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Robust absolute stability criteria for a class of uncertain Lur'e systems of neutral type

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***Abstract*— This paper is concerned with the problem of robust absolute stability for a class of uncertain Lur'e systems of *neutral type*. Some *delay-dependent* stability criteria are obtained and formulated in the form of linear matrix inequalities (LMIs). Neither *model transformation* nor *bounding technique* for cross terms is involved through derivation of the stability criteria. A numerical example shows the effectiveness of the criteria.**

***Index Terms*— Lur'e systems, absolute stability, discrete delay, neutral delay, uncertainty, nonlinearity, robustness**

I. INTRODUCTION

In 1944, when studying the stability of an autopilot, Lur'e and Postnikov [1] introduced the concept of absolute stability and the Lur'e problem. Since then, the problem of absolute stability for Lur'e-type systems has received considerable attention and many fruitful results, such as Popov's criterion, circle criterion, and Kalman-Yakubovih-Popov (KYP) lemma have been proposed [2], [3], [4], [5]. From the view of modern robustness theory, absolute stability theory can be considered as the first approach to robust stability of nonlinear uncertain systems.

Time-delays are frequently encountered in many fields of science and engineering, including communication network, manufacturing systems, biology, economy and other areas [6], [7]. During the last two decades, the problem of stability of time-delay systems has been the subject of considerable research efforts. Many significant results have been reported in the literature. For the recent progress, the reader is referred to [7] and the references therein.

Due to time-delay occurred in practical systems, the problem of absolute stability for Lur'e systems of *retarded* type has also been studied. Most of the existing results are *delay-independent* [8], [9], [10], [11], [12], [13] and few are *delay-dependent* [14], [15], [16]. When the time-delay is small, *delay-independent* results are often overly conservative; especially, they are not applicable to closed-loop systems which are open-loop unstable and are stabilized using delayed inputs, due to either delayed measurements or delayed actuator action in the input channels.

For Lur'e systems of *neutral type*, it is also of significance to study the absolute stability since neutral systems can be used to model delay circuits such as the partial element equivalent circuits (PEEC's) [17] and the distributed networks containing lossless transmission lines [18]. However,there exist only a few results available in the literature [19]. These results are *delay-independent*, which are conservative. Deriving some less conservative results motivates the present study.

The purpose of this paper is to investigate the robust absolute stability of uncertain Lur'e systems of *neutral type*. Some *delay-dependent* absolute stability criteria, which will be formulated in the form of LMIs, will be presented without employing any *model transformation* and *bounding technique* for cross terms. As is well known to the area of time-delay systems [7], model transformation sometimes will induce *additional dynamics*. Although a *tighter* bounding for cross terms can reduce the conservatism. However, there is no obvious way to obtain a much *tighter* bounding for cross terms.

II. PROBLEM STATEMENT

Consider the following uncertain Lur'e system of neutral type

$$\begin{cases} \dot{x}(t) = [A+\Delta A(t)]x(t) + [B+\Delta B(t)]x(t-h) \\ \qquad\quad +[C+\Delta C(t)]\dot{x}(t-\tau) + Dw(t), \\ z(t) = Mx(t) + Nx(t-h), \\ w(t) = -\varphi(t, z(t)), \end{cases} \tag{1}$$

with

$$x(\theta) = \phi(\theta), \quad \forall \theta \in [-\max\{h, \tau\}, 0], \tag{2}$$

where $x(t) \in \mathbf{R}^n$, $w(t) \in \mathbf{R}^m$ and $z(t) \in \mathbf{R}^m$ are the state vector, input vector and output vector of the system, respectively; $h > 0$ is the constant discrete delay and $\tau > 0$ is the constant neutral delay; $\phi(.)$ is a continuous vector valued initial function, $A \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times n}$, $C \in \mathbf{R}^{n\times n}$,

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$M \in \mathbf{R}^{m\times n}$ and $N \in \mathbf{R}^{m\times n}$ are constant matrices; $\Delta A(t)$, $\Delta B(t)$ and $\Delta C(t)$ are unknown real matrices of appropriate dimensions representing time-varying parameter uncertainties of system (1) and satisfy

$$\begin{aligned} & \begin{pmatrix} \Delta A(t) & \Delta B(t) & \Delta C(t) \end{pmatrix} \\ = \ & LF(t)\begin{pmatrix} E_a & E_b & E_c \end{pmatrix}, \end{aligned} \tag{3}$$

where L, E_a, E_b and E_c are known real constant matrices of appropriate dimensions, and $F(t)$ is an unknown continuous time-varying matrix function satisfying

$$F^T(t)F(t) \leq I. \tag{4}$$

$\varphi(t, z(t)) : [0, \infty) \times \mathbf{R}^m \to \mathbf{R}^m$ is a memory, possibly time-varying, nonlinear function which is piecewise continuous in t and globally Lipschitz in $z(t)$ and satisfies the *sector* condition, i.e. $\forall t \geq 0$, $\forall z(t) \in \mathbf{R}^m$,

$$[\varphi(t, z(t)) - K_1 z(t)]^T [\varphi(t, z(t)) - K_2 z(t)] \leq 0, \tag{5}$$

where K_1 and K_2 are constant real matrices of appropriate dimensions and $K = K_1 - K_2$ is a symmetric positive definite matrix. It is customary that such a nonlinear function $\varphi(t, z(t))$ is said to belong to a sector $[K_1, K_2]$ [5].

In order to guarantee the existence and uniqueness of the solution of system (1) [6], we assume that $\|C + \Delta C(t)\| < 1-\delta < 1$ for a sufficiently small $\delta > 0$. A sufficient condition is stated as follows.

Lemma 2.1: [20] The condition $\|C + LF(t)E_c\| < 1 - \delta < 1$ is satisfied for a sufficiently small $\delta > 0$ if there exists an $\varepsilon > 0$ such that

$$\begin{pmatrix} -I + \varepsilon E_c^T E_c & C^T & 0 \\ C & -I & L \\ 0 & L^T & -\varepsilon I \end{pmatrix} < 0. \tag{6}$$

We first introduce the following definition

Definition 2.2: The uncertain Lur'e system of neutral type described by (1)-(4) is said to be robustly absolutely stable in the sector $[K_1, K_2]$ if the system is globally uniformly asymptotically stable for any nonlinear function $\varphi(t, z(t))$ satisfying (5) and any uncertainty satisfying (3)-(4).

In this paper, we will attempt to formulate some practically computable criteria to check the robust absolute stability of the system described by (1)-(2). The following lemma is useful in deriving the criteria.

Lemma 2.3: [20] For any constant matrix $W \in \mathbf{R}^{n\times n}$, $W = W^T > 0$, scalar $\gamma > 0$, and vector function $\dot{x} : [-\gamma, 0] \to \mathbf{R}^n$ such that the following integration is well defined, then

$$-\gamma \int_{-\gamma}^{0} \dot{x}^T(t+\xi) W \dot{x}(t+\xi) d\xi$$

$$\leq \begin{pmatrix} x^T(t) & x^T(t-\gamma) \end{pmatrix} \begin{pmatrix} -W & W \\ W & -W \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-\gamma) \end{pmatrix}.$$

III. MAIN RESULTS

The system described by (1), (3)-(4) can be rewritten as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h) + C\dot{x}(t-\tau) \\ \qquad\quad + Dw(t) + Lu(t), \\ z(t) = Mx(t) + Nx(t-h), \\ w(t) = -\varphi(t, z(t)), \\ y(t) = E_a x(t) + E_b x(t-h) + E_c \dot{x}(t-\tau), \end{cases} \tag{7}$$

subject to uncertain feedback

$$u(t) = F(t)y(t), \tag{8}$$

or equivalently, in view of (4) and (7),

$$\begin{aligned} u^T(t)u(t) \leq & [E_a x(t) + E_b x(t-h) + E_c \dot{x}(t-\tau)]^T \\ & \times [E_a x(t) + E_b x(t-h) + E_c \dot{x}(t-\tau)]. \end{aligned} \tag{9}$$

In the following we will employ (7) and (9) to study the robust absolute stability of (1)-(2).

We first consider the case when the nonlinear function $\varphi(t, z(t))$ belongs to a sector $[0, K]$, i.e., $\varphi(t, z(t))$ satisfies

$$\varphi^T(t, z(t))[\varphi(t, z(t)) - Kz(t)] \leq 0. \tag{10}$$

We have the following result.

Proposition 3.1: For given scalars $h > 0$ and $\tau > 0$, the system described by (1)-(2) with nonlinear connection function satisfying (10) and any uncertainty satisfying (3)-(4) is robustly absolutely stable if there exist scalars $\varepsilon > 0$, $\mu > 0$, real matrices $P > 0$, $Q > 0$, $R > 0$, and $S > 0$ such that (6) and

$$\begin{pmatrix} (1,1) & PB + R & PC & PD - \varepsilon M^T K^T & PL & hA^T R & A^T S & \mu E_a^T \\ * & -Q - R & 0 & -\varepsilon N^T K^T & 0 & hB^T R & B^T S & \mu E_b^T \\ * & * & -S & 0 & 0 & hC^T R & C^T S & \mu E_c^T \\ * & * & * & -2\varepsilon I & 0 & hD^T R & D^T S & 0 \\ * & * & * & * & -\mu I & hL^T R & L^T S & 0 \\ * & * & * & * & * & -R & 0 & 0 \\ * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & -\mu I \end{pmatrix} < 0, \tag{11}$$

where

$$(1,1) \triangleq A^T P + PA + Q - R.$$

Proof. Choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V(t, x_t) \ = \ & x^T(t)Px(t) + \int_{t-h}^{t} x^T(\xi)Qx(\xi)d\xi \\ & + \int_{t-h}^{t} (h - t + \xi)\dot{x}^T(\xi)(hR)\dot{x}(\xi)d\xi \\ & + \int_{t-\tau}^{t} \dot{x}^T(\xi)S\dot{x}(\xi)d\xi, \end{aligned} \tag{12}$$

where $P > 0$, $Q > 0$, $R > 0$, and $S > 0$. Taking the derivative of $V(t, x_t)$ with respect to t along the trajectory of (7) yields

$$\dot{V}(t,x_t) = x^T(t)(A^T P + PA + Q)x(t)$$

$$\begin{aligned}&+2x^T(t)PBx(t-h) + 2x^T(t)PC\dot{x}(t-\tau)\\ &+2x^T(t)PDw(t) + 2x^T(t)PLu(t)\\ &-x^T(t-h)Qx(t-h) - \dot{x}^T(t-\tau)S\dot{x}(t-\tau)\\ &-\int_{t-h}^{t} \dot{x}^T(\xi)(hR)\dot{x}(\xi)d\xi + \dot{x}^T(t)(h^2R+S)\dot{x}(t).\end{aligned}$$

Use Lemma 2.3 to obtain

$$-\int_{t-h}^{t} \dot{x}^T(\xi)(hR)\dot{x}(\xi)d\xi \le \begin{pmatrix} x^T(t) & x^T(t-h) \end{pmatrix} \times \begin{pmatrix} -R & R \\ R & -R \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-h) \end{pmatrix}.$$

Noting that (7), the following holds

$$\dot{x}^T(t)(h^2R+S)\dot{x}(t) = q^T(t) \begin{pmatrix} A^T \\ B^T \\ C^T \\ D^T \\ L^T \end{pmatrix} \times(h^2R+S)\begin{pmatrix} A & B & C & D & L \end{pmatrix} q(t),$$

where $q^T(t) = \begin{pmatrix} x^T(t) & x^T(t-h) & \dot{x}^T(t-\tau) & w^T(t) & u^T(t) \end{pmatrix}$. Then we have

$$\dot{V}(t,x_t) \le q^T(t)\Xi q(t),$$

where

$$\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} \\ * & * & * & \Xi_{44} & \Xi_{45} \\ * & * & * & * & \Xi_{55} \end{pmatrix},$$

with

$$\begin{aligned}
\Xi_{11} &= A^TP + PA + Q - R + A^T(h^2R+S)A,\\
\Xi_{12} &= PB + R + A^T(h^2R+S)B,\\
\Xi_{13} &= PC + A^T(h^2R+S)C,\\
\Xi_{14} &= PD + A^T(h^2R+S)D,\\
\Xi_{15} &= PL + A^T(h^2R+S)L,\\
\Xi_{22} &= -Q - R + B^T(h^2R+S)B,\\
\Xi_{23} &= B^T(h^2R+S)C,\\
\Xi_{24} &= B^T(h^2R+S)D,\\
\Xi_{25} &= B^T(h^2R+S)L,\\
\Xi_{33} &= -S + C^T(h^2R+S)C,\\
\Xi_{34} &= C^T(h^2R+S)D,\\
\Xi_{35} &= C^T(h^2R+S)L,\\
\Xi_{44} &= D^T(h^2R+S)D,\\
\Xi_{45} &= D^T(h^2R+S)L,\\
\Xi_{55} &= L^T(h^2R+S)L.
\end{aligned}$$

A sufficient condition for robust absolute stability of the system described by (1)-(2) is that under (6), there exist real matrices $P > 0$, $Q > 0$, $R > 0$, and $S > 0$ such that

$$\dot{V}(t,x_t) \le q^T(t)\Xi q(t) < 0, \tag{13}$$

for all $q(t) \ne 0$ satisfying (9)-(10). Using the $\mathcal{S}$-procedure in [21], one can see that this condition is implied by the existence of scalars $\varepsilon > 0$ and $\mu > 0$ such that

$$\begin{aligned}&q^T(t)\Xi q(t) - 2\varepsilon w^T(t)w(t) - 2\varepsilon w^T(t)K[Mx(t) + Nx(t-h)]\\ &-\mu u^T(t)u(t) + \mu[E_a x(t) + E_b x(t-h) + E_c\dot{x}(t-\tau)]^T\\ &\times[E_a x(t) + E_b x(t-h) + E_c\dot{x}(t-\tau)] < 0,\end{aligned} \tag{14}$$

for all $q(t) \ne 0$. Thus, if there exist real matrices $P > 0$, $Q > 0$, $R > 0$, and $S > 0$, and scalars $\varepsilon > 0$ and $\mu > 0$ such that (11), then (14) holds. Therefore, system (1)-(2) is robustly absolutely stable according to Theorem 1.6 (p. 129, Chapter 4 in [6]).

Remark 3.2: From the proof process of Proposition 3.1, one can clearly see that neither model transformation nor bounding technique for cross terms is involved. Therefore, the stability criterion is expected to be less conservative.

When $\Delta C(t) \equiv 0$, system (1) reduces to the following system

$$\begin{cases} \dot{x}(t) - C\dot{x}(t-\tau) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)] \\ \qquad\qquad \times x(t-h) + Dw(t), \\ z(t) = Mx(t) + Nx(t-h), \\ w(t) = -\varphi(t, z(t)), \end{cases} \tag{15}$$

with uncertainties described by

$$\begin{pmatrix} \Delta A(t) & \Delta B(t) \end{pmatrix} = LF(t)\begin{pmatrix} E_a & E_b \end{pmatrix}. \tag{16}$$

We assume that

Assumption 3.3: $|\lambda_i(C)| < 1$ $(i = 1, 2, \cdots, n)$.

Considering a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}\tilde{V}(t,x_t) = {}& (x(t) - Cx(t-\tau))^T P(x(t) - Cx(t-\tau))\\ &+ \int_{t-h}^{t} x^T(\xi)Qx(\xi)d\xi + \int_{t-\tau}^{t} x^T(\xi)Wx(\xi)ds\\ &+ \int_{t-h}^{t} (h - t + \xi)\dot{x}^T(\xi)(hR)\dot{x}(\xi)d\xi\\ &+ \int_{t-\tau}^{t} \dot{x}^T(\xi)S\dot{x}(\xi)d\xi,\end{aligned}$$

similar to the proof of Proposition 3.1, we have

Proposition 3.4: Under Assumption 3.3, for given scalars $h > 0$ and $\tau > 0$, the system described by (15), (2) with nonlinear connection function satisfying (10) and any uncertainty satisfying (16), (4) is robustly absolutely stable if there exist a scalar $\mu > 0$, real matrices $P > 0$, $Q > 0$,

$W > 0$, $R > 0$, and $S > 0$ such that

$$\begin{pmatrix} (1,1) & PB+R & -A^TPC & 0 & PD-\varepsilon M^TK^T & PL & hA^TR & A^TS & \mu E_a^T \\ * & -Q-R & -B^TPC & 0 & -\varepsilon N^TK^T & 0 & hB^TR & B^TS & \mu E_b^T \\ * & * & -W & 0 & -C^TPD & -C^TPL & 0 & 0 & 0 \\ * & * & * & -S & 0 & 0 & hC^TR & C^TS & 0 \\ * & * & * & * & -2\varepsilon I & 0 & hD^TR & D^TR & 0 \\ * & * & * & * & * & -\mu I & hL^TR & L^TS & 0 \\ * & * & * & * & * & * & -R & 0 & 0 \\ * & * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & * & -\mu I \end{pmatrix} < 0, \quad (17)$$

where

$$(1,1) \triangleq A^TP + PA + Q + W - R.$$

For the nonlinearity $\varphi(t, z(t))$ satisfying the more general sector condition (5), by applying an idea known as *loop transformation* [5], we can conclude that the robust absolute stability of system (1)-(2) in the sector $[K_1, K_2]$ is equivalent to the robust absolute stability of the following system

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t) - DK_1M]x(t) + [B + \Delta B(t) \\ \qquad\quad -DK_1N]x(t-h) + [C + \Delta C(t)]\dot{x}(t-\tau) \\ \qquad\quad +D\tilde{w}(t), \\ z(t) = Mx(t) + Nx(t-h), \\ \tilde{w}(t) = -\tilde{\varphi}(t, z(t)), \end{cases} \quad (18)$$

in the sector $[0, K_2 - K_1]$, i.e., $\tilde{\varphi}(t, z(t))$ satisfies for $\forall t \geq 0$, $\forall z(t) \in \mathbf{R}^m$,

$$\tilde{\varphi}^T(t, z(t))\,[\tilde{\varphi}(t, z(t)) - (K_2 - K_1)z(t)] \leq 0,$$

by Proposition 3.1 we have the following result.

Corollary 3.5: For given scalars $\tau > 0$ and $h > 0$, the system described by (1)-(2) with nonlinear connection function satisfying (5) and any uncertainty satisfying (3)-(4) is robustly absolutely stable if there exist scalars $\varepsilon > 0$, $\mu > 0$, real matrices $P > 0$, $Q > 0$, $R > 0$, and $S > 0$ such that (6) and

$$\begin{pmatrix} (1,1) & (1,2) & PC & (1,4) & PL & (1,6) & (1,7) & \mu E_a^T \\ * & -Q-R & 0 & (2,4) & 0 & (2,6) & (2,7) & \mu E_b^T \\ * & * & -S & 0 & 0 & hC^TR & C^TS & \mu E_c^T \\ * & * & * & -2\varepsilon I & 0 & hD^TR & D^TS & 0 \\ * & * & * & * & -\mu I & hL^TR & L^TS & 0 \\ * & * & * & * & * & -R & 0 & 0 \\ * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & -\mu I \end{pmatrix} < 0, \quad (19)$$

where

$$(1,1) \triangleq (A - DK_1M)^TP + P(A - DK_1M) + Q - R,$$
$$(1,2) \triangleq P(B - DK_1N) + R,$$
$$(1,4) \triangleq PD - \varepsilon M^T(K_2 - K_1)^T,$$
$$(1,6) \triangleq h(A - DK_1M)^TR,$$
$$(1,7) \triangleq (A - DK_1M)^TS,$$
$$(2,4) \triangleq -\varepsilon N^T(K_2 - K_1)^T,$$
$$(2,6) \triangleq h(B - DK_1N)^TR$$
$$(2,7) \triangleq (B - DK_1N)^TS.$$

Similarly, the following corollary is due to Proposition 3.4.

Corollary 3.6: Under Assumption 3.3, for given scalars $h > 0$ and $\tau > 0$, the system described by (15), (2) with nonlinear connection function satisfying (5) and any uncertainty satisfying (16), (4) is robustly absolutely stable if there exist a scalar $\mu > 0$, real matrices $P > 0$, $Q > 0$, $W > 0$, $R > 0$ and $S > 0$ such that

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) & 0 & (1,5) & PL & (1,7) & (1,8) & \mu E_a^T \\ * & -Q-R & (2,3) & 0 & (2,5) & 0 & (2,7) & (2,8) & \mu E_b^T \\ * & * & -W & 0 & -C^TPD & -C^TPL & 0 & 0 & 0 \\ * & * & * & -S & 0 & 0 & hC^TR & C^TS & 0 \\ * & * & * & * & -2\varepsilon I & 0 & hD^TR & D^TR & 0 \\ * & * & * & * & * & -\mu I & hL^TR & L^TS & 0 \\ * & * & * & * & * & * & -R & 0 & 0 \\ * & * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & * & -\mu I \end{pmatrix} < 0 \quad (20)$$

where

$$(1,1) \triangleq (A - DK_1M)^TP + P(A - DK_1M) \\ \quad +Q + W - R,$$
$$(1,2) \triangleq P(B - DK_1N) + R,$$
$$(1,3) \triangleq -(A - DK_1M)^TPC,$$
$$(1,5) \triangleq PD - \varepsilon M^T(K_2 - K_1)^T,$$
$$(1,7) \triangleq h(A - DK_1M)^TR,$$

$$(1,8) \triangleq (A - DK_1M)^T S,$$
$$(2,3) \triangleq -(B - DK_1N)^T PC,$$
$$(2,5) \triangleq -\varepsilon N^T (K_2 - K_1)^T,$$
$$(2,7) \triangleq h(B - DK_1N)^T R$$
$$(2,8) \triangleq (B - DK_1N)^T S.$$

When $C = 0$ and $\Delta C(t) = 0$, system (1) becomes

$$\begin{cases} \dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)] \\ \qquad \times x(t-h) + Dw(t), \\ z(t) = Mx(t) + Nx(t-h), \\ w(t) = -\varphi(t, z(t)), \end{cases} \tag{21}$$

with

$$x(\theta) = \phi(\theta), \ \ \forall \theta \in [-h, 0], \tag{22}$$

By Corollary 3.6, the following result is recovered.

Corollary 3.7: [16] For given scalar $h > 0$, the system described by (21), (22) with nonlinear connection function satisfying (5) and $F(t)$ satisfying (4) is robustly absolutely stable if there exist scalars $\varepsilon > 0$, $\mu > 0$, real matrices $P > 0$, $Q > 0$, and $R > 0$ such that

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) & PL & (1,5) & \mu E_a^T \\ * & -Q-R & (2,3) & 0 & (2,5) & \mu E_b^T \\ * & * & -2\varepsilon I & 0 & hD^TR & 0 \\ * & * & * & -\mu I & hL^TR & 0 \\ * & * & * & * & -R & 0 \\ * & * & * & * & * & -\mu I \end{pmatrix} < 0, \tag{23}$$

where

$$(1,1) \triangleq (A - DK_1M)^T P + P(A - DK_1M) + Q - R,$$
$$(1,2) \triangleq P(B - DK_1N) + R,$$
$$(1,3) \triangleq PD - \varepsilon M^T (K_2 - K_1)^T,$$
$$(1,5) \triangleq h(A - DK_1M)^T R,$$
$$(2,3) \triangleq -\varepsilon N^T (K_2 - K_1)^T,$$
$$(2,5) \triangleq h(B - DK_1N)^T R.$$

IV. A NUMERICAL EXAMPLE

Example 4.1: Consider the system described by (1)-(4) with

$$\begin{aligned} A &= \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix}, \ B = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}, \\ C &= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \ D = \begin{pmatrix} -0.2 \\ -0.3 \end{pmatrix}, \\ M &= \begin{pmatrix} 0.6 & 0.8 \end{pmatrix}, \ N = \begin{pmatrix} 0 & 0 \end{pmatrix}, \\ K_1 &= 0, \ K_2 = 0.5, \ L = \begin{pmatrix} 0.20 & 0 \\ 0 & 0.20 \end{pmatrix}, \\ E_a &= E_b = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ E_c = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

Using the criterion in [15] and Proposition 3.1 in this paper, the maximum allowed time-delay $h_{\max}$ for robust absolute stability is computed as 1.4702 and 1.6528, respectively. It is clear to see that for this example the criterion in this paper can provide a less conservative result than that in [15].

V. CONCLUSION

The problem of robust absolute stability of uncertain Lur'e systems of neutral type has been addressed. Delay-dependent robust stability criteria have been proposed. In order to obtain less conservative criteria, we have avoided using model transformation and bounding technique for cross terms, which are widely used in deriving delay-dependent stability criteria for systems of retarded type and neutral type. A numerical example has shown the effectiveness of the criteria.

REFERENCES

[1] A.I. Lur'e, V.N. Postnikov, "On the theory of stability of control systems," *Prikladnaya Matematika Mehkhanika*, vol. 8, pp. 246-248, 1944

[2] V.A. Yakubovich, "The solution of certain inequalities in automatic control theory," *Doklady Akademii Nauk*, vol. 143, pp. 1304-1307, 1962.

[3] R.E. Kalman, "Lyapunov functions for the problem of Lur'e in automatic control," *Proceedings of the National Academy of Science*, vol. 49, pp. 201-205, 1963.

[4] V.M. Popov, *Hyperstability of control systems*. New York, NY: Springer, 1973.

[5] H.K. Khalil, *Nonlinear systems*. Upper Saddle River, NJ: Prentice Hall, 1996.

[6] V. Kolmanovskii and A. Myshkis, *Applied Theory of Functional Differential Equations*. Boston: Kluwer Academic Press, 1992.

[7] K. Gu, V.L. Kharitonov, and J. Chen, *Stability of time-delay systems*. Boston: Birkhäuser, 2003.

[8] V.M. Popov and A. Halanay, "On the stability of nonlinear automatic control systems with lagging argument," *Automation and Remote Control*, vol. 23, pp. 783-786, 1962.

[9] X.-J. Li, "On the absolute stability of systems with time lags," *Chinese Mathematics*, vol. 4, pp. 609-626, 1963.

[10] A. Somolines, "Stability of Lur'e type functional equations," *Journal of Differential Equations*, vol. 26, pp. 191-199, 1977.

[11] P.-A. Bliman, "Lyapunov-Krasovskii functionals and frequency domain: delay-independent absolute stability criteria for delay systems," *International Journal of Robust and Nonlinear Control*, vol. 11, pp. 771-788, 2001.

[12] Z.X. Gan and W.G. Ge, "Lyapunov functional for multiple delay general Lur'e control systems with multiple non-linearities,"*Journal of Mathematics Analysis and Applications*, vol. 259, pp. 596-608, 2001.

[13] Y. He and M. Wu, "Absolute stability for multiple delay general Lur'e control systems with multiple nonlinearities," *Journal of Computational and Applied Mathematics*, vol. 159, pp. 241-248, 2003.

[14] P.-A. Bliman, "Absolute stability of nonautonomous delay systems: delay-dependent and delay-independent criteria," *Proceedings of the 38th IEEE Conference on Decision and Control*, pp. 2005-2010, Phoenix, Arizona, USA, December 1999.

[15] L. Yu, Q.-L. Han, S. Yu, and J. Gao, "Delay-dependent conditions for robust absolute stability of uncertain time-delay systems," *Proceedings of the 42nd IEEE Conference on Decision and Control*, pp. 6033-6037, Maui, Hawaii USA, December 2003.

[16] Q.-L. Han, "Absolute stability of time-delay systems with sector-bounded nonlinearity," *Automatica*, vol. 41, pp. 2171-2176, 2005.

[17] A. Bellen, N. Guglielmi, and A.E. Ruehli, "Methods for linear systems of circuit delay diferential equations of neutral type," *IEEE Transactions on Circuits and Systems - I: Fundamental Theory Applcations*, vol. 46, pp. 212-216, 1999.

[18] R.K. Brayton, "Bifurcation of periodic solutions in a nonlinear difference-differential equation of neutral type," *Quart. Appl. Math.*, vol. 24, pp. 215-224, 1966.

[19] D.Ya Khusainov, "Absolute stability of regulation systems of neutral type," *Avtomatika*, vol. 2, pp. 15-22, 1992. (in Russian)

[20] Q.-L. Han, "A new delay-dependent stability criterion for linear neutral systems with norm-bounded uncertainties in all system matrices," *International Journal of Systems Science*, vol. 36, pp. 469-475, 2005.

[21] V.A. Yakubovich, $\mathcal{S}$-procedure in nonlinear control theory, *Vestnik Leningradskogo Universiteta, Ser. Matematika, Series 1*, vol. 13, pp. 62-77, 1973.