# Knowledge Building of 3D Geometry Concepts and Processes within a Virtual Reality Learning Environment

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**Abstract**: This paper reports on a pilot study for a prototype VRLE (Virtual Reality Learning Environment) named VRMath. The two primary school students who were involved in this study engaged in two VRMath learning activities designed by the researchers. The results indicated that 3D navigation within the VR 3D space was difficult. However, it could be aided with the navigation aids designed within VRMath. The 3D navigation within the 3D virtual space also caused the participants confusion in terms of their spatial visualisation and orientation abilities. The construction of 3D geometrical objects within VRMath was also difficult especially when the participants were operating the 3D rotation mentally and physically with respect to their body (i.e., the egocentric frame of reference). It was found that the simultaneously use of different frames of reference could help the construction of 3D geometrical objects. During the learning activities, issues about the usability of VRMath were also explored.

# Introduction

This paper reports on a pilot study on a prototype Virtual Reality Learning Environment (VRLE) being researched and developed by Yeh and Nason (2004). Informed by the seminal thoughts in fields such as the semiotic perspectives about mathematics as a meaning-making endeavour, fallibilist philosophies about the nature of mathematics, and constructivist and constructionist perspectives about learning, the researchers have conceptualised and developed a VRLE named VRMath. VRMath is an online web application that integrates desktop VR (Virtual Reality) technology combined with the power of a Logo-like programming language and hypermedia and the Internet to facilitate the learning of 3-dimensional (3D) geometry concepts and processes. It has been designed to enable users to build their own 3D microworlds while operating within a 3D VR space, linking their language to the geometrical Logo-like programming language, and collaborating through its online discussion forum.

VRMath consists of three interfaces: The VR interface, the programming interface, and the hypermedia and forum interface (Fig. 1).

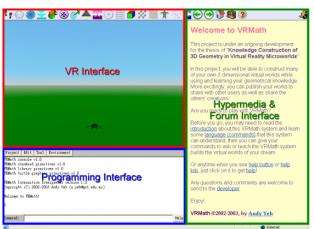


Figure 1: Three Interfaces of VRMath

The VR interface of VRMath allows users to *navigate and view* the geometrical objects within the virtual space from any viewpoints by using the mouse and the keyboard. It also allows users to *create and manipulate* geometrical objects through the Logo-like programming interface according to a 3D turtle's position and orientation within the 3D virtual space. To *navigate and view* is related to the human spatial ability termed *spatial orientation* (McGee, 1979), which is the ability to determine spatial relation with respect to one's body, while to *create and manipulate* is related to another human spatial ability termed *spatial visualisation* (McGee, 1979), which is the ability to mentally rotate, manipulate, and twist two- and three-dimensional stimulus objects. These two spatial abilities are both called into action within VRMath environment. However, the use of 2D input devices such as mouse and keyboard to directly manipulate within a 3D space has a fundamental problem of dimensionality mismatch (Leach, Al-Qaimari, Grieve, Jinks, & Makay, 1997) which may cause difficulties in 3D navigation and confusion about directionality within VR space. The researchers thus developed some navigation aids such as Restore Viewpoint, Fit Screen, Compass, and Coordinate Axis etc. to ease and overcome this problem. These navigation aids were associated with icons and placed in a toolbar above the VR space.

The Logo-like programming interface of VRMath not only inherits the power of Logo programming language, but also includes a set of 3D related turtle graphic commands. For move commands, three frames of reference: *egocentric* (FD, BK), *fixed* (UP, DOWN, EAST, WEST, SOUTH, NORTH), and *coordinate* (SETX, SETY, SETZ, SETXYZ) (Darken, 1996; Piaget & Inhelder, 1956) are employed in VRMath programming language. For turn commands, VRMath also includes four other commands: RU (ROLLUP), RD (ROLLDOWN), TL (TILTLEFT), and TR (TILERIGHT) in addition to RT (Right) and LT (LEFT) from traditional Logo to perform 3D rotation. The inclusion of design features within VRMath such as the navigation aids and the extension of Logo commands etc. enables the investigation in many new aspects of knowledge construction of 3D geometry. In addition to investigating issues such as many new ways of learning 3D geometry knowledge, the researchers also have had to address usability issues with respect to human-computer interaction (HCI) during the research and development of the prototype version of VRMath. Nielsen (1993) listed five attributes of usability:

- 1. Learnability: the ability for users to learn the system easily;
- 2. Efficiency of use once the system has been learned: the ability for users to save time in their work once they've learned the system;
- 3. Memorability: the ability for users to come back to the system and remember how to use it once they've been away from it for some time
- 4. Error recovery and prevention: when the system presents an error message to users, it gives enough information for them to be able to continue with their work. Better yet, the system helps to prevent errors; and
- 5. Subjective user satisfaction: users' overall feelings about the system. Is it pleasant to use?

These five usability attributes were used to inform the investigation of VRMath's usability during this pilot study.

### The Study

The purposes of this pilot study were to investigate the two participants' knowledge construction of 3D geometry concepts and processes within VRMath while at the same time to evaluate the usability of VRMath's human-computer interaction.

#### Participant

The two participants in this study were Emilie (a Grade 6 student) and Anya (a Grade 7 student) who both attended an inner city school in eastern Australia. Neither participant had prior experiences in programming and Logo. Nor had either participant had any experience with 3D computer graphics.

### Procedure

Both participants interacted with VRMath together for six sessions (one hour per session, one session per week). They started from an introduction about VRMath, and then were engaged in two main activities.

Activity 1: The secret of the turtle's eyes. This is a game like activity, in which the researcher placed a proximity sensor near the turtle's eyes. When the users navigate very close (within about 17 cm) to the turtle's eyes, the

eyes will change colour from blue to red. When the users are away from the eyes, the eye colours change back the colour to blue (Fig. 2). In this activity, the participants needed to use their navigation skills and may be some navigation aids in order to discover this secret and also find a "best way" to navigate to the turtle's eyes. The purposes of this activity were to train the participants' 3D navigation skills and for the researchers to investigate the relationship between 3D navigation and learning.

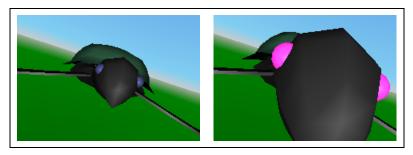


Figure 2: The Secret of the Turtle's Eyes

Activity 2: A frame of a cube. In this activity, the participants were required to construct a frame of a cube (Fig. 3). The necessary information and knowledge (e.g., move and turn commands) were given to the participants. The participants were asked to first experiment with the Quick Command tool (a GUI command centre, see Fig. 4) to construct a frame of a cube. Following this, they were required to mentally imagine and/or physically act out and then to write a procedure for constructing a frame of cube. The procedure then was saved as a public project into online database where it could be examined by other members in the forum community. The purposes of this activity were to investigate the participants' use of different frames of reference commands, and for usability inspections.



Figure 3: A Frame of a Cube

Quick Command					
NextColo:	r 🥅 Jump	Distance:	1 Deg	ree: 30	
∿ Ĉ ⊕ Ŭ ℃ ⊕ W S E ℃ €					
PenUp	PenDown	PenErase	Home	ClearScreen	
CentiMeter	Meter	Point	Line	Face	
Warning: Applet Window					

Figure 4: Quick Command Tool

At the end of each activity, the researchers conducted a focus interview with both participants where usability issues with respect to VRMath were probed.

### Data collection and analysis

During the course of this pilot study, data were collected from three sources:

1. Researchers' observation of the participants' interaction with VRMath interface and intellectual engagement with the activities.

- 2. Videotapes of activities and interviews.
- 3. Computer records of the participants' inputs in VRMath including commands, procedures in their projects, and the geometric objects created.

In order to facilitate the process of analysing data, the videotapes were then transcribed and the data from the transcription, observation, field notes, and artefacts were organised. The analysis of data with respect to learning of 3D geometry knowledge utilised a grounded theory approach (Creswell, 2002). Creswell (2002) defines a grounded theory approach as "a systematic, qualitative procedure used to generate a theory that explains, at a broad conceptual level, a process, an action, or interaction about a substantive topic" (p. 439). The analysis of data with respect to usability issues was informed by Nielsen's five attributes of usability (1993).

## **Results and Discussion**

#### Activity 1: the Secret of the Turtle's Eyes

During this activity, the two participants seemed to find it difficult to navigate in 3D space using a mouse. Two main difficulties were apparent:

- 1. Controlling the speed of navigation: In VRMath, the distance the mouse is dragged determines the speed with which the user navigates around the 3D environment; the further the mouse is dragged, the faster the user moves within the environment. This was explained to the participants at the beginning of this activity. However, during this activity both students had great difficulty in controlling speed as they attempted to navigate within the 3D virtual space.
- 2. Controlling the direction of navigation: To navigate in different directions within the 3D virtual environment requires more than the four directions provided by arrow keys or by the mouse being dragged over the two-dimensional mouse pad. Therefore, to navigate within the 3D virtual world required other keys (e.g., Alt, Space) to be used in conjunction with the arrow keys or the mouse dragging over the mouse pad. This was further complicated by the three modes of navigation within the 3D virtual world: walk, fly, examine. The participants found the coordination of the Alt and Space keys with the dragging of the mouse rather difficult. However, by the end of this session after many trial-and-error explorations, they had mastered the control of navigation.

The completion of the activity (to navigate within 17 cm of the turtle's eyes and see the eyes turn red) was not achieved simply by navigating using the three modes of navigation. The two participants found that they had to experiment with other navigation aids before they were able to satisfactorily complete the activity. These other navigation aids such as Change Navigation Mode, Avatar View, Set Rotation Centre, Align to Ground, Restore Viewpoint, and Fit Screen etc. were displayed as icons above the 3D window. Because it was impossible for these icons to fully deliver their meanings of functionality, a mechanism of showing a Help Text when the mouse was pointing on the icon was built into the design of the system. Both participants found the Help Text very useful, especially when they began the activity. During their navigation in 3D space, the "Restore Viewpoint" icon was found to be used the most often, as the students got lost very easily in 3D virtual space. After investigating all of the other navigation aids, the participants found the best way to complete this task was to use Fit Screen function, which brought all objects into the 3D screen.

Emilie contributed her experience and wrote in the forum: "You make the turtles eyes go red by hitting the fit screenbutten (sic.) and then rotating the turtle by using the examine mode..."

In VRMath's 3D space, the users navigate themselves about the 3D virtual environment with the mouse and the navigation keys. Using the mouse and the navigation keys has no effect on the turtle. The turtle can only be moved by specific written commands (e.g., fd 1, bk 10 etc.). As the participants were navigating towards the turtle using the mouse and navigation keys, they were asked, "Is the turtle moving?" In a very confident manner, both participants replied that the turtle was moving. This indicated that as the participants were navigating within VRMath, their differentiation between spatial visualisation and orientation (McGee, 1979) was lost. Normally in the real world, one can differentiate between spatial visualisation and orientation by referring to points of references located in space or to one's kinaesthetic sense. Thus, if the person perceives that her location with respect to these points of reference are staying constant whilst her perspective of an object is changing and her spatial visualisation of the object is changing. By contrast, if the person perceives that both her location to the points of reference and her perspective of an object is changing, then the person knows that she is moving and thus her spatial orientation with respect to the object is changing. By

changing. Also, in the real world this is further confirmed by kinaesthetic feedback if they are moving. It was conjectured that because neither kinaesthetic feedback nor specific points of reference were provided in this desktop VR, the participants would be unable to differentiate between spatial visualisation and orientation and thus intuitively came to the incorrect conclusion that the turtle was moving. This conjecture was confirmed.

To overcome their incorrect intuition that the turtle rather then they themselves were moving, the researcher (Yeh) asked the participants to pay attention to the compass at the top of the window and the background stimuli in VRMath. When they paid attention to the compass and/or the background stimuli, they soon discovered that in fact they were moving through the virtual 3D world. And because of this, their orientation to the turtle was changing not their spatial visualisation.

### Activity 2: A Frame of a Cube

Emilie first tried on Quick Command tool to construct a cube. She was very confident in using the tool. During the process of using Quick Command, although the construction didn't look like a cube, she didn't navigate to change the viewpoint. Thus, two traditional errors for egocentric reference that have been termed the egocentric bug (Fay & Mayer, 1988) emerged. That is, when the turtle is facing Emilie, she often intuitively clicked on forward for back and back for forward, and left for right and right for left. This didn't happen when the participants were typing commands. It was conjectured that because of the GUI, the arrow icons misdirected her to this egocentric bug.

The GUI interface of Procedure Editor was found to be easy to use. The participants also easily accepted the format of a procedure (e.g., *TO procedure\_name commands END*) and started to write down commands while mentally thinking the position and rotation of the turtle. It was found that the participants had great difficulty remembering the orientation of the turtle especially when more then one dimension of rotation was used (e.g., LEFT and RIGHT is one dimension, ROLLUP and ROLLDOWN is another dimension). The researcher then advised them to use other frames of reference commands such as UP and DOWN to avoid using ROLLUP or ROLLDOWN. It was found as stated in the literature (Yakimanskaya, Wilson, & Davis, 1991) that the use of fixed frame of reference in addition to egocentric frame of reference aided the construction of a cube significantly. The participants then easily completed their procedures (Tab. 1).

Anya's procedure	Emilie's procedure	
TO acube	TO ecube	
cs	cs	
pd	pd	
repeat 4 [fd 1 rt 90]	repeat 4 [fd 1 rt 90]	
up 1	up 1	
repeat 4 [fd 1 rt 90]	repeat 4 [fd 1 rt 90]	
fd 1 down 1	east 1	
rt 90	dn 1	
fd 1 up 1	north 1	
rt 90	up 1	
fd 1 down 1	west 1	
pu	dn 1	
END	bk 1	
	END	

 Table 1: Procedures of a Cube

These two procedures from Anya and Emilie were very similar. Both procedures correctly produced a frame of a cube. However, the researcher found that the use of fixed frame of reference commands instead of egocentric commands actually produced different results. For example, when the turtle is in TILTLEFT 45 orientation, the two procedures above will not create a cube (see Fig. 5). This interesting phenomenon could be further developed in future activities with participants in the main study.

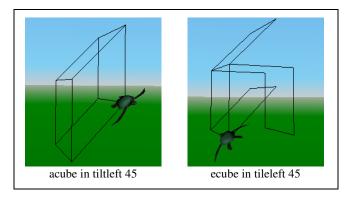


Figure 5: Fixed frame of reference cube

#### **Usability Inspection**

The usability inspection was undertaken through the observation of the participants' use of VRMath system and the interview at the end of each session. The observation and interview focused on the five attributes of usability (Nielsen, 1993).

The researcher observed that the participants had some difficulties in using VRMath system. However, when interviewed the participants claimed that it was easy to learn and remember. For example, Emilie spent about 12 minutes in finding the Fit Screen icon. Despite this, she still thought that it was easy to use and remember. The researcher found that the participants could fluently switch between and utilise the VR interface, programming interface, and hypermedia forum interface. During the five sessions, no systemic errors occurred. Both the participants expressed that they had enjoyed VRMath very much.

However, three main suggestions regarding the interface and activity design of VRMath were made by the two participants:

- 1. The dialogues such as Quick Command should stay in foreground instead of being sent to background when users temporarily switched to navigate in 3D virtual space. This had also been noted by the researcher. However, due to the limitation of the programming language, it was not possible to change this in the short duration of this study.
- 2. The current prototype of VRMath could only save/store the users' constructions of procedures in projects. The participants wished that VRMath could also save/store their constructions of 3D microworlds created in the VR interface, so they could share with each other easier. This is an excellent suggestion as the 3D visualisations are rich and intuitive representations of 3D geometry. The researcher will consider implementing this in the next iteration of the design of VRMath.
- 3. The requirements for more games such as the "turtle's eyes" game to be designed in activities. Their reason for making this suggestion was that they found that "doing" the game was much fun. This suggestion has been adopted by the researcher. However, since the purpose of games such as "turtle's eyes" is to facilitate the students' skill in 3D navigation, the design of game-like activities needs to avoid shifting the students' attention away from learning.

# Conclusion

In this pilot study, VRMath was evaluated with two upper primary school students. Based on the data collected during the enactment and evaluation, findings that reflect on issues of the design of VRMath and learning of 3D geometry within VRMath were generated and reported. Some of these findings such as the emergence of the traditional egocentric bug and the need for the use of multiple frames of reference to aid the construction of 3D geometric objects have confirmed results from past research literature (e.g., Fay & Mayer, 1988; Yakimanskaya et al., 1991). But more importantly, some of the findings from this study have raised issues that hitherto have not emerged in the research literature.

The first of these findings was that the 3D VR environment in VRMath provided users with the opportunity to operationalise both their spatial visualisation and orientation abilities. This is in contrast with most other ICT tools that

enable users only to operationalise their spatial visualisation abilities. However, because kinaesthetic feedback from VRMath only comes from the mouse and keyboard, users often think that they are manipulating the geometric objects as they do with other ICT tools. Therefore, users seem to perceive that the turtle rather than they are moving within the 3D virtual environment. To overcome this problem and to enable users to successfully utilise their spatial orientation abilities within the VRMath environment, environmental cues such as background and compass need to be emphasised to allow users to distinguish between movement by objects and movement by themselves within the 3D virtual environment. In addition, the scaffoldings such as those used by the researcher Yeh during the course of this pilot study can also help. For example, instructors can use "if you can navigate or walk to see the turtle's eyes…" in discourse with users.

The second of these findings was that the participants found the process of navigating in the 3D virtual space of VRMath rather challenging. VRMath enables users to navigate themselves to get multiple viewpoints in the 3D virtual space. This is invaluable in the process of constructing knowledge about 3D geometry. However, users often found that it was difficult to control their navigation speed and direction. To overcome the difficulties in 3D navigation, the three modes of navigation Walk, Fly, and Examine need to be explicitly discussed with the users before giving information about the use of mouse and keyboard. The three modes of navigation are actually informative metaphors for building conceptions about 3D navigation. For example, the walk mode has a gravity effect. These discussions may help alleviate the memory load during 3D navigation. Also, more time should be devoted for users' practice of 3D navigation prior to their engagement with complex design of 3D objects tasks.

The third of these findings was that the use of multiple frames of reference commands could aid the construction of 3D geometric objects significantly. But more importantly, the different frames of reference commands have different effects on the construction of 3D geometric objects. The implications of this finding for later phases or iterations of the study are that the different effects of constructing 3D geometric objects emanating from the use of different frames of reference should be developed and integrated into the learning activities. Also, the users' understanding about the use of different frames of reference are worthy of further investigation.

This pilot study has had great import to the evolution of VRMath. However, it is just a small exploration of VRMath. VRMath will undergo more iteration cycles of design enactment reflection redesign to evolve to a better education tool, and generate more insights for the learning of 3D geometry and the future design of VRLEs.

### References

Creswell, J. W. (2002). *Educational research. Planning, conducting, and evaluating quantitative and qualitative research.* Upper Saddle River, New Jersey: Merrill Prentice Hall.

Darken, R. P. (1996). *Wayfinding in large-scale virtual worlds*. Unpublished Doctor of Science thesis, George Washington University, Washington, DC.

Fay, A. L., & Mayer, R. E. (1988). Learning Logo: A cognitive analysis. In R. E. Mayer (Ed.), *Teaching and learning computer programming: Multiple research perspectives* (pp. 55-74). Hillsdale, NJ: Lawrence Erlbaum Associates.

Leach, G., Al-Qaimari, G., Grieve, M., Jinks, N., & Makay, C. (1997, March 25). *Elements of a three-dimensional graphical user interface*. Retrieved February 12, 2002, from http://goanna.cs.rmit.edu.au/~gl/research/HCC/interact97.html

McGee, M. G. (1979). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin*, 86(5), 889-918.

Nielsen, J. (1993). Usability engineering. Boston: Academic Press.

Piaget, J., & Inhelder, B. (1956). The child's conception of space. London: Routledge & K. Paul.

Yakimanskaya, I. S., Wilson, P. S., & Davis, E. J. (1991). *The development of spatial thinking in schoolchildren* (Vol. 3). Reston, VA: National Council of Teachers of Mathematics.

Yeh, A., & Nason, R. (2004, June). VRMath: A 3D Microworld for Learning 3D Geometry. Paper presented at the World Conference on Educational Multimedia, Hypermedia & Telecommunications, Lugano, Switzerland.