Two Turns Must Take Turns: Primary School Students' Cognition about 3D Rotation in a Virtual Reality Learning Environment

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Abstract: This paper reports on five primary school students' explorations of 3D rotation in a virtual reality learning environment (VRLE) named VRMath. When asked to investigate if you would face the same direction when you turn right 45 degrees first then roll up 45 degrees, or when you roll up 45 degrees first then turn right 45 degrees, the students found that the different order of the two turns ended up with different directions in the VRLE. This was contrary to the students' prior predictions based on using pen, paper and body movements. The findings of this study showed the difficulty young children have in perceiving and understanding the non-commutative nature of 3D rotation and the power of the computational VRLE in giving students experiences that they rarely have in real life with 3D manipulations and 3D mental movements.

Introduction

There are many powerful information communication technology (ICT) tools such as *Logo* and *Geometer's Sketchpad* that provide geometric visualisations and dynamic manipulation of geometric objects (e.g., drag points or lines by mouse) to facilitate the learning of geometry. However, these ICT tools utilise 2D computer graphics for geometric visualisation and thus have limited applications for the learning of 3D geometry especially by primary school students. During the past two years, the author (Yeh, 2004) has developed a Virtual Reality Learning Environment (VRLE) called VRMath that employs virtual reality (VR) 3D computer graphics to facilitate the learning of 3D geometry concepts and processes and which has enabled research into knowledge construction of 3D geometry within VRLE. The purpose of this paper is to report on a study in which the development of primary school students' conceptions of 3D rotation within the VRLE was investigated.

Background

In recent years, there has been a call for explorations within 3D space to become an integral component within elementary school mathematics (Baturo & Cooper, 1993; Bruni & Seidenstein, 1990). Explorations within 3D space are concerned with not only the investigation of 3D shapes but also the investigation of moving, positioning, orientating, constructing and building of objects within 3D space. One important element of these explorations is the study of rotations within 3D space. The Queensland Years 1 to 10 mathematics syllabus (Queensland Studies Authority, 2004) supports and reflects the arguments above. In this new syllabus, the geometry strand has been replaced by space strand, which includes five main topics: shapes, line, location, direction and movement. Activities with 3D rotation fit in the new syllabus and play an important role in linking the geometric concepts and processes about 3D shapes, location, direction, and movement.

However, 3D rotation activities that can be performed in our real environment with concrete objects are limited by the physical condition of the materials and environment, and also by problems with accuracy when performing 3D rotations with concrete objects. A simple question "Will you face the same direction when you turn right 45 degrees first then roll up 45 degrees, or you roll up 45 degrees first then turn right 45 degrees?" puzzles most students and even adults. Intuitively, most people answer that the two 3D rotations end up with the same direction. Unfortunately, this is wrong because 3D rotations are not commutative in nature.

To gain an understanding of the non-commutative nature of 3D rotation in traditional mathematics classroom activities, one generally must have some prior knowledge of the Cartesian 3D coordinate system, trigonometry, and vector and/or matrix notation for 3D translation, scaling, and rotation. These enable accurate operation of 3D rotation and rigorous proof of the nature of 3D rotations. However, this knowledge is far too complex for most primary school children to comprehend. Therefore, if the investigation of 3D rotations is to be integrated into primary school mathematical programs, then new activities which enable young students without knowledge of the Cartesian 3D coordinate system, trigonometry, and vector and/or matrix notation to meaningfully experience 3D rotation need to be designed. VRMath, the computational VRLE being presented in this paper, has been designed to provide young students with first- and third-person experiences (Pasqualotti & Freitas, 2002) within 3D space that cannot be provided by explorations with concrete objects in the real world. It is hypothesised that these first- and third-person experiences within the 3D virtual environment provided by VRMath will enable primary school children to develop new ways of experiencing and thinking about 3D rotations.

The VRLE (VRMath)

Informed by the fallibilist philosophy of mathematics (Ernest, 1994), semiotics (Cunningham, 1992; Lemke, 2001), constructivist and constructionist learning theories (Harel, Papert, & Massachusetts Institute of Technology. Epistemology & Learning Research Group., 1991; Kafai & Resnick, 1996), a VRLE named VRMath has been developed by Yeh (2004) (see Figure 1).

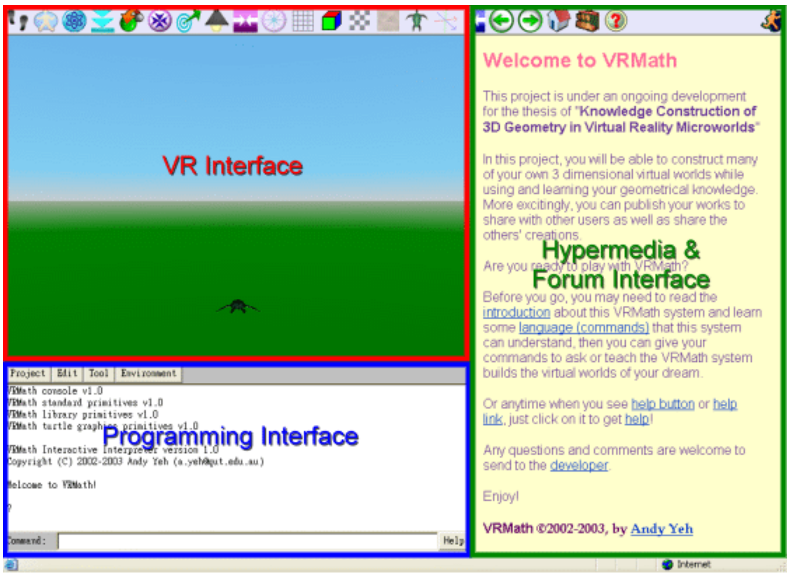
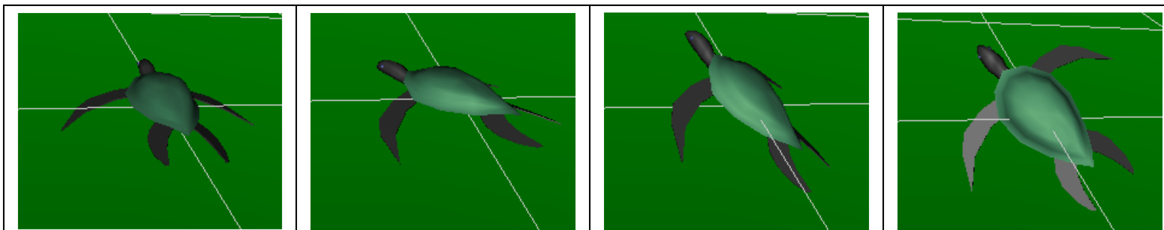


Figure 1. VRMath

VRMath comprises three main interfaces, a *virtual reality (VR) interface*, a *programming interface*, and a *hypermedia and forum interface*.

VR interface: This is the interactive 3D computer graphics that allows real time visualisation of a 3D virtual space. Users can use mouse and/or keyboard to navigate within the 3D virtual space and view the geometrical objects within the 3D virtual space from different and continuous viewpoints. This kind of 3D navigation is a first-person experience (Pasqualotti & Freitas, 2002) in which the users constantly feel that they are moving. The *VR interface* also provides the visualisation of the manipulations (e.g., changing location and orientation) of geometrical objects created through the use of *programming interface*. The manipulation of objects is a third-person experience (Pasqualotti & Freitas, 2002) in which users stay stationarily and the objects are moving. Moving oneself or objects represents two distinguishable human spatial abilities termed spatial orientation and spatial visualisation (McGee, 1979), which can be mapped to first- and third-person imagery respectively. This interface thus enables the cultivation of both spatial orientation and visualisation abilities (Yeh & Nason, 2004a). Amorim, Trumbore, and Chogyen (2000) suggest that giving opportunities to switch between first- and third- person imagery might be of great benefit for the virtual traveller to anticipate new vantage points and appropriate actions. Therefore, VRMath also implements an Avatar View function in which users can view from the turtle's (see *programming interface*) viewpoint. In Avatar View mode, the navigation within VR space can only be controlled by the programming commands such as FORWARD, BACK or turning commands. Thus, when manipulating the turtle through *programming interface*, the Avatar View enables users to switch between first- and third-person experiences. Moreover, when in Avatar View mode and the turtle's orientation is manipulated by a mathematical program through *programming interface*, users can also perceive what has been termed by Elliott and Bruckman (2002) as "mathematical movement" (e.g., the movement of sine wave in parametric equations).

Programming interface: This interface implements a Logo style language with an extended set of 3D related commands. Because of the nature of the VR interface, many geometric concepts in the VRLE environment differ from the traditional 2D Logo environment. For example, VRMath has a 3D turtle in VR space. VRMath uses metre and centimetre as the distance unit while traditional Logo uses pixels on the screen. To enable 3D rotation and movement, VRMath implements another four rotational or turning commands: ROLLUP (RU), ROLLDOWN (RD), TILTLEFT (TL), TILTRIGHT (TR) in addition to the traditional LEFT (LT) and RIGHT (RT). VRMath also has many built-in 3D shape commands such as CUBE, SPHERE, CYLINDER and CONE for easy creation of 3D models in the VR space. Figure 2 presents visual images of the effect of the 3D turning commands.



1. Original orientation	2. After LT 45	3. Then RU 45	4. Then TL 45

Figure 2. 3D Rotation in VRMath

Hypermedia and forum interface: This is the frame on the right side of VRMath containing hypermedia documentations and an online discussion forum. This is designed to provide non-linear and rich information and a channel for users to express and communicate ideas. With proper scaffoldings, this interface can be a pertinent vehicle for collaborative learning (Yeh & Nason, 2004b).

Method

There were five participants involved in this research study, Rosco, Bonbon, Grae, Alekat20, and R2D2 (their pseudonyms), who were aged 9 or 10 years old. They came from two inner city primary schools in eastern Australia. Rosco, Bonbon, and Grae interacted with VRMath as a group of three while Alekat20 and R2D2 interacted as a group of two separately.

The two groups of students were introduced to VRMath through 6 hours of instruction which covered the six rotational or turning commands and 3D navigation within the VR space. The question “Will you face the same direction when you turn right 45 degrees first then roll up 45 degrees, or when you roll up 45 degrees first then turn right 45 degrees?” was then posed to the students.

The two groups of students were videotaped as they experimented with the VRMath environment in order to solve this problem. The two groups spent about one hour each on the problem. The author, the researcher, sat with the students during this time, asking questions to draw out the reasons for any interesting activity. Field notes also were made by the researcher.

The videotapes were transcribed and the students’ posts on the VRMath forum also were collected. The transcriptions and the posts were analysed to provide rich descriptions of the thinking of each student which in turn was analysed for evidence that the students’ experiences on the VRMath environment were assisting them to understand about 3D rotation.

Results

The initial thinking of all participants was that the two 3D rotations (RU 45 RT 45 and RT 45 RU 45) would end up in same direction regardless of the performance sequence. This thinking was challenged when the students interacted with VRMath. The processes by which each student changed their conceptual understanding of 3D rotation will be presented in turn.

Rosco’s experiment: Avatar View

When Rosco was asked to justify his thoughts about the 3D rotation problem, he immediately came up with the idea of using the “Avatar View” in VRMath. Avatar View is a function by which the user temporarily becomes the turtle and views actions within the 3D virtual space from the turtle’s perspective. In this mode, the 3D navigation by mouse and keyboard in VR space are disabled to prevent changing the viewpoint by mouse dragging. And programming commands become the only way to manipulate the turtle’s position and orientation as well as to change the viewpoint. Bonbon suggested that Rosco switched on the Compass in VR space in order to see the degrees. Rosco thus began his experiment as illustrated in Figure 3.

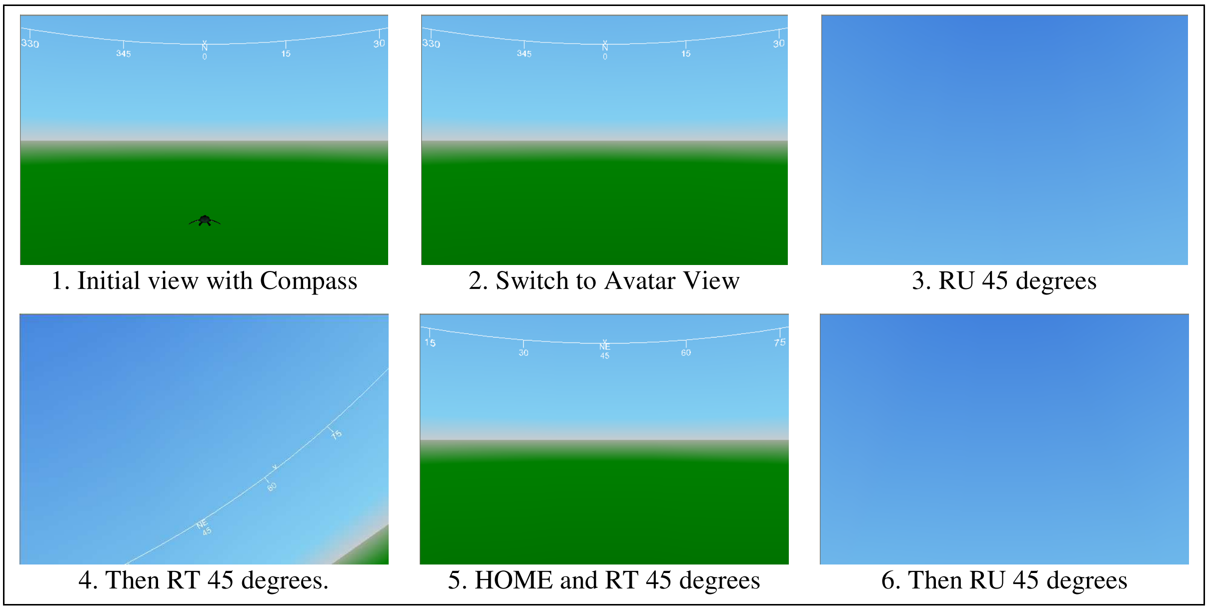


Figure 3. Avatar View experiment about 3D rotation

To his surprise, Rosco found that the views of Picture 4 (RU 45 RT 45) and Picture 6 (RT 45 RU 45) in Figure 3, which he originally thought to be the same, looked different. Because of the different part of the sky he (or the turtle) saw, he then started to think that different order of two 3D rotations may end up with different directions. He also contributed his idea of using Avatar View in the forum in the following posting titled "How to determine if ....":

> *How to determine if ru 45 lt 45*
> *and lt 45 ru 45*
> *have da same veiwpoint*
> *1. go to avatar view*
> *2. copy this text ru 45 lt 45*
> *3. copy this text lt 45 ru 45*
> *4. SEE FOR YOURSELF*
>
> ---
> *Hi peoples im rosco!!! hello*

Bonbon's exploration: Look at the turtle

Bonbon used her hands to simulate the two 3D rotations, and was pretty sure that the two 3D rotations were the same. She did a straight forward experiment by watching the turtle turns, but she decided to try on RU and LT (left) instead of RU and RT (right). The processes of her experiment are illustrated in Figure 4.

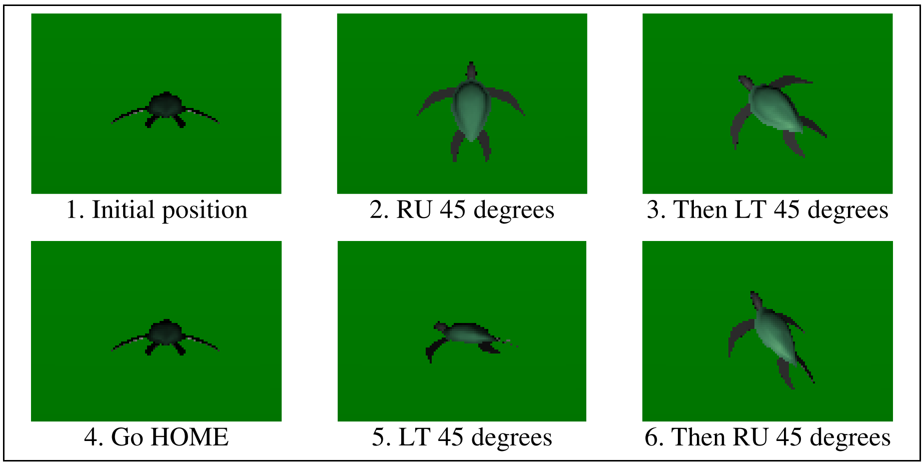


Figure 4. Turtle experiment about 3D rotation

Bonbon carefully compared the two views of Picture 3 (RU 45 LT 45) and 6 (LT 45 RU 45) in Figure 4 and noticed that they were different. However, before she made a conclusion, she also tried tilting rotations (TL and TR) with RU and smaller degrees, and together with Rosco's Avatar View experiment, she convinced herself that the two 3D rotations ended up with different results.

Grae's experiment: Create 3D objects

After seeing Rosco's and Bonbon's experiment, Grae could not think of any idea to show the difference between the two 3D rotations. The author encouraged him to try to create a 3D object after each 3D rotation. Grae then decided to create a sphere after each 3D rotation. He used commands "RU 45 RT 45 BALL" to create the first sphere, and then "HOME RT 45 RU 45 BALL" for the second sphere. The processes are illustrated as in Figure 5.

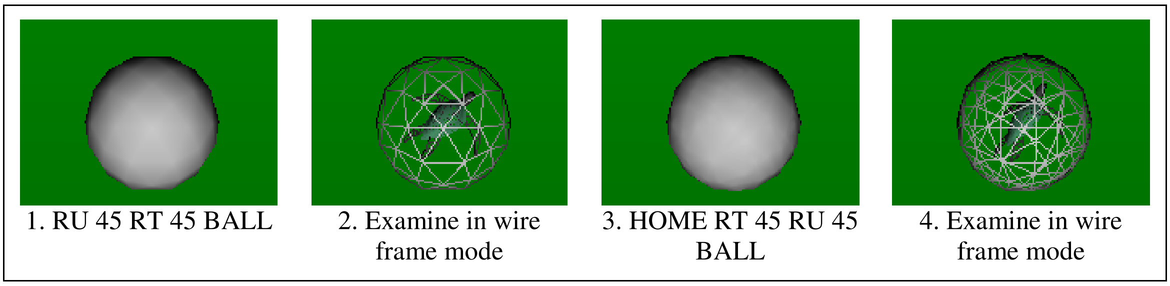
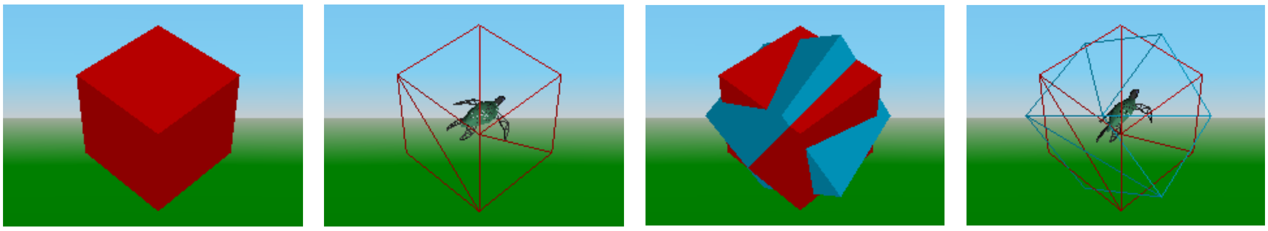


Figure 5. Create 3D Objects experiment about 3D rotation (1)

Grae originally thought that the two spheres should be somewhat overlapped but located at different place. However, he was confused when he navigated to see the two balls from different viewpoints; they seemed to be one ball. The researcher then suggested him to try on CUBE instead of BALL and with different colours. Figure 6 shows the processes of creating cubes after each rotation.



1. RU 45 RT 45 CUBE | 2. Examine in wire frame mode | 3. HOME RT 45 RU 45 CUBE | 4. Examine in wire frame mode

Figure 6. Create 3D Objects experiment about 3D rotation (2)

Grae was then satisfied with this result, and with the help of this researcher, Grae posted a message titled "two turns must take turns" in the forum:

> *Hi,*
> *if you lt 45 ru 45, or if you ru 45 lt 45 Will these be the same?*
> *you can check the answer by doing:*
> *1. home ru 45 lt 45 cube so you have a cube...*
> *2. you pick another color from the material editor.*
> *3. home lt 45 ru 45 cube*
> *so you have another cube but this time the turtle go lt 45 first then ru 45*
> *do you think that the two cubes are in the same place???*
> *--*
> *grae*

After Rosco, Bonbon, and Grae had completed their solutions to the 3D rotation problem, the author challenged their spatial thinking again with another question "if you do RU 45, RT 45, RD 45, and then LT 45, will you go back to the original direction?" Interestingly, even they had found that the two 3D rotations produced different result; they still thought that the turtle could go back to its original direction. They, however, found their prediction was wrong when they carried out the rotations on VRMath.

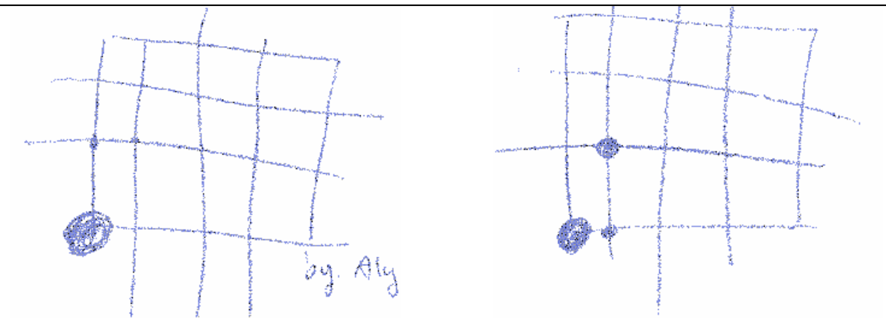
Alekat20 and R2D2's reasoning: Use your head and drawing

Alekat20 and R2D2 were in another group. They tried to solve this 3D rotation problem on the next day. Therefore, they had the chance to see the other three students' posts in the forum and followed their ideas. However, before they used the ideas in the Forum and interacted with VRMath, the researcher challenged their thinking about 3D rotation. Interestingly, they came up with some ideas quite different from the other group.

In addition to the use of hands, or other objects, to simulate the two 3D rotations, Alekat20 and R2D2 used their head movement and eyes to determine whether or not the two 3D rotations would generate the same outcome directionally. They practiced turning their heads RU 45 then RT 45, and tried to remember the spot they saw or faced to. Then they did the same thing with RT 45 first then RU 45. However, both of them claimed that the two 3D rotations would produce the same outcome because they were seeing the same spot.

Alekat20 started to draw grids on paper (see Figure 7). She reasoned that:

Alekat20: If you go FD 1 first and then EAST 1, you will end up with the same position as you go EAST 1 first and then FD 1.
Researcher: But we are talking about turning…
Alekat20: Oh, ya.. but I think they are the same.



1. FD 1 EAST 1 2. EAST 1 FD 1

Figure 7. Reasoning from Moving for Turning

Alekat20 and R2D2 then followed the ideas posted in the forum. After they tried the Avatar View and creating objects, they concluded that the sequence of performing 3D rotations does matter. However, they still had some doubts in mind, as they replied to Grae in the forum:

R2D2: no because me and alekat20 just tried and guess what thier not the same. (strange)
Alekat20: R2D2 And I Found That this is not true!!!!!!!!! How Weird...

Discussion and Conclusion

From "the two 3D rotations are the same" at the beginning to "the sequence of performing 3D rotations does matter" at the end, the five young participants experienced a conceptual shift after their interactions with VRMath.

The non-commutative nature of 3D rotation may be easily understood by one who can perform trigonometry in 3D coordinate system, but it would be very difficult for most people if we can only use our body movements, senses or feelings, mental reasoning, and other concrete objects. It is evident that although students live in a 3D space, they have limitations on manipulating or thinking three dimensionally. In this study, Alekat20 and R2D2 utilised a traditional way to solve the 3D rotation problem by moving their heads to see and feel the direction. Unfortunately, there was a lack of accuracy in this, an insufficiency that became apparent to these students once the rotations were investigated within the VRMath 3D virtual environment.

The VR interface of VRMath which enabled the students to switch between first- and third-person experiences facilitated dynamic visualisations of the 3D rotations. Rosco, for example, utilised the Avatar View to simulate the body movement, which was a typical example of using a computer to address a limitation with real world experiences within 3D space. In the Avatar View, Rosco temporarily became the turtle and viewed the rotations from the turtle's perspective. At the same time, he also manipulated the turtle's orientation by using 3D rotation commands. This operation of switching from third-person experience (watching the turtle turning) to first-person experience (turning himself) allowed Rosco to see different portion of the sky, and as a result, to realise the non-commutative nature of 3D rotations and thus correctly solve the 3D rotation problem posed by the researcher. Rosco's experiences confirmed the benefit of switching between first- and third-person imagery (Amorim et al., 2000), and also addressed the lack of accuracy of using body (e.g., head) movement as performed by Alekat20 and R2D2.

Bonbon and Grae used the Logo-like programming language to manipulate the turtle and build 3D objects in VR space to solve this 3D rotation problem. Bonbon's experiment demonstrated again that the computational environment VRMath easily and accurately showed the two 3D rotations were different, which was in contrast to the use of her hands to simulate the 3D rotation. Grae's experiment of creating objects was another approach to successfully solve this 3D rotation problem. Nevertheless, he also found that creating a sphere after each set of 3D rotation would not show any difference of the two 3D rotations because as long as the turtle doesn't move, the centre for spheres remains the same.

One important misconception about 3D rotation found in this study was Alekat20's reasoning from moving to turning. She argued that changing direction should be the same as changing location. This may be a common misconception about 3D rotation and will need to be further examined in later studies. Another common misconception about 3D rotation found in participants in this study was thinking that a turning could be eliminated by its opposite turning performed later in a series of 3D rotations. For example, in the four rotations RU 45 RT 45 RD 45 LT 45, students with this misconception believe that RU can be eliminated by RD and RT by LT. However, as VRMath showed, a rotation of another dimension in between the two opposite turns means that the two rotations of the same dimension still cannot eliminate each other.

To conclude, VRMath with its computational power provided the young children with new ways of thinking about and doing 3D geometry. The small number of cases reported in this study makes conclusions from this study tentative. Further studies, which are currently in progress, will provide further support for the educational efficacy of VRMath. However, this study does provide initial indications that VRMath, with its VR visualisation interface, fully implemented and extended Logo-like 3D programming language (e.g., mathematical functions and recursive procedures), and online forum for collaborative learning, could be a most powerful environment for young children to experience 3D mathematical modelling, simulation and problem solving.

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