

Refereed conference paper:

Yun, W. Y. and **Ferreira L.** (2000). *Inventory management in railway sleepers: a simulation model for replacement strategies*. Symposium of the International Society of Inventory Research. 20-25 August, Budapest.

## **Inventory Management in Railway Sleepers: A Simulation Model for Replacement Strategies**

Won Young Yun

Department of Industrial Engineering, Pusan National University, Pusan, 609-735, Korea

Luis Ferreira

School of Civil Engineering, Queensland of University of Technology, Brisbane, Australia

### **Abstract**

This paper describes the development of a simulation model to assess the inventory requirements of alternative rail sleeper replacement strategies. The main aim of the model is to determine the optimal replacement strategy, given replacement costs and resultant train operating cost benefits. We consider the replacement problem under the following assumptions: The time to failure under constant stress follows a Weibull distribution and the scale parameter is a function of stress level and the three stress levels under normal (all adjacent units are good), medium-stress (one adjacent unit has failed) and high-stress conditions (two adjacent units are failed) are considered. The cumulative exposure model is used to model the failure distributions.

The operational cost per unit time depends on the maximum number of consecutive failed units. The replacement cost consists of the fixed cost and variable cost proportional to the number of units replaced. A finite horizon is considered and total expected cost is a criterion for comparing the proposed policies. The model has been tested using rail system data and the results are presented in this paper.

### **1. Introduction**

In Australian freight operations, 25-35 percent of total train operating expenses is track maintenance related (Ferreira and Higgins (1998)). Exclusive of rail costs, sleeper replacement represents the most significant maintenance cost for the railways (Hagaman and McAlpine (1991)). For efficient maintenance of sleepers, a railway authority or company needs to plan for a specific amount, and then store the sleepers at regional sites in advance.

For efficient inventory management, firstly it is necessary to predict the demand for sleepers at main location. To forecast future sleeper demand, we need to analyze the trends in demand. In general, the demand for sleepers consists of amounts used with two different activities in maintenance, namely: unplanned and planned maintenance.

Unplanned maintenance is done when the failed sleepers are replaced after accidents or at irregular detection. Planned maintenance is undertaken on regular schedule times for replacement. Thus, the planned maintenance interval or strategy affects the demand process of sleepers and is an important factor for inventory management of sleepers.

In this paper, we consider planned replacement strategies for sleepers and develop a simulation procedure to test the efficiency of proposed replacement policies.

Thus our replacement problem for sleepers belongs to that for replacement policies for system with multi-units or complex systems. For the replacement problem for a complex system, there are some existing studies(Ozekici(1996) and Wlideman and Dekker(1997)). Most of past research assumes that failures of units are independent and only economic dependency is considered. In our case, dependency between sleeper failures is an important characteristic peculiar to railway track maintenance. Finally, we study some field data and compare several replacement policies.

### **Assumptions:**

- 1) The system consists of n sleepers.
- 2) The time to failure under constant stress follows a Weibull distribution with distribution function,

$$F(t) = 1 - \exp\left\{-\left(\frac{t}{\beta}\right)^\alpha\right\} \quad (1)$$

where  $\alpha$  is a shape parameter and  $\beta$  is a scale parameter.

The Weibull distribution is known to produce good results when representing sleeper failure (Tucker (1965)).

- 3) The scale parameter for sleeper failure distribution is a function of the stress level loaded onto sleepers. The scale parameters of under normal (all adjacent units are good), medium-stress (one adjacent unit is failed) and high-stress conditions (two adjacent units are failed) are  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ , respectively.

- 4) After a sleeper is used under normal stress till time t, a sleeper adjacent to it is failed at t and the actual age of the sleeper, s is obtained as follows;

$$F_1(s) = F_0(t), s = t \frac{\beta_1}{\beta_0} \quad (2)$$

The cumulative exposure model is used to model the failure distributions. (Nelson (1990))

- 5) Replacement time is negligible.
- 6) The operational cost per unit time depends on the maximum number of consecutive failed units.
- 7) The replacement cost consists of the fixed cost,  $c_f$  and variable cost,  $c_v$  which is proportionate to the number of units replaced.
- 8) The finite horizon,  $T$  is considered and the salvage cost of sleeper is negligible.
- 9) The status of sleepers (good or failed) is known at no cost.

## 2. Model

We consider a track with  $n$  sleepers of which the failure times follow Weibull distributions. The failure of each sleeper is assumed to be dependent on condition (two adjacent sleepers fail or are working). This dependency is very important and peculiar characteristic in sleeper's failure or deterioration. What make failure modeling of sleepers more complex is how we deal with failure phenomena after replacement of an adjacent failed sleeper. Thus, a relation model in acceleration life testing is adopted to represent the dependency (Nelson (1990)).

In this paper, we consider replacement cost and operation cost. The latter represent the train operation cost which is dependent on track condition. Total replacement cost increases as we replace frequently but the total operation cost decreases when failed sleepers are few. One critical aspect in determining the condition of track with respect to sleepers is dispersion of failed sleepers in the track. A section of railway track with about 20% failed sleepers may be still be safe to operate if each failed sleeper lies between two sound ones, yet the same section of track with only 1% failed sleepers all adjacent to one another would be unusable. Thus, we assume that the operation cost depends on the maximum number of consecutive failed sleepers.

Under previous assumptions about failure processes and cost models, we consider several replacement strategies for comparison by simulation:

### 2.1 Replacement Policies

We can consider four types of replacement strategies for sleepers. However, these are only a subset of all sleeper replacement options.

**Replacement policy A:** We replace the failed sleeper just at failure.

**Replacement policy B:** We replace all the failed sleepers periodically. In this case, replacement interval is a decision variable.

**Replacement policy C:** We replace all the failed sleepers when the  $k$  consecutive sleepers are failed first. In this case,  $k$  is a decision variable.

**Replacement policy D** (Combination of policies **B** and **C**): We replace all the failed sleepers when the  $k$  consecutive sleepers are failed or when time  $t$  is elapsed from last replacement, whichever occurs first. In this case,  $k$  and  $t$  are decision variables.

In the replacement policies A-D, we replace all the failed sleepers at replacement but we can extend this restriction to more general policy. For example, We can delay the replacement point to the first failure time of 3 consecutive sleepers and then replace all consecutive failed sleepers except isolated ones. Until  $t$ , there are no 3 consecutive failed sleepers; we replace all consecutive failed sleepers.

To obtain the optimal replacement policy minimizing the total expected cost during the finite time horizon, we must derive the total expected cost for a given replacement policy. It is very difficult to derive this analytically because of the dependency between failures of sleepers and finite time horizon (Stochastic dynamic programming problem). We formulate the optimization problems by simulation under the given set of assumptions. The total expected cost is a criterion for optimization. For given replacement strategies, the total expected costs are estimated by simulation. Therefore we summarize the simulation procedure to find total expected cost and compare some replacement policies with field data by simulation.

## 2.2 The Procedure of Simulation

In this section, we explain the simulation procedure by pseudo code. As a typical example, we consider replacement policy B with replacement interval,  $T_m$ . It is possible to add other policies by changing some steps. Before we explain the pseudo code, we summarize two methods to obtain random variates related to Weibull distribution.

**Method for generating Weibull failure times:** A sleeper failure time follows Weibull distribution with scale parameter  $\beta$ , and shape parameter  $\alpha$ ,

1. If we want to obtain failure times, the following function is used:

$$NTF = \beta [-\ln U]^{1/\alpha} \quad (3)$$

where U is Uniform variate [0,1].

2. If we know the age of sleeper, x and we want to obtain failure time, t, then the failure time has the following distribution:

$$\Pr\{T \leq t / T > x\} = \frac{\Pr\{x < T \leq t\}}{\Pr\{T \geq x\}} = \frac{\{F(t) - F(x)\}}{\{1 - F(x)\}}$$

where F (t) is distribution function under current stress level. To obtain the next failure time for the used sleepers, we use an inverse transformation method. Let U be the random variate of Uniform [0,1], then.

$$T = \beta_{1or2} [(x / \beta_{1or2})^\alpha - \ln U]^{1/\alpha} \quad (4)$$

where the conditions of two adjacent sleepers determine the scale parameter.

#### **Input Variables:**

Number of sleepers, n; shape parameter,  $\alpha$  ( $>1$ ); scale parameters under normal, medium and high stresses,  $\beta_0$ ,  $\beta_1$ , and  $\beta_2$ ; horizon length, T; simulation replication number, M; cost parameters for operation cost function and replacement cost; parameters for replacement policy, for this policy B, replacement interval,  $T_m$ .

#### **Definition of general variables:**

NT = current time

TC = total cost to current time

MCF = maximum number of consecutive failed sleepers at current time

NF = number of the failed sleepers at current time

NTF = total number of the failed sleepers to current time

Age(j) = age of sleeper, i at current time

NAF(j) = number of the failed sleepers adjacent to sleeper, j at current time

NRT = next replacement time after current time

### Simulation Step

**Step 0 (initialization):** Initialize  $NT = 0$ ;  $TC = 0$ ;  $MCF = 0$ ;  $NF = 0$ ;  $NTF = 0$ ;  $Age(j) = 0$ ;  $NAF(j) = 0$

**Step 1 (generate next failure times):** Generate  $n$  random numbers of Weibull  $(\alpha, \beta_0)$  under normal stress level using equation (3) and let them be  $NFT(j)$ .

**Step 2 (determine next time to which simulation time is moved):** Find the minimum value among  $NFT(j)$  for all  $j$  and next replacement time,  $NRT = \min\{NFT, NRT\}$ ;  
If  $\min\{NFT, NRT\} = \text{next replacement time}$ , **Then** go to **Step 4**,

### Step 3(Change parameters):

$INT = \min\{NFT, NRP\} - NT$ ;  $Age(j) = Age(j) + INT$ , for all  $j$ ;

Change actual ages of the two sleepers adjacent to the failed sleeper using equation (2).

$$Age(j) = Age(j) \beta_{1or2} / \beta_0$$

For the sleepers of which stresses should be changed, obtain next failure times as follows:

When the actual age,  $Age(j) = x$ , the distribution function for next failure is obtained using equation (4). Finally, the next failure times for the two sleepers of which stress levels are changed are

$$NFT(j) = \min\{NFT, NRP\} + \{T - Age(j)\}$$

Update total cost and total number of failed sleepers.

$NT = \min\{NFT, NRP\}$ ; If  $NT = T$ , go to **step 5**.

Go to **Step 2**.

### Step 4(replace all the failed sleepers):

$$INT = NRT - NT; NT = NRT$$

For failed sleepers,  $Age(j) = 0$  and generate new ones using equation (3) and

$$NFT(j) = T + NT$$

For working sleepers adjacent to replaced sleepers, change actual ages using equation (3) and generate

$$T(j) = \beta_0 [(Age(j) / \beta_0)^\alpha - \ln U]^{1/\alpha}$$

Finally, the next failure times are

$$NFT(j) = NT + \{ T(j) - Age(j) \}$$

Update total cost and total number of failed sleepers.

Next replacement time, NRT, is changed; Go to **Step 2**

**Step 5(terminate simulation):** stop simulation and give the estimator of total expected cost.

For each replacement type, we can construct the simulation procedures and run the simulation with several values of decision variable. For example, we can consider 1, 2, 3, 4, 5 years for replacement policy B. Then we compare the total costs and decide on a good planned replacement interval.

### 3. Field Study

In this section, we consider a replacement problem with typical field data related to timber sleepers. It was estimated in 1991 that 75% of the world's railway consists of timber sleepers. Despite the increasing reliability and effectiveness of alternatives such as steel and concrete, Sonti et al. (1995) state that timber has been and will continue to be the most popular material for railway sleepers in the United States. In Gruber (1998), it is stated that well over 90% of maintenance and construction of railway tracks utilize timber sleepers based on cost and benefit.

We consider a railway track with 1000 timber sleepers and the failure distribution is given by a Weibull distribution with shape parameter 3 and scale parameter 20 because the average life is about 20 year in Australia. If an adjacent sleeper is failed, the decay rate for a sleeper increases and usually reduces the remaining life to a half of original remaining life. So we assume that the shape parameter is 10 and 5 after failure of adjacent sleeper. The planning period is assumed to be 20 years.

Replacement cost is estimated as follows:

Cost per sleeper replaced: replacing < 5%: installation cost \$38 + cost of sleeper \$22

- : 5-10%: installation cost \$33 + cost of sleeper \$22
- : 10-50%: installation cost \$28 + cost of sleeper \$22
- : 15-20%: installation cost \$23 + cost of sleeper \$ 22
- ; >20%: installation cost \$ 18 + cost of sleeper \$22

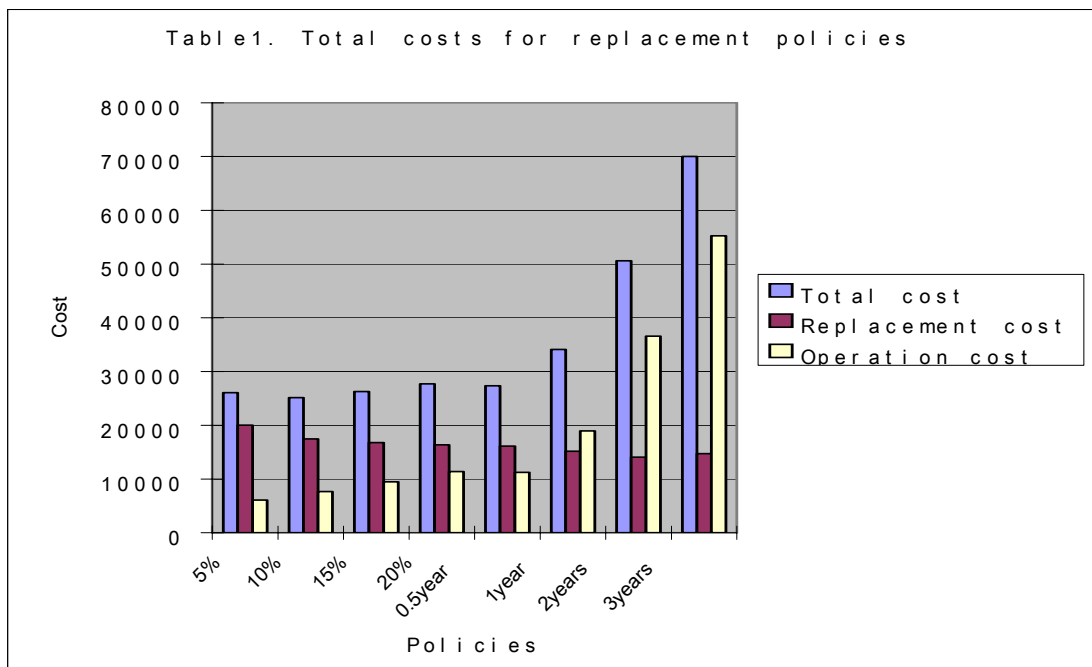
Operation cost is very difficult to estimate exactly and we assume approximately that it is a function:

$$c(m) = \phi m^d$$

where,  $d=2$  and  $\phi=400$ , then if a sleeper fails, then operation cost is \$400 per sleeper a year.

We consider four replacement policies. Replacement every 0.5,1,2,3 years are simulated, the levels of failed sleepers before intervention of >5%, 10%, 15%, 20% are used, and two consecutive failed sleepers are considered as a replacement criterion.

Table 1 shows the simulation results. In this Table, the best policy is 10% policy where we delay replacement until 10% sleepers fail and then replace all the failed ones. And



one-year policy is also an alternative. We considered the additional policies which consider two-consecutive failures as replacement, that is, when we replace all the failed sleepers, we do not replace the isolated and failed ones. 10% and two-consecutive and 0.5-year and two consecutive policies have large values for total cost, 54697 and 27467.



### **3. Conclusion and discussion**

This paper considers replacement policies for prediction of demand for inventory management of sleepers in railway: “ What is the optimal replacement strategy?” We define the problem by the previous assumption and total expected cost to T; time horizon is the optimization criterion. We consider various replacement types in this case. To propose replacement policies, information for replacement should be checked. We are able to know which sleepers fail at any time, the condition of rail system and remaining time to T, life cycle (Which unit is good or failed? And failure distributions of good units), Therefore, we proposed four kinds of replacement policies. Simulation procedure is proposed and a case study has been shown using field data. In this paper, we proposed four policies but for further study, we can consider other types of replacement: We can consider emergency replacement from the safety prospective. For example, if three adjacent sleepers are failed, then it is unsafe to operate a train. This case belongs to replacement policy C.

We can also add replacement condition based on the number of total failed sleepers and for more general cases, we can also consider preventive replacements. In general, when replacement condition based on the proposed replacement policies is satisfied and we replace a failed sleeper, we can consider replacement of working but old sleepers near the failed sleeper ( For opportunistic replacements, refer Wildeman and Dekker (1997)). As another case, when the replacement condition is not satisfied, then we can also consider preventive replacement for old one and some consecutive failed one.

### **References**

1. Adams, J.C.B. (1991), Cost Effective Strategy for Track Stability and Extended Asset Life through Planned sleeper Retention, Demand Management of Assets National Conference Publication, Institution of Engineers, Australia, pp. 145-152.
2. Ferreira, L and A. Higgins(1998), Modelling Rail Track Maintenance Scheduling, Proceedings of International Conference on Traffic and Transportation Studies , American Society of Engineers, Beijing, China, pp820-829

3. Gruber, J.(1998), Making Supply Equal Demand, Railway Track and Structures, October , pp.17-23.
4. Hagman, B.R. and R.J. McAlpine(1991), ROA Timber Sleeper Development Project, Proceedings of the 8<sup>th</sup> International Rail Track Conference, Rail Track Association of Australia, pp. 233-237.
5. Maclean, J.D.(1965), Percentage Renewals and Average Life of Railway Ties, Forest Products Laboratory Report No. 886, Madison, Wisconsin.
6. W. Nelson (1990), Accelerated Testing: Statistical Models, Test Plans, and Data Analysis, John Wiley and Sons, Inc., New York.
7. Ozekici, S. 1996), Reliability and Maintenance of Complex Systems, Springer-Verlag, Berlin.
8. Sonti, S.S., J.F. Davalos, M.G. Zipfel, and H.V.S. Gangarao(1995), A Review of Wood Corsstie Performance, Forest Products Journal, 45, 9, pp. 55-58.
9. Tucker, S.N. (1985), A Reliability Approach Theory to Railway Life, Journal of Wood Science, 10, 3, pp. 111-119.
10. Wildeman, R.E. and R. Dekker (1997), Dynamic Influences in Multi-component Maintenance, Quality and Reliability Engineering International, 13, pp. 199-207.