Reasoning Under Uncertainty During Pre-Negotiations Using a Fuzzy AHP

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Abstract

The paper proposes a new approach for tackling the uncertainty and imprecision of the reasoning process while using decision support tools during pre-negotiations. The pre-negotiation problem is regarded as decision making under uncertainty, based on multiple criteria of quantitative and qualitative nature, where the imprecise decision-maker's judgments are represented as fuzzy numbers. A new fuzzy modification of the Analytic Hierarchy Process is applied as an evaluation technique. The proposed fuzzy prioritization method uses fuzzy pairwise comparison judgments rather than exact numerical values of the comparison ratios and transforms the initial fuzzy prioritization problem into a non-linear program. Unlike the known fuzzy prioritization techniques, the proposed method derives crisp weights from consistent and inconsistent fuzzy comparison matrices, which eliminates the need of additional aggregation and ranking procedures. A detailed numerical example, illustrating the application of the approach to services evaluation is given.

Introduction

The design and implementation of decision support systems that can introduce automation and intelligence to on-line negotiations is currently the focus of intensive research efforts. Various negotiating models and automated trading systems have been produced, addressing different market needs and requirements. Negotiation models, however, are characterized by relatively high complexity, since they involve evaluation and decision making under uncertainty, based on multiple attributes (criteria) of quantitative and qualitative nature, involving temporal and resource constraints, risk and commitment problems, varying tactics and strategies, domain specific knowledge and information asymmetries, etc.

Each negotiation cycle involves a sequence of interdependent activities (decision making and actions) - from preparing to enter the negotiation, through the negotiation *per se*, to the execution of the agreed deal. Since actions and outcomes in one stage may strongly influence and constrain the next, the pre-negotiation phase (Tsvetinov 2002, Tsvetinov 2003) is of a special importance. It sets the scene for the consequent stages and influences in a unique manner the following deliberations. Some authors (Moran and Ritov 2002) even find that in a simulated competitive market the specific composition of the initial offers may influence the final agreements beyond the effect predicted by their overall value. Very few studies address the reasoning and actions that may take place during the pre-negotiation phase (Faratin and Klein 2001; Faratin, Sierra and Jennings, to appear).

While the computational complexities of automating negotiations over multidimensional goods as services have been identified, the concept of preempting some of the decision-making problems and shifting part of the reasoning and deliberations to the pre-negotiation phase has not yet been clearly formulated. Instead, researchers in the area of automated negotiations focus on establishing appropriate tactics and strategies during the exchange of offers and counter-offers (Jennings, Parsons, Sierra and Faratin 2000; Vulkan and Jennings 2000) or the 'negotiation dance', to use the elegant definition of Raiffa (1982).

The current paper addresses the problem of uncertainty related to some of the major evaluation methods used for decision support and automation in the pre-negotiations reasoning process. The proposed approach is intended to overcome difficulties in ranking service package offers by using a modification of the Analytical Hierarchy Process (AHP) as an evaluation tool. The AHP is widely used for multi-criteria decision-making and has successfully been applied to many practical decision-making problems (Saaty 1988). In spite of its popularity, this method is often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision-maker's perception to exact numbers (Deng 1999).

In the traditional formulation of the AHP, human's judgments are represented as exact (or *crisp*, according to the fuzzy logic terminology) numbers. However, in many practical cases the human preference model is uncertain and decision-makers might be reluctant or unable to assign exact numerical values to the comparison judgments. For instance, when evaluating different services, the decision-makers are usually unsure in their level of preference due to incomplete and uncertain information. Since some of the service evaluation criteria are subjective and qualitative, it is very difficult for the decision-maker to express the strength of her preferences and to provide exact pairwise comparison judgments.

The main objective of this paper is to propose a new approach within the AHP framework for tackling the uncertainty and imprecision of service evaluations during the pre-negotiation stage, where the decision-maker's comparison judgments are represented as fuzzy triangular numbers. A new fuzzy prioritization method, which derives crisp priorities (criteria weights and scores of alternatives) from consistent and inconsistent fuzzy comparison matrices, is described. The fuzzy modification of the AHP is applied as an evaluation technique and illustrated by a numerical example.

Statement of the Service Evaluation Problem

Building a utility function as a measure of the goodness of a service package is far from being a straightforward task due to the substantial diversity and complexity in service properties. Adding to the potentially rich variance in service attributes, there are some further generic problems while negotiating over services, such as:

- Since negotiations are typically over a number of issues, a successful outcome will require the whole range of issues to be resolved to the satisfaction of both parties.
- The factors that influence the negotiators' stance and behavior are usually private and not available to their opponents. The negotiating parties are unaware of the other party's utilities, constraints and reasoning models.
- Individual agents can take the role of both a client and a server for different services in different negotiating contexts.

The assessment of the multidimensional service property packages during pre-negotiations involves at least two stages:

- Property discovery and comparison of the services offered in a common ontology framework.
- Using appropriate evaluation methods that can assess both qualitative and quantitative attributes of the offered service packages.

The second stage, a focus of the current study, requires the application of methods that address some intrinsic assessment problems, such as using a common evaluation scheme for qualitative and quantitative criteria (attributes), modelling relationships that may exist among service properties, etc. The main problem, however, consists in the incomplete and imprecise information about possible service providers and attributes of the service offer. Moreover, often the service evaluation criteria are subjective and qualitative, thus it is very difficult for the decision-maker to express the strength of his preferences using exact numerical values.

The AHP is well suited to address some of these problems since the approach is qualitative and easier to implement from both a data requirement and validation point of view than the Multiple Attribute Value Theory (MAVT). Using the AHP means that not all independence conditions of the MAVT need to be verified, nor value functions derived. Thus, the method is appropriate for evaluation of qualitative, related attributes in service offer's packages. However, the standard AHP cannot straightforwardly be applied to solving such uncertain decision-making problems. In order to eliminate this drawback, in the next section we propose a fuzzy modification of the AHP, capable for tackling the uncertainty and imprecision of the service evaluation process.

Fuzzy Analytic Hierarchy Process

The AHP divides the decision problem into the following main steps (Saaty 1988):

1. Problem structuring.
2. Assessment of local priorities.
3. Calculation of global priorities.

The AHP decision problem is structured hierarchically at different levels, each level consisting of a finite number of decision elements. The top level of the hierarchy represents the overall goal, while the lowest level is composed of all possible alternatives. One or more intermediate levels embody the decision criteria and sub-criteria. The relative importance of the decision elements (weights of criteria and scores of alternatives) is assessed indirectly from comparison judgements during the second step of the decision process. The decision-maker is required to provide her preferences by comparing all criteria, sub-criteria and alternatives with respect to upper level decision elements. The values of the weights and scores are elicited from these comparisons and represented in a decision table. The last step of the AHP aggregates all local priorities from the decision table by a simple weighted sum. The global priorities thus obtained are used for final ranking of the alternatives and selection of the best one.

The first and the last steps of the AHP are relatively simple and straightforward procedures, while the assessment of local priorities, based on pairwise comparisons needs some prioritization method to be applied. However, the standard AHP eigenvalue prioritization approach cannot be used, when the decision-maker faces a complex and uncertain problem and expresses the comparison judgements as uncertain ratios, such as 'about two times more important', 'between two and four times less important', etc. A natural way to cope with such uncertain judgements is to express the comparison ratios as fuzzy sets or fuzzy numbers, which reflect better the vagueness of human thinking.

When comparing any two elements at the same level of the decision hierarchy, an uncertain comparison judgement can be represented by a fuzzy number. In this paper we use triangular fuzzy numbers, which are a special class of the L-R fuzzy sets (Dubois and Prade 1980). A triangular fuzzy number $\widetilde{N}$ is defined by three real numbers $a \le b \le c$, and characterized by a linear piecewise continuous membership function $\mu_{\widetilde{N}}(x)$ of the type:

$$\mu_{\widetilde{N}}(x) = \begin{cases} (x-a)/(b-a) & a \le x \le b \\ (c-x)/(c-b) & b \le x \le c \\ 0 & otherwise \end{cases} \quad (1)$$

The fuzzy number $\tilde{N}$ is often expressed as a triple (a, b, c), where b, a, and c are the mean, the lower and the upper bounds, respectively. Such notation will be used further in this paper.

Deriving Priorities from Fuzzy Comparison Matrices

Let us consider a prioritization problem at a level with n elements, where pairwise comparison ratios are represented by fuzzy triangular numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$. As in the traditional AHP method, a fuzzy reciprocal matrix $\widetilde{A} = \tilde{a}_{ij} \in \Re^{n\times n}$ can be constructed, such that:

$$\widetilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & 1 \end{bmatrix} \quad (2)$$

where $\tilde{a}_{ij} = 1/\tilde{a}_{ji}$.

The known fuzzy prioritization methods derive *fuzzy* priorities $\tilde{w}_i, i = 1, 2, \cdots, n$, which approximate the fuzzy ratios $\tilde{a}_{ij}$ so that $\tilde{a}_{ij} \approx \tilde{w}_i/\tilde{w}_j$. These methods are based on fuzzy versions of the logarithmic least squares method (Boender, Graan and Lootsma 1989; van Laarhoven and Pedrycz 1983), fuzzy modifications of the least squares method (Wagenknecht and Hartmann 1983; Xu 2000), fuzzy geometric means (Buckley 1985), or a fuzzy arithmetic mean (Chang 1996). Since all weights and scores derived by these methods are fuzzy numbers or fuzzy sets, their aggregation over the last step of the AHP yields final scores of the alternatives, which are also represented as fuzzy sets.

Due to the large number of multiplication and addition operations, the resulting fuzzy scores have wide supports and overlap over a large range. As it is shown by Gogus and Boucher (1997), the normalization procedure used in some of these methods may even result in irrational final fuzzy scores, where the normalized upper value is smaller than the normalized mean value, which on its turn is smaller than the normalized lower value.

The fuzzy prioritization methods mentioned above all require an additional *fuzzy ranking procedure* for comparing the final fuzzy scores and ranking alternatives. Different ranking procedures, however, often give different ranking results (Bortolan and Degani 1985).

In order to overcome some of the drawbacks of the existing fuzzy prioritization methods, a new approach for deriving priorities from fuzzy pairwise comparison judgments is proposed by Mikhailov (2003), based on α-cuts decomposition of the fuzzy judgments into a series of interval comparisons. The Fuzzy Preference Programming (FPP) method (Mikhailov 2000), which transforms the prioritization task into a fuzzy linear programming problem, is applied to derive optimal crisp priorities.

A non-linear modification of the FPP method is described in the next section, which derives crisp priorities from fuzzy comparison judgements without transforming the judgements into interval series and further aggregation of the priorities. Compared to the known fuzzy prioritization methods in the AHP, the proposed method does not require a fuzzy ranking procedure and is able to derive crisp priorities from an incomplete set of fuzzy judgements.

Fuzzy Prioritization Approach

Consider a prioritization problem with n elements, where the pairwise comparison judgements are represented by normal fuzzy sets or fuzzy numbers. Suppose that the decision-maker can provide a set $F = \tilde{a}_{ij}$ of $m \leq n(n-1)/2$ fuzzy comparison judgments, $i = 1, 2, \cdots, n-1, j = 1, 2, \cdots, n, j > i$, represented as triangular fuzzy numbers $\tilde{a}_i j = (l_{ij}, m_{ij}, u_{ij})$.

The problem is to derive a crisp priority vector $w = (w_1, w_2, \cdots, w_n)^T$, such that the priority ratios w_i/w_j are approximately within the scope of the initial fuzzy judgment, or

$$l_{ij} \tilde{\leq} \frac{w_i}{w_j} \tilde{\leq} u_{ij} \quad (3)$$

where the symbol $\tilde{\leq}$ denotes the statement 'fuzzy less than or equal to'. Each crisp priority vector w satisfies the double-side inequality (3) with some degree, which can be measured by a membership function, linear with respect to the unknown ratio w_i/w_j:

$$\mu_{ij}(\frac{w_i}{w_j}) = \begin{cases} \frac{(\frac{w_i}{w_j} - l_{ij})}{m_{ij} - l_{ij}}, & \frac{w_i}{w_j} \leq m_{ij} \\ \frac{(u_{ij} - \frac{w_i}{w_j})}{u_{ij} - m_{ij}}, & \frac{w_i}{w_j} \geq m_{ij} \end{cases} \quad (4)$$

The membership function is linearly increasing over the interval $(-\infty, m_{ij})$ and linearly decreasing over the interval (m_{ij}, ∞).

The solution to this prioritization problem by the fuzzy preference prioritization (FPP), proposed by Mikhailov (2000), is based on two main assumptions. The first one requires the existence of a nonempty fuzzy feasible area P on the $(n-1)$-dimensional simplex Q^{n-1}

$$Q^{n-1} = \left\{ (w_1, \cdots, w_n) | w_i > 0, \sum_{i=1}^{n} w_i = 1 \right\} \quad (5)$$

defined as the intersection of the membership functions, similar to (4), and the simplex hyperplane (5). The membership function of the fuzzy feasible area P is given by

$$\mu_p(w) = min\{\mu_{ij}(w) | i = 1, \cdots, n-1, ; \\ j = 2, \cdots, n; j > i\} \quad (6)$$

By defining the membership functions (4) as L-fuzzy sets $\{L = [-\infty, 1]\}$, we can relax the assumption of non-emptiness of P on the simplex. If the fuzzy judgements are very inconsistent, then $\mu_p(w)$ could take negative values for all normalized priority vectors $w \in Q_{n-1}$.

The second assumption of the FPP method specifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function (6). It can easily be proved that $\mu_p(w)$ is a convex set, so there is always a priority vector $w* \in Q_{n-1}$ that has a maximum degree of membership:

$$\lambda^* = \mu_p(w^*) = maxmin\{\mu_{ij}(w)\} \quad (7)$$

Solving the Fuzzy Prioritization Problem

The solution procedure of the proposed method is based on the *maximin* decision rule, known from the game theory. The maximin rule has also been applied by Bellman and Zadeh (1970) for solving decision-making problems in uncertain environment. Zimmermann (1990) uses the same decision rule for fuzzy linear problems with soft constraints and shows, that if the membership functions, representing the soft constraints, are linear, the maximin problem can be transformed into a linear programming problem. Similar linear formulations of the prioritization problem are given by Mikhailov (2000, 2003). The maximin prioritization problem (7) can be represented in the following way:

maximize λ
subject to

$$\lambda \leq \mu_{ij}$$

where

$$i = 1, 2, \cdots, n-1; j = 2, 3, \cdots, n; j > i, \qquad (8)$$

$$\sum_{l=1}^{n} (w_l) = 1, w_l > 0, l = 1, 2, \cdots, n.$$

Taking into consideration the specific form of the membership functions (4), the problem (8) can be further transformed into a bilinear program of the type:

maximize λ
subject to

$$\begin{aligned} (\mu_{ij} - l_{ij})\lambda w_j - w_i + l_{ij} w_j &\leq 0, \\ (u_{ij} - m_{ij})\lambda w_j + w_i - u_{ij} w_j &\leq 0, \qquad (9) \\ \sum_{k=1}^{n} (w_k) = 1, w_k > 0, k &= 1, 2, \cdots, n \\ i = 1, 2, \cdots, n-1; j = 2, 3, \cdots, n; j &> i, \end{aligned}$$

The optimal solution to the above non-linear problem (λ^*, w^*) might be obtained by employing some appropriate numerical method for non-linear optimization. The results shown in the next section are obtained by the Excel Solver tool, which is based on a gradient search numerical algorithm.

The optimal value λ^*, if positive, indicates that all solution ratios completely satisfy the fuzzy judgments, which means that the initial set of fuzzy judgements is rather consistent. A negative value of λ^* shows that the solutions ratios approximately satisfy all double-side inequalities (3), i.e. the fuzzy judgements are strongly inconsistent. Therefore, the optimal value λ^* can be used for measuring the consistency of the initial set of fuzzy judgements.

The existence of a consistency index is a very attractive feature of the proposed fuzzy prioritization method, which is illustrated in the next section. It can also be observed, that the non-linear program (9) does not necessarily need a full set of all fuzzy judgements from the upper triangular part of the comparison matrix (2). Therefore, the proposed method can derive priorities from incomplete set of judgements, which is another appealing feature of our approach.

Numerical Example

Suppose that the decision maker has to select a provider for a specific service. Three main criteria have been chosen for evaluation of alternative service providers, namely Pricing, Service Quality and Delivery Time, and each main criterion is additionally divided into two sub-criteria, namely Cost-based and Demand-based Pricing, Reliable and Responsive Service Quality and Immediate and Negotiable Delivery. Three alternative companies have been identified as potential service providers. The goal here is to select a service provider, satisfying all criteria in the best way. The solution process is based on the proposed fuzzy modification of the AHP method. The first step in applying the fuzzy AHP is to construct a (three level) hierarchy of alternative providers and criteria for choice, as shown on Fig.**??**

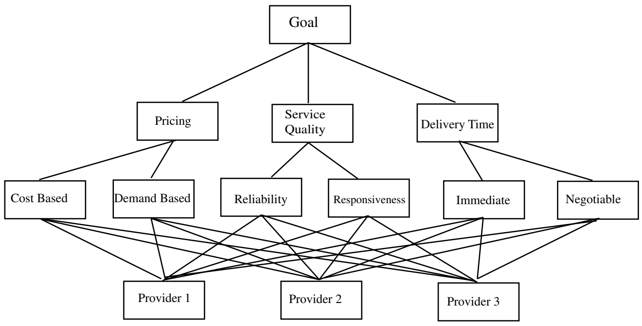

Figure 1: An AHP three level hierarchy for choosing a service provider.

In the next step of the decision-making process, weights of all criteria and scores of alternative providers are to be derived from a fuzzy pairwise comparison matrix of the type (2). In this example, we suppose that all pairwise comparison judgments are represented as fuzzy triangular numbers $\tilde{a}_{ij} = (l_{ij}, m_{ij}, u_{ij})$, such that $u_{ij} > m_{ij} > l_{ij}$.

The fuzzy comparison judgments with regard to the overall goal are shown in Table**??**:

Goal	Pricing	ServQual	DelTime
Pricing	1	(2, 3, 4)	(1, 2, 3)
ServQual	(1/4,1/3,1/2)	1	(1/3, 1/2, 1)
DelTime	(1/3,1/2,1)	(1, 2, 3)	1

Table 1: Fuzzy comparison judgments with regard to the overall goal

As can be seen, Pricing is considered as the most important criterion, since all fuzzy numbers in the first row are greater than one. For example, Pricing is assessed as being about three times more important than Service Quality and about two times more important than Delivery Time. Since the fuzzy pairwise comparison matrix is a reciprocal one, only the elements of the upper right part are used for calculation of the weights by the proposed FPP method. For obtaining crisp weights of these criteria, a non-linear program of the type (9) with one equality and six inequality constraints is to be solved. The weights of the main criteria thus obtained are:

$$\nu_1 = 0.538(Pricing),$$

$$\nu_2 = 0.170(ServiceQuality),$$

$$\nu_3 = 0.292(DeliveryTime).$$

The ratios of the obtained weights are $\frac{\nu_1}{\nu_2} = 3.162$, $\frac{\nu_1}{\nu_3} = 1.838$, $\frac{\nu_2}{\nu_3} = 0.581$, so all initial fuzzy judgements are approximately satisfied. For example, the desired comparison ratio between Pricing and Service Quality, as seen from Table 1, should be about 3, whereas the corresponding solution ratio is $\frac{\nu_1}{\nu_2} = 3.162$. On the other hand, the obtained solution ratios are such that $\lambda = \mu_{12} = \mu_{13} = \mu_{23} = 0.838$, therefore all comparison judgements are equally satisfied with the solution. The positive value of the consistency index $\lambda = 0.838$ indicates that the fuzzy judgements are relatively consistent, which is also seen from the above solution ratios.

By applying the FPP method, the relative weights of all sub-criteria are derived: $\nu_{11} = 0.667$ (Cost-based Pricing); $\nu_{12} = 0.333$ (Demand-based Pricing), etc.

It should be noted that the two-dimensional fuzzy comparison matrices are always consistent. Indeed, in all above cases, the solution ratios are equal to the ratios between the means of the comparison judgments, and the consistency index takes its maximum value .

The three possible providers were further compared with respect to the sub-criteria. By solving a number of optimization problems of the type (9), similar to the first one, we calculated the scores of the alternative providers with respect to all criteria.

The global values of the service providers, calculated by the AHP aggregation rule (weighted arithmetic mean), showed that the first provider is slightly preferable to the second one, while the third provider is ranked last.

In order to verify the obtained results and justify our approach, we solved the same problem using the standard AHP method. Crisp pairwise comparison matrices were constructed from the means of all fuzzy comparison judgments and local weights were found by the eigenvector prioritization method. It was observed that the ranking of the alternative providers is the same as in the fuzzy AHP. However, in comparison to the standard AHP method, the proposed fuzzy approach allowed better modelling of the uncertainty and was cognitively less demanding for the decision-maker.

Conclusions

A new fuzzy programming method was applied for assessment of the evaluation criteria weights and scores during pre-negotiations' reasoning over service selection. A fuzzy modification of the AHP thus obtained was used for finding the global scores of all possible alternatives. A numerical example illustrated the advantages of the proposed fuzzy approach and its applicability to providing valuable decision support in a pre-negotiation process.

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