^\mu_{x_i x_i} \mathbf{R}^{\alpha^T} + \mathbf{R}^\beta \Sigma^\mu_{y_i y_i} \mathbf{R}^{\beta^T} \tag{7}$$

and is formulated under the assumption that errors afflicting corresponding points are independent.

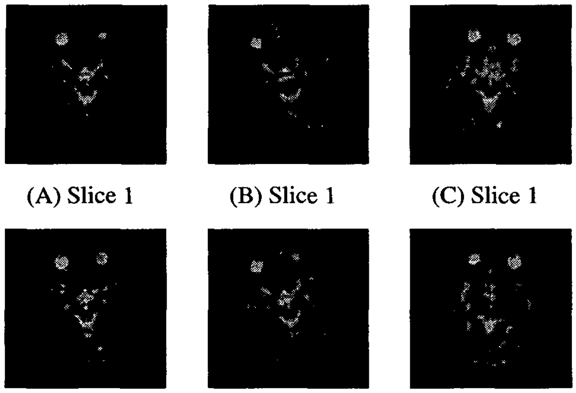
Thus, the registration problem is reduced to the minimisation of a covariance weighted, non-linear least squares objective function, defined by equations 5, 6, 7. We solve the minimisation of equation 6 with the classic Gauss method and hence, solve the registration problem.

For a more in depth description of the problem formulation and solution, see Williams et. al [3]. Note also that by using the Gauss method, not only can we estimate the motions, but we can also characterise the uncertainty in those estimates, as the Gauss method makes this information directly available following the estimation process.

5. RESULTS

This section displays the results obtained after the registration is applied to 3 separate yet successive 3D MR images of the same patient taken at different times (image A, B and C). The images shown in figure 1 are the initial acquired images that have been down-sampled to a size of $102 \times 102 \times 54$ voxels. The first stage in the process was the extraction of the extremal points, as described in section 2. An example of an extracted point set is shown in figure 2 along with its associated smoothed iso-surface that was also extracted during this stage. The resulting 3D point sets were then put into correspondence using the graph matching process described in section 3.

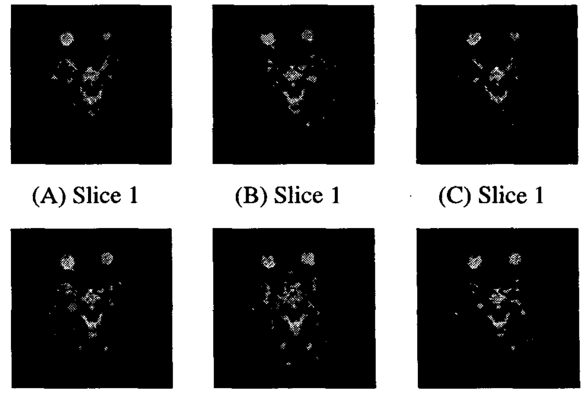
The three point sets could then registered using the method described in section 4. The images shown in figure 3 are representative of preliminary results which have been obtained by applying the transformation matrix acquired from the registration of the point sets to the three MR images. However extensive validification of these results has not yet been completed. Note that only two slices per 3D image are shown in the figures below. The slices in both figures are taken in the transverse plane.



(A) Slice 1 (B) Slice 1 (C) Slice 1

(A) Slice 2 (B) Slice 2 (C) Slice 2

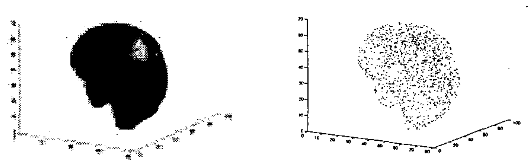
Fig. 1. 3D MR images before registration. Two 2D slices are shown for each of the three volume images (A, B, C).



(A) Slice 1 (B) Slice 1 (C) Slice 1

(A) Slice 2 (B) slice 2 (C) Slice 2

Fig. 3. Preliminary results of 3D rigid registration of MR images. Two slices are shown for each image (A, B, C).



Iso-surface Extremal points

Fig. 2. The extracted smoothed iso-surface and extremal points for image A.

6. CONCLUSION

The method proposed in this paper provides a new approach for global registration of multiple 3D images of the same patient taken at different periods of time, incorporating (2nd order) statistical error models of the extracted feature points. The pre-registration step of extracting the extremal points however, is very computationally extensive. In fact, the extraction of the extremal points and the graph matching that is used to locate corresponding points, constitute the vast majority of the processing power required to complete the overall task.

This registration algorithm was compared with a method that assumes an isotropic error model in Williams et al [3]. Williams demonstrated that the registration algorithm accuracy is improved by the error modelling. The degree of improvement is related to the error model eccentricity, the number of views (or in our case, the number of images), and the number of corresponding point pairs. Thus, an entire set of mono-patient, time-series images that need registration will benefit considerably more from this proposed global registration algorithm as compared to traditional pair-wise registration techniques. The use of error models also provide a confidence measure on the final registration accuracy. This confidence measure is extremely important for clinical validation.

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