INSTANTANEOUS FREQUENCY ESTIMATION USING THE CROSS WIGNER-VILLE DISTRIBUTION WITH APPLICATION TO NON-STATIONARY TRANSIENT DETECTION

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ABSTRACT

This paper proposes a new method for instantaneous frequency (IF) estimation of rapidly time-varying signals, based on an iterative procedure incorporating Time-Frequency Distributions (TFDs). This method is then applied to the problem of adaptively detecting a transient of unknown waveshape in white gaussian noise. For this type of adaptive detection, the signal's time-frequency representation is first estimated and this estimate is used as the true representation in a time-frequency correlator detector. In the methodology proposed in this paper, the IF is assumed to be a critical feature of the transient. Accordingly, the IF is first estimated, and then this estimate is used as an aid to estimate the time-frequency representation of the true signal. This representation is then correlated with the time-frequency representation of the observed signal to provide the appropriate detection statistic.

1. INTRODUCTION

A large number of natural signals exhibit non-stationary spectral behaviour, and for such signals the concept of instantaneous frequency is more useful than the traditional notion of frequency. In Section 2 a new technique is presented for estimating the IF of a signal in a high noise environment. This technique generalises the maximum likelihood procedure for estimating the frequency of a stationary sinusoid in gausssian noise (i.e. extracting the peak magnitude from a periodogram [1]). The procedure for the stationary signal case can be thought of as optimally concentrating the signal energy in frequency so that there is a high likelihood that the true frequency will be correctly estimated as the peak against a broadly dispersed noise background. The generalisation to the non-stationary case involves first projecting the signal onto a time-frequency space where there is high energy concentration, and then extracting the IF estimate as the peak magnitude. The time-frequency space used is the XWVD, a representation which is known to have good noise performance [2] in addition to high energy concentration. Simulations are presented to compare this method with other estimation methods. These other methods include 1) differencing the phase of the analytic signal [3], 2) recursive Least Squares (RLS) adaptive techniques and 3) peak detection from the Short-Time-Fourier Transform (STFT). The XWVD based scheme is seen to compare very favourably with these alternate methods.

This XWVD based IF estimation method is applied to the problem of detecting a signal of unknown waveshape. The solution to this problem involves a three step methodology:

Step 1: Estimation of the IF of the Signal
The XWVD is used to estimate the IF.

Step 2: Formation of the Time-Frequency Signal Estimate
For optimal detection of unknown signals, it is necessary to first estimate the time-frequency representation of the true signal, and to then use this as reference in a conventional detector. This type of detection is refered to as adaptive. Accordingly, from the IF estimate obtained in Step 1, a unit amplitude "reference" signal is estimated. This signal is then used, along with the observed signal, to form a XWVD exhibiting high signal energy concentration. This time-frequency representation can then be windowed or filtered to produce a time-frequency estimate of the reference signal.

Step 3: Correlation of Time-Frequency Representations to Form the Detection Statistic
The 'reference XWVD' estimate is correlated with the 'observed XWVD' to yield the required detection statistic. Where the reference XWVD is estimated without error, this is equivalent to matched filtering for white Gaussian noise [2]. The particular practical implementation presented in this paper, however, can be shown to be equivalent to a selective energy detector (i.e. a detector based on energy measures from selected regions of the time-frequency plane [4]). Simulations are presented to show that the XWVD based scheme performs better than classical methods (i.e. the energy detector) for the non-stationary signal used.

If the signal is multicomponent, then the signal can be broken down into approximately monocomponent parts, and each of the parts can be processed along the lines of the above steps.

2. IF ESTIMATION PROCEDURE

The XWVD between a reference signal, s(t), and an observed signal, r(t), is defined as:

$$W_{z_s z_r}(t,f)=\int_{-\infty}^{+\infty} z_s(t+\tau/2)\, z_r^*(t-\tau/2)e^{-j2\pi f\tau}d\tau \qquad (1)$$

where $z_s(t)$ and $z_r(t)$ are analytic signals corresponding to s(t) and r(t) respectively. Both are time limited between -T/2 and +T/2.

The proposed IF estimation procedure is as follows:

1. Obtain an initial IF estimate and from it form a unit amplitude estimate, $\hat{s}(t)$, of the reference signal according to:

$$z_{\hat{s}}(t) = \Pi_T(t)\, e^{j2\pi\int_{-T/2}^{t}\hat{f}_i(\alpha)d\alpha} \qquad (2)$$

where $\hat{s}(t) = \mathrm{Re}\{ z_{\hat{s}}(t) \}$, and $\hat{f}_i(t)$ is the IF estimate.

2. Form a XWVD between the estimated reference signal, $\hat{s}(t)$, and the observed signal, r(t). Extract the peak magnitude of this representation as the new IF estimate.

3. Repeat from Step 1 until the difference in IF estimates from successive iterations is less than a specified amount.

The idea behind the method is that each time a new XWVD is estimated the signal energy concentration should increase, so that the probability of correctly estimating the IF in a noise background should also increase. Note that a number of estimators could be used as the starting value for the IF. An STFT based peak detection estimate was chosen for simulations in this paper since its comparatively low variance would be likely to reduce the number of iterations required for convergence.

2.1 Performance of IF Estimation using the XWVD

Consider a XWVD formed from the observed signal, r(t), and the unit amplitude estimated reference signal, $\hat{s}(t)$. Both signals are time limited between -T/2 and +T/2:

$$W_{z_r z_{\hat{s}}}(t,f) = \int_{-T/2}^{+T/2} z_r(t+\tau/2) z_{\hat{s}}^*(t-\tau/2)\, e^{-j2\pi f\tau}\, d\tau \tag{3}$$

$$= \underset{\tau\to f}{\mathcal{F}} \left[z_r(t+\tau/2) z_{\hat{s}}^*(t-\tau/2)\right] \tag{4}$$

where $\underset{\tau\to f}{\mathcal{F}}$ signifies Fourier Transformation in the τ variable, and the synthesised signal is formed from the initial IF estimate, $\hat{f}_i$, according to:

$$z_{\hat{s}(t)} = \Pi_T(t)\, e^{j2\pi\int_{-T/2}^{t} \hat{f}_i(t)dt} \tag{5}$$

The initial IF estimate will actually differ from the true IF by some error term. The IF estimate, then, may be considered to be the sum of a true term and an error term:

$$\hat{f}_i(t) = f_i(t) + f_e(t) \tag{6}$$

Then, in the time domain $z_{\hat{s}}(t)$ could consequently be written as the product of a "true" complex signal term and an "error" complex signal term both exhibiting unit amplitude. Thus,

$$z_{\hat{s}}(t) = z_s(t).z_e(t),$$

where $z_s(t) = \Pi_T(t)\, e^{j2\pi\int_{-T/2}^{t} f_i(t)dt}$ and

$$z_e(t) = \Pi_T(t)\, e^{j2\pi\int_{-T/2}^{t} f_e(t)dt} \tag{7}$$

Eqn.(4) may then be rewritten:

$$W_{z_r z_{\hat{s}}}(t,f) = \underset{\tau\to f}{\mathcal{F}} \left[z_r(t+\tau/2) z_s^*(t-\tau/2).z_w(t+\tau/2) z_e^*(t-\tau/2)\right] \tag{8}$$

where $z_w(t)$ is a unit amplitude rectangular window of the same extent as $z_r(t)$. Rearrangement gives

$$W_{z_r z_{\hat{s}}}(t,f) =$$

$$\underset{\tau\to f}{\mathcal{F}} \left[z_r(t+\tau/2) z_s^*(t-\tau/2)\right] \underset{f}{*} \underset{\tau\to f}{\mathcal{F}} \left[z_w(t+\tau/2) z_e^*(t-\tau/2)\right] \tag{9}$$

$$= W_{z_r z_s}(t,f) \underset{f}{*} W_{z_w z_e}(t,f) \tag{10}$$

From (10) it can be seen that the XWVD formed with an estimated reference signal is the "true" XWVD smeared by the XWVD formed from the error signal term and a unit rectangular window function. The latter is essentially the running frequency scaled Fourier Transform of the error component term, $z_e(t)$.

Where the initial IF estimation error has a constant bias and zero variance, the resulting XWVD estimate will simply be a shifted version of the true XWVD, the shift being equal to half the bias. Thus at each new iteration of the proposed scheme the bias will be reduced to half its previous value. The XWVD estimate, then, will eventually converge to the true value.

Where the initial IF estimate has zero bias but non-zero variance, general expressions of the smearing factor in (10) are difficult to obtain. Some expressions for limiting cases, however, are provided in [5]. It is also shown in [5] that for the case of linearly modulated chirp signals, one can expect the XWVD scheme to perform better than peak detection from the STFT, under a certain condition. This condition is that the bandwidth of the error component term, $z_e(t)$, is less than twice the bandwidth of the chirp signal within the window used for the STFT. This fits intuitively with what one might expect. Clearly, if the observed signal is stationary, the XWVD will not provide any better energy concentration than the STFT, and additionally, errors would be introduced in estimating the reference signal for the XWVD. One would therefore not expect the XWVD to perform better than the STFT for stationary signals. As the frequency content of the signal varies significantly, though, improvements in energy concentration will override the effects of the uncertainty in the reference. Thus, for signals with significant non-stationarity, the XWVD scheme would be expected to perform better than the STFT.

2.2 Computer Simulations

Example - Zero bias, non-zero variance on the initial IF estimate:

For these examples the initial IF estimates were obtained using the STFT, with the bias being very close to zero. Due to difficulties in obtaining reliable fourier based estimates for small data lengths, the leading and trailing 10 points were determined by extrapolation. A typical initial IF estimate for a sinusoidally modulated FM signal in -1 dB noise is shown in Fig.1. The estimate was obtained from the peak of the STFT. The estimate after 2 iterations of the XWVD scheme is seen in Fig.2, and the true FM law is shown in Fig.3. The updated estimate is significantly enhanced over the original.

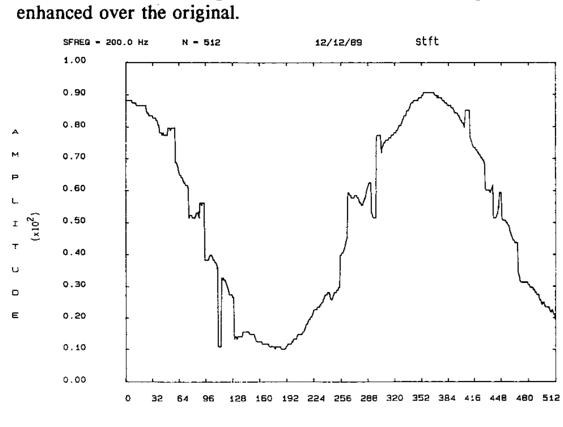

Fig.1 IF Estimate Using STFT Peak

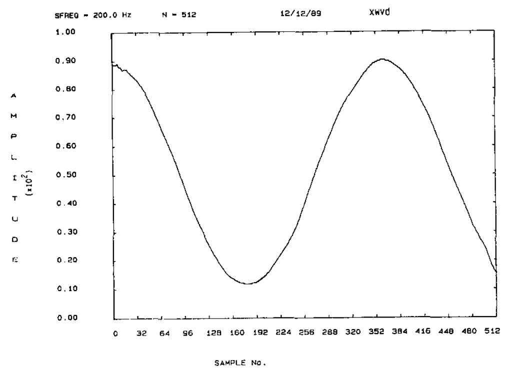

Fig.2 IF Estimate Using XWVD Scheme

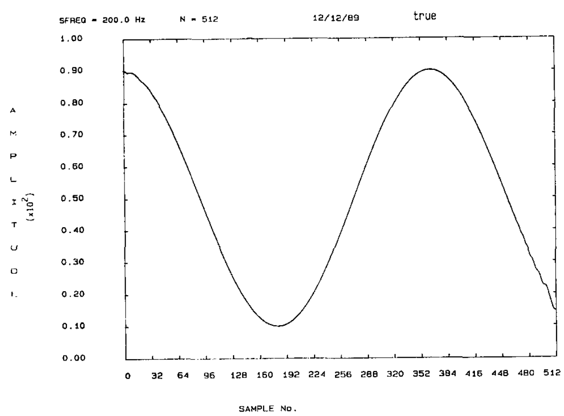

Fig.3 True IF Law

For comparison, estimates of the IF using Ville's direct definition [3] and the Recursive Least Squares adaptive estimator [6] are shown in Figs.4 and 5 respectively. It is seen that these latter two estimators exhibit considerably poorer performance than both the STFT and XWVD based estimators.

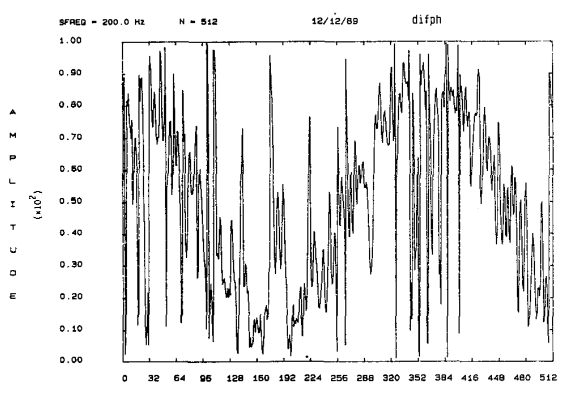

Fig.4 IF Estimate Using Analytic Signal Phase Difference

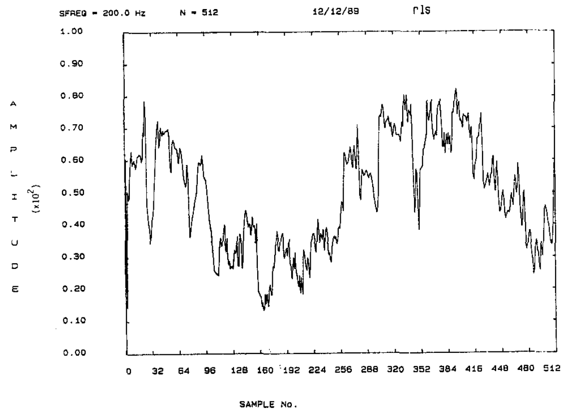

Fig.5 IF Estimate Using RLS Adaptive Algorithm

It should be noted that this scheme is particularly effective for signals exhibiting a frequency law which is close to linear. For the general case in which there is significant non-linearity in the IF, best results are likely to be achieved by adjusting the window length so that the IF variation is approximately linear within the window. This was the approach adopted for the simulations presented in Figs.1-5. A formula for optimally adjusting the window length may be found in [7] and [8].

If there is more than one signal component present in the signal, then after obtaining the IF estimate for the highest energy component, one can filter out this component and process for the next highest energy component. The filtering can be done by performing a dechirping transform on the signal to make the first component "stationary". This transformed signal can easily be filtered, and then an inverse or rechirping transform can be applied.

3. DETECTION OF AN UNKNOWN NON-STATIONARY SIGNAL

For a reference signal, s(t), and an observed signal, r(t), a detection statistic which is equivalent to the matched filtering detection statistic, is [9]:

$$\eta_{xwvd} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} W_{z_r z_x}(t,f)\, W_{z_s z_x}(t,f)\,dtdf \qquad (11)$$

where $z_x(t)$ is an arbitrary signal and $W_{z_r z_x}(t,f)$ is a time-frequency representation of the observed signal. The time-frequency representation of the reference signal may be estimated from $W_{z_r z_x}(t,f)$ by retaining the significant features through time-varying filtering. To do this most effectively the signal energy should be as concentrated as possible, and so as to achieve this goal $z_x(t)$ is constructed from the estimated IF of the observed signal, according to (2). Fig.6 shows the true time-frequency representation of an FM signal (modulated by an segment of a sinusoid). Fig.7 shows the XWVD of the same signal in noise, and Fig.8 shows the XWVD estimate of the same signal, the estimation being affected by time-varying filtering to eliminate the noise. The two representations shown in Figs.7 and 8 may then be correlated to yield the detection statistic described by (11).

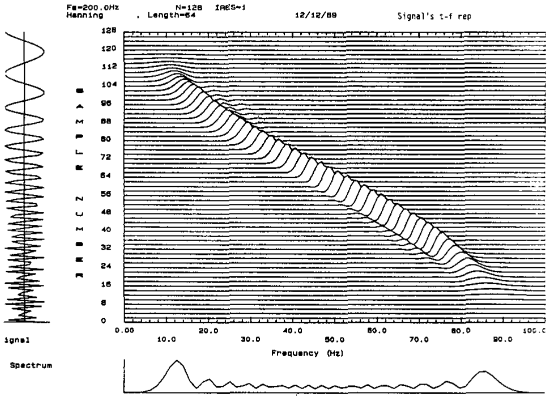

Fig.6 T-F Representation of Signal

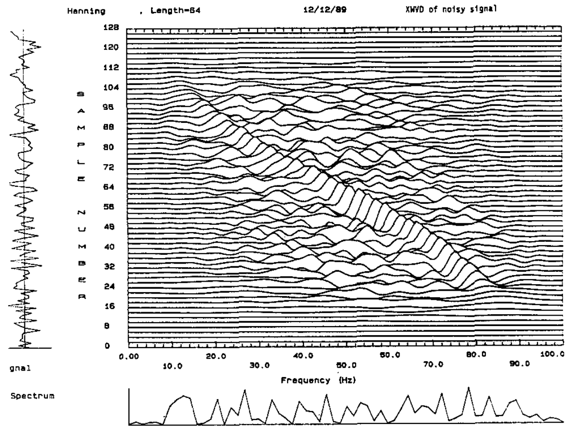

Fig.7 XWVD of Observed Signal

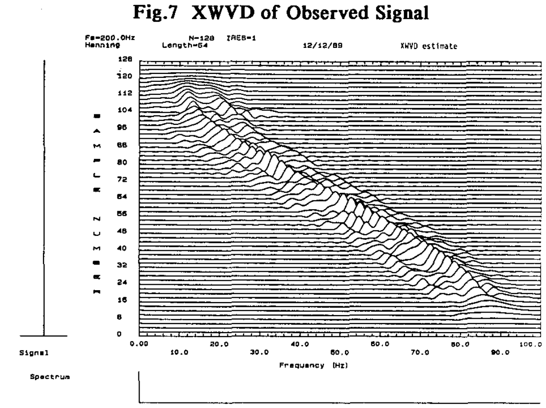

Fig.8 XWVD Estimate of Signal

The approach adopted for estimating the reference XWVD from the observed XWVD was to try to reject the noise away from the vicinity of the IF, by a windowing. The detector resulting from this type of estimation can easily be shown to be equivalent to a "selective" energy detector. i.e. a detector in which only energy from a selected region of the time-frequency plane contributes to the detection statistic. Fig.9 shows simulations comparing the XWVD based detector with the energy detector for the sinusoidally modulated signal shown in Fig.6. The XWVD based scheme is seen to perform better in this case, with the improvement arising from the energy "selectivity" factor.

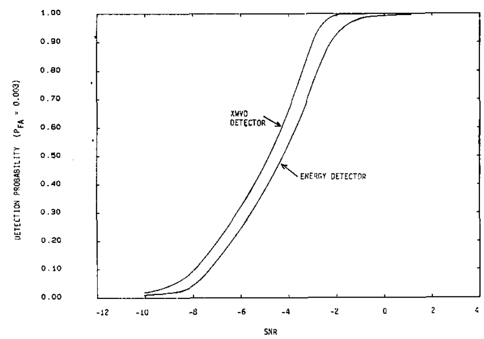

Fig.9 Comparison of XWVD Based Detector and Energy Detector for Observed Signal

4. CONCLUSION

A new method for estimating instantaneous frequency has been proposed, based on the Cross Wigner-Ville Distribution. The scheme is seen to perform very well for non-stationary signals in high noise environments. The scheme has been applied to adaptive detection of an unknown non-stationary signal for which the instantaneous frequency was a significant feature. The detection has been seen to be more effective than the classical energy detector for the signal considered.

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