¹*Transmission of returns and volatility in art markets: A multivariate GARCH analysis*

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This paper examines the transmission of returns and volatility among eight major art markets. The art indices included in the analysis are Contemporary Masters (CM), 20th Century English (TE), 19th Century European (NE), French Impressionist (FI), Modern European (ME), Modern US Paintings (US), Old Masters (OM) and Surrealists (SR). A multivariate generalised autoregressive conditional heteroskedasticity (MGARCH) model is used to identify the source and magnitude of spillovers. The results indicate the presence of large and predominantly positive mean return and volatility spillovers, though the spillovers between art markets are not homogeneous.

I. INTRODUCTION

It goes without saying that art markets differ substantially from financial markets. Most art markets would appear to be characterised by product heterogeneity, illiquidity, market segmentation, information asymmetries, behavioural abnormalities, and monopolistic price setting. And there is no doubting the fact that a substantial component of the return from art investment is derived not from financial returns rather from intrinsic aesthetic qualities. However, in recent years it has been widely accepted that most art markets have moved closer to the ideals set by financial markets. Turnover has increased dramatically in a globalised setting, information on alternative art investments is now more accessible, and the publishing and dissemination of catalogues has increased the amount of information available to both buyers and sellers.

The growing interest in the economics of art markets [see, for instance, Pesando (1993), Goetzmann (1993), de-la Barre et al. (1994), Frey and Eichenberger (1995a; 1995b), Curry (1998), Frey and Pommerehne (1998), Flores et al. (1999) and Pesando and Shum (1999)] has

¹ This is the authors' version of a paper that was later published as:

Higgs, Helen and Worthington, Andrew (2004) Transmission of returns and volatility in art markets: a multivariate GARCH analysis. *Applied Economics Letters* 11(4):pp. 217-222.

prompted a number of studies to examine the mechanism by which art market returns are transmitted amongst each other, analogous to the well-investigated linkages among financial markets. Most art market studies have relied upon zero-order correlation analysis and just a few studies of market interdependencies have employed Granger-causality testing of market indices [see, for example, Ginsburgh and Jeanfils (1995) and Chanel (1995)]. No work to date into art market interrelationships has availed itself of the sizeable advances in modeling that take into account the time-varying properties of art market data. To meet this deficiency, this note examines the relationships between eight major art markets using a multivariate generalised autoregressive conditional heteroskedasticity (MGARCH) approach (Engle and Kroner 1995; Gallagher and Twomey 1998; Dunne 1999).

II. DATA AND DESCRIPTIVE STATISTICS

The data employed in the study is composed of indices for eight major categories of paintings, namely: Contemporary Masters (CM), 20^{th} Century English (TE), 19^{th} Century European (NE), French Impressionist (FI), Modern European (ME), Modern US Paintings (US), Old Masters (OM) and Surrealists (SR). All art index data is obtained from UK-based Art Market Research (AMR) and encompasses the period January 1976 to February 2001. All monthly index data is specified in US dollars with the return in market *i* represented by the continuously compounded return or log return of the index at time *t*. Selected descriptive statistics of the monthly returns for the eight art indexes are presented in Table 1.

<TABLE 1 HERE>

Table 1 presents the summary of descriptive statistics of the annualised returns for the eight art markets. Samples means, medians, maximums, minimums, standard deviations, skewness, kurtosis and the Jacque-Bera statistic and *p*-value are reported for the annualised art returns. The highest mean annual returns are in Contemporary Masters and French Impressionist and the lowest are in Modern European and Surrealists. Of the eight art markets, 19th Century European and Old Masters are the least volatile, while French Impressionist and Modern US Paintings are the most volatile. Using the coefficient of variation (standard deviation divided by the mean return) the degree of risk in relation to the mean return is lower for Contemporary Masters and Old Masters than Modern European and Surrealists. Jarque-Bera statistics fail to reject the null hypotheses that the monthly

distributions of returns are normally distributed for all markets with the exception of French Impressionists.

III. MULTIVARIATE GARCH MODEL

The following model is developed to examine the joint processes relating the monthly rates of return for the eight art markets. The sample period is chosen on the basis that it represents the longest common time period over which data for most of the major art markets is available. The following conditional expected return equation accommodates each market's own returns and the returns of other markets lagged one period:

$$R_t = \alpha + AR_{t-1} + \varepsilon_t \tag{1}$$

where R_t is an $n \times 1$ vector of weekly returns at time t for each market and $\varepsilon_t | I_{t-1} \sim N(0, H_t)$. The $n \times 1$ vector of random errors, ε_t is the innovation for each market at time t with its corresponding $n \times n$ conditional variance-covariance matrix, H_t . The market information available at time t - 1 is represented by the information set I_{t-1} . The $n \times 1$ vector, α , represent long-term drift coefficients. The elements a_{ij} of the matrix A are the degree of mean spillover effect across markets, or put differently, the current returns in market i that can be used to predict future returns (one month in advance) in market j. The estimates of the elements of the matrix, A, can provide measures of the significance of the own and cross-mean spillovers. This multivariate structure then enables the measurement of the effects of the innovations in the mean returns of one series on its own lagged returns and those of the lagged returns of other markets.

For the purposes of the analysis, the BEKK (Baba, Engle, Kraft and Kroner) model is employed, whereby the variance-covariance matrix of equations depends on the squares and cross products of innovation ε_t and volatility H_t for each market lagged one period. One important feature of this specification is that it builds in sufficient generality, allowing the conditional variances and covariances of the stock markets to influence each other, and, at the same time, does not require the estimation of a large number of parameters (Bollerslev 1990; Bollerslev et al. 1992; Karolyi 1995). The model also ensures the condition of a positive semi-definite conditional variance-covariance matrix in the optimisation process, and is a necessary condition for the estimated variances to be zero or positive. The BEKK parameterisation for the MGARCH model is written as:

$$H_{t} = B'B + C'\varepsilon_{t}\varepsilon_{t-1}C + G'H_{t-1}G$$
⁽²⁾

where b_{ij} are elements of an $n \times n$ symmetric matrix of constants *B*, the elements c_{ij} of the symmetric $n \times n$ matrix *C* measure the degree of innovation from market *i* to market *j*, and the elements g_{ij} of the symmetric $n \times n$ matrix *G* indicate the persistence in conditional volatility between market *i* and market *j*. This can be expressed for the bivariate case of the BEKK as:

$$\begin{bmatrix} H_{11t}H_{12t} \\ H_{21t}H_{22t} \end{bmatrix} = B'B + \begin{bmatrix} c_{11}c_{12} \\ c_{21}c_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1}^2 \varepsilon_{2t-1} \\ \varepsilon_{2t-1}\varepsilon_{1t-1}\varepsilon_{2t-1}^2 \end{bmatrix} \begin{bmatrix} c_{11}c_{12} \\ c_{21}c_{22} \end{bmatrix} + \begin{bmatrix} g_{11}g_{12} \\ g_{21}g_{22} \end{bmatrix} \begin{bmatrix} H_{11t-1}H_{12t-1} \\ H_{12tt-1}H_{22t-1} \end{bmatrix} \begin{bmatrix} g_{11}g_{12} \\ g_{21}g_{22} \end{bmatrix}$$
(3)

In this parameterization, the parameters b_{ij} , c_{ij} and g_{ij} cannot be interpreted on an individual basis: "...instead, the functions of the parameters which form the intercept terms and the coefficients of the lagged variance, covariance, and error terms that appear are of interest" (Kearney and Patton 2000: 36). With the assumption that the random errors are normally distributed, the log-likelihood function for the MGARCH model is:

$$L(\theta) = -\frac{Tn}{2} + \ln(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \left(\ln \left| H_t \right| + \varepsilon_t' \left| H_t^{-1} \right| \varepsilon_t \right)$$

$$\tag{4}$$

where *T* is the number of observations, *n* is the number of markets, θ is the vector of parameters to be estimated, and all other variables are as previously defined. The BHHH (Berndt, Hall, Hall and Hausman) algorithm is used to produce the maximum likelihood parameter estimates and their corresponding asymptotic standard errors. Overall, the proposed model has sixty-four parameters in the mean equations, excluding the eight constant (intercept) parameters, and thirty-five intercept, thirty-five white noise and thirty-five volatility parameters in the estimation of the covariance process, giving one hundred and seventy-seven parameters in total. The Ljung-Box *Q* statistic is used to test for independence of higher relationships as manifested in volatility clustering by the MGARCH model [Huang and Yang 2000:329]. This statistic is given by:

$$Q = T(T+2)\sum_{j=1}^{p} (T-j)^{-1} r^{2}(j)$$
(5)

where r(j) is the sample autocorrelation at lag *j* calculated from the noise terms and *T* is the number of observations. *Q* is asymptotically distributed as χ^2 with (p - k) degrees of freedom

and k is the number of explanatory variables. The test statistic in (5) is used to test the null hypothesis that the model is independent of the higher order volatility relationships.

IV. EMPIRICAL RESULTS

The estimated coefficients and standard errors for the conditional mean return equations are presented in Table 2. All art markets examined exhibit significant own mean return spillovers with values ranging from 0.3644 (OM) to 0.5675 (NE). Just four of the eight art markets, namely CM, OM, NE and FI exhibit significant mean return spillovers from lagged returns in other markets. The mean return for CM is influenced by its own lagged returns and those of SR, OM is influenced by its own lagged returns and also CM and NE, the mean return for NE is influenced by the lagged returns in CM and ME along with its own lagged returns, and FI is influenced by OM and its own lagged returns. The mean returns in ME, SR, TE and US are influenced only by their lagged returns.

<TABLE 2 HERE>

Importantly, the mean return spillovers from lagged art markets are not homogeneous. For example, a one percent increase in the lagged NE art market will Granger-cause the OM art market to increase by 0.103 percent over the following month, while the lagged return in CM Granger-causes an increase in the order of 0.119 percent. However, in reference to NE a one percent increase in CM will Granger-cause a 0.234 percent increase while ME will Granger-cause a 0.3803 percent increase. While innovations from most of the markets do get eventually incorporated into other markets with a lag, across all markets it is generally found that the magnitudes of causation for own mean return spillovers are larger than those for the cross mean return spillovers. This suggests that art markets, at least those examined in the current analysis, are relatively isolated from each other in terms of mean return spillovers.

Table 3 presents the estimated coefficients for the variance covariance matrix of equations. The conditional variance covariance equations effectively capture the volatility and cross volatility spillovers among the eight art markets. These quantify the effects of the lagged own and cross innovations and lagged own and cross volatility persistence on the present own and cross volatility of the eight art markets, indicating the presence of a weak ARCH effect, but a strong GARCH effect. There is then little evidence of innovation spillover effect but strong

evidence of conditional volatility. Overall, there is strong evidence of time-variation in market risk.

<TABLE 3 HERE>

The coefficients of the variance covariance equations are insignificant for own and cross innovations and significant for own and cross volatility spillovers to the individual returns for all art markets. This suggests the presence of weak ARCH effects. Own-innovation spillover effects range from 0.0093 (ME) to 0.0975 (CM). In terms of cross-innovation effects in the art markets, past innovations in TE have the greatest effect on future volatility in CM from among past innovations in other art market returns. However, in the case of TE and US past innovations in CM have the greatest influence on future volatility.

In the GARCH set of parameters, seventy-three percent of the estimated coefficients are significant. For NE the lagged volatility spillover effects range from 0.61 for TE to 0.94 for OM. This means that past volatility shocks in OM have a greater effect on future NE volatility over time than past volatility shocks in other art returns. Conversely, in US the past volatility shocks range from 0.31 for SR to 0.90 for NE. In terms of cross-volatility for the GARCH parameters, the most influential market would appear to be NE. That is, past volatility shocks in the NE art market have the greatest effect on future volatility in the remaining seven art markets.

The sum of the ARCH and GARCH coefficients measures the overall persistence in own and cross conditional volatility. There is evidence of a weak own persistent volatility of 0.1078 for ME while the remaining seven markets exhibit strong own persistent volatility ranging from 0.8723 for TE to 0.9521 for CM. Thus, CM has a lead-persistence volatility spillover effect on the remaining markets. The cross-volatility persistence spillover effects range from 0.3934 for SR to 0.9697 for NE to US. Finally, the Ljung-Box (LB) Q statistics for the standardised residuals in Table 4 reveal that all the art markets are significant (all have p-values of less than .05) with the exception of US (a p-value of 0.6891). Significance of the Ljung-Box (LB) Q statistics for the return art series indicates linear dependences due to the strong conditional heteroskedasticity. The seven art markets with Ljung-Box statistics at 12 degrees of freedom are significantly greater than the Ljung-Box for US. These Ljung-Box statistics suggest a strong linear dependence in seven out of eight art markets estimated by the MGARCH model.

<TABLE 4 HERE>

V. CONCLUSION

This paper examines the transmission of art returns and volatility among eight major art markets during the period 1976 to 2001. A multivariate generalised autoregressive conditional heteroskedasticity (MGARCH) model is used to identify the source and magnitude of spillovers. The estimated coefficients from the conditional mean return equations indicate that all eight art markets exhibit significant own mean return spillovers. Own-volatility spillovers are also generally higher than cross-volatility spillovers for all markets, indicating the presence of strong GARCH effects. Strong own and cross-persistent volatility are also evident in the art markets examined.

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	СМ	FI	ME	NE	OM	SR	TE	US
Mean	4.2090	3.7045	2.1398	2.4645	2.8132	2.0307	2.5541	3.3180
Median	3.9766	6.0617	2.7789	1.3484	3.1091	3.0168	3.2300	1.5463
Maximum	29.7088	34.2042	21.6283	17.0330	18.2180	22.6799	13.4611	26.4745
Minimum	-15.2562	-40.5108	-23.7159	-16.0957	-8.1525	-29.3774	-12.1199	-27.4104
Std. Deviation	10.5006	13.6610	11.2592	7.1423	7.5800	11.2977	7.6386	12.7691
Skewness	0.4858	-0.9286	-0.4300	-0.0624	0.3586	-0.6922	-0.2471	-0.1118
Kurtosis	3.4723	5.9786	2.5919	3.5388	1.9856	3.8568	2.0100	2.8911
CV	2.4948	3.6876	5.2618	2.8981	2.6944	5.5636	2.9908	3.8485
Jarque-Bera	1.2643	13.3475	0.9818	0.3313	1.6718	2.8718	1.3265	0.0670
<i>p</i> -value	0.5314	0.0013	0.6121	0.8473	0.4335	0.2379	0.5152	0.9670

Table 1. Descriptive statistics of annual returns for eight art markets, 1976-2001

CM – Contemporary Masters, FI – French Impressionists, ME – Modern European, NE – 19th Century European, OM – Old Masters, SR – Surrealists, TE – 20th Century English, US – Modern US Paintings.

Table 2. Estimated coefficients for conditional mean return equations

	Estimated coefficient	Standard error	Estimated coefficient	Standard error	Estimated coefficient	Standard error	Estimated coefficient	Standard error	
	CM (i	= 1)	FI (i	= 2)	ME (i	i = 3)	NE (<i>i</i> = 4)		
CONS.	0.0367	0.1116	0.0334	0.1529	-0.0487	0.1550	0.0055	0.0780	
a_{i1}	***0.5083	****0.5083 0.1266 0		0.0845	0.0438	0.1213	*0.2341	0.1749	
a_{i2}	0.0815			**0.4525 0.1407		0.1761	0.3682	0.3232	
a_{i3}	0.0405 0.1416		-0.0810 0.1345		**0.5229	0.1495	*0.3803 0.27		
a_{i4}	0.0309 0.0780		0.0083 0.0748		0.0015	0.0884	****0.5675 0.1529		
a_{i5}	-0.0334	-0.0334 0.0803		*0.1487 0.0905		0.0884	0.0793	0.1621	
a_{i6}	*0.1769	*0.1769 0.1182		0.1199	0.1103	0.1460	0.0865	0.2718	
a_{i7}	0.0712 0.1118		-0.0314	0.1201	-0.0648	0.1389	0.1903	0.2245	
a_{i8}	0.1470 0.1975		-0.0171 0.2185		-0.0400 0.2428		0.2351	0.4347	
	OM (a	<i>i</i> = 5)	SR (i	= 6)	TE (<i>i</i>	= 7)	US (<i>i</i> = 8)		
CONS.	0.0660 0.1154 -0.0586		-0.0586	0.1297	0.1153	0.1166	0.0019	0.2624	
a_{i1}	*0.1195	0.0836	0.0418	0.1130	0.0126	0.1325	0.0098	0.0780	
a_{i2}	0.1594	0.1756	0.0991	0.1476	0.0009	0.1859	0.0432	0.1130	
a_{i3}	0.1246	0.1220	0.0909	0.1413	-0.0030	0.1714	0.0057	0.0807	
a_{i4}	*0.1030	0.0684	0.0131	0.0733	0.0717	0.0716	0.0206	0.0502	
a_{i5}	***0.3644	0.1299	0.0207	0.0827	-0.0050	0.1011	0.0018	0.0705	
a_{i6}	0.1405	0.1228	***0.4095	0.1240	0.1473	0.1442	-0.0532	0.0958	
a_{i7}	0.0418	0.1127	0.0928	0.1146	***0.4137	0.1635	-0.0310	0.0848	
a_{i8}	0.1716	0.2088	-0.0680	0.2103	0.2326	0.2640	****0.5070	0.1538	

Asterisks indicate significance at the * - 0.10, ** - 0.05 and *** - 0.01 level

Table 4. Ljung-Box tests for standardized residuals

	СМ	FI	ME	NE	OM	SR	TE	US
Statistic	43.8000	78.9670	38.2040	48.7630	65.4240	65.5530	61.2360	9.1620
<i>p</i> -value	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.6891

	CM (j	= 1)	FI (j =	= 2)	ME (j	= 3)	NE (j	= 4)	OM (j	= 5)	SR (j	= 6)	TE (j	= 7)	US (j	= 8)
	Estimated	Standard	Estimated	Standard	Estimated	Standard		Standard		Standard	Estimated	Standard	Estimated	Standard		Standard
	coefficient	error	coefficient	error	coefficient	error	coefficient	error	coefficient	error	coefficient	error	coefficient	error	coefficient	error
$b_{1\mathrm{j}}$	*0.0376	0.0263	0.0118	0.0228	0.0511	0.1029	0.0229	0.0311	0.0135	0.0676	0.0176	0.0295	0.0624	0.0744	0.0749	0.0663
$b_{2\mathrm{j}}$	0.0118	0.0228	0.0922	0.1461	0.0572	0.1073	0.0110	0.0195	-0.0010	0.0792	0.0651	0.1216	0.0127	0.0399	0.1001	0.1135
$b_{3\mathrm{j}}$	0.0511	0.1029	0.0572	0.1073	0.8897	1.6014	0.0123	0.0280	-0.0026	0.0109	0.1284	0.2266	0.0524	0.1122	0.0788	0.1665
$b_{ m 4j}$	0.0229	0.0311	0.0110	0.0195	0.0123	0.0280	0.0441	0.0597	0.0013	0.0065	0.0101	0.0320	0.0487	0.0868	0.0079	0.0217
$b_{5\mathrm{j}}$	0.0135	0.0676	-0.0010	0.0792	-0.0026	0.0109	0.0013	0.0065	0.0755	0.1252	0.0012	0.0350	0.0049	0.0229	-0.0054	0.0608
$b_{ m 6j}$	0.0176	0.0295	0.0651	0.1216	0.1284	0.2266	0.0101	0.0320	0.0012	0.0350	0.0729	0.0872	0.0054	0.0193	0.1753	0.3285
$b_{7\mathrm{j}}$	0.0624	0.0744	0.0127	0.0399	0.0524	0.1122	0.0487	0.0868	0.0049	0.0229	0.0054	0.0193	0.1105	0.1805	0.1040	0.1679
$b_{8\mathrm{j}}$	0.0749	0.0663	0.1001	0.1135	0.0788	0.1665	0.0079	0.0217	-0.0054	0.0608	0.1753	0.3285	0.1040	0.1679	0.2284	0.2780
c _{1j}	**0.0975	0.0533	0.0485	0.0387	0.0500	0.0730	0.1061	0.1012	0.0740	0.1164	0.0901	0.0724	*0.1166	0.0902	$^{*}0.0886$	0.0655
c_{2j}	0.0485	0.0387	0.0249	0.0271	0.0247	0.0323	*0.0679	0.0449	0.0747	0.0644	0.0344	0.0523	0.0364	0.0607	0.0768	0.0696
c _{3j}	0.0500	0.0730	0.0247	0.0323	0.0093	0.0722	0.0532	0.0845	0.0254	0.0549	0.0489	0.0751	0.0655	0.1337	0.0635	0.0873
c_{4j}	0.1061	0.1012	*0.0679	0.0449	0.0532	0.0845	0.0757	0.0973	0.0266	0.0624	0.0640	0.0925	0.0768	0.1226	0.0656	0.0692
c _{5j}	0.0740	0.1164	0.0747	0.0644	0.0254	0.0549	0.0266	0.0624	0.0325	0.0339	$^{*}0.0792$	0.0524	0.0528	0.0698	0.1210	0.1708
C _{6j}	0.0901	0.0724	0.0344	0.0523	0.0489	0.0751	0.0640	0.0925	$^{*}0.0792$	0.0524	0.0473	0.0473	0.0165	0.0503	0.0825	0.0953
<i>c</i> _{7j}	*0.1166	0.0902	0.0364	0.0607	0.0655	0.1337	0.0768	0.1226	0.0528	0.0698	0.0165	0.0503	0.0565	0.0904	0.0768	0.0886
\mathcal{C}_{8j}	$^{*}0.0886$	0.0655	0.0768	0.0696	0.0635	0.0873	0.0656	0.0692	0.1210	0.1708	0.0825	0.0953	0.0768	0.0886	0.0578	0.0488
$g_{1\mathrm{j}}$	***0.8546	0.0619	***0.8749	0.1148	*0.6987	0.4947	****0.7529	0.2082	0.4637	1.2772	0.8230	0.2847	*0.5853	0.4041	***0.7718	0.1573
$g_{2\mathrm{j}}$	****0.8749	0.1148	***0.9136	0.1021	****0.8730	0.2156	****0.8946	0.0826	0.5288	0.6382	***0.8230	0.2847	****0.9059	0.2348	****0.6935	0.2642
$g_{ m 3j}$	*0.6987	0.4947	***0.8730	0.2156	0.0985	1.5498	****0.8613	0.2203	****0.9386	0.1251	0.5948	0.6858	0.6784	0.5512	0.6700	0.5618
$g_{4\mathrm{j}}$	***0.7529	0.2082	***0.8946	0.0826	****0.8613	0.2203	****0.7969	0.2357	***0.9421	0.1090	***0.8257	0.3490	0.6050	0.5919	***0.9041	0.1629
g_{5j}	0.4637	1.2772	0.5288	0.6382	***0.9386	0.1251	***0.9421	0.1090	***0.8804	0.1541	***0.7766	0.2733	*0.7863	0.4798	**0.7497	0.3861
$g_{ m 6j}$	0.8489	0.1263	***0.8230	0.2847	0.5948	0.6858	****0.8257	0.3490	***0.7766	0.2733	***0.8925	0.0954	***0.9330	0.1663	0.3109	1.2645
$g_{7\mathrm{j}}$	*0.5853	0.4041	***0.9059	0.2348	0.6784	0.5512	0.6050	0.5919	*0.7863	0.4798	***0.9330	0.1663	***0.8158	0.2723	*0.6496	0.4447
g_{8j}	***0.7718	0.1573	****0.6935	0.2642	0.6700	0.5618	****0.9041	0.1629	**0.7497	0.3861	0.3109	1.2645	*0.6496	0.4447	***0.8422	0.1546

Table 3. Estimated coefficients for variance covariance equations

Asterisks indicate significance at the * - 0.10, ** - 0.05 and *** - 0.01 level