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Formulation of an uncertainty model relating modal parameters and environmental factors by using long-term monitoring data

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ABSTRACT

From the point of view of structural health monitoring, it is extremely important to discriminate alteration in structural behavior/response attribute due to damage from that due to environmental and operational fluctuation. In this paper, the correlation between natural frequencies and temperature is investigated for the cable-stayed Ting Kau Bridge by using measurement data from a long-term monitoring system installed on this bridge. One-year continuously acquired data from 45 accelerometers (a total of 67 channels) and 83 temperature sensors are used for this study. The data from 20 temperature sensors at the locations susceptible to temperature are first selected for the correlation analysis. Natural frequencies of the first 10 modes are identified by spectral analysis of the acceleration data at one-hour intervals. In order to ensure the identification accuracy, the natural frequency for a specific mode is determined using only the data from the accelerometers which produce large spectral peaks at that mode. The identified natural frequencies for each hour are used to correlate with the one-hour average temperatures measured from the 20 sensors during the same time. Based on the one-year measurement data which cover a full cycle of varying environmental and operational conditions, a four-layer perceptron neural network with 20 input nodes and 1 output node is trained for each mode to represent the relation between the measured temperatures (input) and the corresponding natural frequency (output). The configured neural networks for the 10 modes show excellent capabilities for mapping between the temperatures and natural frequencies for all the one-year measurement data.

Keywords: cable-stayed bridge, on-line monitoring, environmental variability, temperature, modal frequency

1. INTRODUCTION

The vibration-based structural health monitoring approach uses measured change in modal parameters to evaluate change in physical properties that may indicate structural damage or degradation. In reality, however, a civil structure is subjected to varying environmental and operational conditions such as traffic, humidity, wind, solar-radiation and most important, temperature. These environmental effects also cause changes in modal properties which may mask the changes caused by structural damage. Environmental variability in modal parameters must be considered for reliable performance of modal-based damage detection algorithms. It is necessary to discriminate abnormal changes in dynamic features resulting from structural damage from normal changes due to the natural variability for avoiding false positive alarming in health monitoring and for accurate identification of damage location and severity.

Considerable research efforts have been devoted to investigating the variability in modal frequencies of bridges caused by environmental conditions. In order to distinguish between normal structural changes arising from environmental causes – most notably temperature – and abnormal structural changes due to damage, Roberts and Pearson [1] made field measurements for a nine-span box girder bridge over a twelve month period with attempts to understand and hence isolate temperature effect on the modal frequencies. Abdel Wahab and De Roeck [2] conducted dynamic tests for a prestressed concrete bridge in the spring and winter respectively and a change of 4 to 5 % in the natural frequencies was observed. Farrar et al. [3] and Cornwell et al. [4] studied the variability of modal properties of the Alamosa Canyon Bridge caused by different environmental factors. Based on the measurement data from the Alamosa Canyon Bridge, Sohn et al. [5] subsequently proposed a linear adaptive model to discriminate the changes of modal parameters due to temperature changes from those caused by structural damage or other environmental effects. Alampalli [6] conducted several tests over nine months on a steel-stringer bridge with concrete deck to examine the sensitivity of measured modal parameters with respect to variations resulting from test and in-service environmental conditions. Lloyd et al. [7] presented the correlation of modal frequencies with seasonal temperature variations during a seven month period of observation for a prestressed segmental concrete bridge and documented the temperature sensitivities for the first four

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vertical modes. Rohrmann et al. [8] studied in detail the thermal effects on modal frequencies of an eight-span prestressed concrete bridge by the use of three-year continuous monitoring data in an attempt to establish functional proportionality between the temperature variations and changing natural frequencies. Peeters and De Roeck [9] reported one-year monitoring of a four-span post-tensioned concrete box girder bridge and presented a method to distinguish normal modal frequency changes due to environmental effects from abnormal changes due to damage. Most of the above investigations indicated that the most significant source of modal variability was temperature.

All the reported studies addressing this issue were conducted on general highway bridges. In this paper, the effects of environmental temperature on modal frequencies of the cable-stayed Ting Kau Bridge are investigated by using measurement data from a long-term monitoring system installed on the bridge. Based on one-year continuously collected data from 45 accelerometers (a total of 67 channels) and 20 temperature sensors, the natural frequencies of the first ten modes of the bridge are identified at one-hour intervals and correlation with the corresponding temperatures at different bridge locations is obtained. Ten multi-layer perceptron neural networks, each representing one of the ten modes, are then trained by using the measurement data to correlate the 20 temperature measurements with the corresponding modal frequencies. The configured neural networks will act as non-parametric models for the use of removing or isolating the temperature-related normal variations of natural frequencies in modal-based structural damage identification. Some methods are already available for eliminating or explicitly taking into account the environmental variability in damage identification in terms of parametric or non-parametric models [10-12].

2. TING KAU BRIDGE AND SENSORY SYSTEM

The Ting Kau Bridge is a 1177 m long cable-stayed bridge with two main spans of 448 m and 475 m respectively, and two side spans of 127 m each [13]. A unique feature of the bridge is its arrangement of three single-leg towers which are strengthened by longitudinal and transverse stabilizing cables. As part of a sophisticated long-term monitoring system devised by the Hong Kong SAR Government Highways Department, over 230 sensors have been permanently

Sensor number	Sensor name	Description	Location				
Deck steel at chain	age 12217.5m (L)		1				
3	P1GLE01C	East side deck	Deck 2 side				
6	P1GLE02C	East side deck	Under deck				
9	P1GLW01C	West side deck	Deck 2 side				
12	P1GLW02C	West side deck	Under deck				
Air temperature							
16	P3GHT01	North side outer tower	Main Tower +73m				
22	P3GHT07	South side outer tower	Main Tower +143m				
24	P3GLE01	Chainage L	Under east side deck				
25	P3GLE02	Chainage L	Above east side deck				
Tower concrete at chainage 11953.0m (H)							
30	P4GHT01C	North side inner tower	Tower +73m				
33	P4GHT02C	East side inner tower	Tower +73m				
36	P4GHT03C	South side inner tower	Tower +73m				
39	P4GHT04C	West side inner tower	Tower +73m				
Deck concrete at chainage 12217.5m (L)							
55	P4GLE01D	East side deck	Above deck				
63	P4GLE03D	East side deck	Above deck				
67	P4GLW01D	West side deck	Above deck				
75	P4GLW03D	West side deck	Above deck				
Deck asphalt at cha	ainage 12217.5m (L)						
77	P5GLE01B	East side deck	Above deck				
79	P5GLE01D	East side deck	Above deck				
81	P5GLW01B	West side deck	Above deck				
83	P5GLW01D	West side deck	Above deck				

Table 1. Temperature sensors selected for correlation analysis

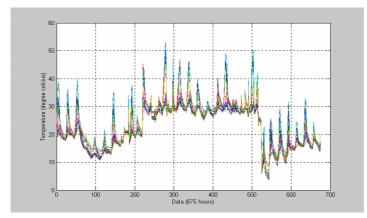


Figure 1: Time histories of temperature records from 20 selected temperature sensors

installed on the bridge immediately after completion of its construction [14, 15]. The sensors include accelerometers, strain gauges, displacement transducers, anemometers, temperature sensors, weigh-in-motion sensors, and recently deployed global positioning system (GPS) [16]. A total of 83 temperature sensors are installed at different positions of the bridge to measure steel-girder temperature, temperature inside concrete deck and tower legs, temperature in asphalt pavement, and air temperature. For the dynamic response measurement, twenty-four uni-axial accelerometers, twenty bi-axial accelerometers and one tri-axial accelerometer (totally 67 accelerometer channels) are permanently deployed at the deck of the two main spans and the two side spans, the longitudinal stabilizing cables, the top of the three towers, and the base of central tower to monitor seismic excitation and dynamic response of the bridge.

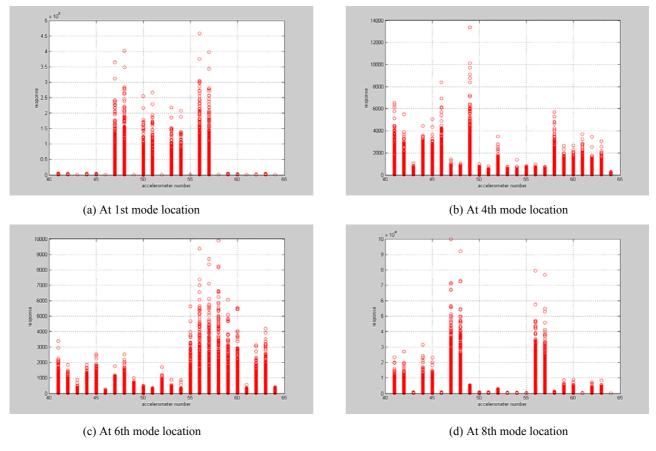


Figure 2: Auto-spectrum peaks at modal frequency locations

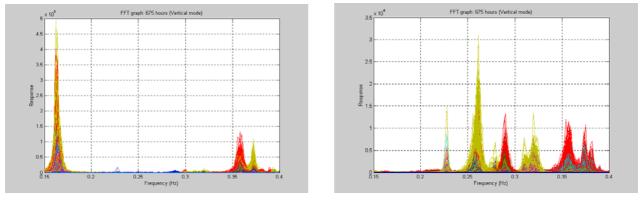
One-year continuous measurement data from all sensors in the bridge have been collected to establish a database in the Hong Kong Polytechnic University for validating a baseline finite element model and observing variability of the modal parameters due to different environmental factors (temperature, wind and traffic). The present study explores the correlation of modal frequencies with temperature from the measurement data. For each of the five temperature measurement categories (steel-girder, concrete deck, tower legs, asphalt, and air), only the data from four sensors which indicate most noticeable temperature variations at the measurement positions are utilized. As a result, a total of 20 temperature sensors as shown in Table 1 are picked up to provide temperature data for the correlation analysis. Figure 1 illustrates the time histories of temperature records from the 20 selected temperature sensors.

The first ten modes of the bridge are firstly determined and classified by applying a CMIF-based modal parameter identification program [17] to several-day data of all the accelerometers in the bridge. Then the natural frequencies are identified at one-hour intervals through auto-spectrum analysis of the data from a few (2 to 6) accelerometers for each mode. For different modes, the selected accelerometers to provide data for spectral analysis are different. Figure 2 illustrates the auto-spectrum peaks at the modal frequency locations obtained from different measurement points (due to space limitation only the 1st, 4th, 6th and 8th modes are plotted). For a specific mode, only the data from the sensors which produce large spectral peaks at the corresponding frequency location are used for frequency identification. In this way, the identification accuracy of modal frequencies is ensured.

3. IDENTIFICATION RESULTS OF MODAL FREQUENCIES

Spectral analysis for each one-hour acceleration data is conducted with the following parameters: sampling rate in the time domain: 25.6 Hz; frequency resolution in spectrum: 0.00139 Hz; number of data for each FFT: 92160; overlap in FFT: 75%; number of FFT averages: 17; window: Hanning. Figure 3 illustrates the auto-spectra obtained from the vertically and laterally oriented accelerometers at different times. It is observed that the spectral peaks 'move' as the environmental conditions change. Figure 4 shows the variations in measured modal frequencies from the vertically and laterally oriented accelerometers, respectively. Because of adopting extremely long longitudinal stabilizing cables (up to 465 m), slender monoleg towers and separated deck system, the bridge exhibits many modes with coupled modal components in vertical and lateral directions. As a result, the natural frequencies of these coupled modes can be obtained from both vertically and laterally oriented accelerometers. Figure 5 gives a comparison of the identified modal frequencies obtained from six accelerometers for two modes (the 3rd and 6th modes respectively). It is observed that for each mode the modal frequencies identified from different accelerometers exhibit considerable consistence.

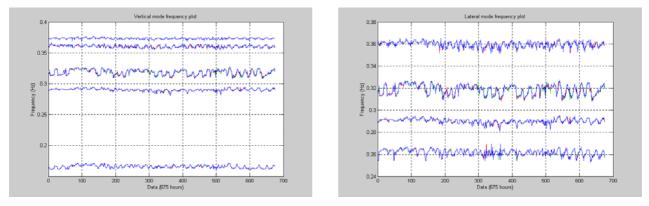
Table 2 shows statistical information of the modal frequencies measured from one-year data representing a full cycle of in-service/operating conditions. All the ten identified natural frequencies are less than 0.4 Hz, indicating closely spaced modes in this bridge. Apart from the first mode being an almost purely vertical mode, the other nine modes are all coupled modes with notable modal components in both vertical and lateral directions. The variance error of the measured natural frequencies for the ten modes is between 0.20% and 1.52%. The maximum relative discrepancy occurs at the first mode. Because the fundamental modal frequency (0.166 Hz) of the Ting Kau Bridge is extremely low (the corresponding frequency of the world's longest cable-stayed Tatara Bridge is 0.199 Hz [18]), the relative error of



(a) From vertically oriented accelerometers

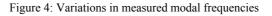
(b) From laterally oriented accelerometers

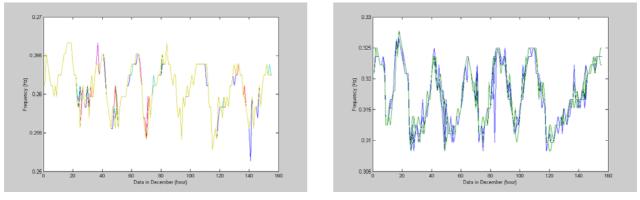
Figure 3: Auto-spectra of acceleration responses obtained at different times



(a) From vertically oriented accelerometers

(b) From laterally oriented accelerometers





(a) 3rd modal frequency

(b) 6th modal frequency

Figure 5: Comparison of measured modal frequencies obtained from six accelerometers

Mode	Frequency Range (Hz)	Average Frequency (Hz)	Standard Deviation (Hz)	Variance	Remark	
1	0.155-0.175	0.16605	0.00253	1.52%	Predominantly vertical mode	
2	0.221-0.232	0.22744	0.00081	0.36%	Coupled torsional & lateral mode	
3	0.250-0.271	0.26193	0.00262	1.00%	Predominantly lateral mode	
4	0.279-0.296	0.29029	0.00209	0.72%	Coupled lateral & torsional mode	
5	0.294-0.306	0.29983	0.00085	0.28%	Predominantly vertical mode	
6	0.308-0.328	0.31861	0.00450	1.41%	Coupled torsional & lateral mode	
7	0.350-0.365	0.35973	0.00267	0.74%	Coupled lateral & torsional mode	
8	0.351-0.367	0.36039	0.00212	0.59%	Predominantly vertical mode	
9	0.369-0.377	0.37733	0.00142	0.38%	Predominantly vertical mode	
10	0.380-0.390	0.38502	0.00074	0.20%	Predominantly torsional mode	

Table 2	Statistics	of identified	modal	frequencies
1 auto 2.	Statistics	of fuentified	mouar	nequencies

the measured natural frequencies for the first mode is thus large. The standard deviation given in Table 2 represents the absolute standard error (variability) of the measured natural frequencies. It is seen that the variability level is different for different modes. Some damage identification methods allow environmental variability/uncertainty to be accounted for as random error corrupted in the modal parameters. When using such a method, different variability levels obtained for at least one complete cycle of environmental/operational conditions as given in Table 2 should be incorporated in

the corresponding modes to achieve reliable damage detection. Figure 6 illustrates the distribution of the measured natural frequencies for different modes. It is observed that the measured natural frequencies do not meet the normal distribution very well. This may be due to non-uniform thermal conditions during the measurement period.

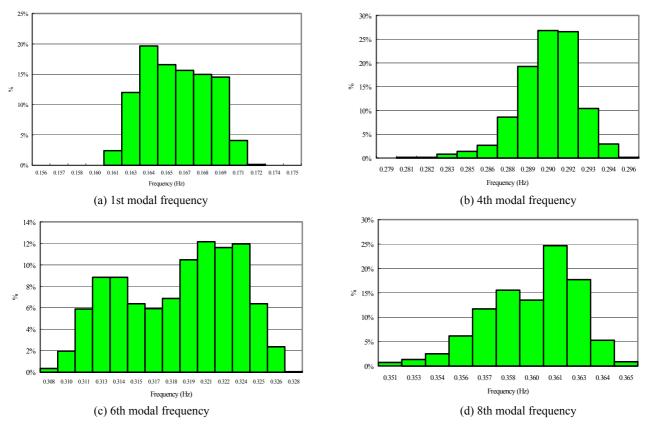


Figure 6: Distribution of measured modal frequencies

4. CORRELATION OF MODAL FREQUENCIES WITH TEMPERATURE

Natural frequencies of the first ten modes of the bridge have been identified at one-hour intervals and their correlations with the corresponding one-hour average temperatures from 20 selected temperature sensors are obtained as plotted in Figure 7. In these diagrams, the ordinate represents the averaged modal frequencies measured from different accelerometers for each one-hour interval, and the abscissa denotes the corresponding one-hour average temperatures respectively recorded from the 20 sensors. The temperature ranges from 5°C to 55 °C. For all the measured modes an overall decrease in modal frequencies is observed with the increased temperature of the bridge. But the rate of the inverse proportion is slightly different for different modes.

Although the effect of temperature on natural frequencies of the 4th mode looks reasonably linear as shown in Figure 7, it is obvious that changes in the modal frequencies reply primarily on the temperature variations, but are not completely due to temperature. Other environmental and operational factors, such as wind and traffic loading, also affect the modal properties. Because the Ting Kau Bridge is instrumented as well with anemometers for wind measurement and weight-in-motion systems for traffic loading measurement, it is possible to analyze correlation of the modal frequencies with all the temperature, wind and traffic. Based on simultaneously measured modal frequencies, temperature, wind and traffic loading, the principal component analysis and independent component analysis [19, 20] can be employed to find the maximum-variability components and extract unique characteristic features. Then a reduced-order multivariate model can be established to characterize modal uncertainty arising from all environmental and operational factors. Towards this direction, correlations between the modal frequencies and wind loading and between the modal frequencies and traffic loading are also under development by the authors with the use of measurement data.

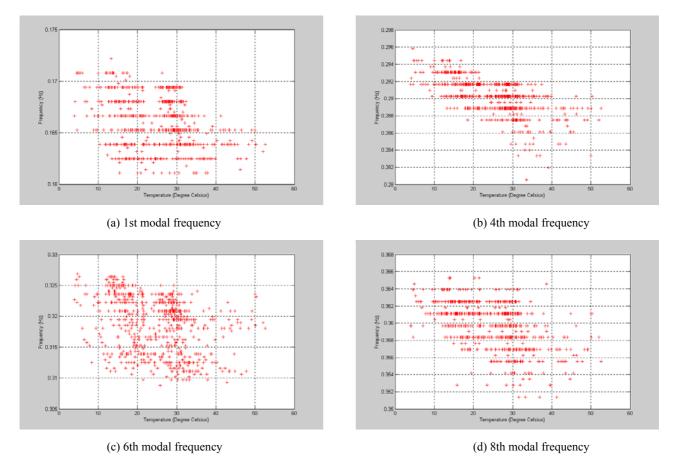


Figure 7: Measured modal frequencies versus temperature

5. NEURAL NETWORK MODELS OF ENVIRONMENTAL VARIABILITY

After obtaining the correlated measurement data of the modal frequencies and temperature, a total of ten neural networks, each representing one mode, are trained to generate mapping between the 20 measured temperatures (input) and the corresponding natural frequencies (output). Each neural network is configured as a four-layer feed-forward perceptron with the node structure 20-15-10-1. The twenty input nodes represent the 20 measured temperatures, and the one output represents the natural frequency of a specific mode. The activation functions are taken as the tan-sigmoid

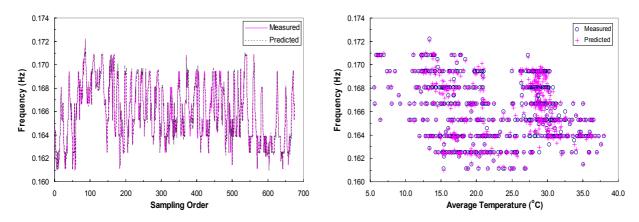


Figure 8: Comparison of measured and predicted results for 1st mode

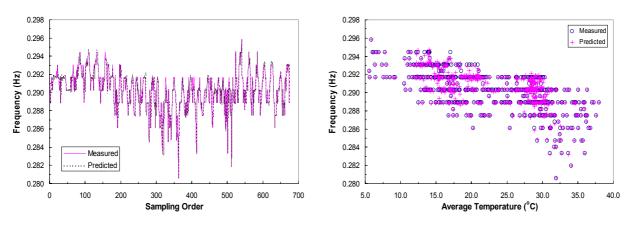


Figure 9: Comparison of measured and predicted results for 4th mode

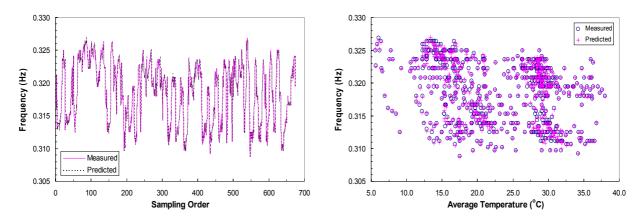


Figure 10: Comparison of measured and predicted results for 6th mode

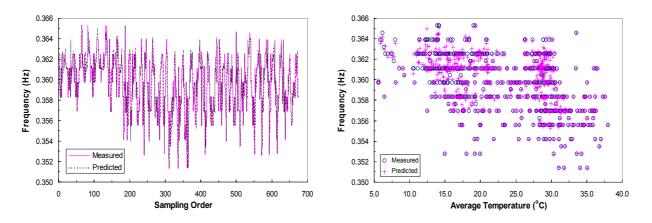


Figure 11: Comparison of measured and predicted results for 8th mode

function between the input and second layers and between the second and third layers, and the linear transfer function between the third and output layers. The number of hidden layers and the number of hidden nodes are determined by trial-and-error. All the one-year measurement data are used in training the neural networks. Before the training, the input datasets are normalized to the range of [-1, 1]. Back-propagation in terms of the Levenberg-Marquardt algorithm is used for the neural network training. It is found that for all the ten neural networks the training achieves a stable squared error in the level of 10^{-7} after 1500 iterations.

After completion of the training, all the measurement temperature data used in training are fed again into the trained neural networks to predict the corresponding modal frequencies. Figures 8 to 11 show a comparison of the measured and predicted natural frequencies for the 1st, 4th, 6th and 8th modes. The measured and predicted frequency sequences coincide very well for all the ten modes. In the diagrams of frequency versus temperature, the measured temperatures from the 20 sensors are averaged as the abscissa for clarity, while the measured and predicted modal frequencies are plotted as the ordinate. Excellent agreement is observed again for all the ten modes. These results validate strong capabilities of the configured neural networks for mapping between the temperatures and modal frequencies for all the one-year measurement data.

6. CONCLUSIONS

In this paper the change of modal frequencies with normal environmental conditions has been studied for the cablestayed Ting Kau Bridge based on one-year continuous measurement data. It is found that the normal variations in the bridge modal properties, due to environmental changes, show an inversely proportional trend in natural frequencies with temperature for all the ten measured modes. From one-year measurement data covering a full cycle of in-service/ operating conditions, it is found that the normal environmental changes can account for changes in modal frequencies of variance error 0.20% to 1.52% for the first ten modes. This level of variations may mask the modal changes caused by structural damage. It is also noted that the variations in modal frequencies due to environmental changes are in different levels for different modes. In feasibility study of vibration-based methods, the damage detectability can be accurately evaluated only when the true variability level, which may be different for different modes, has been incorporated in the identification methods.

Neural networks have been trained from the measurement data as non-parametric models to correlate the structural temperatures and the corresponding modal frequencies for the Ting Kau Bridge. The configured neural networks show good capabilities for mapping between the temperatures and modal frequencies for all the one-year measurement data. The well-defined nature of the temperature effect on the modal properties means that this effect can be eliminated or separated from the measurements, giving the potential for subtle structural damage to be detected. A thorough understanding of the environmental variability is necessary for discriminating changes in modal properties resulting from damage from changes resulting from such variability. For the Ting Kau Bridge and other structures instrumented with a wide variety of sensors, it is possible to quantitatively understand the effects of various environmental and operational factors (temperature, wind, traffic, etc.) on the modal properties, and a multivariate correlation analysis can formulate a variability model to simultaneously account for all the environmental effects. Such a variability model is extremely useful for vibration-based damage detection of real-world civil structures with uncertainty.

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