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MACHINE PROGNOSTICS BASED ON HEALTH STATE ESTIMATION USING SVM

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The ability to accurately predict the remaining useful life of machine components is critical for continuous operations in machines which can also improve productivity and enhance system safety. In condition-based maintenance (CBM), effective diagnostics and prognostics are important aspects of CBM which provide sufficient time for maintenance engineers to schedule a repair and acquire replacement components before the components finally fail. All machine components have certain characteristics of failure patterns and are subjected to degradation processes in real environments. This paper describes a technique for accurate assessment of the remnant life of machines based on prior expert knowledge embedded in closed loop prognostics systems. The technique uses Support Vector Machines (SVM) for classification of faults and evaluation of health for six stages of bearing degradation. To validate the feasibility of the proposed model, several fault historical data from High Pressure Liquefied Natural Gas (LNG) pumps were analysed to obtain their failure patterns. The results obtained were very encouraging and the prediction closely matched the real life particularly at the end of term of the bearings.

Key Words: Prognostics, Degradation State, Support Vector Machines (SVM), Remaining Useful Life (RUL), High Pressure LNG Pump

1 INTRODUCTION

An important objective of condition-based maintenance (CBM) is to determine the optimal time for replacement or overhaul of a machine. The ability to accurately predict the remaining useful life of a machine system is critical for its operation and can also be used to improve productivity and enhance system safety. In condition-based maintenance, maintenance is usually performed based on an assessment or prediction of the machine health instead of its service time, which leads to intended usage of the machine, reduced down time and enhanced operation safety. An effective prognostics program will provide ample time for maintenance engineers to schedule a repair and to acquire replacement components before catastrophic failures occur. Recent advances in computing and information technology have accelerated the production capability of modern machines and reasonable progress has been achieved in machine failure diagnostics but not in prognostics.

Prognosis is considerably more difficult to formulate since its accuracy is subjected to stochastic processes that are yet to occur. In general, although many diagnostic engineers have lots of information and experience about machine failure and health states by continuously condition monitoring and analysing of machine condition in industry, there are still no clear systematic methodologies on how to predict machine remnant life. The task still relies on human expert knowledge and experience. Therefore, there is an urgent need to continuously develop and improve effective prognostic models which can be implemented in intelligent maintenance systems for industrial applications. In order to conduct effective prognosis, performance assessment, degradation models, failure analysis, health management and prediction, feature extraction and knowledge base of faults are required [1]. To prognosis accurately, one needs to conduct prior analysis of the system degradation process, failure patterns and event history of the machine as well as use a data driven approach.

In this paper, for accurate assessment of the remnant life of machine, the authors proposed a machine prognostics model based on health state estimation using SVM. In this model, prior expert knowledge embedded in the closed loop prognostics system together with the SVM for classification of faults were used to evaluate the health states. Nambura et al. [2] presented the possibility of fault severity estimation via SVM for the mode-invariant fault diagnosis of automotive engines.

In our study, historical failure data and events were analysed to identify failure patterns using an expert knowledge system to extract effective features and construct the fault degradation steps for certain impending faults. To validate the feasibility of the proposed model, bearing fault cases of High Pressure Liquefied Natural Gas (LNG) pumps were analysed to obtain the failure degradation process of bearing failure. Then, pre-determined failure stages were employed for the estimation of the machine's remaining useful life (RUL) by using the SVM classifier. The results showed that the proposed prognosis system has the potential to be used as an estimation tool for machine remnant life prediction in real life industrial applications.

The remaining part of the paper is organised as follows. Section 2 presents the proposed prognosis system based on health state estimation with embedded expert knowledge. In Section 3, the basic principle of SVM employed in this research is described briefly. Section 4 presents the result of bearing failure cases for high pressure Liquefied Natural Gas (LNG) pumps. We conclude the paper in Section 5 with a summary for future research work.

2 PROGNOSTICS SYSTEM BASED ON HEALTH STATE ESTIMATION

In this research, a new prognostics system based on health state estimation with embedded expert knowledge is proposed. In terms of design and development of intelligent maintenance systems, effective intelligent prognostics models using condition monitoring techniques and failure pattern analysis for a critical dynamic system can lead to robust prognostics system in industry. Furthermore the combined analysis of event data and condition monitoring data can be accomplished by building a mathematical model that properly describes the underlying mechanism of a fault or a failure.

For an accurate assessment of machine health, a significant amount of a priori knowledge about the assessed machine or process is required because the corresponding failure modes must be known and well-described in order to assess the current machine or process performance [3]. In general, each machine system has inherent characteristics that could be used to determine the entire life cycle of machine. Therefore, prior analysis and knowledge failure pattern could lead to more accurate prediction of remnant life. For accurate prognosis, one requires expert knowledge about the machinery degradation, failure patterns and maintenance history. The objective of prognosis is to predict when the machine is likely to fail or degrade. Figure 1 illustrates the closed loop architecture of the prognostics system with embedded expert knowledge. The entire sequence for diagnosis and prognosis are related with expert knowledge. Historical failure data and events are analysed for identifying the failure patterns of the system making use of expert knowledge. Expert knowledge could also be used for effective feature extraction, selection, condition monitoring and construction of fault degradation steps of certain impending faults, as depicted in Figure1.

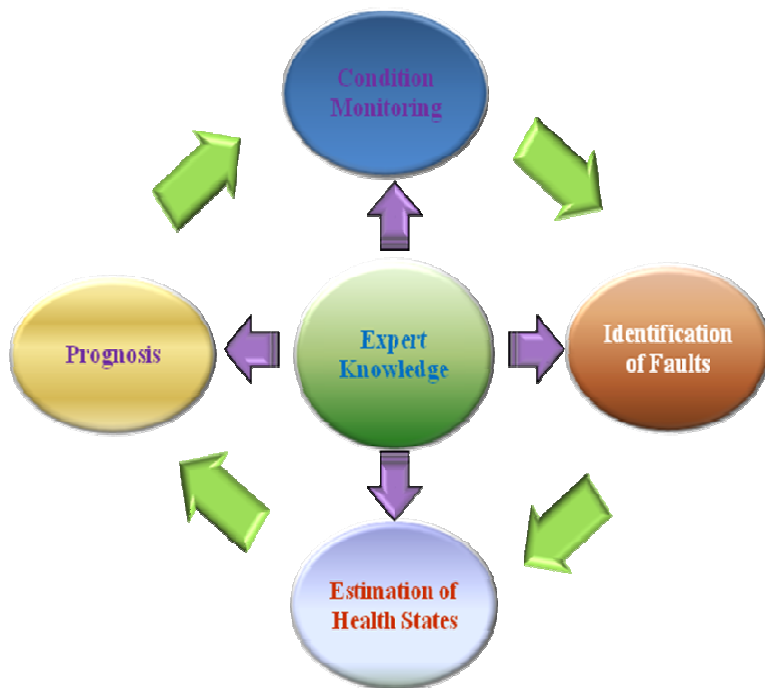


Figure 1. Closed Loop Architecture of the Prognostics System.

In this research, the authors aimed to develop a practical prognostics model which has the capability for application in current on-line condition monitoring systems. Figure 2 shows the flow chart of a prognostics system based on health state estimation using SVM. This system consists of three sub-systems, namely, expert knowledge, diagnostics and prognostics. Through failure pattern analysis of the historical data and events, failure degradation stages can be determined to estimate the health stage of the machine. This type of prior expert knowledge is also related with signal processing, feature extraction and selection in diagnosis and prognosis as depicted in Figure 2. In this paper, the prognostic sub-system is used to estimate the RUL since the feasibilities of SVM for the fault classification have been introduced in several recent literatures.

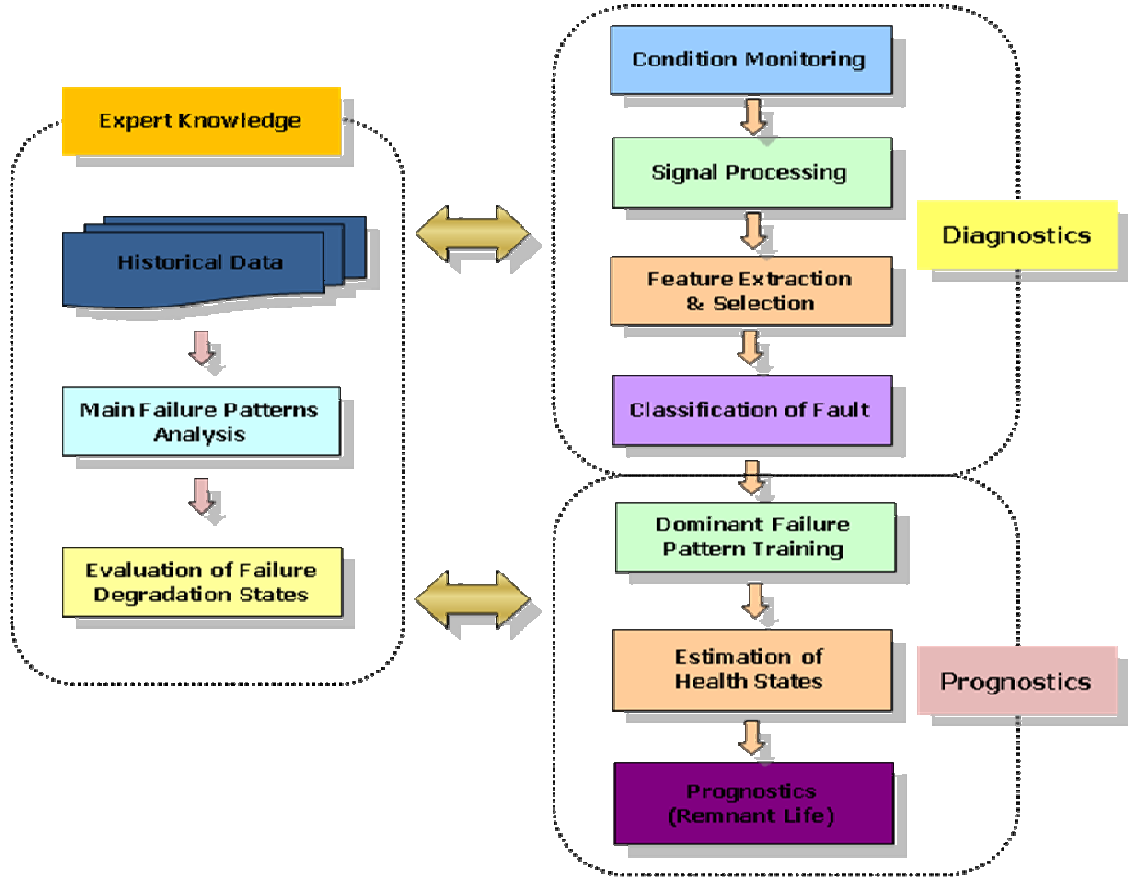


Figure 2. Flow Chart of the Prognostics System Based on Health State Estimation.

3 SUPPORT VECTOR MACHINES

Support vector machines (SVM) is based on the statistical learning theory introduced by Vapnik and his co-workers [4]. SVM is also known as maximum margin classifier with the abilities of simultaneously minimizing the empirical classification error and maximizing the geometric margin. Due to the excellent generalization ability, a number of applications have been addressed with the machine learning method in the past few years. This section provides a brief summary of the standard SVM for pattern recognition. Given data input \mathbf{x}_i ($i = 1, 2, \dots, M$), where M is the number of samples. The i th sample $\mathbf{x}_i \in \mathbb{R}^n$ in an n -dimension input space belongs to one of two classes labelled by $y_i \in \{-1, 1\}$ that have two classes, namely, positive class and negative class. In the case of linear data, it is possible to determine the hyperplane $f(\mathbf{x}) = 0$ that separates the given input data.

$$f(\mathbf{x}) = \mathbf{w}^T \mathbf{x} + b = \sum_{j=1}^M w_j x_j + b = 0 \quad (1)$$

Where \mathbf{w} is a coefficient vector and b is a bias of the hyperplane. The vector \mathbf{w} and scalar b are used to define the position of the separating hyperplane. The decision function is made using $\text{sign}(f(x))$ to create separating hyperplane that classify input data in either positive class and negative class.

A distinctly separating hyperplane should satisfy the constraints

$$\begin{aligned} f(x_i) &= 1 & \text{if } y_i &= 1 \\ f(x_i) &= -1 & \text{if } y_i &= -1 \end{aligned} \quad (2)$$

or it can be presented in a complete equation

$$y_i f(\mathbf{x}_i) = y_i (\mathbf{w}^T \mathbf{x}_i + b) \geq 1 \quad \text{for } i = 1, 2, \dots, M \quad (3)$$

The separating hyperplane that creates the maximum distance between the plane and the nearest data, i.e., the maximum margin, is called the optimal separating hyperplane (OSH). By taking into account the noise with slack variables ξ_i and the error penalty C , the optimal hyperplane separating the data can be obtained as a solution to the following optimization problem

$$\text{minimize } \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^M \xi_i \quad (4)$$

$$\text{subject to } \begin{cases} y_i (\mathbf{w}^T \mathbf{x}_i + b) \geq 1 - \xi_i, & i = 1, \dots, M \\ \xi_i \geq 0 & i = 1, \dots, M \end{cases} \quad (5)$$

Where ξ_i is the measured distance between the margin and the examples \mathbf{x}_i that lying on the wrong side of the margin. The calculation can be simplified by converting the problem with the Kuhn-Tucker condition into the equivalent Lagrangian dual problem, which will be

$$\text{minimize } L(\mathbf{w}, b, \boldsymbol{\alpha}) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^M \alpha_i y_i (\mathbf{w}^T \mathbf{x}_i + b) + \sum_{i=1}^M \alpha_i \quad (6)$$

The task is minimizing Eq. (6) with respect to \mathbf{w} and b , while requiring the derivatives of L to $\boldsymbol{\alpha}$ to vanish. At the optimal point, following saddle point equations are applied.

$$\frac{\partial L}{\partial \mathbf{w}} = 0, \quad \frac{\partial L}{\partial b} = 0 \quad (7)$$

which can be replaced by

$$\mathbf{w} = \sum_{i=1}^M \alpha_i y_i \mathbf{x}_i, \quad \sum_{i=1}^M \alpha_i y_i = 0 \quad (8)$$

From Eq. (8), \mathbf{w} is contained in the subspace spanned by the \mathbf{x}_i . By substitution Eq. (8) into Eq. (7), the dual quadratic optimization problem is obtained.

$$\text{maximize } L(\boldsymbol{\alpha}) = \sum_{i=1}^M \alpha_i - \frac{1}{2} \sum_{i,j=1}^M \alpha_i \alpha_j y_i y_j \mathbf{x}_i^T \mathbf{x}_j \quad (9)$$

$$\text{subject to } \alpha_i \geq 0, \quad i = 1, \dots, M. \quad \sum_{i=1}^M \alpha_i y_i = 0 \quad (10)$$

Thus, by solving the dual optimization problem, one obtains the coefficients α_i which is required to express \mathbf{w} to solve Eq. (4). This leads to the non-linear decision function.

$$f(\mathbf{x}) = \text{sign} \left(\sum_{i,j=1}^M \alpha_i y_i (\mathbf{x}_i^T \mathbf{x}_j) + b \right) \quad (11)$$

SVM can also be used in non-linear classification tasks with application of kernel functions. The data to be classified is mapped onto a high-dimensional feature space, where the linear classification is possible. Using the non-linear vector function $\Phi(\mathbf{x}) = (\phi_1(\mathbf{x}), \dots, \phi_l(\mathbf{x}))$ to map the n -dimensional input vector \mathbf{x} onto l -dimensional feature space, the linear decision function in dual form is given by

$$f(\mathbf{x}) = \text{sign} \left(\sum_{i,j=1}^M \alpha_i y_i (\Phi^T(\mathbf{x}_i) \Phi(\mathbf{x}_j)) + b \right) \quad (12)$$

Working in the high-dimensional feature space enables the expression of complex functions, but it also generates other problems. Computational problem can occur due to the large vectors and overfitting can also exist due to the high-dimensionality. The latter problem can be solved by using the kernel function. The Kernel is a function that returns a dot

product of the feature space mappings of the original data points, stated as $K(\mathbf{x}_i, \mathbf{x}_j) = (\Phi^T(\mathbf{x}_i)\Phi(\mathbf{x}_j))$. When applying a kernel function, learning in the feature space does not require explicit evaluation of Φ and the decision function will be

$$f(\mathbf{x}) = \text{sign} \left(\sum_{i,j=1}^M \alpha_i y_i K(\mathbf{x}_i, \mathbf{x}_j) + b \right) \quad (13)$$

Any function that satisfies Mercer's theorem [5] can be used as a kernel function to compute a dot product in feature space. There are different kernel functions used in SVM, such as linear, polynomial and Gaussian RBF. The kernel defines the feature space in which the training set examples will be classified. In this research, the polynomial function $((\gamma \mathbf{x}^T \cdot \mathbf{x}_j + r)^d, \gamma > 0)$ was employed for classification of health states.

Support vector machines were originally designed for binary classification and there are several methods that have been addressed for multi-class classification, such as "one-against-one", "one-against-all", and directed acyclic graph (DAG) where Hsu and Lin [6] presented a comparison of these methods and pointed out that the "one-against-one" method is more suitable for practical use than other methods. Consequently, in this study, the authors have adopted the "one-against-one" method to classify the six failure degradation stages.

4 VALIDATION OF MODEL USING HP LNG PUMP

4.1 High Pressure LNG Pump

Liquefied natural gas (LNG) takes up six hundredths of the volume of natural gas to be reached below the boiling temperature (-162°C), which can make storage and transportation much easier. In an LNG receiving terminal, high pressure LNG pumps are used to boost the LNG pressure to 80 bar for evaporation into highly compressed natural gas in order to be

sent out as highly compressed natural gas via a pipeline network across the nation. The numbers of high-pressure LNG pumps determine the amount of LNG at the receiving terminal. It is a critical equipment in the LNG production process and should be maintained at optimal conditions. Therefore, vibration and noise of high-pressure LNG pumps are regularly monitored and managed based on predictive maintenance techniques.

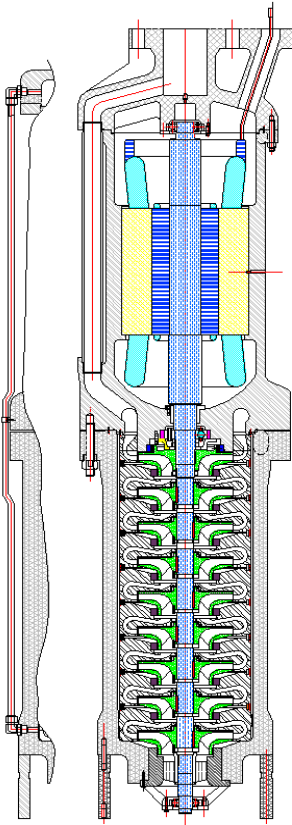


Table 1 shows the pump specifications. These high-pressure LNG pumps are submerged and operate at super cooled temperatures. They are self-lubricated at both sides of the rotor shaft and tail bearings using LNG. Due to the low viscous value (about 0.16cP) of LNG, the three bearings of the high-pressure LNG pump are poorly lubricated and the bearing must be specially designed. There are some difficulties in detecting the cause of pump failure at an early stage because of certain bearing components which can result in rapid bearing failure due to poor lubricating conditions and a high operating speed (3,600rpm). In other words, in case of abnormal problems happening, one would not have sufficient time to analyze the possible root cause before pump failure. Especially, due to the material property variations of cryogenic pumps at super low temperatures and some difficulties in measuring the vibration signals on the submerged pump housing, there are some restrictions for the diagnosis of pump health and the study of vibration behaviour. Hence there is a need to use the expert knowledge of the failure patterns for accurate estimation of remnant life. Long term prediction of certain failures for safe operation and Condition Based Maintenance is also highly recommended in case of these pumps.

Figure 3. Pump schematic and vibration measuring points

Table 1. Pump Specification

Capacity	Pressure	Impeller Stage	Speed	Voltage	Rating	Current
241.8 m ³ /hr	88.7 kg/cm ² . g	9	3,585 RPM	6,600V	746 kW	84.5 A

As shown in figure 3, high-pressure LNG pumps are enclosed within a suction vessel and mounted with a vessel top plate. Three ball bearings are installed to support entire dynamic load of the integrated shaft of the pump and motor. The submerged motor is cooled and the bearings lubricated by a predetermined portion of the LNG being pumped. For condition monitoring of pumps, three accelerometers are installed on housing near the bearing assembly in horizontal, vertical and axial directions respectively.

4.2 Vibration Data Acquisition of Bearing Failure

For machinery fault diagnosis and prognosis, signals such as vibration, temperature and pressure are commonly used. In this research, the authors only collected vibration data because the other data had no relationship with bearing failure directly and they were simply process information. Vibration data was collected through three accelerometers installed on the pump housing. In this paper, we focused on bearing failure cases for validation of the proposed model. Therefore, data from two pumps with the same specifications were used for prediction of the remaining useful life. Due to the random operation of the pumps to meet the total production rate of LNG supply, there were some restrictions to collect full data for the entire pump life. The acquired vibration data are summarized in Table 2.

Table 2. Acquired vibration data of the LNG pump

Machine No	Total operation hours	Reason of remove & Root cause	No of sample data	Sampling frequency
P301 C	4,698Hr's	High Vibration & Outer raceway spalling	136	12,800 Hz
P301 D	3,511Hr's	High Vibration & Inner raceway flaking	120	12,800 Hz

As shown in Table 2, a total 136 and 120 vibration samples were collected during full pump life for training and testing of the proposed prognosis model respectively.

Figure 4 shows the damage of the outer raceway spalling and inner raceway flaking, respectively. Although these two bearing faults had different fault severities on the inner race and outer race, these faults occurred on the same bearing of the pump.

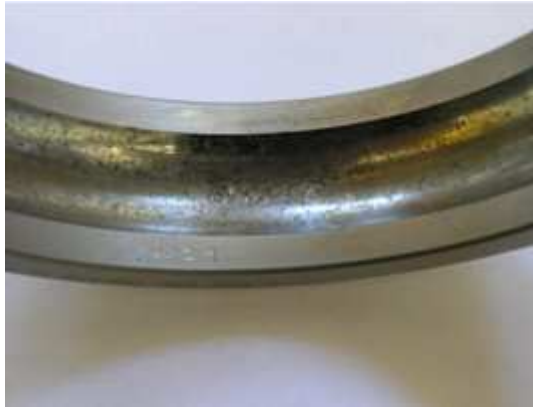
4.3 Features Calculation and Selection

In this paper, the authors calculated 10 statistical parameters from the time domain data. These feature parameters were mean, rms, shape factor, skewness, kurtosis, crest factor, entropy estimation, entropy estimation error, histogram lower and upper. In addition to these parameters, four parameters (rms frequency, frequency centre, root variance frequency and peak) in the frequency domain were calculated. A total of 42 features (14 parameters, 3 positions) were calculated as shown in Table 3.

Table 3. Statistical feature parameters

Position	Time Domain Parameters	Frequency Domain Parameters
Radial(A)	Mean, RMS, Shape factor, Skewness, Kurtosis, Crest factor, Entropy estimation, Entropy estimation error, Histogram lower and Histogram upper	Root mean square frequency,
Radial(B)		Frequency centre,
Axial		Root variance frequency Peak

In general, effective selection of features is required to avoid the problem of dimensionality and high training error value for the estimation of health states. In this paper, the authors divided the bearing failure process into six stages that could minimise the classification training error of each bearing degradation stage. For better training and testing of bearing failure degradation steps, four features that represent the degradation of bearing failure among the 14 features were selected.



(a) Outer raceway spalling of P301 C



(b) Inner raceway flaking of P301 D

Figure 4. Outer and inner race bearing failures

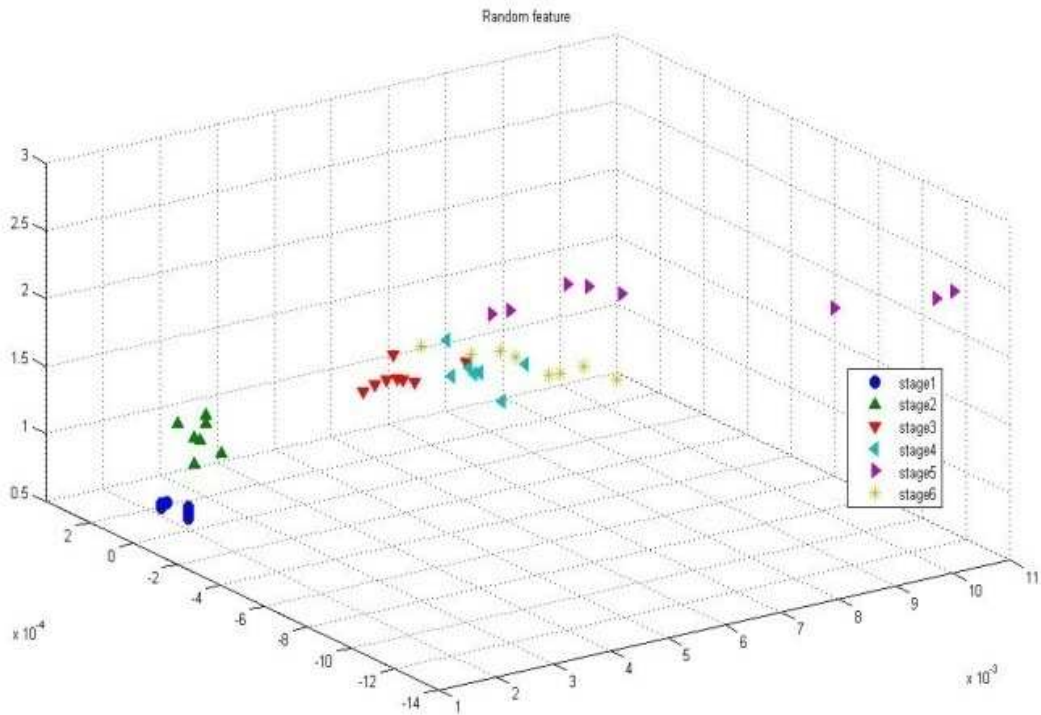


Figure 5. The mapping of random selected features.

Figure 5 shows the feature mapping of random selected features. Although some of health stages are well separated from other stages, the figure also shows some overlap clusters of health stages (stage 4, 5 and 6).

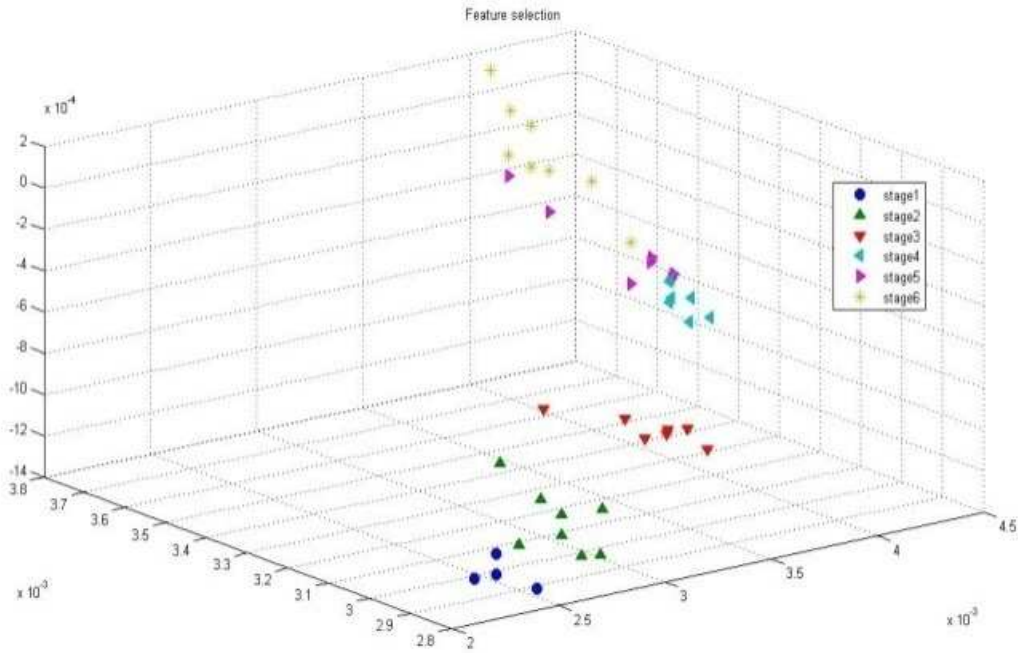


Figure 6. The mapping of selected features.

Figure 6 shows the feature mapping of kurtosis, entropy estimation and entropy estimation error value. The feature mapping of selected features indicates the failure degradation stages are well grouped in clusters as compared with the random selected features.

4.4 Training and Classification of Degradation Stages

In this paper, a polynomial was used as the basic kernel function of SVM. Multi-class classification using one-against-one was applied to perform the classification of degradation. Sequential minimal optimization (SMO) proposed by Platt [7] was employed to solve the SVM classification problem. For selection of optimal kernel parameters (C , γ , d), the authors used the cross-validation technique in order to obtain good performance of classification suggested by Hsu [7] so as to avoid overfitting or underfitting. For training and testing of the six stages of failure degradation, six data sets of P301 D were employed to perform the classification of health stages, which consisted of eight sets of samples with four selected features as shown in Table 4. The test data consisted of eight sets of samples in order following the next sample. The percentage of training error was 18.75% for classification of the six classes.

Table 4. Training data sets for classification of degradation stages [P301D, Radial (a)]

Stage	Training data set	Average operation hours	Remaining Life (%)	No of features
1	1 ~ 8	4 Hr's	99.89%	4
2	25 ~ 32	503 Hr's	85.67%	4
3	41 ~ 48	843 Hr's	75.99%	4
4	81 ~ 88	2,501 Hr's	28.77%	4
5	105 ~ 112	2,897 Hr's	17.49%	4
6	121 ~ 128	3,405 Hr's	3.02%	4

4.5 Classification Result and Useful Remaining Life Prediction

Once the six stages features were trained according to above training data sets, the full data sets of P301 D (136 data sets) were tested to obtain the probabilities of the six degradation stages using test error values of each stages. Figure 7 shows the probabilities of each stage of P301 D that was also used for training of the six degradation states. The first stage probability started with 100% and decreased as long as next stage probability increased. Some overlaps between the two stages could be explained due to the uncertainty of machine health condition or inappropriate data acquisitions in real environment. The entire probabilities of each stage well explain the sequence of six degradation stages, which are distinctly separated.

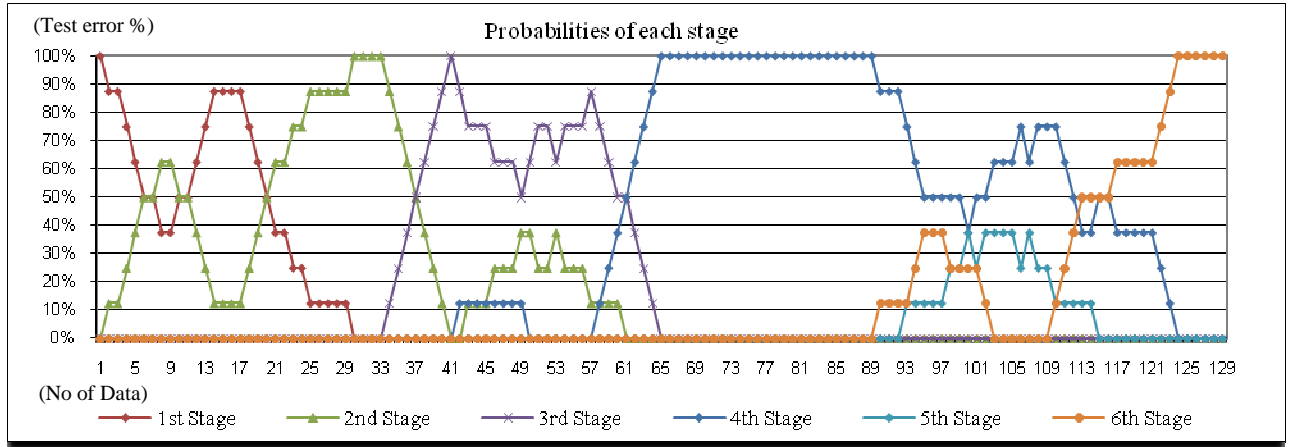


Figure 7. Probabilities of bearing failure (P301D).

For the estimation of remaining useful life (RUL), the expected life of the machine was calculated by using the operation hours for each training data and their probabilities. The average operation hours at each training sets are described in Table 4. Figure 8 shows the result of estimated remnant life and the comparison between real remaining life and estimated life.

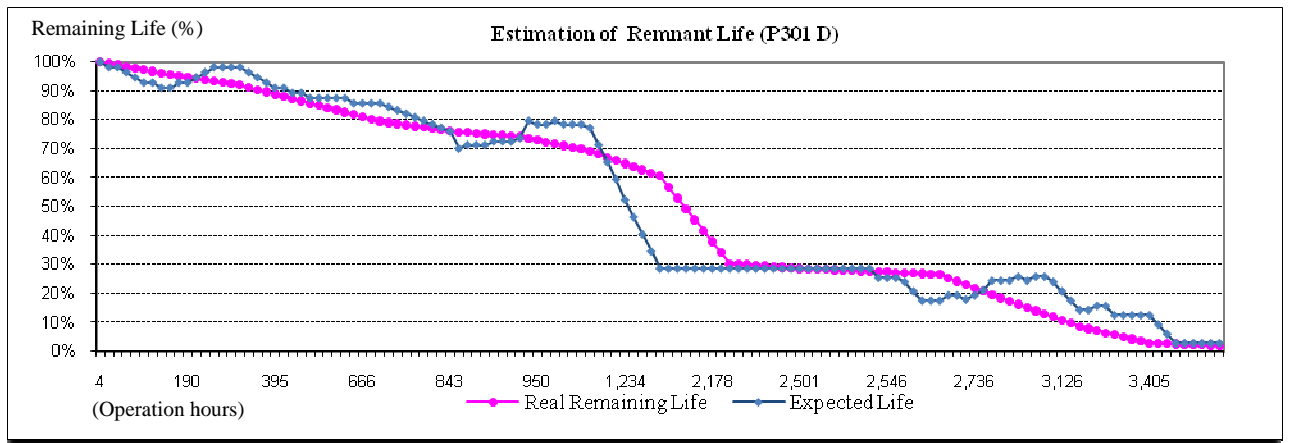


Figure 8. Comparison of real remaining life and estimated life (P301D).

As shown in Figure 8, although there are some discrepancies in middle zone of the display, the overall trend of the estimated life follows the real remaining life of the machine. Furthermore, the estimated life of the final stage more closely matched the real remaining life with less than 1% of remaining life.

Similar bearing fault data (P301 C) which consisted of 120 sample sets were also used to validate the proposed model using the same training data sets (P301 D) as described above. Figure 9 shows the result of the probabilities of each stage. With those probabilities of six degradation stages, the remaining life of the P301 D pump was also estimated.

In Figure 10, the result of estimated remaining life indicated that the expected remaining life (%) also matched closely with the real remaining life (%) of P301 C pump. However, there are some differences between the calculated remaining life time (Hr's) and real remaining life time (Hr's) as shown in Figure 11.

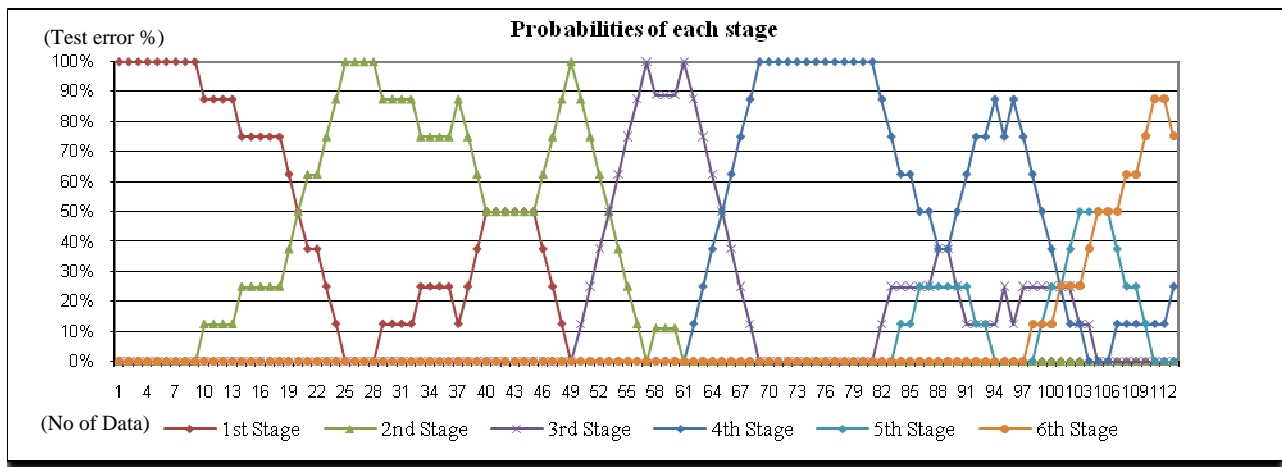


Figure 9. Probabilities of bearing failure (P301C).

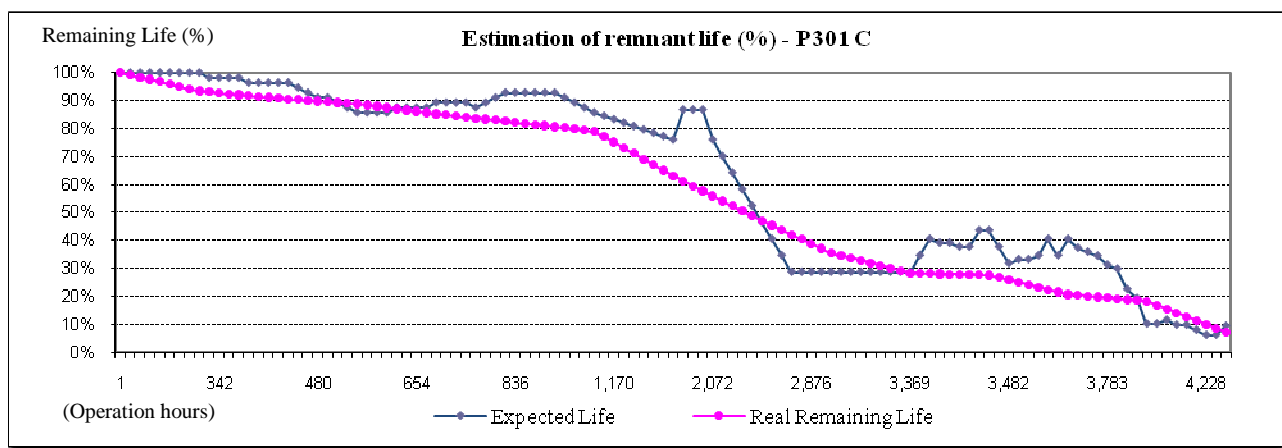


Figure 10. Comparison of real remaining life and estimated life (P301C).

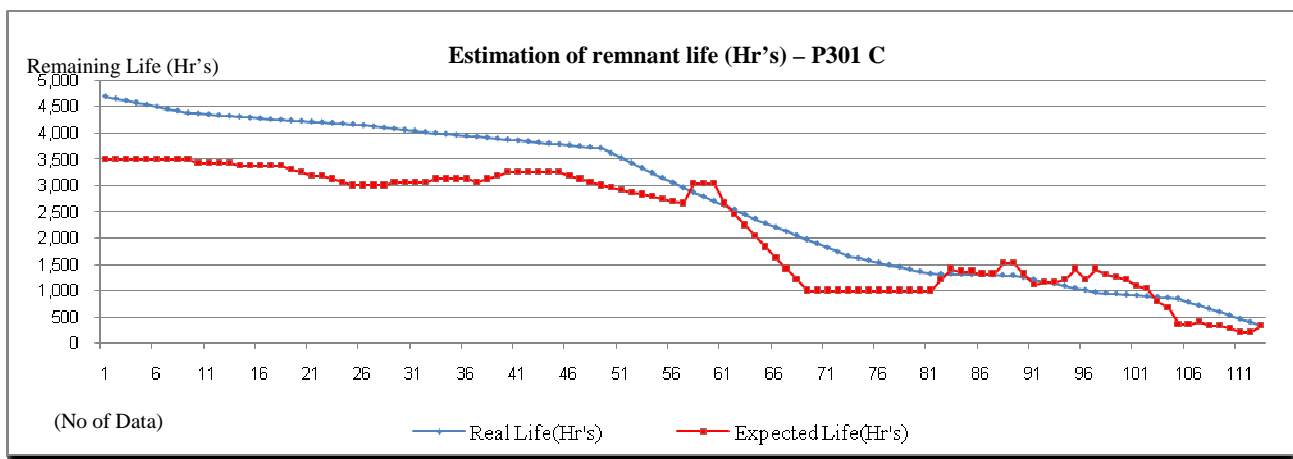


Figure 11. Comparison of real remaining life time and estimated life time.

As mentioned above, the estimated time was calculated from training data sets (P301 D) which had 3,511 hours in total operation. Therefore, this difference at initial degradation stages was caused by the discordance of total operation hours between training and test data.

5 CONCLUSIONS

Machine prognostics based on health state estimation using SVM has been presented. Through the failure pattern analysis of historical data in terms of expert knowledge, failure degradation stages were determined for the estimation of degradation stages of a machine. To verify the proposed model, bearing failure data of High Pressure LNG pump was used to extract features and determine the probabilities of six degradation stages using an SVM classifier. Although the training error value was about 18.75% for classification of the six classes respectively, the result of estimated remaining useful life followed closely with the real remaining life of machine. Finally, in the second case with similar failure pattern, the estimated life probability matched closely with the real remaining life, especially in the final stage of bearing failure. These results indicate that the proposed concept has the potential for further study and application in industry. However, effective feature extraction techniques for a variety of faults are still needed. Accurate training of failure states and the time prediction using the probabilities of each stage still needs investigation.

6 ACKNOWLEDGEMENTS

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