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MOVING FORCE IDENTIFICATION FROM BRIDGE STRAIN RESPONSES

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ABSTRACT

Accurate identification of moving forces in bridge is an important issue from the aspects of design, control and diagnosis of bridges. Previous studies show that the identification results will often be highly sensitive to changes in parameters of both the bridge and vehicle. The different solutions to an equation $Ax = b$, the over-determined system in the force identification, give results with different accuracy, particular when the systematic matrix A is close to rank deficient. Based on the time domain method (TDM) and frequency-time domain method (FTDM), this paper aims to evaluate the two solutions to the system equation, i.e. direct pseudo inversion (PI) solution and the singular value decomposition (SVD) solution. The effects of various parameters on both the TDM and FTDM are also discussed. Assessment results show that the moving force identification from the bridge strain responses is feasible and acceptable, but there exists different influence on the TDM and FTDM by means of the two solutions. The SVD solution can effectively improve the identification accuracy of the two methods, particular for the FTDM.

1 INTRODUCTION

Accurate identification of moving forces is often vital for accurate and cost effective design and maintenance of bridges. In some situations the forces can be measured directly using a force transducer in the load path. In other situations this is impossible or difficult. In such cases, it would be very convenient to resort to an indirect identification technique. Therefore, the indirect identification of the forces appears as a valuable alternative. In this context, "indirect force identification" refers to the process of deducing time histories of moving axle loads from measurements of bridge responses due to the passages of vehicles. This process is also frequently referred to as "moving force identification" or "moving force reconstruction".

Moving force identification has been studied extensively. Fryba [1] presented a comprehensive survey of the references and methods for solving the problems involving moving loads on structures. Stevens [2] provided an overview of the general problems involved in the identification of unknown stationary and transient forces. Dobson and Rider [3] reviewed different techniques and applications of indirect force identifications reported in the literature. A number of different applications of force identification were described, and the computational procedures and difficulties involved were outlined. However, the results from indirect force identification will often be highly sensitive to measurement noise and errors in the model of the structure. An interesting observation was made by Hillary and Ewins [4] who found that measurements of strain might lead to more accurate results than measurements of acceleration for beam-like structures. This is explained by the fact that there are generally more vibration modes significantly contributing to the strain response than to the acceleration response. This sensitivity of the results to the number of participating structural modes has been investigated in detail by Fabunmi [5], who suggested a scalar measure of the sensitivity based on this modal participation. Measures of the sensitivity have also been suggested by Starkey and Merrill [6] and by Hansen and Starkey [7]. The precision problems of dynamic load identification in time domain have been discussed by Tang [8]. Based on a system identification theory, the authors have developed another two moving force identification methods, namely the time domain method (TDM) [9] and frequency time domain method (FTDM) [10]. Comparative studies [11] showed that the two methods could identify moving forces with acceptable accuracy to some extent, the TDM was the best one but the FTDM suffered from several constraints. The results are sensitive to changes in both bridge and vehicle parameters. This is mainly associated with the solution by direct using the *pseudo-inverse* technique to the over-determined equation with the rank deficient coefficient matrix. The singular value decomposition (SVD) technique should be used to calculate the pseudo inversion of the system matrix if the system matrix is close to rank deficient [12]. The SVD technique is one of the most important tools in numerical analysis [13].

In this paper, both the TDM and FTDM are briefly described first. Laboratory experiments are then introduced. The results of an error study are evaluated, which show the effect of the change of bridge and vehicle parameters on the TDM and FTDM. They also show the robustness of the SVD technique that is adopted to solve the over-determined system equation in the TDM and FTDM.

2 TECHNICAL BASIS

2.1 Equation of Motion

Referring to Figure 1, the bridge superstructure is modeled with a simply supported beam. The effects of shear deformation and rotary inertia are not taken into account (Bernoulli-Euler beam). If the force P moves from left to right at a speed c, then an equation of motion in term of modal coordinate $q_n(t)$ can be expressed as

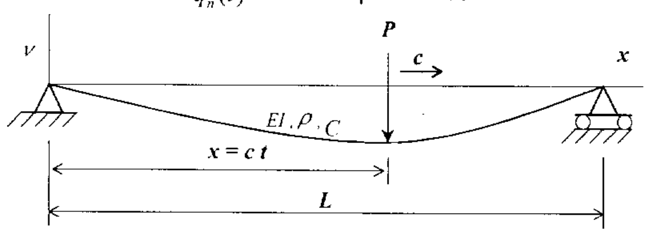

Figure 1. Moving force on a steel beam bridge

$$\ddot{q}_n(t)+2\xi_n\omega_n\dot{q}(t)+\omega_n^2 q_n(t)=\frac{2}{\rho L}p_n(t) \qquad (n=1,2,\cdots\infty) \tag{1}$$

Where

$$\omega_n=\frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{\rho}},\ \xi_n=\frac{C}{2\rho\omega_n},\ p_n(t)=P(t)\sin(\frac{n\pi ct}{L}) \tag{2}$$

are the nth modal frequency, the modal damping ratio and the modal force respectively. If the time-varying force $P(t)$ is known, equation (1) can be solved to yield $q_n(t)$. The dynamic deflection $v(x,t)$ can then be obtained from the $q_n(t)$ and the nth mode shape $\Phi_n(x)$. This is a forward problem. The moving force identification is an inverse problem in structural dynamics, in which the unknown time-varying force $P(t)$ is identified using the measured displacements, accelerations or bending moments of real structures. The TDM and FTDM are developed here for moving force identification.

2.2 Time Domain Method (TDM)

Equation (1) can be solved in time domain by the convolution integral and the dynamic deflection $v(x,t)$ of the beam at point x and time t can be obtained as

$$v(x,t)=\sum_{n=1}^{\infty}\frac{2}{\rho L\omega_n'}\sin\frac{n\pi x}{L}\int_0^t e^{-\xi_n\omega_n(t-\tau)}\sin\omega_n'(t-\tau)\sin\frac{n\pi c\tau}{L}P(\tau)\,d\tau \tag{3}$$

Where $\omega_n'=\omega_n\sqrt{1-\xi_n^2}$, therefore, the bending moment of the beam at point x and time t is

$$m(x,t)=-EI\frac{\partial^2 v(x,t)}{\partial x^2}$$
$$=\sum_{n=1}^{\infty}\frac{2EI\pi^2 n^2}{\rho L^3\omega_n'}\sin\frac{n\pi x}{L}\int_0^t e^{-\xi_n\omega_n(t-\tau)}\sin\omega_n'(t-\tau)\sin\frac{n\pi c\tau}{L}P(\tau)\,d\tau \tag{4}$$

Assuming that both the time-varying force $P(t)$ and the bending moment $m(x,t)$ are step functions in a small time interval Δt, equation (4) can be rewritten in discrete terms and rearranged into a set of equations as follows

$$\mathbf{B}_{N\times N_B}\,\mathbf{P}_{N_B\times 1}=\mathbf{R}_{N\times 1} \tag{5}$$

Where, **P** is the time series vector of time-varying force $P(t)$, **R** is the time series vector of the measured response of the bridge deck at the point x, says the bending moment $m(x,t)$. The system matrix **B** is associated with the system of bridge deck and the force. The subscripts $N_B=L/(c\Delta t)$ and N are the number of data samples for the force $P(t)$ and measured response **R** respectively when the force goes through the whole bridge deck.

If $N=N_B$, P can be found by solving the N order linear equations. If $N>N_B$ or l ($l>1$) responses are measured, the least squares method can be used to find the time history of the moving forces $P(t)$.

2.3 Frequency Time Domain Method (FTDM)

Equation (1) can also be solved in the frequency domain. Performing the Fourier transform for Equations (1) and $v(x,t)=\sum_{n=1}^{\infty}\Phi_n(x)q_n(t)$, the Fourier transform of the dynamic deflection $v(x,t)$ is

$$V(x,\omega)=\sum_{n=1}^{\infty}\frac{2}{\rho L}\Phi_n(x)H_n(\omega)P_n(\omega) \tag{6}$$

Where $H_n(\omega)$ are the Fourier Transform of $q_n(t)$, and

$$H_n(\omega)=\frac{1}{\omega_n^2-\omega^2+2\xi_n\omega_n\omega} \tag{7}$$

$$\Phi_n(x)=\sin(n\pi x/l) \tag{8}$$

$$P_n(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}p_n(t)e^{-i\omega t}dt \tag{9}$$

Substituting equations (7)-(9) into Equation (6), and assuming the Fourier transforms of both the force and the bending moments are step functions in a small frequency interval, Equation (6) can be rewritten in discrete terms and rearranged into a set of equations as

$$A_{(N+2)\times(N+2)}F_{(N+2)\times 1} = V_{(N+2)\times 1} \tag{10}$$

where V and F are Fourier transform of the dynamic deflection vector V and the force vector $P(t)$ respectively. The matrix A is associated with the bridge-vehicle system. Because variables in Equation (10) are complex number, Equation (10) are often written in terms of real and imaginary parts of complex number as follows

$$\begin{bmatrix} A_{RR} & -A_{II} \\ A_{RI} & A_{IR} \end{bmatrix}_{(N+2)\times(N+2)} \begin{Bmatrix} F_R \\ F_I \end{Bmatrix}_{(N+2)\times 1} = \begin{Bmatrix} V_R \\ V_I \end{Bmatrix}_{(N+2)\times 1} \tag{11}$$

or

$$A_{RI}\, F_{RI} = V_{RI} \tag{12}$$

Components F_R and F_I can be found from Equation (12) by solving the N order linear equations after taking account of initial conditions. The time history of the moving force $P(t)$ can then be obtained by performing the inverse Fourier transformation. However, the computation cost for solving Equation (12) is high in finding the inverse of a full matrix, and therefore the following procedure is developed to overcome these difficulties.

If the DFTs are expressed in matrix form, the Fourier transform of the force vector F will be written as follows

$$F = \frac{1}{N} W P \tag{13}$$

where $W = e^{-i2k\pi/N}$ and all terms in F are real [14]

$$k = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 2 & \cdots & N-2 & N-1 \\ 0 & 2 & 4 & \cdots & N-4 & N-2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & N-2 & N-4 & \cdots & 4 & 2 \\ 0 & N-1 & N-2 & \cdots & 2 & 1 \end{bmatrix}_{N\times N} \tag{14}$$

The matrix W is an unitary matrix, which means

$$W^{-1} = (W^*)^T \tag{15}$$

where W^* is a conjugate of W. Substituting Equation (13) into Equation (12), it yields

$$\underset{N\times N}{(W^*)^T}\ \underset{N\times N}{A}\ \underset{N\times N_B}{W_B}\ \underset{N_B\times 1}{P_B} = \underset{N\times 1}{V} \tag{16}$$

W_B and P_B are the sub-matrix of the W and P respectively. Similarly, the relationships between bending moment m and the moving force vector P can be described as follows

$$\underset{N\times N}{(W^*)^T}\ \underset{N\times N}{B}\ \underset{N\times N_B}{W_B}\ \underset{N_B\times 1}{P_B} = \underset{N\times 1}{m} \tag{17}$$

If $N = N_B$, P_B can be found by solving the N order linear equations in Equation (16) or (17). If $N > N_B$ or l ($l > 1$) responses are measured, the least squares method can be used to find the time history of the moving forces $P(t)$.

The above procedure is derived for one single force identification in TDM and FTDM methods. They can be modified for multi-force identification using the linear superposition principle.

2.4 Solutions

As mentioned in the previous sections, it is easy to see that the TDM or the FTDM will usually result in a system of equation that is often of the form

$$Ax = b \tag{18}$$

Assuming the size of Matrix A belongs to $k \times n$, if $k > n$ then the system $Ax = b$ is an over-determined system of equation. In principle, Equation (18) will have a solution given by the least-squares method as

$$x = A^+ b = [(A^T A)^{-1} A^T]\, b \tag{19}$$

where A^+ denotes the *pseudo-inverse* (PI) of matrix A. The solution vector x is called PI solution. This definition requires A to have full rank. A^+ is an $n \times k$ array that is unique. If matrix A is square and non-singular then $A^+ = A^{-1}$, Equation (18) becomes a linear equation system, and the force vector x can be directly found by solving Equation (18). If matrix A is singular, Equation (18) is ill-posed and the elements of the solution vector x will be sensitive to small changes in both matrix A and vector b. If matrix A is close to rank deficient then A^+ is best calculated from the singular value decomposition (SVD) of A [12].

The SVD technique, applied to structural dynamics problems in the last fifteen year, is one of the most important tools in numerical analysis [13]. If matrix A is real, the SVD of A is USV^T, its inverse can easily be calculated from $A^+ = VS^{-1}U^T$. For simplicity, assuming that A has no exact zero singular values, it can be shown that the least-squares solution vector x is given by

$$x = \sum_{i=1}^{\min(k,n)} \frac{u_i^T b_i}{\sigma_i} v_i \tag{20}$$

The solution vector x here is called SVD solution. Equation (20) clearly illustrates the difficulties associated with standard matrix solutions of Equation (18). If the numerator does not decay as fast as the singular value σ_i of the denominator, the solution is dominated by terms containing the smallest σ_i. Consequently, the solution x may have many sign changes and thus appears to be random. When A is rank deficient, only the r $(r \leq \min(k,n))$ non-zero singular values of the matrix are taken into account so that S is a $r \times r$ matrix where r is the rank of A. To make the multiplication of Equation (20) conformable, the first r columns of V and the first r columns of U in Equation (20) are used.

3 EXPERIMENTS

Both the model car and model bridge deck were constructed in laboratory. An Axle-Spacing-to-Span-Ratio (ASSR) is defined as the ratio of the axle spacing between two consecutive axles of a vehicle to the bridge span length. Here, the ASSR was set to be 0.15. The model car had two axles at a spacing of 0.55 m and it ran on four rubber wheels. The static mass of the whole vehicle was 12.1 kg in which the mass of rear wheel was 3.825 kg. The model bridge deck consisted of a main beam, a leading beam and a trailing beam as shown in Figure 2. The main beam with a span of 3.678 m long and $101\,mm \times 25\,mm$ uniform cross-section, was simply supported. It was made from a solid rectangular mild steel bar with a density of 7335 kg/m^3 and a flexural stiffness $EI = 29.97kN/m^2$.

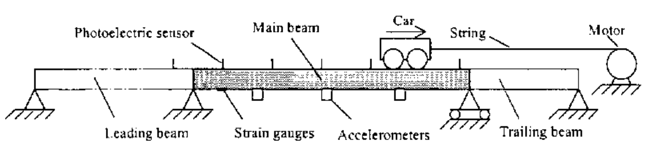

Figure 2. Experimental setup in laboratory

A U-shape aluminum track was glued to the upper surface of the main beam as a guide way for the model car, which was pulled along by a string wound around the drive wheel of an electric motor. The speed of the motor could be adjusted. Seven photoelectric sensors were mounted on the beams to measure and check the uniformity of moving speed of the model car. Seven equally spaced strain gauges were mounted on the lower surface of the main beam to measure the bending moment response. A system calibration of the strain gauges was carried out before the actual testing program by adding masses at the middle of the main beam. A 14-channel tape recorder was employed to record the response signals. The software Global Lab from the Data Translation was used for data acquisition and analysis in the laboratory test. Before exporting the measured data in ASCII format for identification, the Bessel IIR digital filter with lowpass characteristics was implemented as cascaded second-order systems. The Nyquist fraction value was chosen to be 0.05.

4 ERROR STUDIES

In the moving force identification, many parameters affect the identification accuracy, such as the sampling frequency, the mode number, the speed of the vehicle, the measuring stations and locations. The error study is aimed to investigate the effects of the change of these parameters on the TDM and FTDM by means of comparing the SVD solution with the PI solution.

4.1 Definition of Error

In practice, the parameters were studied one at a time. The procedure was to examine each parameter in cases and to isolate the case with the highest accuracy for the corresponding parameter. The accuracy is quantitatively defined as follows, which is called a relative percentage error (RPE).

$$RPE = \frac{\sum |f_{true} - f_{ident}|}{\sum |f_{true}|} \times 100\% \tag{21}$$

Since the true forces are unknown, Equation (21) is not practical. The true force (f_{true}) and identified force (f_{ident}) are here replaced by the measured response $(R_{measured})$ and rebuilt response $(R_{rebuilt})$ respectively. The RPE between the measured and rebuilt responses are calculated instead of comparing the identified forces with the true forces directly. In the present error studies, the results were based on the measured bending moments. The maximum acceptable RPE value adopted here is 10% [11].

4.2 Effect of Sampling Frequency

In the laboratory experiment, all the responses were acquired at a sampling frequency of 1000 Hz per channel. To obtain new sequential data samples at a lower sampling frequency, the sequential data samples acquired at 1000 Hz were sampled again at a few intervals. New sequential data at the sampling frequencies of 333, 250 and 200 Hz would be obtained by sampling the data again at every third, fourth and fifth point respectively. Because there is a computer memory problem in the computation of the inverse of a large matrix, the maximum sampling frequency is limited to be within 500 Hz, the sampling frequencies of 200, 250 and 333 Hz were set here. The case used here is for mode number MN=5, the sensor number is seven and the car speed is 15 $Units$ (1 $Unit \cong 0.102\ m/s$).

f_s	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
333	4.62	2.62	1.68	1.91	1.78	2.58	3.08
	4.63[1]	2.63	1.68	1.95	1.80	2.58	3.08
250	5.43	3.09	1.80	2.56	1.94	3.43	4.83
	5.42	3.08	1.80	2.58	1.95	3.44	4.83
200	8.55	4.00	2.63	3.71	2.99	4.72	9.54
	8.55	4.00	2.62	3.72	2.99	4.72	9.55

[1]Underlined data are for the PI solution, the same applies to the following tables.

TABLE 1. Effect of sampling frequency on TDM

Table 1 shows the identification accuracy that is acceptable for the TDM under all cases. The higher the sampling frequency, the lower are the RPE values for all stations. The TDM is suitable for the higher sampling frequency case. It can be observed from Table 1 that there is almost no difference for the RPE data at each station using whether the SVD or the PI technique.

f_s	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
333	4.51	2.53	1.87	1.95	1.82	2.25	3.17
	205	167	157	148	154	162	187
250	4.53	2.52	1.87	1.95	1.82	2.24	3.17
	5.74	2.80	2.15	2.08	2.14	2.41	4.74
200	4.51	2.53	1.87	1.95	1.82	2.25	3.17
	4.58	2.54	1.87	1.98	1.83	2.26	3.18

TABLE 2. Effect of sampling frequency on FTDM

For the FTDM, Table 2 clearly shows the comparison of the identified results by both the SVD and PI solution. When using the PI, the effect on the identification accuracy increases with increase in the sampling frequency. When the sampling frequency is 333 *Hz*, the FTDM failed. However, when decreasing the sampling frequency to 250 *Hz*, further to 200 *Hz*, the FTDM is acceptable. This shows that the FTDM is suitable for a lower sampling frequency when the PI technique is used to solve the equation. Significantly, the identified results by the SVD are clearly different from that by the PI. They are all acceptable and almost constant at each station for the different sampling frequencies. This means that use of the SVD technique is independent to the sampling frequency, and use of the SVD can effectively improve the identification accuracy, especially when the sampling frequency is in the highest case 333 *Hz*.

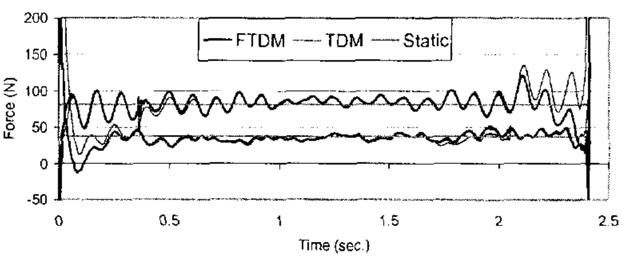

Figure 3. Comparison of identified forces

Comparing Table 2 with Table 1, it can be found that the RPE data due to the SVD are much closer when the highest sampling frequency $f_s = 333\ Hz$ is used. Figure 3 shows the comparison of the identified forces by the TDM and FTDM respectively when the SVD is used. It clearly illustrates the identified results are feasible and in a good agreement with each other, especially during both the two axles of the car are on the beam.

4.3 Effect of Mode Number (MN)

The case $f_s = 250Hz$, $c = 15\ Units$ was chosen in this section. The mode number was varied from three to seven. Table 3 lists the RPE values by the TDM using the SVD. The results using the PI are not listed here because the accuracy is almost the same as that using the SVD. When MN=3, the identified results are acceptable except there are bigger RPE values than 10% at the 1st and 7th stations. If the mode number is larger than 3, the RPE values increase gradually with an increase in the mode number. If the mode number increases up to MN=7, the RPE data at the 1st and 7th station are bigger than 10% again. This shows that the TDM is unstable using whether the SVD or the PI.

MN	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
3	11.6	3.36	5.03	2.66	5.23	3.72	12.2
4	5.80	3.33	1.89	2.85	1.98	3.76	5.76
5	5.43	3.09	1.80	2.56	1.94	3.43	4.83
6	8.09	3.79	2.57	2.42	2.75	4.65	7.12
7	14.3	6.46	6.89	3.95	6.58	5.38	11.0

TABLE 3. Effect of mode number (MN) on TDM

MN	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
3	12.1	3.51	5.26	2.61	5.21	3.70	12.9
	Fail	Fail	Fail	Fail	Fail	Fail	Fail
4	5.88	3.24	2.03	2.64	1.97	3.55	5.83
	13.6	7.19	5.69	5.93	5.55	6.75	12.9
5	4.53	2.52	1.87	1.95	1.81	2.24	3.17
	5.74	2.80	2.15	2.08	2.14	2.41	4.74
6	3.74	2.57	1.77	1.95	1.58	1.62	2.06
	4.91	2.72	1.89	2.05	1.73	1.87	3.45
7	3.69	2.36	1.44	1.55	1.15	1.32	1.82
	4.79	2.44	1.54	1.73	1.39	1.57	3.07

TABLE 4. Effect of mode number (MN) on FTDM

The results by the FTDM are given in Table 4. They show that the RPE values decrease slightly with an increase in the mode number using whether the SVD or the PI technique. Further, comparing the data at each station, it can be found that the SVD result is clearly improved in identification accuracy, particular for the lower mode number cases, says MN=3 and MN=4. When MN=3 and using the PI technique, the FTDM failed, but using the SVD the identified results are acceptable except there are bigger RPE values than 10% at the 1st and 7th stations. When MN=4, using the SVD can effectively improve the identification accuracy more than 50% at each station. These show using the SVD technique to solving the system equation can clearly improve the FTDM.

If the mode number is less than 3, both the RPE values and the identified forces become badly terrible and both the TDM and FTDM fail to identify the two moving forces. The above fact shows the two methods are effective only if the required mode number is achieved or exceeded but otherwise fail. Further, using the SVD can effectively improve the identification accuracy of the two methods, especially for the FTDM.

4.4 Effect of Vehicle Speed

Three vehicle speeds were set manually at 5,10 and 15 *Units* respectively in the experiments. If the speed was stable, the experiment was repeated five times for each speed case. The average speed of the vehicle on the whole beam is used to identify the moving forces in the TDM and FTDM. If the mode number, the sampling frequency and bridge span length are not changed in this case, a change of the vehicle speed would mean a change of the data samples, which will in turn change the dimensions of matrix A in Equation (18). Therefore, in order to make the TDM and FTDM effective and to analyze the effects of various vehicle speeds on the identified results, the case with MN=4, $f_s = 200\ Hz$ was selected. The RPE values are calculated and tabulated in Table 5 for the TDM and Table 6 for the FTDM under cases 5-2, 10-4 and 15-2. Where, case "5-2" means the *second* set of data was recorded when the vehicle moved across the bridge at the speed *5 Units*. Others are similarly identified. The data in Table 5 show that the TDM is effective for all three various vehicle speeds. The RPE data at each station are almost same under each speed using whether the SVD or the PI technique. Although the change in the RPE value is not so significant, the RPE values tend to be reduced with the increase in the vehicle speed, especially for those at the middle measuring stations.

Speed	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
5-2	5.18	3.46	2.87	3.30	2.78	4.20	5.75
	5.23	3.48	2.88	3.31	2.80	4.22	5.78
10-4	5.89	2.66	2.95	3.19	2.76	3.91	7.38
	5.89	2.66	2.95	3.20	2.76	3.91	7.38
15-2	6.72	3.31	2.43	2.99	2.58	3.96	6.29
	6.71	3.30	2.42	3.01	2.58	3.96	6.29

TABLE 5. Effect of vehicle speed on TDM

Speed	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
5-2	23.7	13.0	6.79	7.90	6.93	12.4	26.4
	Fail	Fail	Fail	Fail	Fail	Fail	Fail
10-4	23.3	12.6	7.99	8.32	7.79	11.8	26.3
	110.	50.0	25.9	48.2	24.8	47.6	102.
15-2	5.87	3.24	2.04	2.65	1.98	3.55	5.84
	5.94	3.29	2.05	2.66	2.01	3.57	5.89

TABLE 6. Effect of vehicle speed on FTDM

The data in Table 6 show that the FTDM failed to identify the forces when the PI technique is adopted and the vehicle speed is lower, says 5 *Units*. But the identified results are getting better and better when the vehicle speed increases. Fortunately, the identified result is acceptable at last in the case of 15 *Units*. However, once using the SVD the situation is completely changed. It predicts that it is better to use the SVD technique to solve the system equation so that the FTDM can be effective and can identify the moving forces with a higher accuracy. Besides, these results also show that the identification accuracy at a faster vehicle speed is higher than that at a lower vehicle speed for both TDM and FTDM.

4.5 Effect of Measuring Station

The station number N_l was set to 3, 4, 5, and 7 respectively while the other parameters MN=5, $f_s = 250\ Hz$, $c = 15\ Units$ were not changed for all study cases in this section. The RPE values by using the SVD and the PI technique are given in Table 7 for the TDM and Table 8 for the FTDM, respectively. When using the PI technique, the underlined RPE data show that the TDM requires at least three, best having four measuring stations to obtain the two correct moving forces. However, the FTDM should have at least one more measuring station than using the TDM, i.e. 4, to obtain the same number moving forces. However, the RPE errors are increased obviously when the measuring station number is equal to 5 for the FTDM. This is because the addition of the fifth station is placed on the $1/2L$ point, which is the node of the second and fourth modes of the supported beam. Nevertheless, when $N_l = 7$, i.e. put two more stations at the $1/8L$ and $7/8L$ respectively, the RPE values by the FTDM recover normal level to within 10%. It indicates that the FTDM is sensitive to the locations of measuring station, and they should be selected carefully. In general, for the TDM and FTDM, the identification accuracy increases with increase in the measuring station number, but if the increased station is put on any node of modes, it will make the identified results worse, especially for the FTDM.

N_l	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
3	*	*	1.26	1.73	1.39	*	*
	*	*	15.8	13.4	8.33	*	*
4	*	1.52	1.42	*	1.82	1.59	*
	*	1.48	1.44	*	1.85	1.56	*
5	*	2.17	1.82	2.63	1.95	1.95	*
	*	2.17	1.80	2.67	1.94	1.98	*
7	5.43	3.09	1.80	2.56	1.94	3.43	4.83
	5.42	3.08	1.80	2.58	1.95	3.44	4.83

Asterisk * indicates the station is not chosen.

TABLE 7. Effect of measuring stations on TDM

N_l	RPE (%)						
	Sta.1	Sta.2	Sta.3	Sta.4	Sta.5	Sta.6	Sta.7
3	*	*	1.24	1.38	1.05	*	*
	*	*	70.2	77.3	73.9	*	*
4	*	1.67	1.50	*	1.51	1.68	*
	*	2.90	2.09	*	2.11	2.74	*
5	*	2.14	1.71	1.77	1.71	1.63	*
	*	34.6	17.3	34.7	16.7	32.9	*
7	4.53	2.52	1.87	1.95	1.81	2.24	3.17
	5.74	2.80	2.15	2.08	2.14	2.41	4.74

TABLE 8. Effect of measuring stations on FTDM

According to the data in Table 7, when the station number is equal to or bigger than four, there is a little difference in the RPE values for the TDM, using whether the SVD or the PI. When the station number equals to three, using the SVD can dramatically improve the identification accuracy. Same situation also appears in Table 8 for the FTDM when the station number equals to three and five. Obviously, when $N_l = 3$ and $N_l = 5$, the FTDM is failed to identify the two moving forces if using the PI to solve the system equation. However, if using the SVD the FTDM can effectively identify the two moving forces and the identification accuracy is feasible and acceptable. This shows that both the TDM and FTDM can effectively identify the two moving forces when using the SVD technique to solve the over-determined system equation. Particular for the FTDM, it is really important to adopt the SVD to solve the system equation.

5 CONCLUSIONS

In this paper, the time domain method (TDM) and frequency-time domain method (FTDM) are presented to identify the axle loads of a moving vehicle on bridges. A bridge-vehicle system model is made in laboratory for purpose of moving force identification. A series of experiments on measurement of bending moment responses caused by the vehicle moving across the bridge are conducted. Error studies on moving force identification have been carried out. The effects of bridge-vehicle system parameters have been investigated. The singular value decomposition (SVD) solution and the pseudo-inverse (PI) solution used in both the time domain method and the frequency-time domain method have been compared. The following conclusions are drawn: 1) Both the TDM and FTDM are successful in the moving force identification from the measured bending moment responses caused by the vehicle moving across a bridge. 2) The effects of the change of bridge-vehicle system parameters on the TDM and FTDM are obvious and dependent on the solution to the over-determined system equation. 3) The use of the SVD technique can effectively improve the identification accuracy for both the TDM and the FTDM, particular for the FTDM. 4) The SVD technique is recommended as the first solution in over-determined system equation involved in both the TDM and FTDM.

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