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### STATISTICAL MODELING OF MULTIPLE ACCESS INTERFERENCE POWER: A NAKAGAMI-M RANDOM VARIABLE

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#### ABSTRACT

This paper proposes a statistical model for the total multiple access interference (MAI) power for both Direct-Sequence Code Division Multiple Access (DS-CDMA) and Multicarrier Code Division Multiple Access (MC-CDMA) systems. We consider the use of both Walsh-Hadamard (WH) and Gold spreading codes transmitting over the asynchronous uplink channel. Detailed signal models of both CDMA systems are derived illustrating the production of MAI under asynchronous conditions. The paper demonstrates the Gaussian nature of the total MAI and shows that the probability density function (pdf) of the total MAI power can be very accurately characterized by the Nakagami-m distribution.

#### 1. INTRODUCTION

Multiple access interference (MAI) is a major contributor to reception errors and limits the overall capacity of Code Division Multiple Access (CDMA) systems. Direct-Sequence CDMA (DS-CDMA) is a popular multiple access technique and features predominantly in 3rd Generation cellular technologies, namely UMTS, W-CDMA, cdma2000 [1]. The combination of Orthogonal Frequency Division Multiplexing (OFDM) with CDMA, referred to as Multicarrier CDMA (MC-CDMA) [2], has sparked great interest in the future of wireless communications. This work provides a method for the approximation and statistical modeling of the resulting MAI power experienced by mobile users in both DS-CDMA and MC-CDMA systems.

MAI arises from the use of nonorthogonal code sets or from code misalignments upon reception due to timing offsets experienced during transmission. Asynchronous transmission (such is the case over the uplink) is a major cause of MAI in CDMA systems and is therefore the focus of this work. The amount of MAI produced in a CDMA system is code dependent and both Walsh-Hadamard (WH) and Gold codes (deterministic spreading codes) are considered.

Bit error rates (BERs) are highly influenced by the amount of interference and noise power incurred during transmission and as such, it is beneficial to model this interference power. Previous DS-CDMA studies have established MAI to be Gaussian distributed and have lead to the standard Gaussian approximation method in which the total MAI is modeled as an additive white Gaussian noise process [3]-[6]. More recently, BER approximations have been proposed for both synchronous and asynchronous MC-CDMA. The standard Gaussian approximation method was used in determining the upper and lower bounds of asynchronous MC-CDMA [7]. Exact MC-CDMA BERs were calculated

under synchronous conditions without the assumption of the total MAI distribution in [8][9], however, such methods are limited by computational complexity. Alternative approximation methods have seen the use of Monte Carlo integration and moment-generating function analysis [10]. This paper proposes a statistical model of the total MAI power probability density function (pdf) for both asynchronous DS-CDMA and MC-CDMA. With this model, the BER can be accurately modeled without computational complexity.

This paper is organized as follows. Section 2 provides the signal models for both CDMA systems considered and section 3 details the nature of the MAI. Section 4 analyzes the characteristics of the MAI power and explains how the total MAI power pdf is approximated. Section 5 shows accurate fittings of the total MAI power pdf to the Nakagami-m distribution and offers an explanation to this modeling. Section 6 provides Monte Carlo simulation results reinforcing the findings and the paper is concluded in section 7.

#### 2. SIGNAL MODEL

We consider a single cell consisting of *K* users  $(k = 1, \dots, K)$  transmitting over the uplink channel. The *ith* data symbol of user *k* is denoted by  $b_{ki}$  and all data symbols are *i.i.d.* and BPSK modulated such that  $b_{ki} = \pm 1$  and  $E[b_{ki}b_{ji}] = E[b_{ki}]E[b_{ji}]$  for  $k \neq j$ . Representing the *kth* data stream in a continuous form gives

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_{ki} \cdot u_{T_s}(t - iT_s)$$
(1)

where

$$u_T(t) = \begin{cases} 1, & 0 \le t < T \\ 0, & \text{elsewhere} \end{cases}$$
(2)

is the square pulse shaping function. Each user is assigned a unique spreading code of spreading factor  $N = T_s/T_c$ ; here  $T_s$  and  $T_c$  denote the symbol and chip durations respectively. The *kth* code vector is represented as  $\underline{c_k} = [c_{k1}, ..., c_{kN}]$  or temporally as  $c_k(t) = \sum_{n=1}^{N} c_{kn} \cdot u_{T_c}(t - nT_c)$ , where *n* is the chip index. To focus on the analysis of MAI only (resulting from the asynchronous nature of the subscribers) the assumption of perfect power control at the BS is made and the *kth* user's channel is modeled as  $h_k(t) = \delta(t - \tau_k)$ . This is to say the only signal distortion introduced over the channel emanates from the timing offsets originating from the asynchronous transmission<sup>1</sup>. All offsets are made with re-

<sup>&</sup>lt;sup>1</sup>Other distortions introduced by the channel, i.e. Rayleigh/Rician fading and AWGN will be considered in future research

spect to the reference user (denoted by *x*) and are *i.i.d.* over one symbol duration, taking values of integer multiples of  $T_c$  only. The *kth* user signal is offset by  $\tau_k \in [0, (N-1)T_c]$ where  $\tau_x = 0$ .

#### 2.1 DS-CDMA Signaling

The DS-CDMA system structure used is conventional; data signals are spread and up-converted at the transmitter and down-converted and despread at the receiver, followed by matched filtering shaped to the rectangular pulse waveform [11]. The transmission signal of the kth user is expressed as

$$s_k^{ds}(t) = b_k(t) c_k(t) \cos(\omega_c t)$$
(3)

where  $f_c = \omega_c/2\pi$  is the carrier frequency. Ignoring any random phase offsets, the received signal at the BS is expressed as

$$r^{ds}(t) = \sum_{k=1}^{K} b_k (t - \tau_k) c_k (t - \tau_k) \cos(\omega_c t - \phi_k)$$
(4)

where  $\phi_k = \omega_c \tau_k$ . Performing the required frequency downconversion and despreading (with the user of interest being the reference user *x*) yields the following decision variable at the output of the matched filter

$$Z_{x}^{ds}(i) = \frac{1}{T_{s}} \int_{iT_{s}}^{(i+1)T_{s}} r^{ds}(t) c_{x}(t) \cos(\omega_{c}t) dt$$
(5)

Substituting (4) into (5) gives

$$Z_{x}^{ds}(i) = \frac{1}{2T_{s}} \int_{iT_{s}}^{(i+1)T_{s}} \sum_{k=1}^{K} b_{k}(t-\tau_{k}) c_{k}(t-\tau_{k}) c_{x}(t)$$
$$\cdot [\cos(2\omega_{c}t-\phi_{k})+\cos(\phi_{k})] dt$$
(6)

Under the assumption that  $f_c \gg T_s^{-1}$ , we have  $\tau_k \gg 1/f_c$ and as such the double frequency component above is ignored. Moreover, given that  $\tau_k > 0$ , the resulting  $Z_x^{ds}(i)$  becomes conditioned on  $\{b_{k(i-1)}, b_{ki}\}$  over the interval  $iT_s \leq t \leq (i+1)T_s$ . The decision variable therefore becomes

$$Z_{x}^{ds}(i) = \sum_{k=1}^{K} \frac{\cos(\phi_{k})}{2T_{s}} \left[ b_{k(i-1)} R_{kx}(\tau_{k}) + b_{ki} \hat{R}_{kx}(\tau_{k}) \right]$$
(7)

where the partial cross-correlation functions,  $R_{kx}(\tau_k)$  and  $\hat{R}_{kx}(\tau_k)$ , are defined in [12] as

$$R_{kx}(\tau_k) = \int_0^{\tau_k} c_x(t) c_k(t - \tau_k) dt \tag{8}$$

$$\hat{R}_{kx}(\tau_k) = \int_{\tau_k}^{T_s} c_x(t) c_k(t - \tau_k) dt$$
(9)

Equation (7) finally reduces to

$$Z_{x}^{ds}(i) = \frac{1}{2}b_{xi} + \sum_{\substack{k=1\\k \neq x}}^{K} M_{kxi}^{ds}$$
(10)

where  $b_{xi}$  is the desired information symbol and

$$M_{kxi}^{ds} = \frac{\cos(\phi_k)}{2T_s} \left[ b_{k(i-1)} R_{kx}(\tau_k) + b_{ki} \hat{R}_{kx}(\tau_k) \right]$$
(11)

is the corresponding MAI incurred by user *x* from user *k*.

#### 2.2 MC-CDMA Signaling

The MC-CDMA system considered has been previously presented in [2] in which the number of subcarriers equals the spreading factor, N. The same data symbol,  $b_{ki}$ , is transmitted over all N subcarriers where single chips are modulated onto successive subcarriers (making  $T_c = T_s$ ), for which the subcarrier frequencies are assigned via an IFFT block at the transmitter. Summing all subcarriers prior to transmission gives the following transmission signal representation for the *kth* user

$$s_{k}^{mc}(t) = \sum_{n=1}^{N} b_{k}(t) c_{kn} \cos(\omega_{n} t)$$
 (12)

where  $\omega_n = \omega_c + \frac{2\pi n}{T_s}$  is the *n*th subcarrier frequency. Reception over the asynchronous channel gives

$$r^{mc}(t) = \sum_{k=1}^{K} \sum_{n=1}^{N} b_k (t - \tau_k) c_{kn} \cos(\omega_n t - \phi_k)$$
(13)

Subcarriers are demodulated by an FFT block at the receiver, followed by despreading and matched filtering. The decision variable at the output of the matched filter for the MC-CDMA system is expressed as

$$Z_{x}^{mc}(i) = \frac{1}{T_{s}} \int_{iT_{s}}^{(i+1)T_{s}} r^{mc}(t) \sum_{n'=1}^{N} c_{xn'} \cos(\omega_{n'}t) dt \qquad (14)$$

Substituting (13) into (14) yields

$$Z_{x}^{mc}(i) = \frac{1}{2T_{s}} \int_{iT_{s}}^{(i+1)T_{s}} \sum_{k=1}^{K} \sum_{n=1}^{N} \sum_{n'=1}^{N} b_{k}(t-\tau_{k}) c_{kn} c_{xn'}$$

$$\{\cos\left[(\omega_{n}+\omega_{n'})t-\phi_{k}\right] + \cos\left[(\omega_{n}-\omega_{n'})t-\phi_{k}\right]\} dt$$
(15)

Once again the high frequency components in (15) can be ignored as it is assumed  $f_c \gg T_s^{-1}$ . Equation (15) becomes

$$Z_{x}^{mc}(i) = \frac{1}{2T_{s}} \sum_{k=1}^{K} \left[ b_{k(i-1)} V_{kx}(\tau_{k}) + b_{ki} \hat{V}_{kx}(\tau_{k}) \right]$$
(16)

where the partial spectral correlation functions [13],  $V_{kx}(\tau_k)$  and  $\hat{V}_{kx}(\tau_k)$ , are given by

$$V_{kx}(\tau_k) = \int_0^{\tau_k} \sum_{n'=1}^N \sum_{n=1}^N c_{xn'} c_{kn} \cos\left[(\omega_n - \omega_{n'})t - \phi_k\right] dt \quad (17)$$

$$\hat{V}_{kx}(\tau_k) = \int_{\tau_k}^{\tau_s} \sum_{n'=1}^N \sum_{n=1}^N c_{xn'} c_{kn} \cos\left[(\omega_n - \omega_{n'})t - \phi_k\right] dt \quad (18)$$

The final MC-CDMA decision variable can be expressed in terms of the desired information and the corresponding interference incurred as

$$Z_{x}^{mc}(i) = \frac{1}{2} b_{xi} \sum_{n=1}^{N} c_{xn}^{2} + \sum_{\substack{k=1\\k \neq x}}^{K} M_{kxi}^{mc}$$
(19)

where

$$M_{kxi}^{mc} = \frac{1}{2T_s} \left[ b_{k(i-1)} V_{kx}(\tau_k) + b_{ki} \hat{V}_{kx}(\tau_k) \right]$$
(20)

is the MAI contribution of user k in the MC-CDMA system.

#### 3. MULTIPLE ACCESS INTERFERENCE: A GAUSSIAN RANDOM VARIABLE

As seen from (10) and (19) of the previous section, a general form of the decision variables can be expressed as

$$Z_x(i) = \alpha_x b_{xi} + M_{xi} | \underline{\tau}$$
(21)

where  $\alpha_x$  is the attenuation of the corresponding system. By definition, the total MAI incurred by the reference user is given by the sum of individual interferer MAI contributions as

$$M_{xi}|_{\underline{\tau}} = \sum_{\substack{k=1\\k\neq x}}^{K} M_{kxi}|_{\tau_k}$$
(22)

where  $\underline{\tau} = [\tau_1 \cdots \tau_K]$  is the offset vector for which  $\tau_x = 0 \quad \forall \underline{\tau}$  and *i* is the symbol index.

**Remark**:  $M_{kxi}|_{\tau_k}$  is conditioned on  $\underline{b}_k = \{b_{k(i-1)}, b_{ki}\}$  and therefore can only take four values for a given offset,  $\tau_k$ . For example, Fig.1 illustrates MAI as a random process taking 4 possible values,  $\pm 0.3 \& \pm 0.5$ . As  $M_{kxi}|_{\tau_k}$  depends on  $\{b_{k(i-1)}, b_{ki}\} \& M_{kx(i+1)}|_{\tau_k}$  depends on  $\{b_{ki}, b_{k(i+1)}\}$ , we can say the four values are correlated.

*Remark*:  $M_{kxi}(\tau_k) \& M_{jxi}(\tau_j)$  are not necessarily characterized by the same pdf for  $k \neq j$ .

**Remark**:  $M_{kxi}(\tau_k)$  is a zero-mean random variable and it follows that  $M_{xi}$  is also a zero-mean random variable.

Assumption:  $M_{kxi}(\tau_k) \& M_{jxi}(\tau_j)$  are independent if  $k \neq j$ . This is to say that interferers produce independent MAI contributions. This holds as  $M_{kxi}(\tau_k) = f(b_{k(i-1)}, b_{ki})$   $\& M_{jxi}(\tau_j) = f(b_{j(i-1)}, b_{ji})$  and  $b_{ki} \& b_{ji}$  are independent. Moreover, all offsets are independently generated and distributed over  $T_s$  with equal probability.

It follows that (22) is a sum of K - 1 independent random variables and by the Central Limit Theorem (CLT) the pdf of the resulting total MAI approaches the Gaussian distribution for a sufficient K; in this work the total MAI is found to be Gaussian for  $K \ge 8$ . For example, Fig. 2 shows the histogram of the total MAI for K = 64. The histogram was obtained via Monte Carlo simulations consisting of 10,000 realizations for WH codes of N = 64 and demonstrates the Gaussian nature of the total MAI incurred by the reference user.

#### 4. CONDITIONAL MULTIPLE ACCESS INTERFERENCE POWER

All MAI contributions are zero-mean random variables and it follows that the corresponding MAI powers are equal to  $M_{kxi}^2$ . For a given offset, the MAI power is dependent on the random variable  $b_{k(i-1)}b_{ki}$  (observed by squaring (11) and (20)) and therefore can only equate to one of two values; these two values correspond to the scenarios  $b_{k(i-1)} = b_{ki}$ and  $b_{k(i-1)} \neq b_{ki}$ . We therefore consider the conditional MAI power in our system performance analysis which is the conditional expectation of the MAI power for a given offset value. As the conditional MAI is a zero-mean random variable, the conditional MAI power is equal to the variance of

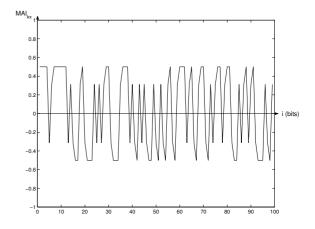


Figure 1: MAI Contribution as a Random Process

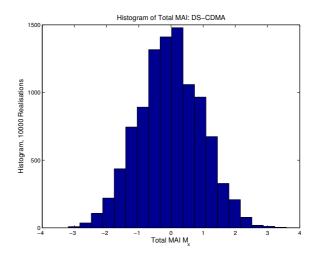


Figure 2: Histogram of Total MAI; 10,000 Monte Carlo Realizations

the conditional MAI. This conditional MAI power can be expressed as

$$\sigma_{kx}^{2}(\tau_{k}) = E\left[M_{kxi}^{2}(\underline{b}_{k},\tau_{k}) \,|\, \tau_{k}\right]$$
(23)

The total interference power incurred by the reference user is expressed as

$$\sigma_x^2(\underline{\tau}) = \sum_{\substack{k=1\\k \neq x}}^K \sigma_{kx}^2(\tau_k)$$
(24)

given the assumption of independent MAI contributions.

In order to make a performance measurement of the two CDMA systems under consideration, it is desirable to calculate the pdf of the incurred interference power and in turn, the BER pdf. We will limit this paper to model the interference power only. The BER pdf is easily obtained through a change of variable transformation  $p(P_e)|dP_e| = p(\sigma_x^2)|d\sigma_x^2|$  where  $P_e$  denotes the BER [14].

Since (24) is a sum of independent random variables, the pdf of  $\sigma_x^2$  can be computed by K - 1 convolutions, i.e.

$$p_{\sigma_x^2}(u) = \left(p_{\sigma_{1x}^2} * p_{\sigma_{2x}^2} * \dots * p_{\sigma_{Kx}^2}\right)(u)$$
(25)

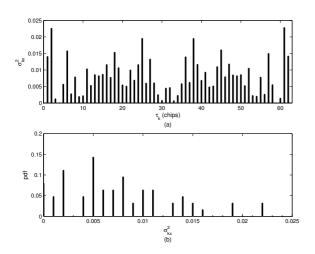


Figure 3: (a) Conditional MAI Power as a Function of Timing Offset  $\tau_k$ ; (b) Discrete pdf of Individual Interferer Conditional MAI Power

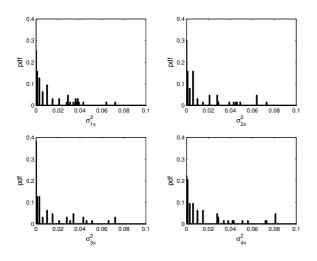


Figure 4: Pdfs of Individual Conditional MAI Power

where *u* represents the convolution variable and  $p_{\sigma_{vv}^2} = \delta$ .

Figure 3.(*a*) shows an example of the conditional MAI power of a selected user as a function of offset. This plot is obtained by evaluating (23) at all possible offset values (integer multiples of  $T_c$ ). The corresponding pdf (obtained by taking the histogram of the function in Fig. 3.(*a*)) of these power values is given in Fig. 3.(*b*).

Due to the code properties, the interference contributions of different users are not characterized by the same pdf. For example, Fig. 4 shows the conditional MAI power pdfs of 4 selected users in a system of K = 64, N = 63. This figure corresponds to the use of Gold codes for which the pdf displays numerous unique values of  $\sigma_{kx}^2$ ; this is in contrast to the use of WH codes, for which few unique values are evident.

Once the pdf of the conditional MAI power contributed by each interferer has been computed, the pdf of the total conditional MAI power received by user *x* is computed using (25). The pdfs for both systems and code sets are shown in Fig. 5 by the solid curves for which K = 64.

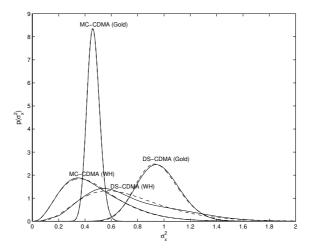


Figure 5: Total Conditional MAI Power pdfs fitted against Nakagami-m pdfs

#### 5. MODELING OF INTERFERENCE POWER

## 5.1 Statistical Modeling Using The Nakagami-m Distribution

In [15], the Nakagami-m pdf for a random variable r is given as

$$p(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} exp\left(-\frac{mr^2}{\Omega}\right)$$
(26)

where  $m = E[r^2]^2 / var(r^2)$ ,  $\Omega = E[r^2]$  and  $\Gamma(m)$  is the Gamma function given in (27). As in [15], by putting  $w = r^2$ , we can make use of the Nakagami-m pdf of the power w. This pdf is obtained through the transformation p(w) |dw| = p(r) |dr| and is given in (28), where  $\Omega$  is replaced by  $\bar{w} = E[w]$ .

$$\Gamma(m) = \int_0^\infty x^{m-1} \exp\left(-x\right) dx \tag{27}$$

$$p(w) = \left(\frac{m}{\bar{w}}\right)^m \frac{w^{m-1}}{\Gamma(m)} exp\left(-\frac{mw}{\bar{w}}\right)$$
(28)

Substituting our power variable,  $\sigma_x^2$ , into (28) as  $w = \sigma_x^2$  and replacing  $\bar{w}$  by  $\zeta = E[\sigma_x^2]$ , yields the expression of  $p(\sigma_x^2)$  in (29).

$$p\left(\sigma_{x}^{2}\right) = \left(\frac{m}{\zeta}\right)^{m} \frac{\left(\sigma_{x}^{2}\right)^{m-1}}{\Gamma(m)} exp\left(-\frac{m\sigma_{x}^{2}}{\zeta}\right)$$
(29)

Normally, due to the CLT, the interference power is expected to be Gaussian distributed. Obtaining the interference power pdfs from the convolutions in (25) reveals that for the use of Gold codes, the convergence towards the Gaussian distribution is rapid (CLT is applicable), however, for the use WH codes, this is not the case. For spreading factors of  $N \ge 31$ ,  $p_{\sigma_x^2}$  can be almost perfectly fitted to both the Gaussian and Nakagami-m distributions under Gold code utilization, however, the Nakagami-m distribution offers a more accurate fitting. As the convergence towards the Gaussian distribution is not evident from the convolutions of (25) for WH codes, the CLT is not applicable, even for spreading

factors as high as N = 128. The resulting pdf,  $p_{\sigma_x^2}$ , can however be very accurately fitted to the Nakagami-m distribution for spreading factors of  $N \ge 32$ . The pdf fittings of both systems under the use of both code sets can be seen in Fig.5 for WH codes of N = 64 and Gold codes of N = 63. The pdfs obtained by the convolutions in (25) are represented by the solid curves while the dashed curves represent the Nakagami-m power pdfs (29). These Nakagami-m distributions were obtained by taking the first and second order moments and calculating *m* and  $\zeta$  in (29). These results suggest that MAI power is a Nakagami-m random variable for both DS-CDMA and MC-CDMA systems in asynchronous environments. Monte Carlo simulations offer strong accordance with the Nakagami-m fittings (see section 6).

#### 5.2 Discussion

We find that the CLT is applicable for identically distributed interference powers. Performing the K - 1 convolutions in (25) with the same pdf (*k* fixed),  $p_{\sigma_{kx}^2}$ , yields

$$p_{\sigma_{\kappa}^{2}}(u) = \left(p_{\sigma_{kx}^{2}} * p_{\sigma_{kx}^{2}} * \dots * p_{\sigma_{kx}^{2}}\right)(u)$$
(30)

which is a Gaussian random variable. This is true for both WH and Gold codes. It is when the interference powers are not identically distributed that the pdf,  $p_{\sigma_x^2}$ , does not converge to the Gaussian distribution, but fits the Nakagamim distribution (characterized by the parameters *m* and  $\Omega$ ); this is the case even for large spreading factors (as high as N = 128).

As is evident from Fig. 4, the conditional MAI power of each interferer is not characterized by identical pdfs. We take a closer look at the statistics of the individual interferer pdfs,  $p_{\sigma_{kx}^2} \quad \forall k$ , to determine the degree in which the pdfs vary from user to user. Figure 6 shows histograms of the means taken from each interferer's pdf,  $p_{\sigma_{kx}^2}$ . It can be clearly seen that the first order statistics are widely spread for WH codes (Fig. 6(a)). Given the pdf laws from interferer to interferer are not identically or even similarly distributed for WH codes, it is not clear that the CLT will be applicable. In contrast, the means are closely distributed (clustered) for Gold codes (Fig. 6(b)) which suggests the pdfs are similarly distributed. This would suggest that the CLT is more applicable for the use of Gold codes, meaning the convergence towards the Gaussian distribution is more rapid. This is in fact the case as previously stated.

Through further analysis of the distribution of mean pdf values for the WH case, we can divide the interferers into groups based on their corresponding mean interference power pdf value. Taking the N = 64 case as an example, it is noticed that the majority of interferers have their pdf mean clustered in the region  $0 \le E \left[ p_{\sigma_{kx}^2} \right] \le 0.003$ . Separating the interferers in this group from the other interferers and performing the convolution of their power pdfs results in a Gaussian shaped distribution. As the statistics are similar in this case, this is analogous to the convolutions in (30) and hence the convergence towards the Gaussian distribution is rapid. For this specific case, there exists 41 out of the 63 interferers in this clustered group. In the approximation of the total interference power pdf, the convolution of all interference pdfs must be performed (25). The interferers whose pdfs have a mean in the higher end of the scale add a tail to the Gaussian

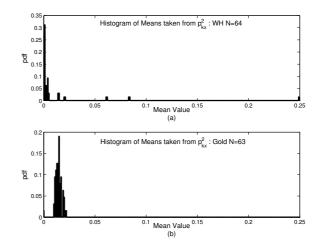


Figure 6: Mean pdf Values, (a) WH N = 64, (b) Gold N = 63

distribution of the clustered group during the convolutions of (25). The resulting distribution is Nakagami-m as previously discussed.

#### 6. SIMULATION RESULTS

Monte Carlo simulations were run to verify the accuracy of the theoretically obtained pdfs and to validate the assumption of independence amongst the interferers. Both WH and Gold codes were used in each CDMA system taking spreading factors of N = 32,64,128 and N = 31,63,127 respectively. Moreover, the number of users was chosen as K = N for WH codes and K = N + 1 for Gold codes; this ensures a consistent capacity irrespective of the code set used. Simulations of 10,000 realizations were run varying the offset vector,  $\underline{\tau}$ , for each realization. The histograms of the observed  $\sigma_x^2$  values are displayed by the jagged curves in Fig. 7 and have been superimposed over the Nakagami-m distribution represented by the bold curves; the Nakagami-m distributions were obtained by calculating m and  $\bar{\zeta}$  from the observed data and applying them to (29). These curves correspond to the use of WH codes of length N = 64 and results of the same accuracy can be seen for the use of Gold codes. From such results, it is evident that the interference power for both CDMA systems can be accurately characterized by the Nakagami-m distribution. For spreading factors of  $N \ge 31$ , all pdfs can be accurately fitted to the Nakagami-m distribution.

#### 7. CONCLUSION

This paper has presented a statistical model for the MAI power of both DS-CDMA and MC-CDMA systems incurred during asynchronous uplink transmission. A method for obtaining the total MAI power pdf was presented and these pdfs were successfully fitted to the Nakagami-m distribution, suggesting the incurred interference power to be a Nakagami-m random variable.

#### REFERENCES

 J. S. Blogh and L. Hanzo, *Third-Generation Systems and* Intelligent Wireless Networking: Smart Antennas and Adaptive Modulation, John-Wiley & Sons, 2002.

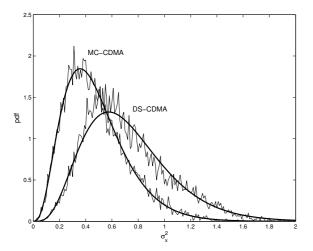


Figure 7: Monte Carlo Conditional MAI Power pdfs

- [2] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Communications Magazine*, vol. 36, pp. 126-133, Dec. 1997.
- M. B. Pursley, "Error probability for direct-sequence spread-spectrum multiple-access communications-Part I: Upper and lower bounds," *IEEE Transactions on Communications*, vol. COM-30, NO. 5, pp. 975-984, May 1982.
- [4] J. S. Lehnert and M. B. Pursley, "Error probabilities for binary direct-sequence spread-spectrum communications with random signature sequences," *IEEE Transactions on Communications*, vol. 35, NO. 1, pp. 87-98, January 1987.
- [5] J. S. Lehnert, "An efficient technique for evaluating direct-sequence spread-spectrum multiple-access communications," *IEEE Transactions on Communications*, vol. 37, NO. 8, pp. 851-858, August 1989.
- [6] P. K. Enge and D. V. Sarwate, "Spread-spectrum multiple-access performance of orthogonal codes: linear receivers," *IEEE Transactions on Communications*, vol. COM-35, NO. 12, pp. 1309-1319, December 1987.
- [7] K. W. Yip and T. Ng, "Tight error bounds for asynchronous multicarrier CDMA and their application," *IEEE Communications Letters*, vol. 2, NO. 11, pp. 295-297, November 1998.
- [8] Q. Shi and M. Latva-aho, "An exact floor for downlink MC-CDMA in correlated rayleigh fading channels," *IEEE Communications Letters*, vol. 6, NO. 5, pp. 196-198, May 2002.
- [9] Q. Shi and M. Latva-aho, "Exact bit error rate calculations for synchronous MC-CDMA over a rayleigh fading channel," *IEEE Communications Letters*, vol. 6, NO. 7, pp. 276-278, July 2002.
- [10] Q. Shi and M. Latva-aho, 'Spreading sequences for asynchronous MC-CDMA revisited: accurate bit error rate analysis," *IEEE Transactions on Communications*, vol. 51, NO. 1, pp. 8-11, Jan 2003.
- [11] S. Glisic and B. Vucetic, Spread Spectrum CDMA Systems For Wireless Communications, Artech

House, 1997.

- [12] M. B. Pursley, D. V. Sarwate and W. E. Stark, "Performance evaluation for phase-coded spread-spectrum multiple-access communication-Part I: System analysis," *IEEE Transactions on Communications*, vol. COM-25, NO. 8, pp. 795-799, Aug. 1977.
- [13] B. M. Popovic, "Spreading sequences for multicarrier CDMA systems," *Electron. Lett.*, vol. 35, pp. 1797-1798, Oct. 14, 1999.
- [14] D. Carey, B. Senadji and D. Roviras, "Statistical modeling of bit-error-rates in asynchronous multicarrier CDMA and direct-sequence CDMA systems," *in Proc. EUSIPCO '04*, Sept. 2004, pp. 2083-2086.
- [15] M. D. Yacoub, J. E. V. Bautista and L. Guerra de Rezende Guedes, "On higher order statistics of the Nakagami-m distribution," *IEEE Transactions on Vehicular Technology*, vol. 48, pp. 790-794, May 1999.