Reliability Prediction of Complex Repairable Systems: an engineering approach

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ABSTRACT

This research has developed several models and methodologies with the aim of improving the accuracy and applicability of reliability predictions for complex repairable systems.

A repairable system is usually defined as one that will be repaired to recover its functions after each failure. Physical assets such as machines, buildings, vehicles are often repairable. Optimal maintenance strategies require the prediction of the reliability of complex repairable systems accurately. Numerous models and methods have been developed for predicting system reliability. After an extensive literature review, several limitations in the existing research and needs for future research have been identified. These include the follows: the need for an effective method to predict the reliability of an asset with multiple preventive maintenance intervals during its entire life span; the need for considering interactions among failures of components in a system; and the need for an effective method for predicting reliability with sparse or zero failure data.

In this research, the Split System Approach (SSA), an Analytical Model for Interactive Failures (AMIF), the Extended SSA (ESSA) and the Proportional Covariate Model (PCM), were developed by the candidate to meet the needs identified previously, in an effective manner. These new methodologies/models are expected to rectify the identified limitations of current models and significantly improve the accuracy of the reliability prediction of existing models for repairable systems.

The characteristics of the reliability of a system will alter after regular preventive maintenance. This alternation makes prediction of the reliability of complex repairable systems difficult, especially when the prediction covers a number of imperfect preventive maintenance actions over multiple intervals during the asset's lifetime. **The SSA uses a new concept to address this issue effectively and splits a system into repaired and unrepaired parts virtually.** SSA has been used to analyse system reliability at the component level and to address different states of a repairable system after single or multiple preventive maintenance activities over multiple intervals. **The results obtained from this investigation demonstrate that**

SSA has an excellent ability to support the making of optimal asset preventive maintenance decisions over its whole life.

It is noted that SSA, like most existing models, is based on the assumption that failures are independent of each other. This assumption is often unrealistic in industrial circumstances and may lead to unacceptable prediction errors. To ensure the accuracy of reliability prediction, interactive failures were considered. The concept of interactive failure presented in this thesis is a new variant of the definition of failure. **The candidate has made several original contributions such as introducing and defining related concepts and terminologies, developing a model to analyse interactive failures quantitatively and revealing that interactive failure can be either stable or unstable.** The research results effectively assist in avoiding unstable interactive relationship in machinery during its design phase. This research on interactive failures pioneers a new area of reliability prediction and enables the estimation of failure probabilities more precisely.

ESSA was developed through an integration of SSA and AMIF. ESSA is the first effective method to address the reliability prediction of systems with interactive failures and with multiple preventive maintenance actions over multiple intervals. It enhances the capability of SSA and AMIF.

PCM was developed to further enhance the capability of the above methodologies/models. It addresses the issue of reliability prediction using both failure data and condition data. **The philosophy and procedure of PCM are different from existing models such as the Proportional Hazard Model (PHM).** PCM has been used successfully to investigate the hazard of gearboxes and truck engines. The candidate demonstrated that PCM had several unique features: 1) it automatically tracks the changing characteristics of the hazard of a system using symptom indicators; 2) it estimates the hazard of a system using symptom indicators without historical failure data; 3) it reduces the influence of fluctuations in condition monitoring data on hazard estimation.

These newly developed methodologies/models have been verified using simulations, industrial case studies and laboratory experiments.

The research outcomes of this research are expected to enrich the body of knowledge in reliability prediction through effectively addressing some limitations of existing models and exploring the area of interactive failures.

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Notations

$[0]$	The null matrix
a, b,…	Constants
A, B, …	Events or systems
Aav	Average vibration amplitude
b_{am}	The slope of the fit-line
C_k	A value of baseline covariate function
$Det[\bullet]$	The determinant of matrix $[\bullet]$
$f(t)$	Failure density function
$F(t)$	Failure distribution function
F_0	The predefined control limit of failure probability
$F_A(t)$	The failure distribution function of System A
$F_B(t)$	The failure distribution function of System B
$F_{Ii}(t)$	The independent failure distribution function of Component i
$F_s(t)$	The general failure distribution function of a system during the entire life span
$F_s(\tau)_i$	The failure distribution function of a system after the ith PM action
$F_{sb}(\tau)_i$	The failure distribution function of a subsystem after the ith PM action
$F_{sbi}(\tau)_i$	The failure distribution function of Component i in a subsystem after ith PM action
$h(t)$	Hazard function

$\tilde{h}(t)$	The estimated hazard function of a system
$h_0(t)$	Baseline hazard rate (function)
$H_1(\tau)_1$	The Integrated Interactive Hazard (IntH) of Component 1 after the 1st PM action
$h_i(t)$	The hazard function of Component i
$h_{IC}(t)$	The Independent Hazard (IndH) function of a "virtual" Component C – a common failure cause
$h_{Ii}(t)$	The IndH function of Component i
$h_{Isb}(\tau)_i$	The IndH function of a subsystem after the ith PM action
$h_{Isb}^e(\tau)_i$	The equivalent IndH function of a subsystem after ith PM action
$h_{in}(t)$	The initial estimation of a hazard function
$\vec{h}_{j_i}(t)_B$	The all hazard functions of the influencing components of Component i before an interaction
$\vec{h}_{sb}(\tau)$	The IntH vector of a subsystem
$\vec{H}_{sb}(\tau)_1$	The IntH vector of a subsystem after the 1st PM action
$\{h(t)\}$	Interactive hazard vector
$\{h(t)\}_B$	The hazard vector before an interaction
$\{h_I(t)\}$	Independent hazard vector
$\{h^{(n)}(t)\}$	The jth state of failure interaction
$[I]$	Identity matrix
j_i	The subscripts of the influencing components of Component i

L_k	The number of times of PM action when Component k ($k \le m$) receives its last repair
m	The number of repaired components
M	The number of components in a system
m_c	The number of condition monitoring data
m_f	The number of failure data
m_n	The number of new condition monitoring data
n	The number of PM actions
p	The failure probability of a component due to the effect of a common cause shock
$P(\bullet)$	Probability of $(\bullet)$
$P(B_k \mid A)$	The conditional probability that event B_k occurs at the occurrence of event A
$R(t)$	Reliability function
R_0	Predefined reliability control level
$R_1(\tau)_i$	The reliability functions of repaired Component 1 after the i^{th} PM action
$R_{I1}(t)_0$	The independent reliability function of Component 1
$R_{Is}(t)_0$	The independent reliability function of an original system
$R_{kc}(\tau)_i$	The cumulative reliability of Component k after the i^{th} PM action
$R_s(t)$	The reliability of a repairable system
$R_s(t)_i$	The reliability of a system after the i^{th} PM action
$R_{sc}(\tau)_n$	The cumulative reliability of a system after the n^{th} PM action

$R_{sb}(\tau)_i$	The reliability functions of a subsystem after the i[th] PM action
$R_{sb}^{e}(\tau)_i$	The equivalent reliability calculated based on a subsystem after i[th] PM action
RCP_i^{in}	The initial value of relevant condition parameter for the i[th] item
$RCP_i^{\lim}$	The limit value of relevant condition parameter for the i[th] item
t	The absolute time scale
t_c	The time when the characteristic of the hazard of a system changes
t_i	The i[th] failure time
t_p	Required minimum operating time
T	Time period
T_i^1	The time to the first examination of the i[th] item
$tr([\theta(t)])$	The trace of matrix $[\theta(t)]$
$\{t_i\}$	A set of historical failure times
x_{lbh}	The displacement of the test bearing housing from its central position
y_a	The average acceleration amplitude of the test bearing
y_{a0}	The initial value of the average acceleration amplitude of the test bearing
$Z(t)$	Covariate function
$Z_e(t)$	Environmental covariates
$Z_r(t)$	Responsive covariates
$\{Z_r(t_j)\}$	A set of condition monitoring data
$[\alpha]$	The Sate Influence Matrix (SIM)

$\vec{\alpha}_i$	A partition matrix in the SIM $[\alpha]$
α_{ij}	The i[th] row j[th] column element of SIM $[\alpha]$
α_{sb1}^{e}	An equivalent state influence coefficient to represent the effect of the failure of Component 1 on a subsystem
β	Shape parameter in the Weibull distribution
β_c	Common cause factor
γ	Weighting parameter
Δt_i	Time Between Two Failures (TBTF)
$\varepsilon(t)$	The difference between two hazard functions
$\varepsilon_{b_{am}}$	The relative estimation error of the slope
$\varepsilon_{y_{a0}}$	The relative estimation error of the initial values of the average acceleration amplitude y_{a0}
η	Scale parameter in the Weibull distribution
$[\theta(t)]$	Interactive coefficient matrix
$\vec{\theta}_i$	A partition matrix in the interactive coefficient matrix $[\theta(t)]$
$\theta_{ij_i}(t)$	The Interactive Coefficient (IC) that represents the degree of the effect of failure of Component j_i on Component i
ϑ_{sm}	The degree of angular misalignment of the shaft in test rig
λ	Constant failure rate
λ_e	Eigenvalue
λ_{Ii}	The independent constant failure rate of Component i
ν	The occurrence rate of a common cause shock

$\nu(t)$	Intensity function
ξ_1	The time when the hazard function curve shows the random failure phase of its life cycle
ξ_2	The time when the hazard function curve shows the wear-out phase of its life cycle
$\rho([\theta(t)])$	The spectral radius of matrix $[\theta(t)]$
τ	The relative time scale
$\phi_i(t)$	The independent hazard function of Component i
$\psi(Z,\gamma)$	The function of covariates
$\Psi(Z_r(t))$	The function of responsive covariates
$\lvert \bullet \rvert$	The absolute value of $(\bullet)$

Glossary

Affected component: a component whose failure likelihood increases by the failures of other components in a system.

Average acceleration vibration amplitude: the mean acceleration amplitude value of a vibration process of a system over time.

Baseline covariate function: a function that describes the relationship between covariates and hazard.

Baseline hazard function: a function that represents the hazard without the influence of the covariates.

Cascading failure: multiple sequential failures that are initiated by the failure of one component, which leads to sequential failures of other components.

Common cause failure: failures of different items resulting from the same direct cause, occurring within a relatively short time, where these failures are not consequences of another (ISO14224).

Complex system: a system composed of multi-components which can be connected with each other in either series or parallel or in a complex way.

Corrective maintenance: maintenance that is carried out on an item after fault recognition to return it to a state in which it can perform the required function.

Covariate:	a parameter that measures the conditions of an asset.
Cumulative reliability:	the probability of survival of a system over its whole life time with consideration of the cumulative effect of the repaired components over time.
Dependent failure:	a failure that leads to an increased or a reduced tendency of another failure.
Environmental covariate:	a type of condition parameter whose changes will cause the characteristics of the hazard of a system to change.
Extended split system approach:	the split system approach without using the independent failure assumption.
Failure:	termination of the ability of an item to perform a required function (ISO/DIS14224).
Fault tree:	a diagram that logically represents the various combinations of possible events, both fault and normal, occurring in a system that leads to the top event.
Gradual degraded interactive failure:	a failure due to the interactions among gradually deteriorating components.
Hazard:	the probability that a system or a component will fail in the next interval $(t, t+\Delta t]$ under the condition that this system or component has survived until time t.
Immediate interactive failure:	the failure of the influencing component will cause its affected components to fail immediately.
Imperfect repair:	a repair that returns the state of a system between "as good as new" and "as bad as old".

Independent failure:	a failure that does not affect or is not affected by another failure.
Interactive failure distribution function:	the failure distribution function of a system or a component if its failures are independent.
Independent hazard function:	the hazard function of a system or a component if its failures are independent.
Interactive hazard function:	the hazard function of a system or a component with failure interaction.
Influencing component:	a component whose failure leads to an increased tendency of failures of other components in a system.
Interactive coefficient:	a parameter that is used to represent the degree of the effect of failure of one component on another component.
Interactive coefficient matrix:	a matrix whose elements are interactive coefficients.
Interactive failure:	mutually dependent failures, that is, the failures of some components will affect the failures of other components and vice versa.
Interactive failure distribution function:	the failure distribution function of a system or a component with failure interaction.
Interactive hazard:	the increased hazard due to failure interactions.
Maintenance:	the combination of all technical and associated administrative actions intended to retain an item or system in, or restore it to, a state in which it can perform its required function.

Maintenance framework:	a conceptual model or process guideline on how to conduct maintenance effectively through proper integration of various maintenance models and methodologies.
Markovian process:	a type of stochastic process whose future probability behaviour is uniquely determined by its present state and not dependent on its previous state.
Monte Carlo method:	numerical analysis method using random simulations.
Negative dependency failure:	a failure that can prevent other components in a system from failing further.
Poisson point process:	a special type of stochastic process in which the failures are independent of each other and the number of failures in each time interval follows a Poisson distribution.
Predictive maintenance:	maintenance that is carried out based on the condition of a system.
Preventive maintenance:	maintenance that is carried out at scheduled and fixed intervals based on time or duty.
Proactive maintenance:	maintenance that aims much more at avoiding or reducing the consequences of failure than at preventing the failure themselves.
Reliability:	ability of a functional unit to perform a required function under stated conditions for a stated period of time (ISO 2382-9).

Reliability based preventive maintenance:	a preventive maintenance policy in which a control limit of reliability is defined in advance. Whenever the reliability of a system falls to this predefined control limit, the system is maintained.
Reliability block diagram:	a logic network used to describe the function of a system.
Reliability function:	the probability that a system or a component will function over a period of time t.
Renewal process:	a sequence of independent, identically distributed non-negative random variables which are not all zero and with probability 1.
Repair:	an action to recover the function of a failed system.
Repairable system:	a system which will be repaired to recover its functions after each failure rather than to be discarded during continuous operation.
Required minimum operating time:	a minimum operating period of time demanded between two PM actions due to maintaining production and cost effectiveness.
Responsive covariate:	a type of condition parameter whose changes are caused by the changes of the hazard of a system.
Split system approach:	an approach modelling the reliability of a system after PM activities. In this approach repaired and unrepaired components are separated within a system virtually.

Stable failure interaction: in the case of considering interactive failures only, the interactions among some surviving components increase deterioration of these components rather than leading to immediate failure of any these components.

State influence matrix: a matrix derived from the interactive coefficient matrix. It can determine the degree of influence of failure interactions on stable interactive failure uniquely.

Time based preventive maintenance: a preventive maintenance policy in which a system is maintained based on scheduled PM times.

Unstable failure interaction: In the case of considering interactive failures only, the interactions among some surviving components cause at least one of them to fail in a very short time.

Abbreviations

ACM	Availability Centred Maintenance
ALM	Accelerated Life Model
AMIF	Analytical Model for Interactive Failures
AMRL	the Aeronautical and Maritime Research Laboratory
ARC	the Australian Research Council
BCM	Business-Centred Maintenance
BFR	Binomial Failure Rate
BSC	the British Steel Corporation
CAD	Computer Aided Design
CBM	Condition Based Maintenance
CIEAM	Cooperative Research Centre on Integrated Engineering Asset Management
CM	Corrective Maintenance
CMFD	Condition Monitoring and Fault Diagnosis
CRC	Cooperative Research Centre

DTA	Delay Time Analysis
DWT	Discrete Wavelet Transform
ETA	Event Tree Analysis
ESSA	Extended Split System Approach
FFT	Fast Fourier Transform
FMEA	Failure Mode and Effect Analysis
FMECA	Failure Modes, Effect and Criticality Analysis
FTA	Fault Tree Analysis
GPR	Ground Penetrating Radar
HPP	Homogeneous Poisson Process
i.i.d.	independent, identical distribution
IC	Interactive Coefficient
IndFDF	Independent Failure Distribution Function
IndH	Independent Hazard
IntF	Interactive Failure
IntFDF	Interactive Failure Distribution Function
IntH	Interactive Hazard
IntIH	Integrated Interactive Hazard

JIT	Just In Time
LM	Lean Maintenance
LMDA	Linear Multivariate Discriminant Analysis
MCS	Monte Carlo Simulation
ME	Maintenance Excellence
MLE	Maximum Likelihood Estimation
MSI	Maintenance Significant Item
MTP	Maintenance Tasks Priorities
MTTF	Mean Time To Failure
NHPP	Non-Homogeneous Poisson Process
PCM	Proportional Covariate Model
PHM	Proportional Hazard Model
PIM	Proportional Intensities Model
PM	Preventive Maintenance
QFD	Quality Function Deployment
RBD	Reliability Block Diagram
RBPM	Reliability Based Preventive Maintenance
RCM	Reliability-Centred Maintenance

RCP	Relevant Condition Predictor
RCP	Relevant Condition Parameter
RIF	Risk Influencing Factors
ROCOF	Rates of OCcurrence Of Failures
ROI	Regions Of Interest
SDM	Success Diagram Method
SFL	Sequential Failure Logic
SIM	State Influence Matrix
SSA	Split System Approach
TBPM	Time Based Preventive Maintenance
TBTF	Time Between Two Failures
TPM	Total Productive Maintenance
TQM	Total Quality Management
TTT	Total Time on Test

Statement of Original Authorship

The work contained in this thesis has not been previously submitted for a degree or diploma at any other higher education institution. To the best of my knowledge and belief, the thesis contains no material previously published or written by another person except where due reference is made.

Signature: ___________________________

Date: _______________________________

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Chapter 1

INTRODUCTION

1.1 INTRODUCTION OF RESEARCH

This thesis presents for improving the accuracy of reliability prediction of complex repairable systems. The methodologies/models have been developed specifically for practical applications in the industrial environment.

The majority of assets in industry are repairable systems. The performance of these assets can influence the quality of product, the costs of business, the service to the customers, and thereby the profit of enterprises directly. Asset management has two major objectives: (1) to maintain the availability and quality of assets at a required performance level using the lowest possible cost; (2) to use these assets efficiently. The activity related to the first objective is asset maintenance management. The concerns about asset maintenance management are (1) reliability predictions of assets and (2) the optimal maintenance policy for assets. The former lays a critical foundation for the latter. Hence, it is essential to make an accurate reliability prediction for an asset. Nowadays, Preventive Maintenance (PM) is often conducted by companies to reduce unexpected failures and overall costs. A company can optimise its maintenance strategy according to the prediction of remaining useful life and effectiveness of PM actions. With increasing complexity of machines and competition among business, the need to formulate changes in reliability of a complex repairable system with PM becomes pressing.

Currently, the most common techniques used to model the reliability prediction of a repairable system are based on stochastic or statistical analysis, including the Markov chain (process), the Poisson point process, the Bayesian method, condition based models, Monte Carlo simulations and combinations of those models. After an extensive literature review, several limitations of existing models have been identified. For example, much of the existing literature focused on analysing the Mean Time To Failure (MTTF) or/and expected number of failure times of a repairable system. An effective model for explicit prediction of reliability of a complex system with imperfect multiple PM actions is still not available. The research on the interaction among failures of components in a system and on reliability prediction with spares or zero failure data is adequate. This research is aimed at developing new models and methodologies to address these limitations in an effective manner.

In this chapter, the objectives of the research program and the research methods will be surveyed. The outcomes of the research and the relationship among the developed models will be overviewed. The original contributions made by the candidate will also be identified.

1.2 OBJECTIVES AND METHODS OF THE RESEARCH

1.2.1 Objectives

The overall research objective in this thesis is to develop new models and methodologies for the reliability prediction of a repairable system in order to improve the accuracy of prediction using condition monitoring data and historical failure information for engineering application. The detailed objectives of the research are as follows:

(1) Development of a new reliability prediction approach for complex repairable systems with multiple PM intervals

The first objective of the research program is to develop a new approach to predict the reliability of complex repairable systems with multiple PM actions. This new approach extends the current research in two ways: releasing the assumption that treats the states of a system after repairs as being “as good as new”, and predicting reliability of a complex system with multiple PM actions over multiple intervals. Most existing models/methods have only focused on the case of “as good as new” after repair [1-5]. Imperfect repairs have not been modelled effectively. Currently most modelling techniques based on statistical analysis applied in maintenance cannot accurately estimate the effect of individual repair on the performance of a system. These models were often applied to predict the next repair activity or the expected failure times over a period [6, 7] rather than explicit prediction of reliability of a system after multiple PM actions. The effectiveness of long-term prediction of these models is questionable. This research addresses these issues and suggests remedies. The reliability prediction of a system with multiple PM intervals over its whole life was investigated on the assumption that failures of components are independent of each other.

(2) Development of an analytical reliability prediction model for repairable systems with interactive failures

The second objective of the research program is to remove the assumption that failures of components are independent of each other from the reliability prediction models. Industrial experiences have shown that there are a number of situations where the assumption of independent failures is unrealistic and will lead to unacceptable analysis errors although this assumption has been adopted in the most of existing models [8]. Percy et al [9] have also indicated that a prediction approach is dangerous if interactions between different components in a system are not directly considered. To address the dependency among the failures of components, a concept of dependent failures was introduced [8, 10]. However, the conventional models of dependent failures do not cope at all with interactive failures, which are the failures caused by interactions between different components particularly in industry. It appears that research on interactive failures has not been addressed in the literature to date although the term “failure interaction” has been used by Murthy and Nguyen [11, 12] and Lewis [13]. The failures described in the literature [11, 12] can fall into the classical definition of common cause failure. Lewis analysed some special cases using Markovian theory. In this research, an analytical reliability prediction model for repairable systems with interactive failures was developed. The proposed research therefore significantly advances the knowledge in analytical reliability prediction modelling.

(3) Development of a failure prediction methodology using both failure data and condition monitoring data, especially when historical failure data are sparse

The third objective of the research program is to develop a new model for the prediction of the dynamic failure trend of a system with condition monitoring data. The model can predict the failure time when historical failure information is not adequate for statistical analysis while condition monitoring data is available. These condition monitoring data can describe the condition changes of a system. Existing researchers have not successfully modelled this case. While condition monitoring and diagnosis is playing a more and more important role in maintenance [14], the use of condition monitoring data to predict future failure times is still a challenge. Currently the most frequently adopted model is the Proportional Hazard Model (PHM) [4, 15]. However, this model has several unavoidable disadvantages. For example, historical hazards estimated using different covariates are often different. Fluctuations of covariates can affect hazard estimation greatly, which makes reliability prediction difficult. PHM needs sufficient failure data for parameter estimations. In practice, failure data are not always available, and sometimes difficult to obtain due to quality improvement and design changes of equipment.

(4) Verification of models/methodologies

Another objective of the research is to verify the above models and methodologies using appropriate experimental analysis methods. The verification includes designing and conducting numerical simulation experiments and laboratory experiments, collecting real data from industry, as well as analysing experimental and industrial data. The data should include failure time, failure modes, working hours and condition of assets, corresponding parameters used for condition monitoring such as particles in oil and vibration signal. The configuration and properties of repaired assets also need to be identified.

The above proposal models realistic scenarios and deals with the identified limitations in current research. Objective (1) and Objective (2) focus on the reliability prediction of a repairable system with multiple PM intervals. Objective (1) concentrates on the reliability prediction of repairable systems with independent failures whereas Objective (2) on interactive failures. Objective (3) is about improving the reliability prediction of a system using both condition monitoring data and historical records, especially for sparse historical failure data.

1.2.2 Research Methods

To achieve these objectives, both theoretical modelling and experimental analysis were used. The entire research was divided into three stages. In Stage 1, multiple PM actions on a complex system were considered. However, the failures among components were assumed to be independent. In Stage 2, the model developed in Stage 1 was extended to the reliability prediction of a system with interactive failures. The models developed in the previous two stages assume adequate available failure data. In Stage 3, both condition monitoring data and failure data were used to improve the accuracy of prediction, especially when historical failure data were sparse. During these three stages of research, simulations, laboratory experiments and industrial case studies were conducted to verify the developed models and methodologies. More details about the research methods are presented as follows:

(1) Stage 1

The research in this stage is related to the first objective of the research program, i.e., to develop a new approach to predict the reliability of complex repairable systems with multiple PM actions. This approach is used to explicitly predict the reliability of a complex system after each PM action and the cumulative reliability of a system.

To achieve this goal, a Split System Approach (SSA) was developed based on Ebeling’s heuristic approach [16] and Reliability Block Diagram (RBD) [8, 17]. The basic concept of SSA is to separate repaired components from the unrepaired components of a system virtually when modelling the reliability of the system with PM. After the theoretical methodology was developed, Monte Carlo simulations and case studies, with real life data from industry, were used in its justification.

(2) Stage 2

In the first stage of the research, the failures of components were assumed to be independent of each other. As mentioned in Section 1.2.1, the assumption of independent failures is not always adequate for modelling the true state of a repairable system in practice. In the second stage of the research, the situations where the failures of certain components are not independent were investigated and an analytical reliability prediction model for repairable systems with interactive failures was developed.

The research methods used to achieve the goals of Stage 2 were as follows:

At first, the phenomena of interactive failures were comprehensively investigated. Considering the complexity of stochastic theory, Taylor's expansion approach was used to develop an Analytical Model for Interactive Failures (AMIF) from aspects of engineering application.

Secondly, a solution of AMIF was derived and the theorems for determining the conditions of stability for interactive failures were proposed and proved using the matrix theory, the limitation theory and the Principle of Mathematical Induction [18].

Thirdly, the properties of interactive failures and the effects of interactive failures on the reliability of components and systems without repairs were analysed based on the solutions of the model.

Fourthly, AMIF was combined with the Split System Approach (SSA) to predict the reliability of repairable systems with interactive failures and multiple PM actions.

Finally, the newly developed models and methodologies were verified using Monte Carlo simulation, laboratory experiments and case studies.

(3) Stage 3

In the third stage of the research, a new model was developed to predict dynamic failure trends of a system using condition monitoring data and historical maintenance data. This new model improved existing condition based hazard prediction models such as PHM.

In Stages 1 and 2, historical failure data were assumed to be sufficient for parameter estimations. However, in practice, failure data are not always available, and are sometimes difficult to obtain. Effective models are needed for this situation in order to predict failure time when historical failure information is not adequate for statistical analysis, where condition monitoring programs can be made available. Condition monitoring data describes the change in the condition of a system. While condition monitoring and diagnosis plays an important role in maintenance [14], the use of condition monitoring data to predict failure time is still a serious challenge. Little research has been done to date. There has been an attempt made to use PHM [4, 15]. However, as indicated in Section 1.2.1, the disadvantages in PHM affect the effectiveness of its application in industry. On the other hand, Al-Najjar [19] introduced a mechanistic model to predict the vibration level of rolling element bearings based on online vibration signals. This method can be used to improve an understanding of the deterioration process of a bearing although it only ensures a reasonable level of confidence for prediction over a very short time period.

The research methods in Stage 3 include a comprehensive investigation of PHM, development of a Proportional Covariate Model (PCM), justification of the reasonableness of the assumption used for developing PCM and investigation of the robustness of PCM in practical applications theoretically and experimentally. The advantages of Cox's PHM [4] and Al-Najjar's mechanistic model [19] were considered in the development of PCM.

(4) Validation of Methodologies and Models

The newly developed models/methodologies have been verified using both experimental data from numerical simulation and laboratory experiments, as well as the real life data from industry. The verification of the newly developed reliability models was mainly conducted using simulation experiment and maintenance data from industry. However, the data from industry cannot meet all needs of the model verification. Laboratory experiments have also been conducted using the mechanical test rig and corresponding condition monitoring measurement instruments in the School of Engineering Systems. This experimental system was available for the experiments on condition monitoring and on failure interactions among components.

In addition, some laboratory test failure data and condition monitoring data of gearboxes have been collected from the Aeronautical and Maritime Research Laboratory (AMRL), Australia and Condition Based Maintenance (CBM) Lab, Canada to enhance these evaluations.

The field data include the maintenance data of truck engines, the maintenance data of pipelines and failure data from pump stations. The Corporative Research Centre (CRC) on Integrated Engineering Asset Management (CIEAM) has provided partial funding to support the experiments and data collection phases for this project.

1.3 OUTCOMES OF THE RESEARCH

The research in this thesis explored two new research areas - the research on interactive failure and the reliability prediction of a system with zero failure data. The research composed mathematical modelling, theoretical analysis and the proof of theorems, as well as validation of the developed models using numerical simulation, laboratory experiments and life data from industry.

1.3.1 Research Results Achieved

The important contributions of the work in this thesis are as follows:

(1) Development of a Split System Approach (SSA)

SSA is linked to the first objective of the research program. SSA models the reliability of complex systems with multiple PM actions over multiple intervals using a new concept that splits a system into repaired and unrepaired two parts within a system virtually. It models system reliability at the component level and addresses different states of a repairable system after single or multiple PM actions such as “as good as new”, “imperfect repair”, “as bad as old” and “better than new”. A heuristic approach has been derived for the implementation of SSA. The formulae for special scenarios have been also derived.

(2) Development of an Analytical Model for Interactive Failures (AMIF)

AMIF is linked to the second objective of the research program. AMIF is used to analyse Interactive Failure (IntF) quantitatively. IntF is caused by the failure interactions among components in a system. The research introduced a series of new concepts and investigated the properties of IntF. The research indicated that IntF mainly depends on interactive relationship of components rather than the topology of a system. The Interactive Hazard (IntH) of a system can be calculated by its Independent Hazard (IndH) plus some portion of the IntHs of its influencing components. The degrees of the failure interactions among components are measured by interactive coefficients. IntF can be either stable or unstable. The conditions that IntF is stable have been identified.

(3) Extension of the above two models to the reliability prediction of repairable systems with interactive failures – development of Extended SSA (ESSA)

ESSA is also linked to the second objective of the research program. ESSA integrates AMIF with SSA to remove the assumption of independent failures which is adopted by SSA. The assumption of independent failures is unrealistic in numerous industrial cases and interactive failures need to be considered. When interactive failure exists, Interactive Hazards (IntHs) of repaired and unrepaired components after a PM action will change. The candidate has derived the formulae to calculate these changeable IntHs. An extension of the heuristic approach for SSA has been derived to model the reliability of a complex system with or without interactive failures after single or multiple PM intervals.

(4) Development of the Proportional Covariate Model (PCM)

PCM is linked to the third objective of the research program. PCM was developed to use both condition monitoring data (condition indicators) and historical failure data for hazard prediction. It models the covariates of a system as the product of baseline covariate function and the hazard function of the system. The procedure of PCM and the corresponding formulae were developed. The robustness of PCM was also addressed. The application of PCM for the hazard estimation of a system with zero failure data was demonstrated.

(5) Validated the newly developed methodologies and models using Monte Carlo simulation and the data collected from industries and laboratories.

This work included designing and implementing laboratory experiments, as well as collecting and handling life data. The statistical analyses conducted in this thesis were based on a 95% confidence level. However, for simplicity, when a parameter or a multi-dimensional parameter was estimated, the point estimation of the parameter [12], rather than a 95% confidence interval for this parameter, was presented.

1.3.2 Relationship of the Developed Models and Methodologies

SSA, AMIF, ESSA and PCM have been developed in this research.

SSA is a basic methodology that models system reliability at the component level and addresses different states of a repairable system after single or multiple PM intervals. The characteristics of the reliability of a system will alter after repairs. This alternation makes it difficult to predict the reliability of complex repairable systems, especially when the prediction covers a number of imperfect PM actions over multiple intervals. SSA was developed to redress this difficulty effectively. However, SSA was developed under the assumption of independent failures. This assumption is often unrealistic and may lead to unacceptable prediction errors although it was adopted by the most existing reliability prediction models and methods. To ensure the accuracy of reliability prediction, Interactive Failures (IntFs) need to be considered. AMIF incorporates failure interactions of components into reliability prediction models, but it does not consider the effect of repairs. ESSA integrates SSA and AMIF to the reliability prediction of systems with PM. SSA, AMIF and ESSA all need sufficient historical failure data to estimate the original Independent Failure Distribution Function (IndFDF) of a system. PCM improves the accuracy or enhances the capability of reliability prediction for these three models. PCM uses condition monitoring data to conduct reliability predictions with or without historical failure data and thus overcomes difficulties of reliability predictions when historical data are sparse or zero.

These new methodologies/models enhance the capability or improve the accuracy of reliability prediction of complex repairable systems. The methodologies and models developed in this thesis can be related based on the assumptions used and their applications.

1.4 ORIGINALITY AND INNOVATION

The two new approaches and two new models – SSA, AMIF, ESSA and PCM are the major contributions of this research. These new approaches/models are expected to enhance the capability and improve the accuracy of the reliability prediction of existing models for repairable systems significantly.

SSA was developed to predict the reliability of complex repairable systems, which can cover a number of PM actions using a new concept - to split a system into repaired and unrepaired parts within a system virtually. SSA provides more realistic and accurate prediction of reliability compare with the fixed deterioration rate model [20] and Ebeling's heuristic approach [16]. In SSA, the changes of reliability is calculated based on the individual system and repair condition rather than assumed or estimated by human's experience. Therefore, the rate of change is no longer constant.

Generally, SSA has the following major advantages:

(1) Ability to explicitly predict the reliability of a repairable system with multiple PM intervals over a long term and ability to decide when the system is unworthy of further PM from reliability aspects. SSA is more suitable for supporting a long term PM decision making of complex repairable systems in industry than the renewal process model and the Non Homogeneous Poisson Process (NHPP) model.

(2) Ability to deal with the individual contributions of different parts in a system and the influence of system structures on the reliability of a repairable system. This ability provides an understanding of PM on a system in more depth.

(3) Ability to model different states of a system after single or multiple PM actions such as "as good as new", "imperfect repair" and "as bad as old".

(4) No restrictions on the forms of failure distribution.

The research on SSA has resulted in the publication and submission of the following refereed international journal:

- Sun, Y., Ma, L., and Mathew, J., Reliability prediction of repairable systems for single component repair, Journal of Quality in Maintenance Engineering, in press.

- Sun, Y., Ma, L., Mathew, J., Morris, J. and Zhang, S., A practical model for reliability prediction of repairable systems, The Journal of Quality and Reliability Engineering International, submitted.

AMIF was developed to analyse interactive failures quantitatively. The research on interactive failures is a new area. Despite an intensive literature review, the candidate was not able to find any related research reported to date. The candidate has made the following original contributions:

(1) Introduced and defined related new concepts and terminologies such as interactive failure, influencing components, affected components and interactive coefficient for the analysis of interactive failure.

(2) Identified that interactive failure can be either stable or unstable. The candidate proposed and proved two theorems to justify stable interactive failures. These theorems effectively assist in analysing and avoiding potential unstable interactive relationship in machinery during its design phase. The research outcomes on stable and unstable interactive failures can benefit to designing more maintainable and reliable machines.

(3) Developed an analytical model for analysing interactive failure. Based on this model, the candidate derived a formula to calculate the failure distribution functions of systems with stable interactive failures and successfully investigated the effects of interactive failures on components and systems. The investigation results can be significant to improving risk management of assets with interactive failures.

The research on interactive failures has resulted in the publications of the following refereed international journal and conference papers:

- Sun, Y., Ma, L., Mathew, J., and Zhang, S., An analytical model for interactive failures, Reliability Engineering and System Safety, in press, available on ScienceDirect in May 2005.
- Sun, Y., Ma, L., Mathew, J. and Zhang, S., Experimental research on interactive failures, Proceedings of International Conference of Maintenance Societies, Sydney, Australia, 25-28 May 2004: p.04073.
- Sun, Y., Ma, L., and Mathew, J., On stable and unstable interactive failures, Proceedings of the 10^{th} Asia-Pacific Vibration Conference, ed. J. Mathew, Gold Coast, Australia, 12-14 November 2003: p.664-668.
- Sun, Y., Ma, L., and Mathew, J., A descriptive model for interactive failures, Proceedings of International Conference of Maintenance Societies, Perth, Australia, 20-23 May 2003: p.03-078.

ESSA integrates SSA and AMIF to the reliability prediction of systems. It is used to model the reliability of complex system with interactive failures after single or multiple PM intervals. The reliability prediction of repairable system with interactive failures is also a new research area. Unlike a system with independent failure, when IntF exists, the Interactive Hazards (IntHs) of both repaired and unrepaired components in a system will change. The candidate has derived the formulae to effectively calculate these changeable IntHs for a system after PM and demonstrated that ESSA enhanced the capability of SSA and AMIF.

The research on ESSA has resulted in the publication of the following refereed international journal paper:

- Sun, Y., Ma, L., Mathew, J., and Zhang, S., Determination of preventive maintenance lead time using hybrid analysis, International Journal of Plant Engineering and Management, 2005. 10(1), p13-18

PCM was developed to enhance the capability of SSA, AMIF and ESSA. It addresses the issue of reliability prediction using both failure data and condition monitoring data. The philosophy and procedure of PCM are different from existing condition-based models such as PHM. PCM predicts the hazard of a system using the covariates caused by the deterioration of a system and is therefore suitable for situations where symptoms of a system are monitored. PCM is shown to be more effective than existing condition based reliability prediction models when using condition monitoring data to predict the reliability of a system without historical failure data. It is also more effective than existing condition based reliability prediction model when using responsive covariates (symptom indicators) of a system to track the changes of hazard of the system.

The research on PCM has resulted in the publications of the following refereed international journal and conference papers:

- Sun, Y.; Ma, L., Mathew, J., Wang, W.Y., and Zhang, S., Mechanical systems hazard estimation using condition monitoring, Mechanical Systems and Signal Processing, in press, available on ScienceDirect in December 2004.

- Sun, Y., Ma, L., Mathew, J. and Zhang, S., Estimation of hazards of mechanical systems using on-line vibration data, Proceedings of International Conference on Intelligent Maintenance System, Arles, France, 15-17 July 2004: p.S3-B

- Zhang, S., Mathew, J., Ma, L., and Sun, Y., Best basis based intelligent machine fault diagnosis, Mechanical Systems and Signal Processing, 2005. 19: p357-370

- Sun, Y., Ma, L., and Mathew, J., Alarming limits for preventive maintenance using both hazard and reliability functions, Proceedings of the 10th Asia-Pacific Vibration Conference, ed. J. Mathew, Gold Coast, Australia, 12-14 November 2003: p.669-703.

- Sun, Y., Ma, L., and Mathew, J., Maintenance frameworks: A survey and new extension, Proceedings of International Conference of Maintenance Societies, Perth, Australia, 20-23 May 2003: p.03-077.

The new methodologies and models developed in this research are expected to enrich the knowledge of reliability engineering through effectively addressing some significant limitations of existing models and exploring the area of interactive failures. The research outcomes are of significance to the reliability prediction of repairable systems. The new methodologies and models developed in this research have been chosen for use in the Intelligent Maintenance Decision Support System for the Water Utility Industry and will become one of the unique features of this advanced software. The research on the Intelligent Maintenance Decision Support System for the Water Utility Industry is funded by the Australian Research Council (ARC) and supported by the CRC on Integrated Engineering Asset Management (CIEAM).

Due to the innovative and significant outcomes from this research, the candidate has received 2004 Student Award from the Maintenance Engineering Society of Australia. This national award is presented annually to only one student throughout Australia.

1.5 THE STRUCTURE OF THE THESIS

The entire thesis is mainly composed of nine chapters.

In Chapter 1, as it has been shown, the general information of the research is delivered. The topic and the scope of the research program are presented. The objectives of the research program and the methods used to achieve the research objectives are described. The outcomes of the research and the innovative contributions made by the candidate are identified.

The rest of this thesis is organised as follows:

In Chapter 2, a literature review is presented. The literature review includes two parts. At first, an overall survey on maintenance is carried out to identify possible research topics. Then an intensive literature review is conducted to focus on the research topic of this thesis.

In Chapter 3, the Split System Approach (SSA) is developed. The concept of SSA is presented. According to this new concept, different formulae and a heuristic approach for reliability prediction of a repairable system with PM are derived based on three different scenarios. An example with Monte Carlo simulations and a case study are used to demonstrate and verify SSA.

In Chapter 4, an Analytical Model for Interactive Failure (AMIF) is developed. The new concepts and terms related to IntF are defined. An analytical model - AMIF is derived to describe interactive failure. Two theorems to identify stable IntF are proposed and proved. The methods to calculate the IntFDF of systems with stable IntF based on AMIF are presented. Some properties of interactive failures are investigated. Four case studies are used to demonstrate and justify AMIF.

In Chapter 5, an Extended Split System Approach (ESSA) is developed. The ESSA integrates SSA with AMIF to predict the reliability of complex systems with interactive failures after single or multiple PM intervals. The method to calculate the changeable IntH of repaired and unrepaired components is presented. An example is used to demonstrate ESSA, and several Monte Carlo simulations are used to verify ESSA.

Chapter 6 focuses on the development of the Proportional Covariate Model (PCM). It contains two parts. The strategy of determining PM leading time using hazard function and reliability function is investigated in the first part because PCM is developed to estimate the hazard of a system rather than the reliability of a system directly. The PCM is developed in the second part. The concept and procedure of PCM are presented. The corresponding equations to estimate the baseline covariate function and hazard function are derived. The robustness of PCM is also addressed. Simulation experiments and two case studies are used to demonstrate and verify this model.

Chapter 7 is used to present laboratory experiments. The verification of the newly developed methodologies/models is mainly located in the last part of the above each chapter, just following the corresponding theoretical derivations and analysis. However, laboratory experiments are described in an independent chapter because they involved designing testing systems and were used for different verification purposes.

Chapter 8 presents the conclusions of the thesis while the directions for future research are briefly identified in Chapter 9.

The publications contributed by the candidate are listed in Appendix A.

Chapter 2

LITERATURE REVIEW

2.1 INTRODUCTION

Numerous papers on the topic of maintenance engineering have been published. However, the history of vigorous studies into maintenance is quite brief. Parkes [21] stated that maintenance has been with us longer than operational research - but despite this, maintenance has probably achieved less respectability than operational research. The earliest publication that the candidate found was published in 1952 [22].

Maintenance can be defined as the combination of all technical and associated administrative actions intended to retain an item or system in, or restore it to, a state in which it can perform its required function [23]. Commonly maintenance is categorized into four strategies: corrective, preventive, predictive and proactive ones [24, 25].

Corrective Maintenance (CM) strategy is the first generation of maintenance. The period of time is about 1940 to 1950. The strategy of corrective maintenance is to fix a system when it breaks.

Preventive Maintenance (PM) strategy is the second generation of maintenance. Its origins can be dated back to the 1960's. The strategy of preventive maintenance mainly consists of asset overhauls done at scheduled and fixed intervals based on time or duty. The main aims are higher plant availability, longer equipment life and lower costs.

Predictive Maintenance strategy belongs to the third generation of maintenance, which started in the mid 1970's. The aims of maintenance management became higher plant availability and reliability, greater safety, better product quality, longer equipment life and greater cost effectiveness.

Proactive Maintenance strategy aims much more at avoiding or reducing the consequences of failure than at preventing the failure themselves.

Wang [2] provided a survey of existing maintenance models in terms of maintenance policies. He classified maintenance policies of deterioration systems in the following categories: age replacement policy, random age replacement policy, block replacement policy, periodic preventive maintenance policy, failure limit policy, sequential preventive maintenance policy, repair cost limit policy, repair time limit policy, repair number counting policy, reference time policy, mixed age policy, preparedness maintenance policy, group maintenance policy, and opportunistic maintenance policy.

There are other classification schemes. Maintenance is widespread. It appears in almost all industries or assets, from steelworks [26] to power plant [27] to nuclear power plant [3, 28, 29], from software maintenance [30, 31] to hardware maintenance [32], from machines [33] to buildings [34-36], from offshore platform to bridges [37, 38], from railways [39, 40] to aircraft [41, 42] and the space shuttle [43].

The maintenance concept was first identified by Gits and Geraerds [44, 45]. It is concerned with implementing maintenance, training maintenance staff, integrating maintenance with enterprise management [46] and spare parts inventory [47-49]. It is also concerned with developing repairing materials and techniques [50, 51].

This survey will be conducted in terms of the research purpose of maintenance science, which can be categorized into three major classes: reliability assessment models and methodologies, maintenance optimization policies and maintenance frameworks. Maintenance optimization is the objective of maintenance while reliability prediction and risk assessment lays a basis for optimal maintenance decision making. Maintenance frameworks are concerned with applying these models, methodologies and policies effectively. Although there are numerous of publications on maintenance research, they can be classified into one of these three categories. Figure 2-1 shows an overview of the research on maintenance science.

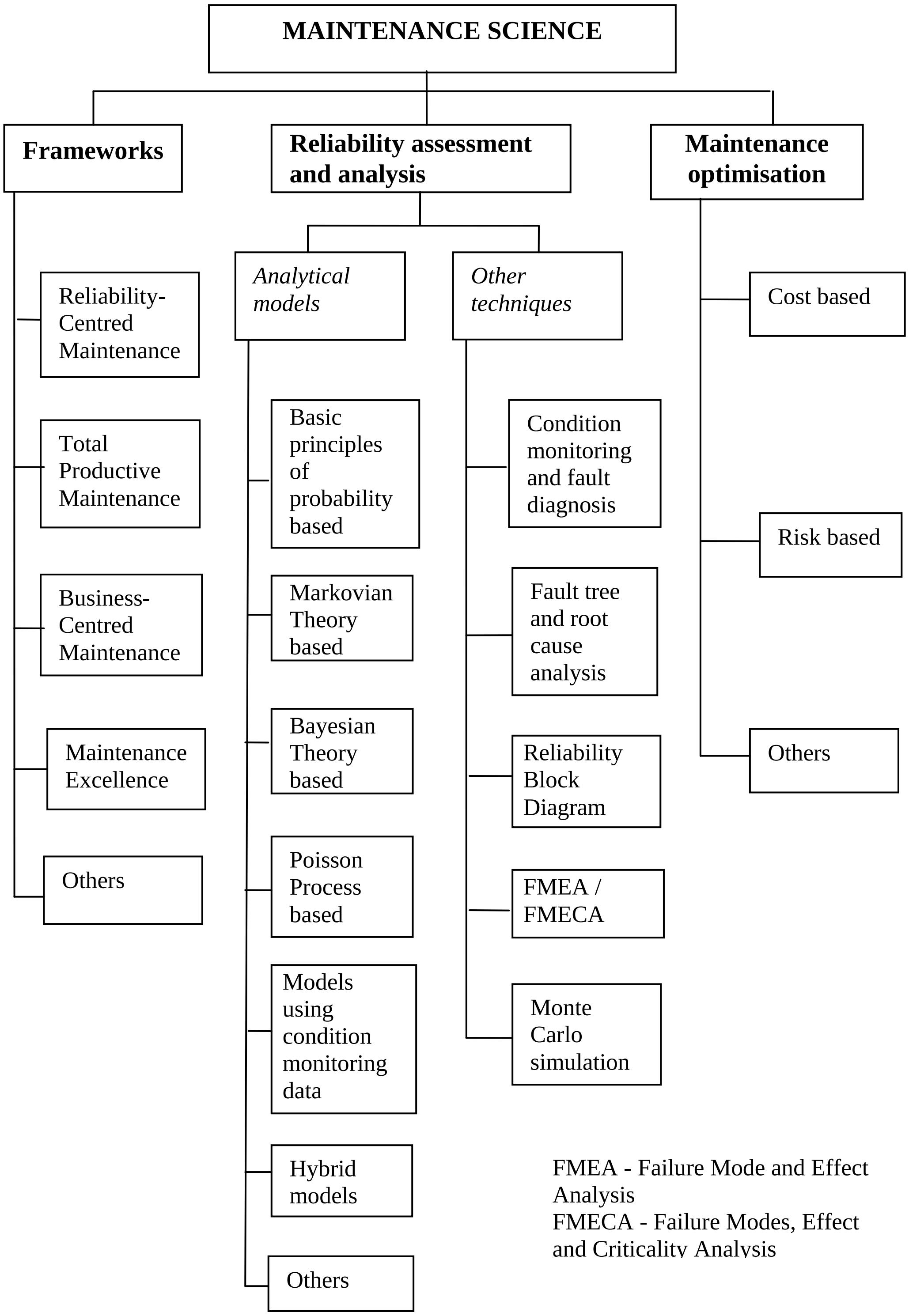

Figure 2-1. An overview of the research on maintenance

2.2 GENERAL REVIEW

2.2.1 Frameworks

A maintenance framework is a conceptual model or process guideline on how to conduct maintenance effectively through proper integration of various maintenance models and methodologies. This subsection summarizes, classifies, and compares the characteristics, general ideas and processes of different maintenance frameworks. The first four subsections discuss the most common used frameworks currently, i.e., Reliability-Centred Maintenance (RCM), Total Productive Maintenance (TPM), Business-Centred Maintenance (BCM) and Maintenance Excellence (ME). The subsection 2.2.1.5 provides a general survey of some other maintenance frameworks and new maintenance philosophies.

2.2.1.1 Reliability-Centred Maintenance (RCM)

The RCM [52-55] philosophy has been developed over a period of thirty years. The first industry involved in RCM was the international civil aviation industry [56] with MSG3 [25] framework. Moubray and his colleagues' pioneering work [57] resulted in the development of RCM2 for industries other than aviation in 1990.

The RCM process starts with significant functions and failure modes selection. It classifies the consequences of failure into four groups: hidden failure consequence, safety and environmental consequence, operational consequence and non-operational consequence. Maintenance decisions are made on the basis of these four categories so that the operational, environmental and safety, and cost effective objectives can be integrated. Figure 2-2 shows the basic structure of RCM [58].

2.2.1.2 Total Productive Maintenance (TPM)

TPM was initially developed in Japan and rose in popularity in the 1990's [59-61]. It is a strategy to maximize equipment effectiveness, to assure the life of equipment, to cover all departments and staff, and to improve maintenance through small group autonomous activities. Figure 2-3 shows an overview of TPM [61].

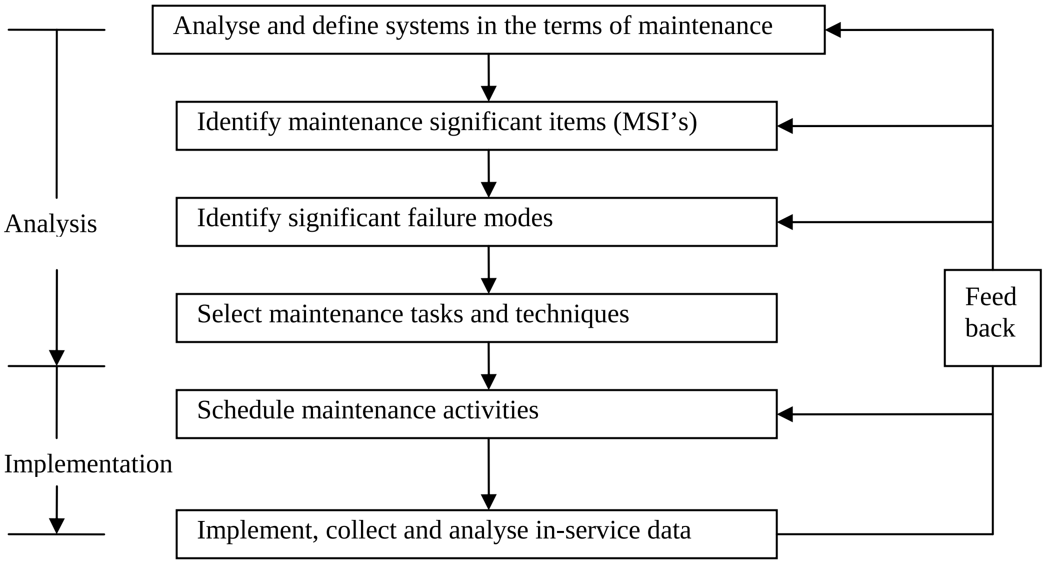

Figure 2-2. Structure of RCM (modified from: A. Kelly, Maintenance Strategy, 1997, Oxford: Butterworth-Hernemann, p. 220)

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Please consult the hardcopy thesis
available from the QUT Library

Figure 2-3. An overview of TPM (source: A. Kunio Shirose, TPM for Operators, 1992, Cambridge: Productivity Press, p.12)

2.2.1.3 Business-Centred Maintenance (BCM)

BCM was introduced by Kelly [58]. Unlike RCM and TPM, BCM is driven by the identification of the business objective, and then translated into maintenance objectives. Figure 2-4 shows the thought process of the BCM strategy [58].

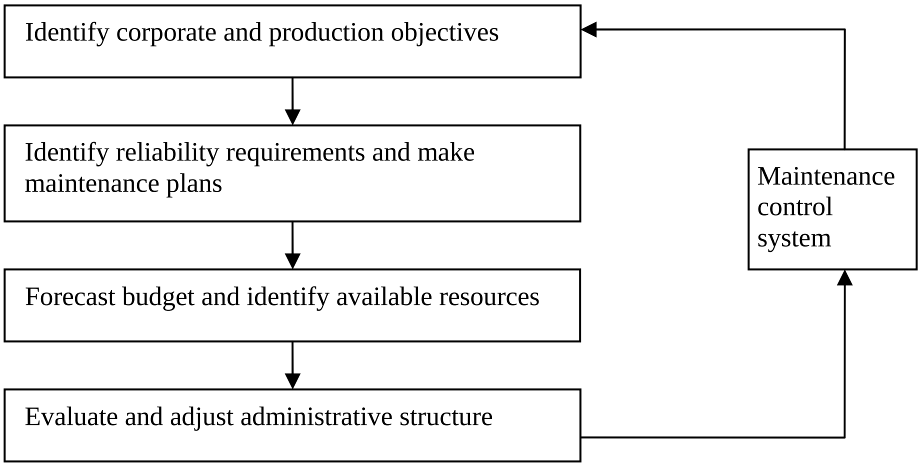

Figure 2-4. BCM strategy (modified from: A. Kelly, Maintenance Strategy, 1997, Oxford: Butterworth-Hernemann, p. 18)

2.2.1.4 Maintenance Excellence (ME)

ME was recently proposed by Campbell and Jardine [62]. In strict terms ME is not a new framework for maintenance. However, it does present some new ideas to conduct maintenance effectively. Figure 2-5 shows the implementation steps for ME [62].

At this point, it is worth introducing the holistic approach to the maintenance "problem" as proposed by Coetzee [63]. He pointed out that a typical approach towards increasing the efficiency of the maintenance function is to implement some highly publicised philosophy or maintenance techniques such as RCM, TPM, and BCM. Coetzee was of the opinion that these conventional frameworks were not effective due to lack of proper integration. The correct method of addressing the need for a very effective maintenance function in the organisation is to have a more integrated view of the maintenance function. The maintenance management process consists of two cycles. The outer cycle is a descriptive model. This model describes the overall managerial planning and measurement process. The inner cycle is a descriptive model. This model describes the maintenance plan and the maintenance operation itself (Figure 2-6) [63, 64].

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Figure 2-5. Steps to implement ME (source: J.D. Campbell and A.K.S. Jardine, Maintenance Excellence, 2001, New York: Marcel Dekker, p.369)

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available from the QUT Library

Figure 2-6. Coetzee's maintenance cycle model (source: J.L. Coetzee, *A holistic approach to the maintenance "problem"*. J. Quality in Maint. Eng., 1999. 5(3): p. 276-280)

RCM, TPM, and CBM are all aimed at the inner cycle of the maintenance cycle and will thus not produce the results envisaged. TPM is a philosophy addressing the total complexity but it has had limited success in the western world due to a difference of managerial outlook. The only solution is to apply a variety of techniques to a small part of the organisation instead of applying one technique over the total organisation, to touch and to take a cross-section of all the critical parts of the maintenance organisation simultaneously.

Coetzee [63] pointed out that a maintenance policy must consider the operation, the procedure, the workforce, hence, a down-top-down requirements' analysis would be more suitable. However, he does not address where the maintenance (management) policy comes from. In addition, there feedback from the inner cycle to the outer cycle is not apparent in Coetzee's maintenance framework.

It is not easy to determine a suitable maintenance strategy for a specific problem. Martorell and his colleagues [65] optimized maintenance by comparing effectiveness and efficiency in technical specifications and maintenance. Starr [66] identified that corrective maintenance (CM) is at best only suited to non-critical areas whereas Jardine [67] furthermore indicated that CM may be an appropriate strategy when the hazard rate is constant. Al-Najjar and Alsyouf [68] indicated that the most important criteria are (i) possibility to model the time to failure, or monitor damage initiation and its development and (ii) the cost effectiveness of CM. Preventive maintenance (PM) is best suitable for failures with a clear wear-out characteristic. Time-based preventive maintenance is performed on a scheduled basis with scheduled intervals, which are often based on experience or manufacture's recommendations [67, 69]. Statistics-based preventive maintenance is more advanced [70-72]. Valdez-Flores and Feldman [72] reviewed the preventive maintenance models for single-unit systems whereas Cho and Parlar [70] for multi-unit systems. Matched and composite components which are always renewed together can be treated as a single item with a combined distribution [73]. Swanson [74] applied the exploratory factor analysis to determine whether RCM, TPM or CBM can explain a specific practice.

Although RCM, TPM, BCM and ME are currently very common and have found wide applications in industry, generally speaking, all of them seem too complex for industrial applications.

2.2.1.5 Other frameworks

Some effort has gone into enhancing these common frameworks [29, 55, 75, 76]. New philosophies to enrich these frameworks have also been developed in recent years as itemised below:

- A framework for maintenance concept development [77];
- An optimal inspection and diagnosis policy for a multi-mode system[78];
- Availability Centred Maintenance (ACM) [79];
- A synchronous Quality Function Deployment (QFD) over the world wide web [80];
- A double critical age policies model applied to make age repair policies for the machine repair problem of m identical machines serviced by n identical technicians, $n < m$ [81];
- A method to study scheduling problems involving repair and maintenance rate-modifying activities with objective functions such as expected make-span, total expected completion time, maximum expected lateness, and expected maximum lateness, respectively [82].

Integration of maintenance is a necessary method to improve maintenance frameworks. A variety of automated inspection and maintenance integration systems, usually combined with condition monitoring and fault diagnosis or automated manufacturing system, have been developed [83-85]. The research on integration of maintenance includes:

- The knowledge based process monitoring system [86-88];
- The integration of predictive maintenance in manufacturing systems [33];
- The synergy of combined technologies for pipeline evaluation [89];

- Computer Aided Design (CAD)-integrated reliability evaluation and calculation for automotive systems [90];
- An integrated approach linking the Charles Kepner and Benjamin Tregoe methodologies (K-T) [91];
- The integration of Total Quality Management (TQM) with Root Cause Analysis (RCA) to TPM [92];
- The establishment of relationships between implementation of TQM, Just In Time (JIT) and TPM and manufacturing performance [93];
- The stopping time optimisation in condition monitoring with expert judgements involved [94];
- The integrated system which can deal with the analysis of deterioration due to corrosion, finite element analysis of load, on the repair scheme with a cost estimate, condition monitoring and audible warnings [95].

2.2.2 Reliability Assessment and Analysis

In order to reduce maintenance costs and to optimize a maintenance strategy, it is necessary to understand reliability and its variations, the consequences of failures, the factors affecting maintenance and the relationship between the maintenance tasks and production or other performance of assets to be maintained [96]. Reliability is the ability of a system to perform a required function under stated conditions for a given period of time [8]. It is usually measured by determining the probability that a system survives in a time interval (0, t]. The most direct expression to describe the properties of reliabilities of systems is the reliability function $R(t)$. The reliability function is also called as survivor function [4]. Another mathematically equivalent way of specifying the reliability of systems is in terms of failure distribution function $F(t)$ or failure density function $f(t)$. “Failure” in this thesis means that a system or a component fails to meet its performance requirement. This “failure will naturally lead to a need for maintenance.

The models and techniques for reliability assessment and analysis can be classified into two categories: The mathematical models and the conceptual models and techniques. This subsection summarizes, classifies, and discusses the characteristics, advantages and disadvantages of various models, techniques as well as methodologies of the conceptual reliability models and techniques; whereas the mathematical models for reliability analysis will be presented in Section 2.3.

2.2.2.1 Condition Monitoring and Fault Diagnosis (CMFD)

CMFD has been playing an increasing role in maintenance research [14] so that a new term - condition based maintenance (CBM) is now used. CBM is currently the best preventive maintenance strategy because it enables maintenance decisions to be made based on the current status of the equipment, thus avoiding unnecessary maintenance and thus facilitating timely maintenance when there is a strong indication of impending failure [97].

Condition monitoring is popular and has a wide range of applications. In techniques, CMFD are concerned with vibration detection, lubricants analysis, infra-red scanner, ultrasonic-pulse echo technique in data processing, with Fast Fourier Transform (FFT), Discrete Wavelet Transform (DWT), demodulation, debris counting, data fusion, image processing, etc, and in measurements, with vibration, wearing debris, acoustic emission, temperature, strain, torque, power. New methodologies or philosophies continue to emerge. For instance, Chanda et al's [98] wavelet multi-resolution analysis for location of faults on transmission lines and the knowledge-based diagnosis used in a case study on rolling bearing of a pump [99]. The US Navy is currently developing a new ship structural health monitoring system based on fibre optic technology [100]. The determination of the best sensor positions is one of the main research goals in the field of CMFD [85, 101]. Roberts, et al [40] demonstrated that the distributed method of fault diagnosis can reduce the cost of maintenance through a railway junction case study. Image processing techniques were used for identifying frequency regions which have a high discriminative power between the different classes, or Regions Of Interest (ROI) [102]. Recently it was reported that infrared thermograph is an appropriate method to identify the condition of railway track ballast [103], and a Ground Penetrating Radar (GPR) can be thought of as a suitable and economical alternative to the other methods [38, 104]. The internal condition of a line can be assessed by a combination analysis of its dynamic response and temperature and pressure readings [105].

It should be noted that cost-effective and reliable damage detection is critical for the utilization of monitoring techniques. For example, non-destructive evaluation techniques (e.g. ultrasound, radiography, infra-red imaging) are available for use to composite materials during standard repair and maintenance cycles. However by comparison with the techniques used for metals these are relatively expensive and time consuming [106].

2.2.2.2 Fault tree and root cause analysis

Root cause analysis is used to find out causes of failures [25, 107]. The classic technique is Fault Tree Analysis (FTA). A related technique is Event Tree Analysis (ETA) [108-112]. “A fault tree is a model that graphically and logically represents the various combinations of possible events, both fault and normal, occurring in a system that leads to the top event.” [111] FTA was introduced at Bell Telephone Laboratories in 1961 [113] and was used in the aerospace industry in the early 1960’s. It can be used for qualitative analysis, quantitative analysis or both. FTA enables one to find the most likely causes of system failure, but it is costly and time consuming. This method will also fail to identify some important causes and effects. It is difficult to apply Boolean logic to describe failures of items that can be partially successful in operation and thereby have effects on the performance of the system. It is also difficult to have pertinent failure rate data to conduct quantitative fault tree evaluation. Classic FTA describes the effects of failures at lower levels on those at upper levels. It does not model the effects of failures at upper levels inversely on those at lower levels and the effects among the same levels. Some new applications are found in [114, 115].

2.2.2.3 Reliability Block Diagram (RBD)

The method of RBD, also called as Success Diagram Method (SDM), was the first method used for analysing system and assessing reliability in the history of reliability research [116]. RBD is a logic network used to describe the function of a system. For a system with multiple functions, different RBD might be established. In most cases, a fault tree can be converted into a RBD, and vice versa. Generally, fault tree is more suitable for root cause analysis and RBD is more suitable for quantitative analysis. When used for qualitative analysis, RBD can be used to identify whether a system is in a functioning state or in a failed state under a given conditions. The state of a system is often described by the structure function of the system. The structure function is a binary function. When used for quantitative analysis, RBD can be used to calculate exact system reliability at a given time t. Many methodologies have been developed to analyse and calculate RBD [8, 116, 117]. RBD is a powerful tool for reliability calculations. However, when RBD is used to calculate the reliability of a system, the reliability function of each individual component in this system must be known and these components are assumed to be independent [8].

2.2.2.4 Failure Modes, Effect and Criticality Analysis (FMECA)

The FMECA is a combination of Failure Mode and Effect Analysis (FMEA) and criticality analysis [118, 119]. The basic task of FMEA is to identify and list the modes of failures and the consequences [120, 121]. FMEA is very important in the application of RCM [25].

Criticality analysis is generally used to evaluate the severity of harmful effects of a failure on the function and operation of a system, on other components, on the environment, and more importantly on mankind so that the most suitable maintenance policies can be made [53]. Starr [66] defined the term Plant Criticality to determine areas which are likely to be cost effective in terms of safety, capital value and the value of production.

The knowledge of historical failure and plant criticality is required before CBM can be applied. Three popular techniques are used to assess the criticality in CBM: FTA, FMECA and RCM [122]. These three techniques have become popular because they can be used to detect a range of failures in a machine by vibration, thermal and lubricant analysis [123].

El-Haram and Saranga [124, 125] used identification of the Maintenance Significant

Items (MSIs) to do similar work. They believed only MSIs would be considered for a Relevant Condition Parameter (RCP) based maintenance.

Another important concept is the Maintenance Tasks Priorities (MTP). A example to assign priorities for maintenance can be found in [126]. FMECA can be utilised to decide MTP [127]. Gopalakrishnan et al [128] have noticed this problem too. They used a Multi-Logit Regression Model (MLRM) [129, 130] to decide MTP. The maintenance tasks for the current time-bucket are rescheduled to maximize PM effectiveness subject to workforce availability and to yield an adaptive and effective PM schedule for each time-bucket. In Gopalakrishnan's model, the following five factors were considered: Cumulative machine utilization; Current machine utilization; PM delay; Comparative machine failure rate associated with the PM task, and severity of the last repair action. MTP of a task is assumed to be proportional to its expected contribution to PM effectiveness.

Both the Markov analysis [131] and Linear Multivariate Discriminant Analysis (LMDA) [130] are also available for the calculation of the expected contribution to PM effectiveness.

Recently, Hokstad, etc. [132] presented an approach to relate the risk of an activity to so-called Risk Influencing Factors (RIFs), in which, the overall picture of the factors at all levels can be easily found and quantitatively analysed.

FMECA can be used to determine the modes of failures and their effects on system operation and to discover potential critical failure areas. It is performed using the system's functional tree. It includes three elements: (1) Failure mode analysis: to study a system and the working relationship of components under various anticipated conditions of operation; (2) Failure effect analysis: to study the potential failure in any section of the system; (3) Failure criticality analysis: to study and determine the severity of each failure in terms of probable safety hazard, unacceptable deterioration in the performance of the system [133]. However, classical FMEA or FMECA is difficult to conduct even for relatively straightforward systems.

2.2.2.5 Monte Carlo methods

Monte Carlo methods are based on random simulation. It was said that the earliest documented application of Monte Carlo method is that of Comte de Buffon in 1777 [134]. Monte Carlo methods are possible to be used to solve the reliability prediction problems that cannot be solved analytically. With increasing computing speed and memory size of computers, Monte Carlo methods have received more attention from maintenance researchers. Some applications in reliability and maintenance analysis can be found in [17, 135, 136]. However, efficient Monte Carlo algorithms are often difficult to develop.

2.2.3 Maintenance Optimization Policies

The optimization of maintenance decision-making is defined as an attempt to resolve the conflicts of a decision situation in such a way that the variables under the control of the decision-maker take their best possible value [20, 62, 137]. This subsection reviews maintenance optimization policies in three classes: cost based optimal policy, risk based policy and combined optimal policy.

2.2.3.1 Cost based optimal policy

Whatever maintenance strategy is chosen, its goal is to minimize overall cost. Cost based optimal policy is aimed at reducing the costs related to the maintenance activities.

The calculation of overall cost and benefit of PM is still a big challenge to scientists and engineers. The typical techniques include optimal maintenance costs based on failure prediction and life-cycle cost analysis [58, 138]. Figure 2-7 shows an example of life cycle cost profile [58].

Lean Maintenance (LM) is also a popular strategy. It emphasises efficient maintenance management in order to reduce waste in maintenance activities [139, 140]. This policy does not analyse the problems quantitatively. Therefore, it is unknown if a LM based policy is optimal or not.

Starr [66] formalised a structured approach to the selection of condition based maintenance. In his formalised procedure, major factors were taken into account. However, he only provided a general direction (or basic rules). He also reviewed a method to calculate the production losses due to unexpected failure. This method simply uses the value of production at a normal rate to multiply the potential hours of downtime. Actually, even though this method is adopted, the time of stoppage for repair or replacement should be reduced from the potential time of downtime.

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available from the QUT Library

Figure 2-7. A life cycle cost profile (source: A. Kelly, Maintenance Strategy, 1997, Oxford: Butterworth-Hernemann, p. 9)

Today more and more attention is paid to the maintenance optimizations when two or more factors are taken into account [141-145]. For systems that are not normally in continuous operation, the maintenance should be scheduled or planned to be done when the system is idle. It is more cost-effective to do the inspection in an opportunity (i.e., the system should stop) than the conventional PM, in which the system stops for the purpose of inspection [146]. The management of maintenance in a large plant involves numerous factors. Sherwin [147] proposed eight important rules and assumptions for practical optimal maintenance and presented a formula to calculate the age-optimised residual value. The costs of failure and PM of each failure mode (or combined PM operation considered as a separate, independent and indivisible event) can be estimated according to Glasser and Sherwin [146, 148].

Artana and Ishida [20] presented a method for determining the optimum maintenance schedule for components in the wear-out phase. The interval between maintenance for the components is optimized by minimizing total cost. The total cost consists of maintenance cost, operational cost, downtime cost and penalty cost. Nakanishi and Nakayasu [149] proposed a new expected total cost concept including initial cost, cost of reliability test, annual maintenance cost, penalty cost for designer's faults and losses by structural failure to make reliability design of structural system with cost effectiveness during its life cycle.

Tadashi, et al [150, 151] derived an optimal model for the order quantity and safety stock so as to minimize the expected cost per unit time in the steady-state under somewhat different restrictive assumptions from the model by Cheung and Hausman [150]. A case study shows that Lagrangian relaxation method can be applied to find an optimal solution for the net benefit of pipe repair maintenance in water distribution networks [152]. Jardine et al [67] applied PHM to optimize PM cost based on the change of covariates.

Delay Time Analysis (DTA) is also an important tool to model maintenance decision problems. The delay-time concept was introduced by Christer [153]. "Attention of DTA is focused upon the maintenance engineering decisions of what to do, as opposed to the logistical decisions of how to do it." [60] The delay-time concept regards failure propagation as a two-stage process. It is assumed that a component can be in one of three states: non-defective, defective and failed. The sojourn in the defective state is called the delay-time. Wang and Christer [154, 155] presented three solution algorithms for an established multi-component inspection system model. This model is based upon the delay time concept and used to solve the multiple-decision problem with a possible large number of decision variables depending upon the number of inspections. Earlier papers related to inspection maintenance based on the delay-time model are based on either the classical approach or the combined classical Bayesian approach, and are mainly concerned with saying something about presumed true parameters, like average costs per unit time and failure rates. However, often relevant objective data ("hard data") is typically not sufficient in practice (It is even true today due to the short renewal period of equipment).

Some researchers considered the change of a system after maintenance and introduced an imperfect maintenance concept which deals with the economic production problems with imperfect production processes under assumption that the age of the system is reduced in proportion to the PM level [156-158].

2.2.3.2 Risk based optimal policy

Although generally it is a common goal to minimize the costs in industry, in some cases more attention may be placed on increasing reliability whenever a failure will cause a disaster consequence to the human being or environment. In these cases, a criticality based optimal policy should be used. Little research has been conducted specifically on this policy. Some related research can be found in [43, 96, 131, 159-162].

2.2.3.3 Combined optimal policy

The cost related to the maintenance activities should be carefully considered even though under criticality based policy. Some combined optimal policies have been developed for an overall maintenance optimization through a comprehensive consideration of several different factors such as costs, reliability requirements, and availability.

The Relative Condition Parameter (RCP)-based maintenance policy is a combined optimal policy. RCP-based maintenance was proposed by Knezevic [163]. El-Haram and Saranga [69, 124, 125] have further developed this policy in recent years. The model requires that a minimum required level of system reliability must be maintained when optimizing maintenance costs.

RCP-based maintenance does not deal directly with the nature of the failure mechanisms like wear and fatigue crack, but instead depends on the sophistication of condition monitoring devices to take these factors into account. Under RCP-based policy, Maintenance Significant Items (MSIs) must be identified. Only these MSIs will be considered for maintenance. The Relevant Condition Predictor (RCP) is a key factor in the RCP-based maintenance. RCP is a condition parameter to describe and quantify the direct condition of the item at every instant of operating time. If a

RCP is not available for a particular MSI, then RCP-based maintenance is not applicable to that particular item. Once RCP s are determined for all the MSIs, suitable condition monitoring techniques are selected, in order to monitor the condition of the item. The same idea was put forward by Starr [66]. In general, RCP is directly related to the shape, geometry, weight and other characteristics of the item. The basic principle behind this mathematical implementation is the assumption that as long as the RCP lays within the prescribed limits RCP^{in} and $RCP^{\lim}$, the item or system will function satisfactorily. RCP^{in} and $RCP^{\lim}$ are set by the manufacturers. Once RCP exceeds these two limits, a failure occurs. The principle can be represented in the following equation:

$$R(T_i^1) = P(RCP_i^{in} < RCP_i(T_i^1) < RCP_i^{\lim}) = R_i^r \,. \tag{2-1}$$

where, RCP_i^{in} is initial value of relevant condition parameter for ith item; $RCP_i^{\lim}$ is the limit value of relevant condition parameter for ith item; T_i^1 is the time to the first examination of ith item, which is defined as the time up to which the required probability of reliable operation is maintained; R_i^r is the minimum required level of the item. For a system connected in series, the time to the first inspection should be the shortest one in all first inspection time of all items, that is

$$T_s^1 = \min_{i=1,2,\ldots n}(T_i^1)\,, \tag{2-2}$$

where, T_s^1 is the time to the first examination of the system.

RCP_i^{cr} is the critical value of the relevant condition predictor RCP_i. If RCP_i is above RCP_i^{cr}, maintenance tasks should be performed. RCP_i^{cr} exists objectively, while $RCP_i^{\lim}$ is set by people. The difference between RCP_i^{cr} and $RCP_i^{\lim}$ represents the length of time during which the major maintenance preparation activities can be conducted. RCP-based maintenance was claimed to be able to reduce the maintenance costs because it shortens the duration of maintenance task by the prior condition information, and reduces the duration of support task by the proper selection of $RCP^{\lim}$. The cost benefits of RCP-based maintenance can be summarized in following six characteristics:

(1) Reduction in maintenance induced failures;

(2) Reduction in planned / scheduled maintenance;

(3) Reduction in repair time and costs;

(4) Elimination of unexpected failures;

(5) Increase in the realisable operating life of components;

(6) Increase in the coefficient of life utilisation, which is the ratio of the average realisable operating life to its expected operating life.

Neither El-Haram nor Saranga considered the effects of different MSI on the maintenance plan. They failed to match the different numerical value of RCPs with different monitoring techniques. The assumptions that production is in continuous operation and the cost of lost production and the revenue are directly proportional to the length of time are questionable. Comparing RCP-based maintenance policy with RCM, it can be identified that this policy actually corresponds to the RCM framework.

Other policies include Jiang and Ji's [164] multi-attributes model which considered four attributes: cost, availability, reliability and lifetime when making an optimal age replacement policy, and Stewart's [165] applications of risk ranking and life-cycle cost analysis to assess the reliability of a bridge. Strouvalis, et al [166] applied an accelerated Branch-and-Bound algorithm for assignment problems of utility systems to find out the appropriate sequence of switching off turbines and boilers for preventive maintenance, which contributes to the reliability, availability and profitability of the entire system.

2.2.4 Advanced Tools and Methodologies

Some maintenance research uses advanced tools and methodologies which have found wide applications in other fields such as fuzzy logic [167-169], neural network [170, 171], the Kalman filter [172], the genetic algorithm [173, 174], data fusion [175], Monte Carlo [176] or combination of those techniques [177]. The application of data fusion techniques in maintenance is attractive, because there is an increasing demand for the accuracy of prediction and decision.

Using computer techniques to enhance maintenance analysis ability is another attractive respect of maintenance research. The computer was used to study maintenance problems as early as in 1963 [178]. In 1974, the British Steel Corporation (BSC) [179] started using computers to manage maintenance. However, only in recent decades, have some commercial practical software for maintenance become available[180]. Software packages such as EXAKT [67] and RELCODE [181] are programmed to determine the failure model and to carry out maintenance optimization. Relax (Relax software corporation) and Reliability Workbench integrate the performance of reliability prediction, maintainability prediction, FMECA, RBD analysis, FTA, ETA and Markov analysis [182]. There are other software which is used for management of human competencies [183], or simulating the deterioration system using Monte Carlo simulation [176], or enhancing the efficient exchange of relevant information [184], or taking advantage of the Internet [185].

2.2.5 Comments and Discussion

The models and methods mentioned above have found their applications in maintenance. However, they have fallen short of finding practical applications

Dekker [144] conducted a literature survey on the real world applications of current models in industries. He found a total of 112 applications of maintenance optimization models. Most of them were used between 1985 and 1989 (45 cases). Strangely enough, there were only 25 cases found after 1990, and indicates that current maintenance optimization models cannot meet the demands of today's industry.

There is a lack of effective methodology to analyse the relationship between a failure and its root causes quantitatively, especially when reliability information is incomplete, e.g., new equipment.

Improper maintenance activities such as repeatedly deferred inspections or repairs result in very costly failure. On the other hand, too often inspections or unnecessary monitoring may also cause high cost. One needs to estimate the states of a system more accurately. Current maintenance models including PHM, FMECA and FTA usually do not specify which items fail. However, the real situation is, more often, that a system fails because some and not all items fail. One therefore may not need to repair the entire system or all of items in the system. In order to carry out actions particular to business goals, one needs to get information which is perception, or recognition and localization, of structures. It involves the spatial-temporal form of components and their relationships [186, 187].

It is a challenge to scientists to develop an appropriate model which can take account of historical failure records, monitoring data and other available information to enhance the accuracy of predictions.

Historical records are valuable, but they are often incomplete and inaccurate. The records normally contain the activities of maintenance rather than the causes of failures. They may have erroneous records [25]. On the other hand, condition monitoring is more expensive and in many cases the monitoring techniques may not be available. Hence new approaches and models are needed to overcome these limitations.

As a result of the above discussion, future research directions are identified as follows:

(1) New methodologies and models need to be developed which can bridge the gap between theoretical research and industry applications. Most of reliability models have been developed for mathematical purpose or computational convenience [144], rather than solutions to real industry problems. Most case based research focus on short term solutions and lack vision on whole life cycle modelling.

(2) A number of topics for complex repairable systems are still in their infancy and need further research, such as, investigating dynamic component-system relationship, releasing the assumption of "as good as new", and predicting multiple failures of whole life.

(3) Models dealing with very small set of data or zero failure data need to be developed more intensively.

(4) The accuracy of reliability prediction needs to be improved. Reliability prediction of systems and maintenance decisions making should be based on comprehensive considerations of current conditions of a system together with historical maintenance/failure records and other information.

(5) Little attention has been paid to integrated spare parts inventory management, which is important especially to asset intensive industries.

(6) The integration of maintenance, monitoring and production is a major issue and needs to be addressed.

2.3 SPECIFIC REVIEW – ANALYTICAL MODELS

A repairable system is usually defined as one which will be repaired to recover its functions after each failure rather than to be discarded during continuous operation [188]. A complex system usually means that it is composed of multi-components which can be connected with each other in either series or parallel or in a complex way. This review is concerned with classifications and characteristics of analytical reliability prediction models of repairable systems. Some major limitations in these models will be identified.

2.3.1 Basic Principles of Probability

Several models for the reliability prediction of a repairable system have been developed using the basic principles of probability. The time-dependent maintenance model mentioned in [189] is an example. According to this model, a system is always replaced at a fixed time T or failure, whichever happens first [2]. The models based on basic probability principles were developed to determine the most appropriate preventive maintaining time T according to the reliability function or failure distribution function of the system. The most common distribution function in use is the Weibull distribution due to its ability to fit a greater variety of data and life characteristics by changing its shape parameter [20, 190]. Normal distribution and exponential distribution [191] are two popular models as well. In some early research, time-dependent maintenance model often assumed that a unit is replaced at its age T or failure, where T is a constant, so it used to be called the age replacement model [192]. Later a block replacement model was developed. Under this model, a unit is replaced at a fixed prearranged time which is also a constant irrespective of the age of the unit, but if the unit fails before the prearranged replacement time, an in-service replacement will be made [2, 193]. If the unit is not replaced but maintained, the block replacement model becomes the periodic preventive maintenance model. Considering the failure rate of a unit generally increases over time and the system often cannot become “as good as new” after repair, the constant fixed maintenance time T is replaced by a time variable T_i, $T_i < T_{i-1}$, and then the periodic preventive maintenance model becomes the sequential preventive model which was introduced by Nguyen and Murthy [194]. Some research has been made to extend this model to a complex repairable system [8, 81, 193, 195-199]. The time-dependent maintenance model was originally developed for the single unit system. Fontenot and Proschan [200] developed several imperfect maintenance models. In each of theses models, they assumed that the state of a system after a planned replacement is as good as new, and the state after an unplanned maintenance have two possibilities: as good as new with probability p and as bad as old with probability $1-p$. Gurov and Utkin [199] presented a model to predict reliability of repairable systems with periodic modifications by arbitrarily distributed times to failure and repair. The application of this model in industry is difficult because the model is represented by the integral equations.

The renewal process model is a generalized classical model. It assumes that whenever a component fails, it is replaced by a new identical one or repaired to the condition of “as good as new” [8]. Mathematically, “a renewal process is defined as a sequence of independent, identically distributed (i.i.d.) non-negative random variables X_1, X_2, …, which with probability 1 are not all zero" [201]. The renewal model basically deals with the renewal function that is defined as the expectation of the random variable $N(t)$ (the number of failures during the time interval (0, t] for fixed time t).

The reliability of repairable standby systems attracts much attention [202]. Narmada and Jacob [203] studied 1-out-of-2 system whereas Dey and Sarmah [204] 1-out-of-N and Wang and Ke [205] W-out-of-W+M.

Due to the inherent difficulty in mathematics, the models were often developed about some special cases, i.e., either system with special structure [206-209] or special process [210-214] or both [215]. Calabria and Pulcini [210] derived the conditional intensity functions introduced by Lawless and Thiagarajah [216] under the assumptions of the Power Law-Weibull Renewal (PL-WR) process and the Log Linear-Weibull Renewal (LL-WR) process separately. When $\beta = 1$ and $\delta = 1$, the PL-WR process reduces to the Homogenous Poisson Process (HPP). When $\beta = 0$ and $\delta = 1$, the LL-WR process reduces to HPP.

Although the research on the classical maintenance model can date back to as early as 1958 [2, 217], this model still attracts the attention of researchers [8, 189]. Significant effort has been made to improve this model such as extend it to a system composed of multiple units and subsystems [8, 81, 193, 195-198]. Models based on the basic principles of probability can cover a wider range of situations. However, some of these models are too mathematical to interpret and to apply. It is still a difficult task to obtain the reliability function for Time Based Preventive Maintenance (TBPM) especially when historical data is sparse. Research activities on the reliability prediction for Reliability Based Preventive Maintenance (RBPM) are scarce.

2.3.2 Markovian Theory

In 1907, the Russian mathematician A.A. Markov introduced a special type of stochastic process whose future probability behaviour is uniquely determined by its present state, that is, with behaviour of non-hereditary or memory-less. The behaviour of a variety of physical systems falls into this category; hence, the Markov model plays an important role in the reliability evaluation of engineering systems [218]. A Markovian stochastic process with a discrete state space and discrete time space is referred to as a Markov chain. If the time (index parameter) space is continuous, it is referred as the Markov process.

The model based on the Markov process assumes that a system has a finite state space and a series of possible transitions between these states. The functions, various failure modes, standby and various maintenance activities all can be described as different states. If the transition between the states can be approximately described by a stochastic process with Markov property, the Markov method can be used to determine the reliability of the system after several states. Therefore, it is fairly common using Markovian theory to model the reliability prediction problem of a repairable system [219-227].

Pham, et al [228] presented a Markov process based model for predicting the reliability of multi-stage degraded systems with partial repairs. Aven [222] used the standard Markov theory to derive an availability formulae for standby systems of similar units that are preventively maintained. Tan [229] used the Markov chain to study the reliability of 1-out-of-2 systems, and Pham [230] extend to K-out-of-N systems. Chen and Trivedi [231] derived a closed-form solution of the underlying Markov chain for the minimal and major maintenance model whereas El-Damcese [232] tried to solve Markov equation for reliability prediction more effectively. Sophie Bloch-Mercier [233] tried to find the degree of the repair of a Markov deteriorating system such that the long-run availability was optimal. She dealt with corrective rather than preventive maintenance. Wang and Sheu [234] used a Markov chain to determine the optimal production maintenance policy with inspection errors, which is an improvement to Lee and Park's method [235].

Sometimes an ordinary Markov process cannot describe a repairable system very well, and hence a semi-Markov process is chosen to model the reliability of a repairable system [236, 237]. A semi-Markov process is an extension of an ordinary Markov process with discrete states and continuous time [236]. Papazoglou [237] derived several approximate equivalent Markov models to decompose a system of dimensionality $N+M$ into two smaller problems of dimensionality N and M. Kim [238] used semi-Markov to reliability modelling of a hard real-time system using the path-space approach. For considering realistic timeframes and for repairable systems in industries, Marquez and Hegueda [1] proposed a model to represent different corrective and/or preventive actions that could take place at different moments, driving the equipment to different states with different hazard rates by the utilization of semi-Markovian probabilistic models.

Markovian method has often been applied to model repairable systems[224, 233, 239] and deteriorating systems [8, 240]. However, it is not easy to find all (sometimes they are numerous.) transition probabilities. The state space method is only suitable for relative small systems and for the prediction of the next failure [1, 8]. Although the Markov model has been used to study problems of a repairable system after repair, it is used under very strict assumptions. For example, the system evolves in time according to the same Markov process as from the beginning [233, 239] or the system has a very special structure with several subsystems in series, each of those subsystem consisting of several parallel identical components [241]. In addition, the Markov equations are often difficult to solve analytically. Some systems do not conform to the Markovian system [242].

2.3.3 Poisson Process

The Poisson point process is a kind of Markov process [8]. This model assumes that the failures are independent of each other and the number of failures in each time interval follows a Poisson distribution [243]. The Homogeneous Poisson Process (HPP) model requires stationary increments whereas a Non-Homogeneous Poisson Process (NHPP) model [4, 188] does not require these increments. Therefore, the NHPP is more favourable for modelling imperfect repairable systems [244]. The NHPP can also be used to study the Rates of OCcurrence Of Failures (ROCOF) when they are time dependent, and the times between failures are neither independent nor identically distributed [243]. Some researchers [245] argued that multi-component repairable systems cannot be modelled by continuous distributions. Failures occurring in repairable systems should be considered as a series of discrete events which occur randomly in a continuum. These situations behave as stochastic point processes and can be analysed by means of the statistics of event series. The log-linear NHPP model and the power law NHPP model are recognized as two widely used models for repairable systems. The power law NHPP model is based on Weibull distribution. It is given by

$$v(t) = \lambda \beta t^{\beta - 1}, \tag{2-3}$$

where, $v(t)$ is the intensity function. λ is the constant failure rate. β is shape parameter and t is the system's age.

One of applications of the power law NHPP was given by Weckman, Shell and Marvel [244] to the reliability modelling of repairable systems in the aviation industry. Coetzee [246] reviewed the NHPP models in the practical analysis of failure data up to 1996 briefly. Guida and Giorgio [247] analysed the reliability of accelerated life-test data from a single-item repairable system moulded by a NHPP. Pulcini [248] applied the NHPP to model the reliability of a complex repairable system with bathtub type failure intensity. Saldanha et al [243] presented a application example to the reliability analysis of service water pumps whereas Bustamante [249] to a software reliability model.

The Poisson process based models are suitable for analysing repairable systems with multi-failures which are stochastic point processes. However, the existing Poisson process based models are only available to the random failure mode but does not appear to subscribe increasing hazard rate. The Poisson process based model assumes that the failure probability of a system follows a Poisson distribution, the number of the failures does not affect the failure probability and the repair does not change the reliability of the system [250]. NHPP model assumes that the reliability immediately after a repair is exactly the same as reliability just before its corresponding failure. It is only suitable for so-called "minimum repair" activities but not general repair.

2.3.4 Condition Monitoring Data Based Models

With increasing applications of condition monitoring techniques, maintenance personnel naturally wish to improve reliability prediction accuracy using monitoring data. The Proportional Hazard Model (PHM) introduced by Cox [4] is currently the most popular condition based model [1, 3-6, 15, 251-254]. Another similar model is Proportional Intensities Model (PIM) [188, 250, 255]. PHM is more flexible and avoids some of the problems related with PIM, but the latter has a clearer mathematical and physical justification [255]. Before the concept of PHM is introduced, the terms reliability function and hazard function are defined mathematically as follows.

The reliability function $R(t)$ is used to decide the distribution of random variable T of a homogeneous population of individuals, each having a "failure time". It is defined as the probability that a system (component) will function over a period of time t [16]:

$$R(t) = P(T \geq t)\,. \tag{2-4}$$

$$R(t) = \int_t^{\infty} f(t)dt\,, \tag{2-5}$$

where $f(t)$ is the failure density function. $P(\bullet)$ is the probability of $(\bullet)$.

On the other hand, the hazard function $h(t)$ is defined as [16]:

$$h(t) = \lim_{\Delta t \to 0} \frac{p(t \leq T < t + \Delta t \mid t \geq T)}{\Delta t}\,. \tag{2-6}$$

Considering Equations (2-4) and (2-5), Equation (2-6) becomes

$$h(t) = \frac{f(t)}{R(t)}\,. \tag{2-7}$$

PHM is used to estimate the hazard of a system based on historical failure data and condition monitoring data [4]. It was developed essentially from Accelerated Life Models (ALM) [256]. In principle, PHM is also a model based on statistical analysis method.

The advantage of PHM is that it includes both the age of a system and its condition in the calculation of the hazard of the system at time t. In this model, the hazard at time t of a system is modelled as a product of the baseline hazard function $h_0(t)$ and a positive function term $\psi(Z,\gamma)$ as follows [4]:

$$h(t) = h_0(t)\psi(Z,\gamma). \tag{2-8}$$

The baseline hazard $h_0(t)$ is the hazard without influence of the covariates. The functional term $\psi(Z,\gamma)$ is dependent on the effects of the different factors that affect the failure of the system through a row vector consisting of the covariates Z and a column vector γ of the weighting parameters. The Maximum Likelihood Estimation (MLE) method is commonly applied to estimate these weighting parameters.

Makis and Jardine [6, 67, 257] studied the problem of optimal replacement using PHM. They defined an optimal replacement rule based on both minimal expected average cost per unit time and the PHM of a system, and then used the values of covariates of the deterioration system to determine the replacement time. Later, Jardine and Banjevic [15] presented an application of this method for optimizing a mine haul truck wheel motor. Kobbacy et al [253] also developed a heuristic approach to scheduling the next PM interval using the semi-parametric PHM and the full condition history of a system. Ansell and Phillips [258] presented a general survey of some practical aspects of using PHM to model repairable systems.

PHM is empirical in nature. Cox [4] summarized seven criteria to assess distributional form, these criteria can help the comparison of those existing distribution models. In order to start the parameter estimation procedure in modelling, at least two histories ending with failure are required, and in addition at least one history ending with failure for each covariate of interest. However, the number of histories is hardly specified since it strongly depends on how covariate information is correlated with failure. This means this technique can only be used in situations where such equipment has run some length of time, and has enough failure records. It is definitely unsuitable for new equipment. The parameters of a PHM based hazard model are estimated according to the historical records. When estimating these parameters, the conditions of current system are not considered. If this PHM based model is used to analyse the hazard of current system (even if the same system as that when the historical records for modelling were taken), the results would be far from accurate because the system may have experienced several different repairs since those historical records were taken. Sometimes, regular maintenance activities such as changing oil may be investigated when a PHM is constructed, but mainly for meeting the requirements of cleaning the historical data to get correct transition path [15]. The effects and the influences of such maintenance work have not been estimated, and hence this PHM based hazard model is not suitable for predicting or optimising these maintenance activities. According to Roberts and Mann [245], classical PHM, as a continuous distribution, cannot be applied for the reliability prediction of a multi-components repairable system in a long-run period. Kumar and Westberg [259] used a linear regression model to find out that the time-invariant assumption of the effect of a covariate in PHM is incorrect. Blischke and Murthy [12] and Ebeling [16] described PHM as an environmental condition based model, but some researchers [257, 260] argued that PHM could be used for both environmental (external) covariates and responsive (internal) covariates.

In condition monitoring and fault diagnosis of a physical asset, often several parameters (termed as covariates in reliability theory) that measure the conditions of the asset are monitored and analysed. As such, several different PHM based models can be formulated by choosing different covariates or combinations of these covariates. For example, Lin [261] used six inspection variables for the condition monitoring of a single reduction helical gearbox to build PHM based models. Six PHM based models are reproduced as follows:

$$h_1(t) = \frac{5.51844}{10319}\left(\frac{t}{10319}\right)^{4.51844} e^{0.388431 FGP1}, \tag{2-9}$$

$$h_2(t) = \frac{1}{2.79213} e^{1.17955 FGP1 - 5.34302 RFM}, \tag{2-10}$$

$$h_3(t) = \frac{4.49062}{56160.6}\left(\frac{t}{56160.6}\right)^{3.49062} e^{2.64606 RFM}, \tag{2-11}$$

$$h_4(t) = \frac{1}{1841840} e^{0.113776RFS}, \tag{2-12}$$

$$h_5(t) = \frac{1}{199259} e^{22.8414RTM}, \tag{2-13}$$

$$h_6(t) = \frac{9.32064}{14929.6}\left(\frac{t}{14929.6}\right)^{8.32064} e^{69.3561RTS}. \tag{2-14}$$

In the above equations, FGP1, RFM, RFS, RTM and RTS are the names of covariates.

The hazard values of the system calculated to these equations can be significantly different. To demonstrate this point of view, part of the data generated through Lin's study [261] was used to conduct a hazard analysis. The original data is reproduced in Appendix B1. Figure 2-8 shows the hazard of the system calculated by Equations (2-9) to (2-14).

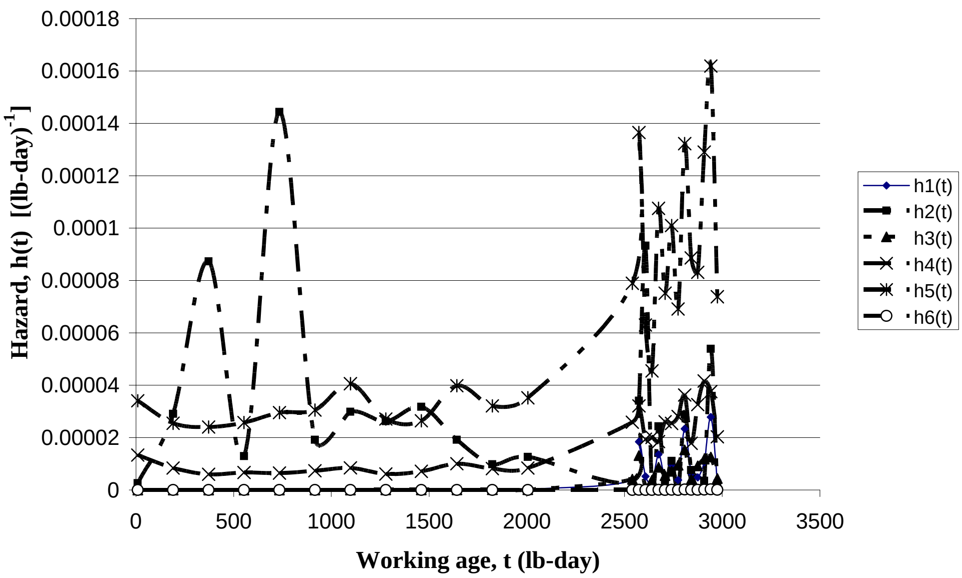


Figure 2-8. The calculated hazards of the system

Figure 2-9 shows the trendlines of the hazard curves in Figure 2-8 in form of the third order polynomials. From these two figures, it can be seen that significant differences among the hazard lines exist. The selection of the most appropriate PHM based model is still a challenge. The optimisation of maintenance costs is currently most used criterion for the selection [15, 261]. In the candidate's view, the first criterion should be the accuracy of the models to represent and predict the hazards of assets rather than optimization of maintenance cost.

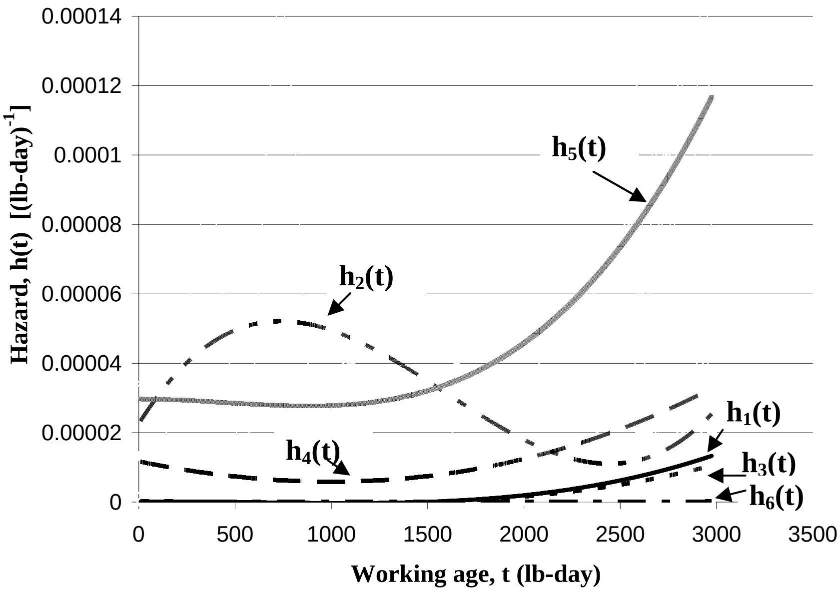


Figure 2-9. Trend lines of the hazard curves in Figure 2-8

In addition, Figure 2-8 indicates clearly that each hazard line fluctuates greatly because the original test data were contaminated by an amount of random noise. The fluctuations in condition monitoring data have significant influence on PHM.

New reliability prediction models using condition monitoring data have also been developed. Al-Najjar [19] developed a mechanistic model to predict the vibration level of rolling element bearings which in turn can be used to assess the conditions of these bearings. Barbera, et al [208] presented a classic RBD based model for a two-unit series system. In this model, a continuous variable (X_{ti}) is adopted to describe the condition of each unit i (i=1, 2) at time t. Condition monitoring data can be used to predict reliability of a system if the probability of failure is given by the exponential distribution and the hazard ($\lambda(X)$) is proportional to the condition. Faber and Sorensen [262] developed a Bayesian formulation of condition indicators for inspection and maintenance planning of concrete structure. These indicators have two states: indicating a defect or not indicating a defect.

2.3.5 Bayesian Theory

The Bayesian model is based on Bayesian theorem which was introduced by Reverend Thomas Bayes in 1763, which can be described as following equation [8]:

$$P(B_k \mid A) = \frac{P(A \mid B_k) P(B_k)}{\sum_{i=1}^{\infty} P(A \mid B_i) P(B_i)}, \tag{2-15}$$

where, $P(B_k \mid A)$ is the conditional probability that event B_k occurs at the occurrence of event A. $P(A \mid B_k)$ and $P(A \mid B_i)$ are the conditional probabilities that event A occurs at the occurrence of event B_k and B_i, respectively. $P(B_k)$ and $P(B_i)$ are the probabilities of event B_k and event B_i occur, respectively.

The Bayesian model allows using the knowledge of designers, operators and maintenance engineers to reduce the uncertainties when modelling the reliability of a system. An observed value is used to update the priori (the prior density) of the Bayesian model. Significant work has been conducted using this model [7, 263-266]. As early as in 1973, Bassin [267] developed a Bayesian block replacement model for a Weibull restoration process under the assumption that repair costs are known. Mazzuchi and Soyer [193] extended this model to the traditional age replacement policy and the block replacement policy with minimal repair under the assumption that repair cost is constant and the scale parameter α and shape parameter β are initially independent. Considering the repair cost for system failures may be random and unknown, Shue, et al. [265] developed an adaptive replacement model using Bayesian approach under the assumption that the hazard $r_i(t)$ of a system is strictly increasing, i.e., $r_{r+1}(t) \geq r_i(t)$ but $r_{i+1}(0) = r_i(0)$. Sheu also applied a Bayesian approach on age replacement with minimal repair when the failure density is Weibull [268].

Percy et al [9, 255, 263] researched the Bayesian approach to enhance preventive maintenance strategic decisions. Apeland [269] tried to use the fully subjective, or fully Bayesian approach to make maintenance decision when objective data are insufficient. However, in Apeland's model, some assumptions are not realistic: (1) Each component has one failure mode; (2) Occurrence of failures and defects related to different types of components are independent; (3) All failures are immediately detected and the corresponding failure components are replaced immediately; (4) The failure components are always replaced by identical new components.

Nootwijk etc [266] extended a Bayesian model to study the structural deterioration problem under the assumption that the amounts of deterioration are exchangeable and isotropic. For small amounts of deterioration, the prior density is evaluated numerically, and for big amounts the inverted gamma distribution is chosen as a good approximation.

The Bayesian model allows adopting the knowledge of designers, operators and maintenance engineers to reduce the uncertainties and using the observed data to update the priori. However, the Bayesian model is not suitable to model reliability function by itself because the Bayesian method is commonly used to update a prior distribution [264]. The prior distribution is difficult to choose. It is complex and difficult for long term prediction [263]. Most of the existing Bayesian models need failure data to update the priori, which might not be available.

2.3.6 Hybrid Models

Naturally, researchers have tried to combine above models, such as combining a Bayesian method with Poisson process [264], combining a Bayesian method with the Markov process [266], combining a Bayesian method with the Weibull distribution [265], combining a Poisson process with PHM [250, 270], combining a Bayesian method with the TARMA (Time-dependent Auto-regressive Moving Average) [7], combining a Bayesian method with Poisson process and PIM [255].

Kawauchi and Rausand [271] proposed a new approach based on two modelling methods: Markov modelling and a rule-based method, and Kumar and Westberg [272] used PHM and the Total Time on Test (TTT) plot to make maintenance scheduling under age replacement policy. The TTT-plots have also been used for condition monitoring of rolling element bearings [273].

Hassett, et al [274] derived a hybrid reliability availability model combing time varying hazard which is characterized by a general polynomial expression and Markov chain analysis. Tractable solutions were found for the 1-component 2-state and the 2-component 4 state configurations.

Gue and Love [250] presented an age model which is based on the non-homogeneous Poisson assumption but combined with a proportional intensities assumption. This model did not regard the reliability of a system as unchangeable but treat the form of intensity function and its parameters' values as unalterable. This model introduces a scalar parameter to reflect the improvement of a system after a repair. This scalar parameter must be estimated by a maintenance engineer. For complicate system, it is too difficult if not impossible to do for an engineer in industry even if he/she is very experienced.

Hybrid models provide a possible direction. However, up to now, a generalized hybrid model has not been derived. Some hybrid models are also very difficult to use.

2.3.7 Other Models

Some reliability prediction models specific for imperfect repaired repairable systems have also been developed. These models often have very restrict and unrealistic assumptions. For example, the fixed decreasing rate model simply assumes that a system after maintenance is subject to a fixed decrease in the reliability index [20]. The proportional reliability deterioration model uses a failure rate deterioration factor (<1) multiplying the original reliability function to describe the system state of somewhere between as good as new and as bad as old after a repair [241, 275]. The failure rate deterioration factor is purely defined by maintenance staff members. On the other hand, Dieulle [276] gave an analytic method for calculating the reliability function, its Laplace transform and the Mean Time To Failure (MTTF). His model allows consideration of an imperfect restoration and even the case where an inspection damages the system. He assumed that restoration time is negligible. Grall, et al [277] established an analytical model using both replacement threshold and inspection schedule as decision variables for the maintenance problem of a condition-based inspection/replacement policy for a stochastically and continuously deteriorating single-unit system. They proposed using a multi-level control-limit rule to implement the maintenance policy.

Most existing models or methodologies have been developed on the assumption that failures among components are independent. However, industrial experiences have shown that the assumption of independent failures has been unrealistic in numerous scenarios and has led to unacceptable analysis errors. Therefore, the concept of dependent failures was introduced, for example as described in Mosleh [10], Hoyland and Rausand [8].

The subject of dependent failures has attracted the interest of researchers for decades. The international journal, Reliability Engineering & System Safety published a special issue on dependent failures in 1991. The most discussed dependent failures are: cascading failure, negative dependency failure and common cause failure [8, 278]. Cascading failure is defined as multiple sequential failures. These failures are initiated by the failure of one component, which leads to sequential failures of other components. Negative dependency failure is defined as failure that can prevent other components in a system from further failing. Common cause failure is defined as multiple related events caused by a single common cause. Cascading failure and negative dependency failure are often analysed using approaches for independent failures such as FTA, RBD and the Markov chain [8]. Greig [279] presented a second moment (covariance) method for estimating the reliability of a system with both common cause and cascading dependency failures. In his study, a component failure changes the system topology, which consequently increases the failure probabilities of remaining components. His case study can fall into the classical definitions of cascading failures. The majority of existing research on dependent failures focuses on common failures [278, 280-284]. Papers in the special journal issue mentioned above mainly concentrated on this type of dependent failure. FMEA and FTA have been extended for the analysis of common cause failures [278]. Mosleh [280] presented a framework for identification, modelling and quantification of common cause failures. Findlay and Harrison [281] identified major common failure modes for an aircraft. Murthy and Nguyen studied different operating policies under the condition that the failure of a component in a system may induce the failure of all other components in the system [11, 12]. Lewis presented a Markovian approach to analysing load-sharing systems [13]. Some methods for analysing common cause failures quantitatively have been developed, such as the square root model [285], β-factor model [286] and Binomial Failure Rate (BFR) model [287].

However, some failures cannot be classified as independent failures nor as a type of the above three dependent failures. One such scenario is Sequential Failure Logic (SFL) [288]. In this scenario, n-cause failures occur in a sequence of $x_1, x_2, \cdots x_n$. A system fails, if and only if these n cause failures occur. The second scenario is the failures due to associate variables, i.e., the state variables of a system are dependent [8]. These scenarios need further research and lie outside of the scope of this thesis.

Another such scenario is that failures of some components can interact with each other. For example, failure of Component A will cause or accelerate the failure of Component B and vice versa. The failure interaction will increase the failure rates (hazards) of both components. In some cases, the increase of failure rates of components due to failure interaction can be significant and cause disastrous consequence. Estimating the failure probability of components subject to failure interaction is imperative. A model or technique used to analyse this failure probability quantitatively and effectively is still unavailable although the term failure interaction has been used in some literature such as [9, 11, 12].

2.3.8 Comments

An intensive literature review has been conducted on the analytical reliability models. Some further literature review specific on repairable systems and condition based reliability prediction models are presented in the following chapters. The literature review indicates that analytical models for reliability were mainly developed based on stochastic process and probability theory. However, analytical reliability models were also empirically developed based on experience or experiments, or derived from failure mechanism [16]. In existing models, the renewal process and minimal repairs are still two basic assumptions [204, 206, 207, 211, 238] although more and more attention has been paid on imperfect repairs in recent years [289]. Pham [290] reviewed several optimal imperfect maintenance models and indicated future research directions on imperfect maintenance. However, he concentrated on maintenance activities rather than reliability prediction.

The literature review indicated that existing models have the following limitations:

(1) Models to calculate the changes of the reliability of a system after imperfect PM actions are inadequate. For example, the imperfect maintenance models presented by Fontenot and Proschan [200] assumed that the state of a system after a planned replacement is as good as new, and the state after an unplanned maintenance have two possibilities only - as good as new with probability p and as bad as old with probability $1-p$.

(2) When analysing the reliability of a repairable system, existing models often consider the entire system rather than the individual contributions of different components of the system to the reliability of the system [1, 8, 15, 81, 266].

(3) Most existing models consider the time to the next failure, MTTF or/and the expected number of failures during a given period. Models for explicitly predicting the changes of the reliability of an asset covering a series of imperfect PM actions need to be developed although Ebeling [16] has presented a heuristic approach for such purpose. Ebeling's approach was developed based on the assumption that a system after a PM action becomes as good as new. This approach was also presented by Lewis [13]. Under the same assumption, Ramakumar [218] modelled the changes of the failure density functions of components with periodic preventive maintenance using the similar approach.

(4) The interactions among failures of components in a system have not been

modelled adequately. Existing models for dependent failures consider single direction effects of failures or some special systems such as a load-sharing system. An effective model for analysing the failures due to continuos interaction among components is yet to be developed.

(5) Inadequacy exists for making reliability predictions given sparse or zero failure data. Some existing models dealing with sparse failure data have been developed based on the Bayesian method [9, 263, 291, 292]. These models need failure data to update posterior distribution function without using condition data [9, 292]. Yet other models have been developed from the failure mechanism of specific assets but these are specific in nature [16, 293].

(6) Systematic consideration of the reliability of repairable systems with all the above aspects such as multiple imperfect repairs, interactive failures and sparse historical failure data is lacking.

(7) Some models are simply theoretical formulations with no real application focus [284].

Chapter 3

RELIABILITY PREDICTION OF SYSTEMS WITH PREVENTIVE MAINTENANCE

3.1 INTRODUCTION

Today, Preventive Maintenance (PM) is often conducted in industries to reduce the probability of unexpected breakdown of assets during a certain period. An asset can be subject to multiple PM actions over its operational life-span. Many companies develop their PM strategies at the stage of acquisition of assets. Observation from industries has revealed that different PM activities can have different effects on the reliability of assets. If PM is conducted at the right time and in the correct way, it can improve the reliability characteristics of assets. Otherwise, PM may not have an effect on the reliability of assets or even worse - decrease the reliability of assets. The majority of physical assets in industries such as machines, buildings and vehicles are repairable. Hence, there is a need to investigate the effects of PM on the reliability of repairable systems comprehensively. This chapter focuses on developing a reliability prediction methodology to quantitatively assess the effectiveness of a PM strategy on the reliability improvement of a complex system, and thus support optimal PM decision making. A particular concern of the research is to explicitly predict the reducing amount of probability of failure of a system over a certain period due to PM, compared with the probability of failure without PM. In this thesis, maintenance includes repair and replacement. From now on, when “repair” is mentioned, it usually indicates maintenance and includes “replacement”.

A complex system is normally composed of several components. These components can have different life cycles - a fact that leads to the result that different components may have different failure patterns and distributions at the same time. The conduct of PM of a system usually comprises PM on individual components in the system according to the states of their conditions. Accurate estimation of the effects of PM of these components on the reliability of systems is essential to the optimal decision making of PM strategy. However, a practical methodology or analytical model for this issue is still not available.

As indicated in Chapter 2, The issues associated with repairable systems have attracted much attention of researchers [1, 8, 81, 188, 250, 263, 266, 294]. The research about repairable systems is focused on two aspects: reliability predictions of repairable systems and the optimal maintenance policy for repairable systems. Different models have been developed to address the reliability prediction of a repairable system with PM. These models have been applied in different scenarios. However, the following three major limitations have affected the effectiveness of these existing models to the reliability prediction of complex systems with PM.

The first limitation is that the different states of repairable systems after multiple repairs have not been adequately modelled. Two common approaches are to assume that a repairable system after repairs becomes “as good as new” [81, 239, 244] or “as bad as old” [8]. Some literature assumed that a system after repairs evolves in time according to the same Markov process as from the beginning [233, 239]. These assumptions are unrealistic in a considerable number of cases. The applications of these models are limited. For example, existing NHPP based models [4, 188] assume that repairs do not change the reliability of a system [250]. These models are only suitable for “minimum repair”. Often a system after a PM action is not as good as new, neither as bad as old, which brings out the concept of imperfect repair. Imperfect repairs are common in industries. Imperfect repairs include the following scenarios (for more details, see [295, 296]).

The first scenario is that the reliability of a system after a repair does not restore to the value of one. This type of imperfect repair occurs when the repaired components may not to function as required just after a repair. This type of imperfect repair can also occur when only some of components in a system are repaired. If some unrepaired components have also failed, the system may not function after a repair even though the repaired components may all work perfectly after this repair.

The second scenario is that the reliability of a system after a repair restores to the value of one, but the system deteriorates faster than before, i.e. the hazard of the system after a repair becomes greater.

The third scenario is a mixture of the above two scenarios.

To date, effective modelling techniques to deal with the reliability prediction of a system with multiple imperfect repairs have yet to be developed [5] although some researchers have noticed the influence of imperfect repairs on the reliability of a system [1-4, 250].

Some models consider the influence of imperfect repairs on the reliability of a repairable system, but have limited applications due to assumptions or methods used in the models. For example, to describe deterioration of reliability of components and systems after repairs, Artana [20] multiplied the original reliability index by a decrease percentage (<100%). Nguyen and Murthy [194] assumed that the failure rate of a system increases with the number of repairs. Monga [275] assumed that the reliability of a system decreased proportionally with repair times which was represented through a scale parameter called failure rate deterioration factor. Later, Monga [241] introduced another time variable parameter to describe the different start points of hazard function of a system after different repairs. Gue and Love [250] introduced a scalar parameter to reflect the degree of improvement of a system after repairs similar to Monga's approach. Their model was based on the non-homogeneous Poisson framework with a proportional intensities assumption. This model treated the form and parameters of intensity function of a repairable system as inalterable. In these models, all parameters or factors employed to describe the changes of reliability function of a system after repairs must be estimated by maintenance engineers (or users). For complicated systems, accurate estimation of these parameters or factors is difficult, if not impossible, even for experienced personnel.

The second limitation is that existing models often treat a repairable system as a "black box", without considering the individual contributions of different components to the reliability of this system [8]. These models often take the entire system into account and do not analyse reliability of repairable systems at component level. As a result, some important information which can assist in improving the accuracy of reliability prediction has been omitted. The following Nelson-Aalen plot can be used to illustrate this argument.

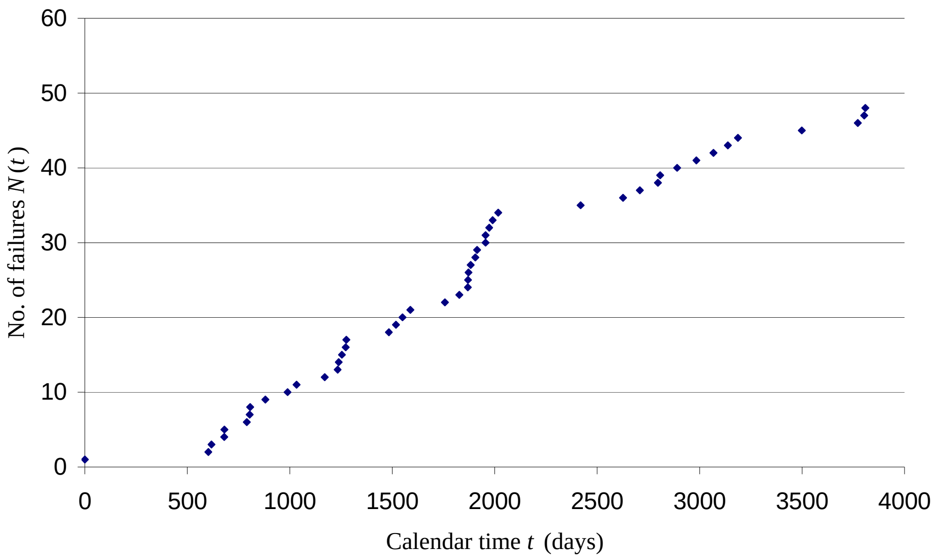


Figure 3-1. Number of failures *N*(*t*) as a function of age of a pump system

The data presented in Figure 3-1 are the times of failure of a pump system over nearly 10 years. From this plot, it can be seen that the Rate of OCcurrence Of Failures (ROCOF) of the pump system can be approximated as constant. However, the determination of a suitable model to analyse these data is very difficult if the pump system is treated as a “black box” because some failure properties can only be identified at the component level. For example, analysis indicated that the 5^{th} failure and the 7^{th} failure were related because they shared the same root cause. In this case, the assumption of independence is not valid. In addition, most of the repairs for these failures were not minimal repairs and this indicates that the NHPP model is not suitable.

The third limitation is that most existing models have been developed based on probability theory and stochastic process as the failure time of an asset is a random variable. These models are often very complex [9], rendering difficulties in engineering applications. These models are normally developed to predict and optimise the next repair event [6, 7] or analyse MTTF or/and the expected number of failures of an assets during a given period [8, 9] rather than explicit reliability changes with multiple PM actions. In contrast, Ebeling [16] presented a heuristic method to predict the reliability of an asset with multiple PM intervals. In this method, PM time is a deterministic variable. This method can produce an intuitive and explicit prediction of reliability and hence is well suited for engineering applications. However, in this model, assets are assumed to have PM actions periodically and the states of the assets after PM activities are assumed "as good as new".

In this chapter, a Split System Approach (SSA) is developed to extend Ebeling's method for a long term prediction that covers a number of imperfect PM intervals during an asset's life time, and attempts to overcome the three limitations mentioned previously. Two types of PM policies are considered. One is the Time Based Preventive Maintenance (TBPM). In this policy, the system is maintained based on scheduled PM times. The intervals between two PM actions may or may not be the same. The other is the Reliability Based Preventive Maintenance (RBPM). In this policy, a control limit of reliability R_0 is defined in advance. Whenever the reliability of a system falls to this predefined control limit, the system is maintained. This thesis focuses on RBPM. There is limited literature on this type of PM strategy. Note that the Ebeling's method was developed based on TBPM.

The rest of this chapter is organised as follows. In Section 3.2, the concepts of SSA and the assumptions used in the SSA are introduced. Section 3.3 consists of three subsections. In Subsection 3.3.1, a basic model to analyse the reliability of the repairable system is developed under the condition that always the same single component is repaired in all PM actions. Subsection 3.3.2 focuses on the scenario that only single but a different component is repaired in each PM action. A heuristic approach is presented in Subsection 3.3.3 for analysing more general cases. In Sections 3.4 and 3.5, an example and a case study are used to demonstrate the applications of the developed models respectively. In Section 3.6, results of simulations to verify the developed model are presented. The chapter concludes in Section 3.7.

3.2 CONCEPTS OF SSA AND ASSUMPTIONS

The basic concept of the SSA is to separate repaired and unrepaired components within a system virtually when modelling the reliability of a system after PM activities. This concept enables the analysis of system reliability at the component level, and stems from the fact that generally when a complex system has a PM action, only some of the components are repaired [194].

In the analysis, the following assumptions were made:

(1) The failure of repaired components is independent of unrepaired components. This assumption means that when a component is repaired, the failure distribution form of the unrepaired components of a system does not change, and the conditions of the unrepaired components do not affect the reliability characteristics of repaired components.

(2) The reliability function of a new repairable system is known. The reliability functions of repaired components are also known.

(3) The topology of a repairable system is known.

(4) The repair time is negligible.

(5) The PM time is a deterministic variable.

The first assumption means that the failures of different components in a system are independent. This assumption has been adopted by most existing models. The assumption of independent failures will be removed in the models developed in Chapters 4 and 5.

The second assumption is reasonable. Several techniques have been developed to determine the original reliability functions if historical data are sufficient. The situation where historical failure data are insufficient will be discussed in Chapter 6.

The third assumption is also reasonable because the configuration of a system is often known.

The fourth assumption is reasonable when repair time is much shorter than the time between two PM actions and has been used previously [8, 221, 292].

The fifth assumption is sustained because PM times considered in this research are either scheduled by maintenance engineers such as in TBPM or dynamically determined based on the requirement for reliability such as in RBPM. PM time is different from failure time which is a random variable.

According to the above assumptions, only the reliability functions of repaired components change when a PM action is conducted on a system. The PM does not change the characteristics of the reliability of the unrepaired components in the system.

3.3 MODELLING

In this chapter, the SSA is developed based on three scenarios. Firstly, a basic model is developed using a simple scenario where always the same single component is repaired in all PM activities. Secondly, this basic model is extended to the scenario where only a single but different component is repaired in each PM action. Finally, a heuristic approach is developed for more general scenarios.

3.3.1 Scenario one: the Same Single Component Repair

In this scenario, the original system can be described using two virtual parts: the repaired Component 1 and the remainder of the system - often referred to as the subsystem. The PM strategy is to repair Component 1 whenever the reliability of the system falls to a predefined control limit of reliability R_0. The term ‘control limit of reliability’ indicates the required minimum reliability level of a system. Although this scenario is mainly used to demonstrate the basic concepts and procedures for SSA, the models based on this scenario can be applied in industrial cases. For example, a system has a vulnerable Component 1, i.e., this component is more likely to fail than the rest of the system. Both series and parallel systems are considered.

3.3.1.1 Series system

A series system is shown in Figure 3-2. The repaired component is connected with the subsystem in series, but the subsystem can be any complex system. In Figure 3-2, $R_1(\tau)_i$ and $R_{sb}(\tau)_i$ are the reliability functions of the repaired Component 1 and subsystem after the i^th^ PM interval, respectively. In this thesis, the second subscript i is used to denote "after the i^th^ PM action". Subscript $i = 0$ stands for no PM. Sometimes, for simplicity, subscript 0 will be omitted if the meaning of no PM is clear. Two time coordinates are used in the modelling (refer to Figure 3-3):

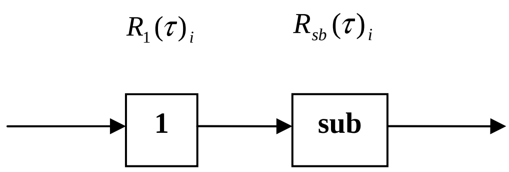


Figure 3-2. Series system

Absolute time scale t: $0 \le t < \infty$.

Relative time scale τ: $0 \le \tau \le t_i$ $(i = 1, 2, \ldots, n)$.

Usually, the reliability of a system after a PM action cannot be restored to its original state, i.e., not "as good as new". The most common phenomenon is that the reliability of a system after a PM action is lower than its original reliability, leading to an imperfect repair. After imperfect repairs, the reliability of a system declines in a manner shown in Figure 3-3.

In Figure 3-3, R_0 is the predefined control limit of the reliability for the system, Δt_i is the interval time between the (i-1)^th^ PM action and the i^th^ PM action ($i = 1, 2, \ldots, n$). Parameter t_i is the i^th^ PM time and also the start time for a system to run again after the i^th^ PM action. Therefore

$$t = \sum_{i=1}^{n} \Delta t_i + \tau . \tag{3-1}$$

Let $R_s(\tau)_i$ represent the reliability function of the system after the i^th^ PM action.

Using reliability theory, the following expression can be obtained:

$$R_s(\tau)_i = R_1(\tau)_i R_{sb}(\tau)_i \qquad (i = 0, 1, 2, \ldots, n) . \tag{3-2}$$

Initially, the reliability function of a system can be expressed as:

$$R_s(\tau)_0 = R_1(\tau)_0 R_{sb}(\tau)_0 . \tag{3-3}$$

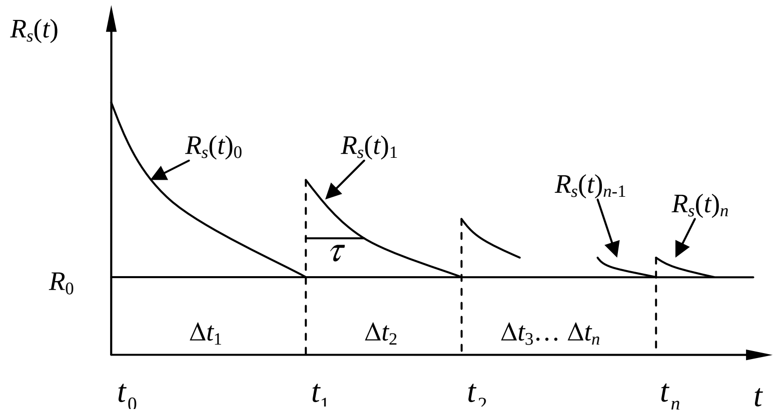


Figure 3-3. Changes of the reliability of an imperfectly repaired system

The reliability function of the subsystem can be derived from Equation (3-3):

$$R_{sb}(\tau)_0 = \frac{R_s(\tau)_0}{R_1(\tau)_0} . \tag{3-4}$$

Equation (3-4) implies that $R_1(\tau)_0 \neq 0$. The reliability functions for typical failure distributions such as exponential distribution, normal distribution, lognormal distribution and Weibull distribution all meet this requirement.

At time t_1, the reliability of the system falls to the control limit R_0 and Component 1 is repaired as requested by the PM strategy. After the first PM action, the reliability function of Component 1 becomes $R_1(\tau)_1$, but the reliability function of the subsystem remains the same since it is not repaired. Considering the cumulative effect of time, the reliability function of the subsystem after the first PM action,

$R_{sb}(\tau)_1$, is $R_{sb}(\tau + \Delta t_1)_0$. Hence, the reliability of the system after the first PM action becomes

$$R_s(\tau)_1 = R_1(\tau)_1 R_{sb}(\tau + \Delta t_1)_0 \,. \qquad (3\text{-}5)$$

If $R_1(\tau)_1 = R_1(\tau + \Delta t_1)_0$, then $R_s(\tau)_1 = R_s(\tau + \Delta t_1)_0$. This indicates that the system is repaired as bad as old.

If Component 1 is repaired or replaced by an identical one so that $R_1(\tau + \Delta t_1)_0 < R_1(\tau)_1 \le R_1(\tau)_0$, then Equation (3-5) represents the situation where the system is repaired imperfectly because $R_s(\tau + \Delta t_1)_0 < R_s(\tau)_1 < R_s(\tau)_0$ in this case.

If the reliability of Component 1 after the repair is better than its original reliability, $R_1(\tau)_1 \ge R_1(\tau)_0$, so that $R_s(\tau)_1 \ge R_s(\tau)_0$, Equation (3-5) then represents the case where the state of a system after repairs is improved to be as good as new or even better than original new one. As a result, Equation (3-5) can describe all possible states of a system after PM (The case that a repair decreases the reliability of a system is not considered in this thesis).

The reliability function of system after the nth PM interval can be derived as:

$$R_s(\tau)_n = R_1(\tau)_n R_{sb}(\tau + \sum_{i=1}^{n} \Delta t_i)_0 \,. \qquad (3\text{-}6)$$

Substituting Equation (3-4) into Equation (3-6) gives

$$R_s(\tau)_n = \frac{R_1(\tau)_n R_s(\tau + \sum_{i=1}^{n} \Delta t_i)_0}{R_1(\tau + \sum_{i=1}^{n} \Delta t_i)_0} \,. \qquad (3\text{-}7)$$

Equation (3-7) can be rewritten using absolute time scale as follows:

$$R_s(t) = \frac{R_1(t - \sum_{i=1}^{n} \Delta t_i)_n R_s(t)_0}{R_1(t)_0} \qquad (t \geq \sum_{i=1}^{n} \Delta t_i \ ). \tag{3-8}$$

where, $R_s(t)$ is the reliability of the system after the n[th] PM interval.

Note that Equation (3-7) and Equation (3-8) both describe the reliability of a system which has been preventively maintained for n times, i.e., these two equations both describe the conditional probability of survival of a system with n PM intervals. Neither of these two equations considers the cumulative effect over time of the repaired components. To predict the probability of survival of a system over its whole life time, these cumulative effects need to be considered, i.e., the probability of survival of these repaired components until their individual repair times need to be considered [16]. The probability of survival of a system over its whole life time is termed as the cumulative reliability of the system. The cumulative reliability function of the system with the first PM action is

$$R_{sc}(\tau)_1 = R_1(\Delta t_1)_0 R_s(\tau)_1, \tag{3-9}$$

where, $R_{sc}(\tau)_1$ is the cumulative reliability of the system after the first PM action. $R_1(\Delta t_1)_0$ is the probability of survival of Component 1 until t_1.

Generally, the cumulative reliability of the system with n PM intervals can be expressed as:

$$R_{sc}(t) = \prod_{i=0}^{n-1} R_1(\Delta t_{i+1})_i R_s(t) \qquad (t \geq \sum_{i=1}^{n} \Delta t_i \ ), \tag{3-10}$$

where $R_{sc}(t)$ is the cumulative reliability of the system with n PM intervals.

A low reliability of the unrepaired components of the system, or poorly repaired components, or both will cause a low $R_s(0)_n$. Obviously, the system should not be repaired any more if

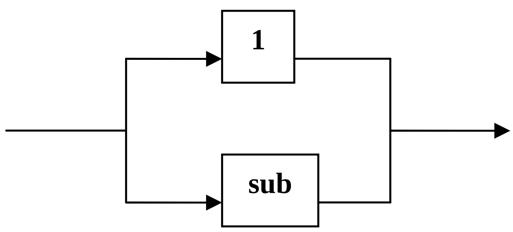
$$R_s(0)_n = \frac{R_1(0)_n R_s(\sum_{i=1}^{n} \Delta t_i)_0}{R_1(\sum_{i=1}^{n} \Delta t_i)_0} \le R_0\,, \qquad (3\text{-}11)$$

i.e., a PM action is unworthy if the reliability of the system after this PM action cannot recover to excess the required reliability level.

3.3.1.2 Parallel system

In this case, the repaired component is connected with the subsystem in parallel as shown in Figure 3-4.

$R_1(\tau)_i$



$R_{sb}(\tau)_i$

Figure 3-4. Parallel system

The relationship of reliability functions $R_1(\tau)_i$, $R_{sb}(\tau)_i$ and $R_s(\tau)_i$ is given by

$$R_s(\tau)_i = R_1(\tau)_i + R_{sb}(\tau)_i - R_1(\tau)_i R_{sb}(\tau)_i \qquad (i = 0,1,2,\ldots,n)\,. \qquad (3\text{-}12)$$

To simplify mathematical operations, let $F_1(\tau)_i$, $F_{sb}(\tau)_i$ and $F_s(\tau)_i$ be corresponding failure distribution functions of Component 1, subsystem and the system after the ith PM action respectively. According to reliability theory, Equation (3-12) becomes

$$F_s(\tau)_i = F_1(\tau)_i F_{sb}(\tau)_i \qquad (i = 0,1,2,\ldots,n)\,. \qquad (3\text{-}13)$$

Based on the same derivation procedure as in Subsection 3.3.1.1, the following results can be obtained (vide Figure 3-5):

$$F_s(\tau)_n = \frac{F_1(\tau)_n F_s(\tau + \sum_{i=1}^{n} \Delta t_i)_0}{F_1(\tau + \sum_{i=1}^{n} \Delta t_i)_0}, \tag{3-14}$$

$$F_s(t) = \frac{F_1(t - \sum_{i=1}^{n} \Delta t_i)_n F_s(t)_0}{F_1(t)_0} \qquad (t \geq \sum_{i=1}^{n} \Delta t_i). \tag{3-15}$$

where, Functions $F_s(\tau)_n$ and $F_s(t)$ are the failure distribution functions of the system after the n^th PM interval described in the relative time scale and the absolute time scale, respectively. Functions $F_1(\tau)_0$ and $F_1(\tau)_n$ represent the failure distribution functions of Component 1 before any PM and after the n^th PM interval, respectively. Function $F_s(t)_0$ is the failure distribution function of the original system. In Figure 5-5, F_0 is a predefined control limit of the failure probability of a system.

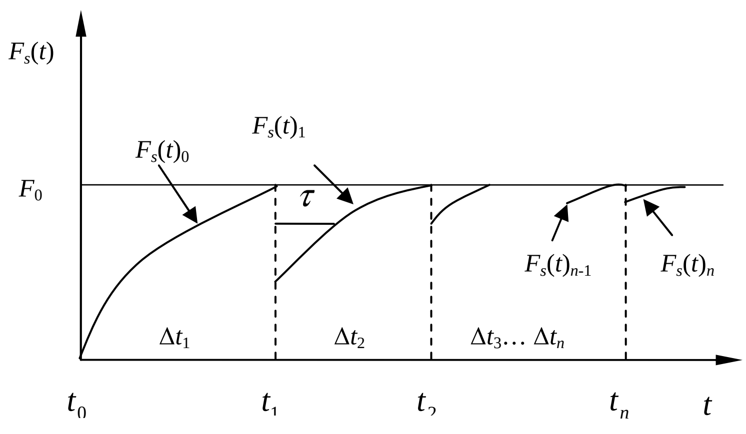


Figure 3-5. Changes of the failure distribution function of an imperfectly repaired system

Equation (3-15) can be rewritten in the term of reliability function as follows:

$$R_s(t) = 1 - \frac{[1 - R_1(t - \sum_{i=1}^{n} \Delta t_i)_n][1 - R_s(t)_0]}{1 - R_1(t)_0} \qquad (t \geq \sum_{i=1}^{n} \Delta t_i). \qquad (3\text{-}16)$$

Generally, $F_1(\tau)_i \leq F_1(\tau + \Delta t_i)_{i-1}$ and $F_{sb}(\tau)_0$ increases monotonously with the increase of operational time, so

$$F_s(\tau + \Delta t_i)_{i-1} > F_s(\tau)_i > F_s(\tau)_{i-1} \qquad (i = 1, 2, \ldots, n). \qquad (3\text{-}17)$$

Equation (3-17) indicates that a system is repaired imperfectly. It is noted that Equations (3-14) and (3-15) or (3-16) can represent all different states of a system after PM due to the similar reasons mentioned in Subsection 3.3.1.1.

The cumulative reliability of the system can be derived as follows:

The cumulative reliability of Component 1 with n PM intervals is

$$R_{1c}(\tau)_n = \prod_{i=0}^{n-1} R_1(\Delta t_{i+1})_i R_1(\tau)_n \ . \qquad (3\text{-}18)$$

The cumulative reliability of the subsystem is $R_{sb}(\tau + \sum_{i=1}^{n} \Delta t_i)_0$ since it is not repaired as assumed by the PM strategy. Hence, the cumulative reliability of the system with n PM intervals is

$$R_{sc}(\tau)_n = 1 - [1 - R_{1c}(\tau)_n][1 - R_{sb}(\tau + \sum_{i=1}^{n} \Delta t_i)_0]\,. \qquad (3\text{-}19)$$

Equation (3-19) can be rewritten using absolute time scale as follows:

$$R_{sc}(t) = 1 - \frac{[1 - \prod_{i=0}^{n-1} R_1(\Delta t_{i+1})_i R_1(t - \sum_{i=1}^{n} \Delta t_i)_n][1 - R_s(t)_0]}{1 - R_1(t)_0} \qquad (t \geq \sum_{i=1}^{n} \Delta t_i). \qquad (3\text{-}20)$$

In Equations (3-19) and (3-20), $R_{sc}(\tau)_n$ and $R_{sc}(t)$ are the cumulative reliability of the system with n PM intervals.

3.3.2 Scenario two: Single but Different Component Repairs

In this scenario, a system has m vulnerable components. The PM strategy is to maintain one of them whenever the reliability of the system falls to the predefined control limit of reliability. Normally, the PM sequence of these components is arranged based on their reliability characteristics to ensure the component with the lowest reliability at each PM time to be repaired. These repaired components will be connected with the subsystems in different ways because both the repaired components and the subsystems will change in each PM action.

3.3.2.1 Multi-series system

In this case, all m repaired components and unrepaired subsystem are connected together serially (see Figure 3-6). Components can be numbered according to their sequences to receive their first repair in n PM intervals so that $m \le n$ without losing any generality.

The situation is exactly the same as Subsection 3.3.1.1 after the first PM action, but is different from Subsection 3.3.1.1 after the second PM action because another component instead of Component 1 may be repaired. Therefore, the subsystem changes after the ith ($i > 1$) PM interval.

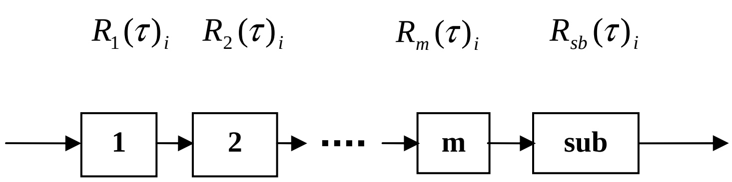


Figure 3-6. Multi-series system

Generally, if m components are repaired in n PM intervals and L_k indicates that Component k ($k \le m$) receives its last repair in the L_k^{th} PM action ($L_k \le n$), then the reliability function of a system after the n[th] PM interval is given by (refer to Appendix B2)

$$R_s(\tau)_n = \frac{R_s(\tau + \sum_{i=1}^{n} \Delta t_i)_0 \prod_{k=1}^{m} R_k(\tau + \sum_{i=L_k+1}^{n} \Delta t_i)_{L_k}}{\prod_{k=1}^{m} R_k(\tau + \sum_{i=1}^{n} \Delta t_i)_0}. \qquad (3\text{-}21)$$

In Equation (3-21), define $\sum_{i=L_k+1}^{n} \Delta t_i = 0$ when $L_k + 1 > n$. The cumulative reliability of the system can be calculated using a heuristic approach which is presented in Subsection 3.3.3.

3.3.2.2 Multi-parallel system and complex system

For a multi-parallel system shown in Figure 3-7, it is straightforward to model the system after the n[th] PM interval using the same method as described in Subsection 3.3.1.2, i.e., using failure distribution functions instead of reliability functions to derive the corresponding formulae. One only needs to replace *R* with *F* in Equation (3-21) in order to model the failure distribution functions of a system after the n[th] PM interval as follows:

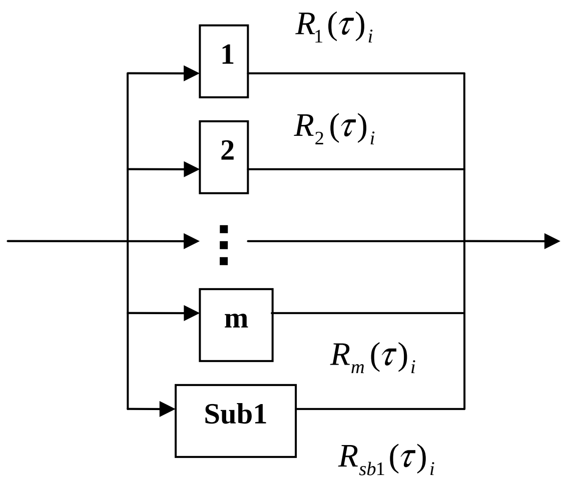


Figure 3-7. Multi-parallel system

$$F_s(\tau)_n = \frac{F_s(\tau + \sum_{i=1}^{n} \Delta t_i)_0 \prod_{k=1}^{m} F_k(\tau + \sum_{i=L_k+1}^{n} \Delta t_i)_{L_k}}{\prod_{k=1}^{m} F_k(\tau + \sum_{i=1}^{n} \Delta t_i)_0}$$

However, derivation of the reliability functions of a complex system after the nth PM action is difficult because numerous possible conditions need to be considered.

During n PM intervals, the repaired components can have either a series relationship or a parallel relationship with the subsystem, or, even worse, a relationship which is neither in series nor in parallel with the subsystem. Figure 3-8 shows one such example. It is impossible to derive a general formula like Equation (3-21) for the case. The reliability of a complex system after the nth PM interval can be calculated using the following heuristic approach.

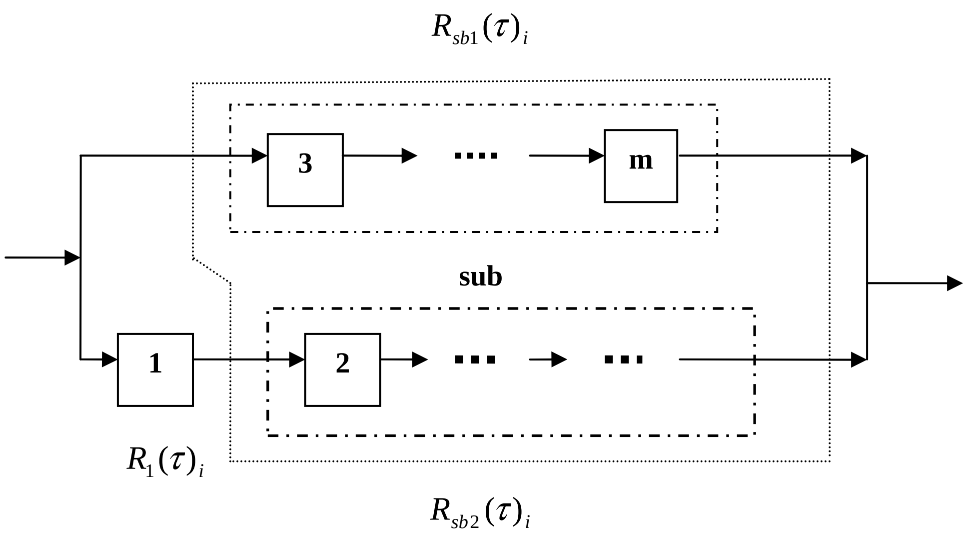


Figure 3-8. An example of complex system

3.3.3 Heuristic Approach

The heuristic approach is described as follows:

(1) Determine the first PM time $t_1 = \Delta t_1$ when the reliability of the system first falls to the predefined control limit of reliability using the original reliability function of the system.

(2) Assume that the system has M components and S_1 components ($1 \le S_1 \le M$) are repaired in the first PM action. The repaired Component k_1 ($k_1 = 1, 2, \ldots, S_1$) is assigned a new reliability function $R_{k_1}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$)

based on the requirement of a PM strategy. The cumulative reliability functions of these repaired components, $R_{k_1c}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$), are $R_{k_1}(\Delta t_1)_0 R_{k_1}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$). The reliability functions of the rest of the components of the system remain the same as before since they are not repaired. However, the cumulative effects of time before the first PM action need to be considered. Hence, $R_{j_1}(\tau)_1 = R_{j_1}(\tau + \Delta t_1)_0$ ($j_1 = S_1 + 1, S_1 + 2, \ldots, M$), which are the same as their cumulative reliability functions with the first PM action.

(3) Calculate the reliability function and the cumulative reliability function of the system after the first PM action, $R_s(\tau)_1$ and $R_{sc}(\tau)_1$, based on the RBD of the system using the reliability functions and the cumulative reliability functions of its components after the first PM action, respectively.

(4) Determine the second PM time t_2 using the reliability function of the system after the first PM action, $R_s(\tau)_1$.

(5) Assume S_2 components are repaired in the second PM action. The repaired Component k_2 is assigned a new reliability function $R_{k_2}(\tau)_2$ (k_2 represents all components repaired in the second PM action) based on the requirement of PM strategy. The cumulative reliability functions of these components $R_{k_2c}(\tau)_2$ (k_2 represents all components repaired in the second PM action) now need to be calculated based on two scenarios: if components have also been repaired in the first PM action, their cumulative reliability functions are $\prod_{i=0}^{1} R_{k_{21}}(\Delta t_{i+1})_i R_{k_{21}}(\tau)_2$. Subcript k_{21} represents all components that are repaired in the first and second PM action. The cumulative reliability functions for those components which are repaired in the second PM action only are $R_{k_{22}}(\sum_{i=1}^{2} \Delta t_i)_0 R_{k_{22}}(\tau)_2$. Subscript $k_{21} \neq k_{22}$ and $k_{21} + k_{22} = S_2$. The reliability functions of the rest of the components of the system remain the same as before this PM action since they are not repaired. However, the

cumulative effects of time on unrepaired components can now be different since some of these components may be repaired in the first PM action. Just like the repaired components, the reliability functions and the cumulative reliability functions of these unrepaired components also need to be calculated based on two scenarios. For components which are never repaired, their reliability functions $R_{j_{21}}(\tau)_2$ and cumulative reliability functions $R_{j_{21}c}(\tau)_2$ both are $R_{j_{21}}(\tau + \sum_{i=1}^{2} \Delta t_i)_0$. Subscript j_{21} represents all components which are never been repaired. For components which have been repaired in the first PM action, their reliability functions $R_{j_{22}}(\tau)_2$ and cumulative reliability functions $R_{j_{22}c}(\tau)_2$ are $R_{j_{22}}(\tau + \Delta t_2)_1$ and $R_{j_{22}}(\Delta t_1)_0 R_{j_{22}}(\tau + \Delta t_2)_1$. Subscript $j_{22} \neq j_{21}$ and $j_{21} + j_{22} = M - S_2$.

(6) Calculate the reliability function and the cumulative reliability function of the system after the second PM action, $R_s(\tau)_2$ and $R_{sc}(\tau)_2$, based on the RBD of the system using the reliability functions and the cumulative reliability functions of its components after the second PM action, respectively.

(7) Continue the above procedure until the nth PM action.

If only one component is repaired in each PM action, the above heuristic approach can often be described using the following recurrence formula:

$$R_s(\tau)_i = R^e_{sb1}(\tau + \Delta t_i)_{i-1} + R_k(\tau)_i R^e_{sb2}(\tau + \Delta t_i)_{i-1} \qquad (i = 1,2,...,n), \tag{3-22}$$

where, subscript $k = 1, 2, \ldots, m$ indicates repaired components in the ith PM action. $R^e_{sb1}(\tau + \Delta t_i)_{i-1}$ and $R^e_{sb2}(\tau + \Delta t_i)_{i-1}$ are the equivalent reliability functions that are calculated based on the subsystem. For example, in the case shown in Figure 3-8,

$$R^e_{sb1}(\tau + \Delta t_i)_{i-1} = R_{sb1}(\tau + \Delta t_i)_{i-1}$$

and

$$R^e_{sb2}(\tau+\Delta t_i)_{i-1} = R_{sb2}(\tau+\Delta t_i)_{i-1}(1-R_{sb1}(\tau+\Delta t_i)_{i-1}) .$$

These equivalent reliability functions can vary when different component is repaired.

For more generalised scenarios - two or more components are repaired in each PM action, the following techniques can be used to simplify calculations.

Case 1: Repaired components can be combined to form a new subsystem, and the new subsystem has a series relationship with original subsystem. This scenario can be treated to be the same as that in Subsection 3.3.1.1, and hence the model in Subsection 3.3.1.1 can be applied.

Case 2: Repaired components can be combined to form a new subsystem, and the new subsystem has a parallel relationship with the original subsystem. This scenario can be treated to be the same as that in Subsection 3.3.1.2, and hence the model in Subsection 3.3.1.2 can be applied.

The SSA is developed to support PM decision making for a repairable system over its lifetime. This capability is demonstrated by the following example and case study.

3.4 An Example: a System with Weibull Failure Distribution

A repairable complex mechanical system is the same as described in Subsection 3.3.1.1. The PM strategy is to replace Component 1 with an identical new one whenever the reliability of the system falls to R_0 - a predefined control limit of reliability. The reliability functions of the original system and Component 1 are Weibull. They are given by

$$R_s(\tau)_0 = \exp\left[-(\frac{\tau}{\eta_s})^2\right] \qquad (3\text{-}25)$$

and

$$R_1(\tau)_0 = \exp\left[-(\frac{\tau}{\eta_1})^2\right], \tag{3-26}$$

where, η_s and η_1 are the characteristic life of the system and Component 1 [16] respectively. Parameter η in the Weibull distribution is also termed as a scale parameter.

When the system receives its first PM action, $R_s(t_1)_0 = R_0$. The first PM time $t_1 = \Delta t_1$ is given by

$$t_1 = \Delta t_1 = \eta_s \sqrt{-\ln R_0} \qquad (1 > R_0 > 0). \tag{3-27}$$

Using Equation (3-7), gives

$$R_s(\tau)_1 = \frac{\exp\left[-(\frac{\tau}{\eta_1})^2\right]\exp\left[-(\frac{\tau+\eta_s\sqrt{-\ln R_0}}{\eta_s})^2\right]}{\exp\left[-(\frac{\tau+\eta_s\sqrt{-\ln R_0}}{\eta_1})^2\right]}$$

$$= \exp\left[-\frac{(\eta_1^2-\eta_s^2)(\tau+\eta_s\sqrt{-\ln R_0})^2+\eta_s^2\tau^2}{\eta_1^2\eta_s^2}\right]. \tag{3-28}$$

The reliability of the system just after the first PM action is

$$R_s(0)_1 = R_0^{(1-\frac{\eta_s^2}{\eta_1^2})}. \tag{3-29}$$

The reliability of the system after the PM increases but is not restored to 1 (the perfect reliability level of the system) since $1 > \frac{\eta_s^2}{\eta_1^2} > 0$, that is, the system has an imperfect repair.

Using Equation (3-7) gives the reliability function of the repairable system after the n[th] PM interval, $R_s(\tau)_n$:

$$R_s(\tau)_n = \exp\left[-\frac{(\eta_1^2-\eta_s^2)(\tau+\sum_{i=1}^{n}\Delta t_i)^2+\eta_s^2\tau^2}{\eta_1^2\eta_s^2}\right]. \tag{3-30}$$

If the absolute time scale is applied, Equation (3-30) can be rewritten as:

$$R_s(t) = \exp\left[-\frac{(\eta_1^2-\eta_s^2)t^2+\eta_s^2(t-\sum_{i=1}^{n}\Delta t_i)^2}{\eta_1^2\eta_s^2}\right] \qquad (t > \sum_{i=1}^{n}\Delta t_i). \tag{3-31}$$

The interval between the (n-1)[th] PM action and the n[th] PM action is given by equation $R_s(\Delta t_n)_{n-1} = R_0$, i.e.,

$$R_0 = \exp\left[-\frac{(\eta_1^2-\eta_s^2)(\Delta t_n+\Delta t_{n-1}+\sum_{i=1}^{n-2}\Delta t_i)^2+\eta_s^2(\Delta t_n)^2}{\eta_1^2\eta_s^2}\right] \tag{3-32}$$

$$\Delta t_n = \left[\frac{\eta_s^2-\eta_1^2+\sqrt{\eta_s^4-\eta_1^2\eta_s^2-\eta_1^4\eta_s^2\ln R_0/(\sum_{i=1}^{n-1}\Delta t_i)^2}}{\eta_1^2}\right]\sum_{i=1}^{n-1}\Delta t_i\,. \tag{3-33}$$

The relationship $\Delta t_n < \Delta t_{n-1}$ can be proved as follows:

When the reliability of the system reaches R_0 after the (n-2)[th] PM action, the time interval Δt_{n-1} can be determined by

$$R_0 = \exp\left[-\frac{(\eta_1^2 - \eta_s^2)(\Delta t_{n-1} + \sum_{i=1}^{n-2} \Delta t_i)^2 + \eta_s^2 (\Delta t_{n-1})^2}{\eta_1^2 \eta_s^2} \right]. \tag{3-34}$$

A combination of Equations (3-32) and (3-34) gives

$$\frac{(\eta_1^2 - \eta_s^2)\left[(\sum_{i=1}^{n} \Delta t_i)^2 - (\sum_{i=1}^{n-1} \Delta t_i)^2 \right]}{\eta_s^2} + (\Delta t_n)^2 = (\Delta t_{n-1})^2 . \tag{3-35}$$

From Equation (3-35), it can be found that $\Delta t_n < \Delta t_{n-1}$ since $\frac{(\eta_1^2 - \eta_s^2)\left[(\sum_{i=1}^{n} \Delta t_i)^2 - (\sum_{i=1}^{n-1} \Delta t_i)^2 \right]}{\eta_s^2} > 0$.

In case Component 1 ceases to be produced, how many spare parts of Component 1 should be kept for the life span of the system? One answer can be found using the following criterion. The interval time between two PM actions must be longer than required minimum operating time t_p, that is

$$\Delta t_n \ge t_p . \tag{3-36}$$

Substituting Equation (3-33) into Equation (3-36), gives

$$\left[\frac{\eta_s^2 - \eta_1^2 + \sqrt{\eta_s^4 - \eta_1^2 \eta_s^2 - \eta_1^4 \eta_s^2 \ln R_0 / (\sum_{i=1}^{n-1} \Delta t_i)^2}}{\eta_1^2} \right] \sum_{i=1}^{n-1} \Delta t_i \ge t_p . \tag{3-37}$$

The maximum number of Component 1 to be stored for PM can be estimated through finding the maximum n from Equation (3-37). The expected life of this repairable system can also be estimated from Equation (3-37). However, Equation (3-37) must be calculated recurrently and numerically. Some examples using Monte Carlo Simulation (MCS) are presented in Section 3.6. The simulations were conducted using Matlab software, and was based on the common knowledge of Monte Carlo simulation which considered the properties of Weibull distribution and series systems, and used the Boolean Algorithms and the empirical cumulative distribution function (CDF) [12]. For more details, please refer to [16] p.90-91, [297] p.400-439 and [298] p.148-150.

To evaluate the effectiveness of the above PM strategy on the reliability of the system over its life span, the cumulative reliability of the system should be calculated. Using Equation (3-10) gives the cumulative reliability of the system with n PM intervals as follows:

$$R_{sc}(t) = \exp\left[-\sum_{i=1}^{n} \left(\frac{\Delta t_i}{\eta_1}\right)^2 \right] R_s(t) \qquad \left(t > \sum_{i=1}^{n} \Delta t_i\right). \tag{3-38}$$

Rewrite Equation (3-38) as:

$$R_{sc}(t) = \exp\left[\frac{2t\sum_{i=1}^{n} \Delta t_i - (\sum_{i=1}^{n} \Delta t_i)^2 - \sum_{i=1}^{n} (\Delta t_i)^2}{\eta_1^2} \right] R_s(t)_0$$

The function $2t\sum_{i=1}^{n} \Delta t_i - (\sum_{i=1}^{n} \Delta t_i)^2 - \sum_{i=1}^{n} (\Delta t_i)^2 > 0$ because $t > \sum_{i=1}^{n} \Delta t_i$ and $\Delta t_i > 0$.

Hence, $R_{sc}(t) > R_s(t)_0$, i.e., in this case, PM reduces the probability of unexpected breakdown of the system.

To investigate the effectiveness of PM further, assume that Component 1 has a constant random failure rate, i.e.

$$R_1(\tau)_i = \exp(-\lambda_i \tau) \qquad (i = 0, 1, 2, \ldots, n) \tag{3-39}$$

where, λ_i is the failure rate of Component 1 after the nth PM action.

Using Equations (3-8) and (3-10) gives the cumulative reliability of the system with n PM intervals:

$$R_{sc}(t) = \exp[\sum_{i=1}^{n}(\lambda_0 - \lambda_{i-1})\Delta t_i]\exp[(\lambda_0 - \lambda_n)(t - \sum_{i=1}^{n}\Delta t_i)]R_s(t)_0 \quad (t > \sum_{i=1}^{n}\Delta t_i). \quad (3\text{-}40)$$

Equation (3-40) indicates that if $\lambda_{i-1} = \lambda_0$ ($i = 1, 2, \ldots, n$), $R_{sc}(t) = R_s(t)_0$, i.e., PM in this case has no effect even though the entire system presents a wear-out characteristic.

3.5 Case Study: a Water Supply Pipeline

The SSA was applied to a water supply pipeline which was made from PVC consisting of 10 segments. The length of each pipe was 6 m. The pipeline was installed on 1 June 1991. A corrective maintenance policy was in force, that is, whenever a pipe failed, it was replaced. During the observed period, the placed pipes where not found to have failed again. (The raw data cannot be presented due to the need for confidentiality.) After a comprehensive investigation, the following assumptions were made in the analysis:

(1) The analysed pipes have an independent, identical failure distribution.

(2) The failed pipes were replaced by identical new pipes.

(3) Repair time is ignored.

(4) All failed pipes started operating at the same time.

(5) All pipes operated under the same conditions.

The scenario in this case study is the same as described in Subsection 3.3.2.1.

3.5.1.1 Failure distribution characteristics of the pipeline

Figure 3-9 shows the assessment of failure distribution of the pipeline. It can be seen that the failure times of the pipeline have a Weibull distribution.

Further analysis using the Mann's Test for the Weibull Distribution indicated that the Weibull hypothesis for the failure time of the pipeline can be accepted at the level of significance 0.05. The Mann's Test is presented in Appendix B3.

The failure distribution of the failure times of the pipeline was obtained using MLE as follows:

$$R_s(\tau)_0 = \exp[-(\frac{\tau}{3573.3})^{5.5923}]. \tag{3-41}$$

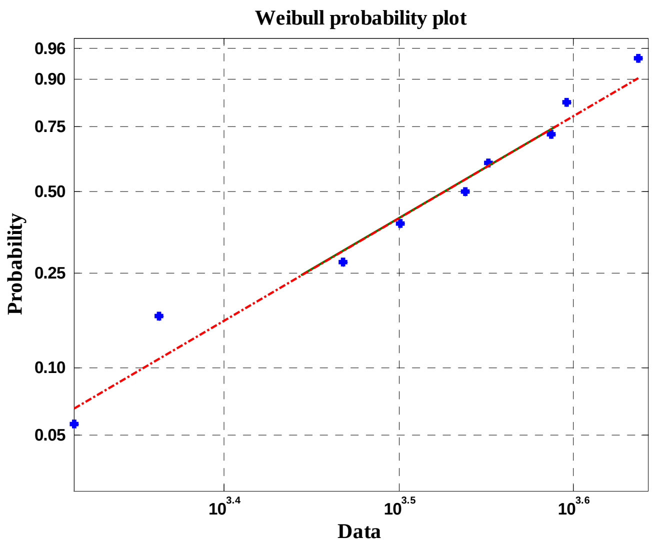


Figure 3-9. Weibull probability plot

The failure distribution function of each pipe was derived from Equation (3-41) since the pipeline was a series system comprised of 10 identical pipes:

$$R_i(\tau)_0 = \exp[-(\frac{\tau}{5393.7})^{5.5923}] \qquad (i = 1, 2, \ldots, 10). \tag{3-42}$$

Failure history indicated that this pipeline has failed frequently after 3000 days under the current corrective maintenance policy. Since the pipes were operating in the wear out stage, a proper PM strategy can be used to improve the overall reliability of the pipeline. SSA was used to investigate the effect of different PM strategies on the reliability of the pipeline. The results are demonstrated in the following subsection.

3.5.1.2 Comparisons between different PM strategies

Both TBPM and RBPM policies were considered. When TBPM policy is applied, pipes are replaced sequentially with an identical new one based on scheduled PM times. The intervals between two PM actions may or may not be the same. When RBPM is applied, a reliability control limit R_0 is defined in advance. Whenever the reliability of the pipeline reaches this predefined control limit, the pipe which has the lowest reliability is replaced with an identical new one.

Figures 3-10 and 3-11 show the reliability prediction of the pipeline. In these figures, the dashed line and the thick continuous line indicate the probability of the pipeline without a failure based on TBPM and RBPM, respectively. The crossed line is the reliability of the pipeline without PM.

In Figure 3-10 (Case1), the predefined control limit of reliability for RBPM is 0.9. The PM interval times for TBPM are unequal. The first PM action is planned at the time of 600 days and then PM is to be conducted every 200 days. From this figure, it can be seen that both TBPM and RTBM improve the cumulative reliability of the pipeline significantly but TBPM is more effective. The cumulative reliability of the pipeline with TBPM is maintained above 0.9 at the time of 4500 days whereas the reliability of the pipeline without PM at the same time will be lower than 0.4. Note that the cumulative reliability of the pipeline with TBPM in this case is much higher than with RBPM but the number of PM times with TBPM is also more than that with RBTM. The former (19 times) nearly doubles the latter (10 times).

Figure 3-11 shows another PM strategy (Case 2). In this strategy, the predefined control limit of reliability for RBPM is still 0.9. However, the first PM time for TBPM changes to 1000 days and the sequential PM intervals also increase to 360 days. Both PM strategies require the same number of PM times (10 times) within 4500 days. The cumulative reliability with TBPM is higher than that with RBPM between 2500 days and 3400 days. After this period, RBPM is more effective. The TBPM was ineffective in the given scenario because several PM actions were conducted after the system reliability had fallen to a very low level.

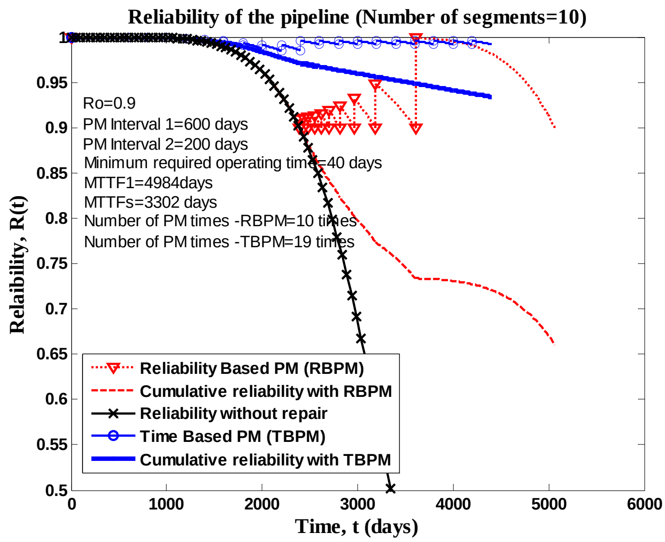


Figure 3-10. The reliability of a pipeline with PM – Case 1

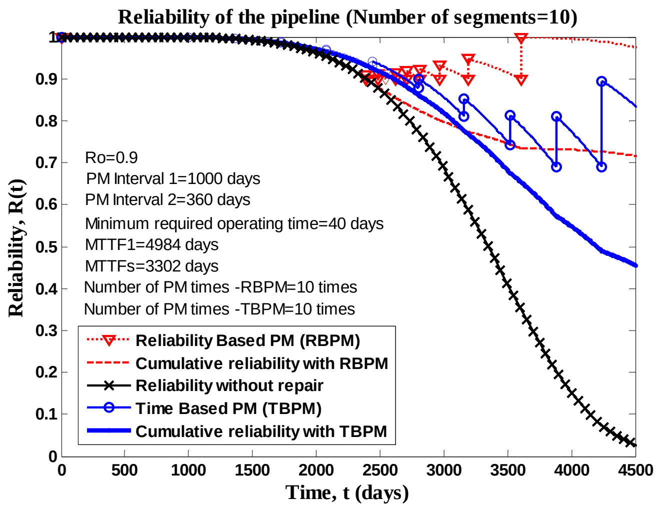


Figure 3-11. The reliability of a pipeline with PM – Case 2

Comparisons can be made not only between different PM policies, but also among different strategies which are developed based on the same PM policy. Look at the cumulative reliability curves with TBPM in Figures 3-11, 3-12 and 3-13. It can be found that different combination of PM times significantly affects the cumulative reliability of the pipeline. All three TBPM strategies require the same number of PM times (10 times), but generate very different cumulative reliability of the pipeline over 4500 days. The TBPM strategy (Case 3) shown in Figure 3-12 has the highest cumulative reliability whereas the TBPM strategy (Case 4) shown in Figure 3-13 generates the lowest cumulative reliability which is 12% lower than the former.

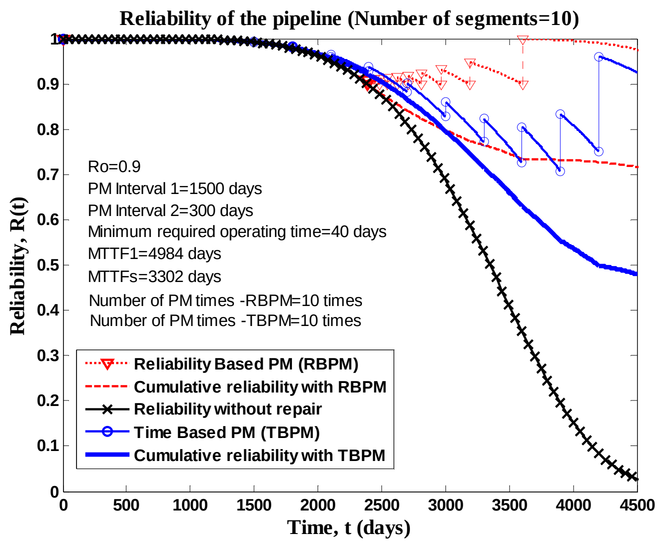


Figure 3-12. The reliability of a pipeline with PM – Case 3

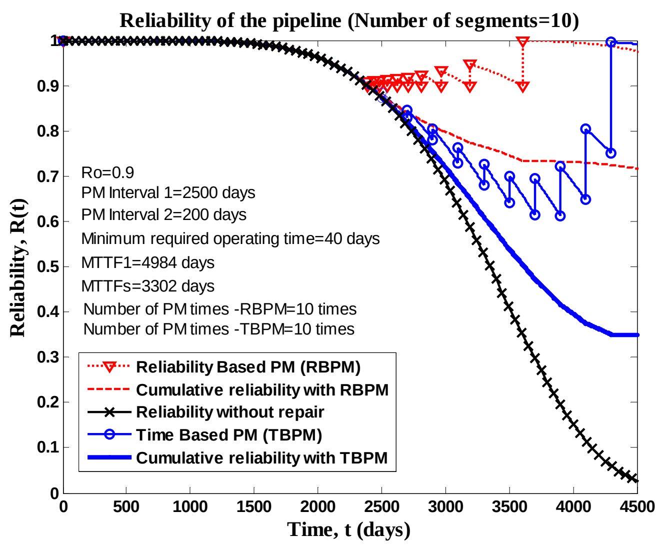


Figure 3-13. The reliability of a pipeline with PM – Case 4

Figures 3-10 to 3-13 demonstrate that the SSA can effectively assist in optimal PM decision making through long term reliability prediction.

3.6 SIMULATIONS

The SSA was also validated by a number of Monte Carlo Simulation (MCS) experiments. Figures 3-14 to 3-16 show the results of the simulations for RBPM. Cumulative reliability was not presented in these figures for simplification. From these figures, it can be concluded that SSA identified the same number of PM times as that demonstrated by the Monte Carlo simulations. The characteristics of the reliability of the system and the PM times predicted by SSA are very close to the results of the MCS experiments. Therefore, SSA has a commendable accuracy of prediction. In Figure 3-16, reliability was also predicted based the fix deterioration rate model for comparison. The deterioration rate was 0.02 which was determined based on the initial reliability of the system after the first PM action. From this figure, it can be seen that the results based on the fix deterioration rate depart from the MCS results significantly.

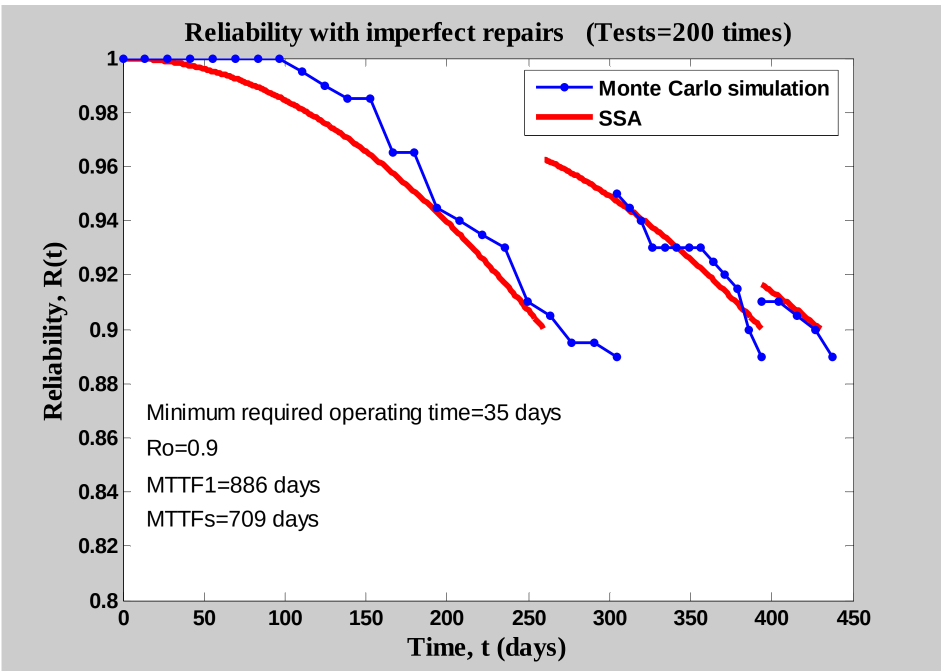


Figure 3-14. Simulation experimental results 1 - the changes of the reliability of a system over the entire life span

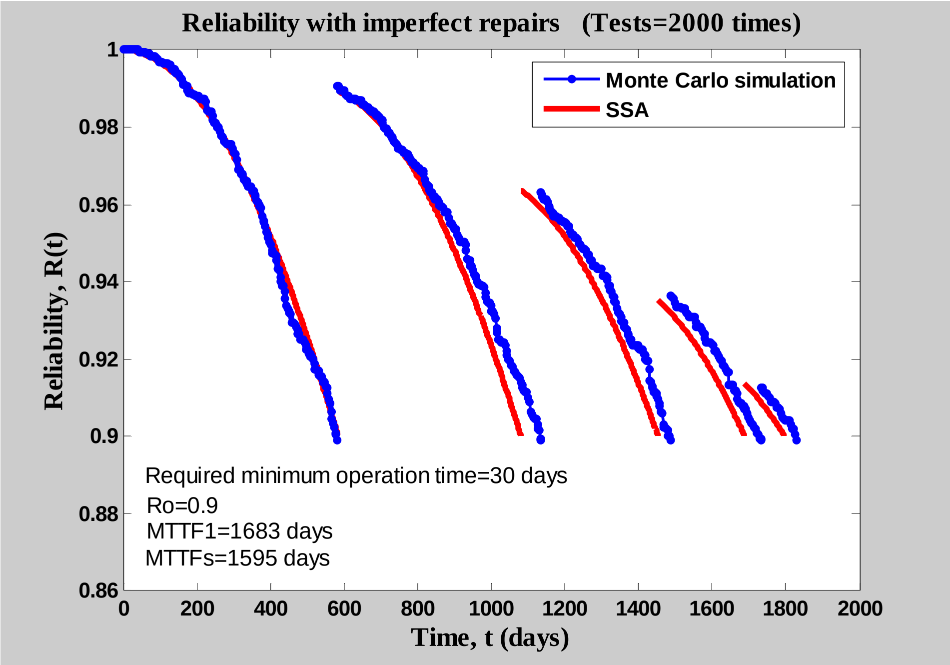


Figure 3-15. Simulation experimental results 2 - the changes of the reliability of a system over the entire life span

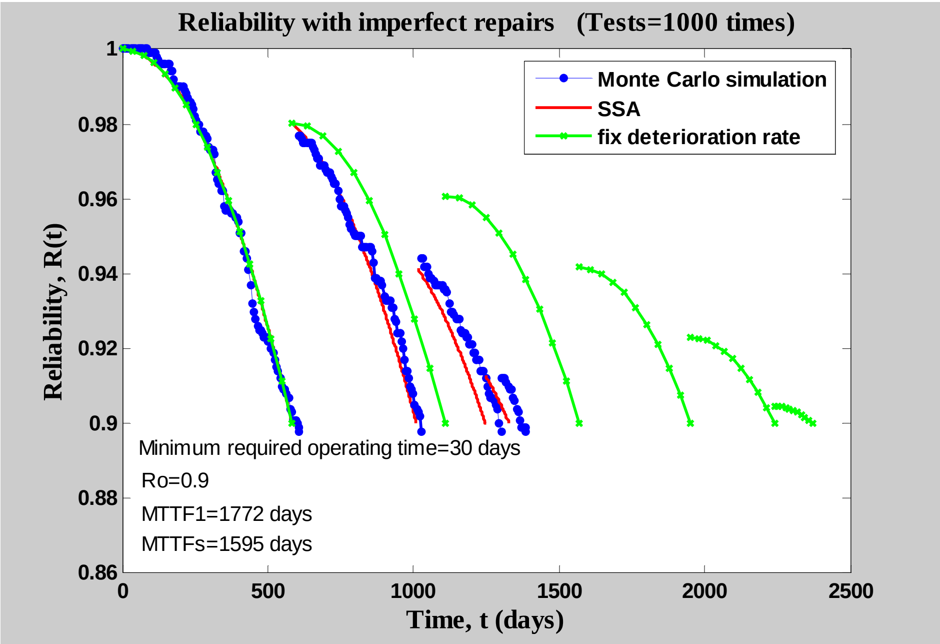


Figure 3-16. Simulation experimental results 3 - the changes of the reliability of a system over the entire life span

3.7 SUMMARY

SSA performs more closely to the real world when compared with Ebeling's method [16] and the fixed deterioration rate model [20]. SSA extended Ebeling method through considering imperfect repairs. In SSA, the changes of reliability are calculated based on individual system and repair condition rather than assumed or estimated by human experience. Therefore, the rate of change may not be constant.

Compared with existing models, the new model developed in this chapter has the following advantages:

(1) Ability to explicitly predict the reliability of a repairable system with multiple PM actions over multiple PM intervals and to decide when the system has deteriorated to a point where it is unworthy of further PM from the reliability view of point. Most of the existing models are applied to predict the next PM time, MTTF or/and the expected number of failures. SSA is hence more suitable for supporting long term PM decision making of complex repairable systems in industry.

(2) Ability to deal with the individual contributions of different parts in a system and the influence of system structures on the reliability of a repairable system. This ability provides an understanding of PM of a system in more depth.

(3) Ability to model different states of a system after PM such as "as good as new", "imperfect repair", "improvement repair" (i.e., better than new) and "as bad as old".

(4) No restrictions on the forms of failure distribution.

The outcomes of the research in this chapter present three important concepts for maintenance decision making.

(1) A PM action for a complex system is often imperfect because normally only some of components are repaired when PM is conducted on a complex system.

(2) An optimal maintenance strategy should consider both the entire system and individual components of a system. For example, in a series repairable system shown in Figure 3-2, maintaining the subsystem to increase its reliability should be considered when the reliability of the subsystem is less than that of Component 1.

(3) The effectiveness of PM is often related to the failure characteristics of repaired components rather than that of a system. If the repaired components have constant failure rates, a PM action, which is to replace these components with new identical ones, has no effect even though the entire system adopts a wear-out characteristic.

The formulae and methods in this chapter have been developed based on RBPM. Extensions of these results to TBPM are straightforward.

In this chapter, the failures of repaired components are assumed to be independent of unrepaired components. This implies that the analysed repairable system has no failure interactions. If the failure interactions between repaired components and unrepaired subsystems are considered, the results would be different. The reliability prediction of systems with failure interaction will be studied in the following chapters.

Chapter 4

ANALYSIS OF INTERACTIVE FAILURES

4.1 INTRODUCTION

As presented in Chapter 2 and Chapter 3, numerous models and methodologies have been developed to describe and predict failures. These models or methodologies have been mainly developed on the assumption that failures are independent. As indicated in Chapter 3, SSA was also developed based on this assumption. However, industrial experiences have shown that the assumption of independent failures has been unrealistic in numerous scenarios and has led to unacceptable errors in reliability analysis. To ensure the accuracy of reliability prediction, the dependency of failures among components needs to be considered.

Currently the most discussed dependent failures include cascading failure, negative dependency failure and common cause failure [8, 278]. Cascading failure is defined as multiple sequential failures. These failures are initiated by the failure of one component, which leads to sequential failures of other components. Negative dependency failure is defined as failure that can prevent other components in a system from failing further. Common cause failure is defined as multiple related events caused by a single common cause. This cause can be the failure of a physical component or an event such as a fire. The latter is often described as the failure of a "virtual" component. Whenever the term "component", is mentioned in this chapter, it usually includes both physical component and virtual component. Cascading failure, negative dependency failure and common cause failure are classified into conventional dependent failures. A common feature of these conventional dependent failures is that failure effect is one directional only, i.e., the failures of some components can affect failures of other components but the latter have no effect on the former. Several models and methodologies have been developed to analyse these conventional dependent failures. However, these models and methodologies cannot be effectively used to analyse the failures due to failure interactions among components.

Failure interaction is common in mechanical engineering and civil engineering. The loss of the Space Shuttle Columbia is such an example. On February 1, 2003, the Space Shuttle Columbia disintegrated on its return to Earth. Seven crew members on board lost their lives (Figure 4-1). The investigation revealed that this disaster was initiated by a large piece of foam which had separated from the external fuel tank. This piece of foam struck Columbia on the underside of the left wing and caused a breach in the thermal protection system on the leading edge of the left wing (Figure 4-2). The breach finally resulted in the burning of the Shuttle including the fuel tank. The failure of Columbia was an interactive failure. The initial failure was not severe, but the consequence of the failure interaction was disastrous. If the foam had not separated or the separated foam did not cause a breach in the thermal protection system, the tragedy of Columbia would have not happened.

This figure is not available online.
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available from the QUT Library

Figure 4-1. The loss of the Space Shuttle Columbia (Source: http://www.evergreen.edu/library/govdocs/hotopics/columbia/ **)**

This figure is not available online.
Please consult the hardcopy thesis
available from the QUT Library

Figure 4-2. The impact of the foam on Columbia (Source: http://www.cbsnews.com/stories/2003/07/10/tech/main562542.shtml **)**

Estimating the failure probability of components subject to failure interaction is imperative. As indicated in Chapter 2, a model or technique used to analyse this failure probability quantitatively and effectively is still unavailable.

In this chapter, a model is developed to analyse interactive failure distribution for a system quantitatively. Several case studies are used to justify the newly developed model. The properties of interactive failures are also analysed.

The rest of this chapter is organised as follows. In Section 4.2, the concepts and definitions of interactive failure and interactive hazard are introduced. In Section 4.3, an analytical model for IntF is derived. In Section 4.4, the determination of interactive coefficients is discussed briefly. In Section 4.5, the stability of IntF is analysed. In Section 4.6, mathematical models and some conditions for existence of stable IntF are presented. Case studies are presented in Section 4.7. In Section 4.8, a methodology to calculate the IntF of components is developed. The properties of IntF are investigated in Section 4.9 and the effects of IntF on systems are analysed in Section 4.10. This is followed by conclusions in Section 4.11.

4.2 INTERACTIVE FAILURE AND INTERACTIVE HAZARD

Definition 4-1: Interactive failure is defined as mutually dependent failures, that is, the failures of some components will affect the failures of other components and vice versa.

Note that the term "components" usually includes subsystems unless specified. This thesis considers positive dependency between failures only.

The simplest case is when only two failures interact. In the case of a gearbox, defects in a bearing will cause it to vibrate. The deterioration of the subsystem that includes related shaft and several gears can accelerate due to the excessive vibration caused by the bearing. Vice versa, a deteriorated subsystem can lead to faster deterioration of the bearings.

The effect of the failure of a component on other components has two consequences:

(1) Failure of one component (influencing component) causes other components (affected components) to fail immediately.

(2) Failure of the influencing component increases the deterioration of affected components instead of causing them to fail immediately.

As a result, these two consequences increase the likelihood of failures of the affected components and accelerate their failure rates.

A component can be either the influencing component or the affected component or both. In the above example, the bearing and the subsystem are both influencing components and affected components.

Interactive failures can be classified into two categories:

(1) Immediate Interactive Failures. The failure of the influencing component will cause its affected components to fail immediately. The conditions of the two components before failure are independent.

(2) Gradual Degradation Interactive Failures. The conditions of two components before failure are dependent. A component deteriorates with time, that is, the failure rate of a component increases with time. The increase of deterioration of this component can result in an increase in deterioration of its affected components. As a result, the failure rate of the “victims” increase, and the system reaches the first state of failure interaction. The increase of deterioration of the “victims” can also increase the failure rate of this component - the original cause, and the system reaches the second state of failure interaction. This interaction can lead to a chain interaction process. As a result of this chain reaction, the two involved components may either achieve a new level of working status or eventually fail.

The second category of interactive failures often occurs in mechanical systems and is the focus of this thesis.

The failure of a component without being affected by the failures of other components is termed as independent failure of the component. Correspondingly, the failure probability of this component in this case is termed as its independent failure probability. The failure probability of a component will be different from its independent failure probability if it is affected by the failures of other components.

The failure likelihood of components with failure interactions will increase. The increased likelihood of failures due to the interactions of components can be considered as the consequences of the increased failure rates due to the same cause. Failure rate is often termed as hazard in reliability theory. For mathematical simplicity in analysing interactive failures of a system, the changes of hazards will be estimated and then the failure distribution functions of the system will be calculated.

Definition 4-2: The increased hazard due to failure interactions is defined as Interactive Hazard (IntH).

Failure probability is represented using failure distribution function. The relationship between the failure distribution function and hazard is [8]:

$$F(t) = 1 - \exp[-\int_0^t h(t)dt], \tag{4-1}$$

where, $F(t)$ is the failure distribution function and $h(t)$ is the hazard function.

Therefore, the failure distribution function of a component can be calculated using Equation (4-1) if its hazard can be estimated.

The failure distribution function and hazard are termed as independent failure distribution function and Independent Hazard (IndH) if the failures are independent. The failure probability and hazard of a component with failure interaction are described using the interactive failure distribution function and interactive hazard function. In this thesis, $F_{Ii}(t)$ and $h_{Ii}(t)$ denote the independent failure distribution function and the independent hazard function of Component i respectively; $F_i(t)$ and $h_i(t)$ denote the interactive failure distribution function and the interactive hazard of Component i respectively.

Independent hazard is either a constant or a function of time, i.e.,

$$h_{Ii}(t) = \begin{cases} \lambda_i & random\ \ failures \\ \phi_i(t) & other\ \ failures \end{cases} \qquad (i = 1, 2, \ldots, M), \tag{4-2}$$

where, M is the number of components in a system.

However, from the Definitions 4-1 and 4-2, it can be seen that the interactive hazard of a component is a function of both its own independent hazard and the hazards of its influencing components. In the case of a system consisting of two components that have interactive failures, the hazards of these two components should be expressed as:

$$h_1(t) = \varphi_1[h_{I1}(t), h_2(t)_B, t], \tag{4-3}$$

$$h_2(t) = \varphi_2[h_1(t)_B, h_{I2}(t), t], \tag{4-4}$$

where, $h_1(t)$ and $h_2(t)$ are the interactive hazards of Component 1 and Component 2 respectively. The functions $h_1(t)_B$ and $h_2(t)_B$ are the hazards of Component 1 and Component 2 before an interaction occurs, while $h_{I1}(t)$ and $h_{I2}(t)$ are the independent hazards of Component 1 and Component 2 respectively.

To generalise the model involving M components, the interactive hazards of M components in a system can be expressed as follows:

$$h_1(t) = \varphi_1[h_{I1}(t), \vec{h}_{j_1}(t)_B, t],$$

$$h_2(t) = \varphi_2[h_{I2}(t), \vec{h}_{j_2}(t)_B, t],$$

$$\vdots$$

$$h_i(t) = \varphi_i[h_{Ii}(t), \vec{h}_{j_i}(t)_B,\ t], \tag{4-5}$$

$$\vdots$$

$$h_M(t) = \varphi_M[h_{IM}(t), \vec{h}_{j_M}(t)_B, t].$$

where $h_i(t)$ and $h_{Ii}(t)$, $i = 1, 2, \ldots, M$, are the interactive hazards and the independent hazard of Component i respectively. $\vec{h}_{j_i}(t)_B$ stands for the all hazards of the influencing components of Component i before an interaction, $i = 1, 2, \ldots, M$. Subscript j_i represents the influencing components of Component i , $i = 1, 2, \ldots, M$. For example, assume that the failure of Component 2 is affected by the failures of Component 1, Component 3 and Component 5. Then $j_2 = 1, 3, 5$ and the second equation in Equation (4-5) now becomes

$$h_2(t) = \varphi_2[h_{I2}(t), h_1(t)_B,\ h_3(t)_B, h_5(t)_B, t]. \tag{4-6}$$

Equation (4-5) contains M coupled equations because the failure of a component is affected by the failures of its influencing components. On the other hand, the failure of this component can also affect the failures of its affected components.

4.3 MATHEMATICAL MODEL FOR INTERACTIVE HAZARD AND INTERACTIVE FAILURE

Different approaches can be used to build a mathematical model to describe the relationship given by Equation (4-5):

(1) Hypothetical method. This approach requires mature knowledge of maintenance engineers and a model developed using this approach is often arbitrary.

(2) Failure mechanism based method. This approach needs to understand the failure and failure interaction mechanism of assets very well and the model is often very specific.

(3) Probability theory and stochastic process based method. This approach can be used to develop a generic model but it is mathematically complex.

(4) Taylor's expansion approach. This approach can be used to derive a generic mathematical model which is more suitable for engineering applications. The approach has been applied to develop a model for the change of the core melt frequency, which is a function of the component unavailability, structure failure probabilities and initiating event frequencies [299]. Taylor's expansion has also been used to obtain an approximate mathematical expression for a random variable which is a function of several mutually independent random variables [293]. Jiang et al [300] used the Taylor expansion of a reliability function to estimate its parameters.

In this chapter, the Taylor's expansion approach is used to derive a mathematical model for interactive failures as follows:

Interactive hazard $h_i(t)$ in Equation (4-5) can be expressed by the Taylor's expansion:

$$h_i(t) = \varphi_i[h_{Ii}(t), \vec{h}_{j_i}(t)_B,\ t]$$

$$= \varphi_i\,|_{h_{j_i}(t)_B=0} + \sum_{j_i} \frac{\partial \varphi_i}{\partial h_{j_i}}\,|_{h_{j_i}(t)_B=0}\ h_{j_i}(t)_B + \sum_{j_i,k_i} \frac{\partial^2 \varphi_i}{2\partial h_{j_i} h_{k_i}}\,|_{h_{j_i}(t)_B=0}\ h_{j_i}(t)_B h_{k_i}(t)_B +$$

$$\sum_{j_i} \frac{\partial^2 \varphi_i}{2\partial h_{j_i}^2}\,|_{h_{j_i}(t)_B=0} h_{j_i}^2(t)_B + \text{higher order terms.} \qquad (4\text{-}7)$$

(Subscripts j_i and k_i represent the influencing components of Component i)

To stress the effect of the hazards of Component j_i, $h_{j_i}(t)_B$ (Subscript j_i represents the influencing components of Component i) on the hazard of Component i, $h_i(t)$ ($i = 1, 2, \ldots, M$), Equation (4-7) can be rewritten as:

$h_i(t) =$

$$\varphi_i \,|_{h_{j_i}(t)_B=0} + \sum_{j_i} [\frac{\partial \varphi_i}{\partial h_{j_i}} \,|_{h_{j_i}(t)_B=0} + \sum_{k_i} \frac{\partial^2 \varphi_i}{2\partial h_{j_i} h_{k_i}} \,|_{h_{j_i}(t)_B=0} \; h_{k_i}(t)_B + \frac{\partial^2 \varphi_i}{2\partial h_{j_i}^2} \,|_{h_{j_i}(t)_B=0} \; h_{j_i}(t)_B$$

$$+ \textit{higher order terms divided by } h_{j_i}(t)_B] \times h_{j_i}(t)_B \,. \qquad (4\text{-}8)$$

(Subscript j_i and k_i represent the influencing components of Component i)

The Component i is not influenced by its influencing components when $h_{j_i}(t)_B = 0$ (Subscript j_i represents the influencing components of Component i). In this case, the hazard of Component i is equal to its independent hazard. Therefore, the first term on the right side of Equation (4-8) represents the independent hazard of Component i, i.e.

$$\varphi_i \,|_{h_{j_i}(t)_B=0} = h_{Ii}(t)\,, \qquad (4\text{-}9)$$

(Subscript j_i represents the influencing components of Component i)

and $\varphi_i \,|_{h_{j_i}(t)_B=0} \geq 0$ according to the properties of hazard.

Therefore, the rest of the terms in Equation (4-8) show the effects of failures of the influencing components on the failure of Component i.

Let

$$\theta_{ij_i}(t) = \frac{\partial \varphi_i}{\partial h_{j_i}} \,|_{h_{j_i}(t)_B=0} + \sum_{k_i} \frac{\partial^2 \varphi_i}{2\partial h_{j_i} h_{k_i}} \,|_{h_{j_i}(t)_B=0} \; h_{k_i}(t) + \frac{\partial^2 \varphi_i}{2\partial h_{j_i}^2} \,|_{h_{j_i}(t)_B=0} \; h_{j_i}(t)_B + \ldots . \qquad (4\text{-}10)$$

(Subscripts j_i and k_i represent the influencing components of Component i)

Substituting Equations (4-9) and (4-10) into Equation (4-8), gives:

$$h_i(t) = h_{Ii}(t) + \sum_{j_i} \theta_{ij_i}(t) h_{j_i}(t)_B, \quad i = 1, 2, \ldots, M \tag{4-11}$$

(Subscript j_i represents the influencing components of Component i)

where the parameter $\theta_{ij_i}(t)$ is the Interactive Coefficient (IC) that represents the degree of the effect of failure of Component j_i on Component i.

Equation (4-11) depicts that the interactive hazard of a component is equal to its independent hazard plus some portion of the hazards of its influencing components. This analytical model has been justified by four special case studies in Section 4.7 and experiments presented in Chapter 7. From Equation (4-11), the following result can be derived in a straightforward manner.

If Component S has the first category of failure interaction with other components, then

$$h_S(t) \geq h_S(t)_B, \tag{4-12}$$

$$h_S(t)_B = h_{IS}(t). \tag{4-13}$$

If Component S has the second category of failure interaction with other components, then

$$h_S(t) \geq h_S(t)_B \geq h_{IS}(t). \tag{4-14}$$

Let $\theta_{ij}(t) = 0$ if the failure of Component j does not affect the failure of Component i, then the subscript i of j_i can be removed and Equation (4-11) can be written in a matrix form:

$$\{h(t)\} = \{h_I(t)\} + [\theta(t)]\{h(t)_B\}, \tag{4-15}$$

where $\{h(t)\}$ is a $M \times 1$ vector representing the interactive hazards and $\{h(t)_B\}$ is the $M \times 1$ hazard vector before an interaction. $\{h_I(t)\}$ is the $M \times 1$ independent hazard vector and $[\theta(t)]$ is an interactive coefficient matrix.

The interactive coefficient matrix $[\theta(t)]$ has the following properties:

(1) It is a non-negative real matrix, i.e., $\theta_{ij}(t) \geq 0$ ($i, j = 1, 2, \ldots, M$). If $\theta_{ij}(t) = 0$, then the failure of Component j has no effect on the failure of Component i. If the failure of Component j will cause Component i to fail immediately, then $\theta_{ij}(t) = 1$.

(2) Its trace is zero, i.e., $tr([\theta(t)]) \equiv 0$. This signifies that a component does not have failure interaction with itself.

(3) In most large complex systems, the interactive coefficient matrix is sparse as a single component usually has direct interactions with only a few other components in a system.

According to the relationship between failure distribution function and hazard, i.e., Equation (4-1), the interactive failure distribution functions of the components are given by:

$$\{F_i(t)\} = \{1 - \exp(-\int_0^t [h_{Ii}(t) + \sum_{j=1}^{M} \theta_{ij}(t) h_j(t)_B] dt)\} \quad (i = 1, 2, \ldots, M). \tag{4-16}$$

where, $F_i(t)$ is the interactive failure distribution function of Component i.

4.4 ESTIMATION OF INTERACTIVE COEFFICIENTS

Interactive Coefficient (IC) is a key parameter in estimating IntF. The determination of IC is not the focus of this thesis. However, selected demonstrations of determining ICs are presented as follows:

(1) ICs can be obtained using probability theory.

Consider a system with M Components 1, 2, …, M, each of which has an independent hazard $h_{Ii}(t)$ ($i = 1, 2, \ldots, M$). The conditions of these components before failure are independent of each other. Failure of any one of these will cause the rest of the components to fail immediately. This case demonstrates an interactive failure with the first category of failure interaction.

Let A_i represent the situation where Component i is fully operational at time t unaffected by any other component or common cause for $i = 1, 2, \ldots, M$. Then the independent reliability of Component i at time t, $R_{Ii}(t)$ is the probability that Component i remains fully operational at time t unaffected by other components or common cause, i.e., $R_{Ii}(t) = P(A_i)$ ($i = 1, 2, \ldots, M$). Based on Equation (4-1) and the relationship between reliability function and failure distribution function, $R(t) = 1 - F(t)$, it can be stated that:

$$R_{Ii}(t) = P(A_i) = \exp[-\int_0^t h_{Ii}(t)dt] \qquad (i = 1, 2, \ldots, M). \tag{4-17}$$

The probability that Component i remains operational at time t, $R_i(t)$ ($i = 1, 2, \ldots, M$), in this case is

$$R_i(t) = P(A_1 \cap A_2 \cap \ldots \cap A_M) \qquad (i = 1, 2, \ldots, M). \tag{4-18}$$

Since events A_1, A_2, …, A_3 are independent of each other,

$$P(A_1 \cap A_2 \cap \ldots \cap A_M) = \prod_{i=1}^{M} P(A_i) . \tag{4-19}$$

Using Equations (4-17) and (4-19) for Equation (4-18), gives

$$R_i(t) = \exp[-\int_0^t \sum_{i=1}^{M} h_{Ii}(t)dt] \qquad (i = 1, 2, \ldots, M). \tag{4-20}$$

Equation (4-20) indicates that the interactive hazard of Component i, $h_i(t)$, is

$$h_i(t) = \sum_{i=1}^{M} h_{Ii}(t) \qquad (i = 1, 2, \ldots, M). \tag{4-21}$$

Considering Equation (4-13) and comparing Equation (4-21) with Equation (4-15), ICs of this system can be obtained as follows:

$$\theta_{ij}(t) = 1 \qquad (i, j = 1, 2, \ldots, M) \text{ and } (i \neq j). \tag{4-22}$$

Probability theory enables interactive hazards and ICs to be calculated accurately. However, this approach is often inapplicable due to its mathematical complexity. In this case, ICs can be determined using the following engineering approaches so that interactive hazards can still be analysed quantitatively. The ability to determine ICs in a pragmatic manner is a major advantage of the newly developed model for IntF.

(2) ICs can be estimated according to the experiences of designers, manufacturers and maintenance staff.

(3) ICs can be calculated based on failure mechanism or/and dynamics. For example, when a bearing has some defects, the related shaft will vibrate. This vibration will increase the failure probability of the shaft. The relationship between the defects of bearing and the failure of the shaft can be determined using dynamics and fatigue failure theory. The IC can then be calculated.

(4) ICs can be determined based on laboratory experiments. An example to determine IC through laboratory experiments is presented in Chapter 7.

4.5 STABLE AND UNSTABLE INTERACTIVE FAILURE

As indicated in Section 4.2, for a system that is composed of *M* components, some of the components (*L*) ($L \leq M$) can be defined as influencing components or affected components or both in reference to their failure relationships. Deterioration in one or more of the influencing components in a system can interact with or cause deterioration of the affected components. As a result, the failure probabilities of the affected components may increase. The interaction between components can lead to a chain interaction process, as shown in Figure 4-3. The superscript (i) $(i = 1, 2, \ldots, n)$ in Figure 4-3 stands for "the ith state of failure interaction". The chain interaction process may involve two or more components (see Figure 4-4).

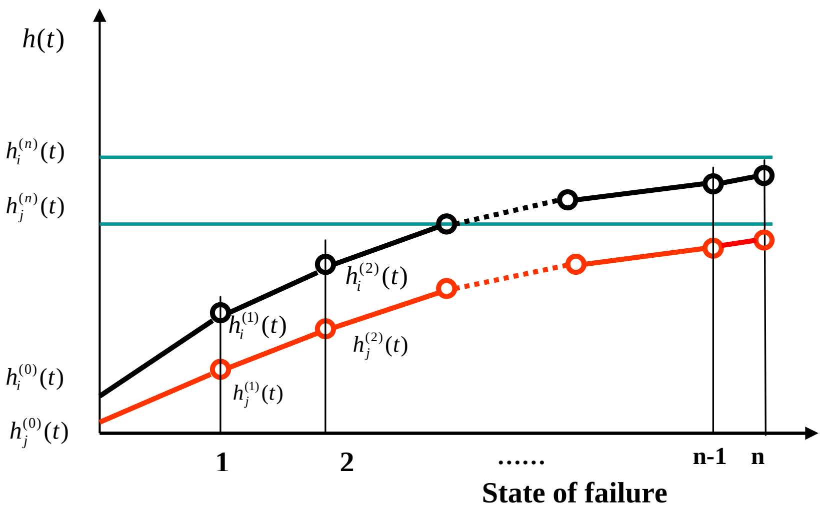


Figure 4-3. The process of failure interaction

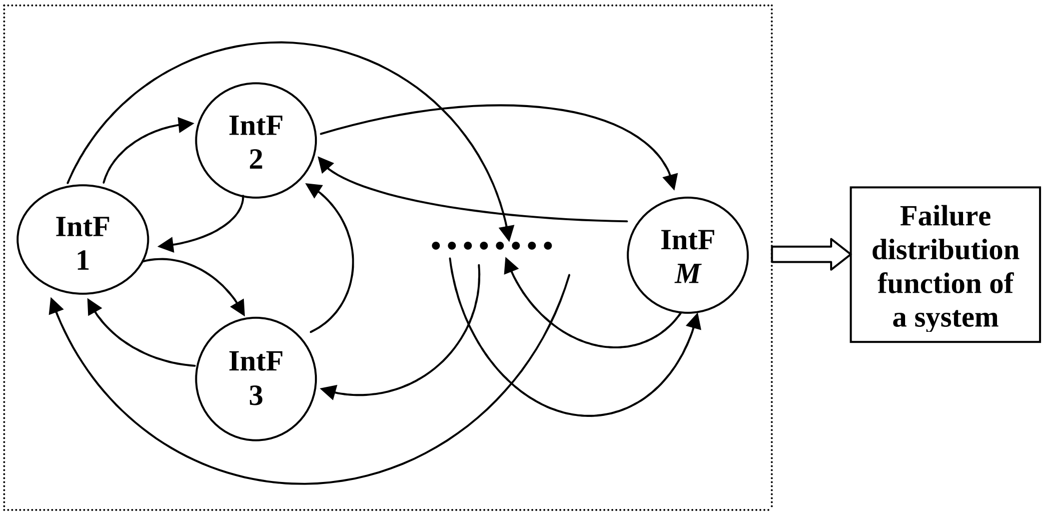


Figure 4-4. Relationship of IntFs in a system

If some components in a system are both influencing components and affected components, the failure of a component can result in an increase in deterioration of the other components. The failure of the "victims" can also increase the failure process of this component which is the original cause. This is called chain reaction of interactive failures which can continue in this manner. As a result of this chain reaction, the system may either achieve a new level of working status or eventually fail. The former is called stable interactive failure and the later, unstable interactive failure.

A stable interaction process occurs when the increment in the hazard due to failure interactions is reducing and finally converges to zero, i.e.,

$$\lim_{n\to\infty}(\sup_{t>0} | h^{(n)}(t) - h^{(n-1)}(t) |) = 0 . \qquad (4\text{-}23)$$

In this case, the hazard of a component remains stable at a new deterioration level as shown in Figure 4-5. In this diagram, $h(t)$ is a hazard function, $h^{(0)}(t)$ is the initial hazard function before interaction and $h^{(n)}(t)$ is a new hazard function after the stable interactions of the components occur. On the other hand, an unstable interaction process occurs when the hazard increases dramatically and the component is very likely to fail immediately. An example of an unstable interaction process is a rotating system which consists of a long flexible shaft and a wheel. The wheel is mounted in the middle of the shaft. The failure modes of this rotating system are unbalanced wheel and bent shaft. These two failures are interactive failures. An unbalanced rotating wheel causes the shaft to bend, and the bent shaft causes eccentricity which increases the unbalance and consequently increases the shaft bend. This chain interaction will continue until the shaft fatigues or breaks down. This failure is unstable IntF. Predictive maintenance can be carried out for stable IntFs, but not usually for unstable IntFs as the hazard increases dramatically.

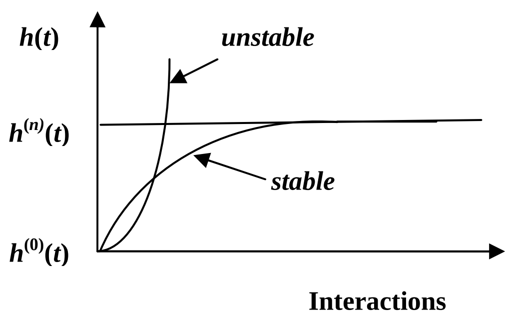


Figure 4-5. Stable and unstable IntF

Definition 4-3: In the case of considering interactive failures only, if interactions among some surviving components cause at least one of them to fail, these interactions are defined as unstable interactions. Otherwise, stable interactions result.

According to Definition 4-3, interactions which cause a cascading failure do not belong to unstable interaction because in this type of failure, the latter failure is caused by the former failure. Due to the same reason, interactions in the common cause failure are not unstable interactions if the common cause event is a failure. However, if a common cause event is not a failure, then the interactions that result in a common cause failure can be classified as unstable interaction.

Definition 4-4: The interactive failure is unstable if it is caused by unstable interactions. Similarly, the interactive failure is stable if it is caused by stable interactions.

In the case of Definition 4-3, if any component deteriorates, then at least one of the components in the system will fail very soon due to the unstable interactions among these components. On the other hand, stable interactions increase the hazard of the components. This failure process will take much longer compared with unstable interaction.

Unstable IntF indicates that the interactive hazard, and thus integrated interactive hazard, increases to an infinite value instantaneously due to the interactions among the components.

4.6 MATHEMATICAL MODELS FOR STABLE INTERACTIVE FAILURES

In Section 4.5, the physical phenomenon of stable and unstable interactive failures in a system has been explained. In this section, mathematical models will be formulated for stable interactive failures and some conditions under which the stable interactive failures exist will also be identified.

In the following derivation, the following assumptions are used.

(1) At least one element in the interactive coefficient matrix of a system is not zero. Note that there is no failure interaction in this system if all elements in the interactive coefficient matrix are zero.

(2) The effects of different components on a component are independent.

(3) A failure interaction occurs so quickly that the increase of time during the interaction can be ignored when the effects of failure interactions on the interactive hazards are considered solely.

(4) The changes of interactive coefficients during the failure interaction are also ignored. This indicates that interactive coefficients are either constants or changes very slowly compared with the changes of the hazard functions.

(5) Components and systems are not repaired. The reliability prediction of systems with PM and IntF will be investigated in the next chapter.

At the time $t\,(t \geq 0)$, the independent hazards of the components in a system are $\{h_I(t)\}$, where $\{\bullet\}$ stands for a $M \times 1$ vector. At this moment, the hazards of some components increase marginally due to their own deterioration or an external event or both. The changes of hazards result in an increase of interactive hazards because of the interactions among the components. The changes of independent hazards of the components can be ignored while failure interaction is being analysed since the time for failure interaction is usually much shorter than the time for natural deterioration of components. An interaction process can be represented by a series of discrete states and the changes of interactive hazards during this interaction process can be treated as state by state (refer to Figure 4-3). According to Equation (4-15), the first state of the interactive hazards can be expressed as:

$$\{h^{(1)}(t)\} = \{h_I(t)\} + [\theta(t)]\{h_I(t)\}\,. \qquad (4\text{-}24)$$

where $\{h^{(1)}(t)\}$ represents the $M \times 1$ interactive hazard vector at the first state of the failure interactions. It is straightforward to prove that $\{h^{(1)}(t)\} > \{h_I(t)\}$ when at least one element in $[\theta(t)]$ is not zero. Hence the failure interactions among the components will occur again and the interactive failures of the system progress to the second state. The expression for the interactive hazards at the second state is given below:

$$\{h^{(2)}(t)\} = \{h_I(t)\} + [\theta(t)]\{h^{(1)}(t)\}\,. \tag{4-25}$$

where $\{h^{(2)}(t)\}$ is the $M \times 1$ interactive hazard vector at the second state of the failure interactions.

The failure interactions among the components will continue because $\{h^{(2)}(t)\} > \{h^{(1)}(t)\}$ when at least one element in $[\theta(t)]$ is not zero. Therefore, the interactive failures of the system will progress to the third state which can be described by an equation similar to Equation (4-25). Continuing the above process, the nth state of the failure interactions is given by

$$\{h^{(n)}(t)\} = \{h_I(t)\} + [\theta(t)]\{h^{(n-1)}(t)\}\,. \tag{4-26}$$

It can also be proved that $\{h^{(n)}(t)\} > \{h^{(n-1)}(t)\}$.

For stable IntF, the increased hazard will converge to a limit. According to Equation (4-23), the following condition holds,

$$\lim_{n\to\infty}\{h^{(n)}(t)\} = \{h(t)\}\,. \tag{4-27}$$

The interactive coefficients can be used to identify whether an IntF is stable or not. If at least one pair of interactive coefficients ($\theta_{ij}(t)$ and $\theta_{ji}(t)$) in a system are equal to or greater than one, then the system has unstable IntF, i.e., whenever interaction occurs, the interacted components will fail very quickly. The above derivation is also correct if an interaction has finite states.

The following theorems for justifying the conditions for stable IntF can be proved:

Theorem 4-1: An IntF is stable, i.e., Limit (4-27) exists, if the interactive coefficient matrix meets the following conditions:

(1) $\max_{i=1,\ldots,M} \sum_{j=1}^{M} \theta_{ij}(t) < 1$

or (4-28)

$\max_{j=1,\ldots,M} \sum_{i=1}^{M} \theta_{ij}(t) < 1$

and

(2) $Det([I]-[\theta(t)]) \neq 0$, (4-29)

where, $[I]$ is a $M \times M$ identity matrix, and $Det(\bullet)$ stands for determinant operation.

Theorem 4-1 can be proved based on the following proposition and lemmas.

Proposition 4-1: For an interaction chain process described by Equation (4-26), the n^th^ state of the interactive chain process is given by

$$\{h^{(n)}(t)\} = ([I] + \sum_{s=1}^{n} [\theta(t)]^s)\{h_I(t)\}. \tag{4-30}$$

The proof of Proposition 4-1 is given in Appendix B4.

If $Det([I]-[\theta(t)]) \neq 0$, the sum of $([I] + \sum_{s=1}^{n} [\theta(t)]^s)$ can be expressed as

$$([I] + \sum_{s=1}^{n} [\theta(t)]^s) = ([I]-[\theta(t)])^{-1}([I]-[\theta(t)]^{n+1}), \tag{4-31}$$

where, $([I]-[\theta(t)])^{-1}$ is the inverse matrix of the matrix $[I]-[\theta(t)]$. The derivation of Equation (4-31) is presented in Appendix B5.

Lemma 4-1: If the interactive coefficient matrix $[\theta(t)]$ meets the conditions:

$$\max_{i=1,\ldots,M} \sum_{j=1}^{M} \theta_{ij}(t) < 1$$

or (4-32)

$$\max_{j=1,\ldots,M} \sum_{i=1}^{M} \theta_{ij}(t) < 1$$

then

$$\lim_{n\to\infty} [\theta(t)]^{n+1} = [0], \qquad (4\text{-}33)$$

where, [0] is the null matrix.

Lemma 4-1 is proved as follows.

According to Lutkepohl [301], for a real $M \times M$ matrix $[\theta(t)] \geq 0$, the following results for the spectral radius of the matrix have been obtained:

$$\rho([\theta(t)]) \leq \max_{i=1,\ldots,M} \sum_{j=1}^{M} \theta_{ij}(t), \qquad (4\text{-}34)$$

and

$$\rho([\theta(t)]) \leq \max_{j=1,\ldots,M} \sum_{i=1}^{M} \theta_{ij}(t), \qquad (4\text{-}35)$$

where, $\rho([\theta(t)])$ is the spectral radius of $[\theta(t)]$ which is defined as

$$\rho([\theta(t)]) \equiv \max\{|\lambda_e| : \lambda_e \textit{ is an eigenvalue of } [\theta(t)]\}. \qquad (4\text{-}36)$$

Substituting Equation (4-32) into Equation (4-34) or (4-35), gives

$$\rho([\theta(t)]) < 1 . \tag{4-37}$$

In line with the properties of matrices, the result that $\rho([\theta(t)]) < 1$ indicates that matrix $[\theta(t)]^n$ is convergent to a null matrix [301], i.e., Equation (4-33) holds.

Theorem 4-1 is proved below:

Proof

The hazards of the components at the nth state of interactions at time t can be rewritten as follows based on Proposition 4-1 and Equation (4-31):

$$\{h^{(n)}(t)\} = ([I] - [\theta(t)])^{-1}([I] - [\theta(t)]^{n+1})\{h_I(t)\} . \tag{4-38}$$

Under conditions (4-28) and (4-29), $\{h^{(n)}(t)\}$ will converge to a stable hazard vector with the increase of states n based on Lemma 4-1, i.e., in this case, the IntF is stable. The new stable IntH is given by

$$\{h(t)\} = [\alpha]\{h_I(t)\} , \tag{4-39}$$

where,

$$[\alpha] = ([I] - [\theta(t)])^{-1} \tag{4-40}$$

is defined as the State Influence Matrix (SIM). The SIM can determine the influence degree of failure interactions on stable IntH uniquely. The elements in SIM are often functions of time. However, for simplicity, expression α instead of $\alpha(t)$ is used in this thesis.

The conditions (4-28) and (4-29) are only sufficient conditions for stable IntF and this can be best demonstrated using the following case study:

Consider a special interactive coefficient matrix $[\theta(t)]$ of the form:

$$[\theta(t)] = \begin{bmatrix} 0 & \theta_{12} \\ \theta_{21} & 0 \end{bmatrix}. \tag{4-41}$$

Then, $[\theta(t)]^2 = \theta_{12}\theta_{21}[I]$,

$$[\theta(t)]^3 = \theta_{12}\theta_{21}\begin{bmatrix} 0 & \theta_{12} \\ \theta_{21} & 0 \end{bmatrix}.$$

$$\vdots$$

$$[\theta(t)]^n = \begin{cases} (\theta_{12}\theta_{21})^{\frac{n}{2}}[I] & n \text{ being even} \\ (\theta_{12}\theta_{21})^{\frac{n-1}{2}}\begin{bmatrix} 0 & \theta_{12} \\ \theta_{21} & 0 \end{bmatrix} & n \text{ being odd}. \end{cases} \tag{4-42}$$

Obviously, only $\theta_{12}\theta_{21} < 1$ is required for the existence of Limit (4-33).

Theorem 4-2: An IntF is stable, i.e., Limit (4-27) exists, if the interactive coefficient matrix $[\theta(t)]$ is triangular.

Proof.

According to the properties of eigenvalues [301], when the interactive coefficient matrix $[\theta(t)]$ is triangular, $\rho([\theta(t)]) = 0$ since all the diagonal elements of $[\theta(t)]$ are zero (the second property of the interactive coefficient matrix). Hence Limit (4-27) exists in this condition based on the property of spectral radii mentioned above.

An upper triangle interactive coefficient matrix indicates the case that the failure of Component *M* can affect all other components in a system but is not affected by any of them. Component *M*-1 can affect all other components in a system except Component *M* but is affected by the failure of Component *M* only. … The failure of Component 1 is affected by the failures of all other components but has no effect on any other component in the system. The case where the interactive coefficient matrix is a lower triangle matrix is the opposite of the above case.

Theorem 4-2 also gives sufficient conditions for stable IntF. In practice, the identification of a stable IntF would be much more straightforward for a specific system.

In accordance with Equation (4-39) and the relationship between failure distribution function and hazard (Equation (4-1)), the Interactive Failure Distribution Functions (IntFDFs) of the components in a system are given by

$$\{F_i(t)\} = \{1 - \exp[-\int_0^t \sum_{j=1}^{M} \alpha_{ij} h_{Ij}(t) dt]\} \qquad (i = 1, 2, \ldots, M), \tag{4-43}$$

where, α_{ij} is the i[th] row j[th] column element in the SIM $[\alpha]$. Equation (4-43) shows that the likelihoods of failures for components with failure interactions have increased because $\alpha_{ii} \geq 1$ and at least one $\alpha_{ij} > 0$ ($i \neq j$), if the interactive coefficients $\theta_{ij}(t)$ ($i, j = 1, 2, \ldots, M$) are not all zero (refer to Appendices B6 and B7). The characteristics of the interactive failure distribution of an affected component can be different from that of its original independent failure distribution.

Equations (4-15), (4-39), (4-40) and (4-43) are integrated as an Analytical Model for Interactive Failures (AMIF).

4.7 MODEL JUSTIFICATION

In this section, AMIF will be justified through the consideration of the following four special case studies. More sophisticated verifications through simulation experiments will be presented in Section 4.8. Laboratory experiments undertaken to verify the model will be presented in Chapter 7.

4.7.1 Special Case 1: Multiple Causes Failure

A system is composed of M components. It is assumed that only one component (Component 1) is affected by its influencing Component j ($j = 2, 3, \ldots, L_1$, $L_1 \leq M$). The failure of Component 1 does not affect other components. Component

j ($j = 2, 3, \ldots, M$) in the system have no failure interaction with each other. In this case, the interaction will stop at the first state of interaction so that the IntHs of all components at n states of interactions among components are the same as their IntHs at the first state of interaction. The interactive failure matrix in this case is

$$[\theta(t)] = \begin{bmatrix} 0 & \vec{\theta} \\ \vec{0}_1 & \vec{0}_2 \end{bmatrix}, \tag{4-44}$$

where, $\vec{\theta}$ is a $1 \times (M-1)$ vector with L_1 non-zero elements and $M-1-L_1$ null elements; $\vec{0}_1$ is a $(M-1) \times 1$ null vector; and $\vec{0}_2$ is a $(M-1) \times (M-1)$ null matrix.

Therefore, according to Equation (4-15), the IntHs of the components at the first state of the interaction is

$$\{h^{(1)}(t)\} = \begin{bmatrix} 1 & \vec{\theta} \\ \vec{0}_1 & \vec{I} \end{bmatrix} \{h_I(t)\}, \tag{4-45}$$

where, $\vec{I}$ is a $(M-1) \times (M-1)$ unit matrix.

It is straightforward to know the inverse matrix $([I] - [\theta(t)])^{-1}$ is $\begin{bmatrix} 1 & \vec{\theta} \\ \vec{0}_1 & \vec{I} \end{bmatrix}$, and

$$[\theta(t)]^n = \begin{bmatrix} 0 & \vec{\theta} \\ \vec{0}_1 & \vec{0}_2 \end{bmatrix}^n = [0] \ \ (\mathit{for\ all}\ n \geq 2). \tag{4-46}$$

Substituting Equation (4-46) into Equation (4-30) and using Equation (4-39), one can conclude that the all states of interaction in this case are the same as the first state, which is described by Equation (4-45). This result is exactly the same as expected.

Specially, if Component 1 is assumed to fail immediately if any its influencing components fail and the conditions of all components before failure are independent, then according to the first property of IC, the L_1 non-zero elements in vector $\vec{\theta}$ in Equation (4-44) all equal one. Using Equations (4-39), (4-40), (4-33) and (4-43), the reliability functions of the components can be obtained as follows

$$R_i(t) = \begin{cases} \exp[-\int_0^t \sum_{j=1}^{L_1} h_{Ij}(t)dt] & i = 1 \\ \exp[-\int_0^t h_{Ii}(t)dt] & i = 2, 3, \ldots, M \end{cases}, \tag{4-47}$$

where, $h_{Ii}(t)$ is the IndH of Component i ($i = 1, 2, 3, \ldots, M$).

Equation (4-47) can be justified using probability theory. Let A_i represent the situation where Component i is fully operational at time t unaffected by all other components or common cause for $i = 1, 2, 3, \ldots, M$. Then the independent reliability of Component i at time t, $R_{Ii}(t)$ is the probability that Component i remains fully operational at time t unaffected by other components or common cause, i.e., $R_{Ii}(t) = P(A_i)$ ($i = 1, 2, 3, \ldots, M$). Based on Equation (4-1) and the relationship between reliability function and failure distribution function, $R(t) = 1 - F(t)$, it can be stated that:

$$R_{Ii}(t) = P(A_i) = \exp[-\int_0^t h_{Ii}(t)dt] \quad (i = 1, 2, 3, \ldots, M). \tag{4-48}$$

The reliability of all components except for Component 1 is the same as their independent reliability since their failures are not affected by other components, i.e.

$$R_i(t) = P(A_i) = \exp[-\int_0^t h_{Ii}(t)dt] \quad (i = 2, 3, \ldots, M). \tag{4-49}$$

The probability that Component 1 remains operational at time t, $R_1(t)$, in this case is

$$R_1(t) = P(\bigcap_{j=1}^{L_1} A_j). \tag{4-50}$$

Since events A_1, A_2, …, and A_{L_1} are independent of each other,

$$P(\bigcap_{j=1}^{L_1} A_j) = \prod_{j=1}^{L_1} P(A_j)\,. \tag{4-51}$$

Substituting Equations (4-48) and (4-51) into Equation (4-50), gives

$$R_1(t) = \exp[-\int_0^t \sum_{j=1}^{L_1} h_{Ij}(t)dt]\,. \tag{4-52}$$

Integrating Equation (4-52) with Equation (4-49), gives Equation (4-47).

4.7.2 Special Case 2: Independent failure

When the failures of the components in a system are independent of each other, all interactive coefficients equal zero.

$$\theta_{ij}(t) = 0 \quad (i, j = 1, 2, \ldots, M\ ). \tag{4-53}$$

Substituting Equation (4-53) into Equation (4-15) gives

$$\{h_i(t)\} = \{h_{Ii}(t)\} \quad (i = 1, 2, ..., M\ ). \tag{4-54}$$

Equation (4-54) shows that the interactive hazard of Component i is determined by its own independent hazard as expected.

4.7.3 Special Case 3: Common Cause Failure

Component K has an independent hazard $h_{IK}(t)$ and its failure is independent of the conditions of other components. It is assumed that whenever Component K fails, Component 1, Component 2…, and Component N in a system all fail at the same time and the failures of Component 1, Component 2…, and Component N do not have interactive relationship. This is defined as a special case of common cause failure, which was studied by Fleming [286] while developing the β-factor model. In this case, Component K is the influencing component of Component 1, Component 2…, and Component N. The interactive coefficient $\theta_{ij}(t)$ is given by

$$\theta_{ij}(t) = \begin{cases} 1 & i = 1, 2, ..., N, \quad j = K \\ 0 & \textit{others.} \end{cases} \qquad (4\text{-}55)$$

Substituting Equations (4-13) and (4-55) into Equation (4-11) gives the interactive hazards of the components in the system as follows:

$$h_i(t) = \begin{cases} h_{Ii}(t) + h_{IK}(t) & i = 1, 2, ..., N \\ h_{IK}(t) & i = K \end{cases} . \qquad (4\text{-}56)$$

Equation (4-56) indicates that the interactive hazard of Component i ($i = 1, 2, \ldots, N$) is greater than its own independent hazard because $h_{IK}(t) > 0$. If $h_{Ii}(t) = \lambda_{Ii}$ ($i = 1, 2, \cdots, N$) and $h_{IK}(t) = \beta_c \lambda$, where β_c is the "common cause factor", Equation (4-56) gives exactly the same result as that obtained using the generalised β-factor model [8]. In particular, when $h_{Ii}(t) = \lambda_I$ ($i = 1, 2, \cdots, N$), Equation (4-56) gives exactly the same result as stated by Fleming [286].

4.7.4 Special Case 4: Common Cause Shock

A system is composed of n identical components with the same independent hazard rate λ_I. The failure time of each component is independent of each other. A common cause shock occurs with an occurrence rate ν. The failure probability of each individual component due to the effect of a common cause shock is p. Shocks and the independent failures of individual components occur independently of each other. This case was investigated by Vesely [287] in 1977 while developing the Binomial Failure Rate (BFR) model. According to his research, the total hazard of one component is equal to

$$\lambda = \lambda_I + p\nu . \qquad (4\text{-}57)$$

Equation (4-57) can also be derived from Equation (4-15). Let $h_i(t)$ denote the total hazard of each component and $h_{Ii}(t)$ denote the independent hazard of each component, then,

$$h_i(t) = \lambda \quad (i = 1, 2, ..., n), \tag{4-58}$$

$$h_{Ii}(t) = \lambda_I \quad (i = 1, 2, ..., n). \tag{4-59}$$

Let $h_{n+1}(t)$ denote the occurrence rate of the common cause shock and let the interactive coefficient denote the failure probability of each individual component due to effect of a common cause shock, then

$$h_{n+1}(t) = \nu . \tag{4-60}$$

and

$$[\theta(t)] = \begin{bmatrix} 0 & 0 & \cdots & 0 & p \\ 0 & 0 & \cdots & 0 & p \\ \vdots & & \cdots & & \vdots \\ 0 & 0 & \cdots & 0 & p \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{4-61}$$

The interactive coefficient matrix $[\theta(t)]$ in this case is an upper triangle matrix with $0 \le p \le 1$. In accordance with Theorem 4-2, the IntF in this case is stable. The SIM is

$$[\alpha] = \begin{bmatrix} 1 & 0 & \cdots & 0 & p \\ 0 & 1 & \cdots & 0 & p \\ \vdots & & \cdots & & \vdots \\ 0 & 0 & \cdots & 1 & p \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}. \tag{4-62}$$

Substituting Equations (4-58), (4-59), (4-60) and (4-62) into Equation (4-39), gives Equation (4-57).

In this section, four special interactive failure cases have been studied using AMIF developed in this chapter. The results justified AMIF comparing with exiting models or methods that have been proved in their specific applications.

4.8 ANALYSIS OF INTERACTIVE FAILURES OF COMPONENTS

To calculate IntF using Equation (4-43) for an engineering system, the interactive relationship among components in the system must be identified. This interactive relationship can be expressed using a relationship chart [302]. Then IC can be determined and furthermore the interactive coefficient matrix can be constructed. After the interactive coefficient matrix has been obtained, the interactive failure distribution functions of these components can be calculated if their independent failure distribution functions are known. The procedures of calculating and analysing IntF of components are best explained through an example as follows:

A system consists of three components with every Interactive Coefficient (IC) having a value less than one. The independent failure distribution function of these three components is assumed exponential and is given by

$$\{F_{Ii}(t)\} = \{1 - \exp(-\lambda_i t)\} \quad (i = 1, 2, 3). \tag{4-63}$$

Therefore, their independent hazards are

$$h_{Ii}(t) = \lambda_i \qquad (i = 1, 2, 3). \tag{4-64}$$

Figure 4-6 is the relationship chart of these three components. In this diagram an oval represents a component. An arrow line represents an interactive relationship. An arrow line starts from Oval i ($i = 1, 2, 3$) and points to Oval j ($j = 1, 2, 3$) if the failure of Component i has an effect on the failure of Component j. Figure 4-6 indicates that there is interactive relationship between Component 1 and Component 2, and between Component 1 and Component 3. However, there is no interactive relationship between Component 2 and Component 3.

Based on the relationship chart, the interaction relationship matrix can be developed (Table 4-1). ICs are assumed to be time independent. In Table 4-1, θ_{ij} is an IC representing the effective degree of the failure of Component j on Component i ($i, j = 1, 2, 3$). That $\theta_{ij} = 1$ means that the failure of Component j has full effect on

Component i. That $\theta_{ij}=0$ indicates that the failure of Component j does not affect Component i directly.

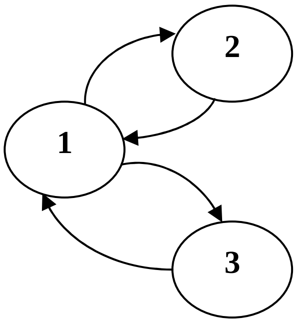


Figure 4-6. Relationship chart

Table 4-1
Relationship matrix

Components	1	2	3
1	0	θ_{12}	θ_{13}
2	θ_{21}	0	0
3	θ_{31}	0	0

Consistent with the relationship table, the interactive coefficient matrix of the system is as follows:

$$[\theta(t)]=\begin{bmatrix}0 & \theta_{12} & \theta_{13}\\ \theta_{21} & 0 & 0\\ \theta_{31} & 0 & 0\end{bmatrix}. \tag{4-65}$$

Hence,

$$[\alpha]=([I]-[\theta(t)])^{-1}=\begin{bmatrix}1 & -\theta_{12} & -\theta_{13}\\ -\theta_{21} & 1 & 0\\ -\theta_{31} & 0 & 1\end{bmatrix}^{-1}. \tag{4-66}$$

Using the Gauss-Jordan reduction method, gives

$$\begin{bmatrix}1 & -\theta_{12} & -\theta_{13}\\ -\theta_{21} & 1 & 0\\ -\theta_{31} & 0 & 1\end{bmatrix}^{-1}=\frac{1}{1-\theta_{12}\theta_{21}-\theta_{13}\theta_{31}}\begin{bmatrix}1 & \theta_{12} & \theta_{13}\\ \theta_{21} & 1-\theta_{13}\theta_{31} & \theta_{13}\theta_{21}\\ \theta_{31} & \theta_{12}\theta_{31} & 1-\theta_{12}\theta_{21}\end{bmatrix}. \tag{4-67}$$

The interactive hazard functions of the components for stable IntF can be calculated by substituting Equations (4-66) and (4-67) into Equation (4-39):

$$\begin{Bmatrix} h_1(t) \\ h_2(t) \\ h_3(t) \end{Bmatrix} = \frac{1}{1-\theta_{12}\theta_{21}-\theta_{13}\theta_{31}} \begin{bmatrix} 1 & \theta_{12} & \theta_{13} \\ \theta_{21} & 1-\theta_{13}\theta_{31} & \theta_{13}\theta_{21} \\ \theta_{31} & \theta_{12}\theta_{31} & 1-\theta_{12}\theta_{21} \end{bmatrix} \begin{Bmatrix} h_{I1}(t) \\ h_{I2}(t) \\ h_{I3}(t) \end{Bmatrix}. \tag{4-68}$$

In the above analysis, the following inequity is implied:

$$1-\theta_{12}\theta_{21}-\theta_{13}\theta_{31} > 0. \tag{4-69}$$

The sufficient condition for Inequity (4-69) is

$$\max\{\theta_{ij} : i, j = 1, 2, 3, \quad i \neq j\} < \frac{1}{\sqrt{2}}. \tag{4-70}$$

According to the relationship between hazard and the failure distribution function, the interactive failure distribution functions of these three components are given by

$$F_1(t) = 1-\exp\left[\frac{-(\lambda_1+\theta_{12}\lambda_2+\theta_{13}\lambda_3)t}{1-\theta_{12}\theta_{21}-\theta_{13}\theta_{31}}\right] \tag{4-71}$$

$$F_2(t) = 1-\exp\left[\frac{-(\theta_{21}\lambda_1+(1-\theta_{13}\theta_{31})\lambda_2+\theta_{13}\theta_{21}\lambda_3)t}{1-\theta_{12}\theta_{21}-\theta_{13}\theta_{31}}\right] \tag{4-72}$$

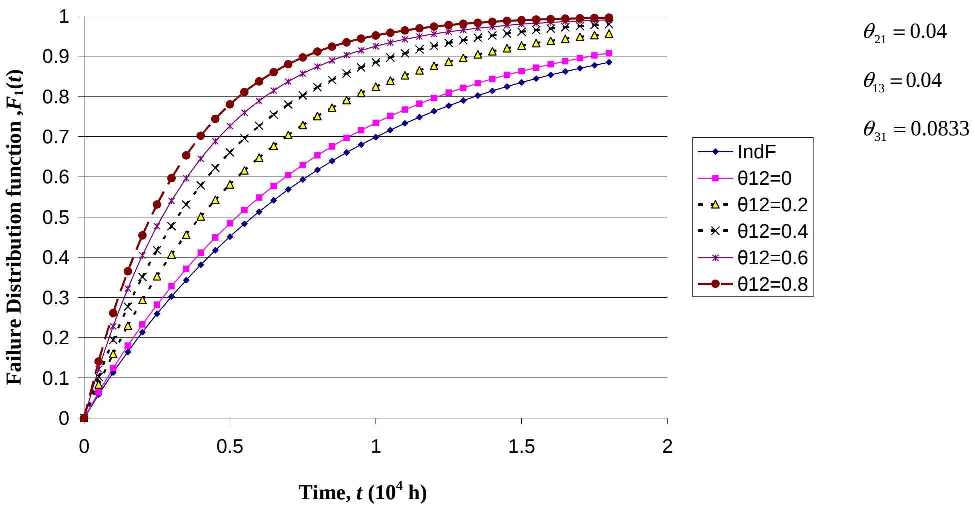
$$F_3(t) = 1-\exp\left[\frac{-(\theta_{31}\lambda_1+\theta_{12}\theta_{31}\lambda_2+(1-\theta_{12}\theta_{21})\lambda_3)t}{1-\theta_{12}\theta_{21}-\theta_{13}\theta_{31}}\right] \tag{4-73}$$

4.9 PROPERTIES OF INTERACTIVE FAILURES

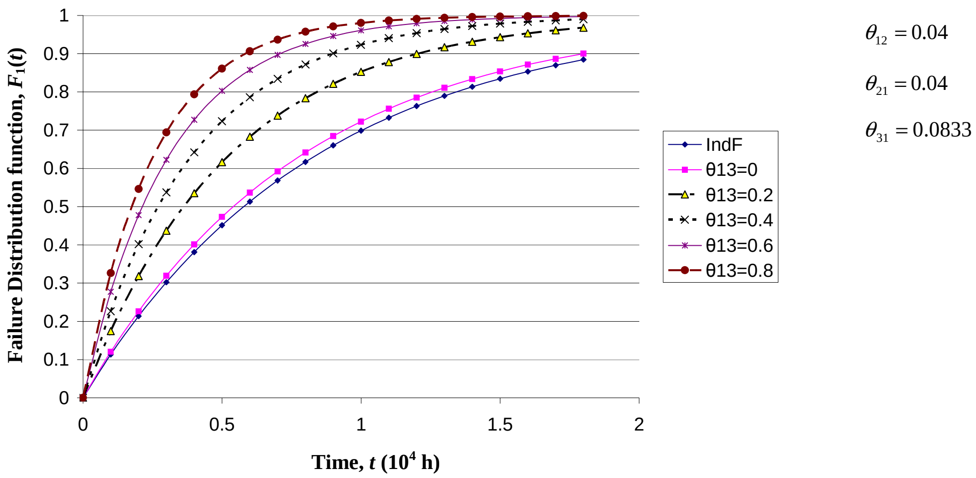
This section focuses on further investigation of the effects of IntF on components. The effects of IntF on systems will be investigated in the next section.

From Equations (4-72) and (4-73), it can be seen that the failures of Component 2 and Component 3 do interact through Component 1, although these two components do not have direct interaction. This phenomenon demonstrates an important property of failure interaction relationship - transmissibility.

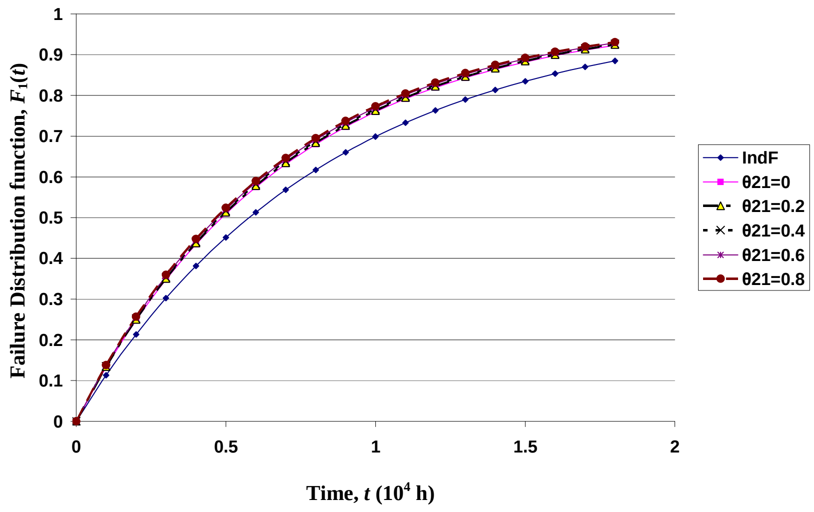
To investigate the other properties of IntF, simulations were conducted using the example presented in the above section. Figures 4-7 to 4-9 show the changes of IntFs of the components with interactive coefficients.



(a) Effects of IC θ_{12} on the IntF of Component 1



(b) Effects of IC θ_{13} on the IntF of Component 1

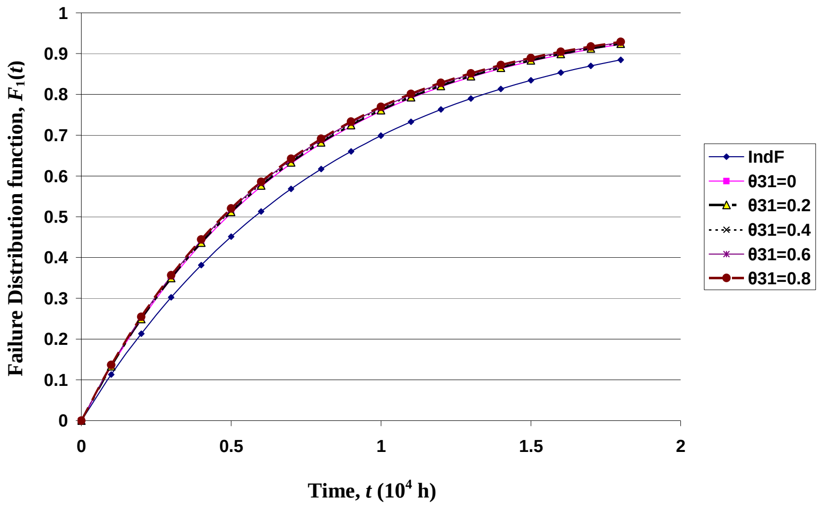


$\theta_{12} = 0.05$

$\theta_{13} = 0.04$

$\theta_{31} = 0.0833$

(c) Effects of IC θ_{21} on the IntF of Component 1



$\theta_{12} = 0.05$

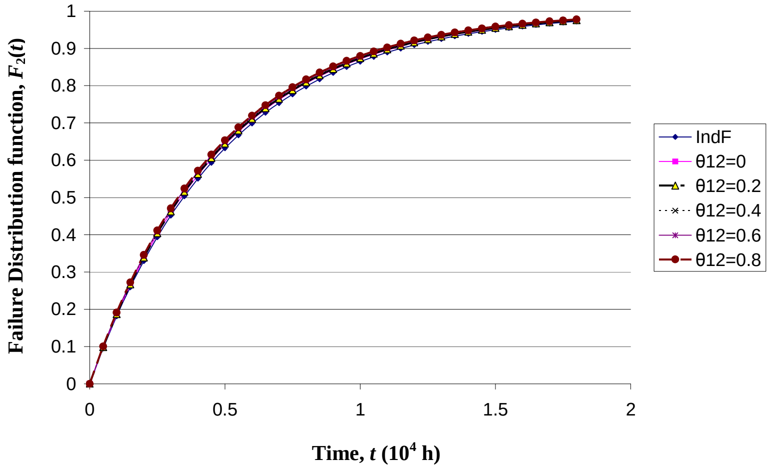
$\theta_{21} = 0.04$

$\theta_{13} = 0.04$

(d) Effects of IC θ_{31} on the IntF of Component 1

$\lambda_1 = 1.2 \times 10^{-4}\ (1/h)$ $\lambda_2 = 2 \times 10^{-4}\ (1/h)$ $\lambda_3 = 3 \times 10^{-4}\ (1/h)$

Figure 4-7. Interactive failure of Component 1 versus ICs



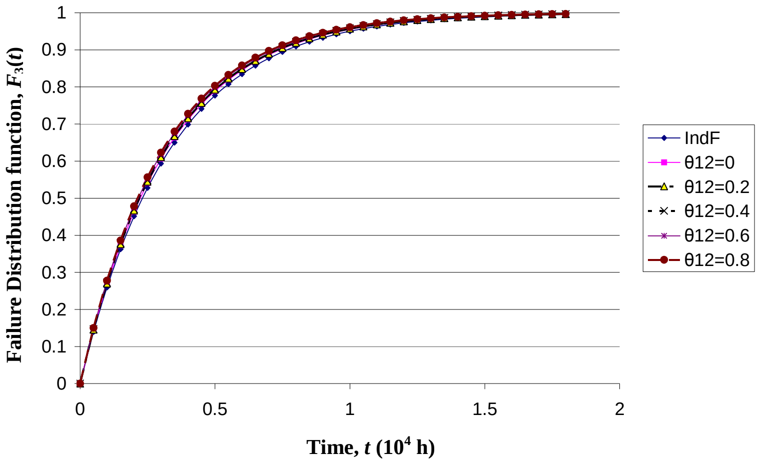
$\theta_{21} = 0.04$

$\theta_{13} = 0.04$

$\theta_{31} = 0.0833$

$\lambda_1 = 1.2 \times 10^{-4}\ (1/h)$ $\lambda_2 = 2 \times 10^{-4}\ (1/h)$ $\lambda_3 = 3 \times 10^{-4}\ (1/h)$

Figure 4-8. Interactive failure of Component 2 versus IC θ_{12}



$\theta_{21} = 0.04$

$\theta_{13} = 0.04$

$\theta_{31} = 0.0833$

$\lambda_1 = 1.2 \times 10^{-4}\ (1/h)$ $\lambda_2 = 2 \times 10^{-4}\ (1/h)$ $\lambda_3 = 3 \times 10^{-4}\ (1/h)$

Figure 4-9. Interactive failure of Component 3 versus IC θ_{12}

Figure 4-7 indicates that the failure likelihood of Component 1 increases with ICs, but different IC has different degree of influence. This characteristic can be applied to other two components. Furthermore, comparing Figure 4-7 (a) with Figures 4-8 and 4-9, one can find that interactive coefficients have different effects on different components. In this example, the interactive coefficient θ_{12} has much greater effect on Component 1 than on the other two components.

Figures 4-10 and 4-11 demonstrate the impact of changing values of θ_{12} and θ_{13} on the Mean Time To Failure (MTTF) of the components. From these two figures, it can be found that the failure interaction between the components will shorten the MTTF of the components. With the increase of θ_{12} or θ_{13}, the MTTF of Component 1 decreases sharply whereas the MTTF of the other two components is not very sensitive to θ_{12} and θ_{13} because Component 1 was affected by θ_{12} and θ_{13} directly.

Figures 4-12 and 4-13 present the influence of the IndF of Component 2 and Component 3 on the IntF of Component 1 respectively. From these two figures, it can be seen that the independent failure distribution of Component 2, $F_{I2}(t)$, has much greater influence on the IntF of Component 1 than the independent failure distribution of Component 3, $F_{I3}(t)$, because θ_{12} is greater than θ_{13}. The failure of Component 2 has almost full effect on Component 1 because θ_{12} is close to 1 (0.8). On the other hand, the failure of Component 3 has little influence on the failure of Component 1 because the value of θ_{13} is very small (0.008).

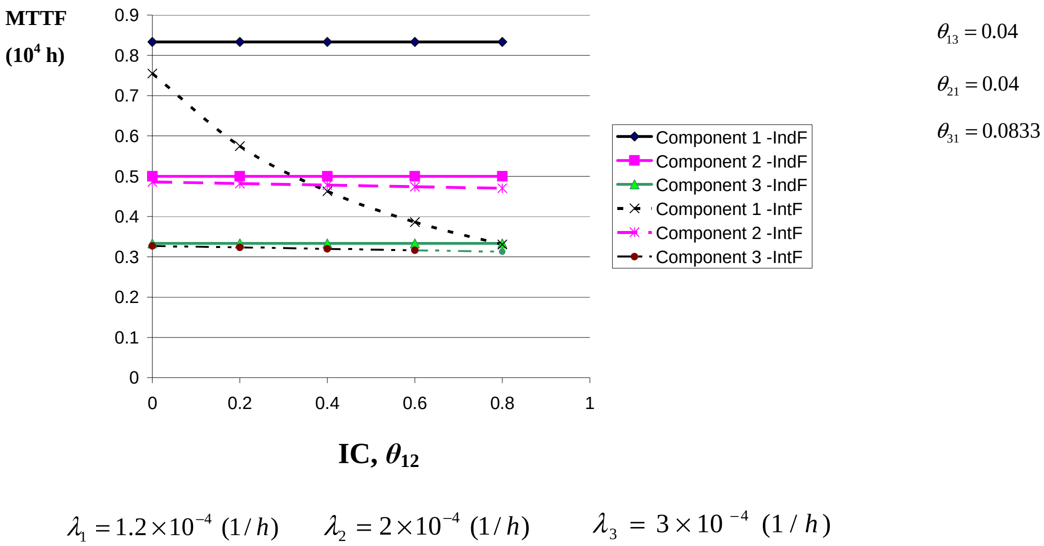
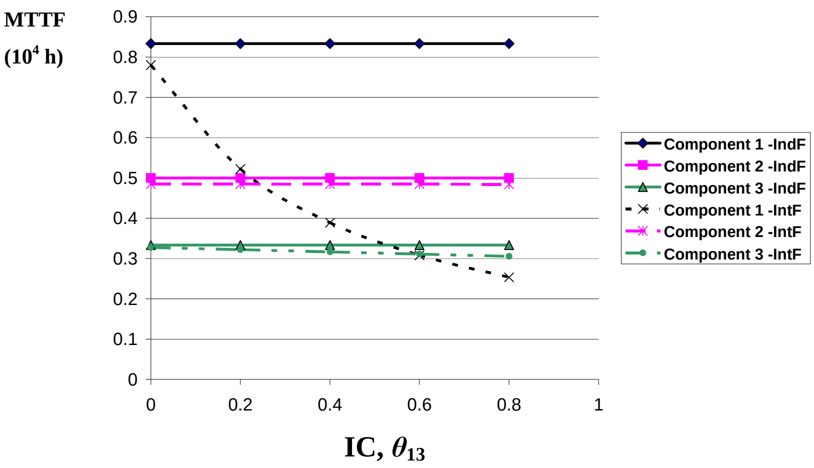


Figure 4-10. Relationship between MTTF and IC θ_{12}



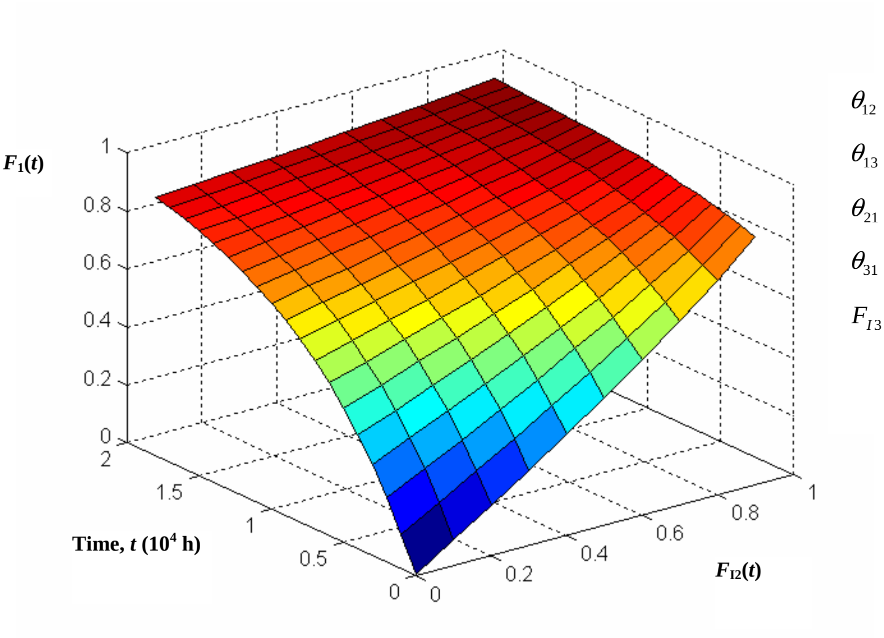
$\theta_{12} = 0.04$

$\theta_{21} = 0.04$

$\theta_{31} = 0.0833$

$\lambda_1 = 1.2 \times 10^{-4}$ $(1/h)$ $\lambda_2 = 2 \times 10^{-4}$ $(1/h)$ $\lambda_3 = 3 \times 10^{-4}$ $(1/h)$

Figure 4-11. Relationship between MTTF and IC θ_{13}



$\theta_{12} = 0.8$

$\theta_{13} = 0.008$

$\theta_{21} = 0.04$

$\theta_{31} = 0.0833$

$F_{I3}(t) = 0.5$

$\lambda_1 = 1.2 \times 10^{-4}$ $(1/h)$

Figure 4-12. Influence of the IndF of Component 2, $F_{I2}(t)$ on the IntF of Component 1, $F_1(t)$

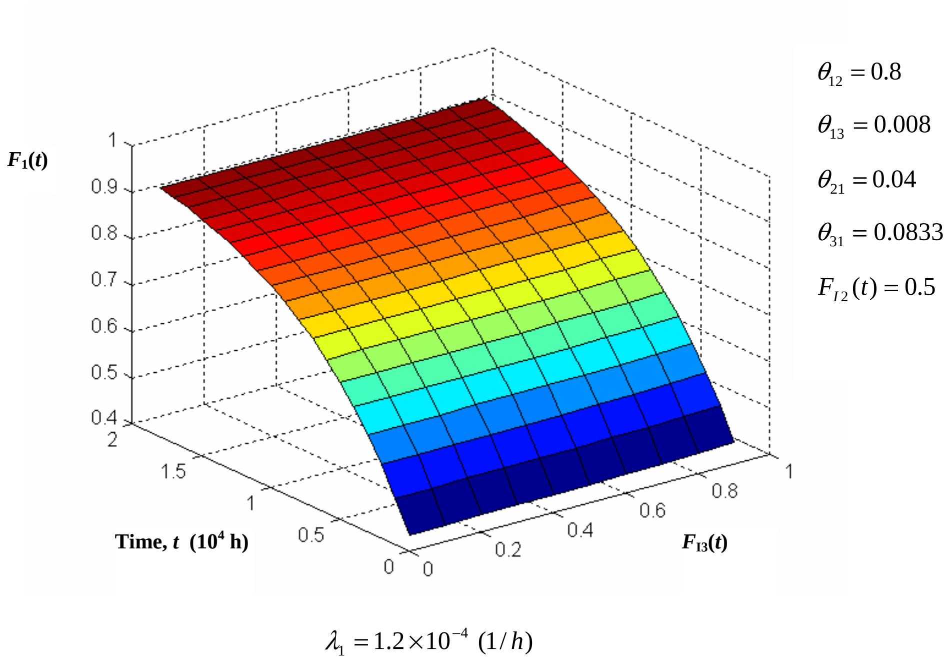


Figure 4-13. Influence of the IndF of Component 3, $F_{I3}(t)$ on the IntF of Component 1, $F_1(t)$

4.10 EFFECTS OF INTERACTIVE FAILURES ON SYSTEMS

As indicated in Section 4.2, interactive failures can be classified into two categories: immediate interactive failure and gradual degradation interactive failure.

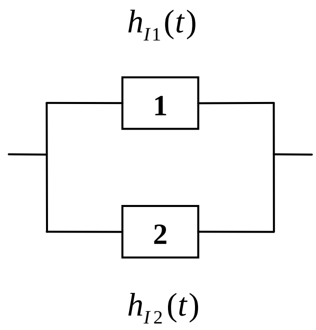
When an immediate interactive failure occurs, the failure of a component is not only related to its own deterioration but also completely dependent on the failure of its influencing components. The affected components either fail simultaneously such as common cause failure or the failure of an influencing component will lead in the failure of its affected component immediately such as cascading failure. The conditions of the influencing components before failure do not affect the failure probability of the affected components. For example, a water supply system consists of a generator and several pumps in a pump station. The generator supplies power for these pumps. A generator is regarded as failed if it is not capable of generating electricity at the same frequency and in a steady state manner. On the other hand, the influence of an unstable power supply of the generator could be ignored. Then when the generator fails, all these pumps will fail to work immediately. However, the condition of the generator before failure usually does not affect the failure of these pumps.

When a gradual degradation interactive failure occurs, the failure interaction among components increases the failure likelihood of the affected components only. The failures of the components are independent. For example, a faulty bearing (Bearing 1) will accelerate the failure rate of another bearing (Bearing 2) on the same shaft. However, when Bearing 1 fails, Bearing 2 may not fail, and vice versa.

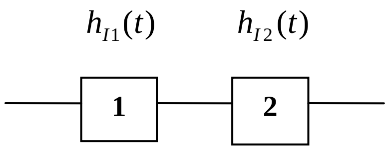
Different techniques are required to analyse the reliabilities of systems with different categories of IntFs. To calculate the reliability of a system with the first category of IntF, the original RBD of this system should be modified. For example, a parallel system shown in Figure 4-14 (a) is composed of two components: Component 1 with an IndH of $h_{I1}(t)$ and Component 2 with an IndH of $h_{I2}(t)$. The failures of these two components are “positive dependent”. The failure of Component 1 will cause Component 2 to fail immediately and vice versa. When the reliability of this parallel system is calculated, the system should be converted to a series system shown in Figure 4-14 (b). If these two components are affected by a common failure cause with an IndH of $h_{IC}(t)$, the original parallel system should be converted into a complex system in which a “virtual” Component C representing the common cause is connected with the original system in series (see Figure 4-14 (c)).

For the reliability of a system with the first category of IntF, the reliability functions of the components in this system do not need to change because failure dependency is considered through changing the RBD of the system. In this case, the reliability functions of the components used to calculate the reliability function of the system are still their original independent reliability functions.

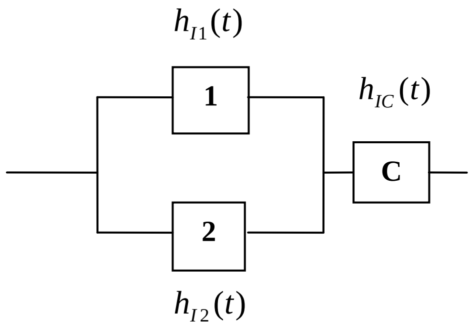
However, when analysing the reliability of a system with the second category of IntF, one should not change the RBD of this system, but needs to use the interactive reliability functions or the interactive failure distribution functions of the components of the system in the analysis. This thesis focuses on the second category of IntFs as mentioned in Section 4.2.



(a) Original parallel system



(b) Equivalent system 1



(c) Equivalent system 2

Figure 4-14. A parallel system and its equivalent system

To demonstrate the effects of the second category of IntF on systems, two different systems consisting of the three components that were described in Section 4.8, System A and System B, are considered. In System A, these three components connect with each other in series as shown in Figure 4-15 and in System B, they connect in a combined way as shown in Figure 4-16.

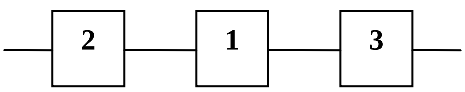


Figure 4-15. System A

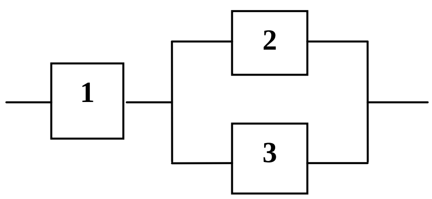


Figure 4-16. System B

The failure distribution function of System A is

$$F_A(t) = 1 - [1 - F_1(t)][1 - F_2(t)][1 - F_3(t)]\,. \qquad (4\text{-}74)$$

The failure distribution function of System B is

$$F_B(t) = 1 - [1 - F_1(t)][1 - F_2(t)F_3(t)]\,. \qquad (4\text{-}75)$$

Figure 4-17 to Figure 4-21 demonstrate the changes of the cumulative interactive failure distributions of these two systems with IC. In Figures 4-17, 4-18 and 4-19, $\lambda_1 = 1.2\times10^{-4}\ (1/h)$, $\lambda_2 = 2\times10^{-4}\ (1/h)$ and $\lambda_3 = 3\times10^{-4}\ (1/h)$.

From Figure 4-17 to Figure 4-19, it can be seen that effects of IC are different if the topologies of systems are different. In this example, failure probabilities of both systems increase with θ_{12}, but θ_{12} has greater influence on the IntF of System A than the IntF of System B. Figures 4-20 and 4-21 present the same properties. The reason is that the failure probabilities of Components 2 and 3 made a larger contribution to the system failure probability in a series system than in a parallel system.

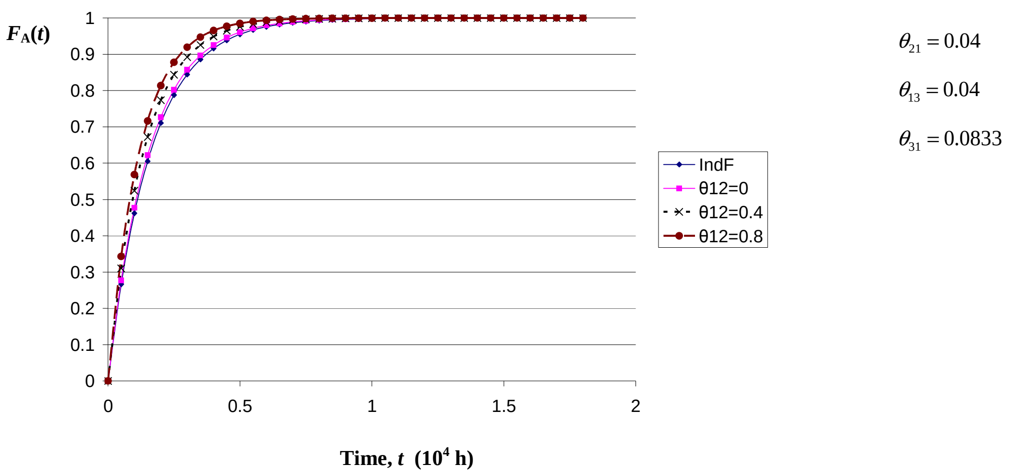


Figure 4-17. Relationship between IntF of System A, $F_A(t)$ and IC θ_{12}

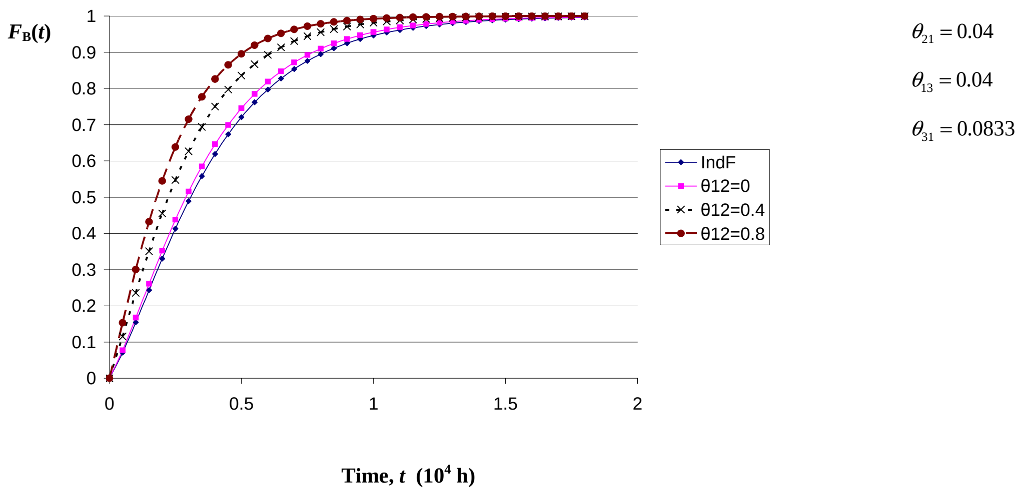


Figure 4-18. Relationship between IntF of System B, $F_B(t)$ and IC θ_{12}

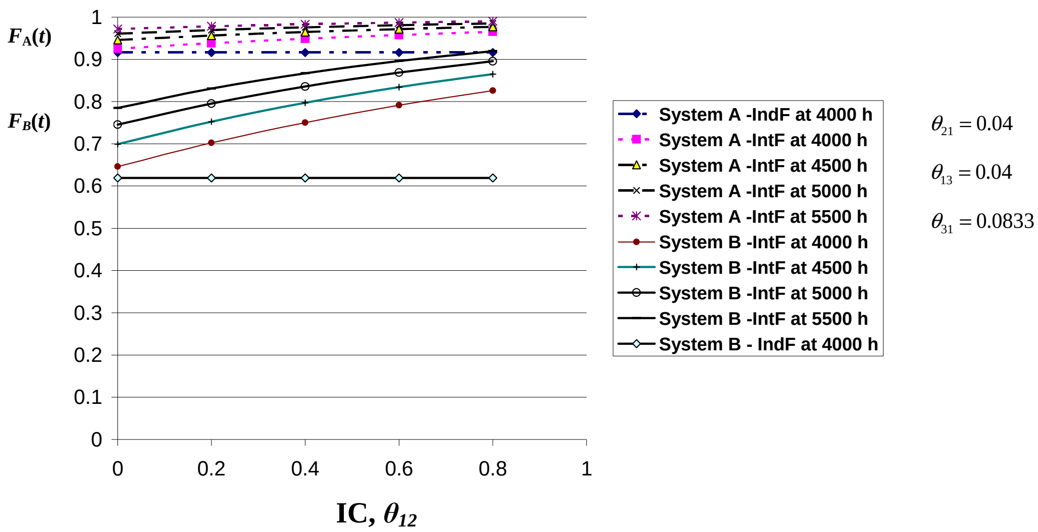
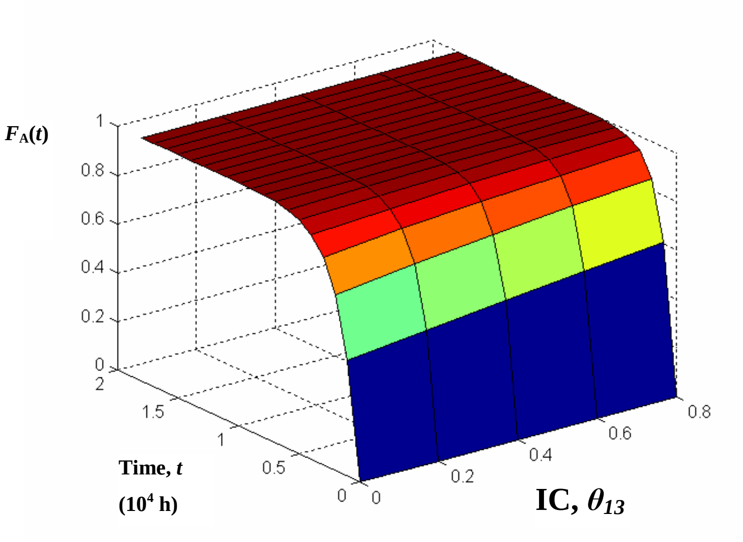


Figure 4-19. IntFs of the systems, $F_A(t)$ and $F_B(t)$, versus IC θ_{12}



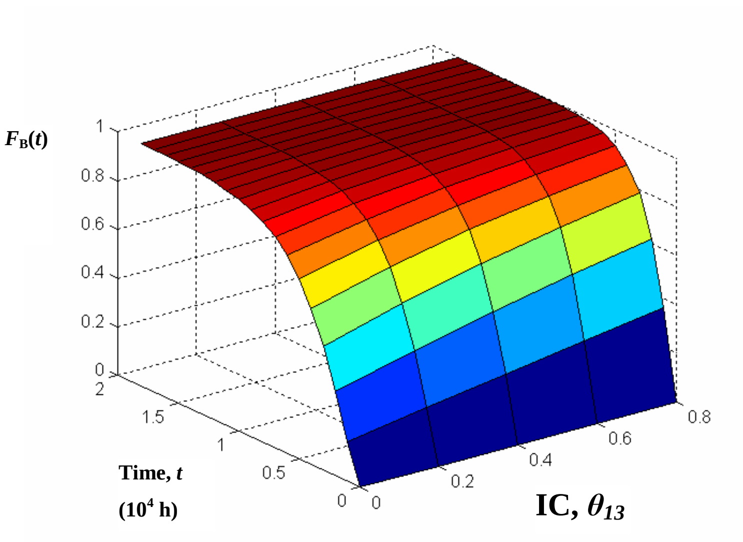
$\theta_{12} = 0.04$

$\theta_{21} = 0.04$

$\theta_{31} = 0.0833$

$\lambda_1 = 1.2 \times 10^{-4}$ $(1/h)$ $\lambda_2 = 2 \times 10^{-4}$ $(1/h)$ $\lambda_3 = 3 \times 10^{-4}$ $(1/h)$

Figure 4-20. Changes of IntF of System A, $F_A(t)$ with IC θ_{13} and time t



$\theta_{12} = 0.04$

$\theta_{21} = 0.04$

$\theta_{31} = 0.0833$

$\lambda_1 = 1.2 \times 10^{-4}$ $(1/h)$ $\lambda_2 = 2 \times 10^{-4}$ $(1/h)$ $\lambda_3 = 3 \times 10^{-4}$ $(1/h)$

Figure 4-21. Changes of IntF of System B, $F_B(t)$ with IC θ_{13} and time t

4.11 SUMMARY

The concept of interactive failure presented in this chapter is a new variant of the definition of dependent failure. Interactive failure provides a measure of accelerated failures due to the failure interactions among different components. In this chapter, an analytical model to describe interactive failure has been developed.

The proposed model can be applied in system failure probability prediction when interactive failures exist. According to the model, the interactive hazard of a component is estimated by its independent hazard plus a portion of the hazards of its influencing components. When the hazards of the influencing components of a component increase, the hazard of this component accelerates. The failure interaction between the components in a system will increase the failure likelihood of the system. Interactive failures should be considered when analysing failures of assets, or otherwise, the probability of failure may be underestimated.

The degree of failure interaction between components is measured by the Interactive Coefficient (IC), which is equal to or greater than zero for positive dependent failures. A greater IC means that the failure of an influencing component has greater effect on the failure of its affected component. An important approach to reducing interactive failures of a system is to reduce its IC. However, interactive coefficients have different effects on different components and different system topologies. Their effects on the interactive failures of a component reach a peak when this component is operating at the midpoint in its life. Different ICs have different sensitivities which can also vary with different system topologies.

Interactive failure can be either stable or unstable. One should attempt to reduce stable interactions and avoid unstable interactions between the components in a system when designing new machines.

When the interactive failure probabilities of the influencing components of an affected component are not all zero, the interactive failure probability of this affected component will be not zero even though its independent failure probability is zero (refer to Equation (4-71) to Equation (4-73)). Therefore, for a repairable system, when a failed component is replaced by an identical new one, its initial hazard will become higher than its original reliability due to the effects of its unrepaired influencing components. This matter has been researched. The methodology and the results are presented in the next chapter.

Chapter 5

RELIABILITY PREDICTIONS OF REPAIRABLE SYSTEMS WITH INTERACTIVE FAILURES

5.1 INTRODUCTION

In Chapter 3, the Split System Approach (SSA) was developed to deal with the reliability prediction of complex repairable systems with multiple PM intervals. In this model the failures of components in a system were assumed to be independent from each other. This assumption has been commonly used in existing reliability prediction models and can meet the requirements of the accuracy of prediction in some industrial scenarios. However, as indicated in Chapter 4, there are also numerous scenarios in industry where the assumption of independent failures is not applicable and Interactive Failure (IntF) must be considered.

IntF occurs commonly in mechanical systems. When repairing a system with failure interactions, one needs to consider IntF; or otherwise the repair may not be complete. This characteristic is best demonstrated with an example. A washing machine was subjected to rotary unbalance and was found to vibrate significantly during its spin cycle. The machine was disassembled and inspected to determine the root cause. The lower bearing (see Figure 5-1) was found to

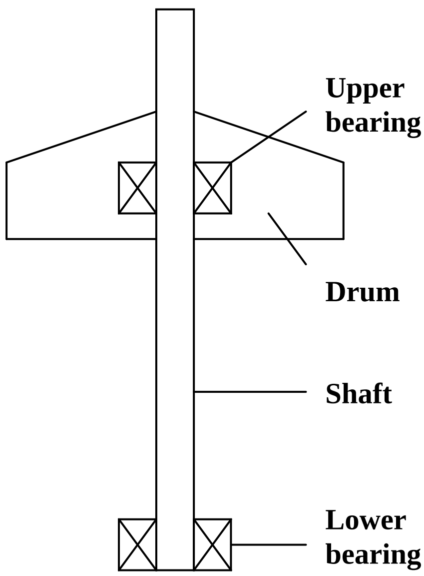


Figure 5-1. Simplified structure diagram of a washing machine

have been damaged. The balls inside the bearing had worn out severely. The clearance between the inner race and outer race became excessive that the shaft experienced eccentricity. The technician suspected that the upper bearing might have also been damaged, but he could not find a suitable tool to tear down the drum. As a result, only the lower bearing was replaced. The machine was assembled and operated smoothly for a short time. However, after three washing cycles the vibration became excessive. The washing machine was disassembled again. An inspection revealed that the new bearing inserted recently was damaged. On this occasion, the technician found a suitable tool to completely disassemble the machine. The inspection confirmed that his previous suspicion was correct - the upper bearing was severely damaged. The machine operated normally after both the upper and lower bearings were replaced.

In this case, the two bearings had failure interactions with the shaft. When only the lower bearing was replaced, the damaged upper bearing still caused the shaft to vibrate. This vibration in turn accelerated the failure of the new lower bearing. This accelerated failure is an interactive failure.

The above case is relatively commonplace in engineering maintenance. In order to maintain a system effectively and efficiently, interactive failures in a system need to be considered. Understanding the characteristics of interactive failures in a system with repairs is desired for optimal maintenance of a repairable complex system.

In Chapter 4, an analytic model, AMIF, to calculate IntF was developed. However, in that chapter, the effects of repairs on the reliability prediction of systems were not considered. The research on the reliability predictions of repairable systems with IntF is still in its infancy. Despite an exhaustive literature review, the candidate was unable to find related research reports to date.

In this chapter, an approach for reliability predictions of repairable systems with IntF is developed. This approach will consolidate both SSA and AMIF, and hence is termed as the Extended Split System Approach (ESSA). The term “component” includes subsystem and the term “repair” includes “replace or replacement” unless specified consistent with nomenclature in Chapters 3 and 4. Stable IntF is the focus of the study in this chapter.

The rest of this chapter is organised as follows. In Section 5.2, the methodology for ESSA is developed. In Section 5.3, the newly developed method is validated using an example and several simulation experiments. Section 5.4 presents the conclusions.

5.2 METHOD DEVELOPMENT

The reliability of a system is expected to increase after a repair because the hazard of this system is reduced [303]. This characteristics has also been observed in experiments conducted by the candidate (refer to Chapter 7). Repairs can improve the reliability of a system in two aspects: reducing the Interactive Hazard (IntH) of unrepaired components and increasing the reliability of repaired components. The improvement of reliability of a system after repairs is analysed below.

Consistent with Chapter 3, this chapter investigates the reliability prediction of assets with specified RBPM strategies only. Hence all assumptions made for SSA, expect the second one – that of independent failures, have been applied to the development of ESSA. Interactive failures among components in a system are considered in this chapter which focused on gradual degradation interactive failures. As analysed in Subsection 4.10, Chapter 4, this type of interactive failure accelerates the hazard of affected components but does not change the RBD of a system. This property enables the reliability prediction of repairable systems with IntF to be analysed in the following two steps:

Step 1. Calculate the changeable IntH and Interactive Failure Distribution Functions (IntFDF) of repaired and unrepaired components using AMIF.

Step 2. Consider the logic position of repaired components in the RBD of the repairable system, and then calculate new interactive reliability function or IntFDF of the system after a PM action and over multiple PM intervals using SSA.

The detailed discussions on these two steps are presented in the following subsections. In the following analysis, interactive reliability function and interactive failure distribution function will be simplified as reliability function and Failure Distribution Function (FDF).

5.2.1 MODIFIED HEURISTIC APPROACH

Since this chapter considers the second category of IntF only and this type of IntF does not change the RBD of a system, a heuristic approach similar to that used in Chapter 3 can be developed to calculate the reliability of a system with IntF over multiple PM intervals. Considering that the hazards of repaired and unrepaired components of the system after a PM action are different from their own independent hazards, the heuristic approach in Chapter 3 is modified as follows:

(1) Determine the first PM time $t_1 = \Delta t_1$ when the reliability of the system first falls to the predefined control limit of reliability using the original reliability function of the system.

(2) Assign the repaired Component k_1 ($k_1 = 1, 2, \ldots, S_1$) a new independent reliability function $R_{Ik_1}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$) based on the requirement of a PM strategy (Assume that the system has M components, and S_1 components ($1 \le S_1 \le M$) are repaired in the first PM action). Calculate the reliability functions of these components, after the first PM action, $R_{k_1}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$), using Equation (4-43). The cumulative reliability functions of these repaired components, $R_{k_1c}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$), are $R_{k_1}(\Delta t_1)_0 R_{k_1}(\tau)_1$ ($k_1 = 1, 2, \ldots, S_1$). The independent reliability functions of the rest of the components of the system remain the same since they are not repaired. However, the cumulative effects of time before the first PM action need to be considered. Hence, $R_{Ij_1}(\tau)_1 = R_{Ij_1}(\tau + \Delta t_1)_0$ ($j_1 = S_1 + 1, S_1 + 2, \ldots, M$). Unlike independent reliability functions, the reliability functions of the unrepaired components after the first PM action, $R_{j_1}(\tau)_1$ ($j_1 = S_1 + 1, S_1 + 2, \ldots, M$) are different from those before this PM action and

need to be calculated using Equation (4-43) based on $R_{Ik_1}(\tau)_1$ and $R_{Ij_1}(\tau)_1$. The cumulative reliability functions of these unrepaired components with the first PM action, $R_{j_1c}(\tau)_1$ need to be calculated using the following equation:

$$R_{j_1c}(\tau)_1 = \exp[-\int_0^{t_1} h_{j_1}(t)_0\,dt - \int_{t_1}^{\tau+t_1} h_{j_1}(t)_1\,dt]$$

$$= \frac{R_{j_1}(\Delta t_1)_0\,R_{j_1}(\tau+\Delta t_1)_1}{R_{j_1}(\Delta t_1)_1} \qquad (\ j_1 = S_1+1, S_1+2, \ldots, M\ ), \qquad (5\text{-}1)$$

where, $R_{j_1}(0)_0$ is assumed to be one for $j_1 = S_1+1,\ S_1+2,\ \ldots,\ M$ and $t_1 = \Delta t_1$ is the first PM time. Functions $h_{j_1}(t)_0$ and $h_{j_1}(t)_1$ ($j_1 = S_1+1,\ S_1+2,\ \ldots,\ M$) are the IntH of the unrepaired components before and after the first PM action in terms of the absolute time scale, respectively.

(3) Calculate the reliability function and the cumulative reliability function of the system after the first PM action, $R_s(\tau)_1$ and $R_{sc}(\tau)_1$, based on the RBD of the system using the reliability functions and the cumulative reliability functions of its components after the first PM action, respectively.

(4) Determine the second PM time t_2 using the reliability function of the system after the first PM action, $R_s(\tau)_1$.

(5) Assume S_2 components are repaired in the second PM action. Reassign the repaired Component k_2 a new independent reliability function $R_{Ik_2}(\tau)_2$ based on the requirement of PM strategy (k_2 represents all components repaired in the second PM action). Calculate the reliability function of these components after the second PM action, $R_{k_2}(\tau)_2$ (k_2 represents all components repaired in the second PM action), using Equation (4-43). The cumulative reliability functions of these components $R_{k_2c}(\tau)_2$ (k_2 represents all components repaired in the second PM action) now need to be calculated based on two

scenarios: if components have also been repaired in the first PM action, their cumulative reliability functions are $\prod_{i=0}^{1} R_{k_{21}}(\Delta t_{i+1})_i R_{k_{21}}(\tau)_2$. Subscript k_{21} represents all components that are repaired in the first and second PM action. The cumulative reliability functions for those components which are repaired in the second PM action only are $\frac{R_{k_{22}}(\Delta t_1)_0 R_{k_{22}}(\sum_{i=1}^{2}\Delta t_i)_1 R_{k_{22}}(\tau)_2}{R_{k_{22}}(\Delta t_1)_1}$. Subscript $k_{22} \neq k_{21}$ and $k_{21} + k_{22} = S_2$. The independent reliability functions of the rest of the components of the system remain the same as before this PM action since they are not repaired. However, the cumulative effects of time on unrepaired components can now be different. For components which are never repaired, their independent reliability functions $R_{Ij_{21}}(\tau)_2$ are $R_{Ij_{21}}(\tau + \sum_{i=1}^{2}\Delta t_i)_0$. Subscript j_{21} represents all components which have never been repaired. For components which have been repaired in the first PM action, their independent reliability functions $R_{Ij_{22}}(\tau)_2$ are $R_{Ij_{22}}(\tau + \Delta t_2)_1$. Subscript $j_{22} \neq j_{21}$ and $j_{21} + j_{22} = M - S_2$.Then the reliability functions of these unrepaired components can be calculated using Equation (4-43). The cumulative reliability functions of the unrepaired components over two PM intervals, $R_{j_2c}(\tau)_2$, are

$$R_{j_2c}(\tau)_2 = \exp[-\int_0^{t_1} h_{j_2}(t)_0\, dt - \int_{t_1}^{t_2} h_{j_2}(t)_1\, dt - \int_{t_2}^{\tau+t_2} h_{j_2}(t)_2\, dt]$$

$$= \frac{R_{j_2}(\Delta t_1)_0 R_{j_2}(\sum_{i=1}^{2}\Delta t_i)_1 R_{j_2}(\tau + \sum_{i=1}^{2}\Delta t_i)_2}{R_{j_2}(\Delta t_1)_1 R_{j_2}(\sum_{i=1}^{2}\Delta t_i)_2} \qquad (j_2 = S_2 + 1, S_2 + 2, \ldots, M).$$

(5-2)

(6) Calculate the reliability function and the cumulative reliability function of the

system after the second PM action, $R_s(\tau)_2$ and $R_{sc}(\tau)_2$, based on the RBD of the system using the reliability functions and the cumulative reliability functions of its components after the second PM action, respectively.

(7) Continue the above procedure until the nth PM action.

5.2.2 COMPONENT INTERACTIVE HAZARDS AND FAILURE DISTRIBUTION FUNCTIONS

This subsection focuses on developing a method for calculating the Failure Distribution Functions (FDF) of the components in a system with IntF after a PM action. Apart from the assumptions mentioned at the beginning of this chapter, the following additional assumptions are made in this subsection:

(1) The system has its first PM action. The case of a system with multiple PM actions will be analysed in the next subsection.

(2) The system is composed of M components and Component 1 is repaired in the first PM action.

(3) The interactive coefficients are constant and independent of repairs.

In the case of repairable systems with IntF, the initial time for calculating the IntH of newly repaired components can be different from that for remaining unrepaired components after a PM action (see Figure 5-2).

As in Chapter 3, parameter t in this chapter represents the absolute time scale and τ represents the relative time scale. Parameter t_n is the nth failure time measured in the absolute time scale. The initial time to calculate the IndH of the unrepaired components after the first PM action is t_1 and the initial time to calculate the IndH of the newly repaired component after the first PM action is zero.

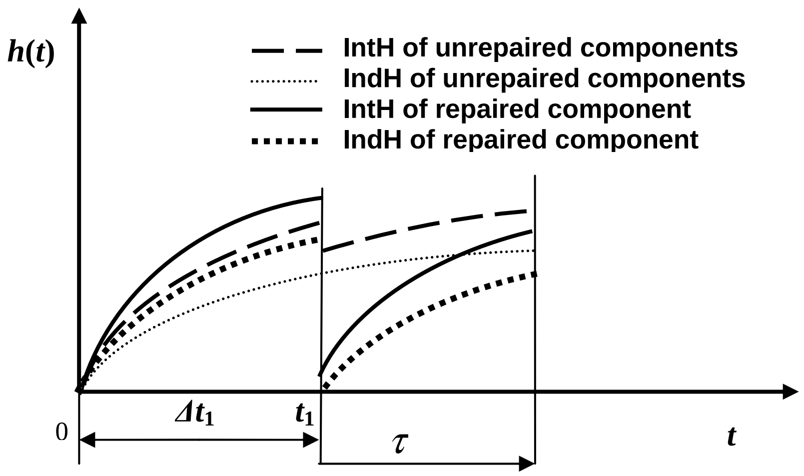


Figure 5-2. The changes of hazard of unrepaired components and repaired component

According to the analysis in Section 4.5 of Chapter 4, the stable IntH of a system is given by Equation (4-39):

$$\{h(\tau)\} = [\alpha]\{h_I(\tau)\}.$$

where, $\{h(\tau)\}$ is the stable IntHs of a system after failure interaction. It is an $M \times 1$ vector. $\{h_I(\tau)\}$ is an $M \times 1$ independent hazard vector of all components due to their own deteriorations. $[\alpha]$ is the State Influence Matrix (SIM) which is given by Equation (4-40):

$$[\alpha] = ([I] - [\theta(t)])^{-1}.$$

where, $[I]$ is an $M \times M$ unit matrix. $[\theta(t)]$ is the Interactive Coefficient (IC) matrix of the system.

Define all unrepaired components as a subsystem. Equation (4-39) can be rewritten using the partition matrix as follows:

$$\begin{Bmatrix} h_1(\tau) \\ \vec{h}_{sb}(\tau) \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \vec{\alpha}_2 \\ \vec{\alpha}_3 & \vec{\alpha}_4 \end{bmatrix} \begin{Bmatrix} h_{I1}(\tau) \\ \vec{h}_{Isb}(\tau) \end{Bmatrix}. \tag{5-3}$$

where, $h_1(\tau)$ is the IntH of Component 1. Vector $\vec{h}_{sb}(\tau)$ is the $(M-1)\times 1$ IntH vector of the subsystem. Parameter α_{11} is the first row first column element of SIM $[\alpha]$; while $\vec{\alpha}_2$, $\vec{\alpha}_3$ and $\vec{\alpha}_4$ are the $1\times(M-1)$, $(M-1)\times 1$ and $(M-1)\times(M-1)$ partition matrix in SIM $[\alpha]$, respectively. Function $h_{I1}(\tau)$ is the IndH of Component 1, and $\vec{h}_{Isb}(\tau)$ is a $(M-1)\times 1$ vector which represents the IndH of the subsystem.

Let $h_{I1}(\tau)_0$ and $\vec{h}_{Isb}(\tau)_0$ denote the IndH of Component 1 and the subsystem before the first PM action respectively.

When the first PM action is conducted, $\tau = t_1 = \Delta t_1$. Hence, just before the first PM action, the IndHs of Component 1 and the subsystem are $h_{I1}(\Delta t_1)_0$ and $h_{Isb}(\Delta t_1)_0$, respectively. Let $h_{I1}(\tau)_1$ be the IndH of Component 1 after the first PM action, then just after the first PM action, the IndH of Component 1 is $h_{I1}(0)_1$. Generally

$$0 \le h_{I1}(0)_1 \le h_{I1}(\Delta t_1)_0 . \tag{5-4}$$

If $h_{I1}(\tau)_1 = h_{I1}(\tau + \Delta t_1)_0$, the state of the system after the first PM action is "as bad as old".

The IndH of the subsystem just after the first PM action is the same as just before this PM action because it has not been repaired, i.e.,

$$h_{Isb}(\tau)_1 = h_{Isb}(\tau + \Delta t_1)_0 , \tag{5-5}$$

where, $h_{Isb}(\tau)_0$ and $h_{Isb}(\tau)_1$ are the IndHs of the subsystem before and after the first PM action respectively.

The IntHs of all components in the system after the first PM action are given by

$$\begin{Bmatrix} h_1(\tau)_1 \\ \vec{h}_{sb}(\tau)_1 \end{Bmatrix} = \begin{bmatrix} \alpha_{11} & \vec{\alpha}_2 \\ \vec{\alpha}_3 & \vec{\alpha}_4 \end{bmatrix} \begin{Bmatrix} h_{I1}(\tau)_1 \\ \vec{h}_{Isb}(\tau + \Delta t_1)_0 \end{Bmatrix}, \tag{5-6}$$

where, $h_{I1}(\tau)_1$ is the IndH of Component 1 after the first PM action. $\vec{h}_{Isb}(\tau)_0$ are the IndHs of the subsystem before the first PM action; while $h_1(\tau)_1$ and $\vec{h}_{sb}(\tau)_1$ are the IntHs of Component 1 and the subsystem after the first PM action respectively.

If IntF is stable and the reliability of Component 1 just after the first PM action has not degraded since just before this PM action, the following inequities can be obtained:

$$h_1(t_1)_0 \geq h_1(0)_1 = \alpha_{11} h_{I1}(0)_1 + \vec{\alpha}_2 \vec{h}_{Isb}(\Delta t_1)_0 \geq h_{I1}(0)_1, \tag{5-7}$$

$$\vec{h}_{sb}(0)_1 = \vec{\alpha}_3 h_{I1}(0)_1 + \vec{\alpha}_4 \vec{h}_{Isb}(\Delta t_1)_0 \leq \vec{h}_{Isub}(\Delta t_1)_0 . \tag{5-8}$$

The above inequities can be proved using the following two propositions and a theorem.

Proposition 5-1: All elements in SIM $[\alpha]$ are nonnegative when $0 \leq \theta_{ij} < 1$.

The proof of Proposition 5-1 is presented in Appendix B6.

Proposition 5-2: All diagonal elements in SIM $[\alpha]$ are greater than or equal to one**.**

The proof of Proposition 5-2 is presented in Appendix B7.

Theorem 5-1: Interactive functions $h_1(\tau)$ and $\vec{h}_{sb}(\tau)$ change monotonously with the change of $h_{I1}(\tau)$.

The proof of Theorem 5-1 is straightforward using Equation (5-3) and Proposition 5-1.

Inequity (5-7) is proved as follows:

According to Proposition 5-1, $\vec{\alpha}_2 \geq 0$. According to Proposition 5-2, $\alpha_{11} \geq 1$. Hence, the following inequity holds because all elements in $\vec{h}_{Isb}(\Delta t_1)_0$ are nonnegative:

$$h_1(0)_1 = \alpha_{11} h_{I1}(0)_1 + \vec{\alpha}_2 \vec{h}_{Isb}(\Delta t_1)_0 \geq h_{I1}(0)_1 \,. \quad (5\text{-}9)$$

If the condition of Component 1 just after the first PM action has not worsened since just before this PM action, i.e., $h_{I1}(0)_1 \leq h_{I1}(\Delta t_1)_0$, the following inequity holds because of Equation (5-6) and Theorem 5-1:

$$h_1(t_1)_0 \geq h_1(0)_1 \,. \quad (5\text{-}10)$$

Inequity (5-7) is obtained by a combination of Inequity (5-9) and Inequity (5-10). Inequity (5-8) can be proved using a similar approach.

Inequity (5-9) indicates that the Interactive Hazard (IntH) of Component 1 can be higher than its original independent hazard due to the effect of the unrepaired subsystem. The inequity symbol in Inequity (5-9) becomes the equality symbol if and only if $\vec{\alpha}_2$ is a null vector. A null vector $\vec{\alpha}_2$ means that the failures of components in subsystem do not affect the failure of Component 1. If $\vec{\alpha}_2$ is a null vector, element α_{11} is equal to one (see Appendix B6). Inequity (5-8) indicates that the IntHs of the components in the subsystem, and hence the subsystem, have been reduced after the first PM action. The inequity symbol in Equation (5-8) becomes equality symbol if and only if $\vec{\alpha}_3$ is a null vector. A null vector $\vec{\alpha}_3$ means that the failure of Component 1 does not influence the failure of components in the subsystem.

The Integrated Interactive Hazards (IntIHs) of Component 1 and the components in the subsystem between the first PM action and the second PM action can be obtained using Equation (5-6), as well as the relationship between hazard and integrated hazard:

$$H_1(\tau)_1 = \int_0^{\tau} [\alpha_{11} h_{I1}(\tau)_1 + \vec{\alpha}_2 \vec{h}_{Isb}(\Delta t_1 + \tau)_0] d\tau \,, \quad (5\text{-}11)$$

$$\vec{H}_{sb}(\tau)_1 = \int_0^{\tau} [\vec{\alpha}_3 h_{I1}(\tau)_1 + \vec{\alpha}_4 \vec{h}_{Isb}(\Delta t_1 + \tau)_0] d\tau \,. \quad (5\text{-}12)$$

The FDFs of Component 1 and the components in the subsystem after the first PM action are

$$F_1(\tau)_1 = 1 - \exp[-H_1(\tau)_1] \tag{5-13}$$

and

$$\{F_{sbi}(\tau)_1\} = \{1 - \exp[-H_{sbi}(\tau)_1]\} \quad (i = 2, 3, \ldots M), \tag{5-14}$$

where, $F_1(\tau)_1$ and $F_{sbi}(\tau)_1$ are the FDFs of Component 1 and Component *i* in the subsystem after the first PM action, respectively; $H_{sbi}(\tau)_1$ is the ith element in the vector $\vec{H}_{sb}(\tau)_1$.

5.2.3 SYSTEM RELIABILITY

Generally, the reliability of a system needs to be calculated based on the above modified heuristic approach by means of a computer. However, in some special scenarios, closed analytical formulae for predicting the reliability of a system after the nth PM action can be obtained. Two such scenarios are analysed as follows.

5.2.3.1 The same single component in a series system is repaired in all PM actions

The system for this scenario has been shown in Figure 3-1. Based on Equation (3-2), the original reliability function of the system before PM can be expressed as:

$$R_s(\tau)_0 = R_1(\tau)_0 R_{sb}(\tau)_0, \tag{5-15}$$

where, $R_s(\tau)_0$, $R_1(\tau)_0$ and $R_{sb}(\tau)_0$ are the original reliability functions of the entire system, Component 1 and the subsystem in this system, respectively.

For the following analysis, a general equation to describe the relationship between integrated hazard and reliability is needed. According to the definition of hazard, the relationship between hazard and reliability is given by [8]

$$h(t) = -\frac{d}{dt}\ln R(t). \tag{5-16}$$

Equation (5-16) leads to the following equation:

$$R(t) = R(0)\exp[-\int_0^t h(t)dt], \tag{5-17}$$

where, $R(0)$ is the initial reliability value. When $R(0)=1$, Equation (5-17) reduces to Equation (4-1).

The original reliability functions of Component 1 and the subsystem can then be expressed using Equation (5-17) as follows:

$$R_1(\tau)_0 = R_1(0)_0 \exp[-H_1(\tau)_0], \tag{5-18}$$

$$R_{sb}(\tau)_0 = R_{sb}(0)_0 \exp[-H_{sb}(\tau)_0], \tag{5-19}$$

where, $R_1(0)_0$ and $R_{sb}(0)_0$ are the initial reliability values of Component 1 and the subsystem before PM, respectively. In most cases, $R_1(0)_0$ and $R_{sb}(0)_0$ are both equal to one. In this thesis, they are always assumed as one. $H_1(\tau)_0$ is the IntIH of Component 1 before PM. It is given by

$$H_1(\tau)_0 = \int_0^\tau [\alpha_{11}h_{I1}(\tau)_0 + \vec{\alpha}_2\vec{h}_{Isb}(\tau)_0]d\tau. \tag{5-20}$$

$H_{sb}(\tau)_0$ is the IntIH of the subsystem before PM and given by

$$H_{sb}(\tau)_0 = \int_0^\tau [\alpha_{sb1}^e h_{I1}(\tau)_0 + h_{Isb}^e(\tau)_0]d\tau, \tag{5-21}$$

where, α_{sb1}^e is an equivalent state influence coefficient to represent the effect of the failure of Component 1 on the subsystem. Function $h_{Isb}^e(\tau)_0$ is the equivalent IndH of the subsystem. The calculation of α_{sb1}^{e} and $h_{Isb}^{e}(\tau)_0$ is dependent on the RBD of a system. When a subsystem is a series system,

$$R_{sb}(\tau)_0 = \prod_{i=2}^{M} R_i(\tau)_0$$

$$= \prod_{i=2}^{M} \exp[-\int_0^{\tau} h_i(\tau)_0 d\tau]. \qquad (5\text{-}22)$$

Then the equivalent state influence coefficient α_{sb1}^{e} is given by

$$\alpha_{sb1}^{e} = \sum_{i=2}^{M} \alpha_{i1}, \qquad (5\text{-}23)$$

where, α_{i1} is the ith row first column element in SIM $[\alpha]$.

The equivalent IndH of the subsystem is given by

$$h_{Isb}^{e}(\tau)_0 = \sum_{i=2}^{M}\sum_{j=2}^{M} \alpha_{ij} h_{Ij}(\tau)_0, \qquad (5\text{-}24)$$

where, α_{ij} is the ith row jth column element in SIM $[\alpha]$. Function $h_{Ij}(\tau)_0$ is the IndH of Component j before PM. In the real world, the calculation of α_{sb1}^{e} and $h_{Isb}^{e}(\tau)_0$ will be more straightforward because Component 1 usually interacts with a few components in the subsystem.

Substituting Equations (5-18) to (5-21) into Equation (5-15) and considering the condition that $R_1(0)_0$ and $R_{sb}(0)_0$ are both equal to one, give

$$R_s(\tau)_0 = \exp[-\int_0^{\tau} [\alpha_{11} h_{I1}(\tau)_0 + \vec{\alpha}_2 \vec{h}_{Isb}(\tau)_0 + \alpha_{sb1}^{e} h_{I1}(\tau)_0 + h_{Isb}^{e}(\tau)_0] d\tau]. \qquad (5\text{-}25)$$

At time t_1, the system has its first PM action and Component 1 is repaired. After the first PM action, the reliability of the system becomes

$$R_s(\tau)_1 = R_1(\tau)_1 R_{sb}(\tau)_1 , \tag{5-26}$$

where, $R_s(\tau)_1$, $R_1(\tau)_1$ and $R_{sb}(\tau)_1$ are the reliability functions of the entire system, Component 1 and the subsystem after the first PM action, respectively.

$$R_1(\tau)_1 = R_1(0)_1 \exp[-H_1(\tau)_1] . \tag{5-27}$$

$H_1(\tau)_1$ is the IntIH of Component 1 after the first PM action. It is given by

$$H_1(\tau)_1 = \int_0^{\tau} \alpha_{11} h_{I1}(\tau)_1 d\tau + \int_{\Delta t_1}^{\Delta t_1+\tau} \vec{\alpha}_2 \vec{h}_{Isb}(\tau)_0 d\tau . \tag{5-28}$$

For a repairable system without failure interaction, the characteristics of the hazard of the subsystem are assumed to be unchangeable just before and just after a PM action. In contrast, when failures of a repairable system have interactions, the characteristics of the hazard of the unrepaired subsystem just after a repair can be different from that just before this repair as analysed previously. These differences are not ignorable in the calculation of the reliability of the system. The reliability of the subsystem after the first PM action needs to be calculated using its new IntH as follows:

$$R_{sb}(\tau)_1 = R_{sb}(0)_1 \exp[-H_{sb}(\tau)_1] , \tag{5-29}$$

where, $R_{sb}(0)_1$ is the initial reliability value of the subsystem, which is equal to its reliability value just before the first PM action:

$$R_{sb}(0)_1 = \exp[-\int_0^{\Delta t_1} [\alpha_{sb1}^e h_{I1}(\tau)_0 + h_{Isb}^e(\tau)_0] d\tau] . \tag{5-30}$$

$H_{sb}(\tau)_1$ is the IntIH of the subsystem after the first PM action. It is given by

$$H_{sb}(\tau)_1 = \int_0^{\tau} \alpha_{sb1}^e h_{I1}(\tau)_1 d\tau + \int_{\Delta t_1}^{\Delta t_1+\tau} h_{Isb}^e(\tau)_0 d\tau \,. \tag{5-31}$$

Rewrite Equation (5-31) as follows:

$$H_{sb}(\tau)_1 = \int_0^{\tau} \alpha_{sb1}^e h_{I1}(\tau)_1 d\tau + \int_{\Delta t_1}^{\Delta t_1+\tau} h_{Isb}^e(\tau)_0 d\tau + \int_{\Delta t_1}^{\Delta t_1+\tau} \alpha_{sb1}^e h_{I1}(\tau)_0 d\tau - \int_{\Delta t_1}^{\Delta t_1+\tau} \alpha_{sb1}^e h_{I1}(\tau)_0 d\tau \,. \tag{5-32}$$

Substituting Equations (5-30) and (5-32) into Equation (5-29), gives

$$R_{sb}(\tau)_1 = R_{sb}(\tau + \Delta t_1)_0 \exp[-\int_0^{\tau} \alpha_{sb1}^e h_{I1}(\tau)_1 d\tau + \int_{\Delta t_1}^{\Delta t_1+\tau} \alpha_{sb1}^e h_{I1}(\tau)_0 d\tau] \,. \tag{5-33}$$

Since only the constant interactive coefficients are considered in this chapter, Equation (5-33) can be rewritten as

$$R_{sb}(\tau)_1 = R_{sb}(\tau + \Delta t_1)_0 \exp[-\int_0^{\tau} \alpha_{sb1}^e [h_{I1}(\tau)_1 - h_{I1}(\tau + \Delta t_1)_0] d\tau] \,. \tag{5-34}$$

Equation (5-34) indicates that the characteristics of the reliability of the subsystem after the first PM action changes unless α_{sb1}^e is zero (the condition of Component 1 does not affect the condition of the subsystem) or $h_{I1}(\tau)_1 = h_{I1}(\tau + \Delta t_1)_0$ (the repair does not change the state of Component 1). If $h_{I1}(\tau)_1 < h_{I1}(\tau + \Delta t_1)_0$ (the repaired Component 1 is better than old one), the reliability of the subsystem after the first PM action is improved. If $h_{I1}(\tau)_1 > h_{I1}(\tau + \Delta t_1)_0$ (the repaired Component 1 is worse than the old one), the reliability of the subsystem after the first PM action decreases. These inferences are also correct when the system has the i^{th} PM action ($i = 2, 3, \ldots, n$).

Substituting Equations (5-27), (5-28) and (5-33) into Equation (5-26), the reliability of a system after the first PM interval is given by

$$R_s(\tau)_1 = \frac{R_s(\tau+\Delta t_1)_0}{R_1(\tau+\Delta t_1)_0} R_1(0)_1 \exp[\int_0^{\tau}[\alpha_{sb1}^{e} h_{I1}(\tau+\Delta t_1)_0 - (\alpha_{11}+\alpha_{sb1}^{e}) h_{I1}(\tau)_1$$

$$-\vec{\alpha}_2 \vec{h}_{Isb}(\tau+\Delta t_1)_0] d\tau]. \quad (5\text{-}35)$$

The reliability function of the system after the nth PM interval can be obtained by continuing the above derivation procedure:

$$R_s(\tau)_n = \frac{R_s(\tau+\sum_{i=1}^{n}\Delta t_i)_0}{R_1(\tau+\sum_{i=1}^{n}\Delta t_i)_0} R_1(0)_n \exp[\int_0^{\tau}[\alpha_{sb1}^{e} h_{I1}(\tau+\sum_{i=1}^{n}\Delta t_i)_0 - (\alpha_{11}+\alpha_{sb1}^{e}) h_{I1}(\tau)_n$$

$$-\vec{\alpha}_2 \vec{h}_{Isb}(\tau+\sum_{i=1}^{n}\Delta t_i)_0] d\tau], \quad (5\text{-}36)$$

where, $R_s(\tau)_n$ is the reliability function of a repairable system with failure interactions after the nth PM interval. $R_1(0)_n$ is the initial reliability value of Component 1 after the nth PM action. Function $h_{I1}(\tau)_n$ is the IndH of Component 1 after the nth PM interval.

Comparing Equation (5-36) with Equation (3-9), one can find that the reliability prediction of repairable systems with IntF is much more complicated.

5.2.3.2 The same single component in a parallel system is repaired in all PM actions

The system for this scenario has been shown in Figure 3-3. The same as in Chapter 3, failure distribution function will be used for derivation in this subsection.

After the first PM action, the reliability of Component 1 is the same as Equation (5-27), but the reliability of the subsystem is different from Equation (5-34).

$$R_{sb}(\tau)_1 = \frac{R_s(\tau+\Delta t_1)_0 - R_1(\tau+\Delta t_1)_0}{1-R_1(\tau+\Delta t_1)_0}\exp[\int_0^{\tau}\alpha_{sb1}^{e}[h_{I1}(\tau+\Delta t_1)_0 - h_{I1}(\tau)_1]d\tau]. \quad (5\text{-}37)$$

Note that $\frac{R_s(\tau)_0 - R_1(\tau)_0}{1-R_1(\tau)_0}$ is the reliability of the subsystem before PM. Hence, the conclusions for Equation (5-34) are also correct for Equation (5-37).

Generally, the failure distribution function of a system with IntF after the nth PM interval is

$$F_s(\tau)_n = [1-\frac{F_1(\tau+\sum_{i=1}^{n}\Delta t_i)_0 - F_s(\tau+\sum_{i=1}^{n}\Delta t_i)_0}{F_1(\tau+\sum_{i=1}^{n}\Delta t_i)_0}\exp[\int_0^{\tau}\alpha_{sb1}^{e}[h_{I1}(\tau+\sum_{i=1}^{n}\Delta t_i)_0 - h_{I1}(\tau)_n]d\tau]]$$

$$[1-R_1(0)_n\exp[-\int_0^{\tau}[\alpha_{11}h_{I1}(\tau)_n + \vec{\alpha}_2\vec{h}_{Isb}(\tau+\sum_{i=1}^{n}\Delta t_i)_0]d\tau]]. \quad (5\text{-}38)$$

where, $F_s(\tau)_n$ is the failure distribution function of a repairable system with IntF after the nth PM interval.

5.3 AN EXAMPLE: A MECHANICAL SYSTEM WITH THREE INTERACTIVE COMPONENTS

A complex repairable mechanical system with IntF is composed of three items. The RBD of the system is shown in Figure 4-15. Item 1 is a single component (Component 1), but both Item 2 and Item 3 can be either a single component or an assembly consisting of several components. The predefined control limit of reliability is R_0 (1> R_0 >0). Component 1 is assumed to be replaced by an identical new one in each PM action. The independent reliability functions of the original system and Component 1 are

$$R_{Is}(t)_0 = \exp(-\lambda_s t) \quad (5\text{-}39)$$

and

$$R_{I1}(t)_0 = \exp(-\lambda_1 t). \tag{5-40}$$

The subsystem is composed of Item 2 and Item 3. According to reliability theory, its reliability function is

$$R_{Isb}(t)_0 = \exp(\lambda_1 - \lambda_s)t, \tag{5-41}$$

where, $R_{Isb}(t)_0$ is the reliability function of the subsystem without failure interactions.

The interactive coefficient matrix of the system is

$$[\theta(t)] = \begin{bmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & \theta_{23} \\ \theta_{31} & \theta_{32} & \theta_{33} \end{bmatrix}. \tag{5-42}$$

The corresponding SIM is

$$[\alpha(t)] = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix}. \tag{5-43}$$

Along with Equation (4-39), the stable IntIHs of the items before any PM are

$$H_i(\tau)_0 = \alpha_{i1}\lambda_1\tau + \alpha_{sbi}\lambda_{sb}\tau \quad (i = 1, 2, 3), \tag{5-44}$$

where, λ_{sb} is the hazard of the subsystem and can be calculated by Equation (5-41). Parameter α_{sbi} is the state influence coefficient that represents the effect of the failure of the subsystem on the failure of Item i, $(i = 1, 2, 3)$. It is given by

$$\alpha_{sbi} = \frac{\alpha_{i2}\lambda_2 + \alpha_{i3}\lambda_3}{\lambda_2 + \lambda_3} \quad (i = 1, 2, 3). \tag{5-45}$$

The FDF of the system before PM is

$$F_s(t)_0 = 1 - \exp(-t\sum_{i=1}^{3}\alpha_{i1}\lambda_1 - t\lambda_{sb}\sum_{i=1}^{3}\alpha_{sbi}) . \tag{5-46}$$

The first PM interval can be calculated using Equation (5-46):

$$\Delta t_1 = \frac{-\ln R_0}{\sum_{i=1}^{3}(\alpha_{i1}\lambda_1 + \alpha_{sbi}\lambda_{sb})} . \tag{5-47}$$

Only Component 1 is repaired in the first PM action. The IntIHs of these three items after the first PM action are

$$H_i(\tau)_1 = \alpha_{i1}\lambda_1\tau + \alpha_{sbi}\lambda_{sb}(\tau + \Delta t_1) \quad (i = 1,\ 2,\ 3) . \tag{5-48}$$

Hence, according to Equations (4-39), (4-40) and (4-43), the FDF of the system after the first PM interval is

$$F_s(\tau)_1 = 1 - \exp(-\tau\sum_{i=1}^{3}\alpha_{i1}\lambda_1 - \tau\lambda_{sb}\sum_{i=1}^{3}\alpha_{sbi} - \Delta t_1\lambda_{sb}\sum_{i=1}^{3}\alpha_{sbi}) . \tag{5-49}$$

Generally, the FDF of the system after the nth PM interval is

$$F_s(\tau)_n = 1 - \exp(-\tau\sum_{i=1}^{3}\alpha_{i1}\lambda_1 - \tau\lambda_{sb}\sum_{i=1}^{3}\alpha_{sbi} - \lambda_{sb}(\sum_{i=1}^{n}\Delta t_i)\sum_{i=1}^{3}\alpha_{sbi}) \tag{5-50}$$

The nth PM interval can be calculated by

$$\Delta t_n = \frac{-\ln R_0 - (\sum_{i=1}^{n-1}\Delta t_i)\sum_{i=1}^{3}\alpha_{sbi}\lambda_{sb}}{\sum_{i=1}^{3}(\alpha_{i1}\lambda_1 + \alpha_{sbi}\lambda_{sb})} . \tag{5-51}$$

Figures (5-3) to (5-7) present the results of Monte Carlo Simulation (MCS) experiments and corresponding theoretical calculation using SSA and ESSA. In these simulations, the interactive coefficient matrix is

$$[\theta(t)] = \begin{bmatrix} 0 & \frac{1}{25} & \frac{1}{50} \\ \frac{1}{20} & 0 & 0 \\ \frac{1}{10} & 0 & 0 \end{bmatrix}. \tag{5-52}$$

Therefore, the corresponding SIM is

$$[\alpha(t)] = \begin{bmatrix} \frac{250}{249} & \frac{10}{249} & \frac{5}{249} \\ \frac{25}{498} & \frac{499}{498} & \frac{1}{996} \\ \frac{25}{249} & \frac{1}{249} & \frac{499}{498} \end{bmatrix}. \tag{5-53}$$

From Figure 5-3 to Figure 5-7, it can clearly be seen that failure interactions shortened the interval between two PM actions of a repairable system. In some cases, failure interaction can reduce the available number of PM actions of a system (see Figures 5-4, 5-6 and 5-9). Figure 5-9 was drawn based on the simulation result 2 (Figure 5-4). The required minimum operating time had a great influence on the available number of PM actions (refer to Figure 5-3 and Figure 5-6). The required minimum operating time is the demanded minimal operating period of time between two PM actions due to maintaining production and cost effectiveness. A system will no longer be maintained if the demanded PM interval to maintain the reliability of this system above a required level is shorter than the required minimum operating time. The available number of PM actions of the system decreased quickly with the increase of the required minimum operating time. Figure 5-8 shows that the interactive failure distribution function of a system is identical to its independent failure distribution function if its interactive coefficient matrix is a null matrix. This result justifies the result shown in Subsection 4.7.2 of Chapter 4.

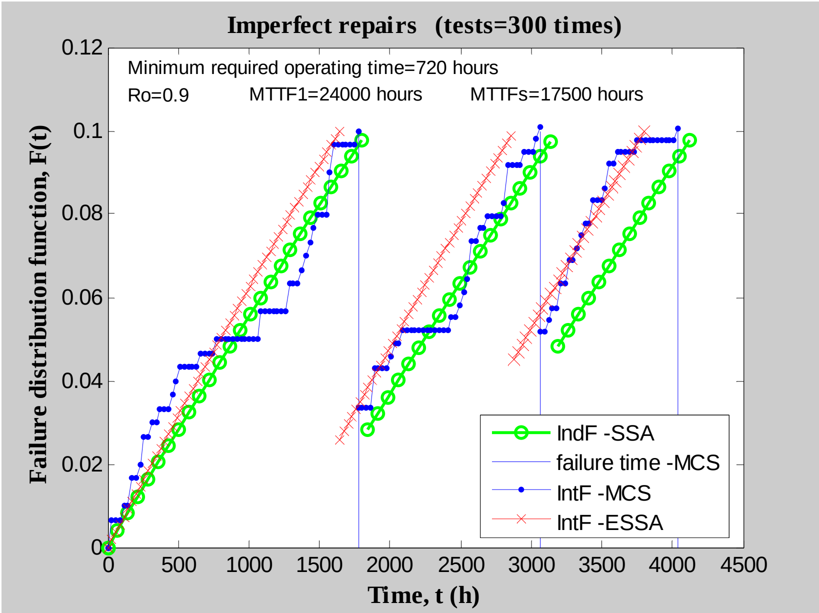


Figure 5-3. Simulation result 1 for the IntF of a repairable system

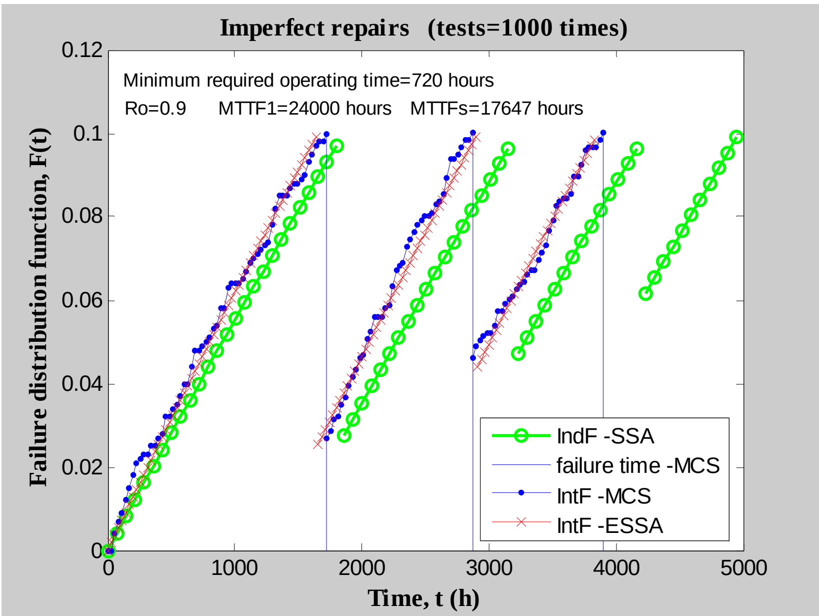


Figure 5-4. Simulation result 2 for the IntF of a repairable system

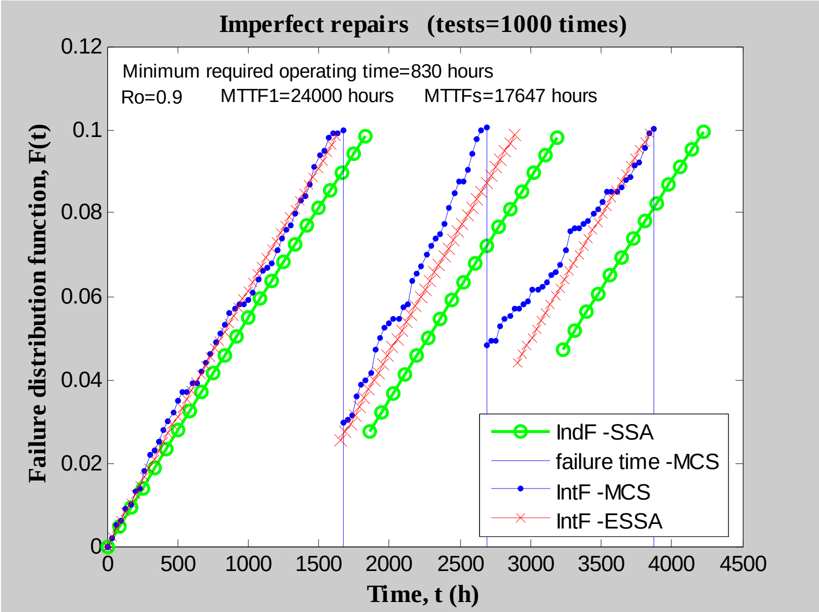


Figure 5-5. Simulation result 3 for the IntF of a repairable system

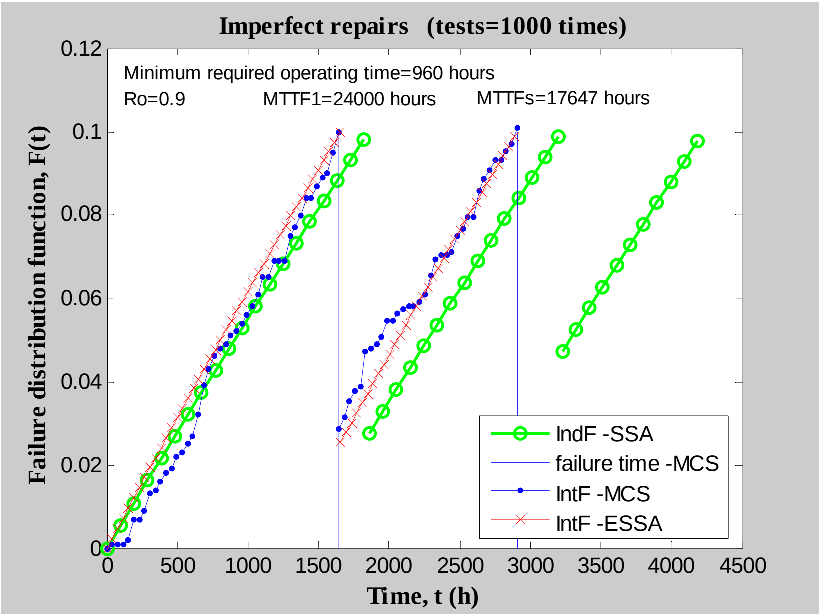


Figure 5-6. Simulation result 4 for the IntF of a repairable system

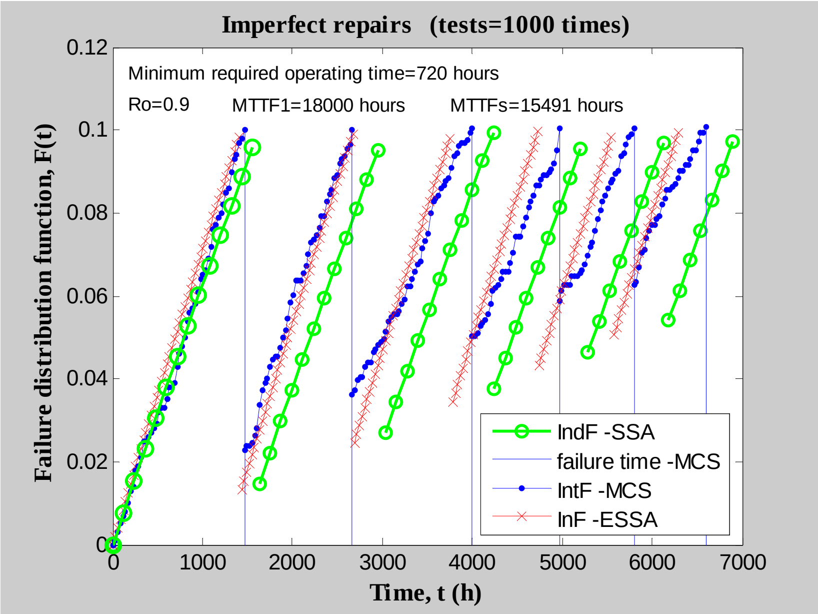


Figure 5-7. Simulation result 5 of the IntF of a repairable system

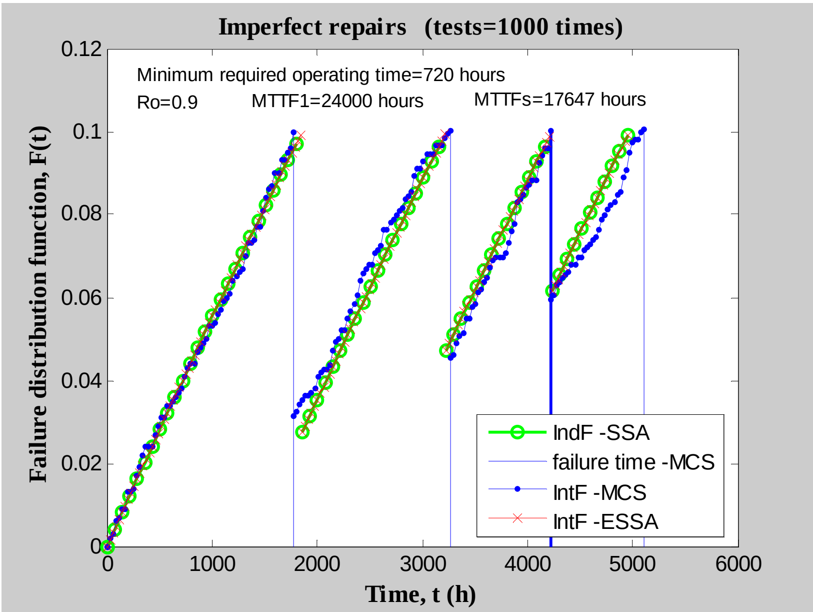


Figure 5-8. Simulation result 6 of the IntF of a repairable system

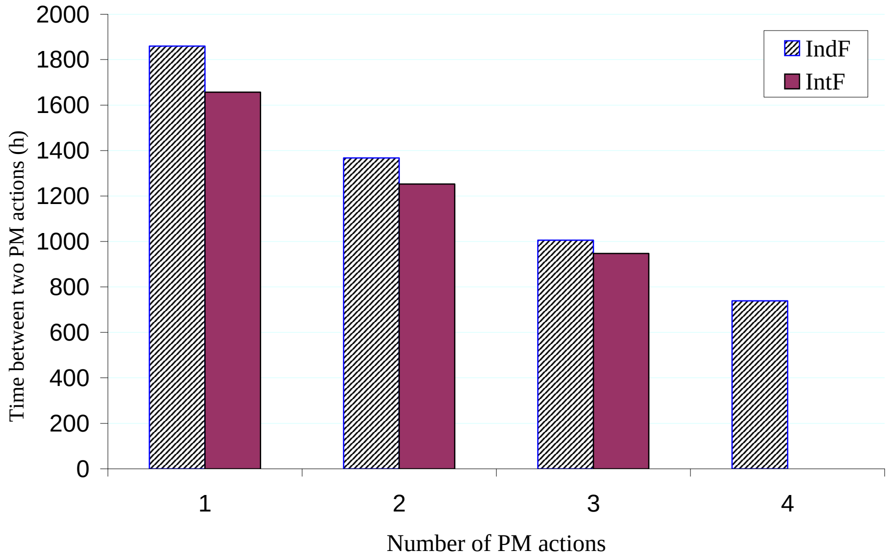


Figure 5-9. Comparison between the time between two PM actions of the system with interactive failures and independent failure

5.4 SUMMARY

In the case of a repairable system with interactive failures, the initial time to calculate the IndHs of components after a repair is different from that of the remaining unrepaired components after this repair. Repair can improve the reliability of a system in two aspects: decreasing IntH of the unrepaired components and increasing the reliability of repaired components.

The calculation of the FDF of a system with IntF under multiple PM intervals includes two steps: firstly, the changeable IntHs of repaired and unrepaired components are calculated using AMIF and then the new interactive reliability function or FDF of the system with multiple PM actions is calculated using SSA. The simulation experiments have shown that ESSA presented in this chapter is accurate.

Failure interactions will shorten the time between two PM actions if the PM strategy is based on the reliability of a system. Interactive failure can reduce the available number of PM actions of a system. When conducting PM, one needs to consider the failure interactions between influencing components and affected components. An affected component in a system should be maintained with its influencing components simultaneously, or otherwise, the deteriorated unrepaired influencing components will accelerate the failure of the repaired components.

Chapter 6

HAZARD PREDICTION USING HISTORICAL FAILURE DATA AND CONDITION MONITORING DATA

6.1 INTRODUCTION

The Extended Split System Approach (ESSA) can be used for predicting the reliability of repairable systems with Preventive Maintenance (PM) and interactive failures. To use this approach for prediction, the independent reliability functions of repaired components and the original system before PM should be known. These reliability functions can be estimated by existing techniques or models if historical failure data are sufficient. However, historical failure data are very difficult to obtain. The challenge is to conduct a reliability prediction when historical data are sparse or even zero. On the other hand, condition monitoring data is often available. A Proportional Covariate Model (PCM) which combines failure and condition monitoring data for hazard prediction is developed in this chapter. In addition, the strategy of determining PM lead time using the hazard function and the reliability function was also studied because PCM was developed to estimate the hazard of a system.

The rest of this chapter is organised in the following manner. In Section 6.2, the method of determining PM lead time is investigated. PCM is developed in Section 6.3, and conclusions are presented in Section 6.4.

6.2 PREVENTIVE MAINTENANCE LEAD TIME DETERMINATION

As mentioned in Chapter 3, this thesis aims to support optimal PM decisions. The objective of PM is to maintain an asset that would perform at a required reliability level and avoid catastrophic failures using the lowest possible cost. To achieve this objective, PM must be conducted at the right time. PM lead time is often determined from the aspect of reliability of a system as demonstrated in the previous chapters. One alternative measurement of reliability is hazard. The hazard function is also often used to predict when PM should be carried out [15, 25, 136, 303]. The hazard function measures the failure rate in a system and is concerned with the probability that a system will fail in the next interval $(t, \Delta t]$ if this system still survives at time *t*. The hazard function is related to the reliability function. There is a need to investigate the relationship of determining PM lead time between using the hazard function and the reliability function before developing PCM because PCM is developed to estimate and present the hazard of a system. In the candidate's view the PM time predicted based on the hazard function needs to be cross-referenced against the reliability function when the failure pattern of a system is composed of several different failure distributions. This section illustrates this argument through some case studies.

6.2.1 Hazard Functions and Corresponding Reliability Functions

General relationship of hazard function and reliability function is well established. In this section, an explicit expression for hazard functions and corresponding reliability functions are presented in order to illustrate the candidate's argument more effectively.

Research and industrial experiences have shown that failure rate or hazard has some common patterns [25]. The bath basin pattern shown in Figure 6-1 is chosen as an example.

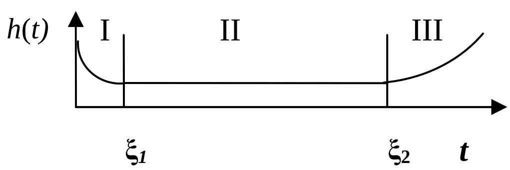


Figure 6-1. Hazard: bathtub curve

The bathtub failure pattern is a typical failure pattern of a mechanical system. It consists of three phases. Phase I represents infant mortality, i.e., the probability of failure declines with age. Phase II represents random failure, i.e., the probability of failure is constant. Phase III represents wear-out, i.e., the probability of failure increases with age. The hazard function of the bath basin failure pattern is given by Equation (6-1) which indicates that in both Phase I and III, the system exhibits Weibull failure distributions with shape parameters $\beta_1 < 1$ and $\beta_2 > 2$ respectively. On the other hand, this system has, in Phase II, an exponential failure distribution with a constant failure rate $\lambda = (\beta_1/\eta_1)(\xi_1/\eta_1)^{\beta_1-1}$.

$$h(t) = \begin{cases} \frac{\beta_1}{\eta_1}(\frac{t}{\eta_1})^{\beta_1-1} & 0 < t < \xi_1 \quad 0 < \beta_1 < 1 \quad \eta_1 > 0 \\ \frac{\beta_1}{\eta_1}(\frac{\xi_1}{\eta_1})^{\beta_1-1} & \xi_1 \le t < \xi_2 \\ \frac{\beta_1}{\eta_1}(\frac{\xi_1}{\eta_1})^{\beta_1-1} + \frac{\beta_2}{\eta_2}[\frac{(t-\xi_2)}{\eta_2}]^{\beta_2-1} & t \ge \xi_2 \quad \beta_2 > 1 \quad \eta_2 > 0. \end{cases} \tag{6-1}$$

The reliability function corresponding to Equation (6-1) is:

$$R(t) = \begin{cases} \exp[(\frac{t}{\eta_1})^{\beta_1}] & 0 \le t < \xi_1 \quad 0 < \beta_1 < 1 \quad \eta_1 > 0 \\ \exp[-\frac{\beta_1}{\eta_1}(\frac{\xi_1}{\eta_1})^{\beta_1-1}(t-\xi_1+\frac{\xi_1}{\beta_1})] & \xi_1 \le t < \xi_2 \\ \exp\{-\frac{\beta_1}{\eta_1}(\frac{\xi_1}{\eta_1})^{\beta_1-1}(t-\xi_1+\frac{\xi_1}{\beta_1})-[\frac{t-\xi_2}{\eta_2}]^{\beta_2}\} & t \ge \xi_2 \quad \beta_2 > 1 \quad \eta_2 > 0. \end{cases} \tag{6-2}$$

Hazard functions and reliability functions can be derived from each other. However, a system that has a low hazard cannot guarantee that it has high reliability. This argument can be illustrated using the following examples.

6.2.1.1 Example 1: Two machines

The following scenarios of two machines are considered

Machine 1: $\eta_1 = 1.25\ years$, $\eta_2 = 1\ year$, $\beta_1 = 0.5$ $\beta_2 = 3$, $\xi_1 = 1.5\ years$ and $\xi_2 = 4\ years$

Machine 2: $\eta_1 = 1.25\ years$, $\eta_2 = 1\ year$, $\beta_1 = 0.8$ $\beta_2 = 3$, $\xi_1 = 0.5\ year$ and $\xi_2 = 8.8\ years$

Substituting the above parameters into Equations (6-1) and (6-2) respectively, the changes of both the hazard and the corresponding reliability can be demonstrated in Figure 6-2 (a) and (b).

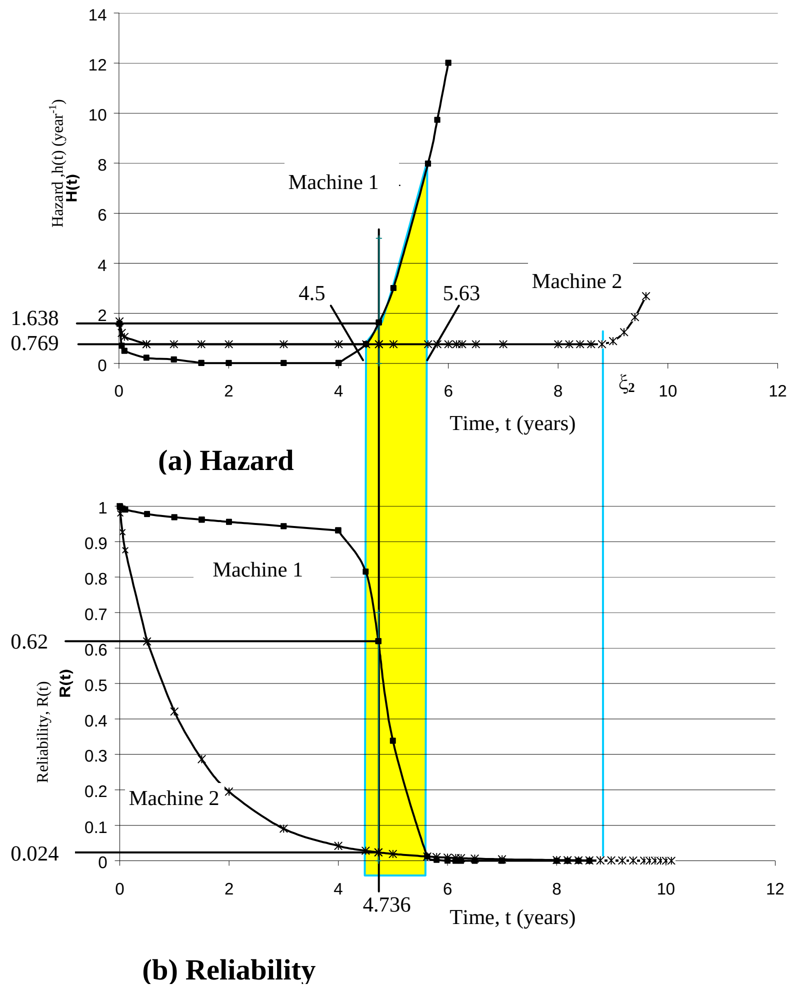


Figure 6-2. Hazard curves (a) and the corresponding reliability curves (b)

Figure 6-2 shows that both the hazard and the reliability of Machine 1 are higher than Machine 2 between 4.5 years and 5.63 years. If the critical limit for the hazard is set to be 1.638, then when the hazard of Machine 1 reaches this level, the hazard of Machine 2 is only 0.769. The hazard of Machine 2 lies below the alarm limit. However, the reliability of Machine 1 at that point is 0.62, whereas the reliability of Machine 2 is 0.024, much lower than that of Machine 1. This indicates that in some cases reducing the hazard does not guarantee an increase in reliability.

Currently, two major methods are used to predict PM time based on hazard functions. The first method establishes a hazard alarm limit in advance. The time when a hazard of an asset reaches this alarm limit is regarded as the time for PM [15]. The second method takes the time when the hazard function curve shows the wear-out phase of its life cycle as the PM time [25]. According to the above analysis, it is shown from the first method that using a predefined alarm limit to predict PM time based on the hazard function can be misleading in some cases.

If the second method to predict PM time using the hazard function is employed, i.e., ξ_2 of about 8.8 years is chosen as an alarm time for PM, it can be found that the reliability of Machine 2 is lower than 0.01 at time ξ_2. In this situation, choosing time ξ_2 as the PM time is certainly inappropriate because the probability of the system failure well before the alarm time is very high.

6.2.1.2 Example 2: Wheel motors

The above analysis method can also be used to study cases where the failure distributions of systems are non-Weibull. For example, in the case given by Jardine [15], the hazard function was derived based on PHM using historical oil monitoring and maintenance data of mine haul truck wheel motors. It was:

$$h(t) = \frac{2.891}{23360}\left(\frac{t}{23360}\right)^{1.891} e^{Z(t)}, \tag{6-3}$$

where, $Z(t)$ is the composite covariate which is composed of significant covariates (here they are the values of different particles in oil) and their associated weights. For application convenience, the hazard control limit was converted into a composite covariate control limit curve shown in Figure 6-3. If the following covariate function $Z(t)$ is used to simulate the monitored composite covariate of a wheel motor, i.e.,

$$Z(t) = \begin{cases} 0.1 & 0 < t \le 10^4 \quad hours \\ 0.1 + \dfrac{1}{1.48745\times10^{11}}(t^{2.891} - 10^{11.564}) & t > 10^4 \; hours \end{cases}, \tag{6-4}$$

then the hazard function of this wheel motor is given by

$$h(t)=\begin{cases}\frac{2.891}{23360}(\frac{t}{23360})^{1.891}e^{0.1} & 0<t\le 10^4 \ hours\\ \frac{2.891}{23360}(\frac{t}{23360})^{1.891}\exp[0.1+\frac{1}{1.48745\times 10^{11}}(t^{2.891}-10^{11.564})] & t>10^4 \ hours\end{cases}. \quad (6\text{-}5)$$

According to Equation (6-2), the reliability function of this wheel motor can be obtained. It is given by

$$R(t)=\begin{cases}\exp(-\frac{e^{0.1}t^{2.891}}{23360^{2.891}}) & 0<t\le 10^4 \ hours\\ \exp\{-\frac{e^{0.1}}{23360^{2.891}}[10^{11.564}+\frac{1.48745\times 10^{11}}{e^{2.463527}}(\exp(\frac{t^{2.891}}{1.48745\times 10^{11}})-e^{2.463527})]\} & t>10^4 \ hours\end{cases}. \quad (6\text{-}6)$$

Figure 6-3 shows the changes of the composite covariate $Z(t)$ and the reliability of the wheel motor (the first wheel motor).

From Figure 6-3 (a), it can be seen that the composite covariate $Z(t)$ had exceeded its control limit (1.21996) in the inspection at working age t =11384 hours. This wheel motor was recommended to be replaced immediately. Figure 6-3 (b) indicates that the reliability of this wheel motor at that moment (t =11384 hours) is 0.84. In addition, it can also be seen from Figure 6-3 that the reliability of the wheel motor fell under 0.91 (0.909) when its composite covariate started to increase at the age of 10000 hours.

Furthermore, in order to make a comparison, the composite covariate of another wheel motor is assumed to be represented by the solid-line in Figure 6-3 (a). This wheel motor is denoted as the second wheel motor in order to distinguish it from the wheel motor mentioned above (the first wheel motor). It can be found from Figure 6-3 (b) that the reliability of the second wheel motor is much lower than the first between 8000 hours and 12000 hours. According to the control limit curve, both wheel motors are recommended to be replaced at the same working age (11384 hours). However, the reliability of the second wheel motor is 0.74 at that moment, much lower than the reliability of the first at the same time (0.84). The solid-line in Figure 6-3 (b) demonstrates that the reliability of the second wheel motor has fallen under 0.84 at working age=10000 hours (0.817). Therefore, if the reliability of the second wheel motor is to be maintained above 0.84, it should be replaced before 10000 hours, 1384 hours earlier than the replacement time suggested by the composite covariate limit curve.

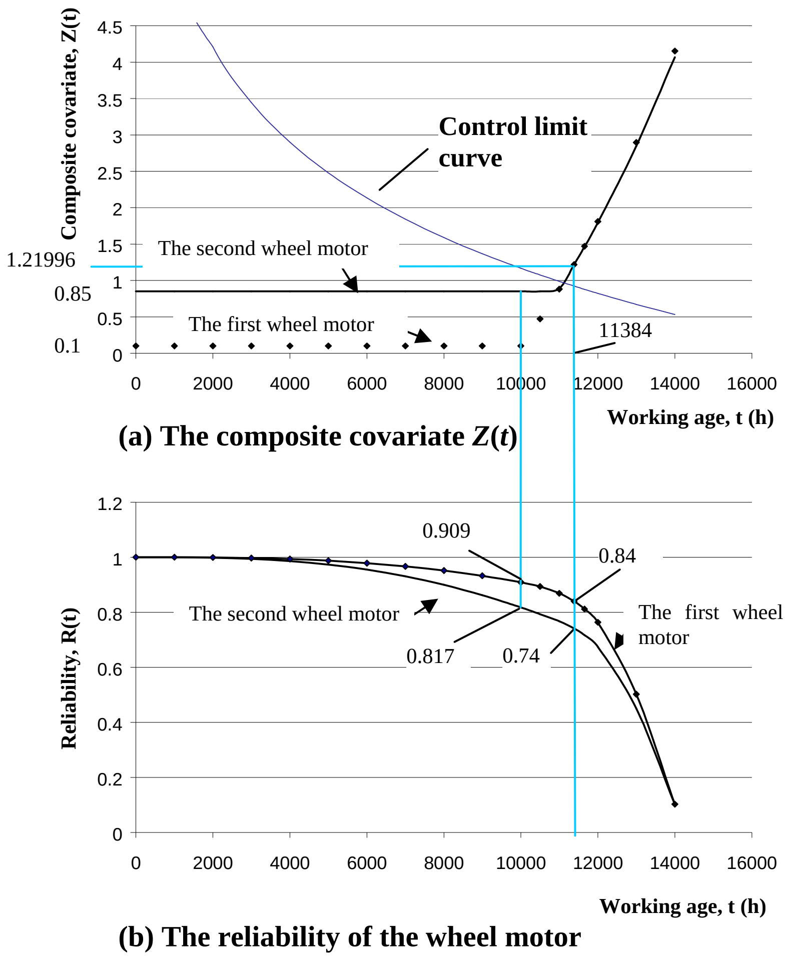


Figure 6-3. The composite covariate *Z*(*t*) (a) and the reliability of the wheel motors (b)

6.2.1.3 Example 3: Mechanical test rig

A system often has different hazard functions under different operation conditions. An example is shown in Figure 7-13 which was obtained using a bearing test rig. The test rig and the experiments will be presented in Chapter 7. Figure 7-13 is reproduced here for convenience. The failure distribution function of the test bearing corresponding to this figure is shown in Figure 7-14 in Chapter 7.

From Figure 7-13, it can be seen that a common hazard alarm limit cannot be predefined for the test bearing under two different conditions. The initial hazard of the bearing under the first condition was higher than the hazard at 1600 hours of the bearing under the second condition. Figure 7-14 indicates that at 1600 hours, the failure probability of the bearing under the second condition was almost 100%. In this case, only the reliability function can be used to determine the time for conducting PM. For example, if the predefined reliability limit is 50%, then the PM time for the bearing under the first condition was 350 hours (20.16 million revolutions) whereas for the bearing under the second condition was 900 hours (50.84 million revolutions).

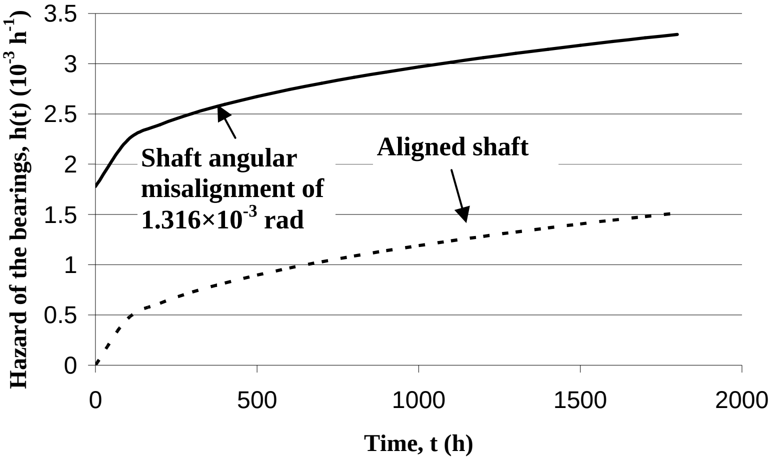


Figure 7-13. Hazard of the right bearing

6.2.2 Comments

Using the hazard function to support PM decision making is not suitable for those failure patterns, in which the failure characteristics of an asset at different stages are represented using several different failure distributions. The resulting PM decisions based on the hazard may not be an accurate reflection of the reliability of assets. The predicted PM time based on the hazard function should therefore be cross referenced against its corresponding reliability functions. However, the investigation of hazards is still very useful because the reliability functions of systems or components can be derived from their corresponding hazard functions.

6.3 PROPORTIONAL COVARIATE MODEL – DEVELOPMENT

A Proportional Covariates Model (PCM) used to predict the hazard of a system using condition data is developed in this section.

Condition data are often termed as covariates in reliability engineering. Covariates can be classified into two categories:

(1) Environmental covariates $Z_e(t)$. The changes of these covariates will cause the characteristics of the hazard of a system to change. In the case study of the motor presented by Ebeling [16], the load placed on the motor was an environmental covariate.

(2) Responsive covariates $Z_r(t)$. The changes of these covariates are caused by the changes of the hazard of a system. Most of condition monitoring data belong to responsive covariates and are symptoms that reflect the deterioration of a system.

This distinction between environmental and responsive covariates is similar to the distinction made for external and internal covariates as discussed, for example, in [260]. Distinguishing environmental covariates from responsive covariates sometimes can be critical to an accurate prediction of the hazard of an asset. This argument can be best demonstrated by the following example:

An oil analysis is often conducted to assess the condition of an engine. Assume that the initial oil entering the engine is clean and all debris coming from the engine enters the oil. Then the metal debris in the oil out of the engine can be used to indicate the wear condition of the engine. For example, a total of X μg metal debris in the oil indicates that this engine has been worn out X μg. In this case, this metal debris is the responsive covariate. If this contaminated oil is not filtered and enters the engine again, this X μg metal debris becomes an environmental covariate because it will generally accelerate the wear of the engine. However, this contaminated oil normally will not cause exactly X μg metal wear from the engine. Hence, to accurately predict the hazard of a system using a covariate, one needs to know not only the value of this covariate, but also the role of this covariate – whether it is an environmental covariate or a responsive covariate. PCM focuses on using responsive covariates for hazard prediction.

It is noted that the Proportional Hazard Model (PHM) also predicts the hazard of a system using historical failure data and condition monitoring data. PHM has been used in various applications [4, 15, 16, 258, 259]. Ebeling [16] presented two case studies. One of these was to analyse the effect of the load placed on a motor on the design life of this motor for a particular reliability level.

The parameters of PHM are normally estimated using the Maximum Likelihood Estimation (MLE) method. PHM needs sufficient failure data to estimate the baseline hazard function $h_0(t)$ and the weight parameters for each covariate. This shortcoming limits the effectiveness of PHM significantly when historical failure data is insufficient. In addition, the accuracy of prediction of PHM can be affected by the fluctuations of covariates greatly. PHM does not reflect the human's general understanding of condition monitoring when it is used to model the relationship between the responsive covariates and the hazard of a system. A general understanding of PHM is that a system has a baseline hazard when the covariates of the system are zero. When the covariates change, the hazard of the system changes correspondently. However, the relationship between responsive covariates and hazard is that the responsive covariates of a system change with the change of its hazard.

The PCM is developed to address these limitations of PHM for the applications in reliability engineering.

6.3.1 Concepts

PCM uses the same assumption as that used in PHM and assumes that covariates of a system, or a function of these covariates, are proportional to the hazard of the system – an assumption that has been supported by empirical evidence [4] and has also been validated by experiments conducted by the candidate (see Chapter 7).

A common understanding of mechanical systems integrity is that increased deterioration more often than not increases the likelihood of failure [176, 177]. Accurate condition monitoring data (covariates) of a system should reflect the degree of the deterioration of the system [304, 305]. Therefore it is reasonable to assume that a covariate of a mechanical system is a continuous and monotonous function of the failure rate (hazard) of the system. The mathematical relationship between these covariates and system hazard can be modelled in different ways, such as a linear function. As a result, the assumption that covariates or their transformed variables, of a system are proportional to the hazard of the system is justified.

This same assumption has been used by Cox [306] while developing PHM. Over last 30 years, PHM has found numerous applications using realistic cases and data. In particular, this assumption has been used to study mechanical systems [16, 67, 258, 307]. Barbera et al [208] developed a condition based maintenance model for repairing equipment based on the same assumption that the hazard of equipment is a linear function of the condition of the equipment. Heyns and Smit [305] demonstrated that the measurement of the natural frequency shift of a fan had a linear relationship with the damage level of the fan throughout his experiments.

In PCM, $\Psi(Z_r(t))$, a function of multiple covariates, is expressed as follows:

$$\Psi(Z_r(t)) = C(t)h(t)\,, \tag{6-7}$$

where, $Z_r(t)$ is the covariate function which is usually time dependent; $C(t)$ is the baseline covariate function which is also usually time dependent and $h(t)$ is the hazard function of a system. Considering the flexibility of Weibull distribution, hazard function $h(t)$ is assumed to have the form of Weibull model in this thesis.

The formulation of the function of covariates $\Psi(Z_r(t))$ plays an important role in improving the accuracy of hazard estimation when using multiple covariates. Due to the limit of candidature, this thesis only investigates the simplest scenario where only one covariate is utilised, and the formulation of the covariate is given by

$$\Psi(Z_r(t)) = Z_r(t)\,. \tag{6-8}$$

The PCM for the simplest case is obtained by substituting Equation (6-8) into Equation (6-7):

$$Z_r(t) = C(t)h(t)\,. \tag{6-9}$$

In PCM, the hazard is the explanatory variable and the covariate is the response variable. The procedure to estimate the hazard function of a system in PCM is different from that in PHM although they have similar function form.

6.3.2 Procedure

The procedure of PCM used in this study is outlined as follows:

(1) Identify failure distribution of a system using its historical failure data $\{t_i\}$ (i =1, 2, …, m_f), where m_f is the number of failure data.

(2) Estimate the initial hazard function $h_{in}(t)$ of the system using the Maximum Likelihood Estimation (MLE) method. The techniques of estimating a hazard function using historical failure data can be found in most books on reliability, for example, in [12].

(3) Analyse the co-relationship between the covariates and the hazard of this system. A covariate should not be used for updating the estimation of hazard if that covariate has a poor relationship with the hazard of a system; or otherwise, updating the estimation of hazard using this covariate will be inaccurate. Correlation analysis is a mature technique and can be found in commercial software such as Matlab.

(4) Estimate the baseline covariate function. From the initial hazard function and

historical covariate data, a set of discrete values for baseline covariate function can be generated:

$$C_k = \frac{Z_r(t_k)}{h_{in}(t_k)} \quad (k = 1, 2, 3, \ldots, m_c). \tag{6-10}$$

where m_c is the number of condition monitoring data.

Then the baseline covariate function can be obtained using the discrete data set $\{C_k, t_k\}$ $(k = 1, 2, 3, \ldots m_c)$ and regression techniques. The recommended functions to represent the baseline covariate functions include the following models:

(a) the polynomial models of various orders,

$$C(t) = a_0 + a_1 t + a_2 t^2 + \cdots, \tag{6-11}$$

(b) the multiplicative model,

$$C(t) = at^b \tag{6-12}$$

and (c) the exponential model

$$C(t) = ae^{bt}, \tag{6-13}$$

where, parameters a_0, a_1, a_2, a, and b are to be identified.

If these nonlinear models can be assumed to be intrinsically linear, standard linear regression procedures can be used to estimate these models, or otherwise nonlinear regression procedures are needed. The required regression techniques can be found in the reference [308].

(5) Update the hazard function of the system using new condition monitoring data $\{Z_r(t_j)\}$ (j =1, 2, …, m_n). Parameter m_n is the number of new condition monitoring data.

$$\tilde{h}_i = \frac{Z_r(t_i)}{C(t_i)} \quad (i = 1, 2, \ldots, m_c, m_c + 1, m_c + 2, \ldots, m_c + m_n) . \tag{6-14}$$

As the hazard function $h(t)$ is assumed to have the form of Weibull model, $h(t) = \frac{\beta}{\eta^\beta} t^{\beta-1}$, then the estimated hazard function of the system $\tilde{h}(t)$ can be obtained using the regression techniques and based on the discrete updated hazard data set $\{ \tilde{h}_i, t_i \}$ $(i = 1, 2, \ldots, m_c, m_c + 1, m_c + 2, \ldots, m_c + m_n)$. Note that in some cases, only the latest condition monitoring data instead of whole condition monitoring data will be used to update the hazard estimation.

(6) Update both $C(t)$ and $\tilde{h}(t)$ using the above steps (1) to (5), if new failure datum is obtained.

(7) Calculate the updated reliability function of the system using the updated hazard function.

(8) Predict the reliability of the system using the updated reliability function and make preventive maintenance decisions.

In the above procedure, steps (1) to (4) are used to estimate the baseline covariate function. These four steps are not applicable if failure data is zero. However, the baseline covariate function can still be estimated under certain conditions (see Subsection 6.3.7).

6.3.3 Comparisons between PCM and PHM

PCM differs from PHM as its principles and methodology are quite different.

In PHM, a baseline hazard rate $h_0(t)$ is used to describe the relationship between covariates and hazard, whereas in PCM, a baseline covariate function $C(t)$ is employed to describe the relationship between covariates and hazard. The baseline hazard rate $h_0(t)$ is the hazard rate without influence of covariates. It is covariate independent. The baseline covariate function $C(t)$ represents the rate of change of covariates when the hazard changes. It is covariate dependent. In PHM, covariate with zero value indicates that the hazard of a system change based on its baseline hazard; whereas in PCM, covariate with zero value indicates that the hazard of a system is zero.

In PCM, the hazard function of a system estimated based on different historical covariate data are consistent, whereas in PHM, the estimated hazard function may change in form when a different covariate is used. This phenomenon can be obtained because different covariates can have different influences on the hazard of a system.

6.3.4 Tracking Changes of the Hazard function

Most statistical models use historical failure data only. These models predicted hazard or reliability using the tendency method, i.e., according to the trend of the hazard function derived from historical conditions of a system. These models can lead to unacceptable errors if the conditions of the system change significantly. To improve the prediction accuracy, on-line condition monitoring data should be used in the prediction models because these data can reflect the latest conditions of a system. PCM predicts hazard using both on-line condition monitoring data and historical data including failure data and condition monitoring data. PCM based hazard estimation can automatically track real changes in the hazard function which can change due to alterations in the operating conditions of a system. This capability of PCM is proved as follows.

In practice, the conditions of a system often change and when a change occurs, the hazard characteristics of the system will change too. Several researchers including Jiang and Murthy [309] have revealed and modelled this change of the hazard characteristics through the investigations of historical failure data of systems. In this case, the overall hazard of the system is often represented using multiple sectional distributions rather than a single distribution [309]. On the other hand, PHM indicates that the hazard characteristics of the system can continuously change with the change of environmental conditions. Suppose the hazard function of a system changes at time t_c. Let

$$Z_1(t) = C(t)h_1(t) \tag{6-15}$$

be the PCM based model of the system derived from historical data. After t_c, the hazard function of the system changes to $h_2(t)$. The hazard function $h_2(t)$ can then be expressed as:

$$h_2(t) = h_1(t) + \varepsilon(t), \tag{6-16}$$

where, function $\varepsilon(t)$ represents the difference between $h_2(t)$ and $h_1(t)$.

Let $Z_2(t)$ be the covariate after t_c. If it is assumed that the relationship between the covariate and the hazard of the system remains the same, the new covariate can be described by the following equation:

$$\begin{aligned} Z_2(t) &= C(t)h_2(t) \\ &= C(t)h_1(t) + C(t)\varepsilon(t). \end{aligned} \tag{6-17}$$

In PCM, the new covariate is used to update the estimated hazard:

$$\tilde{h}(t) = \frac{Z_2(t)}{C(t)}, \tag{6-18}$$

Substituting $Z_2(t)$ with Equation (6-17), gives

$$\tilde{h}(t) = h_1(t) + \varepsilon(t). \tag{6-19}$$

Equation (6-19) indicates that the updated hazard function according to PCM is equal to the new hazard function $h_2(t)$, which is different from the original hazard function $h_1(t)$ due to the change in the operating conditions of the system.

In order to justify the above analysis, a series of simulations were conducted. The simulation results are presented in Figures 6-4 to 6-6. Figures 6-4 and 6-5 describe the random failure data of a system and the normalised covariate data respectively. Figure 6-6 displays the estimation results when different numbers of on-line condition monitoring data were used to update the estimated hazard function.

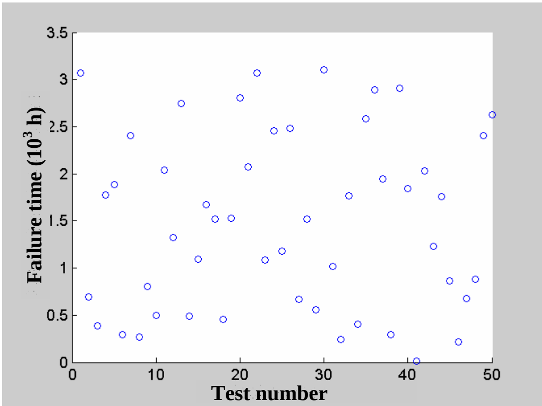


Figure 6-4. The failure times

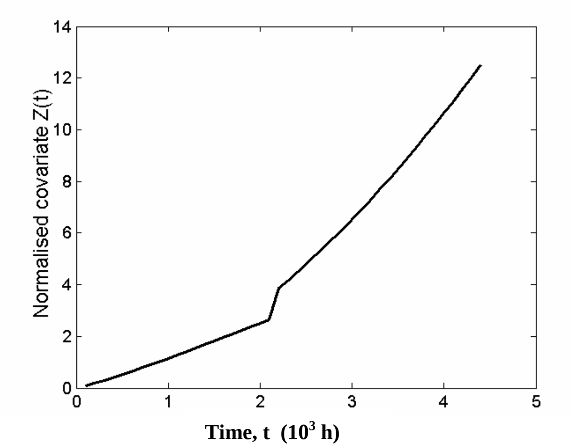


Figure 6-5. Covariate data

Figure 6-6 clearly indicates that the updated hazard estimation automatically tracked real changes in the hazard function of a system. From the figure, it can be seen that the initial hazard predicted using PCM is exactly equal to the initial hazard calculated from the failure times. The reason is that the baseline covariate function is estimated based on this initial hazard function and the corresponding historical responsive covariate data. In PCM, the hazard of a system is an explanatory variable and its change is independent of the responsive covariates of the system, but the changes of these responsive covariates are dependent on the change of the hazard. From Figure 6-6, it can also been seen that the time for the estimated hazard converging to its real hazard became longer when more covariate data were used to update the estimated hazard function. This phenomenon will be analysed in Subsection 6.3.6.

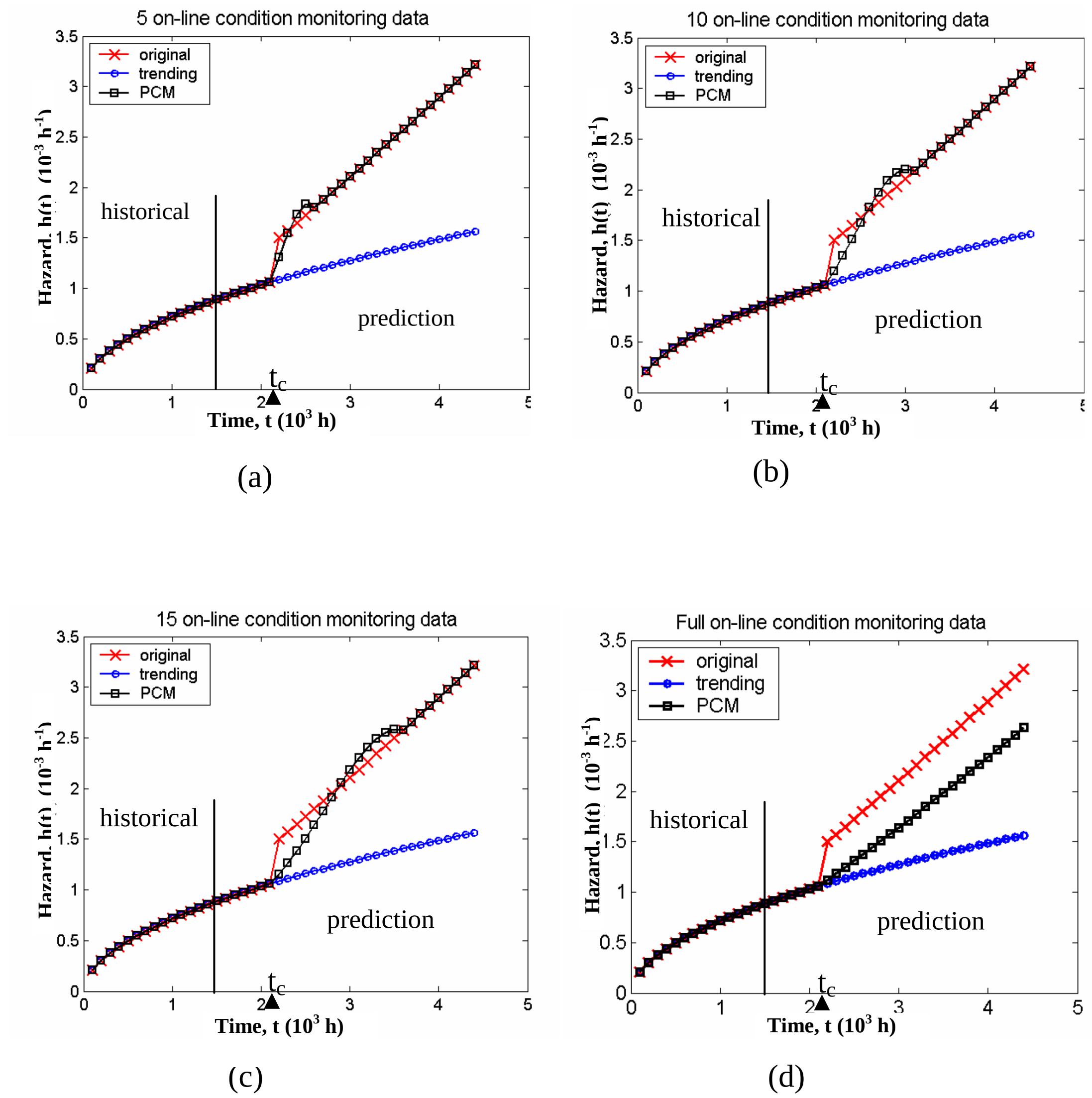


Figure 6-6. The effectiveness of PCM to update the estimated hazard h(t)

6.3.5 Robustness

In Subsection 6.3.4, all covariate data used to update the estimated hazard function were clean and not contaminated by noise – a very unlikely scenario in the real world. To evaluate the efficiency and robustness of PCM, another series of simulations were conducted. In these simulations, different kinds of corrupted covariate data were used to update the estimated hazard function. The results of the simulations indicated that PCM was robust provided that the corrupting noise had a zero mean value. Some results are shown in Figures 6-7 and 6-8. In these figures, the initial hazard function was estimated using 150 historical failure data.

Figure 6-7 shows normalised covariate data which were contaminated by Gaussian random noise. The mean value of the noise was zero and the standard deviation was 0.5. Figure 6-8 shows the simulation results using contaminated covariate data to update the estimated hazard function.

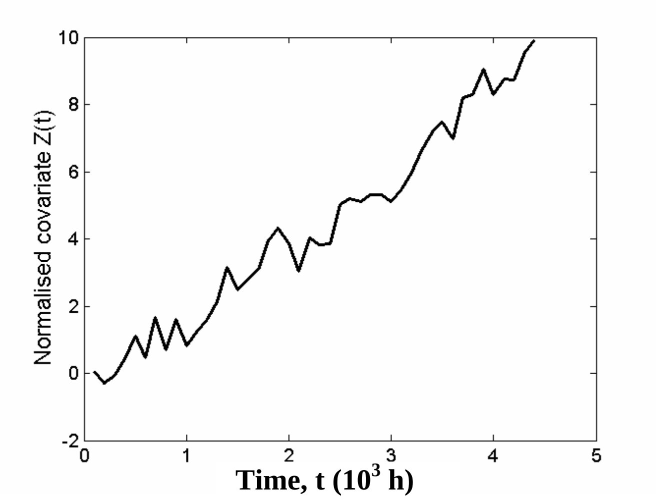


Figure 6-7. Contaminated covariate data

Comparing Figure 6-8 with Figure 6-6, one can find that PCM was robust and can reduce the effects of covariate fluctuations on hazard estimation. Figure 6-8 shows that the influence of corrupting noise decreased with the increasing number of covariate data used for updating the hazard function. The reason - for random noise with zero mean value, the more data used, the less the effects of noise on the estimation results.

At the beginning stage of the prediction, the prediction accuracy of PCM may be lower than tendency method if the hazard function of a system changed only marginally and the covariate data were contaminated by noise (refer to Figure 6-8). The length of this undesirable period depended on the severity of contamination and the data number of the covariate used for updating the estimated hazard function. In fact, the above problem encountered when PCM is used, also exists in other models that predict reliability or hazard using condition monitoring data such as PHM. The reason for this phenomenon was that contaminated condition monitoring data caused estimation errors. When only a minimal set of condition monitoring data were used to estimate the hazard, the effect of the noise contained by the data could not be removed even though this noise had a zero mean value. On the other hand, in a short period at the beginning of the prediction, the hazard did not change much so that the trend of the historical hazard function did not depart much from the real hazard. In this case, the tendency method had higher prediction accuracy.

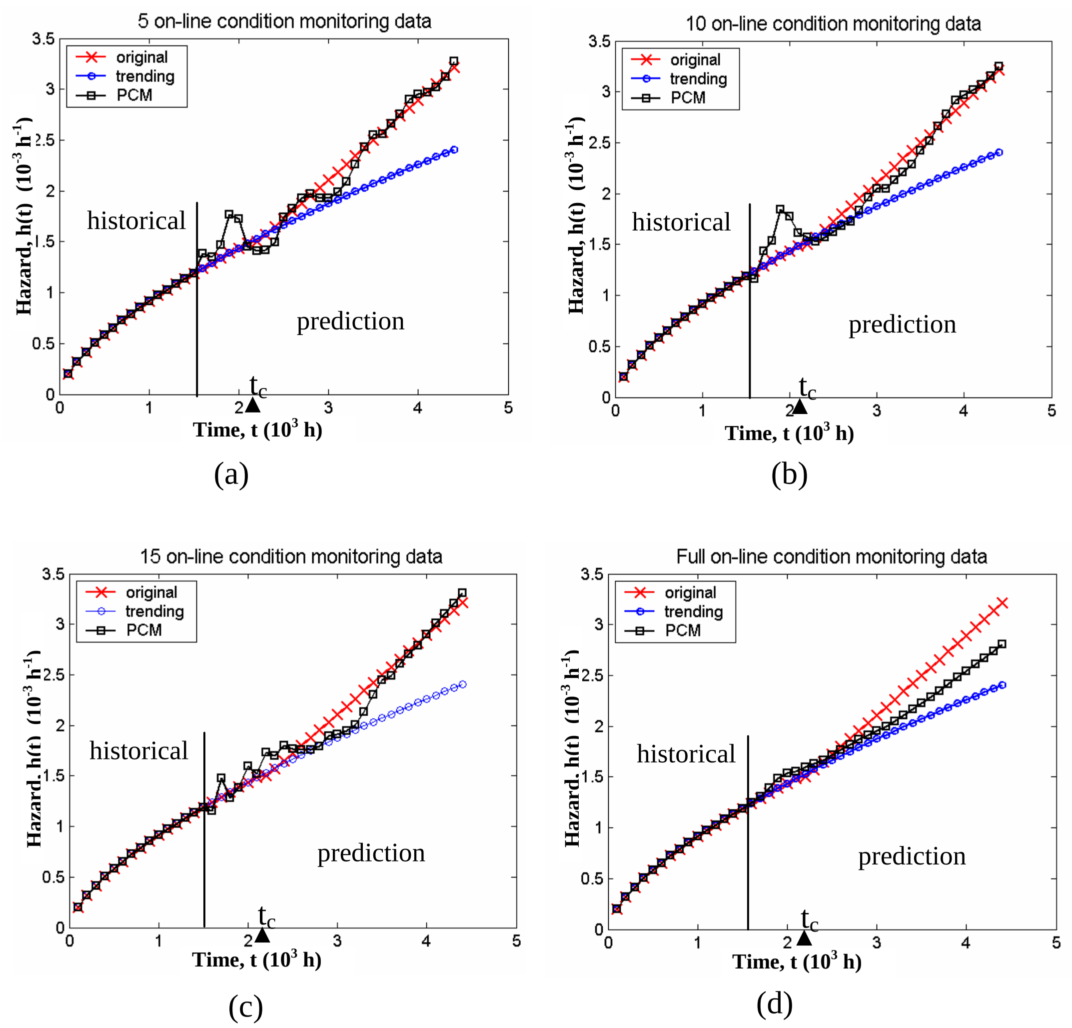


Figure 6-8. Hazard estimated with the contaminated covariate data

One approach to improve the accuracy at the beginning of prediction is to forecast hazard using both PCM and the tendency method, because in reality one cannot predict whether the hazard function of a system does change. Another approach is to increase the frequency of the acquisition of on-line condition data. This approach can shorten the length of the undesirable period where the estimated hazard is severely affected by the noise level in condition monitoring data.

6.3.6 Condition Monitoring Data for Updating Hazard Function

From the analysis in Subsection 6.3.5, one can draw the following conclusion. To reduce the effects of the corrupting noise on the estimated hazard function, the number of covariate data for updating the hazard function should be as large as possible. However, if looking back at Figure 6-6, one can find an interesting phenomenon: the more covariate data used to update the estimated hazard function, the slower the convergence of this estimated hazard function to the real hazard function. This phenomenon can be explained as follows:

After the operating conditions of a system change at time t_c, the covariate data collected before t_c become inaccurate data because the new data does reflect the new conditions of the system. If the old data is used to update the hazard, the estimated hazard function will deviate from the real hazard function. The estimated hazard function will be equal to the real one only after all these "inaccurate" data have been replaced by the new data collected after t_c. The more data used to update the hazard function, the longer time is needed to replace the "inaccurate" data because under a given frequency of data acquisition, collection of more data takes a longer period of time. One should therefore use fewer covariate data to update the estimated hazard function if a quick response of the estimated hazard function to the real hazard function is desired.

The number of covariate data used for updating the hazard function should be determined based on specific cases. Generally speaking, the less the covariate is corrupted by noise, the fewer the number of covariate data should be used, and vice versa. If the hazard characteristics of a system change marginally, the number of covariate data can be larger. In the candidate's study, five to ten data sets were used. When the characteristics of the hazard of a system change, one should avoid using all covariate data for updating the hazard function because the "inaccurate" data will never be replaced and the tracking process will take longer to settle (refer to Figure 6-6 (d) and Figure 6-8 (d)). If both quick tracking process and high prediction accuracy are required in this situation, one needs to increase the frequency of data acquisition – collecting more data within the same or even shorter period of time. However, this approach often means an increase of cost.

6.3.7 Case Studies – Truck Engines and Spur Gearboxes

6.3.7.1 Case study 1: Truck engines

The field data used in this case study were obtained from the maintenance history and the oil analysis report of selected engines from some haul trucks commonly used in mining industry. In the case study, the overall hazard of the truck engines was analysed using PCM.

The condition monitoring covariates presented in the report included the measurements for seven types of metal wear debris in the unit of parts per million (ppm) and the measurements for three types of non-metal materials in percentage of allowable volume. Correlation analysis indicated that the increment of Iron (Fe) debris was sensitive to the changes of the hazards of the engines. The increment of Fe particles was hence used as a covariate in this case study. Figure 6-9 and Figure 6-10 show the changes of the increment of Fe particles from two engines (Engine 1 and Engine 2). The failure data of these two engines collected over time used in this case study. The state of the engines after repairs was assumed to be as good as new. To verify the effectiveness of PCM, the historical data (failure data and the measurement of Fe particles) of Engine 1 were used to estimate the initial hazard function and the baseline covariate function. Based on this estimated baseline covariate function, the prediction on the hazard of Engine 2 is conducted using PCM. The predicted hazard was compared to the real hazard function obtained using the full original failure data of Engine 2 as well as the prediction using a conventional approach. The conventional approach to predicting the hazard of Engine 2 used the estimated hazard function of Engine 1 since they were the same type of engines.

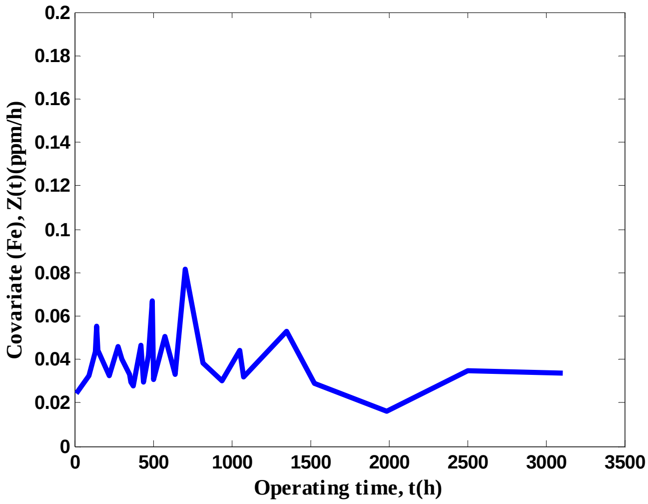


Figure 6-9. The changes of Fe particles – Engine 1

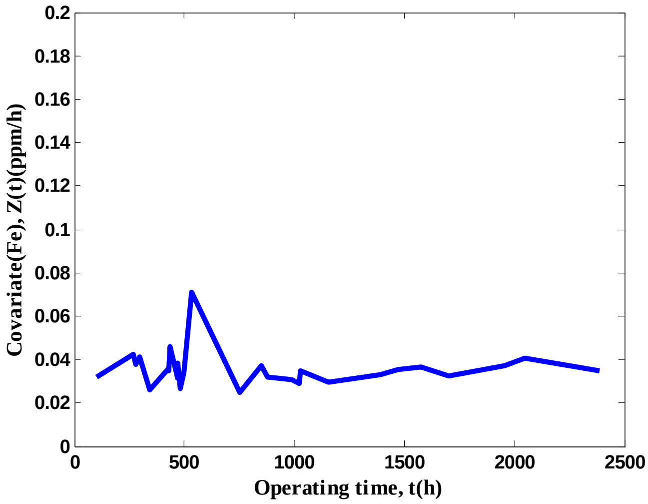


Figure 6-10. The changes of Fe particles – Engine 2

In this case study, the failure times of the engines were assumed to be Weibull distributed as shown in Figure 6-11 and Figure 6-12.

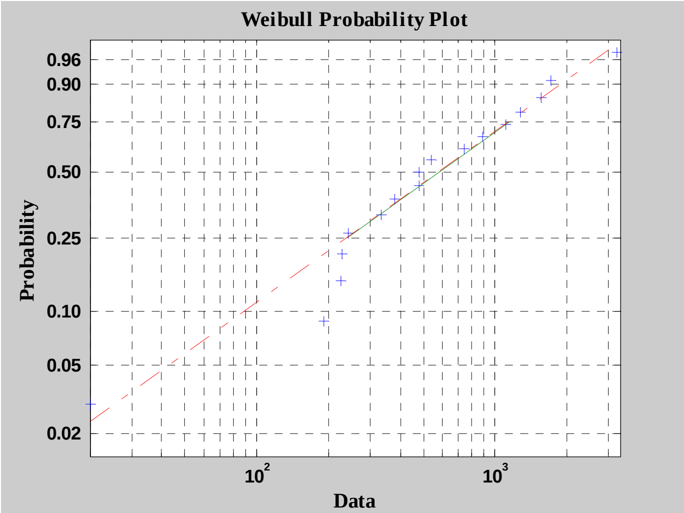


Figure 6-11. Weibull probability plot - Engine 1

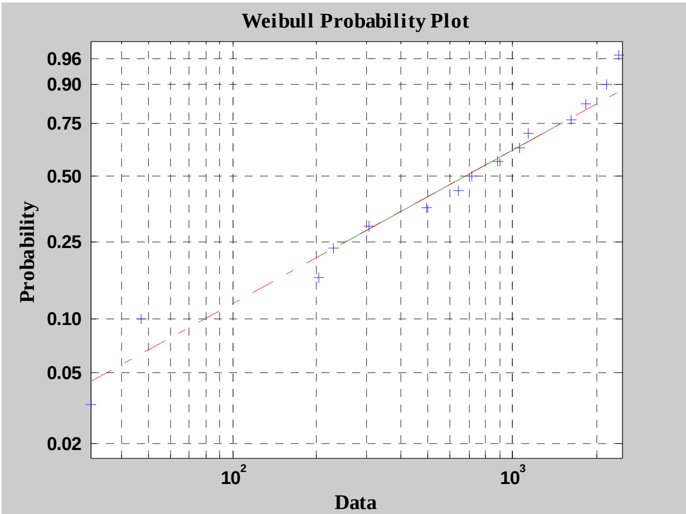


Figure 6-12. Weibull probability plot - Engine 2

The initial hazard function was obtained using the first group of historical failure data.

$$h_{in}(t) = \frac{1.0756}{827}\left(\frac{t}{827}\right)^{0.0756}, \tag{6-20}$$

where, $h_{in}(t)$ is the estimated hazard function for Engine 1.

In Step (4) of the procedure for PCM (see Section 6.3.2), three models were recommended for representing the baseline covariate functions. In this case study, the multiplicative model (Equation (6-12)) was chosen. Using the measurement of Fe particles of Engine 1 and the estimated initial hazard function (6-20), the baseline covariate function was obtained based on Step (4) of the procedure:

$$C(t) = 49.713t^{-0.0827}. \tag{6-21}$$

Assume that the above baseline covariate function is also suitable for representing the relationship between the covariate (the measurement of Fe particles) and the hazard of Engine 2. Therefore the hazard function for Engine 2, $h_e(t)$, can be obtained based on Step (5) of the procedure for PCM and it was given by

$$h_e(t) = \frac{1.0623}{867.34}\left(\frac{t}{867.34}\right)^{0.0623}, \tag{6-22}$$

The full historical measurement of Fe particles of Engines 2 were used for estimating this hazard function because the characteristic of hazard of Engine 2 did not change (refer to Figure 6-12).

Figure 6-13 shows the comparison prediction results of using PCM and the conventional approach, i.e., to predict the hazard of Engine 2 using the hazard function estimated from the historical failure data of Engine 1 (Equation (6-17)). From this figure, it can be seen that the hazard of Engines 2 is lower than that of Engines 1. This difference was caused by different working conditions and can be well explained by PHM. Figure 6-13 indicates that PCM based prediction is more closely matched to the original hazard line than the conventional approach based prediction. The hazard function estimated using PCM certainly more accurately reflects the true hazard than using the conventional approach within the observation period (about 10000 hours).

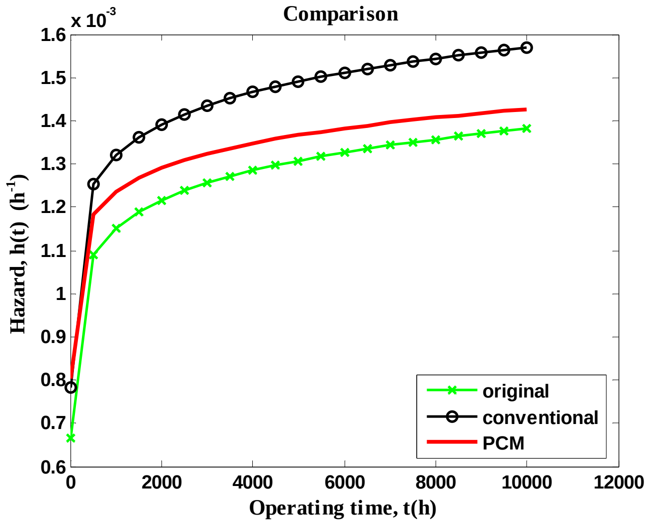


Figure 6-13. The original hazard, the conventional and the PCM based prediction

6.3.7.2 Case study 2: Spur Gearbox

Estimation of the baseline covariate function $C(t)$ is a critical procedure in PCM. The baseline covariate function of a system can be estimated by the following approaches:

(1) The baseline covariate function $C(t)$ is typically estimated based on historical failure data and covariates, which was demonstrated in Case study 1.

(2) In case of sparse or even zero historical data, the baseline covariate function $C(t)$ can also be determined using other information such as accelerated life test data. Hence PCM can be used to estimate hazard functions of systems in this case.

To demonstrate this, a case study was conducted using acceleration life test data on a single stage spur gearbox. Table 6-1 shows the experimental data for operating hours, increments of the crack depth of the test gear and the kurtosis of the residual signal.

Table 6-1. The test gearbox data

Operating hours	0.0917	3.3383	3.7536	4.6383	5.5064	5.6864
Kurtosis of the residual signal	2.2933	2.6934	3.6728	3.5146	3.2240	4.7228
Increments of crack depth (mm)	0	1.57	1.73	2.11	2.81	3.16

A residual signal is obtained from the signal average by filtering out gear meshing harmonics (i.e, using a multi band-stop filter). It represents random transmission errors for healthy gears. For faulty gears, the transmission errors will include a sudden change (eg. a spike) which becomes non-Gaussian. Kurtosis is a good measure of non-Gaussianity (eg. spikiness) in a signal. Tooth cracking and tooth pitting type of faults can be distinguished using the residual signal methods [310].

In this experiment, each test gear was 10 mm wide and had 27 teeth. Its rated load was 24.5 kW at a shaft speed of 2400 rpm, but the gears were overloaded during the tests to “accelerate” the onset of failure. In addition, each gear was initially spark-eroded with a semi-circle notch of 1 mm radius at the root fillet of a tooth, across the middle of the tooth width. When the increment of crack depth of the test gear reached 3.16, the gear box did not operate normally any more.

The vibration of the test gearbox was continuously monitored and recorded. The kurtosis of the residual signal of gear meshing vibration signal was trended and used as a local fault indicator for gear fault diagnosis. In this paper, these test data were used to estimate the trend of the hazard of the test gears, and the hazard functions of the gears. In this case study, the covariate was selected as the kurtosis of the residual signal (the second row in Table 1). Previous research [310, 311] has revealed that the kurtosis of the residual signal has a good co-relationship with the crack of the test gear. The baseline covariate function was estimated using the following two assumptions:

(1) The hazard rate of the test gear is proportional to its crack depth after initiation – a reasonable assumption because a gear with a deeper crack is

likely to breakdown earlier. The assumption was further supported by correlation analysis between the increments of the crack depth of the test gear and its failure rate (see Figure 6-14).

(2) The failure rate of the test gear follows the Weibull distribution (see Figure 6-15). This assumption holds because the test gearbox is a typical mechanical system and the test was conducted to simulate the wear-out stage (crack propagation). This assumption has been supported by Mann's test for the Weibull distribution. The Mann's test statistic M was obtained to be 0.881 which was less than the critical value $F(0.05, 6, 6)$. Hence, the hypothesis that the failure times are Weibull was accepted at the level of significance 0.05.

Using the above two assumptions and Equation (6-15), the baseline covariate function $C(t)$ and the hazard function $h(t)$ were estimated. The multiplicative model (Equation (6-12)) was used to construct the baseline covariate function. In this case, the baseline covariate function $C(t)$ contained the unknown proportional scale which represents the relationship between the hazard rate of the test gear and the increments of its crack depth. Figures 6-16 and 6-17 show the results of the PCM based hazard estimation using 4.47 hours and 5.69 hours online condition monitoring data respectively.

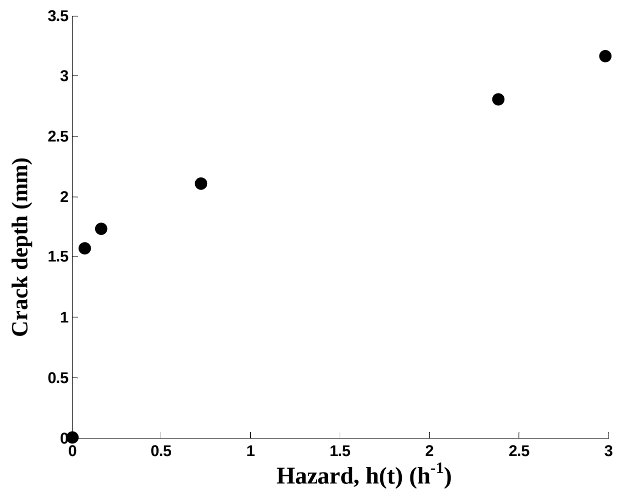


Figure 6-14. Relationship between the increment of crack depth and hazard

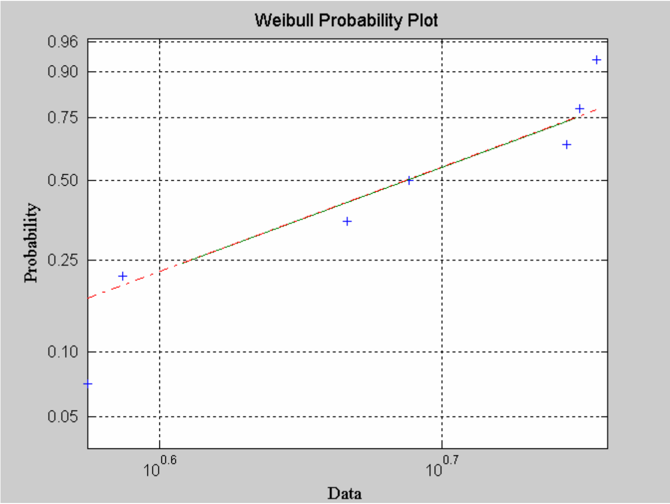


Figure 6-15. Weibull fitness check

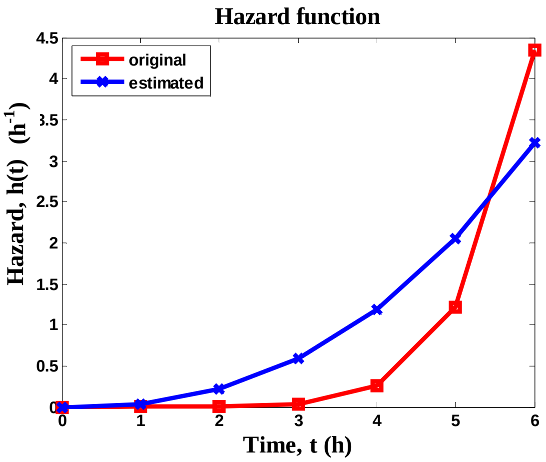


Figure 6-16. Hazard curves of the test gears: 4.47 hours condition monitoring data

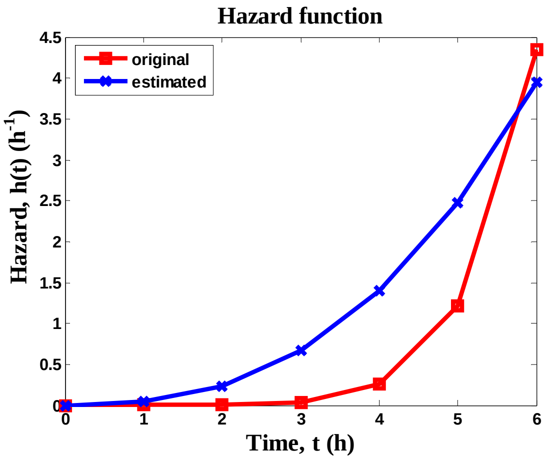


Figure 6-17. Hazard curves of the test gears: 5.69 hours condition monitoring data

The estimated hazard function was

$$h(t) = 0.0403t^{2.5591} . \qquad (6\text{-}23)$$

Figure 6-18 presents a reliability probability distribution of the test gear based on the hazard estimation shown in Figure 6-17. The figure reveals that the reliability of the test gear would be lower than 1% after five and half hours of overloaded operating time. In reality, this low reliability indicated that test gear would certainly operate abnormally after five and half hours of overloaded operating time. The test results confirmed the estimation.

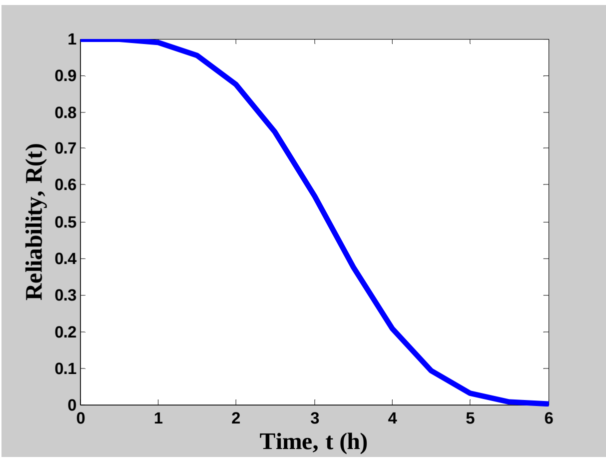


Figure 6-18. Reliability diagram of the test gears

The hazard estimation based on PCM is relatively accurate. Figures 6-16 and 6-17 indicate that the hazard estimation using PCM has the same trend with the original hazard rate. The prediction accuracy increased when more on-line condition monitoring data were used for hazard estimation. The departure between the estimated hazard line and the original hazard line was caused by the departure of the real data from the above two assumptions used to estimate the baseline covariate function. A correlation analysis (Figure 6-14) indicates that the hazard rate of the test gear can be treated as proportional to the increments in crack depth during most of the test period but not at the start of the test because of the initial spark-eroded notch. From the Weibull fitness analysis (Figure 6-15), it can be seen that the failure data is not strictly Weibull distributed although the goodness of fit is reasonable.

6.4 SUMMARY

PCM presents a new approach to predict failure of a system or a component using both condition monitoring data and historical failure data. Compared with PHM, PCM has the following advantages:

(1) In PHM, the baseline hazard function is dependent on historical failure data whereas in PCM, the baseline covariate function can be estimated with even zero failure history. The reason is that the baseline covariate function can be estimated empirically or from accelerated life tests. Hence, PCM can be used to estimate hazard functions of systems in the case of sparse or even zero historical data.

(2) The time for scheduling preventive maintenance can be predicted by PCM, whereas PHM is unable to do so. PHM only triggers an alarm when the hazard of a system has reached a predefined level because it needs covariate data to calculate the hazard values of the system.

(3) The fluctuations in condition monitoring data have much less influence on PCM than on PHM. In PCM, a set of points of a covariate is used to update the estimation of a hazard function at any time, whereas in PHM only single datum of a covariate is used to estimate a single hazard value at each time.

Under the condition that the hazard of a mechanical component or system is proportional to the deterioration of the component or system, the hazard functions of this component or system can be estimated through a combination of PCM and accelerated life tests. In principle, the reliability function of a mechanical system can be estimated by a single accelerated life test when PCM is used. Therefore, the number of accelerated life tests for estimating the reliability of a mechanical system can be significantly reduced by a combination of PCM and accelerated life tests.

In PCM, the hazard function of a system can be updated using on-line condition monitoring data so that the latest changes of the characteristics of the hazard of this system can be determined. PCM based hazard estimation can automatically track real changes in the hazard function which can change due to alterations in the operating conditions of a system, even when condition monitoring data are contaminated by noise (see Figures 6-6 and 6-8). PCM is robust as long as the corrupting noise has a zero mean value.

The number of covariate data for updating the hazard function will affect the accuracy of estimation and the time taken for the estimated hazard to track the real hazard because collecting more data takes a longer period of time under a given frequency of data acquisition. If the covariate is not contaminated by noise, less covariate data, e.g. one or two, are used to update the estimated hazard function in order to ensure a prompt response of the estimated hazard function to the real hazard function. If the covariate of the system is contaminated by zero mean value noise, full covariate data should be used to reduce the effect of the noise on the estimation of the hazard provided that the hazard characteristics of a system do not change. If the hazard characteristics of a system change and the covariates of the system are also corrupted by noise, the number of covariate data used for updating the hazard function is mainly dependent on the severity of noise and the requirement for the tracking time needed for estimating the real hazard. Generally speaking, low noise level and requirement for faster tracking process requires fewer data when updating the estimated hazard. In the case of the simulations as well as the case study presented in Section 6.3.7, seven to ten data produced the best result. When the hazard characteristics of a system changes, one should avoid using all covariate data for updating the hazard function because the tracking process could be extended (refer to Figures 6-6 (d) and 6-8 (d)). If noise level is high and a faster tracking process is required, one needs to increase the frequency of data acquisition so that more data can be collected in a shorter period.

The accuracy of the baseline covariate function is crucial to ensure the accuracy of the updated hazard estimation. A correlation analysis between covariates and the hazard of a system should be conducted to determine which covariate can be used in PCM. Needless to say a covariate with good correlation with the hazard of a system should be used as otherwise it will produce poor estimation result.

Chapter 7

EXPERIMENTS

7.1 INTRODUCTION

The validation of the newly developed methodologies and models was conducted using (a) simulation, (b) laboratory data and (c) field data. Both (a) and (c) were presented earlier. The experiments were conducted with the following objectives:

(1) To validate the Analytical Model for Interactive Failures (AMIF) and demonstrate the estimation of interactive coefficients.

(2) To verify the results described by the Extended Split System Approach (ESSA)

(3) To validate the Proportional Covariate Model (PCM).

The rest of the chapter is organised as follows. In Section 7.2, the test rig and experimental method are described. The test results are presented in Section 7.3 and followed the analysis of the test results in Section 7.4. The conclusions are presented in Section 7.5.

7.2 TEST RIG AND EXPERIMENTAL METHOD

The experimental investigation focussed on using a fault demonstration test rig where a shaft with a wheel was supported by two ball bearings (left bearing and right bearing). The shaft was driven by a motor through a pair of flexible couplings. Failure was categorised as misalignment created by moving the left bearing housing in two opposite directions (forward and back). The movement of the bearing housing was controlled by a screw. A second failure mode was the failure of the bearing. The test rig is shown in Figure 7-1 and Figure 7-2.

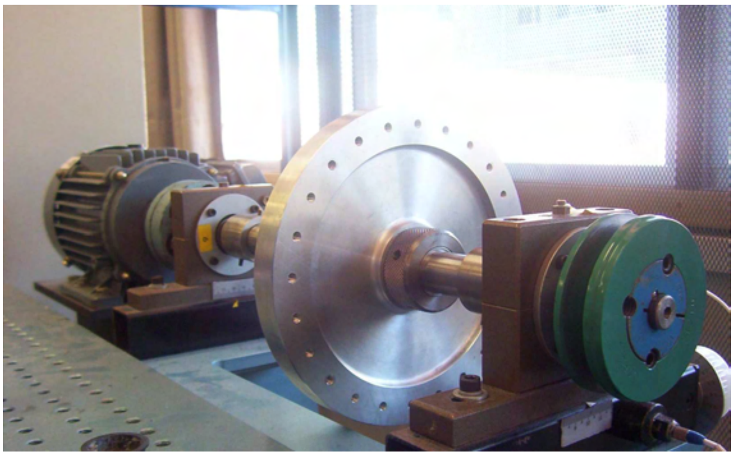


Figure 7-1. Test rig

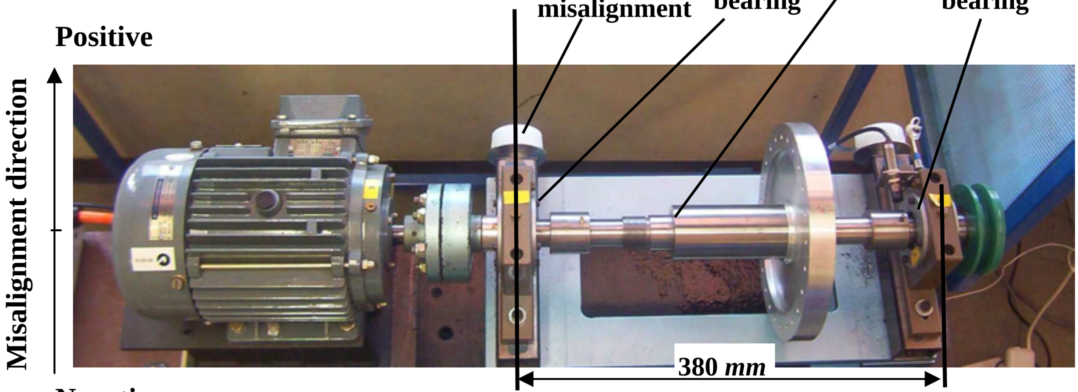


Figure 7-2. The aerial view of the test rig

To address Objective 1 and Objective 2 of the experiments, the effects of the misalignment of the shaft (failure mode 1) on the fatigue failure of the right bearing (failure mode 2) were analysed in the experiments because the shaft and the bearings had direct interactions with each other. Misalignment is a fault, which can be utilised to assess the failure of the shaft when the level of unacceptable misalignment is predetermined. When the shaft rotated, the misaligned shaft caused the bearing to vibrate. The overall vibration level of acceleration of the right bearing was used to indicate its fatigue failure rate. All this information was collected by a data acquisition system shown in Figure 7-3 and Figure 7-4.

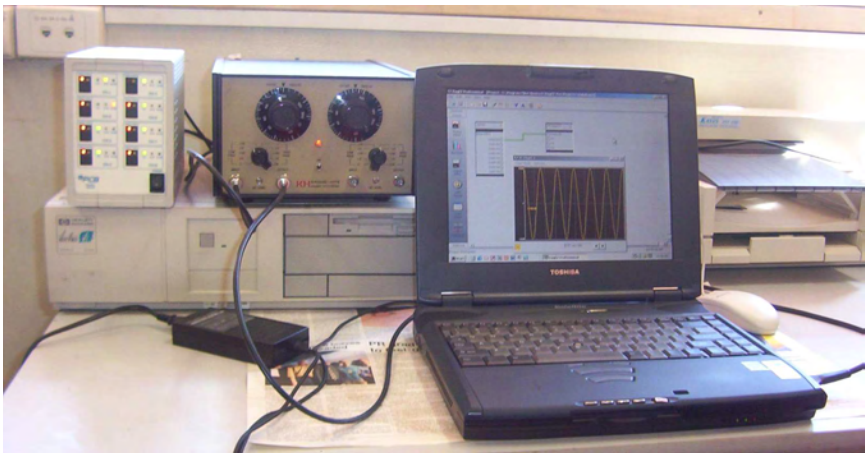


Figure 7-3. Picture of the data acquisition system

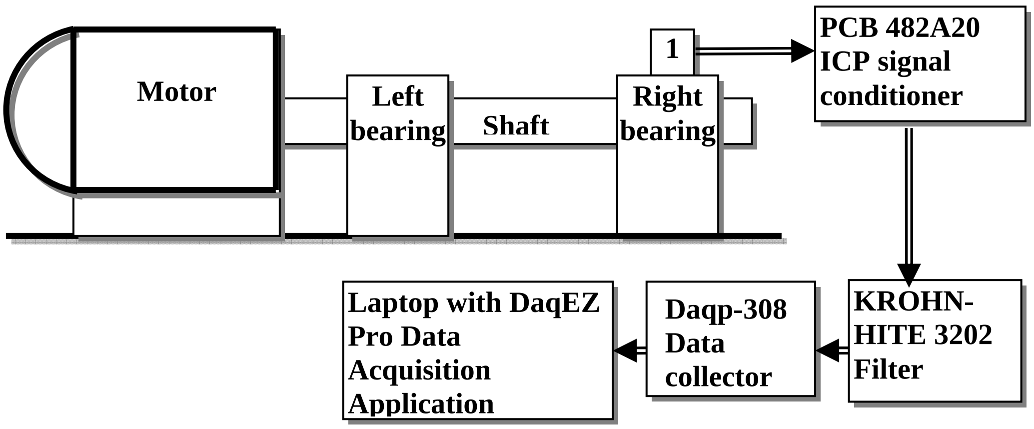


Figure 7-4. Diagram of the test rig and data acquisition system

In Figure 7-4, number 1 was an ENDEVCO 256HX-10 piezoelectric accelerometer (Figure 7-5). The type of the right and left bearings was deep groove ball bearing 6204. Figure 7-6 shows a damaged bearing which was used in the experiments.

Figure 7-5. ENDEVCO 256HX-10 piezoelectric accelerometer

Figure 7-6. The damaged bearing

During the experiments, an accelerometer was mounted on the right bearing housing to detect the vibration signal of the bearing. The speed of the shaft was 960 rpm. The operation load was 0.89 kW. The left bearing was in healthy condition, and both healthy and faulty bearings were used for the right bearing. The faulty bearing was damaged with a notch cut on the inner surface of the outer race (Figure 7-6). The notch extended throughout the cross section of the outer race with a configuration of width × depth =1.8 mm ×0.385 mm respectively. In each test, 20,000 samples of data were collected. The sampling frequency of data acquisition was 10 kHz.

The experimental procedure consisted of assessing the vibration against the misalignment in two opposite directions – forward (positive) and back (negative) (see Figure 7-2) to investigate if the test results were sensitive to the direction of the misalignment of the shaft. A faulty right bearing was used in the experiment initially. The faulty bearing was subsequently replaced by a healthy one to simulate the scenario where a system was repaired. The tests based on the scenario where the shaft was supported by a pair of healthy bearings were also used for achieving Objective 3 of the experiments.

7.3 TEST RESULTS

During the experiments, the degree of angular misalignment of the shaft was less than 0.01rad. For this small degree, the ratio $x_{lbh}/380$ can be used to present the degree of angular misalignment of the shaft ϑ_{sm}, i.e., $\vartheta_{sm} = x_{lbh} / 380$ because

$$arctg(\frac{x_{lbh}}{380}) \approx \frac{x_{lbh}}{380} . \qquad (7\text{-}1)$$

where, x_{lbh} is the displacement of the left bearing housing from its central position and 380 mm is the distance between the two bearings (see Figure 7-2).

Figures 7-7, 7-8 and 7-9 show the part of the test results. Figures 7-7 and 7-8 display the vibration signals (overall vibration level) in the time domain of the faulty bearing when the shaft had different degrees of angular misalignment in the forward (positive) direction and back (negative) direction respectively.

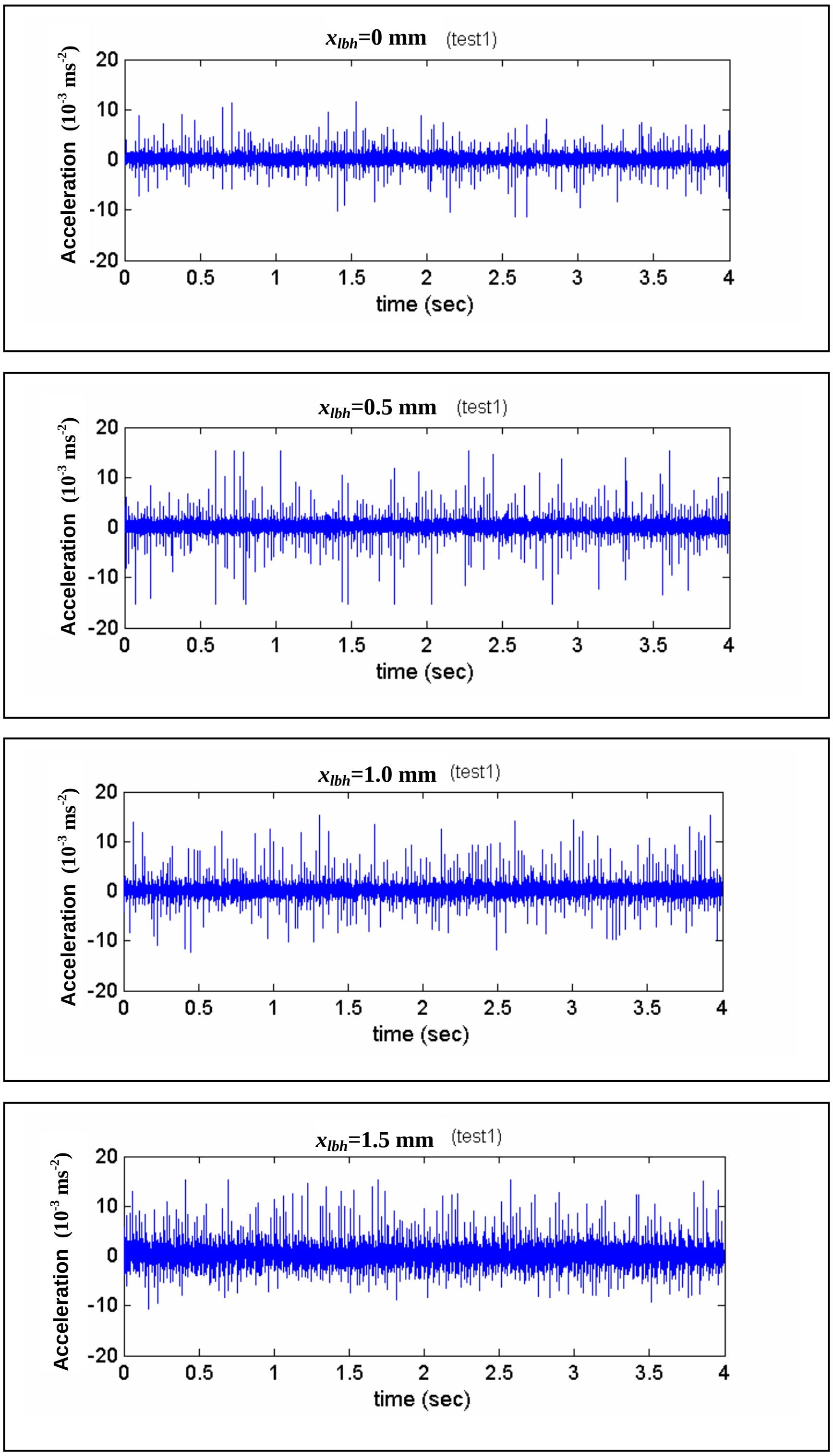


Figure 7-7. The vibration of the faulty bearing under different degrees of misalignment of the shaft in the positive direction

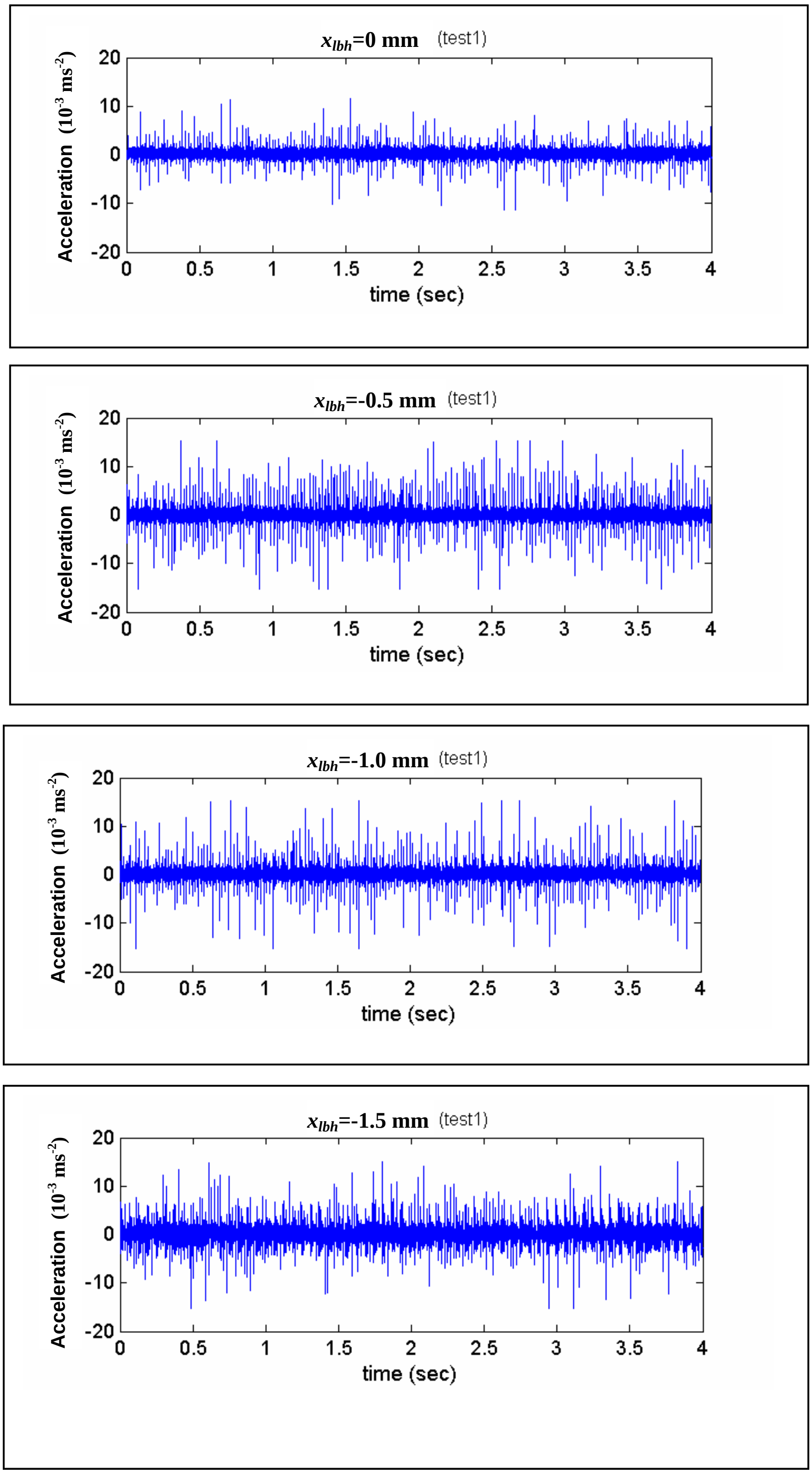


Figure 7-8. The vibration of the faulty bearing under different degrees of misalignment of the shaft in the negative direction

The experiments were also used to analyse the effect of unrepaired subsystem on the repaired component when the subsystem and the component had failure interaction. To do so, the bearing on the right end of the shaft was replaced using a healthy bearing and the experiment was repeated under different degrees of angular misalignment of the shaft. Figure 7-9 shows one set of the test results. It displays the vibration signals in the time domain of the test bearings when the shaft was exposed to different degrees of angular misalignment.

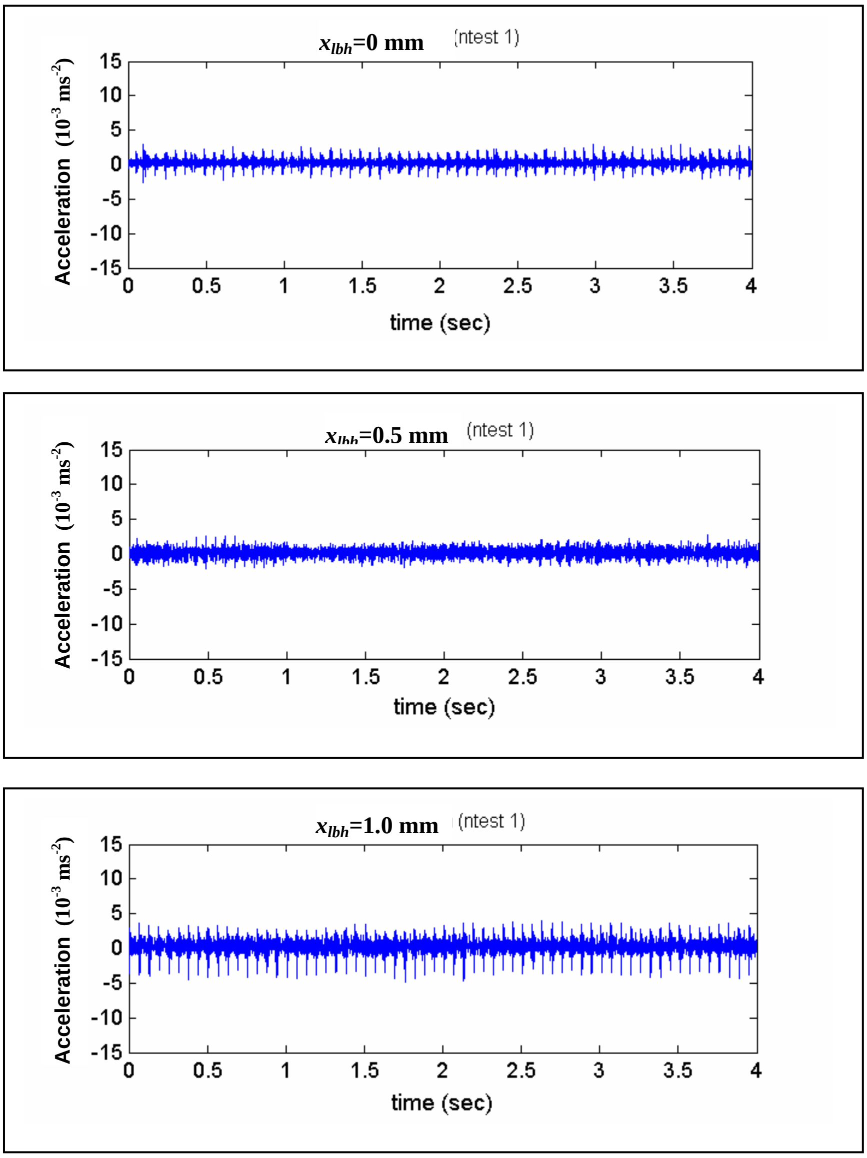


Figure 7-9. The vibration signals in the time domain of the test bearing when two healthy bearings were used

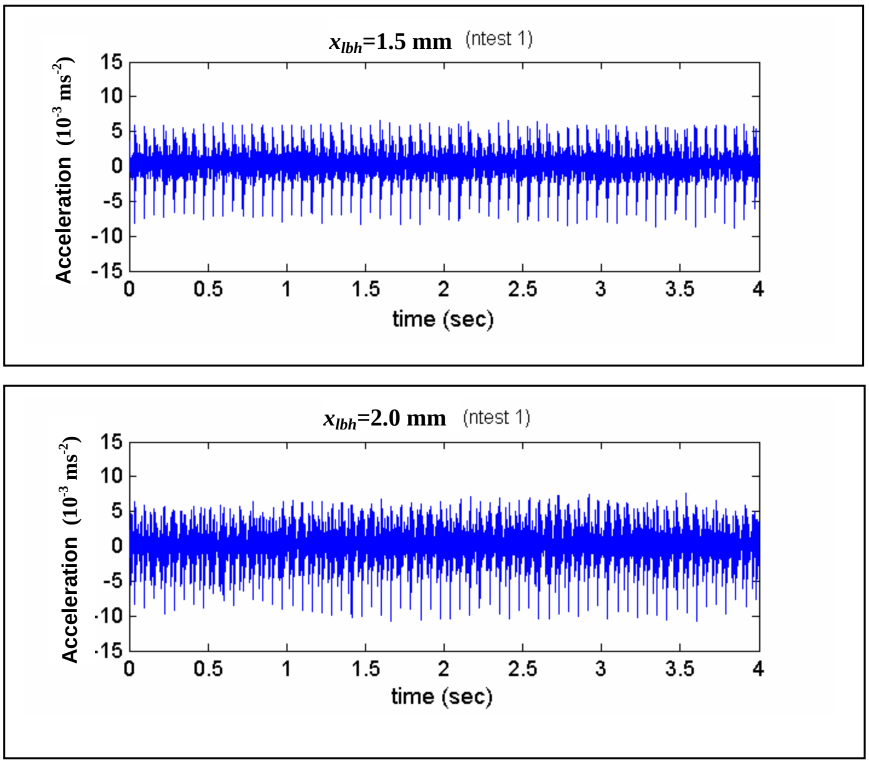
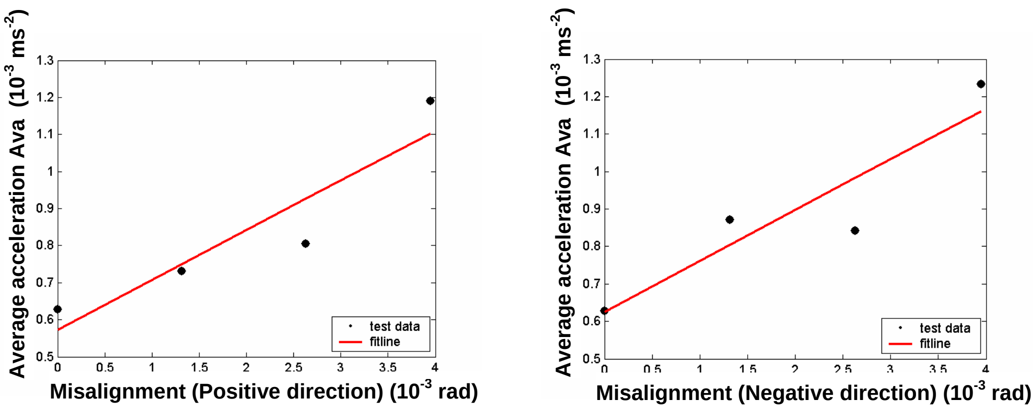


Figure 7-9. The vibration signals in the time domain of the test bearing when two healthy bearings were used (continued)

Figure 7-10 depicts the changes of the average acceleration amplitude of the faulty bearing with different degrees of angular misalignment of the shaft.



(a) **(b)**

Figure 7-10. The average acceleration amplitude of the faulty bearing under different degrees of angular misalignment of the shaft [(a) in the positive direction; (b) in the negative direction]

The average amplitude of acceleration of a bearing is the mean acceleration amplitude value of a vibration process of the bearing over time. Figure 7-11 depicts the relationship between the average vibration amplitude of the test bearing and the overall angular misalignment of the shaft under the condition that the both bearings were healthy.

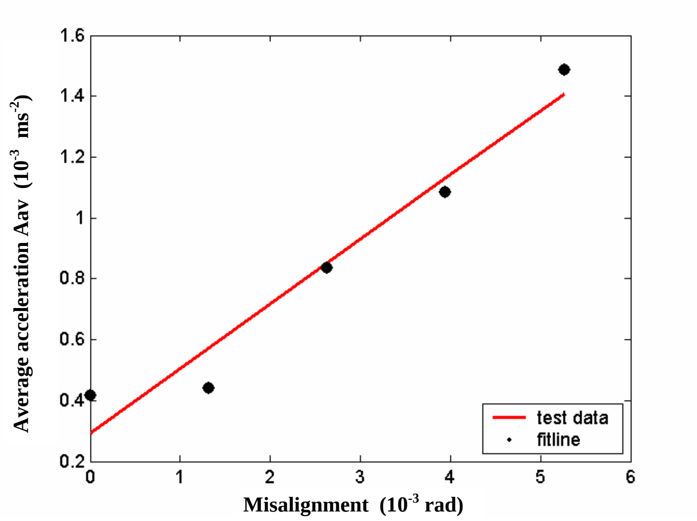


Figure 7-11. The average acceleration amplitude of the healthy right bearing under different degrees of angular misalignment of the shaft

7.4 ANALYSIS OF THE TEST RESULTS

The laboratory experiments were conducted using a mechanical system. The failures of mechanical components generally have the following features:

(1) A mechanical component has several failure modes. The failure of a mechanical component with a specific failure mode is usually defined as its inability to perform its predefined function satisfactorily due to this failure occurring. However, the demarcation line between failure and non-failure is often unclear. Unlike normal failures in electrical components, the failure of a mechanical component usually occurs more gradually rather than a step change.

(2) When a mechanical component fails, it can continue to operate often resulting in this failed component affecting other components in due course.

(3) The failure of a mechanical component usually will not cause its related mechanical components to fail immediately but can accelerate their hazards.

(4) Failure interactions among components in a mechanical system are common. For example, in a mechanical system such as the test rig shown in Figure 7-1, a deteriorated bearing will result in the drift of a shaft supported by this bearing and the misalignment of the shaft will also increase the deterioration of the bearing.

The more deteriorated a mechanical component becomes, the more likely it will fail. The assumption that the hazard of a mechanical component is proportional to the degree of its deterioration is justified. Experiments have supported this assumption (see Figure 6-14 in Chapter 6). From Figure 6-14, it can be seen that the hazard of the test gear can be treated as proportional to its increment of crack depth. Another example is a model for predicting the failure rate (hazard) of ball bearings presented by Ebeling [16]. This model indicates that the hazard of a bearing is proportional to the percentage of water present in its oil lubricant if this percentage is less than 0.2%.

From Figures 7-10 and 7-11, it can be seen that the average acceleration amplitude of the test bearing increases with the increasing degree of angular misalignment of the shaft. This fact indicates that the increased hazard of the shaft (i.e. misalignment) could result in an increase in the hazard of the bearing because the larger vibration amplitude leads to accelerated onset of fatigue failure in a mechanical system [312]. Furthermore, from Figures 7-10 and 7-11, it can be inferred that a linear relationship exists between the degree of angular misalignment of the shaft and the vibration acceleration of the test bearings. The line of best fit in these two figures is described by:

$$y_a = y_{a0} + b_{am}\vartheta_{sm}, \tag{7-2}$$

where, y_a is the average acceleration amplitude of the test bearing and y_{a0} is the initial value of the average acceleration amplitude of the test bearing. Variable ϑ_{sm} is the degree of angular misalignment of the shaft. Parameter b_{am} is the slope of the fit-line.

In reality, y_a, y_{a0} and ϑ_{sm} are usually time dependent. In this case, Equation (7-2) should be rewritten as

$$y_a(t) = y_{a0}(t) + b_{am}\vartheta_{sm}(t)\,. \tag{7-3}$$

In Section 7.2, the candidate indicated the vibration signals of the test bearings were collected against the misalignment of the shaft in two opposite directions (Figure 7-2) to check if the test results were sensitive to the direction of the misalignment. From Figure 7-10 and Table 7-1, it can be seen that the test results were not sensitive to the direction of the misalignment of the shaft, i.e., when testing in two opposite misalignment directions, the relationship between the failure rate of the shaft and the failure rate of the bearing was almost the same.

The analysis of the tests which were conducted when the right bearing was replaced using a healthy bearing also confirmed the above findings (refer to Figures 7-9 and 7-11): (1) the angular misalignment of the shaft increased the vibration of the test bearing; (2) the relationship between the angular degree of misalignment of the shaft and the average acceleration amplitude of the bearing was approximately linear and (3) this relationship was not sensitive to the direction of the misalignment of the shaft.

Each test was repeated five times to ensure the repeatability of the experiments and the accuracy of the experimental analysis. Table 7-1 presents the absolute values of slope $|b_{am}|$ and the initial values of the average acceleration amplitude of the faulty bearing, y_{a0}. Let $\varepsilon_{b_{am}}$ stand for the relative estimation error of the slope and $\varepsilon_{y_{a0}}$ for the relative estimation error of the initial values of the average acceleration amplitude y_{a0}. When the average value $b_{amav} = 135.461 \times 10^{-3}$ ms^{-2}rad^{-1} and $y_{a0av} = 6.01 \times 10^{-4}$ ms^{-2}, $\varepsilon_{b_{am}} \le 5.99\%$ and $\varepsilon_{y_{a0}} \le 6.23\%$. Given that these values lie below 10%, the tests were considered to be relatively accurate and consistent.

Table 7-1

The absolute values of slope | b_{am}| and the initial values of the average acceleration amplitude of the faulty bearing

Test No.	$\lvert b_{am} \rvert$ (10^{-3} ms^{-2}rad^{-1})		y_{a0} (10^{-4} ms^{-2})	
	P	N	P	N
1	134.064	135.926	5.733	6.248
2	135.926	143.374	5.660	6.125
3	130.806	137.323	5.770	6.272
4	129.875	137.788	6.014	6.272
5	136.392	131.271	5.709	6.395
Average	135.461		6.010	

Note: P – Positive direction of misalignment;
N – Negative direction of misalignment (see Figure 7-2)

In the following subsections, the test results presented in Section 7.3 and the above analysis results will be used to justify the new models developed in the previous chapters.

7.4.1 Interactive Failures

A mathematical model for IntF (Equation (4-11)) was derived in Chapter 4and the theoretical model was validated by select case studies. In this subsection, the particular model will be validated by the experiments described above. These test results will also used to estimate the interactive coefficient θ_{12}, where θ_{12} represents the degree of the effect of the misaligned shaft on the fatigue failure of the bearing on the right end of the shaft.

The following assumptions were used in the interpretation of the test results in the above section.

(1) The deterioration of the shaft and the bearings during the experiments are neglected because the experimental time was short compared to the life cycle of the mechanical components.

(2) It is understandable that the shaft will fail to function (rotate) properly when its angular misalignment reaches a threshold. Hence, the failure of the shaft with failure mode 1 was defined as that occurring when the shaft operated abnormally due to the angular misalignment. The greater the angular misalignment, the more likely the shaft operated abnormally. Therefore, for the failure mode 1, the assumption that the hazard of the shaft is proportional to its degree of angular misalignment is justified. As mentioned previously, the assumption that the hazards of mechanical components are proportional to their degrees of deterioration has been supported by other research (refer to Wang [311] and Ebeling [16]). Let $h_1(t)$ represent the hazard of the shaft with failure mode 1, based on this assumption,

$$h_1(t) = b_1 \vartheta_{sm}(t) , \tag{7-4}$$

where b_1 is a coefficient.

(3) The failure of the test bearing with failure mode 2 was defined as that occurring when the bearing could not perform its predefined functionality due to fatigue occurring inside the bearing. The hazard of the test bearing is assumed to be proportional to the average acceleration amplitude of the bearing if the fatigue failure of the bearing is considered solely because the stress of the bearing is proportional to its acceleration and the fatigue hazard is proportional to the stress [312]. Let $h_2(t)$ and $h_{I2}(t)$ represent the interactive hazard and the independent hazard of the bearing respectively. Based on this assumption,

$$h_2(t) = b_2 y_a(t) , \tag{7-5}$$

and

$$h_{I2}(t) = b_2 y_{a0}(t) \, , \tag{7-6}$$

where b_2 is a coefficient.

The Equations (7-4), (7-5) and (7-6) can also be derived using PHM.

Substituting the Equations (7-4), (7-5) and (7-6) into Equation (7-3), gives:

$$h_2(t) = h_{I2}(t) + \frac{b_2 b_{am}}{b_1} h_1(t) \, , \tag{7-7}$$

Let $\theta_{12} = \dfrac{b_2 b_{am}}{b_1}$ (7-8)

be the interactive coefficient that represents the effective degree of the failure of the shaft affecting the failure of the test bearing, then Equation (7-7) can be rewritten as

$$h_2(t) = h_{I2}(t) + \theta_{12} h_1(t) \, . \tag{7-9}$$

Equation (7-9) justifies that the analytical model given by Equation (4-11) can represent the interactive failure relationship between the test bearing and the shaft provided the hazard of a mechanical component is proportional to its degree of deterioration. In a real world application, to reduce the effect of testing errors, the average b_1, b_{1av}, the average b_2, b_{2av} and the average b_{am}, b_{amav} should be used to calculate θ_{12} in Equations (7-7) and (7-8).

Substituting Equation (7-9) into Equation (4-16), gives

$$F_2(t) = 1 - \exp[-\int_0^t h_{I2}(t)dt - \int_0^t \theta_{12} h_2(t)dt] \, , \tag{7-10}$$

where, $F_2(t)$ is the interactive failure distribution function of the test bearing.

According to Equation (4-1) and the relationship between the reliability function

$R(t)$ and the failure distribution function $F(t)$, Equation (7-10) can be rewritten as:

$$F_2(t) = 1 - R_{I2}(t)\exp[-\theta_{12}\int_0^t h_2(t)dt] \quad (7\text{-}11)$$

where, $R_{I2}(t)$ is the independent reliability function of the bearing.

Equation (7-11) indicates that the failure probability of the test bearing affected by the misaligned shaft can be predicted provided the independent reliability function of the bearing and the reliability function of the shaft are known. In this case, interactive coefficient θ_{12} can be calculated using Equation (7-7). At first, the independent hazard of the bearing and the hazard of the shaft can be estimated using Equation (4-1). The average b_1, b_{1av} and the average b_2, b_{2av} can then be calculated using Equation (7-4) and Equation (7-6) respectively.

For simplification, assume that the independent hazard of the faulty bearing is 6×10^{-3} h^{-1} and the hazard of the shaft is 7×10^{-3} h^{-1} with a displacement of 0.5 mm of the left bearing housing. The coefficients b_{1av} and b_{2av} are then 5.319 $\mathrm{rad}^{-1}\mathrm{h}^{-1}$ and 9.983 $\mathrm{m}^{-1}\mathrm{s}^{2}\mathrm{h}^{-1}$ respectively and θ_{12} is 0.254. Equation (7-11) becomes

$$F_2(t) = 1 - R_{I2}(t)\exp[-0.254\int_0^t h_2(t)dt]\,. \quad (7\text{-}12)$$

Note that the coefficients b_{1av} and b_{2av} can vary because they depend on the reliability values of the test bearing and the shaft.

Figure 7-12 shows the comparison between the experimental result and theoretical result using Equation (7-9) and demonstrates the accuracy of the equation.

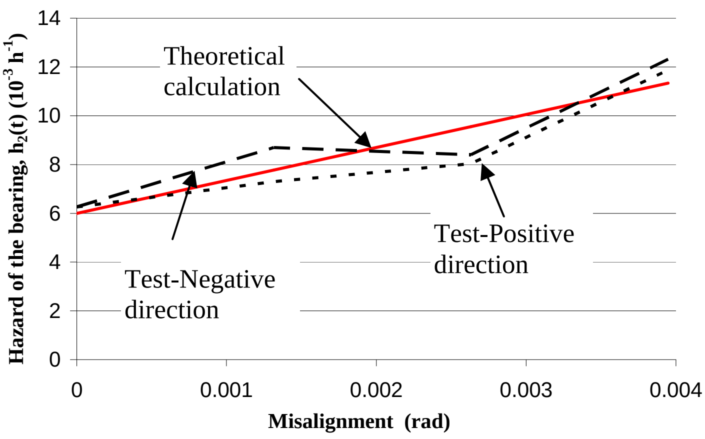


Figure 7-12. Comparison between experimental and theoretical results

7.4.2 Hazard of a Newly Repaired Component

In the development of ESSM, the result from this model indicated that the hazard of a new component used in a deteriorated system would be higher than its original hazard if IntF existed. This result has been demonstrated by the experiment when the faulty bearing was replaced by a healthy bearing.

From Figure 7-9, it can be seen that the acceleration amplitude of the healthy bearing on the right end of the shaft increased with the increasing degree of angular misalignment of the shaft. This result indicates that the new bearing was likely to suffer accelerated wear/damage if a shaft became misaligned and if the misalignment of the shaft was not corrected.

To demonstrate the effect of the misaligned shaft on the failure distribution of the right bearing quantitatively, assume that the degree of angular misalignment of the shaft remained constant during an operation and the independent reliability function of the healthy bearing was obtained from [313] as:

$$R(t) = \exp\left[-\left(\frac{t}{1128}\right)^{1.41}\right]. \tag{7-13}$$

The hazard of the bearing on the right end of the shaft shown in Figure 7-13 was determined under two conditions: angular misalignment of the shaft at 1.316×10^{-3} rad and a well aligned shaft. From the figure, it can be seen that the hazard of the bearing under the first condition was higher than the hazard of the bearing under the second condition, i.e., a misaligned shaft increased the hazard of a new bearing on the shaft. Figure 7-14 shows the failure distribution of the test bearing corresponding to Figure 7-13.

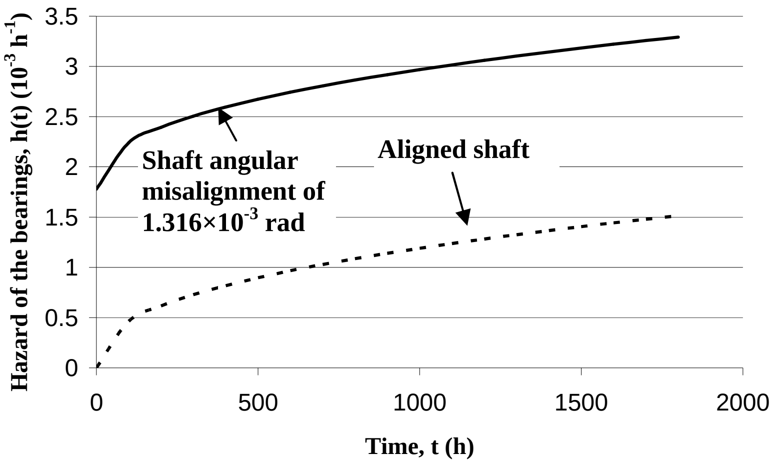


Figure 7-13. Hazard of the right bearing

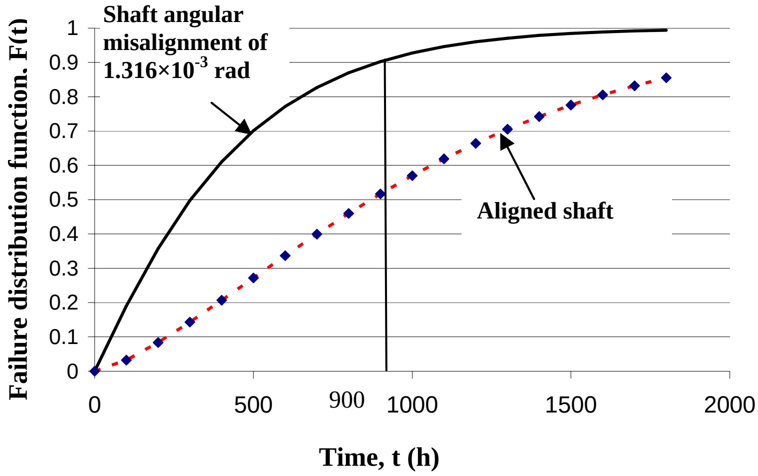


Figure 7-14. Failure distribution of the right bearing

Figure 7-14 indicates that at 900 hours, the failure probability of the bearing under the condition of shaft misalignment was almost 90% whereas the failure probability of the bearing when the shaft was aligned at the same time was just about 50%.

7.4.3 PCM

PCM was developed based on the assumption that covariates of a system are proportional to the hazard of the system. The reasonableness of this assumption has been justified using some existing research results in Chapter 6. In this subsection, the reasonableness of the assumption will be verified using the laboratory experimental results. As a special case, a baseline covariate function is also estimated.

According to the test, the average acceleration amplitude of the vibration of the test bearing was sensitive to the change of the angular misalignment of the shaft (see Figure 7-7 to Figure 7-11). Therefore, the average acceleration amplitude of the vibration of the test bearing was used as a covariate to indicate the degrees of angular misalignment. This covariate was measured and calculated against the different degrees of angular misalignment of the shaft. The result shown in Figure 7-15 was obtained under the conditions mentioned in Subsection 7.4.1 and using two healthy bearings.

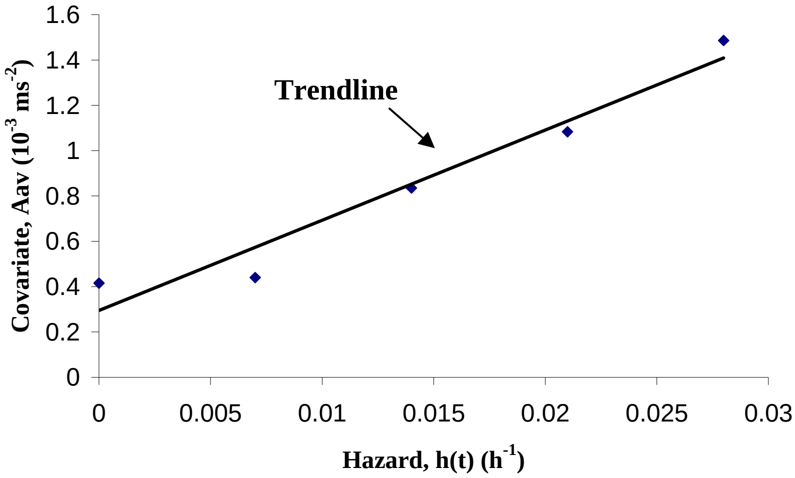


Figure 7-15. The relationship between the hazard *h*(*t*) of the shaft and the average vibration amplitude Aav

Figure 7-15 clearly indicates that the covariate, i.e., the average vibration amplitude Aav, was proportional to the hazard of the shaft. In this experiment, the baseline function, $C(t)$ can be treated as time-independent.

$$C(t) = 1.1052 \times 10^{-4} \quad (\text{ms}^{-3}) \ . \tag{7-14}$$

However, in many scenarios, the baseline functions are time-dependent. In these scenarios, the $C(t) - h(t)$ plot will be a curve instead of a straight-line.

7.5 SUMMARY

A series of laboratory experiments were conducted for validating the newly developed methodologies and models. Through these experiments, the following results have been justified:

Equation (4-11) can be used to describe the interactive failures in a mechanical system. The interactive hazard of a component can be calculated by its independent hazard plus a portion of the interactive hazard of its influencing components.

The hazard of a new component used in a deteriorated system will be higher than its original hazard if this new component has failure interaction with other unrepaired components in the system. The failure likelihood of a component increases when its influencing components deteriorate.

The degree of the failure interaction between two components can be measured by the interactive coefficient. A greater interactive coefficient means that the failure of a component has a greater effect on the failure of its affected component. This experimental study has also provided evidence that the interactive coefficient can be determined through experimentation.

The assumption used to develop PCM is reasonable. The covariates of a system, or a function of these covariates, can be assumed to be proportional to the hazard of the system. This proportional relationship can be represented by a baseline covariate function. The baseline covariate function can be either time independent or time-dependent.

Chapter 8

CONCLUSIONS

This research has developed practical models and methodologies to improve the accuracy of reliability predictions of repairable systems for engineering applications.

After an extensive literature review, the candidate identified the following limitations in existing reliability prediction models:

(1) The different states of repairable systems after multiple repairs were generally inadequately modelled. A common approach is to assume that a repairable system after repairs becomes “as good as new” or “as bad as old”.

(2) Interactive failures have not been modelled previously. Existing models or methodologies have been mainly developed on the assumption of independent failures or unidirectional dependent failures such as common cause failure.

(3) Existing models have not adequately dealt with the reliability prediction of a system using responsive covariates (symptom indicators), especially when historical failure data are sparse or null.

In this thesis, the candidate endeavoured to overcome these limitations and developed the following new methodologies/models:

(1) The split system approach.

(2) The analytical model for interactive failures.

(3) The extended split system approach.

(4) The proportional covariate model.

The detailed conclusions of each of these methodologies /models are presented in the following four sections.

8.1 SPLIT SYSTEM APPROACH (SSA)

The prediction of the reliability of complex repairable systems with multiple PM actions over multiple intervals is difficult because the characteristics of the reliability of a system will alter after each PM. SSA uses a new concept to resolve this difficulty effectively by splitting a system into repaired and unrepaired parts virtually when modelling the reliability of a system with multiple PM actions. SSA possesses the following advanced characteristics:

(1) SSA explicitly predicts the reliability of a repairable system with multiple PM actions over multiple intervals and predicts when the system is unworthy of further PM. Most existing reliability models consider the time to the next failure, MTTF or/and the expected number of failures during a given period.

(2) SSA effectively models all possible states of a system after PM such as “as good as new”, “imperfect repair”, “improvement repair” (better than new) and “as bad as old”. Existing models generally describe imperfect repairs based on the assumption of a fixed deterioration rate of reliability.

(3) SSA considers the individual contributions of different maintained parts in a system and the influence of different system structures on the reliability of a repairable system. This consideration assists in understanding the effects of PM on a system in more depth. Existing models often take the entire system into account.

(4) SSA does not dependent on the restrictions on the forms of failure distribution.

The candidate has derived formulae for reliability prediction of systems for the following scenarios:

(1) The same component is repaired in all PM activities;

(2) A single but different component is repaired in each PM action.

For the scenario where multiple components are repaired in one PM action, the candidate has developed a heuristic approach to predict the reliability of the system.

SSA was shown to be effective in supporting preventive maintenance decision making for a repairable system over its whole life. It can be used to estimate:

(1) The expected life of a repairable system with multiple PM actions.

(2) The available number of PM actions on the system.

(3) The spare parts requirement.

SSA has been effectively used to compare the effectiveness of different PM strategies and assists in making optimal PM decisions.

8.2 THE ANALYTICAL MODEL FOR INTERACTIVE FAILURES (AMIF)

AMIF overcomes the assumption of independent failures and analyses interactive failures of systems without PM or repair.

Existing models or methodologies for the reliability prediction have been mainly developed on the assumption that failures are independent. However, numerous industrial experiences have shown that this assumption is unrealistic and has led to unacceptable errors in failure risk assessment. To ensure the accuracy of reliability prediction, dependent failures need to be considered. Interactive failure is a new category of dependent failure, and is caused by failure interaction among the components in a system.

The research on interactive failures is in its infancy, and the candidate has made the following original contributions:

(1) Introduced new concepts such as interactive failure, influencing components, affected components and interactive coefficient for analysis of interactive

failure.

(2) Identified that interactive failure can be either stable or unstable. The candidate proposed and proved two theorems to justify stable interactive failures. These theorems effectively assist in analysing and avoiding potential unstable interactive relationship in machinery during its design phase. The research outcomes on stable and unstable interactive failures can benefit the design of more maintainable and reliable machines.

(3) Developed a model to analyse interactive failure quantitatively, suitable for engineering application. The candidate derived a formula to calculate the stable interactive failure distribution functions of systems and successfully investigated the effects of interactive failures on components and systems using this new model. The results contribute to improving risk management of assets with interactive failures.

8.3 EXTENDED SPLIT SYSTEM APPROACH (ESSA)

ESSA is an integration of SSA and AMIF, and extends the latter by considering both interactive failures and multiple PM actions over multiple intervals. The reliability prediction of complex reparable systems with interactive failures and multiple PM actions is also a new research area and the candidate has made the following original contributions:

(1) Identified that when the failures of the repaired and unrepaired components in a system have interactions, the hazards of these components after a repair will change. This finding, if taken into account, improves the performance of maintenance on repairable systems with interactive failures.

(2) Developed an effective method to analyze the changed hazards of repaired and unrepaired components in a system after a PM action. The candidate also derived the formulae for calculating the interactive hazards of a system after each PM based on this method.

(3) Extended the heuristic approach for SSA to explicitly predict the reliability of systems with interactive failures and multiple PM actions over multiple intervals.

ESSA enhances the capability of SSA and AMIF and provides an effective tool for optimal PM decision making in more general scenarios.

8.4 PROPORTIONAL COVARIATE MODEL (PCM)

PCM presents a new approach to predicting the hazard of a system with a combination of historical failure data and condition monitoring data (covariates). It uses the same assumption as used in PHM, but the philosophy and procedure of PCM is different from that of PHM.

The research in this thesis has demonstrated the following characteristics of PCM:

(1) PCM automatically tracks the changes of hazard through using responsive covariates.

(2) PCM has much more accurate prediction results than using the conventional approach or tendency method when the characteristics of the hazard of a system alter.

(3) Compared to PHM, PCM has a greater ability to reduce the influence of noise which contaminates covariate data.

(4) PCM is robust even though covariate data can be corrupted by random noise provided the noise has a zero mean value.

(5) PCM is effective in predicting the hazard of a system based on condition monitoring data even though historical failure datum is zero. PHM does not have such ability.

8.5 GENERAL STATEMENTS

The methodologies and models developed in this thesis can be related to each other and applied to predict the reliability of components and systems with multiple PM actions and interactive failures effectively.

The newly developed methodologies and models have been justified through four approaches:

(1) Theoretical proof.

(2) Simulations.

(3) Case studies using field data.

(4) Experiments.

The outcomes of this research are significant to the body of knowledge in reliability engineering.

In total, 15 papers have been published or submitted by the candidate:

- Six in refereed international journals: two published, three in press, and one submitted.
- Nine in refereed international conferences.

In recognition of the significance of this research, the candidate received the 2004 Student Award from the Maintenance Engineering Society of Australia. This national award is presented to only one student throughout Australia each year.

Chapter 9

DIRECTIONS FOR FUTURE RESEARCH

While the candidate has successfully developed four new methodologies/models for predicting reliability of complex repairable systems, this final section of the thesis presents a brief on potential future research directions.

9.1 EXTENSION OF SSA

The candidate developed SSA based on the scenario that PM time is a deterministic variable, and that repair time is negligible. This approach was extended to the reliability prediction of systems with multiple PM actions and interactive failures. SSA can be further extended to predict the reliability of a system in the following scenarios:

- A system with multiple random failures and PM actions. Unlike planned PM time, failure time is a random variable.

- A system with multiple failures and repairs. In this case, repair time is a random variable and cannot be ignored.

- A system with multiple repairs and immediate interactive failures. In this case, the changes of RBD of the system due to interactive failures need to be considered.

9.2 APPLICATION OF SSA FOR PM DECISION MAKING

The candidate demonstrated the application of SSA to support PM decision making for a repairable system during its lifetime in Chapter 3. This case focused on PM decision making based on reliability prediction. In reality, to make an optimal PM decision, one also needs to consider other factors such as:

- Business objectives.
- Maintenance cost.
- Resource constraints.
- Consequences of failures.
- Performance of maintenance personnel.

Further work can lead to an integration of SSA and decision making models, taking into account some, if not all the above factors.

9.3 ENHANCEMENT OF FAULT TREE ANALYSIS

FTA is a useful technique in analysing the relationship between a failure event and its root causes. However, FTA cannot be used to analyse interactive failures. In a fault tree, only the failures at a lower level can affect the failures at a higher level. A failure cannot affect the failures at a level lower. The failures at the same level do not interact with each other. Therefore the fault tree cannot be used to describe interactive failures. To address this issue, a technique that integrates AMIF as developed in this thesis with the conventional FTA technique needs to be developed.

9.4 PCM FOR MULTIPLE COVARIATES

The candidate developed PCM based on a single covariate. PCM can be enhanced through using multiple covariates by:

- Identifying significant covariates.
- Constructing proper functions of covariates based on data fusion techniques, correlation analysis and maximum likelihood estimation.
- Determining different weight/parameter for individual covariates.

The modified Weibull distribution models presented by Murthy and Jiang [314] can be applied in PCM to improve the goodness-of-fit of the model to historical failure data.

9.5 DEVELOPMENT OF SOFTWARE TOOLS TO ENHANCE THE APPLICATION AND TESTING OF THE DEVELOPED MODELS

The candidate has demonstrated that the models developed in this thesis can be beneficial to industries. However, application of these models to industrial problems could be difficult for personnel without sufficient mathematical expertise. Appropriate software tools can be developed to assist in implementing these models.

Appendix A

Publications

1. Refereed International Journals

(1) Sun, Y., Ma, L., Mathew, J., Wang, W.Y., and Zhang, S., Mechanical systems hazard estimation using condition monitoring, Mechanical Systems and Signal Processing, in press, available on ScienceDirect in December 2004.

(2) Sun, Y.; Ma, L., Mathew, J., and Zhang, S., An analytical model for interactive failures, Reliability Engineering and System Safety, in press, available on ScienceDirect in May 2005

(3) Sun, Y., Ma, L., Mathew, J., and Zhang, S., Determination of preventive maintenance lead time using hybrid analysis, International Journal of Plant Engineering and Management, 2005. 10(1), p.13-18

(4) Zhang, S., Mathew, J., Ma, L., and Sun, Y., Best basis based intelligent machine fault diagnosis, Mechanical Systems and Signal Processing, 2005. 19: p357-370

(5) Sun, Y., Ma, L., Mathew, J., Morris, J. and Zhang, S., A practical model for reliability prediction of repairable systems, The Journal of Quality and Reliability Engineering International, submitted.

(6) Sun, Y., Ma, L., and Mathew, J., Reliability prediction of repairable systems for single component repair, Journal of Quality in Maintenance Engineering, in press.

2. Refereed International Conferences

(7) Sun, Y., Ma, L., Mathew, J. and Zhang, S., A Methodology for Analysing Interactive Failures of Components, Proceedings of the 11th Asia-Pacific Vibration Conference, Langkawi, Malaysia, 23-25 November 2005: in press.

(8) Sun, Y., Ma, L., Mathew, J. and Zhang, S., Estimation of hazards of mechanical systems using on-line vibration data, Proceedings of International Conference on Intelligent Maintenance System, Arles, France, 15-17 July 2004: p.S3-B

(9) Zhang, S., Mathew, J., Ma, L., Sun, Y., and Mathew, A., Statistic condition monitoring based on Vibration Signals, A Fusion of Harmonics, Ed. By N.S. Vyas, et al, published by Sunil Sachdev, New Delhi, India, 6-9 December, 2004: p.1238-1243.

(10) Sun, Y., Ma, L., Mathew, J. and Zhang, S., Experimental research on interactive failures, Proceedings of International Conference of Maintenance Societies, Sydney, Australia, 25-28 May 2004: p.04073

(11) Sun, Y., Ma, L., and Mathew, J., On stable and unstable interactive failures, Proceedings of the 10th Asia-Pacific Vibration Conference, ed. J. Mathew, Gold Coast, Australia, 12-14 November 2003: p.664-668.

(12) Sun, Y., Ma, L., and Mathew, J., Alarming limits for preventive maintenance using both hazard and reliability functions, Proceedings of the 10th Asia-Pacific Vibration Conference, ed. J. Mathew, Gold Coast, Australia, 12-14 November 2003: p.669-703.

(13) Sun, Y., Ma, L., and Mathew, J., Maintenance frameworks: A survey and new extension, Proceedings of International Conference of Maintenance Societies, Perth, Australia, 20-23 May 2003: p.03-077.

(14) Sun, Y., Ma, L., and Mathew, J., A descriptive model for interactive failures, Proceedings of International Conference of Maintenance Societies, Perth, Australia, 20-23 May 2003: p.03-078.

(15) Sun, Y., Mathew, J. and Fu, M., The propagation of vibration energy in a forging shop. System Integrity and Maintenance, ed. J Mathew, Cairns, Australia, 25-27 September 2002: p.317-322.

Appendix B1

The Test Data for Gearbox Tooth Failure

Table B1-1. The original test data for gearbox tooth failure

This table is not available online.
Please consult the hardcopy thesis
available from the QUT Library

(Source: D. Lin, Optimizing a condition based maintenance program with gearbox tooth failure, CBM Lab, University of Toronto, 2003)

Appendix B2

Derivation of Equation (3-21)

For convenience, let the subsystem not contain any repaired components (m) in n PM intervals, i.e., the reliability of the subsystem is

$$R_{sb}(\tau)_0 = \frac{R_s(\tau)_0}{\prod_{k=1}^{m} R_k(\tau)_0} \tag{B2-1}$$

After the first PM action, the reliability of the system is

$$R_s(\tau)_1 = \prod_{k=2}^{m} R_k(\tau + \Delta t_1)_0 R_{sb}(\tau + \Delta t_1)_0 R_1(\tau)_1 \tag{B2-2}$$

That is

$$R_s(\tau)_1 = \frac{R_1(\tau)_1 R_s(\tau + \Delta t_1)_0}{R_1(\tau + \Delta t_1)_0} \tag{B2-3}$$

After the second PM action, either Component 2 or Component 1 can be repaired. If Component 1 is repaired again, the reliability of the system after the second PM action is

$$R_s(\tau)_2 = \prod_{k=2}^{m} R_k(\tau + \sum_{i=1}^{2} \Delta t_i)_0 R_{sb}(\tau + \sum_{i=1}^{2} \Delta t_i)_0 R_1(\tau)_2$$

$$= \frac{R_1(\tau)_2 R_s(\tau + \sum_{i=1}^{2} \Delta t_i)_0}{R_1(\tau + \sum_{i=1}^{2} \Delta t_i)_0} \tag{B2-4}$$

If Component 2 is repaired, the reliability function of the system after the second PM action is

$$R_s(\tau)_2 = \prod_{k=3}^{m} R_k(\tau + \sum_{i=1}^{2}\Delta t_i)_0 R_{sb}(\tau + \sum_{i=1}^{2}\Delta t_i)_0 R_1(\tau + \Delta t_2)_1 R_2(\tau)_2$$

$$= \frac{R_2(\tau)_2 R_1(\tau+\Delta t_2)_1 R_s(\tau+\Delta t_1+\Delta t_2)_0}{R_1(\tau+\Delta t_1+\Delta t_2)_0 R_2(\tau+\Delta t_1+\Delta t_2)_0}. \quad \text{(B2-5)}$$

Generally, if m components are repaired in n PM actions and Lk indicates that the component k ($k \le m$) receives its last repair at the Lk[th] PM action ($Lk \le n$), and if one defines

$$\sum_{i=Lk+1}^{n}\Delta t_i = 0 \text{ when } Lk+1 > n, \quad \text{(B2-6)}$$

then the following reliability function for a system after the n[th] PM actions can be proven using the Principle of Mathematical Induction [18].

$$R_s(\tau)_n = \frac{R_s(\tau + \sum_{i=1}^{n}\Delta t_i)_0 \prod_{k=1}^{m} R_k(\tau + \sum_{i=Lk+1}^{n}\Delta t_i)_{Lk}}{\prod_{k=1}^{m} R_k(\tau + \sum_{i=1}^{n}\Delta t_i)_0}. \quad \text{(B2-7)}$$

Proof.

When $n=1$, $k=1$ and $Lk=1$ according to the numbering method defined in Chapter 3. Equation (B2-7) reduces to Equation (B2-3) because $\sum_{i=1+1}^{1}\Delta t_i = 0$ based on Equation (B2-6). Therefore, Equation (B2-7) is true when $n=1$.

Suppose Equation (B2-7) is true when $n=q$, i.e.

$$R_s(\tau)_q = \frac{R_s(\tau + \sum_{i=1}^{q}\Delta t_i)_0 \prod_{k=1}^{m} R_k(\tau + \sum_{i=Lk+1}^{q}\Delta t_i)_{Lk}}{\prod_{k=1}^{m} R_k(\tau + \sum_{i=1}^{q}\Delta t_i)_0}. \quad \text{(B2-8)}$$

Then one needs to prove that Equation (B2-7) is true when $n = q+1$.

There are two possibilities.

(1) A previously repaired Component c ($c \le m$) is repaired again. In this case,

$$R_s(\tau)_{q+1} = \frac{R_c(\tau)_{q+1} R_s(\tau + \sum_{i=1}^{q} \Delta t_i + \Delta t_{q+1})_0 \prod_{\substack{k=1 \\ k \ne c}}^{m} R_k(\tau + \sum_{i=Lk+1}^{q} \Delta t_i + \Delta t_{q+1})_{Lk}}{\prod_{k=1}^{m} R_k(\tau + \sum_{i=1}^{q} \Delta t_i + \Delta t_{q+1})_0} . \qquad \text{(B2-9)}$$

where, $R_c(\tau)_{q+1}$ is the reliability function of Component c after the system has been preventively maintained for $q+1$ times. Write $R_c(\tau)_{q+1}$ as:

$$R_c(\tau)_{q+1} = R_c(\tau + \sum_{i=Lc+1}^{q+1} \Delta t_i)_{Lc} , \qquad \text{(B2-10)}$$

where $Lc = q+1$.

Substituting Equation (B2-10) into Equation (B2-9), gives

$$R_s(\tau)_{q+1} = \frac{R_s(\tau + \sum_{i=1}^{q+1} \Delta t_i)_0 \prod_{k=1}^{m} R_k(\tau + \sum_{i=Lk+1}^{q+1} \Delta t_i)_{Lk}}{\prod_{k=1}^{m} R_k(\tau + \sum_{i=1}^{q+1} \Delta t_i)_0} . \qquad \text{(B2-11)}$$

Equation (B2-11) indicates that Equation (B2-7) is true when $n = q+1$, if a previously repaired Component c ($c \le m$) is repaired again.

(2) A new Component d is repaired. In this condition, the total repaired components represented in Equation (B2-8) are increased by 1, and $d = m$ since Component d is the last repaired component.

$$R_s(\tau)_{q+1} = \frac{R_m(\tau)_{q+1} R_s(\tau + \sum_{i=1}^{q} \Delta t_i + \Delta t_{q+1})_0 \prod_{k=1}^{m-1} R_k(\tau + \sum_{i=Lk+1}^{n} \Delta t_i + \Delta t_{q+1})_{Lk}}{R_m(\tau + \sum_{i=1}^{q+1} \Delta t_i)_0 \prod_{k=1}^{m-1} R_k(\tau + \sum_{i=1}^{q} \Delta t_i + \Delta t_{q+1})_0} . \qquad \text{(B2-12)}$$

where, $R_m(\tau)_{q+1}$ is the reliability function of Component d after the system has been preventively maintained for $q+1$ times. Write $R_m(\tau)_{q+1}$ as:

$$R_m(\tau)_{q+1} = R_m(\tau + \sum_{i=Lm+1}^{q+1} \Delta t_i)_{Lm} , \qquad \text{(B2-13)}$$

where $Lm = q+1$.

Substituting Equation (B2-13) into Equation (B2-12), one has the same result as Equation (B2-11), i.e., Equation (B2-7) is true when $n = q+1$, if a new component is repaired.

A combination of the conclusions of (1) and (2) proves that Equation (B2-7) is true.

Appendix B3

The Mann's Test for the Weibull Distribution of the Pipeline Failure Data

The Mann's Test [16] for the Weibull Distribution was applied as follows. The hypotheses are

H_0 : The failure times are Weibull.

H_1 : The failure times are not Weibull.

The test statistic is

$$M = \frac{k_1 \sum_{i=k_1+1}^{r-1} [\ln t_{i+1} - \ln t_i] / M_i]}{k_2 \sum_{i=1}^{k_1} [\ln t_{i+1} - \ln t_i] / M_i]} \qquad \text{(B3-1)}$$

where, k_1 and k_2 are the integer portion of the number $\frac{r}{2}$ and $\frac{r-1}{2}$. Number r is failure times.

$$M_i = \ln[-\ln(1 - \frac{i+0.5}{n-0.25})] - \ln[-\ln(1 - \frac{i-0.5}{n-0.25})] \quad i = 1, 2, \ldots, r. \qquad \text{(B3-2)}$$

where, n is the test number.

If α stands for the level of significance of the test and $M \le F(\alpha, 2k_2, 2k_1)$, then H_0 is accepted. $F(\bullet)$ is the F -distribution function.

The test that the failure times of the pipeline are Weibull distributed is shown in Table B3-1.

Table B3-1. Mann's Test for the Weibull Distribution of the failure times of the pipeline

<table>
<tr><th>i</th><th>M_i</th><th>$\ln t_{i+1} - \ln t_i$</th><th>Numerator</th><th>Denominator</th></tr>
<tr><td>1</td><td>1.124371</td><td>0.11232312</td><td>4.9328954</td><td>4.251268767</td></tr>
<tr><td>2</td><td>0.537753</td><td>0.242214656</td><td rowspan="7" colspan="2">$r = 9$
$k_1 = 4$
$k_2 = \frac{9-1}{2} = 4$
$\alpha = 0.05$
$n = 10$
$F(0.05, 4, 4) = 3.44$
$M = 1.1603 < F(0.05, 4, 4)$

The hypothesis H_0 is accepted.</td></tr>
<tr><td>3</td><td>0.364689</td><td>0.07505569</td></tr>
<tr><td>4</td><td>0.280963</td><td>0.086169006</td></tr>
<tr><td>5</td><td>0.231918</td><td>0.029682544</td></tr>
<tr><td>6</td><td>0.200101</td><td>0.083584063</td></tr>
<tr><td>7</td><td>0.178189</td><td>0.020482027</td></tr>
<tr><td>8</td><td>0.16259</td><td>0.093096055</td></tr>
</table>

Appendix B4

The Proof of Proposition 4-1

Proposition 4-1: For an interaction chain process described by Equation (4-26), the n[th] state of the interactive chain process is given by

$$\{h^{(n)}(t)\} = ([I] + \sum_{s=1}^{n} [\theta(t)]^s)\{h_I(t)\}. \quad (4\text{-}30)$$

Proof.

This proposition is proved using the Principle of Mathematical Induction [18] as follows.

When $n = 2$, substituting Equation (4-24) into Equation (4-25), gives

$$\{h^{(2)}(t)\} = [I]\{h_I(t)\} + [\theta(t)]\{h_I(t)\} + [\theta(t)]^2\{h_I(t)\}$$

$$= ([I] + \sum_{s=1}^{2} [\theta(t)]^s)\{h_I(t)\}. \quad (B4\text{-}1)$$

Proposition 4-1 is true.

Assume that when $n = k$, Proposition 4-1 is true, i.e.,

$$\{h^{(k)}(t)\} = ([I] + \sum_{s=1}^{k} [\theta(t)]^s)\{h_I(t)\}. \quad (B4\text{-}2)$$

Then when $n = k + 1$, the following equation can be obtained using Equation (4-26):

$$\{h^{(k+1)}(t)\} = [I]\{h_I(t)\} + [\theta(t)]\{h^{(k)}(t)\}. \quad (B4\text{-}3)$$

Substituting Equation (B4-2) into Equation (B4-3), gives

$$\{h^{(k+1)}(t)\} = ([I] + [\theta(t)][I] + [\theta(t)]\sum_{s=1}^{k}[\theta(t)]^{s})\{h_I(t)\}$$

$$= ([I] + \sum_{s=1}^{k+1}[\theta(t)]^{s})\{h_I(t)\}\,. \tag{B4-4}$$

Therefore, Proposition 4-1 is true.

Appendix B5

The Derivation of Equation (4-31)

Let

$$[S] = [I] + \sum_{s=1}^{n} [\theta(t)]^s \ . \qquad \text{(B5-1)}$$

Then

$$[S] - [I] = \sum_{s=1}^{n} [\theta(t)]^s \ . \qquad \text{(B5-2)}$$

The following equation can be obtained from Equation (B5-2):

$$[S] - [I] = [\theta(t)]([I] + \sum_{s=1}^{n-1} [\theta(t)]^s) \ . \qquad \text{(B5-3)}$$

Note that

$$\sum_{s=1}^{n-1} [\theta(t)]^s = \sum_{s=1}^{n} [\theta(t)]^s - [\theta(t)]^n \ . \qquad \text{(B5-4)}$$

Substituting Equation (B5-4) into Equation (B5-3) and rearranging the result, gives

$$([I] - [\theta(t)])[S] = [I] - [\theta(t)]^{n+1} \ . \qquad \text{(B5-5)}$$

Left-multiplying the inverse matrix $([I] - [\theta(t)])^{-1}$ to the both sides of Equation (B5-5) if the determinant $Det([I] - [\theta(t)]) \neq 0$, the following expression can be obtained:

$$[S] = ([I] - [\theta(t)])^{-1}([I] - [\theta(t)]^{n+1}) \ . \qquad \text{(B5-6)}$$

Appendix B6

The Proof of Proposition 5-1

Proposition 5-1: All elements in the State Influence Matrix $[\alpha]$ are nonnegative when $0 \le \theta_{ij} < 1$.

Proof

Proposition 5-1 is proved using the Principle of Mathematical Induction [18] as follows.

According to Chapter 4, SIM $[\alpha]$ is the inverse matrix of $([I]-[\theta(t)])$:

$$[\alpha] = ([I]-[\theta(t)])^{-1}, \qquad \text{(B6-1)}$$

where,

$$([I]-[\theta(t)]) = \begin{bmatrix} 1 & -\theta_{12}(t) & \cdots & -\theta_{1M}(t) \\ -\theta_{21}(t) & 1 & \cdots & -\theta_{2M}(t) \\ \vdots & \cdots & \ddots & \vdots \\ -\theta_{M1}(t) & -\theta_{M2}(t) & \cdots & 1 \end{bmatrix}. \qquad \text{(B6-2)}$$

M is the number of components in a system. Matrix (B6-2) has the following properties:

(1) All diagonal elements are equal to 1.

(2) All non-diagonal elements are either negative or zero because

$$0 \le \theta_{ij}(t) < 1 \quad (i, j = 1,2,...,M;\ i \ne j). \qquad \text{(B6-3)}$$

When $M = 2$,

$$[\alpha] = ([I] - [\theta(t)])^{-1} = \begin{bmatrix} 1 & -\theta_{12}(t) \\ -\theta_{21}(t) & 1 \end{bmatrix}^{-1}$$

$$= \frac{1}{\det[([I] - [\theta(t)])]} \begin{bmatrix} 1 & \theta_{12}(t) \\ \theta_{21}(t) & 1 \end{bmatrix}. \qquad \text{(B6-4)}$$

The proposition is true because $\det[([I] - [\theta(t)])] > 0$.

Suppose that the proposition is true when $M = K$, i.e.,

$$\alpha_{ij} \geq 0 \quad (i, j = 1,2,\ldots,K). \qquad \text{(B6-5)}$$

When $M = K + 1$, rewrite matrix $[\alpha]$ in the form of partition matrix:

$$[\alpha] = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1k} & \alpha_{1K+1} \\ \vdots & \ddots & \vdots & \vdots \\ \alpha_{K1} & \cdots & \alpha_{KK} & \alpha_{KK+1} \\ \alpha_{K+11} & \cdots & \alpha_{K+1K} & \alpha_{K+1K+1} \end{bmatrix} = \begin{bmatrix} \vec{\alpha}_{11} & \vec{\alpha}_{12} \\ \vec{\alpha}_{21} & \alpha_{K+1K+1} \end{bmatrix}. \qquad \text{(B6-6)}$$

In Equation (B6-5) and Equation (B6-6), the variable t is omitted for simplicity. From now on, variable t will not be written in expressions.

In Equation (B6-6),

$$\vec{\alpha}_{11} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1K} \\ \vdots & \ddots & \vdots \\ \alpha_{K1} & \cdots & \alpha_{KK} \end{bmatrix}, \qquad \text{(B6-7)}$$

$$\vec{\alpha}_{12} = \{\alpha_{1K+1}, \alpha_{2K+1}, \ldots, \alpha_{KK+1}\}^T, \qquad \text{(B6-8)}$$

$$\vec{\alpha}_{21} = \{\alpha_{K+11}, \alpha_{K+12}, \ldots, \alpha_{K+1K}\}. \qquad \text{(B6-9)}$$

Rewrite the matrix $([I] - [\theta])$ into the same sized partition matrix. Let $[\nu] = [I] - [\theta]$, then

$$[\nu]=\begin{bmatrix} 1 & -\theta_{12} & \cdots & -\theta_{1K+1} \\ -\theta_{21} & 1 & \cdots & -\theta_{2K+1} \\ \vdots & \cdots & \ddots & \vdots \\ -\theta_{K+11} & -\theta_{K+12} & \cdots & 1 \end{bmatrix}=\begin{bmatrix} \vec{\nu}_{11} & \vec{\nu}_{12} \\ \vec{\nu}_{21} & 1 \end{bmatrix}, \qquad \text{(B6-10)}$$

where,

$$\vec{\nu}_{11}=\begin{bmatrix} 1 & -\theta_{12} & \cdots & -\theta_{1K} \\ -\theta_{21} & 1 & \cdots & -\theta_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ -\theta_{K1} & -\theta_{K2} & \cdots & 1 \end{bmatrix}, \qquad \text{(B6-11)}$$

$$\vec{\nu}_{12}=\{-\theta_{1K+1},-\theta_{2K+1},\ldots,-\theta_{KK+1}\}^T, \qquad \text{(B6-12)}$$

$$\vec{\nu}_{21}=\{-\theta_{K+11},-\theta_{K+12},\ldots,-\theta_{K+1K}\}. \qquad \text{(B6-13)}$$

The following equation can be obtained by using the equation $[\alpha][\nu]=[I]$ and matrix multiplying rules:

$$\vec{\alpha}_{11}\vec{\nu}_{12}+\vec{\alpha}_{12}=\{0\}, \qquad \text{(B6-14)}$$

where, $\{0\}$ is a $1\times K$ null vector.

From Equation (B6-14), one can obtain the following equations:

$$-\sum_{s=1}^{K}\alpha_{is}\theta_{sK+1}+\alpha_{iK+1}=0 \quad (i=1,2,\ldots,K). \qquad \text{(B6-15)}$$

The first term in Equation (B6-15) is equal to or less than zero because of Equations (B6-3) and (B6-5). Therefore,

$$\alpha_{iK+1}\geq 0 \quad (i=1,2,\ldots,K). \qquad \text{(B6-16)}$$

On the other hand,

$$[\nu][\alpha]=[I]. \tag{B6-17}$$

Then the following result can be gained by using the same inference as mentioned above:

$$-\sum_{s=1}^{K}\theta_{K+1s}\alpha_{sj}+\alpha_{K+1j}=0 \quad (j=1,2,\ldots,K). \tag{B6-18}$$

From Equation (B6-18), one has

$$\alpha_{K+1j}\geq 0 \quad (j=1,2,\ldots,K). \tag{B6-19}$$

Furthermore, from

$$\alpha_{K+1K+1}-\sum_{s=1}^{K}\theta_{K+1s}\alpha_{sK+1}=1, \tag{B6-20}$$

the following conclusion can be drawn:

$$\alpha_{K+1K+1}\geq 1. \tag{B6-21}$$

A combination of Inequities (B6-16), (B6-19) and (B6-21) gives

$$\alpha_{ij}\geq 0 \quad (i,\,j=1,2,\ldots,K+1). \tag{B6-22}$$

That is, when $M=K+1$, the Proposition 5-1 is also true.

Appendix B7

The Proof of Proposition 5-2

Proposition 5-2: All diagonal elements in the State Influence Matrix $[\alpha]$ are greater than or equal to one.

Proof

According to Equation (B6-17),

$$\alpha_{ii} - \sum_{\substack{s=1 \\ s \neq i}}^{M} \theta_{is} \alpha_{si} = 1 . \tag{B7-1}$$

The second term on the left side of Equation (B7-1) is not negative according to the properties of Interactive Coefficient (IC) and Proposition 5-1. Therefore,

$$\alpha_{ii} \geq 1 \quad (i = 1, 2, \cdots, M) . \tag{B7-2}$$

The inequity symbol becomes equal symbol if all θ_{is}=0 ($s = 1, 2, \ldots, M$).

Propositions 5-1 and 5-2 have explicit physical meanings. Proposition 5-1 indicates that components in a system are subject to stable IntF. Proposition 5-2 indicates that the IntHs of the affected components in a system are greater than their Independent Hazards (IndHs) due to failure interactions. The failure likelihoods of these affected components also increase. The IntH of a component will be equal to its IndH if the failures of other components do not affect it.

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