

Knowledge Construction of 3D Geometry Concepts and Processes within a Virtual Reality Learning Environment

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ABSTRACT

A consensus has emerged within the mathematics education community about the limitations of traditional approaches for teaching and learning 3D geometry. Therefore, it has been suggested that new approaches based on the use of computers need to be adopted. One such new approach that has been proposed utilises Virtual Reality Learning Environment (VRLE). This paper reports on the initial phases of a research study whose major aim is to design and evaluate a VRLE to facilitate the construction of knowledge about 3D geometry concepts and processes. This research study investigates two primary school students' construction of 3D geometry knowledge whilst engaged within a VRLE developed by the researcher. A design experiments research methodology was employed in this study. This is research that iterates through cycles of design and research with the objective of arriving at theoretical and design principles that will have application both within and beyond the immediate research study. Therefore, the results being reported in this paper will be used to inform the modification not only of the VRLE but also of theoretical frameworks underlying the design and implementation of VRLEs.

Keywords: Knowledge Construction, 3D geometry, Virtual Reality, Microworlds, Virtual Reality Learning Environment (VRLE), Constructivism, Constructionism, Semiotics, Logo, Design experiments, Design-based Research.

In recent years, much disquiet has been expressed about the limitations of traditional approaches to the teaching and learning of geometry (Andrews, 1999; Mason, 1998; Papert, 1993; Rahim, 2002; Song, Han, & Lee, 2000), leading to the development of alternative ways of teaching and learning geometry. Some of the alternative approaches have been based on van Hiele's (1999) cognitive stage development theory whilst many others have utilised information and communication technologies (ICT). Included in the latter are Logo (Clements & Sarama, 1997; Hoyles & Noss, 1992; Papert, 1993), Cabri-Géomètre (Laborde, 2000) and Geometer's Sketchpad (Jackiw, 1995). Unfortunately, most of the ICT tools such as Logo, Cabri-Géomètre, and Geometer's Sketchpad that have been utilised to reform the teaching and learning

of geometry operate within a two-dimensional (2D) environment. Therefore, it can be argued that the number and types of investigations of three-dimensional (3D) shape, position and orientation concepts and processes that can be investigated in these environments is rather limited. This is in marked contrast to the number and types of investigations that can be carried out in Virtual Reality (VR) environments.

A recent development of VR technology, often described as desktop VR (Pimentel & Teixeira, 1995), has significantly changed the feasibility of having VR on personal computers. Desktop VR utilises real-time 3D graphics to define a virtual 3D space in which users can freely manipulate their viewpoints and objects within the 3D space. This 3D environment provides many advantages for learning including: (a) real world like experiences for learners (M. Bricken, 1991a, 1991b; W. Bricken, 1990; Byrne, 1996; Heudin, 1999; Pantelidis, 1996; Winn & Jackson, 1999), (b) simulation of impossible scenes (Dede, Salzman, Loftin, & Ash, 1999; Hedberg & Alexander, 1994), (c) extension of the microworlds paradigm (Barnett, Keating, Barab, & Hay, 2000; Salzman, Dede, Loftin, & Chen, 1999; Winn & Jackson, 1999), (d) high motivation and engagement in students (Barab et al., 2000; Barnett et al., 2000), and (e) affordability in common places on personal PCs (Cronin, 1997; Pasqualotti & Freitas, 2002).

Because of the enhanced capabilities of VR environments for 3D exploration, it has been suggested that VR learning environments (VRLE) are worthy of investigation as tools for facilitating the teaching and learning of 3D geometry (Ainge, 1996; Barab et al., 2000; Kwon, Kim, & Kim, 2002; Pasqualotti & Freitas, 2002; Song et al., 2000). This paper reports on the initial phase of a research study whose major aim is to design and evaluate a VRLE to facilitate the construction of knowledge about 3D geometry concepts and processes.

VRLE PROTOTYPE (VRMath)

The VRLE prototype named VRMath (Figure 1) was developed from an analysis and synthesis of the research literature from fields such as fallibilist philosophy about the nature of mathematics (Ernest, 1994, 1999), constructionist perspectives about learning (Kafai & Resnick, 1996; Papert, 1993), and semiotic perspective about mathematics being a meaning-making activity (Lemke, 1999, 2001).

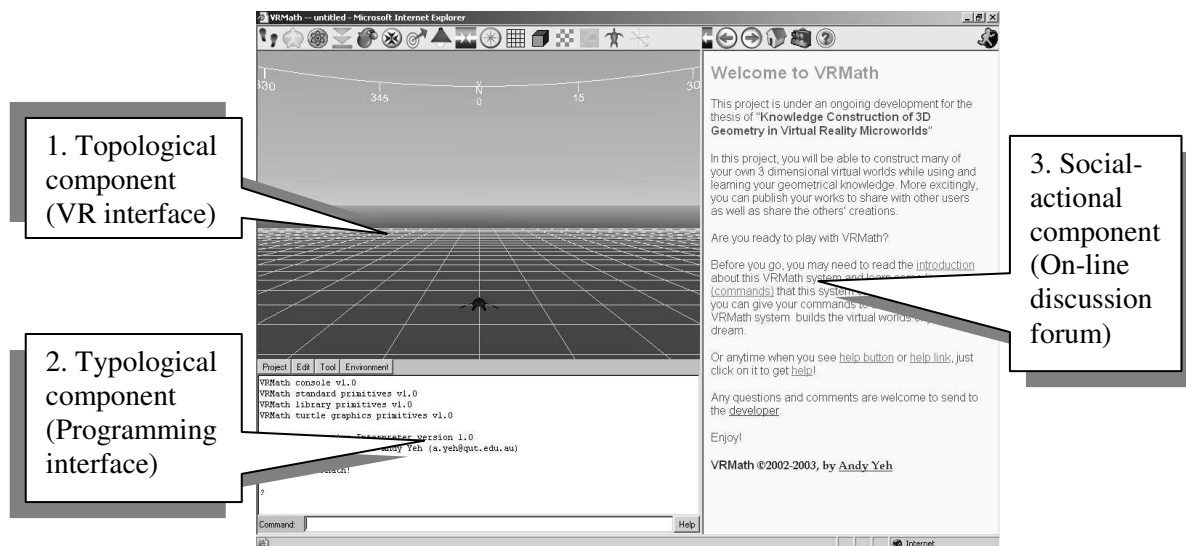


Figure 1. Prototype of VRLE

VRMath consisted of three components namely the topological, typological and socio-actional components. It was informed by a semiotic perspective which advocated multiple semiotic resources (i.e., different types of representations such as dynamic 3D graphics, symbolic language, natural language) for mathematical meaning-making (Lemke, 1999, 2001).

1. *Topological component*: This component refers to any meaning by degree or continuous representations of geometry such as digitalised graphics, pictures of real-world objects and 3D shapes and space. The main part of this component is the virtual reality interface that provides a rich representation of colours, textures, geometric objects and behaviours, and allows for real-time navigation within the 3D virtual space.
2. *Typological component*: This component refers to any meaning by kind or discrete representations of geometry such as language, texts, numbers, icons and buttons. The main part of this component is the Logo-like programming language interface that engages students in logical procedural thinking and the tutee mode of computer using (Taylor, 1980). In the tutee mode of computer use, students “teach” the computer to generate an artefact rather than being taught by the computer as occurs in the tutor mode. This links to topological component while the students are programming to manipulate and build within the 3D virtual reality microworld.

3. *Social component*: This component refers to many facilities such as online discourse that stimulates thinking, and the provision and sharing of information. The main part of this component is the web-based online discussion forum that aggregates information and scaffolds discourse. Students thus can contribute ideas, search for information and ask for help from more knowledgeable peers.

These three components were considered pertinent to reflect the analysis and synthesis of the research literature from the fields of fallibilist philosophy about mathematics, constructionism, and semiotics (e.g., multiple solutions and representations of mathematics, learning by making, and social and collaborative construction of knowledge). Moreover, VRMath was implemented in a web environment so that users could easily access it in a web browser, and as a result, it utilised to the full potential of multimedia, hypermedia and Internet.

METHOD

A design experiments methodology (Bereiter, 2002; Brown, 1992; Hsi, 1998; The Design-Based Research Collective, 2003) is being employed in this research study. Consistent with this methodology, the VRLE prototype is being iteratively designed, enacted and evaluated in order to: (a) evolve into a better tool for facilitating the learning of 3D geometry by primary school children, and (b) advance theoretical knowledge about computers and the learning of 3D geometry. In addition, usability inspections (Conyer, 1995; Nielsen & Mack, 1994) focusing on human-computer interaction (HCI) issues such as those identified by Nielsen (1993) are being undertaken throughout the design experiments. According to Nielsen (1993), the following five HCI issues with respect to usability need to be addressed:

1. Learnability: Ability for users to learn the system easily.
2. Efficiency of use once the system has been learned: Ability for users to save time in their work once they've learned the system.
3. Memorability: Ability for users to come back to the system and remember how to use it once they've been away from it for some time.
4. Error recovery and prevention: When the system presents an error message to users, it gives enough information for them to be able to continue with their work. Better yet, the system helps to prevent errors.

5. Subjective user satisfaction: Users' overall feelings about the system. Is it pleasant to use?

Participants

The participants in this phase or iteration of the design experiment were Emilie (a Grade 6 student) and Anya (a Grade 7 student) who both attended an inner city school in eastern Australia. Neither participant had prior experiences in programming and Logo. Nor had either participant had any experience with 3D computer graphics.

Procedure

In the initial session, the participants were given a brief introduction about VRMath. During the next five sessions, they engaged in the following five pre-designed activities:

1. *Become a member*: In this activity, participants registered themselves into VRMath forum. This enabled them to participate in online discussions and to create projects within the learning environment.
2. *Turtle's eyes*: In this activity, the participants were presented with a game in which they had to discover the secret about the turtle's eyes. That is, the turtle's eyes only change colour when they navigate close to the turtle using the mouse. The purpose of this activity was to develop the participants' skills in 3D navigation (walk, fly, examine) within VRMath using the mouse and keyboard.
3. *Turtle dance*: This activity focused on developing an understanding of basic commands for changing the turtle's position and orientation within VRMath. These commands included six turning commands (*left*, *right*, *rollup*, *rolldown*, *tiltleft*, and *tiltright*) and position changing commands (Egocentric: *forward*, *back* Fixed: *east*, *west*, *north*, *south*, *up*, *down*). *Repeat* command was introduced in this activity for animating the turtle.
4. *Formula of polygon*: The purpose of this activity was to have the participants develop a pattern or command for constructing a polygon by using the repeat command. The desired formula or command for a polygon is *repeat side [forward 1 right 360/side]*.
5. *Creating a cube*: In this activity, participants were required to write a procedure for constructing a frame of cube, and then save it as a public

project where it could be examined by other members in the learning community.

At the beginning of each session, the researcher explained the purpose of the activity (e.g., “We are going to find a secret about the turtle’s eyes,” or “We are going to ask the turtle to dance”) to both students. Then the researcher introduced information about the VRLE necessary for the successful completion of tasks to the students (e.g., the three modes of navigation, the semantics and syntax of commands). This was done by reading through with the students the information pages for the particular activity. These information pages were located in the VRMath forum. Each student then went to her own computer and worked on the assigned activity. When any difficulty was encountered, the students were able to seek help from the researcher or from the other student. Rather than merely showing the student how the difficulty could be overcome, the researcher actively tried to scaffold student resolution of the difficulty by replying to their questions with questions of his own in a manner suggested by Papert (1993). At the end of each session, the researcher and the two students met in a focus group to discuss what they had learnt. Each focus group session concluded with the administration of a set of questions that focused on the usability of VRMath and other HCI issues with respect to VRMath.

Data collection and analysis

As the students participated in each of the five activities, data were collected from three sources:

1. Participant observations of the students’ interaction with VRMath interface and intellectual engagement with the five activities.
2. Focus interviews with the two students after each session on the usability issues of VRMath.
3. Computer records of the participants’ inputs in VRMath including commands, procedures in their projects, and the geometric objects created.

In order to facilitate data analysis, the students’ sessions at VRMath and the focus interviews were videotaped. The videotapes were then transcribed and the data from the transcription, observation, field notes, and artefacts were organised. Then using a grounded theory approach (Strauss & Corbin, 1990, 1998), themes with respect to differentiating between spatial orientation (i.e., the ability to determine spatial relation

with respect to one's body) and spatial visualisation (i.e., the ability to mentally rotate, manipulate, and twist two-and three-dimensional stimulus objects) (McGee, 1979), frames of references (e.g., egocentric vs. fixed), and 3D navigation were then identified and conclusions were drawn.

FINDINGS

The findings from each of the five activities will be reported on in turn. The findings reflect both on the issues of the design of VRMath and learning of 3D geometry within VRMath.

Activity 1: Become a member

To become a member of the VRMath forum community, participants have to register themselves by filling in an online form. The two participants found the process of filling in the online form was easy. They were excited when choosing a nickname and in particular were very intrigued by being able to select a personal Avatar (i.e., picture image that represents themselves).

After becoming a member, the participants were asked to post messages in the forum. They learnt how to post and send messages very quickly using the information given on the webpage with very few verbal instructions from the researcher. When writing a message, they also utilised the small 'smilies' (e.g., 😊, 🤔), which are termed 'emoticons' to express emotions. They did not post public messages; instead they preferred to communicate by sending private messages to one another and to the researcher. The messages sent to the researcher did not focus on the geometry but instead were mainly social in nature. Because VRMath was just in its rudimentary stage, there were only few casual users in the forum community. Therefore, there was no effective social construction of knowledge evident in the forum. The small number of participants in this study may explain the participants' preference for communicating through private messages. With a larger number of participants, the efficacy of using the VRMath forum to engage in knowledge-building discourse would probably have become apparent.

Activity 2: Turtle's eyes

During this activity, the two participants seemed to find it difficult to navigate in 3D space using a mouse. Two main difficulties were apparent:

1. *Controlling the speed of navigation:* In VRMath, the distance the mouse is dragged determines the speed with which the user navigates around the 3D environment; the further the mouse is dragged, the faster the user moves within the environment. This was explained to participants at the beginning of this activity. However, during this session both students had great difficulty in controlling speed as they attempted to navigate within the 3D virtual environment.
2. *Controlling the direction of navigation:* To navigate in different directions within the 3D virtual environment requires more than the four directions provided by arrow keys or by the mouse being dragged over the two-dimensional mouse pad. Therefore, to navigate within the 3D virtual world required other keys (e.g., Alt, Space) to be used in conjunction with the arrow keys or the mouse dragging over the mouse pad. This was further complicated by the three modes of navigation within the 3D virtual world: *Walk*, *Fly*, and *Examine*. The students found the coordination of the Alt and Space keys with the dragging of the mouse rather difficult. However, by the end of this session after many trial-and-error explorations, they had mastered the control of navigation.

The completion of the activity, which was to navigate within 0.5 metre (with respect to the virtual 3D environment) of the turtle's eyes and see the eyes turn red, was not achieved simply by navigating using the three modes of navigation. The two participants found that they had to experiment with other navigation aids before they were able to satisfactorily complete the activity. These other navigation aids such as *Change Navigation Mode*, *Avatar View*, *Set Rotation Centre*, *Align to Ground*, *Restore Viewpoint*, and *Fit Screen* etc. were displayed as icons above the 3D window. Because it was impossible for these imagery icons to fully deliver their meanings of functionality, a mechanism of showing a *Help Text* when the mouse was pointing on the icon was built into the design of the system. Both participants found the *Help Text* very useful, especially when they began the activity. During their navigation in 3D space, the *Restore Viewpoint* icon was found to be used the most often, as the students got lost very easily in 3D virtual space. After investigating all of the other navigation aids, the participants found the best way to complete this task was to use *Fit Screen* function, which brought all objects into the 3D screen. Emilie contributed her

experience and wrote in the forum, “You make the turtles eyes go red by hitting the fit screenbutton (sic) and then rotating the turtle by using the examine mode”.

In VRMath’s 3D space, the users navigate themselves about the 3D virtual environment with the mouse and the navigation keys. Using the mouse and the navigation keys has no effect on the turtle. The turtle can only be moved by specific written commands (e.g., “*fd 1*” to command the turtle forward 1 metre). As the participants were navigating towards the turtle using the mouse and navigation keys, they were asked, “Is the turtle moving?” In a very confident manner, both participants replied that the turtle was moving. This indicated that as the participants were navigating within VRMath, their differentiation between spatial visualisation and orientation (McGee, 1979) was lost. Normally in the real world, one can differentiate between spatial visualisation and orientation by referring to points of references located in space or to one’s kinaesthetic sense. Thus, if the person perceives that her location with respect to these points of reference are staying constant whilst her perspective of an object is changing and her kinaesthetic sense indicates that she is not moving, then the person knows that the object is moving and her view of the object is changing. In contrast, if the person perceives that both her location to the points of reference and her perspective of an object is changing, then the person knows that she is moving and thus her spatial orientation with respect to the object is changing. Also, in the real world this is further confirmed by kinaesthetic feedback if they are moving. It was conjectured that because neither kinaesthetic feedback nor specific points of reference were provided in this desktop VR, the participants were unable to differentiate between spatial visualisation and orientation and thus intuitively came to the incorrect conclusion that the turtle was moving.

To overcome their incorrect intuition that the turtle was moving rather than they themselves, the researcher asked the participants to pay attention to the compass at the top of the window and the background stimuli in VRMath. When they paid attention to the compass and/or the background stimuli, they soon discovered that in fact they were moving through the virtual 3D world in which their orientation to the turtle was changing not their spatial visualisation.

Activity 3: Turtle dance

In this activity, the participants were introduced to VRMath’s Logo-like programming language that enabled changes to be made to the turtle’s position and orientation. To facilitate learning of these commands, the researcher designed a tool named *Quick Command*, which included these commands within a GUI (Graphic User Interface) dialogue (Figure 2).

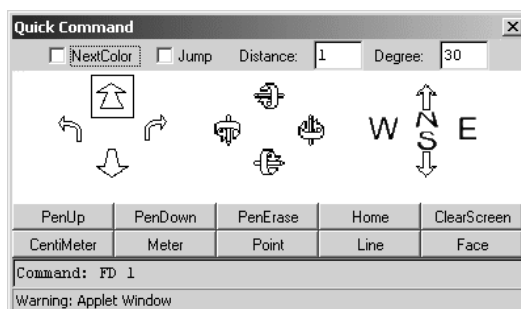


Figure 2. Quick Command tool

In *Quick Command*, any icons or buttons clicked will produce the corresponding command sent to *Command Input Box*. After a few tries on *Quick Command* and watching the effect on the turtle, the participants built up some basic knowledge about these commands. Then the participants tried these commands again by typing commands in *Command Input Box*. Some general mistakes were found when the participants were typing these commands. These mistakes included misspelling of commands, no space between command and argument, and lost focus of *Command Input Box*.

During their practice in using VRMath’s commands, the researcher found that Emilie preferred using GUI *Quick Command*, while Anya preferred typing directly into *Command Input Box*. They however, both preferred typing when they knew that they could recall the command history by using Up and Down arrow keys. The function of recall command was particularly useful when more complex commands such as repeat command were introduced.

The researcher was very cautious when introducing the semantics and syntax of VRMath’s commands to the participants. The semantics of command were introduced and discussed before the syntax of command. For example, when introducing repeat command for the purpose of animation (i.e., the turtle dance), the researcher explained, “If we want to repeat some actions, we need to tell the turtle how many times to repeat with some commands” and then brought in the syntax “*repeat counts [commands list]*.” Both participants were able to make sense of the syntax and build

VRMath commands such as *repeat 12 [rt 30]*, which made the turtle repeat twelve times turning right 30 degrees.

The participants were able to substitute the *rt* command in *repeat 12 [rt 30]* with other turning commands such as *lt*, *ru* (roll up), *rd* (roll down), *tr* (tilt right), and *tl* (tilt left) to animate the turtle. They also tried to change the counts and degrees in that command to play with the turtle. During their play with the turtle dance, they were able to predict the turtle movements before they inputted the commands. This showed that the participants could use the turtle as the reference point for performing rotation in 3D virtual space. However, the researcher also found that the participants couldn't mentally think of the all rotations in 3D. When he asked about the directions of 3D rotation without using computer, they talked about turning *left* and *right* and *rollup* and *rolldown* but both ignored the rotations of *tiltleft* and *tiltright*.

Activity 4: Formula of polygon

In the previous activity, the participants learnt about *repeat* command. In this activity, the researcher began with a discussion about how a square could be constructed by *repeat 4 [fd 1 rt 90]* commands. The two participants physically paced out a square to generalise the commands. They were then asked to use the similar syntax to construct polygons.

Anya first tried to create a pentagon. She drew a pentagon on a piece of paper and tried to measure the degrees by estimation. Because the pentagon she drew was not equilateral, she found that there were two right angles, two obtuse angles and one acute angle in the pentagon. The researcher then advised her that she could make all angles the same in the pentagon. She then started to guess the degree of the angle and replace the degree in the following formula:





repeat 5 [fd 1 rt degree]

Anya tried degrees such as 110 120, 180, 190, and 280 etc. After a shape was drawn by the turtle, she navigated to view the shape. She found that 290 degrees was almost right for a pentagon.

Emilie also tried to generate a formula for drawing a pentagon on VRMath. She was unsuccessful in this endeavour. However, she found that if 120 degrees was the input to the procedure *repeat 5 [fd 1 rt degree]*, she ended up with an equilateral triangle.

The researcher then generated a table to organise their exploration and stimulate their thinking (Table 1).

Table 1
Generalisation of the Polygon

Name	Shape	Sides (repeat)	Degree	Relationship
Triangle		3	120	$3 \times 120 = 360$
Square		4	90	$4 \times 90 = 360$
Pentagon		5	290 (?)	$5 \times ? = 360$
Hexagon		6	?	$6 \times ? = 360$

Emilie used a calculator to get 72 degrees for a pentagon. Anya also calculated 45 degrees for an octagon. The researcher then introduced that the mathematical operation could be written in the formula. Thus the final formula for polygon was formed as *repeat sides [fd 1 rt 360/sides]*. The researcher then posed a question “What will it look like if there are 360 sides?” The participants answered “a circle,” which they verified with the final formula within VRMath.

Activity 5: Creating a cube

In this activity, the two participants were asked to create a frame of cube, write commands in a procedure, and save it as a public project into online database. Emilie first tried on *Quick Command* tool to construct a cube. She was very confident in using the tool. During the process of using *Quick Command*, although the construction didn’t look like a cube, she didn’t navigate to change the viewpoint. Thus, the typical error for egocentric reference that has been termed the egocentric bug (Fay & Mayer, 1988) emerged. That is, when the turtle was facing Emilie, she often intuitively clicked on forward for back and back for forward and left for right and right for left. This didn’t happen when the participants were typing commands to make the turtle move. It was conjectured that because of the GUI, the arrow icons misdirected her, which resulted in this egocentric bug.

The GUI interface of *Procedure Editor* was found to be easy to use. The participants also easily accepted the format of a procedure and started to write down commands while mentally thinking the position and rotation of the turtle. The participants had great difficulty remembering the orientation of the turtle especially when more than one dimension of rotation was used (e.g., *left* and *right* is one dimension, *rollup* and

rolldown is another dimension). The researcher then advised them to use other frame of reference commands such as *up* and *down* to avoid using *rollup* or *rolldown*. The finding that the use of fixed frame of reference in addition to egocentric frame of reference aided the construction of a cube significantly is consistent with other studies (Yakimanskaya, Wilson, & Davis, 1991) that. The participants then easily completed their procedures (Table 2).

Table 2
Procedures of a Cube

Anya's procedure	Emilies procedure
<i>TO acube</i>	<i>TO ecube</i>
<i>cs</i>	<i>cs</i>
<i>pd</i>	<i>pd</i>
<i>repeat 4 [fd 1 rt 90]</i>	<i>repeat 4 [fd 1 rt 90]</i>
<i>up 1</i>	<i>up 1</i>
<i>repeat 4 [fd 1 rt 90]</i>	<i>repeat 4 [fd 1 rt 90]</i>
<i>fd 1 down 1</i>	<i>east 1</i>
<i>rt 90</i>	<i>dn 1</i>
<i>fd 1 up 1</i>	<i>north 1</i>
<i>rt 90</i>	<i>up 1</i>
<i>fd 1 down 1</i>	<i>west 1</i>
<i>pu</i>	<i>dn 1</i>
<i>END</i>	<i>bk 1</i>
	<i>END</i>

These two procedures created by Anya and Emilie were very similar. Both procedures correctly produced a frame of a cube. However, the use of fixed frame of reference commands instead of egocentric commands actually produced different results. For example, when the turtle is in tilt left 45 orientation, the two procedures in Table 2 will not create a cube (Figure 3). This could be further developed in a new activity.

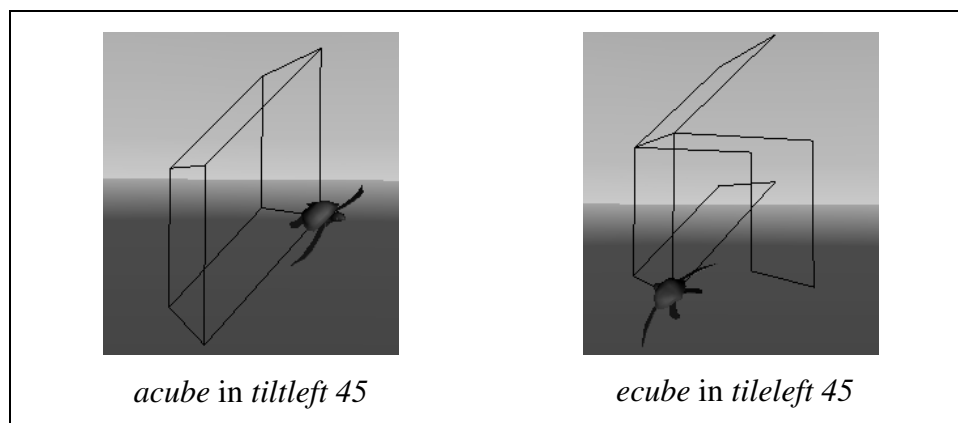


Figure 3. Fixed frame of reference cubes

Usability inspection

The usability inspection was undertaken through the observation of the participants' interactions with the VRMath system and the focus interview at the end of each session. The usability inspection was informed by Nielsen's (1993) criteria of learnability, efficiency of use once the system has been learned, memorability, error recovery and prevention, and subjective user satisfaction.

The participants had some difficulties in using VRMath system. However, when interviewed they claimed that it was easy to learn and remember. For example, Emilie spent about 12 minutes in finding the *Fit Screen* icon. Despite this, she still thought that it was easy to use and remember. The researcher found that the participants could fluently switch between and utilise the VR interface, programming interface, and hypermedia forum interface. During the five sessions, no systemic errors occurred. Both the participants expressed their enjoyment of VRMath.

However, the two participants made three suggestions regarding the interface and activity design of VRMath.

Firstly, the dialogues such as *Quick Command* and *Material Editor* should stay in foreground instead of being sent to background when users temporarily switched to navigate in 3D virtual space. This had previously been noted by the researcher. However, due to the limitation of the programming language, it was not possible to change this in the short duration of this study. It is anticipated that this issue will be addressed in future iteration of the research program.

Secondly, the current prototype of VRMath could only save/store the users' constructions of procedures in projects. The participants wished that VRMath could also save/store their constructions of 3D microworlds created in the VR interface, so they could share these with each other more easily. This is an excellent suggestion as the 3D visualisations are rich and intuitive representations of 3D geometry. The researcher will consider implementing this in the next iteration of design of the VRLE.

Finally, they suggested that more games such as the "turtle's eyes" game could be designed in activities. Their reason for making this suggestion was that they found that "playing" the game was much fun. This suggestion has been adopted by the researcher. However, since the purpose of a game such as "turtle's eyes" is to

facilitate the students' skill in 3D navigation, the design of game-like activities needs to avoid shifting the students' attention away from learning.

SUMMARY AND DISCUSSION

During this phase or iteration of the design-experiment, a prototype of VRLE (VRMath) first was developed based on a conceptual framework derived from the review of research literature. The prototype then was enacted and evaluated with two upper primary school students. Based on the data collected during the enactment and evaluation, findings that reflect on issues of the design of VRMath and learning of 3D geometry within VRMath were generated and reported. Some of these findings such as the emergence of the traditional egocentric bug and the need for the use of multiple frames of reference to aid the construction of 3D geometric objects have confirmed results from past research literature (e.g., Fay & Mayer, 1988; Yakimanskaya et al., 1991). However, three findings from this study have raised issues that hitherto have not emerged in the research literature on the learning of 3D geometry in 3D VR environments.

Firstly, the 3D VR environment in VRMath provided users with the opportunity to operationalise both their spatial visualisation and orientation abilities. This is in contrast with most other ICT tools that enable users only to operationalise their spatial visualisation abilities. However, because kinaesthetic feedback from VRMath only comes from the mouse and keyboard, users often think that they are manipulating the geometric objects as they do with other ICT tools. Therefore, users seem to perceive that the turtle rather than they are moving within the 3D virtual environment. To overcome this problem and to enable users to successfully utilise their spatial orientation abilities within the VRMath environment, environmental cues such as background and compass need to be emphasised to allow users to distinguish between movement by objects and movement by themselves within the 3D virtual environment. Scaffolding can also help the students. For example, the researcher can use "if YOU can navigate or walk to see the turtle's eyes..." in discourse with users. Scaffolding comments such as this help students to remind themselves that they rather than the turtle are moving.

Secondly, the participants found the process of navigating in the 3D virtual space of VRMath rather challenging. VRMath enables users to navigate themselves to get

multiple viewpoints in the 3D virtual space. This is invaluable in the process of constructing knowledge about 3D geometry. However, users often found that it was difficult to control their navigation speed and direction. To overcome the difficulties in 3D navigation, the three modes of navigation *Walk*, *Fly*, and *Examine* need to be explicitly discussed with the users before giving information about the use of mouse and keyboard. In this study, these three modes of navigation were introduced simultaneously. However, in future studies, they probably will be introduced separately. The three modes of navigation are actually informative metaphors for building conceptions about 3D navigation. For example, the walk mode has a gravity effect. These discussions may help alleviate the memory load of 3D navigation. Also, more time should be devoted for users' practice of 3D navigation prior to their engagement with complex design of 3D objects (e.g., the frame of a cube).

Finally, the use of multiple frames of reference commands could aid the construction of 3D geometric objects significantly. But more importantly, the different frames of reference commands have different effects on the construction of 3D geometric objects. The implications of this finding for later phases or iterations of the study are that the different effects emanating from the use of different frames of reference should be developed and integrated into the learning activities. Also, the users' understanding about the use of different frames of reference are worthy of further investigation.

In summary, an open, generative, and semiotic rich VRLE (VRMath) informed by fallibilist notions about the nature of mathematics, constructivist/constructionist perspectives about learning, and semiotic perspective about mathematics being a meaning-making activity has been developed. The participants were able to utilise and perceive rich semiotic resources (i.e., topological, typological, and social-actional) to develop many new ways about thinking and doing 3D geometry within VRMath. Based on the analysis of the findings from this iteration of the design experiment, it was confirmed that VRMath has great potential to facilitate the knowledge construction of 3D geometry concepts and processes. However, because only a small portion of this prototype VRMath has been explored, more iterations of design experiments will need to be carried out to further examine the conceptual framework and the design of VRMath. Within the cyclic iterations of design experiments, VRMath will keep evolving into a sustainable innovation that provides contexts for

unveiling new and innovative ways for facilitating children's knowledge construction of 3D geometry.

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