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THE DIFFICULTIES STUDENTS EXPERIENCE IN GENERATING DIAGRAMS FOR NOVEL PROBLEMS

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Abstract *Although "draw a diagram" is advocated as a useful problem-solving strategy, generating an appropriate diagram is problematic for many students. This case study explored primary-aged students' difficulties in generating diagrams for novel problems. Three categories of difficulties were identified: (1) non-use of diagrams; (2) generic difficulties with diagrams and (3) idiosyncratic difficulties that were related to specific diagrams. The results suggest that there is a need for instruction in diagram use to empower students and address their difficulties. Specific attention needs to be given to: (1) the diagram-picture distinction; (2) the ambiguity of diagrams; and (3) the dynamic feature of diagrams.*

Background

"For all a rhetorician's rules; Teach nothing but to name his tools." *Hudibras 1663*

The use of the strategy *draw a diagram* is strongly advocated as a tool for problem solving (e.g., National Council of Teachers of Mathematics, 1989). A diagram is a particularly effective problem representation because it exploits spatial layout in a meaningful way, enabling complex processes and structures to be represented holistically (Winn, 1987). Generating a diagram facilitates the conceptualization of the problem structure and is the first step towards a successful solution (van Essen & Hamaker, 1990). However, it is fallacious to assume that diagrams are spontaneously effective tools for students (e.g., Dufoir-Janvier, Bednarz, & Belanger, 1987). As inadequate diagrammatic representations limit students' problem-solving capabilities (Klahr 1978), it is important to investigate factors that influence problem representation (Goldman, 1986). The purpose of this paper is to explore students' difficulties in generating diagrams for novel problems.

Representing problem information on a diagram involves the decoding of linguistic information and the encoding of visual information. During this translation process, there is the potential for knowledge acquisition (Karmiloff-Smith, 1990) through the reorganisation of information (Weinstein & Mayer, 1986) and subsequent inference-making (Lindsay, 1995). Knowledge acquisition depends on three components, namely, selective encoding, selective combination and selective comparison (Sternberg, 1990). Selective encoding relates to the relevance of the information that is represented. Some students' representations are unhelpful for problem solving because relevant problem information is not included (Dufoir-Janvier et al., 1987). Selective combination refers to how new information is integrated as a discrete entity. The diagram is an effective problem representation because problem information is indexed by location on a plane, which supports a large number of inferences (Larkin & Simon, 1987). Selective comparison focuses on the relationship between new knowledge about the content of the problem or diagram and prior knowledge. These components highlight the importance of knowledge about the types of diagrams that have applicability in problem solving.

General purpose diagrams (i.e, networks, hierarchies, matrices, and part-whole diagrams) are particularly important in problem solving because they provide representational frameworks that are applicable to a range of problem structures. For example, a matrix can be used to represent the problem structure in combinatorial tasks or in deductive reasoning tasks. Novick (1996) developed a theoretical framework for spatially-oriented diagrams, namely, matrices, networks, and hierarchies, that describes the conditions of applicability and distinguishing properties for each of these diagram types. Her framework does not include part-whole diagrams, which are conceptually-oriented diagrams and have no unique external form.

Design and Methods

The hypothesis that there would be an improvement in students' generation of diagrams after diagram-related instruction was tested using an explanatory case study design (Yin, 1994) and subsequently supported (Diezmann, 1999). As knowledge of students' difficulties is a key facet of pedagogical content knowledge (Carpenter, Fennema, & Franke, 1996), data from the case study was used in an inductive theory-building approach (Krathwohl, 1993) to develop a list of students' difficulties in generating a diagram to further inform instructional practice.

The participants in the case study were 12 Year 5 students with a mean age of 10 years 3 months from a moderately sized school in a lower socio-economic suburb in Brisbane, Australia. The participants comprised a cross-section of students, who were high and low performers in problem solving, and had high and low preferences for a visual method of solution. Isomorphic sets of five novel problems were presented to each participant, by an interviewer who was known to them, during 30 minute interviews conducted before and after instruction. The interviews were video-taped and subsequently transcribed. As the participants were not specifically instructed to use a diagram, those participants who did not spontaneously use a diagram were given further opportunities to generate a diagram.

Selective Results

The students experienced three categories of difficulties generating effective diagrams: (1) non-use of a diagram; (2) generic difficulties; and (3) idiosyncratic difficulties. Examples of difficulties in each category follows. By necessity, examples are illustrative rather than comprehensive due to space limitations.

Category 1: Non-use of a Diagram

The first category focuses on the reasons why students did not use a diagram. *Draw a diagram* is only one of many strategies that students might use in problem solving. However of particular concern are reasons why *draw a diagram* might not be part of students' repertoire of problem solving strategies.

A lack of understanding of the mathematical use of the term "diagram": Although the term "diagram" is commonly used in mathematics it cannot be assumed that it is understood by students as indicated by comments, such as "What's a diagram?" Further indication of a lack of understanding of the term can also be inferred from students' "diagrams" which depicted pictorial elements of a problem but lacked a representation of the relational elements of the problem (e.g., Figure 3). Confusion about the term is also demonstrated by students' synonymous use of the terms "diagram", "picture" and "drawing" without any qualification.

A lack of understanding of the concept of a "diagram": When students were unable to proceed with a non-diagrammatic strategy, they were encouraged to generate a diagram. However there was no recognition by some students that a diagram might assist them in problem solving. When the students were asked, "Would a diagram help?" their responses generally ranged from "no" to "maybe". Even when students responded positively, a reluctance to generate a diagram was still apparent. For example, while Frank conceded that a diagram might help, he baulked at drawing a diagram due to the time he perceived it would take. On a farmyard problem, he commented, "Yeh but it'd take ages because you'd have to draw a lot of chickens and a lot of pigs". Frank's comment suggests that he did not view the diagram as a means to the solution even when unsuccessful with another strategy.

A lack of understanding of the diagram as a representation that utilises scale: A feature of some diagrams is their facility to utilise scale, however a lack of knowledge of scale was one of the reasons why a particular student did not use a diagram. The following interaction between Jon (J) and the researcher (R) shows Jon's concern with depicting the size of the tree and his lack of knowledge of scale.

R:Is there anything that you can think of that you can draw that would help?
J: I thought of drawing a ten metre high, a ten metre tree and then each time going up five metres.
R:Have a quick go and see if you can do that, see if it's possible.
J: I don't have enough room to do a ten metre tree.
R:You don't have enough room for a ten metre tree?
J: (Shook his head.)

Category 2: Generic Difficulties in the Generation of a Diagram

The second category provides examples of generic difficulties that students experienced in generating diagrams. When the same difficulty was identified for a range of general purpose diagrams it was also included in this category.

Creating a diagram that was unusable: The reasons why students self-generated diagrams were unusable included: (a) being too small to represent all the relevant information; (b) having insufficient space around the diagram to extend the diagram; and (c) being too untidy to clearly see the elements of the diagram. For example, untidiness was a problem for Candice (C) and led to her discarding her diagram (See Figure 1). Drawing an unusable diagram is particularly problematic because students generally abandoned the strategy rather than redrew their diagrams.

R: And I saw that you also had a diagram there but you scribbled it out. What happened?

C: It (the diagram) didn't work because it was really messy and I couldn't do it because um too complicated (pointed to the diagram).

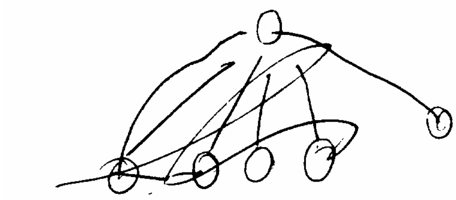
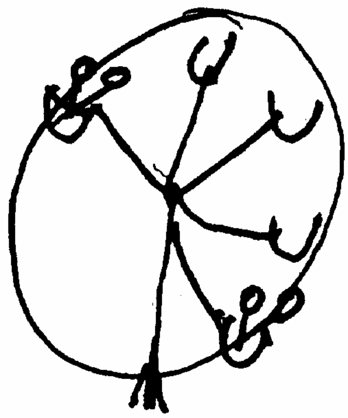
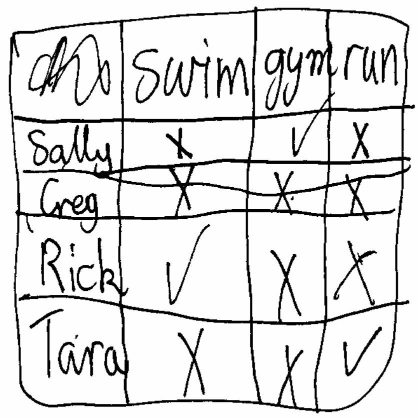


Figure 1. An untidy diagram.

Incorrect representation of quantity: A common error made by students was to represent quantities incorrectly. Examples of this error were evident in a variety of diagrams, such as those on Figure 2. While this error may simply reflect carelessness, it is an irrecoverable error because it was generally undetected by students and resulted in an incorrect answer. For example, in the matrix on Figure 2, Gemma did not detect her error even when she experienced difficulty utilising the clues in a deductive problem about a group of friends playing sport.



“Six” people on a ride



Four people and “four” sports

Figure 2. Incorrect representations of quantity.

Category 3: Idiosyncratic Difficulties in the Generation of a Diagram

The third category comprises the idiosyncratic difficulties that students experienced with networks, hierarchies, matrices, and part-whole diagrams. For illustrative purposes, examples of students' difficulties with networks and hierarchies are described for *The Koala* task and *The Party* respectively.

> **The Koala**: A sleepy koala wants to climb to the top of a gum tree that is 10 metres high. Each day the koala climbs up 5 metres, but each night, while asleep, slides back 4 metres. At this rate, how many days will it take the koala to reach the top?

A lack of precision in network diagrams: Network diagrams are useful for representing location. However students' diagrams often lack the requisite information to ascertain precise locations. In *The Koala* task, the koala's location can be identified by the number of metres the koala is above the ground. The lack of metre marks on Helen's (H) diagram became problematic when she tried to ascertain the exact location of the koala at a certain point in time (See Figure 3).

H: … he had to he climbed up another five and then he slept again and the second day when he climbed up I mean the third day he climbed up to the top.

R: How do you know he climbed up to the top are you sure or could he have been just a bit lower?

H: He might have been a bit lower.

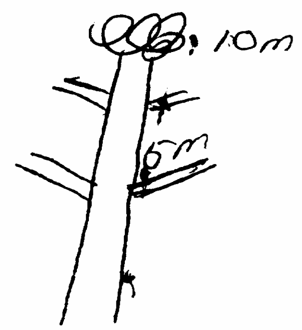


Figure 3. Difficulty identifying a precise location.

Overlooking the constraints in a network diagram: Locations on network diagrams are both provided in the problem information and produced as the student generates the diagram. In *The Koala* task, some students overlooked the height constraint of 10 metres as they progressively generated diagrams and tracked the koala's changing location. For example, Kate (K) produced the diagram in Figure 4 through a process of repeatedly moving up five metres and down four metres. However, she overlooked the goal of reaching 10 metres.

K: … and then he'd climb back up to number nine and slide back down to five and then he'd climb up to ten and slide back to six and then he'd climb up to eleven ...

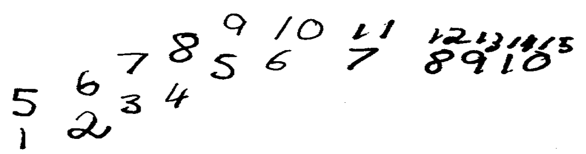


Figure 4: Extending beyond the constraints of the problem.

Labelling the starting position incorrectly on the network diagram: Measurement can also feature on network diagrams. A common error in *The Koala* task was to identify the base of the tree as one metre, as shown in the interaction between Damien (D) and the researcher (R). This error was not restricted to the use of standard measures but also occurred when students used non-standard measures. In another network problem, a number of students identified the base of a well as one brick high.

R: So where does he start?
D: He starts here. (Pointing to a line at the base of the tree)
R: He starts on that mark. What mark is that? What number?
D: The first one.
R: The first what?
D: Um. The first part of the tree.
R: Okay. If you told me in metres how many metres would it be?
D: One metre.

In summary, the difficulties students experienced with network diagrams tended to relate to location and measurement. Similarly, the difficulties students experienced with other general purpose diagrams were related to the uniqueness of each of those types of diagrams. Students' difficulties with matrices were associated with two-dimensional arrays; their difficulties with part-whole diagrams were related to determining the parts and wholes of sets; and their difficulties with hierarchies occurred in the representation of hierarchical relationships. An example of a difficulty unique to hierarchies follows.

The Party: At a party 5 people met for the first time. They all shook hands with each other once. How many handshakes were there altogether?

R: If you were telling me what to do how would I know which ones (lines representing people) to join?

G: Um. These two together (1-2 moving from left to right), those two together (2-3), those two together (3-4), and those two together (4-5) and those ones (Gemma retraced the lines from 5-4, 4-3, 3-2 but from right to left) and that (tracing over 1-5).

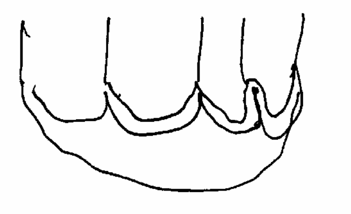


Figure 5. An incorrect relationship of a hierarchical situation.

An incorrect relationship on a hierarchy: Hierarchies represent information that either increases (e.g., a family tree) or decreases (e.g, a knockout tennis competition). In *The Party* task, there are progressively less people who need to shake hands so there is a hierarchical relationship among the number of handshakes that might be initiated by each individual. While Gemma (G) has some understanding that people need to shake hands with one another, the hierarchical relationship is neither apparent in her diagram nor in her explanation (See Figure 5).

Discussion and Conclusions

Students' difficulties in generating a diagram indicate that, despite its potential, the strategy *draw a diagram* was initially not an effective problem-solving tool for many students. While explanations for students' non-use of diagrams, and their generic and idiosyncratic difficulties differed from student to student, their difficulties were essentially related to a lack of knowledge about the affordances and constraints of diagrams as tools for problem solving. Clearly, to be empowered, students need to be educated in the use of the diagram as a problem-solving tool. Students need to know why a diagram can be useful in problem solving, which diagram is appropriate for a given situation, and how to use a diagram to solve a problem. In addition, to this explicit content, there is a need to address three further issues related to diagram use.

First, students need to distinguish diagrams from other pictorial representations and understand their relative purposes. There are substantive differences between diagrams and pictures or drawings. Surface details are generally important in a picture, while structural features are important in a diagram (Dufoir-Janvier et al, 1987). Additionally, a picture is a static knowledge-representation system — a snapshot — while a diagram is a knowledge-generating system that is designed to support inference making (Lindsay, 1995). Using the terms "diagram", "picture" and "drawing" synonymously fails to distinguish diagrams from other pictorial representations and may lead to confusion.

Second, students need to understand the nuances of ambiguity associated with diagrams. Diagrams by their nature are an ambiguous representation and problem information can be variously depicted. However, what is important is that the arrangement of the information on the diagram represents the structure of the problem. While some representation of the surface features can be helpful as a "reminder" about particular elements on the diagram, a focus on representing surface features is counterproductive if the student is distracted from representing the problem structure.

Third, students need to develop awareness that diagrams are dynamic rather than static representations. Diagrams are "physical" working spaces for trialing relationships among the elements of the problem, and, hence, need to be sufficiently large and relatively neat. As an understanding of the problem can evolve through the generation of a diagram (Nunokawa, 1994), it can be beneficial to produce more than one diagram.

Diagrams are an important tool for problem solving, however the benefits of any tool are closely associated with the users' knowledge of the tool, their opportunities to observe master craftspersons, and the development of their skill in using the tool. Advocating that students *draw a diagram* without addressing their difficulties and educating them about diagrams is quite simply the waste of a very good tool!

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