COVER SHEET

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Instruction to support mental computation development in young children of diverse ability

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Fostering young children's mental computation capacity is essential to support their numeracy development. Debate continues as to whether young children should be explicitly taught strategies for mental computation, or be afforded the freedom to develop their own. This paper reports on teaching experiments with two groups of students in their first year of schooling: those considered 'at-risk', and those deemed mathematically advanced. Both groups made considerable learning gains as a result of instruction. Importantly, the gains of the at-risk group are likely to renew both their own, and their teacher's confidence in their ability to learn. In this paper, the instructional programs are documented, highlighting the influence of instruction upon the children's development.

LITERATURE AND BACKGROUND

While young children bring powerful mathematical knowledge to their formal school settings, early number teaching where the emphasis is on manipulation of symbols and materials ignores children's powerful mathematics and results in both the teacher's and children's perceptions of failure (Dole & Beswick, 2001). Several research studies investigating successful instructional programs (e.g., Blöte, Klein, & Beishuizen, 2000; Gravemeijer, Cobb, Bowers, & Whitenack, 2000) have indicated that the emphasis of instruction should be strategic flexibility and students' exploring, discussing, and justifying their strategies and solutions. In addition, teacher competence is also an important factor in successful instruction (e.g., Brown, Askew, Rhodes, Denvir, Ranson, & Wiliam, 2001). Important factors in effective teaching include teacher expectations, and instruction as systemised and connected.

With the current emphasis on numeracy in schools, mental computation is regarded as one of the basic components of numeracy (Steen, 1999). There is some debate about the place of basic facts knowledge in mental computation. Some research indicates that one of the pre-requisites for mental computation facility is instant recall of basic facts (Sowder, 1988). Other research (e.g., Hope & Sherrill, 1987) suggests that basic facts knowledge is a related skill to mental computation, not a prerequisite. Research has shown that through the provision of rich learning environments, where explorations of number combinations and arrangements are encouraged, children spontaneously derive their own strategies for basic fact combinations (Fuson, 1992) as well as develop number sense through exploration of number relationships (Wright, 1996). However, other research has shown that many children do not develop efficient strategies for basic facts. Poor mental computers in primary grades predominantly use inefficient counting strategies (McIntosh & Dole, 2000; Heirdsfield & Cooper, 2004a). In contrast, Heirdsfield and Cooper (2004b) found that proficient mental computers used efficient number facts strategies when number facts were not known by recall.

Research into addition and subtraction has found that children frequently will use knowledge of facts (that is, those that are retrieved automatically) to derive answers to addition and subtraction problems involving quantities related to known facts (Heirdsfield, 1999; Steinberg, 1985). These strategies have been reported by several names, including *derived facts strategies* (DFS) (Steinberg, 1985). One particular group of facts that has been identified as necessary to both assist DFSs and mental computation of larger amounts is the combinations of ten facts (Ruthven, 1998; Steinberg, 1985). For developing tens facts, the use of a ten-frame has been suggested (Bobis, 1996). Another model that has been suggested for developing DFS, in particular, tens facts (build to ten and through 10) is the empty number line (Blöte, Klein, & Beishuizen, 2000).

AIM

This paper reports on two case studies of contrasting groups of children in their first year of formal schooling. One group was identified as underachieving mathematically, and the other group was identified as mathematically very able. While it is essential to build on students' prior knowledge (Steffe & Gale, 1995), there is a delicate balance between the need for explicit instruction, and the freedom for students to think for themselves and value their own thinking strategies. In light of this the study sought to address the following two research questions:

How and to what extent are models such as ten frames useful in developing mental computation in young children of widely differing mathematical ability?

To what extent is explicit teaching of strategies effective for mathematically high achieving and at risk students?

THE STUDY

The research adopted a case study design (Yin, 1994) in which teaching experiments (Steffe & Thompson, 2000) were conducted with the aim of developing young children's number understanding and computational performance. The studies proceeded in three steps, commencing with pre-interviews, followed by instruction, and finally post-interviews. Data comprised the researchers' field notes, student work samples, and the pre- and post-interviews. Data were analysed to identify emerging issues related to the students' number reasoning.

Participants

Children in their first formal year of schooling from two sites participated in the study. At site A, the top eight performing children (five boys and three girls) in a class of 26 participated in the study during the fourth (final) term of school. At site B, four children (two girls, two boys) identified by their classroom teacher as displaying the most 'at risk' behaviours in mathematics of all 28 children in the class, participated in the study in the final weeks of the school year.

Pre-interviews

Pre-interviews were utilised to establish children's prior knowledge in number and number facts strategies. Interview items were drawn from *Schedule for early number assessment (SENA 1)* (from *Count me in too professional development package*, NSW Department of Education and Training, 2003), and included items on numeral identification, counting (forwards and backwards), immediate sight recognition of dot patterns (subitising), and basic addition and subtraction. Additional items were developed by the researchers as extension tasks for children at site A, and as further probes for children at site B. At site A, addition and subtraction examples were presented in picture form and accompanied by an oral question (e.g., "You've won 5 marbles, and then later you win 2 more. How many have you won altogether?"). At site B, further probes asked children to visualise the relative size of 15 M&Ms held in a hand (i.e., is it possible?); to describe 4 in relation to 7 (i.e., which is bigger? How do you know?); and to state the effect of addition and subtraction upon collections.

All children at site A were able to count forwards and backwards to 100, although 4 of the children were not confident counting down from 103. All could subitise to 10. Most addition and subtraction examples were solved successfully, although the predominant strategy was counting. Two children were presented with 2-digit computational examples, as they were successful with most one-digit (number facts) examples. Two of the children also exhibited DFS: *use doubles* and *through 10*. These children were presented with more complex addition and subtraction examples, presented in picture form and accompanied by an oral question (e.g., 23+20, 25+23, 36+99, 46-20, 38-14, 134-99). These examples proved quite challenging to them, but some questions were successfully solved using knowledge of place value (addition and subtraction examples) and using ten as a reference point (addition examples only). At site B, all four children could competently count both forwards and backwards to 20, and displayed understanding of the value of numbers to 10, but not beyond (i.e., they had difficulty visualising the size of 15 smarties). Only one child could confidently state that addition resulted in a larger amount and subtraction a smaller amount. In terms of basic facts, counting strategies were used, often in conjunction with fingers as concrete materials, and counting occurred in the order of which the numbers were presented. For example, for 2 plus 8, all children counted on from 2, and often lost track of the count.

Instructional Program

The development of the instructional programs for both groups of students was guided by their prior knowledge of number as identified in the pre-instruction interviews. The programs were constructivist in nature (Steffe & Gale, 1995) with the focus on assisting students make connections and communicate their thinking. At both sites, the researchers implemented the teaching episodes, reflecting upon the outcomes of each episode to inform the planning of subsequent episodes.

<u>Site A</u>. A two-week instructional program was developed, with three foci: (1) development of DFS (*facts to 10*, *through 10*, and *doubles*); (2) development of basic mental computation strategies (counting forward and backward in tens from 2- and 3-digit numbers); and (3) development of number sense of numbers to 1000. Visualisation of numbers 1-10 was developed using the ten-frame. Questioning served to consolidate number facts to 10; for instance, how

many can you see? (8); how do you know? (It’s less 2, so 10 take 2 is 8. It’s 5 and 3, so 5 and 3 is 8). Visualisation of numbers 11-20 was developed using two ten-frames. In conjunction with the two ten-frames, the empty number line was introduced to develop the *through 10* DFS. The ten-frame and domino dot patterns were used to develop *doubles*. A hundred-chart was introduced as a visual aid for developing counting forwards and backwards in tens. However, most work with larger numbers and counting forwards and backwards in tens was conducted orally. At no stage were children asked to write number sentences or to solve addition and subtraction examples on paper. Instead, children were encouraged to share their thinking with each other and with the researcher.

Site B. A two-week instructional program was designed, with its prime focus on the development of children’s facility with tens facts. Instruction was based around the use of the ten frame to promote visualisation of combinations to 10. Three main episodes were planned: (1) orientation to the ten frame and visualisation of numbers 1-10; (2) development of connections between number combinations to 10 through focusing on the number of counters on the ten frame and number of counters missing (commutativity); and (3) subtraction facts to 10 through relating to known addition facts.

Post-interviews

Given the differences in focus and direction of the two teaching experiments between the two sites, different post-interviews were conducted with the children at each site. The post-interviews were designed to serve as a measure of the effectiveness of each instructional sequence, and to align more closely the goals of each program. At site A, SENA 1 items were presented again in the post interviews. As all children were successful on these tasks, extension items (SENA 2) were also presented. As in the pre-interviews, additional addition and subtraction examples were presented. Four children were also presented with the more complex addition and subtraction examples (e.g., 23+20, 25+23, 36+99, 46-20, 38-14, 134-99). At site B, two post-instruction instruments were used. The first was a basic addition and subtraction fact test. This test consisted of 35 items organised into four categories: (1) addition tens facts (11 items); (2) subtraction tens facts (11 items); (3) doubles using digits 2-9 (7 items); and (4) addition count-ons using digits less than 10 with counting on 0, 1, and 2 (6 items). The second instrument was an individual oral test consisting of tens facts (both addition and subtraction) in symbolic form printed on individual cards.

RESULTS

Site A. All children progressed from high levels of achievement in SENA 1 to medium to high levels of success at SENA 2. This indicated progress from working with 2-digit numbers to working with 3- and 4-digit numbers. All children used some number facts strategies, including *doubles* and *through 1*0, although counting was still used. Further, more children were successful with the complex addition and subtraction examples; for instance, 2 children solved 36+99 using 100 as a reference.

Site B. The children who participated in the study undertook the basic addition and subtraction fact test in a similar manner to other class tests: all children seated at their desks, facing the teacher. All children in the class participated in this test, not just the children in the study. A secondary purpose for this test was to explore children's performance in a formal test situation. Collectively the four site B students performed well on both the addition (82%) and subtraction tens facts (64%), a heartening result as tens facts were the focus of the teaching experiment. On the doubles facts, performance was unsurprisingly low (7%), as there was no direct teaching of doubles in the instructional program. Scores for add-on facts also were relatively low (33%). In comparison to the performance of the rest of the class, the four students' scores were higher for the addition and subtraction facts, but considerably lower for the doubles and count-on facts. Class mean percentage scores for each section of the test were 41%, 47%, 39% and 64% respectively. When tested individually, all four children were able to recall all addition and subtraction tens facts without resorting to counting.

DISCUSSION

Results of these two case studies highlight the importance of tailoring instruction to the individual needs of learners, and the delicate balance between explicit instruction and fostering children's own strategic thinking. During the instruction at site A, it was noted that explicit instruction was required to assist children to make connections in their thinking. Even though the children could count forward and backward in tens, they could not readily link this knowledge to adding or subtracting 10, 20 and so forth. Specific teacher questioning assisted the children to make these links. Further, although both the empty number line and two ten-frames were used to develop *through 10* strategy, one child explained that he did not "see" 9+6 as (9+1)+5; rather, he "saw" the solution strategy as (10+6)-1. The child did not use a model – he verbalised a strategy that he had developed "in his head". While models (e.g., 100 chart, empty number line, ten-frame) were introduced to support the development of computational strategies, it is acknowledged that not all children needed to use these (or possibly any) models. The "big picture" was the development of number sense; therefore, encouraging the self development of strategies was more important than learning how to use the models. As Beishuiszen (2001, p. 130) explained

> What is important is the development of abstract thinking on different levels that will progress and become more curtailed, otherwise, the expected advantages of an earlier start on fostering the general aspects of mental strategies might get lost in the prolonged use of 'modelling strategies', even on the empty number line.

For children at site B, posttest results indicated how explicit instruction supported children in their capacity to compute tens facts in a test situation. The posttest results also indicate that further explicit instruction is required to develop competence in other fact groups, particularly doubles. The ten frame model was predominantly used to promote tens facts for these children, and appeared to provide a valuable model for visualisation, as suggested in the literature (e.g., Bobis, 1996; Van de Walle, 1988). However, the ten frame was not particularly valuable in supporting subtraction facts. Two different coloured counters on two ten frames (e.g., 2 of one colour on one and 8 of a different colour on the other) were presented to the children to help connect 2 and 8 as a combination to 10 to provide a scaffold to promoting 10 take 8 is 2. The children appeared confused. Putting the two different coloured counters on one ten frame was not particularly helpful either and actually appeared to "muddy" children's understanding of 8 as a ten frame with 2 missing and the connection between 10, 8 and 2. One child took a set of cards on which the numbers 0-10 were individually displayed. He placed all cards in a pattern in front of him (10-0;

9-1; 8-2, etc), and then promptly started chanting: “10 plus 0 is 10; 10 take 0 is 10; 9 plus 1 is 10; 10 take 9 is 1…). This child was making connections in his own mind without the assistance of the ten frame. Following this revelation, the researchers changed the instructional approach for the other three children, and asked them to visualise each number that “goes with” a given number (e.g., what goes with 3 to make 10?). From this point, children were then given a subtraction fact (e.g., 10 take 3), and asked to think “what goes with 3 to make 10?). This thinking strategy appeared to be a successful step in developing subtraction tens facts, and posttest results tend to confirm this. Due to time constraints, little consolidation time was available. Hence, with both groups of students, teacher flexibility and contingent instruction can be seen to be extremely vital in catering to students’ individual learning styles.

One of the significant outcomes of the research conducted at site B was the importance of specific, individual and immediate support for very young children in their first year of formal schooling. After one year of schooling, these children had been categorised as seriously mathematically challenged. As the majority of their peers were moving ahead, these children appeared to be falling further and further behind. In just 2 weeks, children at site B demonstrated remarkable achievements in recall of tens facts as a result of explicit and deliberate instruction. Recall of basic facts is a key pre-requisite for successful mental computation (Heirdsfield & Cooper, 2004b; Sowder, 1992), and children who have only access to counting strategies are hampered as they progress through school (Ostad, 1998). The results at site B suggest that major and specialised support is required to “boost” children’s thinking about number. Clearly further research is needed, particular in determining the extent to which instruction has supported addition and subtraction facility in general, and the pathway for further basic fact instruction that fosters children’s own strategies.

CONCLUDING COMMENTS

While the teaching experiments at both sites represented small samples, combining these studies revealed important findings concerning engaging all children (mathematically gifted and mathematically challenged) in developing their own thinking about number and the foundation for mental computation, and the part that focussed and fine-grained teacher input played in this development. Importantly, explicit and individually tailored instruction was necessary for both groups of children to achieve the gains that they demonstrated. For both groups visualising and modelling strategies were useful but children also needed to be free to develop strategies for themselves.

This teaching experiment has given insight into the potential for young students of widely differing abilities to develop and efficiently use a range of computational strategies. Since this thinking also underpins the development of number sense, place value and the use of the operations, its achievement needs to be pursued in order to give students the foundational mathematics understanding necessary for them to confidently proceed into higher mathematics.

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