Toward a Semiotic Framework for Using Technology in Mathematics Education: The Case of Learning 3D Geometry

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Abstract: This paper proposes and examines a semiotic framework to inform the use of technology in mathematics education. Semiotics asserts that all cognition is irreducibly triadic, of the nature of a sign, fallible, and thoroughly immersed in a continuing process of interpretation (Halton, 1992). Mathematical meaning-making or meaningful knowledge construction is a continuing process of interpretation within multiple semiotic resources including typological, topological, and social-actional resources. Based on this semiotic framework, an application named VRMath has been developed to facilitate the learning of 3D geometry. VRMath utilises innovative virtual reality (VR) technology and integrates many semiotic resources to form a virtual reality learning environment (VRLE) as well as a mathematical microworld (Edwards, 1995) for learning 3D geometry. The semiotic framework and VRMath are both now being evaluated and will be re-examined continuously.

Background

The use of information communication technology (ICT) tools is burgeoning in mathematics classrooms throughout the world. Unfortunately, most of these ICT-based tools for "learning math" can be criticised as being ineffective as supports for the construction of mathematical knowledge. According to Papert (1996a), the ideas about what mathematics is and why the students should learn mathematics implicit in these ICT tools are "flimsy". Therefore, in Papert's opinion, many ICT-based tools end up in teaching "junk maths" to the students. Other critics such as Heid (1997) and Roschelle et al. (2001) have suggested that the ineffectiveness of many ICT-based tools can be attributed to factors such as: (a) use of computers as computational tools rather than knowledge-building tools, (b) use for inappropriate topics, (c) technology issues overwhelming mathematical issues to such an extent that students' focus of attention is on the technology rather than on the mathematics.

In order to address the limitations of most existing ICT-based tools for learning mathematics, the authors have generated some principles from a review of the research literature to inform the use of ICTs in mathematics education:

1. Computers should be used not only just to amplify what we currently do with mathematics, but to reorganise our mental functioning that creates new ways of thinking and doing mathematics (Kilpatrick & Davis, 1993; Pea, 1985).
2. Computer use should focus on creating knowledge-building environments rather than knowledge-reproduction environments (Scardamalia & Bereiter, 2002).
3. Computers should be used as tools and tutees rather than as tutors. This way, the focus of learning can be shifted from end product to process and from acquiring facts to manipulating and understanding the knowledge (Papert, 1996b; Resnick, 1996).
4. Computers should be used to create environments (e.g., CSCL environments) that can be utilised to enhance learning in a social constructivist manner (Bruckman & Bandlow, 2002).

In the field of teaching and learning geometry, many attempts have been made to operationalise these four principles. A review of research literature identified four categories of ICT use for the teaching and learning of geometry that have in part attempted to operationalise these principles.

1. Turtle Geometry: This utilises a programming language (e.g., Logo) to link between language and visualisation, and promotes logical and procedural thinking. The Logo microworld is a form of tutee mode of computer use in which learners actively instruct the turtle to construct geometric figures with understanding (Abelson & DiSessa, 1981; Maddux & Johnson, 1997; Noss, 1999).
2. Dynamic Geometry: The dynamic geometric figures created by mouse dragging within dynamic geometry environments such as Cabri-Géomètre (Laborde, 2000) and Geometer's Sketchpad (Jackiw, 1995) have provided new ways of thinking and doing mathematics and thus have facilitated the reorganisation of users' mental functioning about geometry (Goldenberg & Cuoco, 1998; Hoyles & Noss, 1994).
3. Virtual Reality: This utilises real-time 3D graphics to simulate dangerous or impossible scenes and provides users with realistic and/or authentic experiences. The experience with VR also enables users to generate different ways for learning and knowledge construction (Barab et al., 2000; Elliott & Bruckman, 2002; Kwon, Kim, & Kim, 2002; Song, Han, & Lee, 2000).
4. Multimedia, Hypermedia and Internet: This utilises the organisation of thoughts about geometry with hypermedia and networked collaboration through Internet web environments. These environments provide a channel for users to be involved in knowledge-building activities within online communities (Dede & Palumbo, 1991; Hidaka, 1994; Teixeira, Silva, & Silva, 1999).

These four categories of ICT use have significantly advanced the teaching and learning of geometry. However, in terms of 3D geometry, each of these four categories of ICT use has serious limitations. Each seems to be beset by limitations when it comes to the task of exploring and investigating 3D shapes, and positions, orientations and directions within 3D environments. For example, turtle geometry has traditional 2D graphics and the Logo language. Though it is rich in geometric semantics, turtle geometry lacks 3D geometric language. Dynamic geometry tools are also limited because they are based on 2D geometry and graphics. VR systems provide more natural 3D environments. However, most VR environments are not open environments and need to be designed by professional computer engineers, not primary school students. CSCL environments mainly focus on text and image representation of knowledge and communication.

The authors therefore believe that there is a need for the development of a new generation of ICT tools for teaching and learning 3D geometry that overcomes the limitations of current ICT tools. The design of a new generation of ICT tool to facilitate the teaching and learning of 3D geometry has been the focus of their research program during the past two years. The design of this new generation ICT tool has been informed by a semiotic framework consisting of a set of four guidelines that we have developed to inform the design of ICT tools in mathematics education.

The Semiotic Framework

Semiotics is the study of signs and sign-using behaviours. It has its roots in both the European tradition founded by de Saussure and the American tradition based on the works of Peirce and Morris (Andersen, 1990). These two traditions can be synthesised into Pierce's semiotic triad consisting of *representamen* (sign), *object* and *interpretant* (Fig. 1). This triad identifies the three structural components of meaningful knowledge construction (Osberg, 1997).

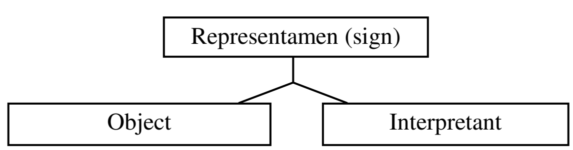

Figure 1: Peirce's semiotic triad

Cunningham (1992) elaborated on Peirce's semiotics triad and noted that a sign mediates between the object and its interpretant. However, a sign is not the object itself; a sign is only an incomplete representation of the object. A sign can only represent certain aspects of the object and in addition, it has aspects that are not relevant to the object. According to Peirce, signs can be classified into three categories: *icon*, *index* and *symbol*. An *icon* stands for an object by resembling or imitating it, in particular, in a visual way such as a map visually resembles some characteristics of the territory. An *index* refers to the sign which is the effect produced by the object. An example is that when a fire produces smoke, smoke becomes the index of the fire. The last category *symbol* refers to objects by virtue of a law, rule or convention. In this case, language could be a prototype of symbols. In Cunningham's opinion, sign theory offers a significant advantage into the analysis of mind, which is to consider systems of signs as acting like a code for some system of objects. The systems of codes are used to structure our experience. Codes are not equivalent to the objects they represent, but it is possible to specify rules for those equivalences. The rules are referred to as syntax, which allows the manipulation of signs in a potentially indefinite number of ways. The implications of this, according to Cunningham, are that we don't finalise a single correct way of manipulating syntax and don't limit our conception of the structure of signs to any particular type of linguistic model or only to linguistic models.

When discussing sign-using behaviours, Cunningham (1992) identified a special type of inference called abduction. Abduction is a type of thinking or reasoning that invents signs to make sense of some new experience. Abductive reasoning provides us with ways to question, invent and alter our beliefs (and cognitive structures) about reality when we confront some experience not accounted for by our existing beliefs. Hence, abductive thinking reveals inquiry as an on-going process of life. This notion, according to Cunningham, challenges the assumption held by absolutists that the objective knowledge of few privileged people is unchallengeable.

Lemke (2001) stated that a semiotic perspective helps us understand how natural language, mathematics and visual representations form a single unified system for meaning making in which mathematics extends the typological resources of natural language to enable it to connect to more topological meanings made with visual representations. He argued that mathematics is an on going process of "semiosis" or meaning making of mathematical symbols (signs), for those signs form special relationships. Based on Peirce's semiotics triad, Lemke interpreted that in mathematics, there is a material signifier or "representamen" (R) that we encounter on page, the "object" (X) or signified which could be a real object, a concept, a quantity, an abstraction or another sign, and the "interpretant" (I) which connects the R to X. Lemke pointed out was there has to be some system of interpretance (SI) in which the R gets interpreted as X. He restated Peirce's idea that we have a sign when something (R) stands for something else (X) for somebody in some context (SI).

Lemke (1999) identified two types of semiotics in mathematics, which he termed typological and topological semiotics. Typological semiotics represents meanings by types or categories such as spoken words, written words, mathematical symbols and chemical species. They are discrete, point-like and distinctive signs. In contrast to this, topological semiotics makes meaning by degree such as size, shape, position, colour spectrum, visual intensity, pitch, loudness and quantitative representation in mathematics. There is no continuum between typological signs (e.g., word "GOOD" and "GOAD", variable "X" and "Y", atomic species "Carbon" and "Nitrogen"), but there is a continuum in topological signs (e.g., the acoustical spectrum of sounds, between any two real numbers). The differences between topological and typological semiotics identified by Lemke are illustrated in Figure 2.

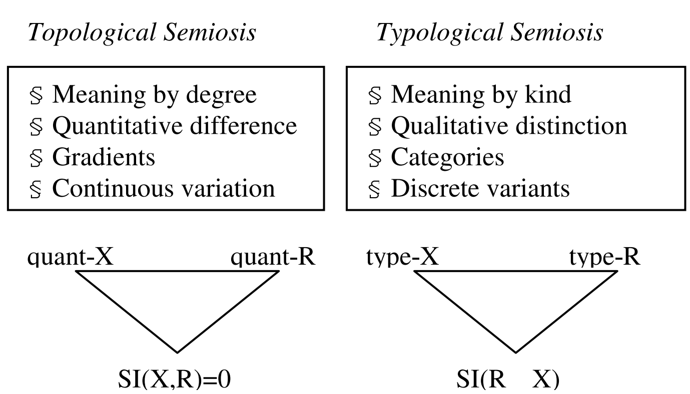

Figure 2: Topological versus Typological Semiosis
(Figure adopted from Lemke, 2001)

In addition to his semiotic perspective of mathematical meaning-making within typological and topological resources, Lemke also took a view that mathematics is a system of related social practices, a system of ways of doing things. He argued that mathematics is not in the "how-to" handbook or textbook, but is embedded in writing about physics, engineering, accounting, surveying, and so forth. Meaning-making is always a material process as well as a social semiotic practice. Therefore, Lemke proposed that we should integrate multiple semiotic resources in classroom learning with both actional-typological and actional-topological resources.

The following four guidelines to inform the design of a new generation of ICT tool for mathematics education have been derived from the above literature about semiotic perspectives towards meaningful knowledge construction or meaning-making:

1. Because all cognition is irreducibly triad, we should take the three structural components: *sign*, *object*, and *interpretant* into account for meaning-making or meaningful knowledge construction when designing new generation of ICT tools for mathematics education.
2. Knowing that there are three categories of signs (i.e., icon, index, and symbol), we should carefully consider the meaning emission capacity of each sign category when integrating a variety of signs (e.g., pictorial icons, GUI widgets, 3D space and objects, and text and emotions etc.) into the interface of an ICT tool. Unwanted interference from signs can be reduced and learners can effectively make sense of and manipulate user-friendly interfaces. Furthermore, the design of the ICT tools should allow users multiple ways to manipulate the signs to express and achieve ideas. For example, users can build 3D objects through manipulating icons and/or buttons, or by specifying language commands and/or programming procedures.
3. The new generation of ICT tools for mathematics learning should encourage and engage learners in abductive thinking. The ICT environment should confront users with some new experiences (e.g., design of 3D artefacts and explorations in 3D virtual space) and enable users to create and invent signs (e.g., naming a new command) to apply to new experiences. Moreover, the new invented signs need to be ready to be modified again to make sense of further new experiences. If so, the learners will be ready to alter and challenge their's and others' beliefs.
4. Multiple semiotic resources including typological, topological, and social-actional resources should be integrated into new generation of ICT tools. With multiple semiotic resources at users' disposal, they can then approach the object or its meaning easier.

The Case of 3D Geometry

3D geometry can be conceptualised within a semiotic framework as consisting of three indispensable components namely external material world, internal spatial ability, and communication (Fig. 3). The 3D geometry as defined in this study therefore is not limited on the textbook or school mathematics. Instead, it refers to the real world 3D space in which we live, move, and perceive.

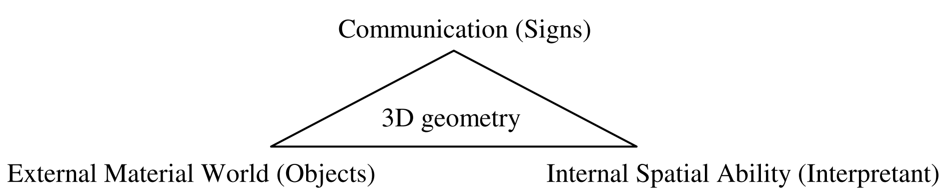

Figure 3: Semiotic Triad of 3D Geometry

The external material world represents all geometric objects including ill-structured natural objects (e.g., a shell, a growing tree) and simplified idealised objects (e.g., shapes of triangle and cube) with their behaviours (e.g., the growing recursive fern leaves) and properties (e.g., angle, height, and length). These objects are often found with mathematical patterns and relationships embedded within them.

Internal spatial ability (Interpretant) is the human potential and capacity to perceive and know the external geometric objects. Many human spatial abilities have been identified in research literature such as McGee's (1979) categorisation of spatial visualisation and orientation, Hoffer's (1977) identification of seven spatial perception abilities, and Gutiérrez's (1996) framework for the visualisation in 3D geometry consisting of mental images, external representation, process of visualisation, and abilities of visualisation. The development of these spatial abilities as the internal understandings of external world relies heavily on the mediation of sign systems (communication).

The communication component of 3D geometry refers to the language that in a broad sense includes the spoken and written language, mathematical notation, pictures, diagrams, kinaesthetic body movements, and even geometric objects themselves etc.

Because signs are incomplete representations of objects (Cunningham, 1992; Lemke, 2001), these varieties of communication will result in different understandings of the external world forming within the minds of learners. For example, the geometric language in describing position and direction is classified into three frames of reference: egocentric (e.g., forward, back, left, and right with reference to own body), fixed (e.g., above, between, east, and west with reference to other objects), and coordinate (e.g., x, y, and z with reference to coordinate system) (Darken, 1996; Piaget & Inhelder, 1956). The egocentric frame of reference is developed earlier in children. However, Yakimanskaya et al. (1991) pointed out that the predominant use of one particular frame of reference (most often the human body) often impedes successful problem solving particularly in descriptive geometry. They suggested that it is necessary to use several frames of reference simultaneously.

This semiotic triad of 3D geometry presented in Figure 3 indicates that the knowledge construction of 3D geometry concepts and processes requires the development of internal spatial abilities to reflect on the external world through the mediation of various communications. With this in mind, the new generation of ICT tool will need to take into account all three components of the triad.

The Application: VRMath

Informed by the guidelines subsumed within the semiotic framework, the authors have developed a new generation ICT tool named VRMath (Yeh & Nason, 2004b) to facilitate the learning of 3D geometry. VRMath utilises innovative virtual reality (VR) technology and integrates many semiotic resources to form a virtual reality learning environment (VRLE) as well as a mathematical microworld (Edwards, 1995) for learning 3D geometry. VRMath consists of three components namely topological, typological, and social-actional components (Fig. 4) which now will be discussed in turn.

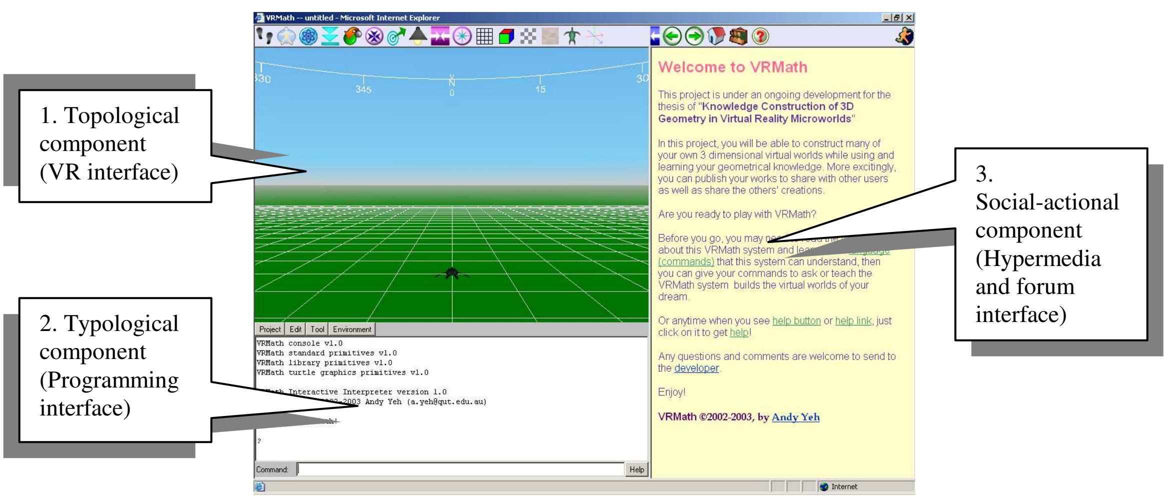

Figure 4: Three Components of VRMath

The topological component: The main part of this component is the *virtual reality (VR) interface* that provides rich representations in colours, textures, geometric objects and behaviours, and allows for real-time navigation within the 3D virtual space. This enables students to engage in the exploration of visual graphics, images and 3D shapes and space, meanings by degree and/or continuous representations of 3D geometry.

The typological component: The main part of this component is the Logo-like *programming interface* that engages students in logical procedural thinking and the tutee mode of computer use, and links to the topological component when the students are programming to manipulate and build objects within 3D virtual reality microworlds. This component refers to any meaning by kind or discrete representations of geometry such as language, texts, numbers, icons and buttons.

The social-actional component: The main part of this component is the *hypermedia and forum interface* that aggregates information and scaffolds discourse. Students thus can contribute ideas, search for information and ask for help from more knowledgeable peers. This component includes facilities such as online discourse that stimulate thinking, and allows for the provision and sharing of ideas.

These three components in many ways reflect and implement the semiotic triad of 3D geometry presented in Figure 3. The three components can all act as signs that mediate and communicate between objects (e.g., knowledge of 3D geometry, external material world) and interpretant (e.g., users' cognitive structure, internal spatial ability). For example, The *VR interface* provides topological visualisations of the external material world. It also enables real time navigation in its 3D space. Thus, users are provided with an opportunity to develop their spatial visualisation and orientation abilities (McGee, 1979). However, the 3D virtual world doesn't equate to the real world. The topological signs are still incomplete representations of the objects. To have better understandings about knowledge of 3D geometry, users can create and design their own 3D microworlds through the manipulation of *programming interface*. By manipulating the typological language or icons (e.g., Procedure Editor and Quick Command tool), users can link the discrete geometric vocabularies or commands to continuous geometric figures (Fig. 5), and make meaning of 3D geometry concepts through the construction processes within *VR interface* and *programming interface*.

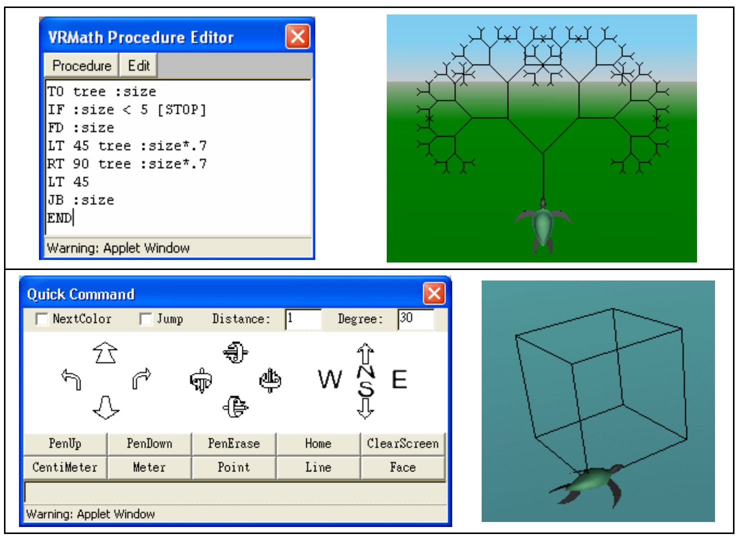

Figure 5: Typological Signs link to Topological Signs

The *programming interface* employs a Logo-like programming language with many built-in 3D turing (e.g., LEFT, RIGHT, ROLLUP, ROLLDOWN, TILTLEFT, and TILTRIGHT) and moving commands (e.g., egocentric FORWARD and BACK, fixed EAST, WEST, UP, and DOWN etc.). The programming language also engages users in the creation of their own commands (i.e., procedures) and the construction of new 3D objects in an abductive manner. Moreover, the *hypermedia and forum interface* organises thoughts in the forms of multimedia and hyperlinks, and implements the social-actional communications, which allow users to interact with each others through the Internet. Users are able to express, exchange, search, retrieve and discuss their ideas about 3D geometry in the forum.

Discussion

Initial evaluation of VRMath with primary school children indicates that it is a very effective tool for facilitating construction of knowledge about 3D geometry concepts and processes (Yeh & Nason, 2004a). The design and exploration of 3D geometric objects within the VRMath environment seems to enable primary school children to gain deep insights into the anatomy of 3D geometry. That is, it not only helps them to construct knowledge about specific 3D geometry concepts and processes, but also to construct knowledge about the relationships between the 3D geometry concepts and processes. This may be attributed by the rich multiple semiotic resources provided in VRMath system. For example, during the collaborative construction of a spiral staircase (Fig. 6), a group of six 9-10 years old students learnt much new knowledge about the relationship between the cube, notions of scale, and location and direction in 3D virtual space.

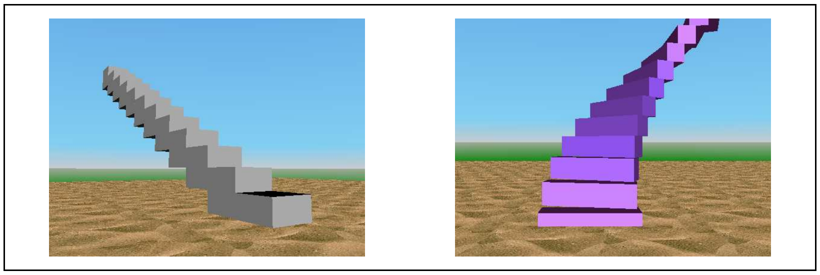

Figure 6: Collaborative Construction of a Spiral Staircase

In the collaborative construction of a spiral staircase, the students constructed their artefact by consolidating their geometric language (e.g., Logo-like programming language) with geometric concepts about scaling a cube in three dimensions (width, height and depth), positioning and rotating in 3D space (moving and turning the turtle), and collaborating in online discussion. The rich semiotic resources including topological (e.g., the spectrum of colours, continuous visualisation and navigation), typological (e.g., geometric terms and programming commands), and social-actional (e.g., discourse and online forum communication) enabled them to construct an effective and deep understanding of 3D geometrical knowledge.

This paper reports on research in progress. As of the essence of semiotics, meaning has to be constantly qualified and challenged. The semiotic framework and its application VRMath are continuously being re-examined, and together, they are both evolving continuously. For example, a pilot study (Yeh & Nason, 2004a) evaluating the design of VRMath and children's learning about 3D geometry within VRMath has identified many interesting usability (Nielsen & Mack, 1994) issues and issues concerning children's spatial abilities and use of different frame of reference commands in constructing 3D artefacts that need to be investigated. Based on the findings and feedback from this pilot study, VRMath has been redesigned and now is being re-examined in more empirical studies. Concurrent with the re-design of VRMath, the semiotic framework utilised to inform the design of VRMath also is being revised and modified based on findings from the pilot study.

In this paper, the semiotic framework was applied to inform the design of an ICT tool focusing on the learning of 3D geometry. However, the authors believe that as more insights are derived from evaluations and ensuing modifications of VRMath, the scope of the semiotics framework can be extended so that it can contribute to the research literature pertaining to the use of technology in mathematics education in general.

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