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# LONG TERM SALES FORECASTS OF INNOVATIONS – AN EMPIRICAL STUDY OF THE CONSUMER ELECTRONIC MARKET

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## ABSTRACT

This paper empirically examines models of replacement sales for six electronic consumer durables – TVs, VCRs, DVD players, Digital Cameras, personal and notebook computers – using data from a large survey of 8077 German households. A new replacement model is developed that fits the empirical "lifetables" better than existing models. This said, fitting to replacement sales data was not substantially improved as these fits are not particularly sensitive to mis-specification of the shape of the underlying distribution. Since many product innovations can be targeted at replacement rather than first purchase buyers – this improved understanding of replacement behaviour helps entrepreneurs identify new opportunities.

## INTRODUCTION

Since the pioneering work of Bass (1969) (Bass, 1969), sales models of innovation diffusion have been an important theme in the technological innovation field. Understanding and forecasting technological trajectories, the resulting product sales and their components play an important role in opportunity recognition and evaluation for entrepreneurs. While forecasting innovation adoption has received the most attention both in the diffusion literature and by entrepreneurs, the inclusion of a replacement-purchase component greatly enhances the managerial utility of such models for several reasons (Mahajan, Muller, & Wind, 2000). Indeed, the recognition of opportunities presented by the trends of replacement purchases and consumer tastes has been largely neglected in both the entrepreneurship literature and practice.

For durable goods (the focus of this paper) the replacement component of sales First, replacement sales are a significant component of total sales even for moderately new products, and account for the bulk of sales for mature durable products. Second, the managerial interest in sales forecasting is not confined to new product categories. Third, by extending the time horizon over which the approach can be used to represent sales, the well documented timeliness problem (Hyman, 1988) of diffusion models is largely overcome. Moreover, it is difficult to obtain stable parameter estimates for the diffusion model based on a product's early sales history.

The consumer electronic industry is characterized by high development and launch costs of technological innovations as well as high failure rates. Moreover, technological innovation tends to be an ongoing pursuit of consumer electronic companies. Accurate long term forecasting of sales, and understanding the drivers of those sales, assists managers to better recognise and exploit opportunities over the life cycle of a product category. The paper is organised as follows.

The paper starts with a theoretical and empirical review of the published aggregate level sales aggregate models for durables. Then the models are tested with empirical data. Based on the insights of this review process a new forecasting model is developed that builds on data obtained from a large survey of 8077 German households. The paper concludes with a discussion of the implications for both theory and practice.

## AGGREGATE LEVEL SALES MODELS FOR DURABLES

### Diffusion Models

Diffusion models were popularised by the seminal article by Bass (Bass, 1969). From the perspective of durable products, these models are useful for describing first purchase sales. Due to their aggregate nature, the traditional and primary use of diffusion models is sales forecasting. While the models might also be used for descriptive or normative purposes (Mahajan, Muller, & Bass, 1990), these applications are far less common. In any event, normative applications require forecasting accuracy as a pre-requisite condition. Unfortunately, forecasting applications of traditional first purchase diffusion models have met with only limited success (Mahajan et al., 1990)

More recently, aggregate sales modelling has paid some attention to incorporating replacement purchases thereby increasing the time period for which the models are applicable. This essentially allows the models to be adequately estimated using early sales data, and subsequently forecast a reasonable period of the product's life. Most of the earlier efforts in this regard are directed towards models appropriate for frequently purchased products where replacement purchase sales at any time are approximately proportional to the number of consumers currently using the product. Others have studied replacements of durable products using a distribution of replacement ages approach. These are discussed in more detail below.

**Replacement Sales Model**

Replacement models are diffusion models which explicitly incorporate replacement sales component. Most of them build on the Bass model. The main focus is on forecasting goals. The primary difference between the models are different distributions and assumptions to model the replacement behaviour. The basic idea of these approaches is to model the expected service lifetime L via the replacement distribution of a product and so to forecast the replacement sales. All models use aggregate sales data of the durable consumer market.

The distinction developed by Bayus (1988) between forced replacements (replacement of a failed unit) and unforced replacements (discretionary replacements of a working unit) provides a useful starting point for the discussion of durable product replacements. The drivers of each type of replacement are likely to be considerably different. A further important distinction is between replacement and scrapping. Some unforced replacements may result in a second hand transaction. In this case, the timing of the replacement of the unit by the original purchaser and the ultimate scrapping of the unit are clearly different. It is also noted that on occasion, a unit might be scrapped and not replaced. In this paper, the focus is aggregate sales forecasts. Hence, the complications of the second hand market are avoided by considering only aggregate replacements (replacements of vehicles retired from service) rather than individual replacements (replacements of a vehicles no longer used by the incumbent owner).

Numerous articles have posited that unforced replacements can be influenced by several factors such as price, advertising, promotion, residential moves, product features, product styling and colours and newer technologies (Bayus, 1988, 1991). Since many of these factors change over time it is reasonable to assume that aggregate replacement also varies. However, this has not been examined empirically. Nevertheless, a number of empirical cross sectional studies have investigated closely related issues such as variations in the timing of replacements across brands or product type and the impact of individual characteristics on individual replacement timing. These studies are discussed below.

**Modelling Approach**

Long term sales S(t) of a durable consists of first (adoption), replacement and multiple sales. We define:

$$S(t) = Y(t) + R(t) + M(t) \tag{1}$$

where S(t) are the total sales at time t, Y(t) the first purchase sales, R(t) the replacement sales and M(t) multiple purchases (purchase of an additional unit – e.g. a second TV for the bedroom – rather than a replacement of an existing unit). The discrete, deterministic form of this approach may be written as (value of replacement sales at time t):

$$R_t = \sum_{i=1}^{t-1} S_i \left[F(t-i) - F(t-i-1)\right] \tag{2}$$

where,

| | | |
|---|---|---|
| St | = | Total sales in year t |
| Rt | = | Replacement sales in year t |
| F(a) | = | Cumulative replacement distribution. |

Here F(a) is a standard probability cumulative density function – the probability that a products is replaced before age a. The related probability density function is written as f(a). An important concept in modelling replacement behaviour, is the idea of the replacement density function, or hazard rate.

Replacement rate r(a), also known as the hazard rate, refers to the probability that a unit is replaced at age a, given that has survived until then. The replacement rate is the probability that a unit would be replaced at age a, given that it survived until that age. Mathematically it is related to the distribution function as:

$$r(a) = f(a) / [\ 1 - F(a)\ ] \tag{3}$$

Existing replacement models differ in their choice of probability distribution. A number of different specifications of the replacement distribution have been suggested. Midgley (1981) uses an unspecified distribution, Lawrence and Lawton (1981) employ a constant age approach, Olson and Choi (1985) the Rayleigh distribution, Kamakura and Balasubramanian (1987) the truncated normal distribution and, finally, Bayus et. al. (1989) uses the Weibull distribution. There is no consensus on which distribution function is "best" for modelling consumer durable replacements. Different replacement distributions imply different kind of replacement behaviour which leads to different amount of sales forecasts. The key features of prior research are summarised in Table 1.

The Rayleigh distribution was considered as a model of replacement process by Olson and Choi (1981). They recognise that the product life is not constant but stochastic. Estimation of their model requires information on the populations of units in use ($S_t$) at each time period, aside from sales data. Unfortunately, information on the base $S_t$ is seldom available for most products or expensive to estimate reliably from surveys.

A similar model was used by Kamakura and Balasubramanian (1987) but instead of using the cumulative number of units sold they employ the cumulative number of owners (homes that have purchased the product at least once). Besides that the model uses a more flexible and realistic hazard function for replacement sales, based on the Truncated Normal distribution. The justification of using this distribution is its increasing failure rate with non- zero starting points incorporating replacement due to aging and initial random effects. Kamakura and Balasubramanian consider two possibilities for the estimation of their model. If data are available on replacement purchase over time they can be directly estimated the parameters L and h of the Truncated Normal distribution. L is the average service life of which represents the expected life of a unit, and the shape parameter h which determines the particular type of replacement rate. When replacement data are not available they use information from similar products to infer the shape parameter h and estimate the parameters of the adoption model with the average service lifetime (L) for the replacement model. This last option is appealing because it does not require data on replacement sales which a hardly available. The survival function makes the implicit assumption that the average service life for a product remains constant over time for multiple generations. However changes in with technological development, product reliability tends to increase over time. Further more empirical evidence exists that the average lifetime of certain products has increased over time, like automobiles Steffens (2001) and for many other products it has become shorter (Bayus, 1988). So Past attempts to model replacement sales (e.g., Kamakura and Balasubramanian 1987) typically assume a set "replacement distribution" that remains constant over time. This assumption appears somewhat unrealistic because the mean replacement age of units may vary over time.

An econometric model was proposed by Bayus et al. (1989) which utilize the Weibull distribution to model the replacement sales of color TV. Its replacement rate function can take several forms, depending on the value of its shape parameter. The forecast generated by this model is an account of practical forecasting efforts by RCA’s Consumer Electronics Division. Their primary objective is to develop accurate forecast for managerial use. They employ large-scale survey data as well as historical data on aggregate sales.

Islam and Maede (2000) have been the first who investigated the Gamma distribution in the replacement process context. In total they study seven distributions. They empirically investigate the simultaneous fit of first and replacement sales models to total sales data for 28 household appliances.

Steffens (2001) challenged the notion that replacement distributions remain constant over time. Indeed, this assumption is inconsistent with the notion of Bayus (1988) that non-discretionary replacements can be “accelerated” through the use appropriate marketing efforts. To incorporate this dynamic shift in the replacement life curve Steffens proposes that the average lifetime $L_t$ for a product is sold at time t be expressed as a function of time. He also used the Truncated Normal distribution related on the model. Based on the data of the Austrian automobile market Steffens develops a model which suggests a time variant Truncated Normal distribution.

The model structure for the Steffens (2001) can be expressed as:

$$R_t = \sum_{i=1}^{t-1} S_i \left[F(t-i,t) - F(t-i-1,t-1)\right] \tag{4}$$

where,

F(a, t) is the cumulative replacement distribution at time t.

### A New Replacement Distribution

We propose a new replacement distribution for household electronic products. The basis of our model is that household’s will often own multiple units of electronic products. We distinguish between primary units and secondary units in the household. Primary units are purchased for a specific purpose, and would be replaced if they failed. On the other hand, secondary units are older units (e.g. an older TV) that is kept and still used in the household, although possibly relatively infrequently compared to primary units, and wouldn’t be replaced if it failed. A household may own several primary and/or secondary units.

Using this characterization, we adopt a mixed Gamma distribution as the functional form of our new distribution. We define:

$F_P(a)$ = cumulative probability distribution for replacement of primary units (a standard gamma distribution)

$F_S(a)$ = cumulative probability distribution for replacement of primary units
= 0 (not replaced)

As usual, $f_P(a)$ and $f_S(a)$ are the probability density functions for the two components of the replacement.

Let p be the probability that a unit will be a primary unit. Accordingly, the combined distribution is specified as:

$$F_{MG}(a) = p\,F_P(a)$$
$$= p\,F_G(a) \quad (5)$$

$$f_{MG}(a) = p\,f_P(a)$$
$$= p\,f_{MG}(a) \quad (6)$$

and,

$$r_{MG}(a) = \frac{p\,f_G(a)}{1 - p\,F_G(a)} \quad (7)$$

**Model Estimation and Empirical Findings**

Empirical work has tested these models in three distinct ways. When detailed data is available about the replacement of individual units, the replacement distributions may be directly examined. Otherwise the models are estimated from sales data. Two conditions exist here. If sales data is available for replacement component separately, the replacement sales model can be directly estimated. Alternatively, a first purchase and replacement model must be simultaneously estimated for the total sales data.

Previously reported work for consumer durables that directly investigate the replacement distribution (using life-table or actuarial data) has used the data collected by a USDA survey in Ruffin & Tippett (1975). Data were collected for Ranges, Refrigerators, Freezers, Dishwashers, Clothes Dryers, Washing Machines and Televisions.

In their original paper, Ruffin and Tippet did not fit any distributions, but rather just estimated the average service age. Using this same data, Kamakura & Balasubramanian (1987) found the Truncated Normal distribution superior to the Rayleigh distribution for Ranges, Refrigerators, Washing Machines and B&W Televisions.

Kamakura & Balasubramanian (1987) also found that the Truncated Normal distribution superior to the Rayleigh distribution for both conditions of estimation using sales data (i.e. replacement component available separately and not).

Using only total sales data, Islam and Meade (2000) compare the forecasting performance of seven replacement distributions (simultaneously estimated with the Bass first purchase model) for 28 household appliances.

## DATA AND METHODS

**Data**

A survey has been designed to investigate the purchase history of households for six consumer electronic products. The complete purchase history (i.e. years of each household purchase) was collected. Each purchase was classified into first, replacement and additional unit purchases.13,095 German Households took part in an online-survey which lasted for three months (Oct. till Dec. 2005). Due to non-response, inconsistent responses and failure of control questions 5,018 responses were eliminated from further analysis. 8,077 responses can be used for further analyses. Households were asked about their consumption behaviour concerning six products of the consumer electronic market. The products studied are: TV, VCR, DVD-Player, digital camera, laptop computers and desktop PC. A measurement approach similar to Ruffin and Tippet (1975) was employed. For each product, respondents were asked how many units were in use, a complete ownership history of which years purchases were made, and whether each purchase represented an additional unit for the household, or a replacement. For the most recent purchase of each product, respondents were asked the reason for the replacement.

The data collection method has a significant advantage in that we were able to collect a very large sample of households across Germany. This said it suffers from some drawbacks. Internet surveys suffer from a selection bias in that only household with internet access can participate. In addition asking the entire purchase history leads to a recall problem. Especially the precise year for older purchases becomes more difficult to recall. Often households will not be able to remember when the exact year of purchase was. The graphs indicate data for 2005 are consistently downwardly biased for every product. It is reasonable to assume this is because households took part in the survey at the last three months of 2005. Consequently we eliminated 2005 data from the estimation. Years ending in „0“and to a lesser extent “5“ are clearly upwardly biased. This is a rounding bias by respondents. Hence, it can reasonably be assumed that the surrounding data are also downwardly biased. A further bias is that we only collect a history of the household’s purchases. That is, we of course don’t know their future purchase behaviour. This leads to a technical difficulty that the replacement data is right censored.

Preliminary analysis of the data has been able to compare aggregated estimates of average replacement age with similar estimates derived from sales data in earlier studies. These analyses confirm that these estimates are within the range expected from previous studies, providing confidence that the affect of recall bias is not extreme.

### Fit to Replacement Distribution

Replacement distribution (or Lifetables c.f. Ruffin and Tippet 1975) were derived by considering the time until replacement for each purchase for each household. The aggregate proportion of products in service (at any time) of age, a, that are replaced at that age is calculated. The raw data ("rate") for each product are shown in Figure 1.

Each of the six replacement distributions are fitted to these data using non-linear regression (SPSS NLR routine). For the estimation, the data (percentage replaced for each age) were weighted by the total number of observations.

### Fit to Split-Half Replacement Distribution

We attempt to partially test the assertion of Steffens (2001) that the replacement distribution changes over time. We are able to directly examine long-term shifts for the longer sales history (colour TV) by splitting the data into two time periods: 1967 – 1985 and 1986 – 2004. These two time periods were treated as two entirely different data sets. Only a purchase and its subsequent replacement within the time period were considered. An identical procedure as above is followed to estimate the distributions.

### Fit to Aggregate Sales Data

For each product, the aggregate number of first, replacement and multiple sales were calculated. We used the most common approach of nonlinear least squares regression for parameter estimation initially proposed by Srinivasan and Mason (1986). The parameters of the discrete time replacement sales models (Equation 4 using the different distributions F(a)) were estimated using the total sales and replacement sales data.

## RESULTS

### Replacement Distribution Fits to Life tables

Table 2 displays parameter estimates (with asymptotic standard errors) and goodness-of-fit, $R^2$, of each of the five distributions fitted to the replacement life tables for the six products. These fits are graphically displayed in Figure 1.

The results of these fits are consistent for all six products. They can be summarised as follows:

- The fit for the Rayleigh distribution is clearly inadequate, with $R^2$ values mostly less than zero. It is not capable of even approximately representing the overall shape of the replacement rate distribution.
- The fits for the Truncated Normal and Weibull distributions are fairly similar to each other, with the Weibull distribution performing slightly better for all six products ($R^2$ values in range of 0.02 – 0.85 for Truncated Normal and 0.20 to 0.95 for Weibull). The models are able to follow the empirical replacement rates data for smaller replacement ages, but are unable to "bend down" to follow the empirically observed shape for larger ages.
- Of the existing distributions, the Gamma distribution performs best of the previously reported distributions. Its $R^2$ values are higher (0.31 to 0.99). This distribution is able to follow the empirical replacement rate shape very well for smaller replacement ages, but is still unable to "bend down" quickly enough to follow the empirically observed shape for larger ages.
- The performance of the proposed Modified Gamma distribution is clearly superior. Its $R^2$ values fall in the impressive range of 0.80 to 0.99. The graphs clearly show it is the only model that is able to follow the overall shape of the empirical replacement rates for all product ages.

The superiority of the two parameter distributions (Truncated Normal, Weibull and Gamma) over the one parameter Rayleigh distribution confirms previous empirical studies (Kamakura & Balasubramanian, 1987) Islam & Meade.

This said, the superiority of the Gamma and Modified gamma distributions is in stark contrast to earlier studies. All of these studies used the same data set collected by the USDA Ruffin and Tippett (1975) for major household goods. These are a different kind of product category to the electronic goods reported in this study.

### Fit to Split-Half Replacement Distribution

Table 3 shows parameter estimates (with asymptotic standard errors) and goodness-of-fit, $R^2$, of each of the five distributions fitted to the replacement life tables for the early and late Colour TV data.

The results show that the average replacement age for TV is higher in the later time period and (1986 – 2004) than the earlier time period (1967 – 1985). All but the Rayleigh distribution reveal a statistically significant difference in parameter estimates consistent with this increase. The average replacement age has increased from approximately 10 years to 11 years.

### Fit to Aggregate Sales Data

Parameter estimates and goodness-of-fit measures for the fits of the replacement model using the various distributions to the replacement sales data are shown in Table 4. Measures of fit include mean squared error (MSE), mean absolute percentage error (MAPE) and mean absolute error (MAE), $R^2$ and adjusted $R^2$. The fits are also displayed in Figure 2.

It is immediately clear that all replacement models provide very good fits to the replacement sales data. For most of the six products, $R^2$ values don't fall below 0.9. Even the Rayleigh distribution, that performs dismally in terms of fitting the replacement rate curve, fits the sales data reasonably well, with $R^2$ greater than 0.85 for all but VCRs. Even acknowledging that $R^2$ may provide an artificially high measure of fit for diffusion-type models, a cursory look at the graphs show that all models fit the data very well.

We also note that when fitted to the sales data alone, the parameter estimates for many of the models are unstable. This estimation difficulty has been discussed at length in earlier research when the truncated normal distribution is used (Kamakura & Balasubramanian, 1987; Steffens, 2001). In essence, while the sales data alone provides a good estimate of average replacement age, it does not provide sufficient information to identify the shape of the replacement distribution. Indeed Kamakura and Balasubramanian (1987) suggest that fixing the shape parameter (h) to a fixed value under some estimation conditions (1.75). Steffens also followed this procedure. In preliminary work for this paper, we identified that fixing the h parameter (h = 1.75) had negligible affect on the model fits.

This same estimation problem was identified for both the Gamma and Modified Gamma models. In fact the problem is further exacerbated for these model estimations as there is no parameter that represents the average replacement age (L for truncated normal). For these distributions, simultaneous variation of $\alpha$ and $\beta$ result in negligible changes to the fit – much like a co-linearity problem in linear regression.

Finally we note that the time varying Truncated Normal model performs slightly better than the static Truncated Normal model for Colour TV. This is consistent with the earlier analysis of the split-half replacement distributions that the mean replacement age is increasing. However, the improvement to the sales fit is only marginal ($R^2$ increases from 87.6% to 89.1%).

## CONCLUSIONS

The diffusion literature has paid less attention to replacement purchases than first purchases of new products. However many opportunities for new product variations emanate from replacement purchases because consumers develop more product knowledge and distinctive preferences. This paper has investigated different replacement models for durables with a data set that allows the replacement distributions of consumer electronic products to be investigated for the first time.

Earlier work that empirically investigates the replacement distributions of consumer durable products is confined to major household items (such as white goods, heaters and air conditioners and water heaters). This paper indicates that the models that provided fairly good representations of the replacement distribution for these products (Truncated Normal, Weibull and Gamma distributions) do not as well for these consumer electronic products. A new modified gamma distribution was proposed that provided a substantially better fit to the empirical data. We interpret this difference as being primarily attributed to the characteristics of multiple unit ownership, and the distinction between primary and secondary units in a household being very different for electronic products than for major household items. Older units of electronic products are usually kept when a new unit is purchased (e.g. a TV placed in a bedroom, or DVD placed with another TV set). However the replacement behaviour of these secondary units is quite different to the primary unit in the household. In contrast, multiple unit ownwership of many major household appliances is uncommon (e.g. dishwashers, clothes washers/dryers, water heaters). Even if multiple units are presents (e.g. several room air conditioners), each unit is usually purchased for a specific purpose and undergoes replacement in a similar way to the other units in the household.

This new understanding of multiple units and its impact on the timing of replacements within a household presents new opportunities for entrepreneurial action. The objective should be to accelerate the replacement of all units in the household. In doing so, an entrepreneurial firm could consider several options. First, product variations could specifically target replacements of secondary units in the household. For example, perhaps a household could be encouraged to replace an older TV in a bedroom with a TV designed specifically for bedroom use. Or perhaps a household may replace an older VCR, DVD and stereo taking up space in a smaller second living area with a combination unit.

For the product exhibiting the longest sales history, TVs, we were also able to show that there was a long-term shift in the replacement distribution – confirming the time-varying nature of replacement distributions identified by Steffens (2001) for automobiles.

Despite the superior fit of the modified Gamma model to the replacement distribution data, the fits of all models to the replacement sales data (except the one parameter Rayleigh distribution) were very good. It seems that the sales model is rather insensitive to mis-specification of the underlying replacement distribution. Hence, the new insights in this paper have little value for sales forecasting, but rather generate insights into replacement behaviour as suggested above.

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**Table 1: Summary of Existing Replacement Models**

| Distribution (# parameters) | Paper First Introduced | Replacement Rate Properties | Probability Density Function: f (a) | Replacement rate: r (a) | Key Empirical Findings |
|---|---|---|---|---|---|
| Rayleigh (1) | Olson/Choi (1981) | linear increasing | $f_R(a) = 2\delta a \exp(-\delta a^2)$ <br> $L = \frac{1}{2}\sqrt{\frac{\pi}{\delta}}$ | $r_R(a) = \frac{2a}{\delta}$ | Better fit than the Bass Model (semi logistic) for major household appliances. Inferior fit to truncated normal for both replacement lifetables and replacement data for six major appliances. |
| Truncated Normal (2) | Kamakura & Balasubramanian (1987) | monotonically concave increasing. | $f_{TN}(a) = \frac{\Phi(wa/L - h)}{L\Phi(-h)}$ <br> $w = h + \phi(-h)/\Phi(-h)$ | $r_{TN}(a) = \frac{w\phi(a/L - h)}{L\,\Phi(L - h)}$ <br> $\Phi(x) = \int_{z=x}^{\infty} \phi(z)dz$ | Good fit to replacement lifetables for major household products. Good fits to replacement sales data for six major household appliances and 28 household appliances. |
| Weibull (2) | Bayus, Hong & Labe (1989) | Linearly increasing, concave increasing or convex increasing. | $f_W(a) = L\,\beta\, a^{\beta}\, e^{-La^{\beta}}$ | $r_W(a) = \left[\Gamma\left(\frac{1+1/\beta}{L}\right)\right]^{\beta} \beta a^{\beta-1}$ | Good fits to replacement sales data Color TV and total sales data for 28 household appliances. |
| Gamma (2) | Islam & Maede (1999) | concave increasing or convex increasing or decreasing | $f_G(a) = \frac{L^{\beta} a^{\beta-1} e^{-La}}{\Gamma(\beta)}$ | $r_G(a) = \frac{\left[\beta/L\right]^{\beta} i^{\beta-1}}{\sum_{k=0}^{\beta-1} \frac{\Gamma(\beta)}{\Gamma(k+1)} \left(\beta_i/L\right)^k}$ | Good fit to total sales data for 28 household appliances. |
| Time varying Truncated Normal (3) | Steffens (2001) | As for truncated normal at any time point | As above with <br> $L = L(t) = L + a\,t$ | As above with <br> $L = L(t) = L + a\,t$ | Time variation of replacement distribution identified for Automobiles |
| Modified Gamma (3) | New | concave increasing or convex increasing or decreasing or increasing then decreasing | $f_{MG}(a) = p\, f_P(a) = p\, f_{MG}(a)$ | $r_{MG}(a) = \frac{p\, f_G(a)}{1 - p\, F_G(a)}$ | |

**Table 2: Parameter Estimates and Fits to Replacement Distributions**

| Distribution | Parameter | TV | | | VCR | | | PC | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | Estimate | Asym. Std. Error | $R^2$ | Estimate | Asym. Std. Error | $R^2$ | Estimate | Asym. Std. Error | $R^2$ |
| Rayleigh | δ | 0.1086 | 0.0009 | -0.15 | 0.0093 | 0.0010 | -0.81 | 0.0315 | 0.0046 | -1.61 |
| Weibull | α | 1.43 | 0.4398 | 0.78 | 1.3491 | 0.5822 | 0.67 | 1.2896 | 0.0794 | 0.43 |
| | μ | 10.79 | 0.2435 | | 12.5987 | 0.4243 | | 5.9292 | 0.2700 | |
| Truncated Normal | h | 0.2459 | 0.2159 | 0.51 | 0.1813 | 0.3102 | 0.38 | -0.7467 | 0.6818 | 0.16 |
| | L | 10.6567 | 0.3731 | | 12.3135 | 0.5733 | | 5.872 | 0.3583 | |
| Gamma | α | 1.9442 | 0.7336 | 0.89 | 1.6669 | 0.0920 | 0.78 | 1.7252 | 0.1538 | 0.60 |
| | β | 0.1778 | 0.0074 | | 0.1298 | 0.0090 | | 0.2881 | 0.0281 | |
| Modified Gamma | α | 2.1431 | 0.0763 | 0.93 | 2.0509 | 0.1031 | 0.91 | 2.3911 | 0.1495 | 0.88 |
| | β | 0.2093 | 0.0095 | | 0.2034 | 0.0162 | | 0.4870 | 0.0382 | |
| | p | 0.9622 | 0.0090 | | 0.8415 | 0.0235 | | 0.8800 | 0.0169 | |

**Table 2 (continued): Parameter Estimates and Fits to Replacement Distributions to Lifetables**

| | | Notebooks | | | Digital Cameras | | | DVD | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Distribution** | **Parameter** | **Estimate** | **Asym. Std. Error** | **$R^2$** | **Estimate** | **Asym. Std. Error** | **$R^2$** | **Estimate** | **Asym. Std. Error** | **$R^2$** |
| Rayleigh | δ | 0.0282 | 0.0050 | -1.86 | 0.0363 | 0.0086 | -4.99 | 0.0674 | 0.0083 | 0.24 |
| Weibull | α | 1.2630 | 0.0962 | 0.35 | 1.1573 | 0.0975 | 0.20 | 1.5210 | 0.0586 | 0.95 |
| | μ | 6.7180 | 0.4081 | | 5.8793 | 0.4083 | | 4.9850 | 0.1226 | |
| Truncated Normal | h | -0.6846 | 0.8615 | 0.12 | -2.3701 | 3.4538 | 0.02 | 0.7847 | 0.2395 | 0.85 |
| | L | 6.7368 | 0.5457 | | 6.0352 | 0.6485 | | 4.7075 | 0.1898 | |
| Gamma | α | 1.5939 | 0.1762 | 0.48 | 1.3757 | 01755 | 0.31 | 1.9757 | 0.0640 | 0.99 |
| | β | 0.2363 | 0.0321 | | 0.2337 | 0.0376 | | 0.3800 | 0.0154 | |
| Modified Gamma | α | 2.3803 | 0.2165 | 0.80 | 2.1362 | 0.1315 | 0.90 | 2.1092 | 0.1034 | 0.99 |
| | β | 0.4939 | 0.0630 | | 0.5120 | 0.0457 | | 0.4385 | 0.0406 | |
| | p | 0.8057 | 0.0326 | | 0.8149 | 0.0230 | | 0.9336 | 0.0381 | |

**Table 3: Parameter Estimates and Fits to Replacement Distributions**

| | | TV – First Half (1967 – 1985) | | | TV – Second Half (1986 – 2004) | | |
|---|---|---|---|---|---|---|---|
| **Distribution** | **Parameter** | **Estimate** | **Asym. Std. Error** | $R^2$ | **Estimate** | **Asym. Std. Error** | $R^2$ |
| Rayleigh | $\delta$ | 0.014 | 0.0013 | 0.52 | 0.0121 | 0.0011 | 0.20 |
| Weibull | $\alpha$ | 1.61 | 0.10 | 0.74 | 1.47 | 0.052 | 0.87 |
| | $\mu$ | 10.1 | 0.43 | | 11.3 | 0.28 | |
| Truncated Normal | h | 0.89 | 0.29 | 0.55 | 0.71 | 0.20 | 0.68 |
| | L | 9.89 | 0.54 | | 10.9 | 0.39 | |
| Gamma | $\alpha$ | 2.33 | 0.21 | 0.82 | 1.87 | 0.07 | 0.93 |
| | $\beta$ | 0.227 | 0.023 | | 0.159 | 0.008 | |
| Modified Gamma | $\alpha$ | 3.13 | 0.28 | 0.92 | 2.04 | 0.11 | 0.94 |
| | $\beta$ | 0.362 | 0.043 | | 0.194 | 0.018 | |
| | p | 0.867 | 0.031 | | 0.915 | 0.033 | |

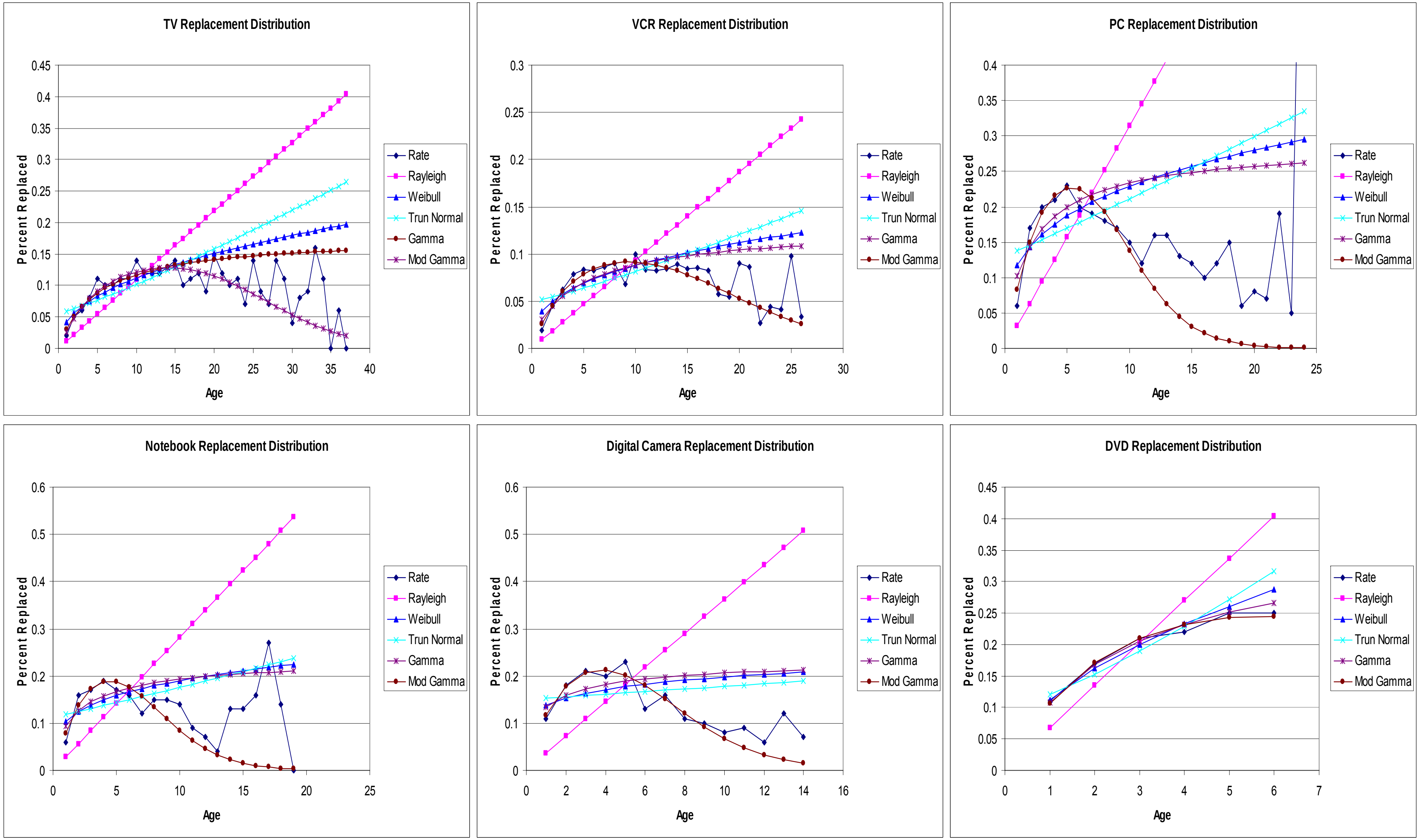


**Figure 1: Replacement Distribution Fits to Life-tables**

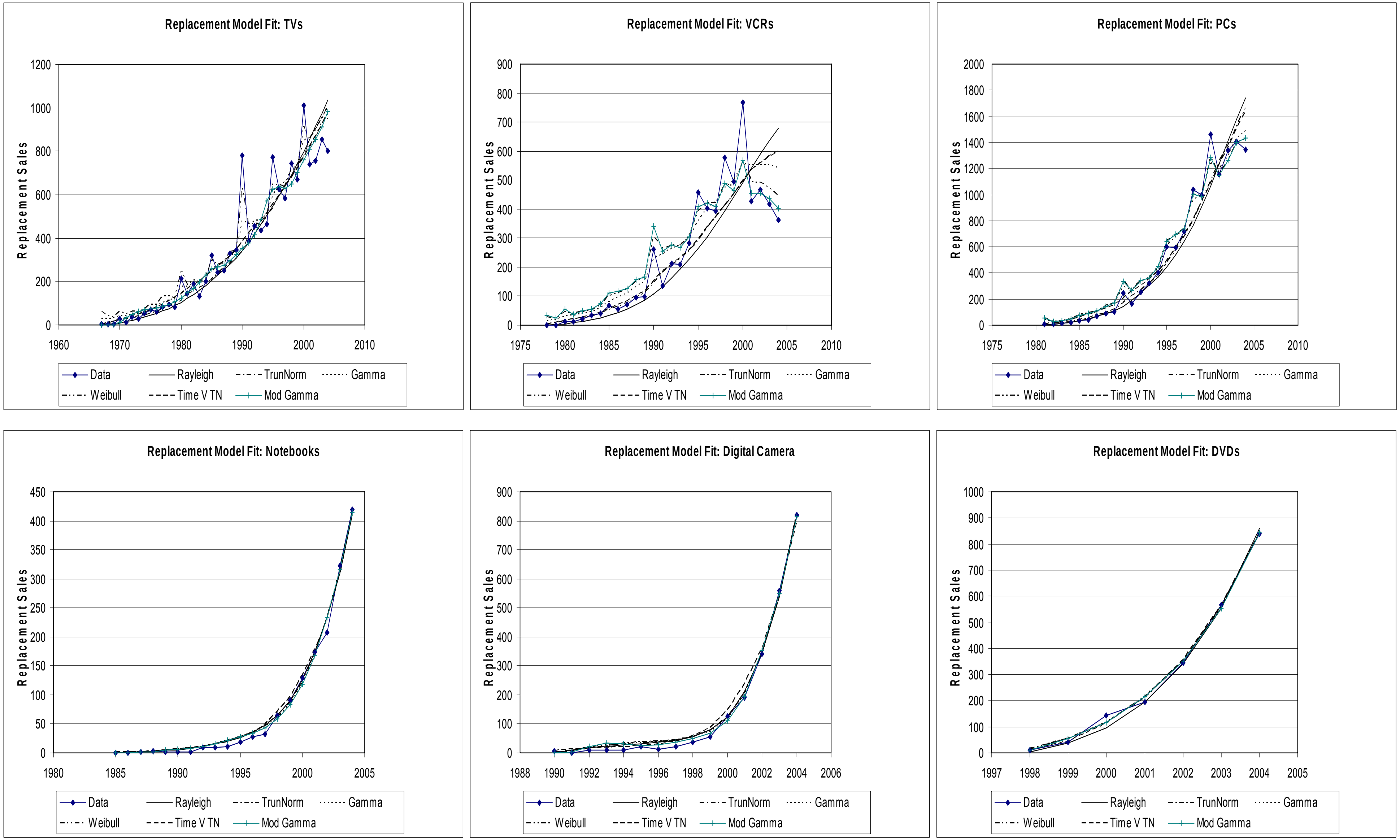


**Figure 2: Replacement Models Fits to Replacement Sales Data**

**Table 4: Parameter Estimates and Fits to Replacement Sales Data**

| | Parameter Estimates | | | Model Fit Statistics | | | | |
|---|---|---|---|---|---|---|---|---|
| Product / Model | 1 | 2 | 3 | MSE | MAE | MAPE | $R^2$ | Adj $R^2$ |
| Colour TV | | | | | | | | |
| Rayleigh | L: 11.49 | | | 1.23E+04 | 6.57E+01 | 19.9% | 86.2% | 86.2% |
| Truncated Normal | L: 12.81 | h: 0.000 | | 1.10E+04 | 6.05E+01 | 28.0% | 87.6% | 87.3% |
| Time Varying Truncated Normal | L: 16.29 | h: 0.765 | a: 0.800 | 9.67E+03 | 5.81E+01 | 28.3% | 89.1% | 88.5% |
| Gamma | L: 6.64 | β: 9.470 | | 1.27E+04 | 7.11E+01 | 74.6% | 85.7% | 85.3% |
| Modified Gamma | L: 4.03 | β: 9.839 | p: 1.000 | 1.15E+04 | 6.93E+01 | 58.4% | 87.1% | 86.3% |
| VCR | | | | | | | | |
| Rayleigh | L: 12.39 | | | 1.43E+04 | 7.88E+01 | 34.0% | 67.1% | 67.1% |
| Truncated Normal | L: 14.00 | h: 0.000 | | 9.85E+03 | 6.25E+01 | 49.4% | 77.3% | 76.4% |
| Time Varying Truncated Normal | L: 14.51 | h: 0.078 | a: 0.763 | 9.74E+03 | 6.28E+01 | 51.8% | 77.6% | 75.6% |
| Gamma | L: 9.99 | β: 9.219 | | 9.55E+03 | 6.80E+01 | 193.6% | 78.0% | 77.1% |
| Modified Gamma | L: 4.49 | β: 9.711 | p: 1.000 | 7.27E+03 | 6.55E+01 | 98.9% | 83.3% | 81.8% |
| PC | | | | | | | | |
| Rayleigh | L: 5.96 | | | 2.13E+04 | 8.66E+01 | 15.3% | 92.0% | 92.0% |
| Truncated Normal | L: 6.44 | h: 0.000 | | 1.53E+04 | 7.35E+01 | 31.3% | 94.2% | 94.0% |
| Time Varying Truncated Normal | L: 6.95 | h: 0.208 | a: 0.800 | 1.47E+04 | 7.70E+01 | 48.4% | 94.4% | 93.9% |
| Gamma | L: 8.65 | β: 9.432 | | 1.34E+04 | 7.76E+01 | 75.9% | 95.0% | 94.7% |
| Modified Gamma | L: 4.58 | β: 9.869 | p: 1.000 | 1.12E+04 | 7.00E+01 | 48.1% | 95.8% | 95.4% |

**Table 4 (continued): Parameter Estimates and Fits to Replacement Sales Data**

| | Parameter Estimates | | | Model Fit Statistics | | | | |
|---|---|---|---|---|---|---|---|---|
| Product / Model | 1 | 2 | 3 | MSE | MAE | MAPE | $R^2$ | Adj $R^2$ |
| Notebooks | | | | | | | | |
| Rayleigh | L: 5.68 | | | 7.88E+01 | 6.09E+00 | 75.5% | 99.4% | 99.4% |
| Truncated Normal | L: 5.39 | h: 2.156 | | 7.12E+01 | 6.12E+00 | 76.2% | 99.5% | 99.5% |
| Time Varying Truncated Normal | L: 7.58 | h: 0.028 | a: 0.856 | 9.72E+01 | 7.38E+00 | 90.4% | 99.3% | 99.2% |
| Gamma | L: 14.99 | β: 8.888 | | 1.10E+02 | 8.70E+00 | 106.5% | 99.2% | 99.2% |
| Modified Gamma | L: 3.93 | β: 9.852 | p: 1.000 | 1.51E+02 | 7.65E+00 | 79.5% | 98.9% | 98.8% |
| Digital Camera | | | | | | | | |
| Rayleigh | L: 4.51 | | | 2.85E+02 | 1.46E+01 | 132.5% | 99.5% | 99.5% |
| Truncated Normal | L: 4.22 | h: 2.165 | | 2.84E+02 | 1.49E+01 | 123.5% | 99.5% | 99.5% |
| Time Varying Truncated Normal | L: 6.19 | h: 0.000 | a: 0.800 | 4.69E+02 | 1.85E+01 | 144.4% | 99.2% | 99.0% |
| Gamma | L: 10.21 | β: 9.141 | | 6.75E+02 | 1.75E+01 | 214.2% | 98.8% | 98.7% |
| Modified Gamma | L: 4.00 | β: 10.020 | p: 0.994 | 3.75E+02 | 1.54E+01 | 134.6% | 99.4% | 99.2% |
| DVD Players | | | | | | | | |
| Rayleigh | L: 4.78 | | | 5.23E+02 | 1.55E+01 | 8.8% | 99.3% | 99.3% |
| Truncated Normal | L: 5.45 | h: 1.009 | | 2.30E+02 | 1.20E+01 | 11.6% | 99.7% | 99.6% |
| Time Varying Truncated Normal | L: 6.51 | h: 1.377 | a: 0.800 | 2.58E+02 | 1.19E+01 | 8.1% | 99.7% | 99.4% |
| Gamma | L: 4.00 | β: 9.877 | | 3.90E+02 | 1.81E+01 | 8.5% | 99.5% | 99.4% |
| Modified Gamma | L: 4.00 | β: 9.870 | p: 1.000 | 3.38E+02 | 1.64E+01 | 8.2% | 99.5% | 99.2% |