Multifractal nature of network induced time delay in networked control systems

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Abstract

When modelling and simulating networked control systems (NCSs) over TCP/IP network protocols, we obtained network traffic data sets with irregular behaviour. Analysing the data sets revealed multifractal network traffic. Typical data sets are given in this paper together with our preliminary analysis. The network architecture and traffic specifications that generated the multifractal traffic are also described in detail.

1 Introduction

Modern large-scale manufacturing and process control systems demand increasing integration of information, communication, and control. As a result, real-time control in these systems is implemented over communication networks, which are used to transmit measurement, control, and management signals [1–6]. This requires highly reliable, flexible, simple, and cost-effective network technologies to replace traditional peer-to-peer interconnection techniques.

Two challenging problems in analysis and design of networked control systems (NCSs) are network induced delays and packet dropouts [5,6]. Both problems

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can significantly degrade the NCS's performance. It has long been realised that network induced communication delay is time-varying and non-deterministic, suggesting that the delay behaviour is unpredictable. Packet dropout occurs when communication networks are unreliable or the communication latency is so big that the packet has to be purposely dropped.

The stochastic behaviour of network induced time delay in NCSs has been modelled and analysed using various techniques. However, direct evidence has not been presented in the open literature to support the fundamental assumption of the randomness of the behaviour of network induced time delay in NCSs. We recently proposed a new real-time communication protocol for NCSs and studied the statistical properties of network induced communication delays [5,6]. Our results have shown that network induced delay exhibits complicated behaviour with significant jitter, which must be compensated for in NCS control design for real-time applications. Our analysis in this paper reveals the multifractal nature of NCS network traffic under certain conditions.

On dynamics of network traffic, early work by Leland et al. investigated the self-similarity nature of Ethernet traffic [7]. Since then, self-similarity behaviour, i.e. long-term memory, has also been observed in several other types of network traffic in general network systems. However, this phenomenon has not been reported for networked control systems in which real-time requirements are essential, and will be addressed in this work for NCSs that employ our recently developed real-time communication protocol [5,6].

When modelling and simulating an NCS over a TCP/IP based communication network, we observed various network traffic. A typical data set is shown in Fig. 1, which depicts network induced communication delay over a certain period of time from sensors to the central control computer and then to actuators. The objective of this paper is to analyse the behaviour of the network induced delay as shown in Fig. 1.

In order to analyse the dynamics of the network traffic shown in Fig. 1, we treat the data set in Fig 1 as a time series. To study time series, Hurst [8] invented a new statistical method — *the rescaled range analysis* (R/S analysis); then Mandelbrot [9] and Feder [10] introduced R/S analysis of fractal records in time into fractal theory. R/S analysis has been applied to many areas in science and engineering. For example, Yu and Chen used R/S analysis to distinguish different DNA functional regions [11].

Multifractal analysis is a useful way to characterise the spatial inhomogeneity of both theoretical and experimental fractal patterns [12]. It was initially proposed to treat turbulence data, and has recently been applied successfully in many different fields including time series analysis [13,14], financial modelling [15], and biological problems [16–22]. Some sets of physical interest have

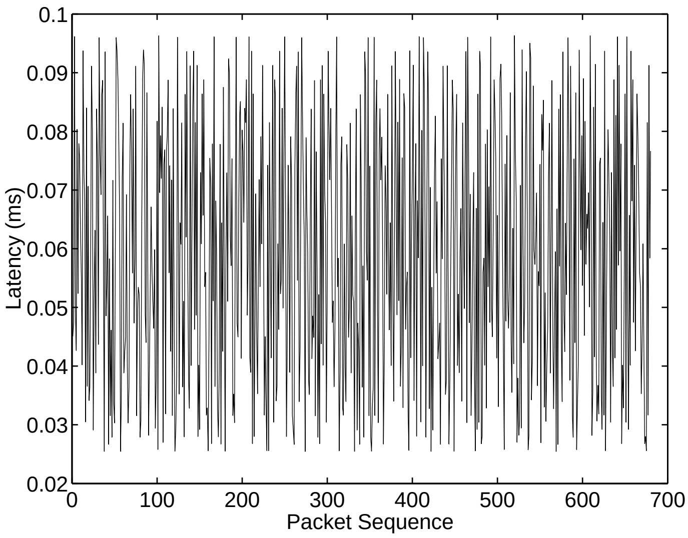

Fig. 1. Irregular Traffic of a TCP/IP based NCS.

a non-analytic dependence of the dimension spectrum D_q on the q-moments of the partition sum of the sequences. Moreover, multifractality has a direct analogy to the phenomenon of phase transition in condensed-matter physics [23]. The existence and type of phase transitions might turn out to be a worthwhile characterisation of universality classes for the structures [24]. The concept of phase transitions in multifractal spectra was introduced in the study of logistic maps, Julia sets, and other simple systems. Evidence of phase transition was found in the multifractal spectrum of diffusion-limited aggregation [25].

In the following, we will first discuss the theoretical background on R/S analysis and multifractal analysis in Section 2. Then, Section 3 will describe the NCS architecture and traffic specifications that generate complicated network traffic. In Section 4, we will focus on the analysis of the inherent complex dynamics of the network traffic shown in Fig. 1. Finally, Section 5 concludes the paper.

2 Theoretical Background on Time Series Analysis

2.1 R/S Analysis

Denote the dynamics of the network traffic shown in Fig. 1 as $x = \{x_k\}_{k=1}^{N}$, where N is the length of the sequence. This sequence can be treated as fractal records in time. Hurst invented the R/S analysis method to study such sequences [8]. Later, Mandelbrot [9] and Feder [10] further developed this method in fractal theory.

For any fractal records in time $x = \{x_k\}_{k=1}^N$ and any $2 \le n \le N$, define

$$< x >_n = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{1}$$

$$X(i,n) = \sum_{u=1}^{i} [x_u - < x >_n] \tag{2}$$

$$R(n) = \max_{1 \le i \le n} X(i,n) - \min_{1 \le i \le n} X(i,n) \tag{3}$$

$$S(n) = [\frac{1}{n} \sum_{i=1}^{n} (x_i - < x >_n)^2]^{1/2} \tag{4}$$

Hurst found that

$$R(n)/S(n) \sim (\frac{n}{2})^H \tag{5}$$

where H is called the *Hurst exponent.*

As n changes from m to N, we obtain $N-m+1$ points in $\ln(n)$ v.s. $\ln(R(n)/S(n))$ plane. Then, we can calculate the Hurst exponent for the time series using the least-square linear fit.

The Hurst exponent is usually used as a measure of complexity. The trajectory of the record is a curve with a fractal dimension $D = 2 - H$ [10, p. 149]. Hence a smaller H means a more complex system. When applied to fractional Brownian motion, if $H > 1/2$, the system is said to be *persistent*, which means that if for a given time period t the motion is along one direction, then in the time succeeding t it is more likely that the motion will follow the same direction. For a system with $H < 1/2$, the opposite holds, that is, the system is *antipersistent*. But when $H = 1/2$ the system produces Brownian motion, which is random.

2.2 Multifractal Analysis

First we define a measure from a positive time series as is done for the length sequence of a genome [26]. Let $T_t, \ t = 1, 2, \cdots, N$, be the time series. We define

$$F_t = T_t / (\sum_{j=1}^{N} T_j) \tag{6}$$

to be the frequency of T_t. It follows that $\sum_t F_t = 1$. Now we can define a measure μ on interval $[0,1]$ by $d\mu(x) = Y(x)dx$, where

$$Y(x) = N \times F_t, \quad \text{when} \quad x \in [\frac{t-1}{N}, \frac{t}{N}]. \tag{7}$$

It is easy to see that $\int_0^1 d\mu(x) = 1$ and $\mu([(t-1)/N, t/N]) = F_t$.

The most common numerical implementations of multifractal analysis are the so-called *fixed-size box-counting algorithms* [27]. In the one-dimensional case, for a given measure μ with support $E \subset \mathbf{R}$, we consider the *partition sum*

$$Z_\epsilon(q) = \sum_{\mu(B)\neq 0} [\mu(B)]^q, \tag{8}$$

$q \in \mathbf{R}$, where the sum runs over all different nonempty boxes B of a given side ϵ in a grid covering of the support E, that is,

$$B = [k\epsilon, (k+1)\epsilon]. \tag{9}$$

The scaling exponent $\tau(q)$ is defined by

$$\tau(q) = \lim_{\epsilon \to 0} \frac{\log Z_\epsilon(q)}{\log \epsilon} \tag{10}$$

and the generalized fractal dimensions of the measure are defined as

$$D_q = \tau(q)/(q-1), \quad \text{for } q \neq 1, \tag{11}$$

and

$$D_q = \lim_{\epsilon \to 0} \frac{Z_{1,\epsilon}}{\log \epsilon}, \quad \text{for } q = 1, \tag{12}$$

where $Z_{1,\epsilon} = \sum_{\mu(B)\neq 0} \mu(B) \log \mu(B)$. The generalized fractal dimensions are numerically estimated through a linear regression of

$$\frac{1}{q-1} \log Z_\epsilon(q)$$

against $\log \epsilon$ for $q \neq 1$, and similarly through a linear regression of $Z_{1,\epsilon}$ against $\log \epsilon$ for $q = 1$. D_1 is called the *information dimension* and D_2 the *correlation dimension*. The D_q of the positive values of q gives relevance to the regions where the measure is large. The D_q of the negative values of q deals with the structure and the properties of the most rarefied regions of the measure.

By following the thermodynamic formulation of multifractal measures, Canessa [14] derived an expression for the "analogous" specific heat as

$$C_q \equiv -\frac{\partial^2 \tau(q)}{\partial q^2} \approx 2\tau(q) - \tau(q+1) - \tau(q-1). \tag{13}$$

He showed that the form of C_q resembles a classical phase transition at a critical point for financial time series. Later, we discuss the property of C_q for the time series of network induced delay in real-time NCSs.

3 Network Architecture and Traffic Specifications

Now let us describe the architecture of the networked control system that we modelled and simulated [5]. A multilevel hierarchy was adopted in the NCS; from top to bottom are management computers, control computers, smart sensors and actuators, and the plant to be controlled. There are 30 smart sensors and 20 actuators, respectively, in the NCS. We used this setting to model a middle-scale industrial process or multiple small-scale industrial processes.

The interconnection of all devices of the NCS using Ethernet-based TCP/IP protocols is shown in Fig. 2. In this logical diagram, notations S1, S2, $\cdots$, and Sn represent n smart sensors, $n = 30$; and A1, A2, $\cdots$, and Am are smart actuators, $m = 20$. All smart sensors and smart actuators are connected to a switch. All hosts in the control computer area and the management computer area are connected to another switch. The two switches are interconnected to one another. Moreover, one control computer was used as the central controller, and other control computers are used for information processing or display. Our real-time communication protocol for NCSs [5] is embedded into this network architecture.

Control tasks are periodic with the control period being 200ms. We use C1 through C5 to represent 5 control computers; and assign C1 to be the central controller and C5 to be the control server, respectively. Furthermore, we use M1 through M5 to denote 5 management computers in the NCS. Traffic specifications of the NCS are summarised below in Table 1 [5].

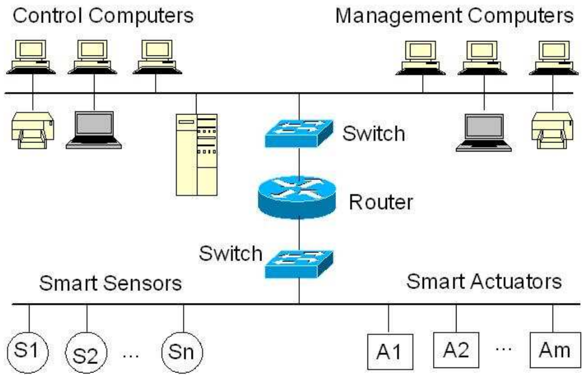

Fig. 2. Network architecture for the TCP/IP based NCS.

Table 1
Traffic flow specifications of the NCS.

No.	Traffic Flow		Flow Rate
TCP1.	0–50ms:	each S1 ∼ Sn ⇒ C1	200 bytes → 8kbps
TCP2.	0–50ms:	each S1 ∼ Sn ⇒ C5	200 bytes → 8kbps
TCP3.	100–200ms:	C1 ⇒ each A1∼Am	200 bytes → 8kbps
TCP4.	100–200ms:	C1 ⇒ C5	1k bytes → 40kbps
TCP5.	0–200ms:	C5 ⇒ each C2∼C4	2k bytes → 80kbps
TCP6.	0–5s:	each C2∼C4 ⇒ C5	1k bytes → 1.6kbps
TCP7.	0–5s:	C5 ⇒ each C2∼C4	10k bytes → 16kbps
TCP8.	0–10min:	each M1∼M5 ⇒ C5	60k bytes → 800bps
TCP9.	0–10min:	C5 ⇒ each M1∼M5	600k bytes → 8kbps
Packet Size (bytes) - TCP1,2,3,6: 200; TCP4,5,7,9: 1k; TCP8: 100			

4 Traffic Data Sets Analysis

We used the open source package ns2 under Unix [28] to simulate the NCS. All traffic flows over the network were monitored and recorded in a trace file, and network performance was then analysed by extracting information from the trace file.

Fig. 1 shows a plot of typical network induced communication delay, which was extracted from the trace file of our ns simulation. For the time series in this figure, we calculated the Hurst exponent. The graph of the R/S analysis of the delay time series is shown in Fig. 3.

Then, the generalized dimensions of the delay time series were computed. The D_q vs q curve is shown in Fig. 4. It is seen from this figure that the D_q

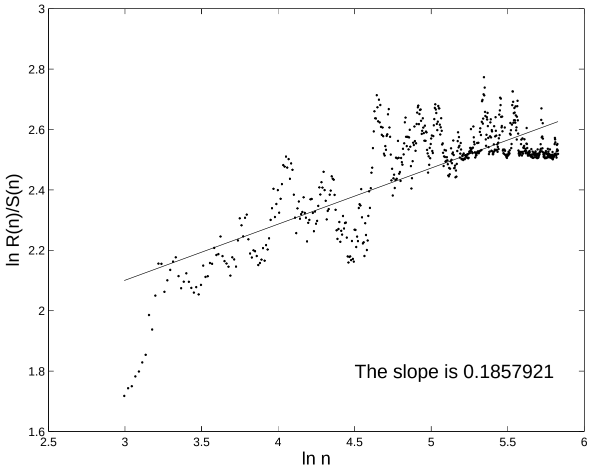

Fig. 3. R/S analysis of the data. $m = 40, N = 680$.

spectra is multifractal-like and sufficiently smooth for the C_q vs q curve to be meaningful. Depicted in Fig. 5 is the C_q vs q curve corresponding to D_q in Fig. 4. It can be seen from Fig. 5 that it resembles a classical phase transition at a critical point.

From the values of the Hurst exponent, D_q spectra and related C_q curve, it can be concluded that the network induced delay has multifractal nature and exhibits long-range correlation.

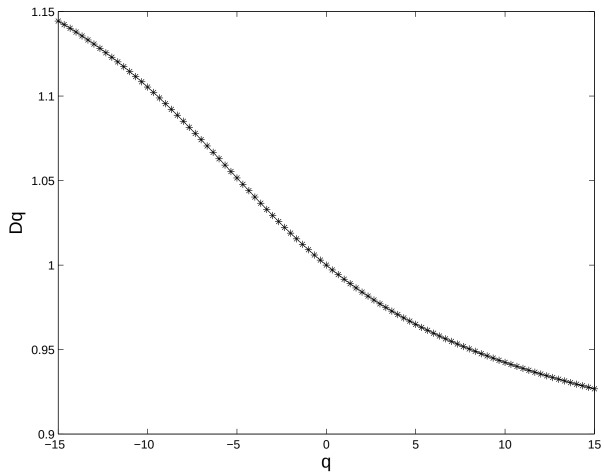

Fig. 4. Generalized dimensions of the data.

5 Conclusion

Network traffic observed from our modelling and simulations of real-time NCSs have been analysed using the techniques of R/S analysis and multifractal analysis. Our analysis results have shown that network induced communica-

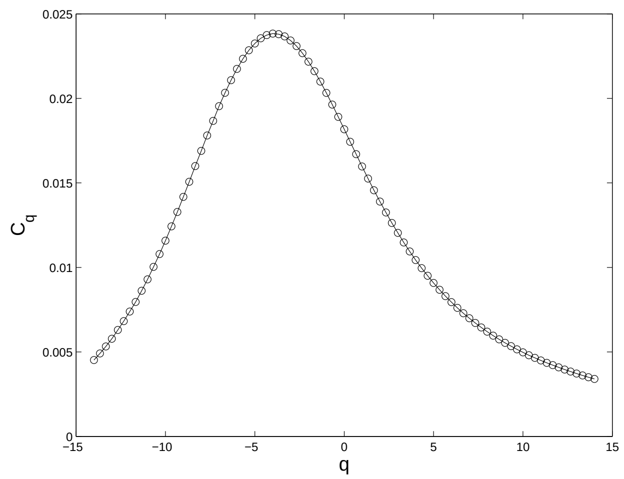

Fig. 5. “Analogous” specific heat of the data.

tion delay in the NCSs has multifractal nature. This implies that the communication delay is long-range correlated, and further suggests that the traffic irregularity we observed does not represent short-term randomness in the networked induced communication delay.

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