Teacher practice and student learning: An 'effective' mental computation lesson

Ann Heirdsfield

Centre for Mathematics, Science and Technology Education, QUT, Australia

This paper describes a mental computation lesson conducted in a Year 3 classroom. The effectiveness of the lesson is gauged using the four key teaching characteristics of effective mathematics lessons devised by Brown, Askew, Rhodes, Denvir, Ranson, and William (2001). Student outcomes are described using pre and post instruction interview data.

Heirdsfield, Ann M. (2004) Teacher practice and student learning: An 'effective' mental computation lesson.. In *Proceedings 10th International Congress on Mathematics Education*, Copenhagen, Denmark.

Background and Purposes

While a growing interest in mental computation as a vehicle for developing number sense has become a focus in many international mathematics curricular (e.g., Maclellan, 2001; McIntosh, 1998; Reys, Reys, Nohda, & Emori, 1995), mental computation is new to the Queensland scene. In fact, many schools in Queensland have not introduced mental computation into their mathematics programs to date, as the new syllabus will not be mandated until the year 2007. However, some schools have been keen to embark on the development of mental computation strategies. One such school was the one described in this paper. In 2003, the researcher worked with two Year 3 teachers during three of the four terms of the school year to develop a program to enhance mental computation. In the context of this study, mental computation refers to efficient mental calculation of two- and three-digit addition and subtraction examples. Mental computation does not refer to the calculation of number facts.

At present in Queensland, children in Year 3 (approximately 8 years of age) are expected to be able to complete addition and subtraction two-digit with and without regrouping and three-digit without regrouping written algorithms. Although the development of these algorithms is assisted by the use of manipulatives and language to enhance understanding of the algorithms, the final product is generally procedural with little understanding. Mental/oral work refers only to calculating number facts. While the development of thinking strategies (derived facts strategies – Steinberg, 1985) for number facts is strongly encouraged, this is not the case for the development of written algorithms.

The purpose of the project was to enhance Year 3 students' mental computation performance. The specific aims were to collaboratively design an instructional program to build on students' existing strategies, and to identify and monitor students' mental computation performance. While the students had not been taught any mental strategies, it was assumed that many could already employ self developed mental strategies, as previous research has shown that students have the ability to develop their own self developed and efficient strategies, often despite classroom teaching and without the teachers' knowledge (Cooper, Heirdsfield, & Irons, 1996; Heirdsfield, 1999). Therefore, the instructional program was based on students' prior knowledge (identified from individual interviews). While two Year 3 teachers volunteered to participate in the project, this paper refers to one lesson taken by one of the teachers.

Description of the project

The researcher was a participant observer who acted as a critical friend to the teacher. The role of the researcher was to provide the teacher with a theoretical background for mental computation, support material for the development of an instructional program, interview materials for individual students (pre and post interviews were conducted with all students), feedback during and after lessons, and any additional support deemed necessary during the project. The teacher assumed responsibility for implementing the program. In conjunction with the researcher, the teacher developed, documented and delivered the instructional program.

The researcher attempted to communicate to the teacher that the emphasis of the instructional program should be strategic flexibility and students' exploring, discussing, and justifying their strategies and solutions (c.f., Blöte, Klein, & Beishuizen, 2000; Buzeika, 1999; Hedrén, 1999; Kamii & Dominick, 1998). However, the teacher reinterpreted the researcher's intentions to providing students with representations to support the development of mental strategies; although, she recognised that there was still to be an emphasis on students' discussing and justifying their strategies.

To familiarise the teacher with the variety of mental strategies, the researcher presented the following table (Table 1), not so these strategies would be taught; rather, so that the teacher could recognise some students' spontaneous strategies. These strategies will be referred to later in this paper.

Table 1. *Mental addition and subtraction strategies*

Strategy		Example
Separation	*right to left*	28+35: 8+5=13, 20+30=50, 63
		52-24: 12-4=8, 40-20=20, 28 (subtractive)
		: 4+8=12, 20+20=40, 28 (additive)
	left to right	28+35: 20+30=50, 8+5=13, 63
		52-24: 40-20=20, 12-4=8, 28 (subtractive)
		: 20+20=40, 4+8=12, 28 (additive)
	cumulative sum or difference	28+35: 20+30=50, 50+8=58, 58+5=63
		52-24: 50-20=30, 30+2=32, 32-4=28
Aggregation	*right to left*	28+35: 28+5=33, 33+30=63
		52-24: 52-4=48, 48-20=28 (subtractive)
		: 24+8=32, 32+ 20=52, 28 (additive)
	left to right	28+35: 28+30=58, 58+5=63
		52-24: 52-20=32, 32-4=28 (subtractive)
		: 24+20=44, 44+8=52, 28 (additive)
Wholistic	*compensation*	28+35: 30+35=65, 65-2=63
		52-24: 52-30=22, 22+6=28 (subtractive)
		: 24+26=50, 50+2=52, 26+2=28 (additive)
	levelling	28+35: 30+33=63
		52-24: 58-30=28 (subtractive)
		: 22+28=50, 28 (additive)

Six lessons aimed at developing mental computation strategies were conducted by the teacher; some were whole class lessons, and others were small group lessons. The representations used by the teacher were the hundred chart and the empty number line (ENL). The students did not document their strategies in symbols (e.g., number expressions/equations); rather, they used the ENL as a means of calculation and communication. Each student was provided with a sheet drawn up with several number lines. The teacher drew several number lines on the blackboard. During the instructional program for mental computation, the traditional written algorithms for addition and subtraction continued to be taught/reinforced.

The teaching episode

The teaching episode is described in relation to the key teaching characteristics of effective mathematics lessons devised by Brown, Askew, Rhodes, Denvir, Ranson, and William (2001). These are summarised below (Table 2).

Table 2.
Teaching Characteristics and their Components (summarised from Brown et al., 2001)

Teaching characteristic	Components
Tasks	• Mathematical challenge • Integrity and mathematical significance • Engage interest
Talk	• Teacher talk that focuses on mathematical meanings and understandings as co-constructed • Teacher-pupil talk about mathematics • Pupil talk – pupils encouraged to talk mathematically and display reasoning and understanding • Management of talk – pupils encouraged by teacher to talk about mathematics
Tools	• Cover a range of modes • Didactically "good" types of models
Relationships and norms	• Teacher and pupils participate as community of learners • Teacher displays empathy with pupils' responses

The lesson commenced with the teacher presenting an example (27 + 28) on the blackboard. The students were provided with open number lines drawn on sheets of paper, and several number lines were drawn on the blackboard (*tools*). Before any calculations were attempted, the teacher initiated discussion of possible calculation strategies (*talk*). One child suggested that there would be more than one way of solving the example (*talk*) and another suggested that they could start with either 28 or 27 (*talk*). The teacher followed up on this last point by confirming that because the example is addition, "you can do a turnaround" (*talk*). Here, the teacher attempted to establish connections between different mathematical ideas and contexts (Brown, Askew, Baker, Denvir, & Millett, 1998). In completing the particular example, the teacher offered support by recording the leaps the children suggested, recording the interim landing places, and drawing the children's attention to what had already been completed and what else still needed to be done (*tasks*). "Now we've added on 23, how much more do we need to add?" "So what does 27, add on 28 equal?" While this last question seemed trivial, it was very important, as one child did not recognise that the last jump on the number line landed on the answer. The children were encouraged to suggest alternative strategies, and the teacher documented the jumps on the number lines. They were also encouraged to suggest why the method worked and why they chose to use the method (*talk*).

For the second example (32 + 43), the children completed the example, and then discussed their strategies (*tools*). While the focus of the discussion was on the strategies the students used, the purpose of the discussion was not merely to find as many different methods as possible; rather, it was to compare and contrast the strategies. The children were encouraged to explain how some strategies were similar and how others were different (*talk*). To do this, they focused on such things as the starting number and the types of jumps (tens/multiples of tens; initial jumps in tens or ones, or combinations). Several different methods are shown in Figure 1.

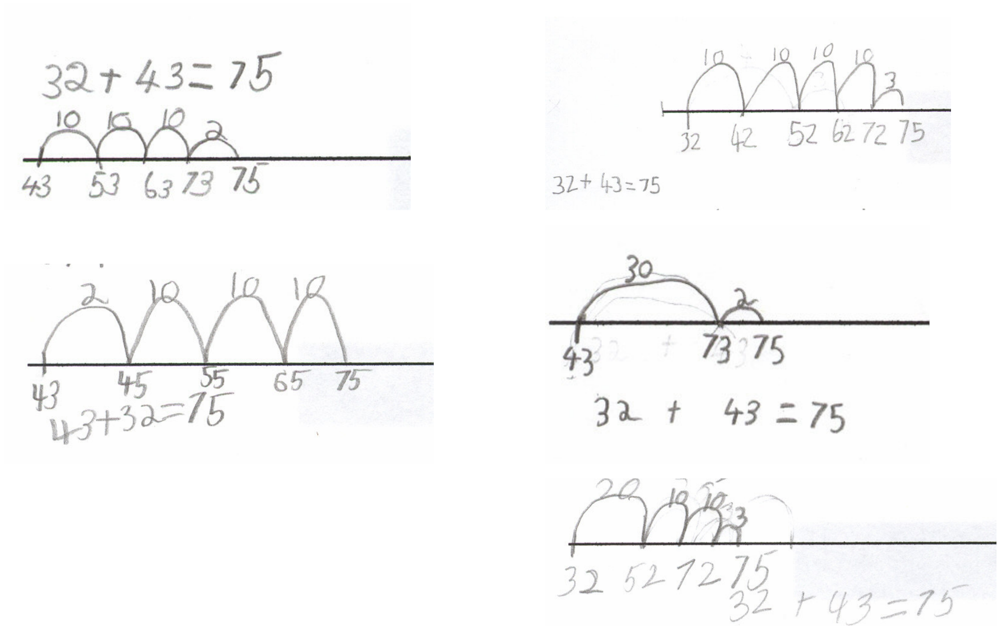

Figure 1. Variety of responses for 32 + 43.

The children were presented with another example (157 + 36), asked to document their solution, and then discuss their methods. Again, the focus was on the description of strategies and the similarities and differences between the strategies (*talk* and *tools*).

Teacher: Who did it this way?

Student: Sort of.

Teacher: Tell me about your 'sort of'.

A number of children explained that they started at 157, added on 30, then 3…(see Figure 2). The teacher asked them why they added 3. Their response elicited a number facts strategy that many use for solving number facts. Many agreed that "going through ten" was a good strategy, but others suggested that they did not need to take the extra step as they knew that seven and six made 13, so it was 193. Therefore, the children were analysing their strategies and making judgements about appropriateness and efficiency (*relationships and norms*).

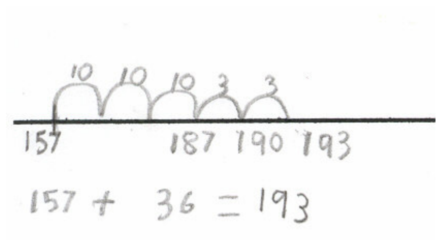

Figure 2. Solution strategy for 157 + 36.

Although there were other examples presented, the final example discussed here was 109 – 47.

Teacher: What's different about this example?

Student: It's a take away.

Ben (student): We'll have to start at the other end.

Teacher: Ben said that we'll have to start at the other end.

Another student: No we don't. We can start at 47 and add on.

As a result of this discussion, the children used both additive and subtractive strategies (see Figure 3). The students explained their strategies, but this time the teacher did not document the children's strategies; rather, the children were required to listen carefully to each others' descriptions to identify similar and different strategies (*talk* and *relationships and norms*).

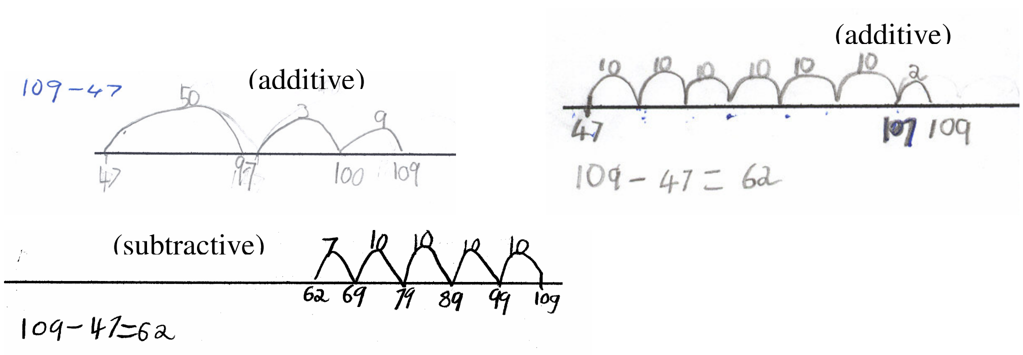

Figure 3. Additive and subtractive solution strategies for 109 – 47.

Final word

While there were examples of good practice through this lesson, there were also deficiencies. There were few links between the examples presented. While there was a general increase in difficulty, the examples were not linked in any way to elicit similar strategies or application of previously used strategies to new situations. So while the *tasks* might have been considered to be of a suitable level of *challenge* and *engage interest*, they would probably fail on *integrity and significance* (see Table 2). Further, there was no 'conclusion' to the lesson, no summing up, and no reflection. There was also an absence of higher order questions. Though, this did not necessarily mean that children were not engaged in higher order thinking. While the analysis of the lesson followed Brown et al.'s (2001) characteristics of effective mathematics teaching, it is recognised that there are varying levels of effectiveness of the components of the characteristics. So, while the episode discussed in this paper featured many components of these characteristics, these components were not always at high levels of effectiveness.

The purpose of the project was to enhance Year 3 students' mental computation performance. Did the students' mental computation performance improve? While it is recognised that one lesson cannot be responsible for improved student outcomes, this lesson was one of several, and might be considered an exemplar. Of the twelve children who participated in this lesson, all but one improved in accuracy in their mental calculations over the period of the project. The child whose accuracy did not improve extended her repertoire of mental strategies; for instance, she started to use *wholistic* strategies. Other students also improved their repertoire of strategies; for instance, half the students employed *separation* or *mental image of pen and paper algorithm* in the pre interviews, but in the post interviews they employed *wholistic* and *aggregation*. Finally, the use of the ENL provided a means of communication for both the children and the teacher. It tended to promote the development of the advanced mental strategies of *aggregation* and *wholistic compensation*. However, some children developed the strategy, *wholistic levelling*, a strategy not developed through the use of the ENL. Was this the result of encouraging children to formulate, discuss and justify their own strategies?

A project to be conducted this year (2004) will focus on enhancing Years 1, 2 and 3 children's mental computation performance. The teachers will receive in and out of class support to develop an instructional program, similar to that offered in 2003. An additional emphasis will be placed on Brown et al.'s characteristics of effective mathematics teaching in the hope of improving classroom practice.

References

Blöte, A. W., Klein, A. S., & Beishuizen, M. (2000). Mental computation and conceptual understanding. *Learning and Instruction, 10*, 221-247.

Brown, M., Askew, M., Baker, D., Denvir, H., & Millett, A. (1998). Is the national numeracy strategy research-based? *British Journal of Educational Studies, 46*(4), 362-385.

Brown, M., Askew, M., Rhodes, V., Denvir, H., Ranson, E., & William, D. (2001). Magic bullets or chimeras? Searching for factors characterizing effective teachers and effective teaching in numeracy. Paper presented at British Education Research Association Annual Conference, Uni. of Leeds, Bath.

Buzeika, A. (1999). Invented algorithms: Teachers face the challenge. In J. M. Truran & K. M. Truran (Eds.), *Making the difference. Proceedings of the Sixteenth Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 128-134). Sydney: MERGA.

Cooper, T. J., Heirdsfield, A., & Irons, C. J. (1996). Children's mental strategies for addition and subtraction word problems. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 147-162). Adelaide: Australian Association of Mathematics Teachers, Inc.

Hedrén, R. (1999). The teaching of traditional standard algorithms for the four arithmetic operations versus the use of pupils' own methods. In I. Schwank (Ed.), *European research in mathematics education 1* (pp. 233-244). [http://www.fmd.uni-osnabrueck.de/ebooks/erme/cerme1-proceedings/cerme1-proceedings.html]

Heirdsfield, A. M. (1999). Mental addition and subtraction strategies: Two case studies. In J. M. Truran & K. M. Truran (Eds.), *Making the difference* (pp. 253-260). Adelaide: MERGA.

Kamii, C., & Dominick, A. (1998). The harmful effects of algorithms in grades 1-4. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics, 1998 yearbook* (pp. 130-140). Reston, VA: NCTM.

Maclellan, E. (2001). Mental calculation: Its place in the development of numeracy. *Westminster Studies in Education, 24*(2), 145-154.

McIntosh, A. (1998). Teaching mental algorithms constructively. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematics, 1998 yearbook* (pp. 44-48). Reston, VA: NCTM.

Reys, R. E., Reys, B. J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6, and 8. *Journal for Research in Mathematics Education, 26*(4), 304-326.

Steinberg, R. T. (1985). Instruction on DFS in addition and subtraction. *Journal for Research in Mathematics Education, 16*(5), 337-355