Modelling within mathematical online knowledge-building communities

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In this paper we present an alternative perspective on interactive and digital media learning environments (IDM) involving mathematical modeling. We argue that we need to move beyond using "add-on" and "up-grading" strategies to enrich learning, towards a focus on the generation of conceptual artifacts and the building of knowledge. To facilitate knowledge building, we advocate extending students beyond basic discourse to include a range of thinking and reasoning processes that can assist them in constructing, analyzing, refining, and conveying mathematical ideas. We also identify a number of knowledge-building enabling characteristics of an IDM-based learning environment and propose two sets of interrelated principles to inform the design of IDM environments and the accompanying ends-in-view activities. We illustrate our ideas with examples of online communities for mathematics education.

A review of the research literature (Attewell, 2005; Brown, 2005; Stead, Sharpe, Anderson, Cych & Philpott, 2006) and a Google search indicate that in many cases, learning environments based on interactive and digital media (IDM) are being used either as add-ons to existing curricula or as means to upgrade curricula to enrich learning. When an Add-On Strategy is utilized, the existing curriculum is kept in place, and activities situated in the IDM learning environments are used as supplements to the existing curriculum. When an Up-Grading Strategy is utilized, activities situated in IDM learning environments are utilized to modify the curriculum to gain greater depth, collaboration, learner autonomy, and constructive activity. We contend that these two strategies are limiting the impact on teaching and learning of environments based on IDM.

Therefore, like Scardamalia (2002), Bereiter (2002) and Lesh and Zawojewski (2007), we are advocating an alternative strategy where there is shift in the focus from simply completing tasks and activities to generating conceptual artifacts[1] (such as mathematical models and models for teaching and learning mathematics etc.), and

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[1] In simplistic terms, **conceptual artifacts** may be defined as "ideas treated as real things." Such artifacts are "human creations intended for some purpose….they have most properties of artifacts in general: they have histories, they can be described and compared, variously used and modified…When treated as real artifacts, ideas can be made objects of inquiry and development, can be adapted to novel purposes…In knowledge-based organizations, conceptual artifacts figure prominently as products and as tools (Bereiter & Scardamalia, 2006, p. 700). Conceptual artifacts help to explain and predict phenomena.

from learning as the goal to knowledge building as the goal. Knowledge building may be defined as the production and continual improvement of ideas of value to a community (Scardamalia & Bereiter, 2003). Participants within knowledge-building communities are engaged in the collaborative production of knowledge artifacts that can be discussed, tested, compared, and hypothetically modified; participants see their main job as producing and improving such artifacts, not simply the completion of tasks (Bereiter, 2002). These pursuits should advance the current understanding of individuals within a group, at a level beyond their initial level of knowledge.

In order for mathematical knowledge building to occur, it is important to extend students beyond basic discourse to include a focus on a range of thinking and reasoning processes that can assist them in constructing, analyzing, refining, and conveying mathematical ideas. Table 1 lists a selection of some of these processes.

Table 1. Knowledge building thinking and reasoning processes

Identifying distinctions and connections	Analyzing concepts and identifying relationships Drawing distinctions and connections among ideas Distinguishing between effective and ineffective cases
Reasoning by analogy	Reasoning with relational patterns Applying existing knowledge and understandings to solving new problem situations
Applying logical reasoning	Reasoning deductively and inductively
Thinking in diverse ways	Reasoning critically, creatively, and flexibly Exploring alternatives and different possibilities
Constructing	Constructing ideas, explanations, reasons, inferences, arguments Generating and testing hypotheses
Predicting	Anticipating, predicting, and exploring consequences
Evaluating	Taking all relevant considerations into account Formulating and applying criteria
Reasoning metacognitively	Acknowledging and respecting different perspectives and viewpoints Developing a commitment to the processes of inquiry and their improvement, including one's own thinking and reasoning processes

The shift in focus to knowledge building that we are advocating has both epistemological and practical implications for the design and implementation of learning environments based on IDM. Epistemologically, this focus extends definitions of understanding beyond traditional notions such as those that view understanding in terms of how one's knowledge corresponds to that of an expert (Nickerson, 1985), or in terms of the ability to perform a number of demanding cognitive tasks in a new way (Perkins & Bylthe, 1994). Instead, our focus on knowledge building suggests that understanding be viewed as a relation between the knower and the knowledge artifact (c.f., Kefai & Resnick, 1996; Bereiter, 2002; Lesh & Zawojewski, 2007).

Learning environments based on the integration of IDM and mathematical modeling provides one possible means of achieving this change in focus to knowledge building (English, 2000; English & Lesh, 2003; Nason & Woodruff, 2003).

Mathematical models and modeling have been defined variously in the literature including with reference to solving word problems, conducting mathematical simulations, creating representations of problem situations (including constructing explanations of natural phenomena), and creating internal, psychological representations while solving a particular problem (e.g., Doerr & Tripp, 1999; English & Halford, 1995; Gravemeijer, 1999; Greer, 1997; Lesh & Doerr, 2003; Romberg, Carpenter, & Kwako, 2005; Van den Heuvel-Panhuizen, 2003). The definition reflected in this paper is of a model as comprising systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behavior of some meaningful system (Doerr & English, 2003). From this perspective, modeling problems are realistically complex situations where the problem solver engages in mathematical thinking beyond the traditional school experience and where the products to be generated often include complex, conceptual artifacts that are needed for some purpose, or to accomplish some goal (Lesh & Zawojewski, 2007). Mathematical modeling is one of many “ends-in-view” problems (English & Lesh, 2003) where students do not know the nature of the products they are to develop; they only know the criteria that have to be satisfied. There are many ways of satisfying these criteria and hence, multiple products are possible. Furthermore, such products are more complex than the usual responses demanded of students in traditional problem-solving situations. In a later section we provide an example of an on-line community in which students worked with ends-in-view problems.

In order to achieve understanding as we define it, learning environments based on an integration of IDM and modeling need to enable the: (1) development of multiple relationships with the knowledge artifact, (2) promotion of thinking and reflection about intelligent actions in relation to the knowledge artifact, (3) building on from intrinsic interest in the knowledge artifact, (4) positioning of the knowledge artifact within a network of relationships, (5) engagement in explanatory discourse about the knowledge artifact, (6) identification/ recognition of misconceptions concerning the knowledge artifact, (7) engagement in actions aimed at advancing understanding of the knowledge artifact, (8) support of construction of a narrative about the knowledge artifact, (9) encouragement of an analyses about deep features of the knowledge artifact, (10) promotion and exploration of insightful solutions concerning the knowledge artifact, and (11) engagement in complex interaction with the knowledge artifact. To achieve most of these ten knowledge-building enabling characteristics in an IDM-based learning environment, one needs to look beyond a simple integration of IDM and modeling.

Principles to Inform Design of IDM-Based Learning Environments

To advance the development of IDM learning environments, we have developed two sets of interrelated principles to inform the design of: (a) IDM-based environments and (b) accompanying ends-in-view activities including modeling problems. These principles are presented in Tables 2 and 3 below.

Table 2. *Principles to inform the design of an IDM environment*

Principles	*Description of Principle*	*Genesis of Principle*

Finger paint	IDM environment should enable members of a learning community to dabble with, create, explore, and share different ways of representing and making sense of mathematics in an authentic context while designing a model or other types of personally meaningful knowledge artifacts	Nason & Woodruff (2003) Resnick (2002) Scardamalia (2002)
Multiple and interlinking representations	IDM environment should enable members of a learning community to generate and link different representations in order to explicitly and dynamically reveal the different facets of the complex idea(s) embedded in the knowledge artifacts they are building.	Nason & Woodruff (2003) NCTM (2000) Kaput (1992)
Knowledge-building discourse	IDM environment should enable members of a learning community to communicate with other learners via iconic models, natural and mathematical language, and mathematical symbols, the thinking that underlies their building of knowledge artifacts	English (2006, 2007 [a], [b]) Nason & Woodruff (2003) Scardamalia (2002)
Scaffolding of thinking and reasoning skills	IDM environment should provide cognitive scaffolding to facilitate the utilization by the learning community of thinking and reasoning skills that can assist in the construction, analysis, refinement, and conveyance of mathematical ideas	English (2000) Nason & Woodruff (2003) Scardamalia (2002)
Beyond amplification	IDM environment should enable members of a learning community to discover new methods of thinking and unanticipated ways of using the technologies.	Pea (1985, 1997) Hoadley & Pea (2002) Yeh & Nason (2003)
Archiving	IDM environment should provide a central clearinghouse to which participants are encouraged to contribute new mathematical models and to build-on, extend, and generalize archived mathematical models	

Table 3. Principles to inform the design of ends-in-view activities including modeling problems

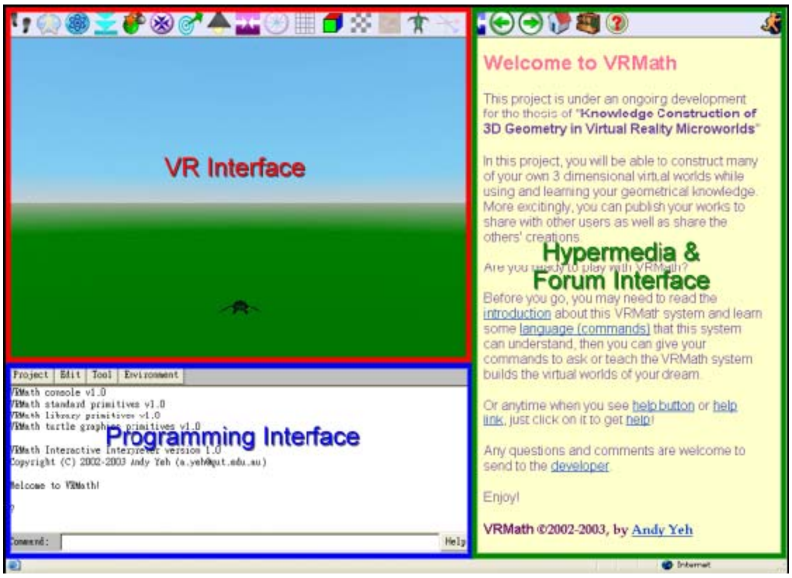
Principles	*Description of Principle*	*Genesis of Principle*
Personal Meaningfulness	Mathematical learning activities should be personally meaningful and ones which learners really care about.	Bereiter (2002) Doerr & Lesh (2000) English (2007, a, b) Kefai & Resnick (1996) Scardamalia (2002)
Improvable Ideas	Ends-in-view activities should enable members of the learning community to value, and to build-on to and improve their repertoires of tacit and “real-world” knowledge.	Scardamalia (2002) Bereiter (2003)
Conceptual Artifact Construction	Mathematical learning activities should create the need for a model or other types of personally meaningful artifacts to be constructed, or modified, extended or refined.	Doerr & Lesh (2000) English (2007, a, b) Kefai & Resnick (1996) Papert (1991, 1993) Scardamalia (2002)

Self- and Peer- Evaluation	When engaged in ends-in-view activities, learners should be able to judge when their responses are good enough, and for what purposes the results are needed.	Doerr & Lesh (2000) English (2007, a, b) Scardamalia (2002)
Powerful Knowledge Building	Ends-in-view activities should entail working toward more inclusive principles and higher-level formulations via the development of useful prototypes (or metaphors) that can be generalized to interpret a variety of other structurally similar situations or be modified and extended to a broader range of situations.	Doerr & Lesh (2000) Papert (1991) Scardamalia (2002)

Principles in Action

To illustrate the design principles for IDM environments summarized in Table 2, we refer to an example of an IDM-based learning environment developed by Yeh and Nason (2003): VRMath. VRMath is an IDM environment that integrates desktop Virtual Reality (VR) technology combined with the power of a Logo-like programming language and hypermedia and the internet to facilitate the collaborative learning of 3-dimensional (3D) geometry concepts and processes. It has been designed to enable users to build their own 3D microworld artifacts while operating within a 3D VR space, linking their language to the geometrical Logo-like programming language, and collaborating through its online discussion forum. VRMath consists of three interfaces: The VR interface, the programming interface, and the hypermedia and forum interface.

Figure 1: Three Interfaces of VRMath



VRMath addresses each of the design principles in the following ways:

- <u>Finger paint principle:</u> VR interface of VRMath enables users to dabble with, create, explore, and share different ways of navigating about, viewing and creating 3D microworld artifacts.

- <u>Multiple and interlinking representations principle</u>: VRMath's three interfaces enable users to generate and link 3D visual, programming/mathematical and natural language representations of important 3D geometry concepts embedded in the 3D microworld artifacts that they are building.
- <u>Knowledge-building discourse principle</u>: The Hypermedia & Forum Interface enables users to communicate with other learners via iconic models, natural and mathematical language, and mathematical symbols, the thinking that underlay their building of the 3D microworld artifacts.
- <u>Scaffolding of thinking and reasoning skills principle</u>: Two important human spatial abilities related to understanding and reasoning within 3D space, spatial orientation, and spatial visualization are scaffolded by icons within the VR interface. Spatial orientation is the ability to determine spatial relation with respect to one's body (McGee, 1979). Spatial visualization is the ability to mentally rotate, manipulate, and twist two- and three-dimensional stimulus objects (McGee, 1979).
- <u>Beyond amplification principle:</u> The inclusion of design features within VRMath, such as the navigation aids and the extension of Logo commands etc., enables the investigation in many new aspects of knowledge construction of 3D geometry.
- <u>Archiving principle</u>: The Hypermedia & Forum Interface provides a central repository where participants are encouraged to archive/display the 3D microworld artifacts they have built and also to build-on, extend, and generalize archived 3D microworld artifacts.

To illustrate many of the design principles for establishing math modeling knowledge-building communities summarized in Tables 2 and 3, , we now describe a cross-cultural virtual community of inquiry that was established with 9^{th-} and 10th -grade students from Australia, Canada, and Zambia (English & Cudmore, 2000; English & Lesh, 2003). Using an interactive, web-based learning environment, the students, their teachers, and the researchers participated in shared data-handling experiences that involved working with ends-in-view situations (namely, developing a data-gathering tool, posing challenging problems, and constructing persuasive, statistical cases). We structured our extranet site to enable interactions of all participants to take place through specifically allocated zones we referred to as "forums." For example, the Discussion Forums enabled each student in each class to share and build on the mathematical, ideas of his or her peers, whether they be local, across the country, or around the globe (Improvable Ideas). Likewise, through our own Discussion Forums, teachers and university researchers shared classroom observations, planned collaborative activities, discussed and monitored students' progress, and assessed and refined the overall program. Our public domain home page acted as a gateway to these Discussion Forums and to other zones we included on the site.

In developing their data-gathering tool (Conceptual Artifact Construction), the students in each country suggested questions to include in an international survey. The students were instructed to include a mix of questions that would produce nominal, ordinal, and interval data. They were also given the criterion to create questions that would have international appeal (Personal Meaningfulness). Following classroom and online discussions, a final survey was created and posted on the site (Conceptual Artifact Construction). Every student then completed the survey online. The raw data were subsequently placed in a new zone on the site that formed the basis of the students' statistical reasoning and problem posing and critiquing (Conceptual Artifact Construction). The students had to decide initially how to analyze the data and whether such analysis would lead to important decisions and discoveries. To assist the students here, we placed two simple questions on their discussion forum, to which a student or pairs of students responded: 1. *In exploring the data, we noticed that….*, 2. *We then wondered…..* The students' responses to these questions stimulated interesting exchanges among all the students especially on issues pertaining to cross-cultural differences (Improvable Ideas).

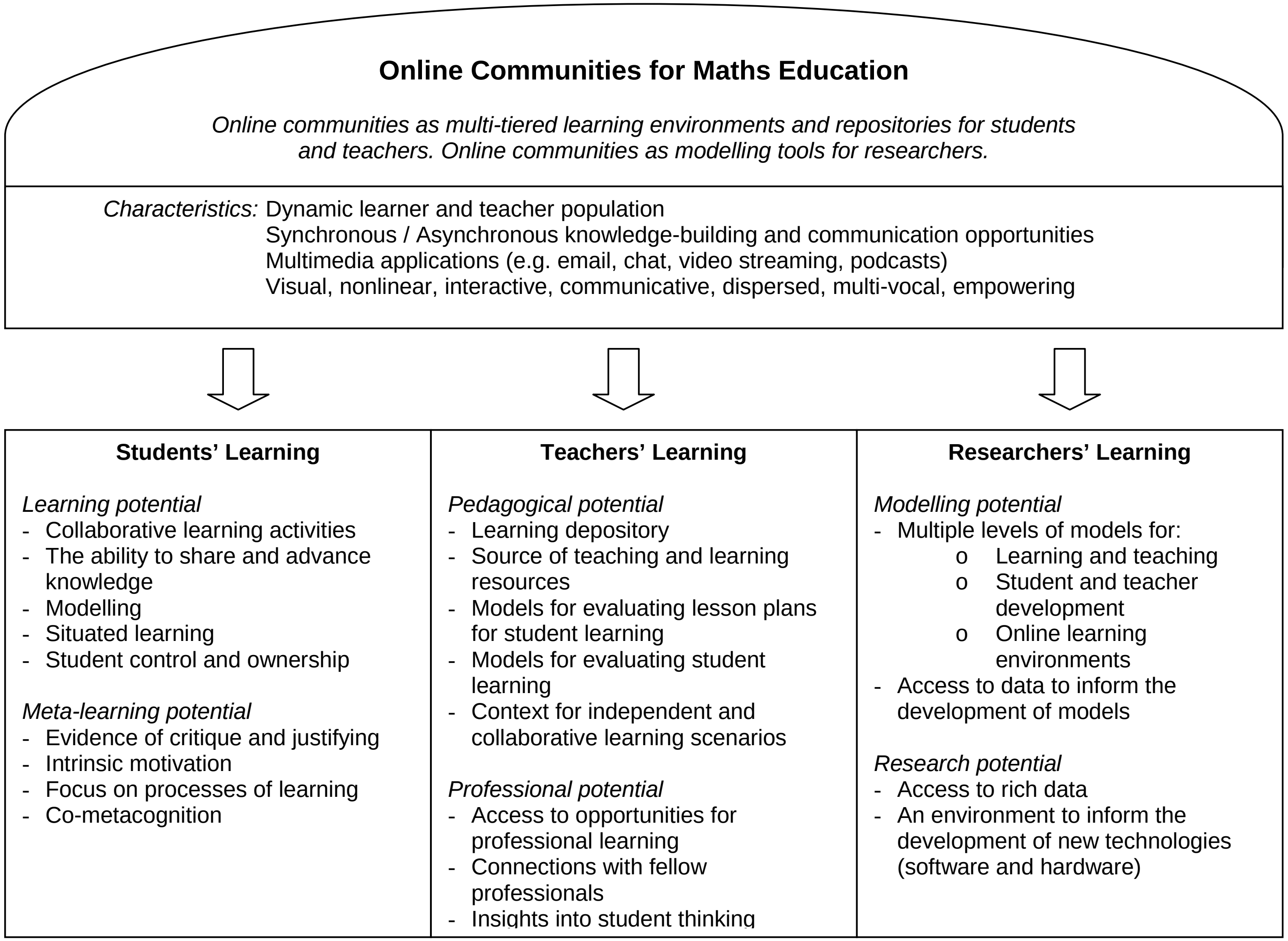
What the students "wondered about" formed the basis of their statistical problem posing. Prior to generating their problems, the students developed their own criteria for determining: 1. *What constitutes a mathematical problem*, 2. *What makes a good mathematical problem*, 3. *What makes a mathematical problem challenging,* and 4. *What makes a problem appealing to the solver*. The criteria that the students developed guided them in posing their initial problems, testing whether one problem was better than another as they refined their problems, and critically assessing one another's completed problems (Improvable Ideas and Self- and Peer-Evaluation), as we indicate next.

The students worked on creating their own problems using the survey data of their choice. Their problems were to require the solver to make a decision or discovery on the basis of the data referred to in the problem (Conceptual Artifact Construction). We also encouraged the students to incorporate thought-provoking questions in their problem creations, that is, questions that required the solver to think beyond the data (Powerful Knowledge Building). The students first shared their problem creations with their classmates and teachers for initial feedback prior to publishing them in the Problem Forum on the website for their international peers to solve (Self and Peer Evaluation). The students in each of the participating classes then selected problems from the Problem Forum, accessing the necessary data to complete them. The students were naturally very keen to try the problems created by their international peers. On solving the problem, the students completed an online critique, which was both highly motivating and important to the students' statistical development (Improvable Ideas and Self- and Peer- Evaluation) In completing the critique, students provided constructive criticism on various aspects of the authors' problem and offered suggestions on ways to improve and extend the problem (Improvable Ideas and Powerful Knowledge Building)

Multiple Knowledge Building Communities

The English and Cudmore (2000) study highlights another important aspect about our perspective on interactive and digital media learning environments (IDM), namely the existence of three knowledge building communities within the online community - one with students, one with teachers and one with researchers (see Figure 2). The major focus of student knowledge building communities is on the building of mathematical subject-matter conceptual artifacts. The major focus of teacher knowledge building communities is on the building of pedagogical content knowledge artifacts. Researcher knowledge building communities focus on the building of both subject-matter and pedagogical content knowledge artifacts. We see teachers being immersed in all three knowledge-building communities— with their students to build mathematical conceptual artifacts to help understand their world, with other teachers to build conceptual artifacts such as models for teaching and learning, and with educational researchers to understand research and reflection on teaching and learning.

Figure 2: Multiple Knowledge Building Communities for Mathematics Education



Concluding Remarks

This paper has presented an alternative perspective on interactive and digital media learning environments (IDM) involving ends-in-view activities including modeling problems. It has been suggested that there is a need to shift the focus from the simple completion of tasks and activities towards the building of conceptual artifacts. The key to achieving this shift lies in the way knowledge is perceived. Traditionally, learning has been the goal of instruction but we propose that the goal should be *knowledge building*. Learning environments based on the integration of IDM and mathematical modeling may provide a possible means of achieving this change in focus towards knowledge building (English, 2000; English & Lesh, 2003; Nason & Woodruff, 2003). In order to facilitate this transition we have developed two sets of interrelated principles to inform the design of: (a) IDM environments and (b) ends-in-view activities including mathematical modeling. The adoption of these principles should facilitate the design of learning environments that move beyond the simple integration of IDM and modeling, towards the development of knowledge building capabilities in learners.

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